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Bidding to lose? Auctions with resale

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A losing bidder can still purchase the prize from the winner after the auction. We show why a strong bidder may prefer to drop out of the auction before the price has reached her valuation and acquire the prize in the aftermarket: a strong bidder may be in a better bargaining position in the aftermarket if her rival won at a relatively low price. So it can be common knowledge that, in equilibrium, a weak bidder will win the auction and, even without uncertainty about relative valuations, resale will take place. The possibility of reselling to a strong bidder attracts weak bidders to participate in the auction and raises the seller's revenue.

1. Introduction

■ Before the UK "third-generation" (3G) mobile-phone licenses auction in 2000, it was known that one of the bidders, Orange, was going to be sold after the auction.[1] All other potential buyers knew that, provided Orange was among the winning firms, even if they lost the auction, they could still obtain a license by acquiring Orange. This is indeed what happened: NTL, a consortium controlled by France Telecom, first raised the auction price and then dropped out, allowing Orange to win one of the licenses on sale.[2] After the auction, France Telecom took over Orange. Similarly, after the European 3G auctions, Telia, the biggest telecom company in northern Europe, took over Sonera, a smaller and debt-burdened telecom company, and obtained the licenses that Sonera had won in Germany, Italy, Spain, and Norway.

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[1] Orange was required to be sold by the European competition authority. Before the auction, Vodafone took over Mannesman, which had previously taken over Orange. Both Vodafone and Orange were incumbent mobile-phone operators in the United Kingdom and were willing to bid for a 3G license. The UK government allowed them to do so, because Vodafone was obliged to sell Orange after the auction and appropriate "Chinese-wall" requirements forbade the coordination of their bidding strategies during the auction. For an analysis of the UK 3G auction, see Binmore and Klemperer (2002).

[2] When Telefonica quit the UK auction in round 133, there were only six bidders left, including Orange and NTL, for the five licenses on sale. At that point, NTL could have ended the auction, making sure that Orange obtained a license (as each bidder could win at most one license). Instead, NTL kept on bidding until the price increased by almost 10%, and only then dropped out in round 150.

Winning an auction is not the only chance for a potential buyer to acquire the object on sale. A losing bidder can also obtain the prize after the auction, by purchasing it from a winning bidder. A weak (i.e., low-value) bidder then has an incentive to bid more than his valuation of the auction prize, in order to win and later resell to a strong (i.e., high-value) bidder. And a strong bidder has a choice between outbidding her competitor during the auction, and letting him win the auction and then purchasing the prize in the aftermarket.[3]

It may be expected that a stronger bidder always prefers to raise the auction price in order to weaken her competitor, and so be able to purchase the prize cheaply after the auction. Furthermore, as a weak bidder knows that the price will rise until his surplus is reduced to zero, it may be expected that he never wants to participate in the auction at all. Neither of these statements is necessarily true, however. We will show that, when wealth constraints matter, a strong bidder is in a better bargaining position in the resale market if the weak bidder has won at a low rather than high price. Therefore, even the weak bidder has an incentive to participate in the auction and bid aggressively, as he knows that the strong bidder will let him win at a low price, rather than outbid him.[4]

The reason is that, when a project with uncertain value is on sale, a wealth-constrained bidder enjoys limited liability (as he cannot lose more than his wealth) and treats the auction prize as an option: if the project turns out to be unprofitable, instead of continuing to invest in it, the bidder can declare bankruptcy and liquidate his wealth.[5] Then a very high auction price, by increasing the potential loss from bad projects, reduces the expected profit of a strong bidder more than the expected profit of a weak and wealth-constrained bidder, and hence reduces the strong bidder's surplus in the resale market. Therefore, in order to purchase in the aftermarket, during the auction the strong bidder does not bid above a certain price, thus allowing the weak bidder to win.

So it can be common knowledge that resale will take place after the auction, even if the order of bidders' valuations—and the fact that the order will not change—is commonly known.

However, there are also reasons why a strong bidder may prefer to raise the auction price at least some distance before dropping out. If a wealth-constrained bidder has to pay a borrowing cost to finance his bid, a higher auction price reduces his profit by a greater amount, and improves the strong bidder's bargaining position in the resale market. So the presence of a borrowing cost pins down a particular price at which the strong bidder chooses to drop out of the auction.

Our broader point is that, when bargaining in the resale market is affected by the price paid by the auction winner, the share of the resale surplus that a strong bidder can appropriate depends on the auction price, and hence a strong bidder is *not* indifferent about the price her rival pays in the auction.[6] The reasons we explore for this are wealth effects due to limited

[3] We adopt the convention of using feminine pronouns for a strong bidder and masculine pronouns for a weak bidder.

[4] In a standard ascending auction with complete information, a strong bidder is indifferent between buying in the resale market and winning the auction at the same price at which he can buy in the resale market (see the example in Section 2). So there is an equilibrium with resale in which the weak bidder bids up to the resale price and the strong bidder drops out at zero. However, this is only one among many possible equilibria, and (unlike the resale equilibrium in our model) it is not robust to slight changes that make the model more realistic—for example, an arbitrarily small cost of resale, or bidders discounting (by even an arbitrarily small amount) the future surplus from the resale market. Moreover, another problem with this equilibrium (but not the equilibrium of our model) is that, if the strong bidder follows her weakly dominant strategy of bidding up to the resale price, the weak bidder cannot obtain a positive profit and, hence, has no incentive to participate in the auction.

[5] The effects of limited liability on bidding strategies in auctions without resale have been analyzed by Che and Gale (1998), Board (2007), and Zheng (2001). Board (2007) and Zheng (2001) show that bidders with limited liability and lower wealth bid relatively more aggressively, and this can raise the seller's revenue. Che and Gale (1998) prove that when bidders face a budget constraint (and there is no uncertainty about profits), first-price auctions yield higher revenue than second-price auctions, because the budget constraint is more likely to bind in a second-price auction.

[6] Other papers (e.g., Bikhchandani and Huang, 1989; Haile, 1999, 2003; Zheng, 2002) show that, when bidders have incomplete information, the resale market can be affected by the auction because bids can signal a bidder's valuation

liability and borrowing cost, but the point is more general. For example, when bidding against a risk-averse rival, a strong bidder may want to raise the auction price if, by reducing the winner's residual wealth, this reduces her rival's bargaining power in the resale market. Or, if the managers of a weak firm are willing to resell the prize at a fixed markup over the auction price (to justify their strategy with shareholders), then a strong bidder may want to drop out of the auction at a lower price, in order to purchase the prize more cheaply from the winner.

Of course, if valuations change after the auction, resale can occur when another potential buyer turns out to have a higher valuation than the winner. This may happen because additional buyers appear after the auction (as in Milgrom, 1987; Bikhchandani and Huang, 1989; Haile, 1999; Bose and Deltas, 1999), or because bidders' valuations change after the auction (as in Haile, 2000, 2001, 2003).[7] By contrast, in our model, the uncertain component of the prize's value is common to all bidders, and all potential buyers can participate in the auction. Therefore, the *ex post* efficient allocation is known before the auction starts. So resale in our model does not arise because of unexpected gains from trade: even with complete information about bidders' valuations, resale may take place in equilibrium.[8] Moreover, our results do not depend on any bidder entering the auction or dropping out of the auction when indifferent about doing so—in our model, resale arises even with bidding and resale costs.

Garratt and Tröger (2006) show that, even without valuations changing after the auction, in a second-price auction there are equilibria in which a weak bidder who has no value for the prize bids a high price (expecting not to pay it) and induces a strong bidder to bid zero, so that the weak bidder wins the auction at price zero and then resells at a profit to the strong bidder.[9] However, in contrast to our model, these equilibria are not unique and rely upon the weak bidder bidding a price higher than the maximum price he would be happy to pay (even after taking into account the surplus he can obtain in the resale market).

Resale increases the seller's revenue by giving even bidders who know they are weak a chance to win the auction, and hence an incentive to participate and bid aggressively. By contrast, resale was not allowed in some of the European 3G mobile-phone license auctions, possibly costing the governments billions of dollars.[10]

Whether resale takes place depends on the borrowing cost and on the weak bidder's initial wealth. A high borrowing cost makes a weak bidder unwilling to bid aggressively, because it reduces his profit when he does not declare bankruptcy. This makes resale harder. And a high initial wealth reduces the effect of limited liability, because it increases the loss of the weak bidder in case of bankruptcy. This also makes resale harder. Therefore, to induce a resale equilibrium,

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to his opponents and, hence, can affect the division of the resale surplus. Notice that, in contrast, we do not require incomplete information for the auction to have effects on the outcome of the resale market.

[7] Haile (1999, 2003) argues that the possibility of resale can induce a common-value element in bidders' valuations, and shows that bidding strategies are determined by the option value of buying and selling in the resale market, whereas Haile (2000) analyzes the effects of a reserve price. Haile (2001), using data from auctions of timber contracts, confirms that bidders' willingness to pay increases with the expected level of competition in the aftermarket, because this increases the option value to resell. Bikhchandani and Huang (1989) analyze multi-unit common-value auctions in which bidders participate only to resell, and compare discriminatory to uniform-price auctions. Gupta and Lebrun (1999) and Hafalir and Krishna (forthcoming) analyze resale in first-price auctions with asymmetric bidders. See also Jehiel and Moldovanu (2000).

[8] Pagnozzi (2007) shows that, in multi-object auctions, resale can arise even with complete information, as a result of demand reduction by strong bidders.

[9] A similar equilibrium is discussed in Zheng (2000). See also footnote 4. Garratt and Tröger (2006) also analyze first-price auctions with resale and show that they are not revenue equivalent to second-price auctions.

[10] For example, in the Netherlands auction, five licenses were on sale in a market with exactly five incumbents (which had a higher valuation than new entrants because of their recognizable brand name, their familiarity with the market, and their customer base). Other bidders did not show up, because they realized they had no chance of winning against incumbents, and the revenue was 70% lower than the government had forecast based on the comparable UK auction (Klemperer, 2002a, 2002b). The Swiss 3G auction, as well as some of the 1995 U.S. PCS auctions (Klemperer and Pagnozzi, 2004), resulted in similar outcomes.

the seller may want to reduce the weak bidder's wealth or improve the terms on which he can finance his bid.[11]

The rest of the article is organized as follows. A numerical example is analyzed in the next section. Section 3 presents the model and discusses the effects of a wealth constraint on a bidder's profit. Section 4 proves that a strong bidder may prefer to drop out of an auction against a weaker competitor and derives conditions under which resale takes place in equilibrium. The strategy that may be adopted by the seller to increase revenue is analyzed in Section 5. Sections 6 analyzes the model with an alternative timing. Sections 7 discusses possible extensions and the last section concludes. All proofs are in the Appendix.

2. An example

■ In this section, we discuss a simple example of an auction with a wealth-constrained bidder which highlights why a strong bidder may prefer to let her opponent win and resale takes place in equilibrium.

