# Multi-Unit Auctions with Resale: An Experimental Analysis

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- Should resale be allowed?
- How should the resale market be structured?



#### Why does resale happen?

- Bidders do not participate in the auction (Milgrom, 1987; Bikhchandani & Huang, 1989)
- Bidders' valuations change after the auction (Haile, 2000, 2003)
- Value uncertainty (in 1<sup>st</sup>-price auctions)
   (Gupta & Lebrun, 1999; Hafalir & Krishna, 2007)
- Auction price affects bargaining in resale market (Pagnozzi, 2007)
- Strategic behavior: demand reduction and speculation (Garratt & Tröger 2006; Pagnozzi, 2009, 2010)



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- Resale induces weak (low-value) bidders to speculate:
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- Resale increases strong bidders' incentive to reduce demand, because they can purchase after the auction the units lost

#### **Overview**

In multi-object uniform-price auctions with asymmetric bidders and resale through bargaining:

Without resale, asymmetry between bidders affects demand reduction

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- ② Demand reduction and speculation emerge when the auction winner can resell
- Effect of resale on efficiency and seller's revenue

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- Openand reduction and speculation emerge when the auction winner can resell
- Effect of resale on efficiency and seller's revenue
- Effects of changing the resale market structure

## THEORETICAL BACKGROUND

#### <u>Model</u>

- 2 units of an identical good for sale
- Uniform-price auction: the 2 highest bids win, and winner(s) pay the 3<sup>rd</sup>-highest bid for each unit
- 2 asymmetric bidders:
- S (strong) demands 2 units and has high value  $v_{s} \sim U$  [30; 50]
- W (weak) demands 1 unit and has low value  $v_{w} \sim U$  [10; 30]
- $\rightarrow$  Either S wins both units or S and W win one unit each

• It is a dominant strategy for W to bid  $v_W$  (as in a single-object  $2^{nd}$ -price auction)

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• S's incentive to reduce demand giving up 1 unit is lower when he has a higher value

#### Resale Market

- ullet After the auction, if W wins a unit, he can resell it to S
- Resale takes place through bargaining
- Gains from trade are v<sub>S</sub> − v<sub>W</sub>
- $oldsymbol{\circ}$  S obtains a share lpha of the gains from trade W obtains a share (1-lpha) of the gains from trade (results are robust to many alternative models of resale market)

• W bids up to the expected resale price

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 W speculates because of the option to resell and bids higher than v<sub>W</sub>

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$$v_s - 0 + v_S - r = 2v_S - r$$
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- $\Rightarrow$  S always reduces demand (for every  $\alpha$  and  $v_S$ )
  - Demand reduction allows S to win 1 unit at price 0 and then purchase the other unit from W in resale (rather than pay  $\mathbb{E}[r]$  for both units)

#### Summing up:

• Without resale, W bids  $v_w$  and S reduces demand if and only if  $v_s < 40$ 

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- Without resale, W bids  $v_w$  and S reduces demand if and only if  $v_s < 40$
- ② With resale, W bids above  $v_w$  and S always reduces demand

## EXPERIMENTAL DESIGN

#### Uniform-Price Ascending Clock Auction

- Bidders choose when to drop out of the auction as the price increases
- When one bidder drops out, the auction is over (# of units on sale = # of units demanded)
- Winner(s) pay the dropout price for each unit

### Treatments - between subjects design

- 1. No Resale
- \_\_\_\_\_
- Complete Information Resale: after the auction, if W won, bidders learn values and participate in resale
- 3. **Incomplete Information Resale:** same as complete info, but bidders do not learn values before resale

**Resale market**: one bidder, chosen with probability  $\frac{1}{2}$ , makes take-it-or-leave-it offer to the other (Calzolari & Pavan '06)  $\Rightarrow$  in expectation, bidders obtain  $\frac{1}{2}$  of gains from trade

4. **Bargain** (unstructured): as Incomp Resale + bidders can make multiple offers and communicate in computerized chat

#### Sessions Information

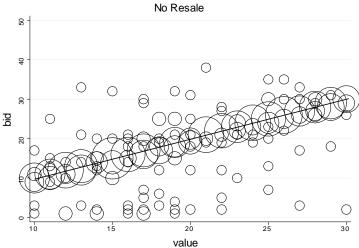
- 3 sessions of 16 subjects per treatment (48 subjects per treatment)
- All sessions had 30 auction periods, except Bargain (20 periods due to 2 hour limit)
- All 12 sessions were run in the xs/fs laboratory at FSU in March and June 2011, and October 2012
- Mostly undergraduate subjects

