Tail Risk and the Macroeconomy*

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Abstract

We empirically investigate how tail risk relates to macroeconomic fundamentals and uncertainty. We introduce a novel univariate time series model to study the dynamics of tail risk in financial markets: we apply results from extreme value theory and build a time-varying peaks over threshold model; and we define the laws of motion for the parameters through the score-based approach. The resulting specification is an unobserved components model. We further propose a connectedness measure for the comovement in tail risk among portfolios. We apply the model to daily returns from U.S. size-sorted decile stock portfolios and show that tail risk is countercyclical: it goes up when fundamentals deteriorate and macroeconomic uncertainty increases; and larger firms tend to respond more than smaller ones to changes in the underlying macroeconomic conditions. We further show that the degree of tail connectedness among portfolios is highly countercyclical. Evidence from international developed markets strengthens our empirical findings and shows that tail connectedness has experienced an upward sloping trend.

JEL classification: C22, C32, G12, G15.

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1 Introduction

Financial risk measurement is an important field of research both in academia and amongst practitioners: it is of central importance to investors in the context of portfolio risk minimization; and to policy makers responsible for monitoring stability in financial markets. It is a challenging and fascinating task, which needs deep knowledge of financial markets on their own and in connection with the broader economic environment. Accurate risk measurement requires processing information from (at least) two sources: econometric modelling of financial markets (see Andersen *et al.* (2013)); and macroeconomic assessment of risk drivers (see Andersen *et al.* (2005)). In this paper we consider both aspects and provide a contribution to the general topic of financial risk measurement.

There exists a voluminous literature providing econometric tools to measure risk with respect to the center of the conditional distribution of financial returns or within a sufficiently small neighborhood about it (see Andersen *et al.* (2013)): those tools provide reliable risk measures during normal or tranquil times. The width of the neighborhood depends on the shape of the tails of the conditional distribution of returns: under gaussianity, the tails are exponentially declining, periods of turmoil are ruled out almost surely and the width coincides with the real line. Financial returns are however conditionally fat-tailed (see Bollerslev (1987)) and risk measures valid during tranquil periods are not informative in periods of turbulent markets. This naturally shifts the attention from the center to the tails of the conditional distribution of returns and to the concept of tail risk: this can be difficult to measure in practice since the conditional distribution of financial returns during periods of distress may not be appropriately described under standard parametric assumptions.

A powerful tool to measure tail risk is extreme value theory (EVT) (see Embrechts *et al.* (1997)): this provides an approximation to the distribution of random variables along the lower and upper tail of the distribution itself. EVT has been widely used in empirical finance to measure tail risk as related to the unconditional distribution of extreme returns. Diebold *et al.* (1998) discuss the application of the power law to estimate tail probabilities for financial risk management. Longin and Solnik (2001) and Poon *et al.* (2004) apply the peaks over threshold (POT) of Picklands (1975) and Davison and Smith (1990) to model the joint distribution of extreme returns in international equity markets: the building block of the POT is the generalized Pareto distribution (GPD).

Estimation of the tails of the unconditional distribution of returns is valid under the maintained assumption of random sampling: the data generating processes of financial returns however exhibit structural breaks and time-varying dynamics, which would lead to model misspecification if neglected (see Kearns and Pagan (1997)). The literature on EVT as related to the unconditional distribution of extreme returns has then been extended along two complementary directions. Quintos *et al.* (2001) build formal tests for the null hypothesis of stability in the index that shapes the distribution of extreme financial returns. Other contributions have modelled time-varying tail behavior within the POT framework. Chavez-Demoulin *et al.* (2014) use Bayesian methods to build a nonparametric POT model for the conditional distribution of extreme returns through regime switches driven by a Poisson process. Massacci (2014) follows a frequentist approach to parameterize the tail index of the GPD as an autoregressive process with innovation equal to the previous period's realized extreme return: the choice of the innovation in his model is *ad hoc* and calls for more generality.

In this paper we build on the dynamic POT of Massacci (2014) and generalize the law of motion for the tail index: we still assume an autoregressive process; and we apply the score-based approach recently proposed in Creal *et al.* (2013) and Harvey (2013). Following the score principle, the innovation in the law of motion of the tail index depends on the previous period's suitably weighted realized score. The new model contributes to the literature in two ways. First, the innovation no longer is *ad hoc* chosen, but it is motivated by statistical arguments (see Blasques *et al.* (2014)). Second, the model turns out to be an unobserved components model (see Harvey (1989, 2013)): it then is a useful tool to investigate the connections between tail risk and the macroeconomic environment. We apply the model to measure tail risk dynamics along the lower and upper tails of the conditional distribution of stock returns. Following the discussion in Gabaix *et al.* (2003), we analyze tail risk across different firm sizes and markets[1]: we focus on daily returns from U.S. size-sorted decile portfolios and equity indices from international developed markets, respectively. We study tail risk in each individual portfolio as well as the comovement in tail risk across portfolios: to achieve the second goal, we introduce the concept of tail connectedness, which builds upon the cumulative risk fraction defined in Billio *et al.* (2012).