Consider an ascending auction with two bidders.[12] It is common knowledge that bidder A has value 10, and bidder B has value 5 with probability $\frac{1}{2}$, and 3 with probability $\frac{1}{2}$. We assume resale can take place before the auction price is paid. We also assume that, if they trade in the resale market, bidders equally share the total resale surplus, which is the difference between the two bidders' (expected) profit.

First suppose that no bidder has a wealth constraint. If bidder A wins the auction at price p, her profit is $\pi_A(p) = 10 - p$, whereas if bidder B wins the auction and keeps the prize, his expected profit is $\mathbb{E}[\pi_B(p)] = 4 - p$. Therefore, if bidder B wins the auction, the total resale surplus is

$$S(p) = \pi_A(p) - \mathbb{E}[\pi_B(p)] = 6.$$

This is independent of the auction price. Hence, if bidder A wants to buy in the resale market, she is completely indifferent about the auction price paid by bidder B.

Moreover, bidder A is also exactly indifferent between winning the auction and buying in the resale market. To see this, notice that in the resale market bidder B obtains a surplus equal to the resale price (i.e., half the resale surplus plus his outside option):

$$\tfrac{1}{2}S(p) + \mathbb{E}[\pi_B(p)] = 7 - p.$$

Anticipating the possibility of resale, bidder B is willing to bid as long as the resale price is positive—that is, up to price 7. Therefore, bidder A can win the auction at price 7 and obtain a surplus of 3 or buy in the resale market and also obtain a surplus of $\frac{1}{2}S = 3$. This is a knife-edged situation and it is not clear why bidder A should prefer to buy in the make smaller like resale market.

Now assume bidder B's wealth is $w_B = 0$, so that he enjoys *limited liability.* In this case, if bidder B wins the auction and keeps the prize, because his final wealth cannot be negative, his expected profit is

$$\mathbb{E}[\pi_B(p)] = \tfrac{1}{2}\max\{5 - p;\ 0\} + \tfrac{1}{2}\max\{3 - p;\ 0\}.$$

[11] There is a recent literature that studies the optimal seller's mechanisms in the presence of resale. Zheng (2002) analyzes when the optimal allocation in an auction without resale can be achieved when repeated resale is permitted, if the auction's winner has all the bargaining power in the resale market. Calzolari and Pavan (2006), on the other hand, assume that the bargaining power in the resale market depends on the identity of the auction's winner, and prove that resale reduces the seller's revenue compared to the revenue-maximizing mechanism without resale. See also Ausubel and Cramton (1999).

[12] In an ascending auction the price is raised continuously by the auctioneer, and bidders who wish to be active at the current price depress a button. When a bidder releases it, he is withdrawn from the auction (and cannot become active again). The price level and the number of active bidders are continuously displayed and the auction ends when only one active bidder is left.

Suppose bidder B wins the auction. Then bidder A's only objective during the auction is to maximize the total resale surplus $S(p)$. In order to find the price at which bidder A drops out of the auction, consider how $S(p)$ changes as the auction price rises:

p	0	1	2	3	4	5	6	...
$\pi_A(p)$	10	9	8	7	6	5	4	...
$\mathbb{E}[\pi_B(p)]$	4	3	2	1	$\frac{1}{2}$	0	0	...
$S(p)$	6	6	6	6	$5\frac{1}{2}$	5	4	...

Because of bidder B's limited liability, inducing him to pay an auction price higher than 3 reduces $S(p)$ because it reduces bidder A's profit more than it reduces B's expected profit. Therefore, bidder A is not indifferent anymore about the auction price and never bids more than 3 if she wants to acquire the prize in the aftermarket.

By contrast, bidder B is willing to bid up to 10, the highest auction price at which the resale price is positive. Hence, bidder A strictly prefers to drop out of the auction at any price up to 3 and purchase the prize from bidder B (obtaining a surplus of 3), rather than winning the auction at price 10 (obtaining a surplus of 0).

Moreover, if bidder A has any reason to want bidder B to pay a higher auction price, she is also not indifferent among prices between 0 and 3. Assume, for example, that bidder B pays a borrowing cost β proportional to the auction price if this is higher than his wealth, where $0 < \beta < \frac{1}{4}$. Consider again the resale surplus at different auction prices:

p	0	1	2	3	4	5	6	...
$\pi_A(p)$	10	9	8	7	6	5	4	...
$\mathbb{E}[\pi_B(p)]$	4	$3-\beta$	$2-2\beta$	$\frac{1}{2}(2-3\beta)$	$\frac{1}{2}(1-4\beta)$	0	0	...
$S(p)$	6	$6+\beta$	$6+2\beta$	$6+\frac{3}{2}\beta$	$5\frac{1}{2}+2\beta$	5	4	...

Now a high auction price increases bidder B's cost relatively more and, hence, it tends to increase the resale surplus. On the other hand, however, a high auction price also increases the effect of limited liability, because it makes it more likely that bidder B's potential profit is negative. It follows that bidder A drops out of the auction at price 2 in order to acquire the prize in the aftermarket (while bidder B is still willing to bid up to 10.)

In the rest of the article, we are going to generalize this example and show under which conditions resale takes place in equilibrium.[13]

3. The model

■ Consider an ascending auction for a project (for example, a license to provide mobile-phone services) with two potential buyers, bidder A and bidder B. Bidder i has initial wealth w_i and values the project v_i, $i = A, B$. Bidders' wealths and valuations are common knowledge. We assume $v_A > v_B$ and $w_A > w_B$: A is a *strong* bidder with both a high valuation and

[13] In the example, without resale, bidder B drops out at price 5—that is, when his expected profit is zero. So, if bidder B always participates in the auction, the presence of resale reduces the seller's revenue. In Section 5, we will show that this depends on bidder B's wealth being zero and that, if B's wealth is sufficiently high or if bidders pay an arbitrarily small cost to participate in the auction, then resale increases the seller's revenue.

a high initial wealth, while B is a *weak* bidder with both a low valuation and a low initial wealth.[14]

In order to run the project, the owner pays an *operating* cost c, drawn from a continuous and strictly increasing distribution function F and realized after the auction is over. To simplify notation, we assume that F is defined on $(-\infty, +\infty)$ but, because c represents a cost, $F(c) = 0$ for $c < 0$. (At the end of Section 4, to obtain a closed-form solution, we consider a uniform distribution on a bounded support for c.)

Therefore, to obtain his valuation of the project, a bidder has to pay the *total* cost of the project, defined as the sum of the auction price p and the operating cost. If his initial wealth is less than the total cost, to pay for the exceeding cost, a bidder borrows money at an additional cost $\beta > 0$ per unit of capital. The borrowing cost can be interpreted as either the interest rate on loans determined exogenously by banks, or as a cost set by the seller for accepting a bid higher than a bidder's wealth. The *operating* profit is the difference between the project's value and the total cost. However, bidders are only liable for the total cost up to their initial wealth, because they cannot end up with negative final wealth. Therefore, if paying the total cost would generate a loss higher than his wealth, a bidder prefers to declare bankruptcy instead.[15] For simplicity, we assume that if the winner declares bankruptcy, the project returns to the seller and is never sold again.[16]

After the auction, and before the operating cost is realized and the auction price is paid, the winner can resell the project to the loser, if both bidders agree.[17] So the auction price is not paid immediately after the auction, but only after resale takes place. This assumption fits many real-life auctions (as well as our example in the Introduction). For instance, winners were required to pay the auction price in yearly installments in the Danish, Italian, and French 3G mobile-phone licenses sales, as well as in many early FCC spectrum auctions in the United States. And after the UK 3G mobile-phone licenses auctions, it was France Telecom that paid the price of the license won by Orange, when it took over Orange, because Orange was only required to pay after being sold by Vodafone.[18]

Our results do not hinge on this last assumption. In Section 6, we analyze an alternative model in which the auction price is paid before resale and show that, even in this case, wealth effects may induce a strong bidder to drop out of the auction and resale may take place in equilibrium.

The timing of the game is the following:

(i) Bidder i wins the auction at price p, which is due to be paid in stage iv;
(ii) Resale can take place;
(iii) The operating cost c is realized;
(iv) The owner of the project, bidder j (which is different from the auction winner if there is resale in stage ii), can:
  (a) obtain v_j by paying for $(p + c)$ out of his wealth w_j and, if this is not sufficient, by paying the borrowing cost β on $(p + c) - w_j$, or
  (b) declare bankruptcy and liquidate his wealth.

[14] We model wealth-constrained bidders as in Zheng (2001), who analyzes auctions without resale.

[15] However, if the total cost is lower than his wealth, a bidder pays for it even if this implies obtaining negative profit. Hence, as is typically the case in spectrum auctions, bidders cannot simply return the project to the seller if it is not profitable to operate it. See also Section 7.

[16] In reality, the seller would probably re-auction the project after a delay. However, our results only require that, after the weak bidder wins the auction, the strong bidder prefers to buy in the resale market, rather than wait for the weak bidder to go bankrupt and for a new auction to take place (e.g., because waiting for a new auction is costly, or because a new auction would attract other weak bidders, so the strong bidder would be unable to buy the project cheaply anyway).

[17] An additional reason for resale arises if resale takes place after the operating cost is realized: a strong bidder may choose to let a weak bidder win and bear the risk of a high cost, waiting until the uncertainty about the project's profitability is resolved before acquiring it. For example, big firms often leave R& D races to small firms with less to lose in case of failure, and then obtain the innovation by taking over the winner.

[18] Because both Orange and Vodafone won a license and a single conglomerate firm was not allowed to own two licenses, the UK government required that Orange was divested before the two firms could claim their licenses.

Bidder A has infinite wealth, so she can pay any total cost and any resale price. Hence, bidder A never pays the borrowing cost if she owns the project, and she also never finds it profitable to declare bankruptcy in stage iv. Her profit if she owns the project, net of the auction price (and neglecting the possible resale price), is

$$\pi_A(p, c) = v_A - (p + c),$$

and her expected profit is

$$\mathbb{E}_c[\pi_A(p, c)] = v_A - p - \mathbb{E}[c].$$

Bidder B's initial wealth is $w_B = w$, so that bidder B faces a wealth constraint but enjoys *limited liability*. Bidder B's profit from owning the project depends on the relation between the total cost, his valuation, and his initial wealth. If the total cost is lower than bidder B's wealth, his operating profit is $v_B - (p + c)$, the same as without a wealth constraint. If the total cost is higher than bidder B's wealth, bidder B pays the borrowing cost to operate the project, and his operating profit is lower than without a wealth constraint, and equal to

$$\underbrace{v_B}_{\text{value}} - \underbrace{(p + c)}_{\text{total cost}} - \underbrace{\beta(p + c - w)}_{\text{borrowing cost}} = v_B + \beta w - (1 + \beta)(p + c).$$

Finally, if c is so high that the above profit is lower than $-w$, bidder B declares bankruptcy, obtains an actual profit of $-w$, and is left with zero final wealth.