/eak Bidding trong Bidding fficiency and Revenue esale Market

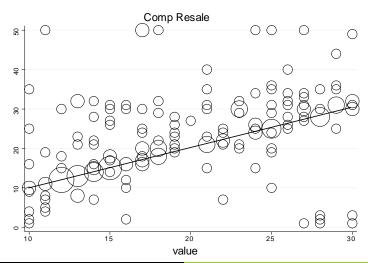
### **RESULTS**

### No Resale: W bids value with high frequency

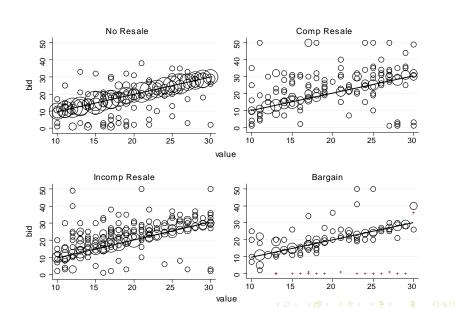
(Weighted scatterplot of observed bids vs. value)



# **Complete Information Resale:** W bids above value with high frequency



- Without resale, W tends to bid value
- With resale, W bids above value much more often



### Bidding by W - Random Effects Tobit (unobserved bids censored at the auction price)

W's Bid	Coefficient	
Constant	0.870	-
$V_W$	0.993***	
Comp Resale	13.248***	- Bids are higher with resale,
Incomp Resale	6.951***	especially in Complete Resale
Bargain	<b>6.747***</b> (2.545)	
$v_w \times Comp$	-0.316*** (0.092)	
$v_w \times Incomp$	-0.117 $(0.084)$	
v <sub>w</sub> ×Bargain	-0.236** (0.108)	_

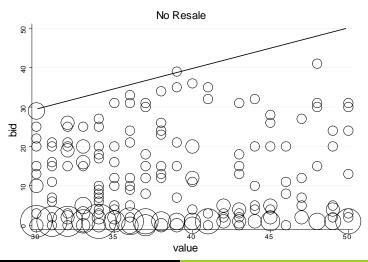
<sup>\*\*\*</sup> and \*\* indicate statistical significance at 1% and 5%

### **Demand Reduction** by S:

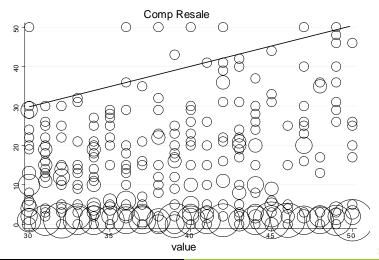
	C' - I-:-	L- / 0	14/ 1	۸/:
	$S$ 's bids $\leq 2$		W Wins	
	$v_s < 40  v_s > 40$		$v_{s} < 40$	$v_s > 40$
No Resale	30%	12%	52%	25%
Comp Resale	37%	43%	77%	72%
Incomp Resale	29%	22%	72%	53%
Bargain	48%	50%	74%	71%

There is evidence of more demand reduction in later periods (learning)

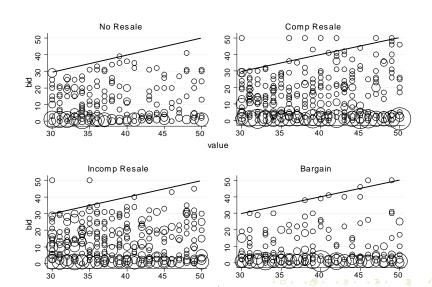
**No Resale:** S reduces demand more frequently when  $v_S < 40$ , less frequently when  $v_S > 40$ 



## **Complete Information Resale:** *S reduces demand with high frequency for all values*



- Without resale, S reduces demand more when  $v_s$  < 40
- ullet With resale, S reduces demand more frequently, for all values
- ullet Uncertainty in resale reduces demand reduction by S



## **Bidding by S** – Random Effects Tobit (unobserved bids censored at the auction price)

Strong Bid	Coefficient
Constant	0.078 (3.705)
$V_S$	0.560***
v <sub>s</sub> >40	4.721*** (1.497)
Comp Resale	-5.894**
Incomp Resale	-2.635 (3.034)
Bargain	-8.615*** (3.065)
$Comp \times v_s {>} 40$	-8.621*** (1.505)
Incomp $\times v_s > 40$	-6.410*** (1.538)
Bargain×v <sub>s</sub> >40	-9.257*** (1.682)