Summing up, bidder B's profit from owning the project is

$$\pi_B(p, c) = \begin{cases} v_B - (p + c) & \text{if } c \leq w - p, \\ v_B + \beta w - (1 + \beta)(p + c) & \text{if } w - p < c < w - p + \frac{v_B}{1+\beta}, \\ -w & \text{if } w - p + \frac{v_B}{1+\beta} \leq c. \end{cases}$$

Figure 1 represents bidder B's profit as a function of the realized cost (for $p < w$).[19]

□ **Effects of a wealth constraint.** A low initial wealth is like a budget constraint because it limits how much a bidder can pay for the project on sale, but it also limits how much a bidder can lose if the projects turns out to be unprofitable. A wealth constraint has two effects (a positive one and a negative one) on bidder B's expected profit from owning the project:

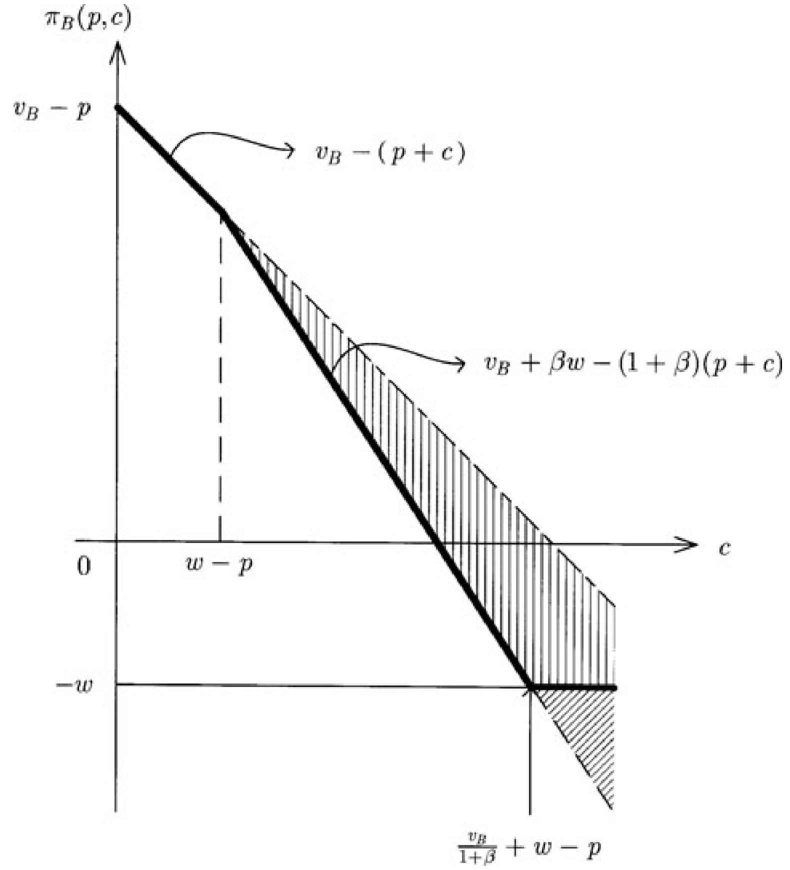
(i) Borrowing cost effect: if the total cost is higher than the bidder's wealth, profits on good projects (i.e., with a low operating cost) are reduced;
(ii) Limited liability effect: losses from bad projects (i.e., with a high operating cost) are reduced, because they are limited by the initial wealth (e.g., Zheng, 2001).

Compared to a situation without a wealth constraint, the borrowing cost effect makes a bidder worse off, whereas the limited liability effect may make a bidder better off. A bidder with high initial wealth does not face a wealth constraint and never pays the additional borrowing cost, but she also always has to pay any realized operating cost. In Figure 1, the vertical shaded area represents the reduction in expected profits due to the borrowing cost effect, while the diagonal shaded area represents the reduction in expected losses due to the limited liability effect.

[19] This framework includes, as special cases, a (standard) model without a wealth constraint (when w is large) and a model with a strictly binding wealth constraint (when $\beta = \infty$ and hence bidder B cannot borrow above his initial wealth).

FIGURE 1

BIDDER *B*'S PROFIT WITH A WEALTH CONSTRAINT



Bidder *B*'s expected profit is

$$\begin{aligned}\mathbb{E}_c[\pi_B(p,c)] = &\int_0^{w-p} [v_B - (p+c)]\, dF(c) \\ &+ \int_{w-p}^{\frac{v_B}{1+\beta}+w-p} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c) \\ &- w\left[1 - F\left(\frac{v_B}{1+\beta} + w - p\right)\right],\end{aligned}$$

and the effect of a change in the auction price is[20]

$$\begin{aligned}\frac{\partial}{\partial p}\mathbb{E}_c[\pi_B(p)] &= -(1+\beta)F\left(\frac{v_B}{1+\beta} + w - p\right) + \beta F(w-p) \\ &= -1 + \Pr[B \text{ goes bankrupt}] - \beta \cdot \Pr[B \text{ borrows}],\end{aligned}$$

where the last equality follows because $[1 - F(\frac{v_B}{1+\beta} + w - p)]$ is the probability of bidder *B* going bankrupt and $[F(\frac{v_B}{1+\beta} + w - p) - F(w-p)]$ is the probability of bidder *B* borrowing to pay the total cost.

[20] To simplify notation, we sometimes write $\mathbb{E}_c[\pi_i(p)]$ or $\mathbb{E}[\pi_i]$ for $\mathbb{E}_c[\pi_i(p,c)]$.

Bidder B's expected profit function is (weakly) decreasing in p. In the absence of a wealth constraint, the marginal negative effect of an increase in the auction price is equal to 1—expected profit falls linearly as p increases. On the other hand, when B faces a wealth constraint,

$$\left|\frac{\partial}{\partial p}\mathbb{E}_c[\pi_B(p)]\right| \geq 1 \Leftrightarrow \beta \cdot \Pr[B \text{ borrows}] \geq \Pr[B \text{ goes bankrupt}].$$

Therefore, an increase in the auction price reduces bidder B's expected profit more than bidder A's expected profit if and only if the borrowing cost effect is stronger than the limited liability effect.

To simplify the analysis, we assume bidder A is sufficiently strong and, specifically, v_A is sufficiently high that, for any equilibrium auction price p, $\mathbb{E}_c[\pi_A(p)] > 0$ and $\mathbb{E}_c[\pi_A(p)] > \mathbb{E}_c[\pi_B(p)]$.[21] This implies that (i) if resale is not possible, in the unique equilibrium in undominated strategies, bidder A wins the ascending auction, and (ii) if bidder B is the winner, there are gains from trade between the two bidders in the resale market. We also assume that $\mathbb{E}_c[\pi_B(0)] > 0$.

□ **Bargaining in the resale market.** Assume bidder B wins the auction at price p. Bidder A can purchase the project from bidder B in the resale market, and then obtain the project's value by paying the total cost. Hence, her valuation of the project is $\mathbb{E}_c[\pi_A(p)]$. We normalize bidder A's outside option in the resale market to zero. Bidder B can keep the project and obtain the project's value by paying the total cost. So his valuation of the project, and hence his outside option in the resale market, is $\mathbb{E}_c[\pi_B(p)]$. The gains from trade resulting from resale are equal to the difference between these two valuations—that is, $S(p) = \mathbb{E}_c[\pi_A(p)] - \mathbb{E}_c[\pi_B(p)]$.

We assume the outcome of bargaining between the two bidders in the resale market is given by the Nash bargaining solution, where the disagreement point is represented by bidders' outside options. This can be interpreted as the limit, as the length of the bargaining periods goes to zero, of a strategic model of alternating offers where parties face a small exogenous risk of breakdown of negotiations that induces them to take their outside options (Binmore, Rubinstein, and Wolinsky, 1986; Sutton, 1986). So bidder B's valuation of the project is relevant in the resale market, and measures his bargaining power.[22]

Therefore, bidders share equally the gains from trade in the resale market.[23] So the resale price is $\frac{1}{2}\left(\mathbb{E}_c[\pi_A(p)] + \mathbb{E}_c[\pi_B(p)]\right)$, and the surplus obtained by each bidder if resale takes place is $\frac{1}{2}\left(\mathbb{E}_c[\pi_A(p)] - \mathbb{E}_c[\pi_B(p)]\right)$. This resale surplus depends on the auction price through its effect on bidders' expected profits from owning the project.

Finally, when evaluated *before* the start of the auction, the total surplus of bidder B if he wins the auction and resells the project is equal to his resale surplus *plus* his outside option in the resale market, that is, the resale price.

We define a *resale equilibrium* as a Nash equilibrium of the auction with the following properties: (i) bidder B bids up to the highest price he is happy to pay, taking into account the

[21] As the auction price increases, B's expected profit, being bounded by $-w$, eventually becomes higher than A's expected profit, which can be arbitrariy small. However, as we are going to show, in equilibrium the auction price is never higher than $\max\{p_B; p^*\}$ (which are defined in Section 4). Therefore, our assumption requires that, at any price up to $\max\{p_B; p^*\}$, bidder A's expected profit is positive and higher than bidder B's expected profit.

[22] This assumption may be justified because, for example, governments auctioning spectrum licenses usually require the winner to start operating the license (and building the necessary infrastructure) by a predetermined date after the auction. So the winner's valuation of the license is relevant in the resale market because, while bargaining, bidders know that, if resale does not take place, the winner will have to take his outside option regardless of his will to do that. Sometimes, while trying to resell a project, the winner is even obliged to start operating it, so that his valuation of the project actually represents his "impasse point" (or "inside option") in case bargaining in the resale market continues forever (Binmore, Shaked, and Sutton, 1989).

[23] There are other ways to model bargaining between bidders—for example, assuming unequal sharing of resale surplus or modelling the outside option differently. Many different assumptions yield results similar to those presented in this article, as long as the auction price affects the outcome of bargaining in the resale market. For example, all our results hold if bidder B obtains a share of resale surplus equal to k, for $0 < k < 1$.

surplus he obtains in the resale market, and (ii) bidder A strictly prefers to drop out of the auction and purchase the project from bidder B.

4. Resale equilibrium

■ In order to have a resale equilibrium, two conditions must be satisfied:

- Condition (A): bidder A must strictly prefer to drop out of the auction and then purchase the project in the resale market, instead of winning the auction;
- Condition (B): bidder B must find it profitable to bid high enough to win the auction and then resell the project.

In this section, we first analyze how bidders' strategies in the auction are affected by the possibility of resale, and then show that, for a wide range of parameters, a resale equilibrium is the only equilibrium in undominated strategies.

□ **Weak bidder bids aggressively.** After winning the auction (at any equilibrium price), bidder B always prefers to resell to bidder A. Hence, bidder B is willing to remain active in the auction as long as his total surplus from winning and reselling the project (i.e., the resale price) is positive, and he drops out when his total surplus is equal to zero. The following lemma describes bidder B's bidding behavior when he can resell the project and compares it with his bidding behavior when he cannot resell the project, but still participates in the auction.

Lemma 1. When resale is possible, it is a weakly dominant strategy for bidder B to bid up to the unique price $p_B > 0$ such that $\mathbb{E}_c[\pi_A(p_B)] + \mathbb{E}_c[\pi_B(p_B)] = 0$. Bidder B bids more aggressively than when resale is not possible.

This captures the idea that, during an auction, a weak bidder does not drop out as soon as the price reaches his valuation of the prize, because even if he wins at a higher price he can still resell to a bidder with a higher valuation, obtaining a positive surplus.[24] Hence, a weak bidder is always willing to pay a price between his own valuation and a strong bidder's valuation (see also Milgrom, 1987).

Moreover, if bidders do not participate in the auction when indifferent (because, for example, they have to pay a small bidding cost to do so), without resale bidder B does not enter an ascending auction at all, because he knows he has no chance of winning against a stronger competitor. By contrast, if he expects to resell the prize, bidder B is willing to enter the auction, even under our extreme assumptions about bidders' valuation and wealth. Therefore, resale induces a weak bidder both to participate in the auction *and* to bid more aggressively.

□ **Strong bidder may drop out.** Bidder A has a choice between two strategies: outbidding bidder B to win the auction, or dropping out of the auction and then purchasing the project from bidder B.

If bidder A allows bidder B to win at price p and purchases the project from him, her surplus depends on the gains from trade in the resale market ($\mathbb{E}_c[\pi_A(p)] - \mathbb{E}_c[\pi_B(p)]$). Therefore, when bidder A buys the project in the resale market, in order to maximize her resale surplus, she raises the auction price only up to[25, 26]

$$p^* \in \arg\max_{p \geq 0} (\mathbb{E}_c[\pi_A(p)] - \mathbb{E}_c[\pi_B(p)]).$$

[24] In the terminology of Haile (2003), bidder B bids more aggressively because of the "resale seller effect."

[25] By assumption, v_A is such that bidder A's expected profit at p^* is positive (see note 21).

[26] This rules out equilibria in which the weak bidder can bid an arbitrarily high price because the strong bidder drops out of the auction at zero, being indifferent about the auction price—bidder A never drops out at a price lower than p^* when bidder B bids a higher price.

This is a result of the two effects of a wealth constraint on bidder B's expected profit. As the auction price rises, the borrowing cost effect, by reducing bidder B's expected profit more than bidder A's, worsens the bargaining position of bidder B and increases the gains from trade; whereas the limited liability effect, by limiting bidder B's loss relative to bidder A's, improves the bargaining position of bidder B and reduces the gains from trade. For a high auction price, the limited liability effect dominates and an increase in price lowers bidder A's resale surplus.[27]

But does bidder A prefer to win the auction at price p_B or drop out at price p^* and purchase from bidder B? Clearly bidder A prefers to purchase in the aftermarket if her resale surplus (after B wins the auction at price p^*) is higher than her profit from winning the auction at price p_B—that is, if $\frac{1}{2}S(p^*) > \mathbb{E}_c[\pi_A(p_B)]$.

Lemma 2. Bidder A strictly prefers to drop out of the auction at price p^* and purchase the project in the resale market if and only if $p_B > p^*$.

As shown in the proof of Lemma 2, at price p_B bidder A is exactly indifferent between winning the auction and dropping out to purchase in the resale market. (And this is true regardless of how bidders share the gains from trade in the resale market.) Because her resale surplus is maximized at p^* (and it is strictly lower at a higher price), if $p_B > p^*$ bidder A strictly prefers to purchase the project in the resale market, because outbidding her rival to win the auction is more costly than sharing the resale surplus with him.

□ **Conditions for resale.** For resale to take place in equilibrium, Condition (B) requires p_B (the price up to which bidder B bids anticipating resale) to be greater than p^* (the price at which bidder A drops out of the auction to purchase in the resale market). And because of Lemma 2, if and only if Condition (B) is satisfied, then Condition (A) is satisfied too. Therefore, the auction has a resale equilibrium if and only if $p_B > p^*$.

In order to obtain explicit conditions on the model's parameters for a resale equilibrium, we now assume that c is drawn from a uniform distribution on $[0, \overline{c}]$. To make the model directly comparable to the alternative model in Section 6, we assume $\overline{c} > v_B$. In the Appendix, we show that the qualitative results of Proposition 1 also hold when $v_B > \overline{c}$. Moreover, to make the model interesting and ensure that $p^* > 0$, we assume $v_B + w > \overline{c}$.[28]

Figure 2 represents bidder B's expected profit and shows that, for a high auction price, the limited liability effect dominates the borrowing cost effect, and an increase in price has a stronger negative effect on bidder A's expected profit than on bidder B's expected profit.[29] Therefore, an increase in the auction price raises bidder A's resale surplus if and only if

$$\left|\frac{\partial}{\partial p}\mathbb{E}_c[\pi_A(p)]\right| < \left|\frac{\partial}{\partial p}\mathbb{E}_c[\pi_B(p)]\right| \Leftrightarrow p < v_B + w - \overline{c}.$$

And when bidder A buys the project in the resale market, she drops out of the auction at price[30]

$$p^* \equiv v_B + w - \overline{c}.$$

A further increase in the auction price reduces the gains from trade in the resale market and makes bidder A worse off.[31] A higher w raises p^* because it makes it more costly for bidder B to declare

[27] When the total cost is so high that bidder B always chooses to declare bankruptcy, an increase in the auction price does not affect his profit at all.

[28] When $p^* = 0$, bidder B always bids more than p^* (because $\mathbb{E}[\pi_B(0)] > 0$ by assumption), and the auction always has a resale equilibrium.

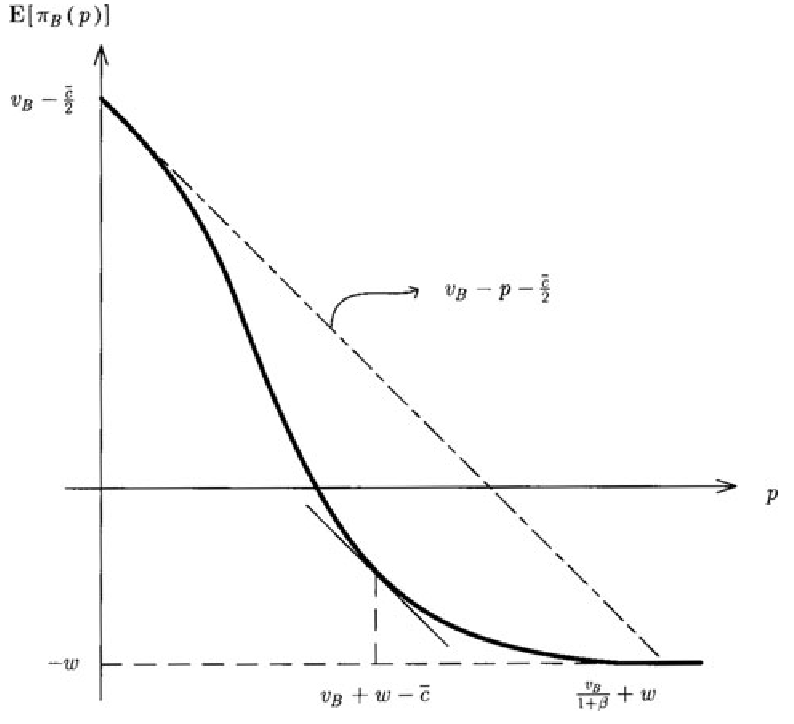
[29] The details of the analysis of the uniform distribution case are in the Appendix.

[30] Bidder A drops out of the auction as soon as the price is such that bidder B's operating profit if he does not have to pay the borrowing cost may become lower than $-w$, and so he may declare bankruptcy.

[31] Our assumption that $\mathbb{E}_c[\pi_A(p^*)] > 0$ requires that $v_A > v_B + w - \frac{\overline{c}}{2}$. Moreover, $\mathbb{E}[\pi_A(p^*)] > v_B - p^* - \mathbb{E}[c] > \mathbb{E}[\pi_B(p^*)]$.

FIGURE 2

BIDDER *B*'S EXPECTED PROFIT FROM OWNING THE PROJECT



bankruptcy, thus reducing the limited liability effect and increasing the incentive for bidder A to raise the auction price.

The following proposition clarifies for which values of the borrowing cost and of B's initial wealth the auction has a unique resale equilibrium.

Proposition 1. Assume $c \sim U[0, \bar{c}]$ and $v_B < \bar{c} < v_B + w$. Bidder A dropping out at price p^* and bidder B bidding up to price $p_B > p^*$ (i.e., a resale equilibrium) is the unique equilibrium in undominated strategies if and only if

$$2w + \frac{v_B^2 \beta}{2\bar{c}(1+\beta)} < v_A + \bar{c} - v_B.$$

So resale takes place if w is low and/or if β is low. In both cases, bidder B is prepared to bid up to the price at which bidder A prefers to let him win the auction and obtain the project in the resale market, rather than outbid him.

Notice that the resale equilibrium does not depend on any bidder entering the auction or dropping out of the auction when indifferent about doing so—resale is robust to the addition of both a (small) bidding cost and a (small) cost to trade in the resale market.

Interpretation. In order to buy the project from bidder B in the resale market, bidder A prefers to drop out of the auction at price p^* instead of bidding up to her expected profit. The reason is that, because of the limited liability effect, after winning at a higher price, bidder B has a relatively stronger bargaining position in the resale market.

So resale takes place if bidder B is prepared to bid up to p^* during the auction (i.e., if Condition (B) holds), which depends on how much he can obtain in the resale market. Specifically, bidder B bids up to p^* if his total surplus when he wins the auction at price p^* and then resells the project—that is, $\frac{1}{2}(\mathbb{E}[\pi_A(p^*)] + \mathbb{E}[\pi_B(p^*)])$—is greater than zero. Therefore, he is willing to bid aggressively if the joint bidders' surplus in case of resale (i.e., $\mathbb{E}[\pi_A(p^*)]$) and his share of it, which depends on his bargaining power (i.e., his outside option $\mathbb{E}[\pi_B(p^*)]$), are large.

As shown in Proposition 1, whether resale takes place depends on w and β. First, a low β increases bidder B's expected profit, because it reduces the cost that bidder B has to pay to obtain the project when he does not declare bankruptcy. Therefore, a low β increases bidder B's outside option in the resale market and makes resale easier.

Second, a low w increases the limited liability effect, because it reduces bidder B's loss in case of bankruptcy and makes it more likely that bidder B chooses to declare bankruptcy if he owns the project. This induces bidder A to drop out of the auction sooner and hence lowers p^*, the auction price in case of resale. And a reduction in p^* increases the joint bidders' surplus and increases B's outside option.[32] Therefore, a low w, through its effect on p^*, also makes resale easier.