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$v_s{>}40$	<b>4.721***</b>	- Without resale $S$ bids higher when $v_s{>}40$
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Comp x $v_s>40$	-8.621*** (1.505)	- When $v_s{>}40$ in all resale treatments
Incomp x $v_s>40$	-6.410*** (1.538)	S bids lower than without resale
Bargain x $v_s>40$	<b>-9.257***</b>	

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Comp Resale	<b>-5.894**</b>	- When $v_s{<}40$ in Comp Resale and Bargain
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Bargain	<b>-8.615***</b>	
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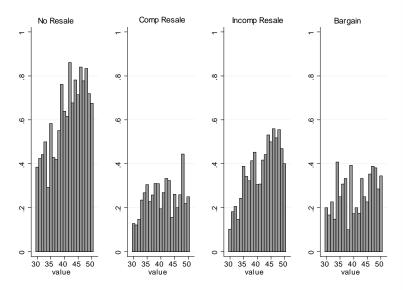
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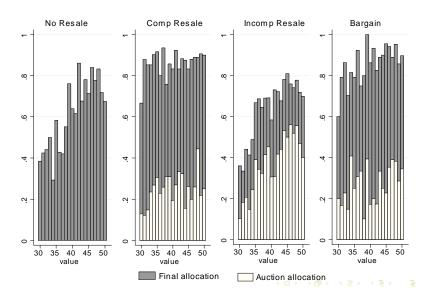
# **Auction Efficiency**: with demand reduction, the auction allocation is inefficient $\Rightarrow$ symmetry and resale reduce auction efficiency

- Average efficiency (winner's value/S's value): No Resale 0.82, Comp 0.64, Incomp 0.71, Bargain 0.65



# **Final Efficiency**: Resale increases efficiency after the auction, but also demand reduction $\Rightarrow$ ambiguous effect on final efficiency

- Average efficiency not significantly different between No Resale (= 0.82) and Incomp Resale (= 0.85)



#### Seller's Revenue:

- Resale reduces revenue because it induces S to reduce demand
- Resale increases revenue (when S does not reduce demand)
   because it induces W to bid aggressively

	No Resale	Comp Resale	Incomp Resale	Bargain
Average Revenue	14.61	11.94	14.05	8.47
Revenue - W wins	8.01	8.64	9.98	5.25
Revenue - S wins	18.81	21.85	21.06	17.22

<sup>-</sup> No significant difference between revenue with No Resale and Incomp Resale (p = 0.319)



<sup>-</sup> Significant difference between revenue with No Resale and either Comp Resale or Bargain (WMW, p < 0.001)

### What are the effects of changing the resale market structure?

- Comp Resale: t-o-l offers with complete information
- **Incomp Resale**: t-o-l offers with incomplete information
- Bargain: multiple offers and communication

	Resale Possible (W won the auction)	Successful Resale
Comp Resale	75%	81.1%
Incomp Resale	63.2%	42.2%
Bargain	73.1%	79.5%
Daigaili	13.170	19.5/0

### Average Resale Price, Earnings, Offer

	Resale Price (Auction Price)	Earnings Weak / Strong	Resale Offer Weak / Strong
Comp Resale	29.56 (11.94)	9.45/10.20	32.47/25.45
Incomp Resale	27.38 (14.05)	8.74/12.59	32.45/17.93
Bargain	27.44 (8.47)	8.35/12.43	-

- Prices are higher in resale than in auction, and highest in Comp
- S earns more than W in resale
- S makes more aggressive offers in Incomp (WMW, p<0.001)



### **Total Earnings: Auction + Resale Profits**

Average Total Earnings	No Resale	Comp Resale	Incomp Resale	Bargain
Weak Bidder	4.61 (8.823)	12.45 (14.273)	7.55 (10.795)	15.83 (12.620)
Strong Bidder (std. dev)	38.33 (17.061)	37.43 (18.348)	<b>34.93</b> <sub>(17.905)</sub>	<b>44</b> .62 (17.147)

cumulative earnings restricted to 20 periods for comparison to bargain treatment

- W obtains higher profits with resale (WMW, p<0.001)
- S obtains higher profits with resale only in Bargain

#### **Conclusions**

- Experiments of multi-object auction with resale and
  - asymmetric bidders
  - resale through bargaining
- Without resale, strong bidders with low values reduce demand more
- With resale, weak bidders speculate and strong bidders reduce demand more frequently
- Resale does not necessarily increase efficiency and reduce the seller's revenue
- More efficient resale market reduces auction efficiency and revenue