5. Seller's revenue

■ The effect of resale on the seller's revenue depends on whether bidders' participation in the auction is completely costless or not.

Assume first that bidders have to pay an arbitrarily small bidding cost to participate in the auction. If resale is not allowed, or if it does not take place in equilibrium, bidder B does not participate in an ascending auction against bidder A because he knows he would lose. If resale is allowed and it takes place in equilibrium, however, bidder A prefers to let bidder B win and then purchase in the aftermarket. This provides bidder B with an incentive to participate in the auction, thus raising the auction price. In this case, allowing resale raises the seller's revenue without reducing efficiency because, even if the auction is won by bidder B, the project is eventually obtained by the bidder with the highest valuation.

By contrast, if there is no bidding cost, bidder B participates in the auction even when resale is not allowed (and hence he has no chance of winning). However, allowing resale still increases the seller's revenue if bidder A wins the auction, because resale induces bidder B to bid more aggressively. On the other hand, if bidding is costless and bidder A loses the auction with resale, allowing resale raises the seller's revenue only when p^*, the price at which bidder A drops out of the auction with resale, is higher than the price that bidder B bids without resale. As the next proposition shows, this only happens if bidder B's wealth is sufficiently high.[33]

Proposition 2. If there is an arbitrarily small bidding cost, resale increases the seller's revenue. If there is no bidding cost and, therefore, bidder B always participates in the auction, (i) when resale does not take place in equilibrium, allowing resale always increases the seller's revenue, and (ii) when resale takes place in equilibrium, allowing resale increases the seller's revenue if and only if $2w > \bar{c} - \frac{v_B^2\beta}{\bar{c}(1+\beta)}$.

To analyze the strategy that the seller can adopt to increase his revenue, we make the natural assumption that, because of a small bidding cost, bidder B does not participate in the auction when he knows he would lose. In this case, the seller's revenue is higher if resale takes place in equilibrium.

Therefore, the seller should allow resale and, if possible, manipulate the conditions under which bidder B finances his bid in order to induce resale after the auction.[34] Suppose bidders'

[32] A low w has also another, direct, effect: it reduces bidder B's operating profit when he does not declare bankruptcy, and this may reduce his expected profit for a given auction price. (Indeed, $\frac{\partial}{\partial w}\mathbb{E}[\pi_B(p)] > 0 \Leftrightarrow p < p^*$.) However, because of its effect on p^*, the net effect of a reduction in w is always to increase bidder B's outside option.

[33] A high w reduces the limited liability effect and increases p^*, whereas its effect on the auction price without resale is more ambiguous.

[34] In our analysis, we assume the seller's only available strategy is to change the borrowing cost. This is an extreme assumption. If the seller knows the exact bidders' valuations and can set a reserve price, his optimal strategy is to set

borrowing cost is determined by the interest rate exogenously chosen by banks. Then the seller can reduce bidder B's financing cost by committing, before the auction, to finance a bid higher than his wealth with a loan at a borrowing rate lower than the bank interest rate. From Proposition 1, this allows bidder B to bid aggressively, hence making resale easier and increasing the competitiveness of the auction.[35]

The seller can also relax the condition for resale by reducing w (for example, by inducing a bidder with a lower wealth to participate in the auction), because this increases the limited liability effect and induces bidder A to drop out of the auction sooner. However, given that resale takes place, the seller's revenue is given by p^*, the price at which bidder A drops out of the auction. Hence, to increase his revenue in case of resale, the seller should increase w to induce bidder A to drop out of the auction later. As argued above, however, precisely because it increases p^*, these strategies also make it less profitable for bidder B to win the auction and, if pushed too far, may prevent resale.

6. Auction price paid before resale

■ In this section, we analyze an alternative model in which the auction price is paid in stage i—that is, right after the auction and before resale takes place—and show that the qualitative results of our main model still hold. In particular, we show that bidder A may still want to drop out of the auction at a relatively low price, and that resale may take place in equilibrium. The reason is that, if bidder B wins the auction, the price he pays changes his residual wealth and, hence, affects the probability of bankruptcy. Therefore, as in our main model, because of the limited liability effect, bidder A may be in a better bargaining position in the resale market if her opponent wins the auction at a relatively low price.

We assume bidder B cannot pay an auction price higher than his wealth (i.e., borrowing during the auction is not allowed). The reason is that, if the auction price is higher than w, an increase in the auction price has no limited liability effect at the margin, because bidder B has no residual wealth.[36] Therefore, after winning the auction and paying the auction price, bidder B is left with residual wealth $(w - p)$. If bidder B retains the project and the operating cost is higher than his residual wealth, in order to pay the operating cost and obtain v_B he has to borrow at cost β per unit of capital. All other assumptions are as in our main model.

The new timing of the game is the following:

(i) Bidder i wins the auction and pays the auction price p;
(ii) Resale can take place;
(iii) The operating cost c is realized;
(iv) The owner of the project, bidder j, can:
  (a) obtain v_j by paying for c out of his residual wealth and, if this is not sufficient, by paying the borrowing cost β on $c - (w_j - p)$, or
  (b) declare bankruptcy and liquidate his residual wealth.

After winning the auction and paying the auction price, bidder B has to pay the borrowing cost if his residual wealth is lower than c, but declares bankruptcy if his operating profit is lower than $-(w - p)$. Therefore, bidder B's profit from owning the project is

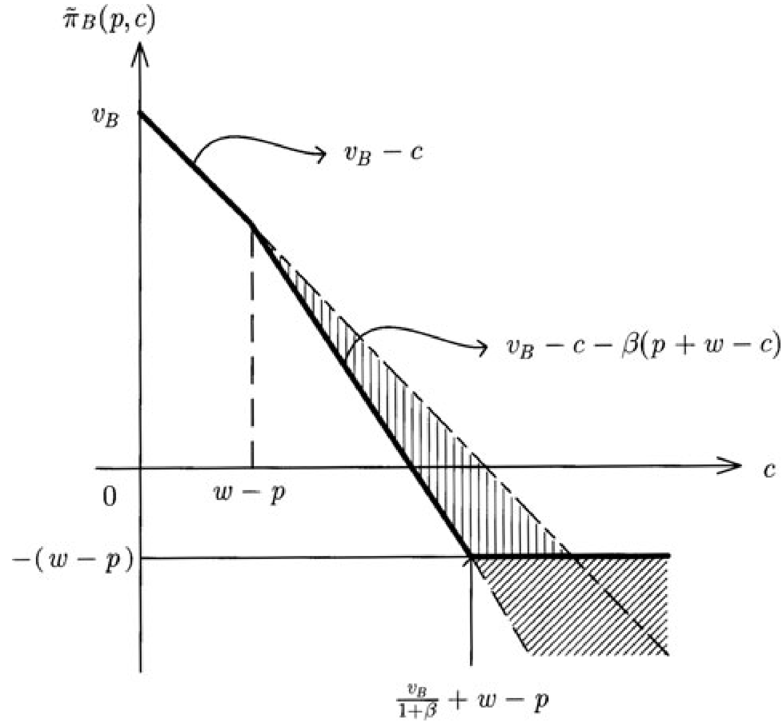
a reserve price equal to $v_A - \mathbb{E}[c]$, and obtain the whole bidders' surplus. In reality, however, the seller's information is much more uncertain. Even if the seller only knows the distribution of bidders' valuations, there are perhaps more complex mechanisms which in theory could extract more of the bidders' surplus. However, in the real world, setting a reserve price is often extremely difficult, and more complex mechanisms are even harder to implement.

[35] Zheng (2001) obtains a similar result in a model without resale. However, in our model, committing to lend money to a weak bidder to induce resale entails no cost for the seller because, due to our assumption that resale always takes place if bidder B wins the auction, it is always bidder A who eventually pays the cost of the project.

[36] We discuss how this assumption affects our results at the end of the section.

FIGURE 3

BIDDER *B*'S PROFIT AFTER PAYING THE AUCTION PRICE



$$\widetilde{\pi}_B(p,c) = \begin{cases} v_B - c & \text{if } c \le w - p, \\ v_B - c - \beta(p + c - w) & \text{if } w - p < c < \frac{v_B}{1+\beta} + w - p, \\ -(w-p) & \text{if } \frac{v_B}{1+\beta} + w - p \le c. \end{cases}$$

Figure 3 represents bidder *B*'s profit. As in our main model, a wealth constraint has both a borrowing cost effect (which reduces profit on good projects) and a limited liability effect (which reduces losses from bad projects).

Bidder *B*'s expected profit is

$$\mathbb{E}_c[\widetilde{\pi}_B(p,c)] = \int_0^{w-p} (v_B - c)\, dF(c) + \int_{w-p}^{\frac{v_B}{1+\beta}+w-p} [v_B - c - \beta(p + c - w)]\, dF(c)$$
$$- (w-p)\left[1 - F\left(\frac{v_B}{1+\beta} + w - p\right)\right],$$

while bidder *A*'s expected profit from the project (after paying the auction price) is

$$\mathbb{E}_c[\widetilde{\pi}_A(c)] = v_A - \mathbb{E}[c].$$

In the absence of a wealth constraint, an increase in the auction price has no effect on a bidder's expected profit in stage ii. By contrast, because bidder *B* faces a wealth constraint,

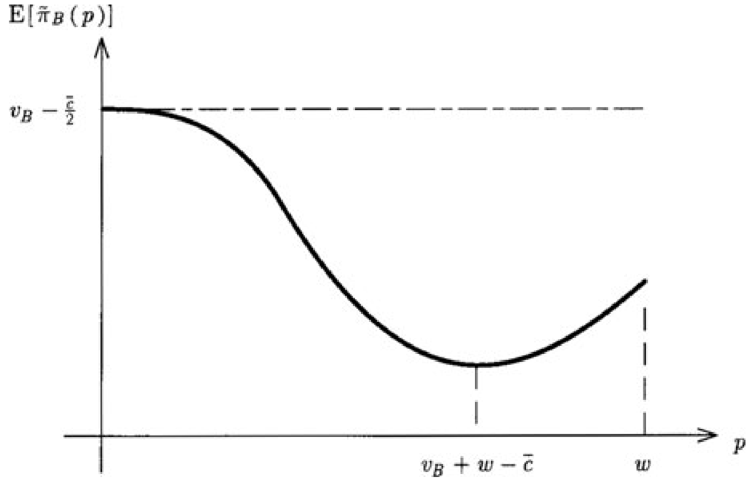
$$\frac{\partial}{\partial p}\mathbb{E}_c[\widetilde{\pi}_B(p)] = -(1+\beta)\cdot F\left(\frac{v_B}{1+\beta} + w - p\right) + \beta \cdot F(w-p)$$

and, hence,

$$\frac{\partial}{\partial p}\mathbb{E}_c[\widetilde{\pi}_B(p)] \le 0 \Leftrightarrow \beta \cdot \Pr[B \text{ borrows}] \ge \Pr[B \text{ goes bankrupt}].$$

FIGURE 4

BIDDER *B*'S EXPECTED PROFIT FROM OWNING THE PROJECT, AFTER PAYING THE AUCTION PRICE



Therefore, as in our main model, an increase in the auction price reduces bidder *B*'s expected profit relative to bidder *A*'s expected profit if and only if the borrowing cost effect is stronger than the limited liability effect.

As in our main model, to obtain a closed-form solution, we assume that c is drawn from a uniform distribution on $[0, \bar{c}]$, where $\bar{c} > v_B$ for limited liability to matter.[37] Figure 4 represents bidder *B*'s expected profit and shows that, at a relatively high price, the limited liability effect prevails.[38]

If bidder *A* buys the project from bidder *B* in the resale market, she wants to maximize her resale surplus $\frac{1}{2}(\mathbb{E}[\tilde{\pi}_A] - \mathbb{E}[\tilde{\pi}_B(p)])$. Because of the effect of the auction price on bidder *B*'s expected profit, bidder *A* is not indifferent about the price paid by her opponent in the auction and, in order to buy in the resale market, she drops out of the auction at price

$$p^* \in \arg\max_{p \geq 0} \mathbb{E}_c[\tilde{\pi}_B(p)] \Leftrightarrow p^* = v_B + w - \bar{c}.$$

As in our main model, to make the analysis interesting and ensure that $p^* > 0$, we assume that $v_B + w > \bar{c}$.

Our general point is that a high auction price can harm bidder *A* in the resale market because of the limited liability effect. When the auction price is paid after resale (as in our main model), a high auction price increases the limited liability effect for bidder *B* because it raises the total cost of the project $(p + c)$. By contrast, when the auction price is paid before resale, a high auction price increases the limited liability effect for bidder *B* because it reduces his residual wealth $(w - p)$.

In order to have a resale equilibrium, bidder *B* must be willing to bid more than p^* and bidder *A* must strictly prefer to let bidder *B* win the auction at price p^* and buy in the resale market, rather than win the auction herself at the price at which bidder *B* drops out, which can be at most equal to w.

Proposition 3. Assume the auction price is paid before resale takes place, $c \sim U[0, \bar{c}]$ and $v_B < \bar{c} < v_B + w$. Bidder *B* bidding up to a price greater than p^* and bidder *A* dropping out at

[37] Limited liability is only relevant if bidder *B*'s operating profit (excluding the auction price) can be negative for some realizations of the operating cost.

[38] The details of the analysis of the uniform distribution case are in the Appendix.

price p^* is an equilibrium if

$$-(\overline{c} - v_B) < v_A - 2w - \frac{v_B^2 \beta}{2\overline{c}(1+\beta)} < (\overline{c} - v_B).$$

As in our main model, a resale equilibrium requires that bidder B is willing to bid up to p^* during the auction, which depends on how much he can obtain in the resale market. Specifically, bidder B bids up to p^* if his surplus when he wins the auction at price p^* and then resells the project—that is, the resale price $\frac{1}{2}(\mathbb{E}[\widetilde{\pi}_A] + \mathbb{E}[\widetilde{\pi}_B(p^*)])$—is greater than the auction price p^*. Whether this condition is satisfied depends on w and β, because a low w reduces p^* and a low β increases bidder B's expected profit, and hence his outside option in the resale market. Therefore, resale takes place in equilibrium only if w and β are sufficiently low.

In addition to this condition, however, when bidder B would like to bid more than w but has to drop out at price w, a resale equilibrium also requires that bidder A prefers to drop out of the auction at price p^* and buy the project in the resale market, instead of bidding up to w to win the auction. And bidder A prefers to drop out at price p^* when bidder B bids up to w if the price she has to pay to buy in the resale market—that is, $\frac{1}{2}(\mathbb{E}[\widetilde{\pi}_A] + \mathbb{E}[\widetilde{\pi}_B(p^*)])$—is lower than w. Whether this condition is satisfied also depends on w and β. Therefore, when bidder B drops out at a price equal to his wealth, resale takes place in equilibrium only if w and β are not too low. Summing up, as proven in Proposition 3, when the auction price is paid before resale and bidder B cannot bid more than w, resale takes place in equilibrium for intermediate values of w and β.

Notice that, if bidder B can bid more than w, bidder A may prefer not to drop out at price p^* and instead raise the auction price above w when she buys the project in the resale market. The reason is that, once bidder B has no residual wealth, an increase in the auction price has no limited liability effect and only reduces bidder B's expected profit.[39] However, even in this case, bidder A drops out at price p^* and resale takes place in equilibrium if, for example, bidder B is only willing to bid up to a price lower than w (and higher than p^*).[40]

□ **Endogenous timing of resale.** If bidders have a choice, do they trade in the resale market before or after the auction price is paid? It can be shown that, when $\overline{c} > v_B$ (i.e., when there is a limited liability effect in both versions of our model) and bidder B cannot bid more than w, after bidder A drops out of the auction at price p^* the two bidders are *indifferent* between trading before or after paying the auction price, because they obtain the same surplus in both cases. The reason is that, if it is bidder A who pays the auction price after resale takes place, she also pays a lower resale price. Similarly, if it is bidder B who pays the auction price before reselling the project, he also obtains a higher resale price. In both cases, the two effects cancel out.

By contrast, bidders are not indifferent about the timing of resale if (i) $\overline{c} < v_B$, because in this case when the auction price is paid before resale there is no limited liability effect and bidder A has no incentive to drop out of the auction, and/or (ii) bidder B can bid more than w, because in this case when the auction price is paid before resale bidder A may want to drop out at a price higher than p^*. In these cases, the two bidders may have contrasting preferences on when to trade in the resale market, and the result depends on the bidders' relative bargaining power.

7. Extensions

■ **More bidders.** Our results do not depend on the presence of only two bidders. Assume there are n bidders and let A be the bidder with the highest valuation, B the bidder with the second-

[39] In Figure 4, bidder B's expected profit decreases monotonically to zero as p increases above w.

[40] More generally, when bidder B can bid more than w, there is a resale equilibrium in which bidder A drops out at price p^* and bidder B bids up to price $p_B > p^*$ such that $\frac{1}{2}(\mathbb{E}[\widetilde{\pi}_A] + \mathbb{E}[\widetilde{\pi}_B(p_B)]) = p_B$ (which is the highest price that bidder B is willing to pay in the auction) if $\mathbb{E}[\widetilde{\pi}_B(p_B)] > \mathbb{E}[\widetilde{\pi}_B(p^*)]$. The reason is that, in this case, bidder A strictly prefers to drop out of the auction and buy in the resale market at price $\frac{1}{2}(\mathbb{E}[\widetilde{\pi}_A] + \mathbb{E}[\widetilde{\pi}_B(p^*)])$, rather than win the auction at price p_B.

highest valuation, and C the bidder with the third-highest valuation. For simplicity, assume that bidder A is the only bidder who is not wealth constrained and that all other bidders have the same initial wealth.

During the auction, the weak bidders (i.e., all bidders who do not have the highest value) compete for a chance to resell to the strong bidder (i.e., bidder A). Because a weak bidder's surplus in the resale market is increasing in his valuation, the price at which he drops out of the auction is also increasing in his valuation. Therefore, bidder B outbids all other weak bidders.[41] And, exactly as with only two bidders, provided bidder B is willing to bid up to p^* (i.e., up to the price that maximizes bidder A's surplus in the aftermarket), bidder A strictly prefers to drop out of the auction and purchase the prize from bidder B.

Because there are other weak bidders, however, in a resale equilibrium bidder B may need to pay more than p^* to win the auction. Specifically, bidder B pays a price equal to the highest of p^* and the price at which bidder C drops out of the auction. Hence, apart from hurting bidder B, competition among weak bidders may also hurt bidder A, because bidder A's surplus in the resale market is maximized when bidder B wins the auction at price p^*. The seller, on the other hand, is better off because the auction price may be higher. However, because all weak bidders other than bidder B know they have no chance of winning the auction, if there is an arbitrarily small bidding cost they do not participate in the auction at all, leaving only two active bidders.

Notice also that, in our analysis, we did not consider that bidders could merge before the auction, or reach an agreement to prevent the participation of the weak bidder in the auction. With such an agreement, bidders could obtain the project at price zero and share the whole surplus. However, this possibility rests on the assumption that there is no other potential buyer for the project on sale. If more buyers are present, an agreement with a weak bidder does not help the strong bidder, as other competitors would participate and raise the auction price anyway.[42]

□ **Returning the prize.** Assume a winning bidder can return the project to the seller without paying the operating cost, if the operating cost turns out to be too high and, hence, the project not to be profitable. This, in practice, introduces a form of limited liability for both bidders, regardless of their initial wealth, because bidders can avoid losses on bad projects when the operating profit is negative.

If bidders are asymmetric, however—that is, if v_A is higher than v_B—limited liability becomes relevant at a lower auction price for bidder B than for bidder A (because, for a given auction price, B's profit is lower than A's profit and, hence, he is more likely than bidder A to want to return the project to the seller). So, even if both bidders have infinite wealth, there is a range of prices (in which bidder B may prefer to return the project to the seller, while bidder A does not) where raising the auction price reduces bidder A's expected profit more than it reduces bidder B's expected profit. Therefore, as in our model, in order to purchase the project from bidder B, bidder A may prefer to drop out of the auction at a low price to maximize the gains from trade in the resale market.

8. Conclusions

■ A strong bidder may prefer to drop out of an auction before the price has reached her valuation, and then purchase the prize from a competitor with limited liability in the aftermarket.

[41] If weak bidders have different initial wealths, the bidder who is willing to bid higher may not be the one with the highest valuation. The rest of our argument remains unchanged, however.

[42] For example, if in the UK 3G mobile-phone licenses auction the number of bidders had been only one more than the number of licenses on sale, bidders would have probably tried to merge before the auction to reduce competition. Because the number of potential buyers was larger, however, this was much more difficult. So only after the auction had been run did the marginal loser take over Orange. On the other hand, before other 3G auctions that were run later (e.g., the Netherlands auction), the number of potential buyers shrank and bidders merged before the auction, reducing competition and the seller's revenue.

The possibility of reselling the prize to a strong bidder makes a weak and wealth-constrained bidder willing to participate in the auction and to bid aggressively. So resale can take place in equilibrium, and a weak bidder can win an auction even against a clearly advantaged competitor.

Thanks to resale, competition in the auction can be stronger and the price higher. The seller can induce resale by improving the conditions at which a weak bidder can finance his bids.

When European governments auctioned 3G mobile-phone licenses, it was not clear whether winners would be allowed to resell the license they acquired. In many countries, resale was explicitly forbidden.[43] This may have induced new entrants, who had a lower valuation of the licenses than incumbents, to bid less aggressively, and may even have discouraged them from participating at all. Indeed, many 3G auctions lacked sufficient competition and generated disappointingly low revenues. Our analysis suggests that, by allowing winners to resell their licenses, governments could have encouraged new entrants and small bidders to participate and bid aggressively, without affecting the efficient allocation of the spectrum.

Our broader point is that there may be a variety of reasons why bargaining in the aftermarket can be affected by the price paid by the auction winner. When this occurs, a strong bidder is not indifferent about the price paid by her competitor during the auction, and may prefer to drop out of the auction in order to purchase the prize in the resale market. We explored some of these reasons, and leave to further research the analysis of more of them.

Appendix

■ Proofs of Lemmas 1 and 2 and Propositions 1–3 follow.

Proof of Lemma 1. By assumption, at any price at which bidder B can win the auction $\mathbb{E}_c[\pi_A(p)] > \mathbb{E}_c[\pi_B(p)]$. Therefore, the value that bidder B attaches to winning is given by the total surplus he can obtain in the resale market, which is the resale price. Because with resale bidder B does not pay the auction price, in an ascending auction it is a weakly dominant strategy for him to bid up to a price p_B such that the resale price is equal to zero—that is, such that $\mathbb{E}_c[\pi_A(p_B)] + \mathbb{E}_c[\pi_B(p_B)] = 0$. The price p_B exists, and is strictly positive and unique because (i) $\mathbb{E}_c[\pi_A(p)] + \mathbb{E}_c[\pi_B(p)]$ is continuous and strictly decreasing, (ii) $\mathbb{E}_c[\pi_A(0)] + \mathbb{E}_c[\pi_B(0)] > 0$ by assumption, and (iii) $\lim_{p\to+\infty}(\mathbb{E}_c[\pi_A(p)] + \mathbb{E}_c[\pi_B(p)]) = -\infty$.

On the other hand, when resale is not allowed, if bidder B participates in the auction, he bids up to his expected profit from operating the project—that is, up to $\widehat{p}$ such that $\mathbb{E}_c[\pi_B(\widehat{p})] = 0$.[44] And because $\mathbb{E}_c[\pi_B(p_B)] < 0$ (as $\mathbb{E}_c[\pi_A(p_B)] > 0$ by assumption) and $\frac{\partial}{\partial p}\mathbb{E}_c[\pi_B(p)] \leq 0$, it follows that $p_B > \widehat{p}$. *Q.E.D.*

Proof of Lemma 2. We prove the result for an arbitrary division of the resale surplus among bidders. Assume bidder B obtains a share k of the gains from trade in the resale market and bidder A obtains a share $(1-k)$, for $0 < k < 1$.

Bidder A can win the auction at the price at which bidder B drops out. This is price p_B such that the resale price is equal to 0—that is,

$$k\,(\mathbb{E}_c[\pi_A(p_B)] - \mathbb{E}_c[\pi_B(p_B)]) + \mathbb{E}_c[\pi_B(p_B)] = 0 \Leftrightarrow \mathbb{E}_c[\pi_A(p_B)] = -\frac{1-k}{k}\mathbb{E}_c[\pi_B(p_B)].$$

If, on the other hand, bidder A drops out at price p_B and purchases the project from bidder B, she obtains a surplus equal to

$$\begin{aligned}(1-k)\,S(p_B) &= (1-k)\,(\mathbb{E}_c[\pi_A(p_B)] - \mathbb{E}_c[\pi_B(p_B)])\\ &= (1-k)\left(\mathbb{E}_c[\pi_A(p_B)] + \tfrac{k}{1-k}\mathbb{E}_c[\pi_A(p_B)]\right)\\ &= \mathbb{E}_c[\pi_A(p_B)].\end{aligned}$$

Therefore, at price p_B, bidder A is exactly indifferent between winning the auction and purchasing in the aftermarket.

Because, by definition, p^* is the price that maximizes bidder A's resale surplus, at any price higher than p^* bidder A's resale surplus is lower than at price p^*. Hence, if $p_B > p^*$ (i.e., if Condition (B) is satisfied),

$$(1-k)\,S(p^*) > (1-k)\,S(p_B) = \mathbb{E}_c[\pi_A(p_B)],$$

and so bidder A strictly prefers to let bidder B win at price p^* and purchase in the resale market, rather than win the auction at price p_B.

[43] Even if direct resale is not allowed, a losing bidder can perhaps still obtain the auction's prize by taking over the winner. However, this is clearly more problematic than just acquiring the prize.

[44] Pagnozzi (2004), working paper version of this article, explicitly derive $\widehat{p}$ and show that, in an auction without resale, bidders with low wealth bid more aggressively than bidders with high wealth because of the limited liability effect, provided the borrowing cost is sufficiently low. This confirms the results obtained by Zheng (2001) for first-price auctions.

By contrast, if $p_B \le p^*$, bidder A has no incentive to drop out of the auction and purchase in the resale market. The reason is that bidder A strictly prefers to win the auction at price p_B, rather than purchase in the resale market after letting bidder B win the auction at a price lower than p_B; and bidder A is indifferent between winning the auction at price p_B and buying in the resale market, after letting bidder B win the auction at price p_B. *Q.E.D.*

□ **Bidder B's expected profit when $c \sim U[0, \overline{c}]$ and p is paid after resale.** Suppose bidder B wins the auction at price p. His expected profit depends on the relation between p, c, and w, and on whether he may choose to declare bankruptcy when the auction price is lower than his initial wealth.

First, if $p + \overline{c} < w$, then B has enough wealth to pay the total cost of the project, regardless of the realized operating cost. In this case (like for a bidder without a wealth constraint), his expected profit is $v_B - p - \frac{\overline{c}}{2}$. Second, if $w - \overline{c} < p < \min\{w; \frac{v_B}{1+\beta} + w - \overline{c}\}$, bidder B has to pay the additional borrowing cost if the total cost is high (i.e., for $c + p > w$). Moreover, in this case, B does not declare bankruptcy and always pays the whole total cost, as his operating profit is never lower than $-w$. In this case, B's expected profit is

$$\int_0^{w-p} [v_B - (p+c)]\, dF(c) + \int_{w-p}^{\overline{c}} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c)$$
$$= v_B - (1+\beta)\left(p + \frac{\overline{c}}{2}\right) + \beta w - \frac{\beta (w-p)^2}{2\overline{c}}.$$

Third, if $\frac{v_B}{1+\beta} > \overline{c}$ and $w < p < \frac{v_B}{1+\beta} + w - \overline{c}$, bidder B always pays the additional borrowing cost in order to obtain v_B because the auction price is higher than his wealth, and he never declares bankruptcy because his operating profit is still never lower than $-w$. Therefore, in this case, B's expected profit is

$$\int_0^{\overline{c}} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c) = v_B - (1+\beta)\left(p + \frac{\overline{c}}{2}\right) + \beta w.$$

By contrast, if $\frac{v_B}{1+\beta} < \overline{c}$ and $\frac{v_B}{1+\beta} + w - \overline{c} < p < w$, bidder B pays the additional borrowing cost only if the total cost is higher than his wealth, but, at the same time, his loss is limited to w when the total cost is very high (i.e., when $p + c > w + \frac{v_B}{1+\beta}$). Therefore, in this case, B's expected profit is

$$\int_0^{w-p} [v_B - (p+c)]\, dF(c) + \int_{w-p}^{w-p+\frac{v_B}{1+\beta}} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c)$$
$$- \int_{w-p+\frac{v_B}{1+\beta}}^{\overline{c}} w\, dF(c)$$
$$= \frac{v_B}{\overline{c}}(w-p) + \frac{(w-p)^2}{2\overline{c}} + \frac{v_B^2}{2\overline{c}(1+\beta)} - w.$$

Fourth, if $\max\{w; \frac{v_B}{1+\beta} + w - \overline{c}\} < p < \frac{v_B}{1+\beta} + w$, bidder B always pays the additional borrowing cost, but he declares bankruptcy and limits his actual loss to w when the operating profit is lower than $-w$. Therefore, in this case, B's expected profit is

$$\int_0^{w-p+\frac{v_B}{1+\beta}} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c) - \int_{w-p+\frac{v_B}{1+\beta}}^{\overline{c}} w\, dF(c)$$
$$= \frac{1}{2\overline{c}(1+\beta)} [v_B - (1+\beta)(p-w)]^2 - w.$$

Finally, if $p > \frac{v_B}{1+\beta} + w$, bidder B declares bankruptcy and loses his initial wealth regardless of the operating cost; hence, his expected (and actual) profit is $-w$.

Summing up, after winning the auction at price p, the expected profit of bidder B is

$$\mathbb{E}_c[\pi_B(p,c)]$$
$$= \begin{cases} v_B - \left(p + \frac{\overline{c}}{2}\right) & \text{if } p < w - \overline{c}, \\ v_B - (1+\beta)\left(p + \frac{\overline{c}}{2}\right) + \beta w - \frac{\beta(w-p)^2}{2\overline{c}} & \text{if } w - \overline{c} \le p < \min\left\{w; \frac{v_B}{1+\beta} + w - \overline{c}\right\}, \\ v_B - (1+\beta)\left(p + \frac{\overline{c}}{2}\right) + \beta w & \text{if } \frac{v_B}{1+\beta} > \overline{c} \text{ and } w \le p < \frac{v_B}{1+\beta} + w - \overline{c}, \\ \frac{v_B}{\overline{c}}(w-p) + \frac{(w-p)^2}{2\overline{c}} + \frac{v_B^2}{2\overline{c}(1+\beta)} - w & \text{if } \frac{v_B}{1+\beta} < \overline{c} \text{ and } \frac{v_B}{1+\beta} + w - \overline{c} \le p < w, \\ \frac{1}{2\overline{c}(1+\beta)}[v_B - (1+\beta)(p-w)]^2 - w & \text{if } \max\left\{w; \frac{v_B}{1+\beta} + w - \overline{c}\right\} \le p < \frac{v_B}{1+\beta} + w, \\ -w & \text{if } \frac{v_B}{1+\beta} + w \le p. \end{cases}$$

Bidder B's expected profit is decreasing in β because of the borrowing cost effect. The effect of an increase in w, however, is ambiguous: on the one hand, an increase in w reduces the need to obtain outside financing, which increases bidder B's expected profit for a relatively low auction price; on the other hand, an increase in w increases the liability of bidder B and his loss in case of bankruptcy, which reduces his expected profit for a relatively high auction price.

Bidder B's expected profit function is (weakly) decreasing in p. It is straightforward to check that[45]

$$\left|\frac{\partial}{\partial p}\mathbb{E}_c[\pi_B(p)]\right| > 1 \Leftrightarrow \begin{cases} p < v_B + w - \overline{c} & \text{if } \overline{c} > v_B, \\ p < \frac{v_B - \overline{c}}{1+\beta} + w & \text{if } v_B > \overline{c}. \end{cases}$$

Proof of Proposition 1. By Lemma 2, bidder A drops out of the auction at price $p^* = v_B + w - \overline{c} > 0$ and resale is an equilibrium if bidder B is willing to bid more than p^*. Because bidders' expected profits are decreasing in the auction price, this is true if and only if bidder B's surplus if he wins the auction at price p^* and resells the project is positive, that is,

$$\begin{aligned} & \mathbb{E}_c[\pi_A(p^*)] + \mathbb{E}_c[\pi_B(p^*)] > 0 \\ \Leftrightarrow & \left\{ v_A - p - \frac{\overline{c}}{2} + \frac{v_B}{\overline{c}}(w-p) + \frac{(w-p)^2}{2\overline{c}} + \frac{v_B^2}{2\overline{c}(1+\beta)} - w \right\}\Bigg|_{p=v_B+w-\overline{c}} > 0 \\ \Leftrightarrow & \; v_A - v_B + \overline{c} - \frac{v_B^2\beta}{2\overline{c}(1+\beta)} - 2w > 0. \end{aligned}$$

It is straightforward to check that, under this condition, a resale equilibrium is the only equilibrium in undominated strategies. On the other hand, when the condition is not satisfied (i.e., when $p^* > p_B$), bidder A bidding up to p^* and bidder B dropping out at p_B is an equilibrium without resale in undominated strategies. *Q.E.D.*

□ **Resale equilibrium when $c \sim U[0, \overline{c}]$ and $v_B > \overline{c}$.** When $v_B > \overline{c}$, bidder A drops out at price $p^* = \frac{v_B - \overline{c}}{1+\beta} + w$ if she wants to buy the project in the resale market. Bidder B is willing to bid more than p^* if and only if

$$\begin{aligned} & \mathbb{E}_c[\pi_A(p^*)] + \mathbb{E}_c[\pi_B(p^*)] > 0 \\ \Leftrightarrow & \left\{ v_A - p - \frac{\overline{c}}{2} + \frac{1}{2\overline{c}(1+\beta)}[v_B - (1+\beta)(p-w)]^2 - w \right\}\Bigg|_{p=\frac{v_B-\overline{c}}{1+\beta}+w} > 0 \\ \Leftrightarrow & \; 2w + \frac{v_B - \overline{c}}{1+\beta} + \frac{\overline{c}\beta}{2(1+\beta)} < v_A. \end{aligned}$$

This condition has a similar interpretation to the condition of Proposition 1. (However, when $v_B > \overline{c}$ there is an additional effect of β on p^*, which is discussed in the working paper version of this article.)

Proof of Proposition 2. The first part of the statement is obvious. If there is no bidding cost and resale does not take place, bidder B follows his weakly dominant strategy of dropping out of the auction when his expected value from winning is equal to zero. Therefore, when resale is not allowed, bidder B drops out at price $\widehat{p}$ such that $\mathbb{E}_c[\pi_B(\widehat{p})] = 0$ and the effect of allowing resale on the seller's revenue depends on whether resale takes place in equilibrium or not. When resale does not take place in equilibrium, the auction price is higher with resale because bidder B bids more aggressively and up to $p_B > \widehat{p}$.

By contrast, when resale takes place in equilibrium, the auction price is p^*. Therefore, the auction price and the seller's revenue are higher with resale if and only if $p^* > \widehat{p}$, that is,

$$\mathbb{E}_c[\pi_B(p^*)] < 0 \Leftrightarrow 2w > \overline{c} - \frac{\beta v_B^2}{\overline{c}(1+\beta)}. \qquad Q.E.D.$$

[45] Indeed,

$$\frac{\partial}{\partial p}\mathbb{E}_c[\pi_B(p)] = \begin{cases} -1 & \text{if } p < w - \overline{c}, \\ -(1+\beta) - \frac{\beta}{\overline{c}}(p-w) & \text{if } w - \overline{c} \le p < \min\{w; \frac{v_B}{1+\beta} + w - \overline{c}\}, \\ -(1+\beta) & \text{if } \frac{v_B}{1+\beta} > \overline{c} \text{ and } w \le p < \frac{v_B}{1+\beta} + w - \overline{c}, \\ -\frac{1}{\overline{c}}(v_B + w - p) & \text{if } \frac{v_B}{1+\beta} < \overline{c} \text{ and } \frac{v_B}{1+\beta} + w - \overline{c} \le p < w, \\ -\frac{1}{\overline{c}}[v_B - (1+\beta)(p-w)] & \text{if } \max\{w; \frac{v_B}{1+\beta} + w - \overline{c}\} \le p < \frac{v_B}{1+\beta} + w, \\ 0 & \text{if } \frac{v_B}{1+\beta} + w \le p. \end{cases}$$

Moreover, for $p < \min\{w; \frac{v_B}{1+\beta} + w - \overline{c}\}$, the expected profit function is concave in p, whereas for $p > \min\{w; \frac{v_B}{1+\beta} + w - \overline{c}\}$ it is convex.

□ **Bidder B's expected profit when $c \sim U[0, \bar{c}]$ and p is paid before resale.** Suppose bidder B wins the auction and pays p. His expected profit in stage ii depends on whether he may choose to declare bankruptcy.

First, if $w - p > \bar{c}$, bidder B has enough residual wealth to pay any realized cost and, hence, his expected profit is $v_B - \frac{\bar{c}}{2}$. Second, if $\bar{c} - \frac{v_B}{1+\beta} \leq w - p < \bar{c}$, bidder B pays the additional borrowing cost β if $c > w - p$ but never declares bankruptcy because his operating profit is always higher than $-(w - p)$. Therefore, in this case, bidder B's expected profit is

$$\int_0^{w-p} (v_B - c)\, dF(c) + \int_{w-p}^{\bar{c}} [v_B - c - \beta(p + c - w)]\, dF(c)$$
$$= v_B - \frac{(1+\beta)\bar{c}}{2} - \beta(w-p) - \frac{1}{2\bar{c}}\beta(w-p)^2 .$$

Third, if $0 \leq w - p < \bar{c} - \frac{v_B}{1+\beta}$, bidder B pays the borrowing cost if $c > w - p$, but he declares bankruptcy and loses his residual wealth when the operating profit is lower than $-(w - p)$. Therefore, in this case, bidder B's expected profit is

$$\int_0^{w-p} (v_B - c)\, dF(c) + \int_{w-p}^{\frac{v_B}{1+\beta}+w-p} [v_B - c - \beta(p + c - w)]\, dF(c)$$
$$- \int_{\frac{v_B}{1+\beta}+w-p}^{\bar{c}} (w-p)\, dF(c)$$
$$= \frac{1}{2\bar{c}}\left[(w-p)^2 - 2(w-p)(\bar{c} - v_B) + \frac{v_B^2}{1+\beta}\right].$$

It is straightforward to show that bidder B's expected profit is minimized at $p^* = v_B + w - \bar{c}$.

Proof of Proposition 3. If bidder B wins the auction at price p^*, the resale price is equal to

$$\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A] + \mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right) = \frac{1}{2}\left[v_A + v_B - \bar{c} - \frac{v_B^2 \beta}{2\bar{c}(1+\beta)}\right].$$

This is the lowest price that bidder A can pay to buy the project in the resale market. There may be a resale equilibrium in which bidder A drops out at price p^* in two possible cases: (i) bidder B bids up to a price higher than p^* and lower than w and (ii) bidder B bids up to w.

First, bidder B bids up to a price higher than p^* and lower than w if: (i) the resale price at which bidder B can resell the project after winning the auction at price p^* is higher than p^*—that is,

$$\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A] + \mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right) > p^* \Leftrightarrow v_A - 2w - \frac{v_B^2 \beta}{2\bar{c}(1+\beta)} > -(\bar{c} - v_B); \tag{A1}$$

and (ii) the resale price at which bidder B can resell the project after winning the auction at price w is lower than w—that is,

$$\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A] + \mathbb{E}_c[\widetilde{\pi}_B(w)]\right) < w \Leftrightarrow v_A + \frac{v_B^2}{2\bar{c}(1+\beta)} < 2w + \frac{1}{2}\bar{c}. \tag{A2}$$

When these conditions are satisfied, bidder B bids up to the price at which he is indifferent between winning and losing the auction—that is, he bids up to $p_B > p^*$ such that $\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A] + \mathbb{E}_c[\widetilde{\pi}_B(p_B)]\right) = p_B$. At price p_B bidder A is indifferent between winning the auction and dropping out to buy in the resale market, because the auction price is equal to the resale price. And because $\frac{\partial}{\partial p}\mathbb{E}_c[\widetilde{\pi}_B(p)] > 0$ for $p > p^*$, bidder A strictly prefers to drop out of the auction at price p^* and buy in the resale market at price $\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A] + \mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right)$, rather than win the auction at price p_B. Therefore, there is a resale equilibrium in which bidder B bids up to a price p_B higher than p^* and lower than w and bidder A drops out at price p^* if condition (A1) and condition (A2) are satisfied—that is,

$$-(\bar{c} - v_B) < v_A - 2w - \frac{v_B^2 \beta}{2\bar{c}(1+\beta)} < \frac{\bar{c}}{2} - \frac{v_B^2}{2c}. \tag{A3}$$

Second, bidder B bids up to w if the resale price at which bidder B can resell the project after winning the auction at price w is lower than w—that is, if condition (A2) is not satisfied. In this case, bidder A drops out of the auction at price p^* and buys in the resale market if and only if the resale price after bidder B wins the auction at price p^* is lower than w—that is,

$$\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A] + \mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right) < w \Leftrightarrow v_A - 2w - \frac{v_B^2 \beta}{2\bar{c}(1+\beta)} < \bar{c} - v_B. \tag{A4}$$

Notice that, if bidder B is happy to win the auction at price w, then he is also happy to win at price p^* because

$$\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A] + \mathbb{E}_c[\widetilde{\pi}_B(w)]\right) > w \Rightarrow \frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A] + \mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right) > p^*. \tag{A5}$$

Therefore, there is a resale equilibrium in which bidder B bids w and bidder A drops out at price p^* if conditions (A4) and (A5) are satisfied—that is,

$$\frac{\overline{c}}{2} - \frac{v_B^2}{2c} < v_A - 2w - \frac{v_B^2 \beta}{2\overline{c}(1+\beta)} < \overline{c} - v_B. \tag{A6}$$

Summing up, resale takes place in equilibrium if either condition (A3) or condition (A6) is satisfied. Rearranging yields the result. *Q.E.D.*

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Erratum

Due to an error in the *RAND Journal of Economics* 38:4 on page 1105, an equation printed incorrectly.
The corrected equation appears below.

$$p^* \in \arg\min_{p \geq 0} \ \mathbb{E}_c[\widetilde{\pi}_B(p)] \Leftrightarrow p^* = v_B + w - \overline{c}.$$

Reference

PAGNOZZI, M. "Bidding to Lose? Auctions with Resale." *RAND Journal of Economics*, Vol. 38 (2007), pp. 1090–1112.

*Please note that an erratum for this error was printed incorrectly in Vol. 39 (2008), pp. 327. The publisher apologizes for this mistake.