Our empirical findings are as follows. Within the U.S. stock market, tail risk is highly persistent across all firm sizes: this is analogous to what found in relation to returns volatility (see Andersen *et al.* (2003) and references therein); and the degree of persistence increases with firm size. We then

[1] See Gabaix *et al.* (2003), p. 267.

investigate the connections between the dynamics of tail risk and the macroeconomic environment. We proceed by considering three classes of macroeconomic indicators: stock market volatility (see Schwert (1989)), business cycle coincident indicators (see Stock and Watson (2014)) and the uncertainty measures constructed in Jurado *et al.* (2014). We carry out a non-causal analysis based on sample correlations (see Andersen *et al.* (2013)), which shows that tail risk as measured by our model is countercyclical and increases with macroeconomic uncertainty; and a causal analysis based on a vector autoregression (see Bloom (2009), Kelly and Jiang (2014) and Jurado *et al.* (2014)), which is able to quantify the effect of macroeconomic uncertainty on tail risk. Consistently with the persistency pattern we document, we show that the macroeconomic environment affects large firms more than smaller ones: this complements the results of Cenesizoglu (2011), who shows that at daily frequency large firms react more than small ones to macroeconomic news. Using our proposed measure of tail connectedness, we uncover the linkages between the comovement in tail risk across portfolios and the macroeconomy: a negative shock to the fundamentals or a higher level of uncertainty (or both) are associated to a higher level of connectedness.

Our findings from international developed markets confirm those from the U.S.. At market level, tail risk is countercyclical and increases with macroeconomic uncertainty. Tail connectedness among markets exhibit a similar pattern. Interestingly, tail connectedness also displays an upward trend: this is in line with the findings in Christoffersen *et al.* (2012), who show that pairwise correlations in developed markets have increased through time.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 lays out the econometric model for the univariate conditional distribution of extreme returns and introduces the idea of tail connectedness. Section 4 presents empirical results from portfolios of U.S. stocks. Section 5 studies international stock markets. Finally, Section 6 concludes.

2 Related Literature

This paper relates to several strands of economic literature, which are not independent of each other. Our econometric model for the conditional distribution of extreme returns builds upon results from EVT through the POT of Picklands (1975) and Davison and Smith (1990). Balkema and De Haan (1974) and Picklands (1975) provide the mathematical foundation of the POT in a static framework. Diebold *et al.* (1998) discuss the pitfalls of static EVT with dependent data and volatility clustering and suggest to estimate the time-varying tail of the conditional distribution of returns; Kearns and Pagan (1997) confirm through a Monte Carlo analysis the inadequacy of the static framework when financial data are in use. Chavez-Demoulin *et al.* (2014) work in a Bayesian context and model the conditional distribution of extreme returns through regime switches governed by an underlying Poisson process. We work in a frequentist environment that allows to develop an unobserved components model suitable to link tail risk to the macroeconomy (see Harvey (1989, 2013)). We also relate to Engle and Manganelli (2004): rather than using results from EVT, they rely on a quantile autoregression with *a priori* specified observable innovation; thanks to our approach based on the POT, we build a more flexible data-driven unobserved components model. We describe the econometric model for the conditional distribution of extreme returns in Section 3.1.

The laws of motion we choose for the time-varying parameters link our work to a voluminous literature in econometrics. Cox (1981) distinguishes between two classes of models with dynamic parameters: observation-driven models and parameter-driven models. In the observation-driven approach, the laws of motion for the parameters are specified in terms of functions of observable variables: popular examples are the ARCH model by Engle (1982), the GARCH by Bollerslev (1986, 1987), the autoregressive conditional density by Hansen (1994), and the recent score-based autoregressive models by Creal *et al.* (2013) and Harvey (2013). In parameter-driven models, the parameters are seen as stochastic processes with an error component: examples are stochastic volatility models (see Shephard, 2005). As in Massacci (2014), we follow the observation-driven approach: rather than *a priori* specifying an *ad hoc* updating mechanism in the laws of motion of the time-varying parameters, we apply the more general data-driven score-based principle of Creal *et al.* (2013) and Harvey (2013). We describe in details our contribution to this stream of literature in Section 3.2.

In Section 3.4 we introduce the concept of tail connectedness: this is informative about the comovement in the probability of tail events in financial returns. Our measure differs from what proposed in Billio *et al.* (2012) and Diebold and Yilmaz (2014), who focus on returns and realized volatilities as risk drivers, respectively. Tail connectedness cannot be measured from the dynamic power law of Kelly and Jiang (2014) and Kelly (2014), who very interestingly estimate tail risk from the cross section of returns under suitable assumptions.

Finally, we relate to a large empirical literature on modelling the distribution of financial returns.

Stochastic modelling of returns traces back to Bachelier (1900), whose work implies that their unconditional distribution is Gaussian and the tails are exponentially declining. In later work, Mandelbrot (1963) and Fama (1965) have empirically rejected the Gaussian assumption and argued that the unconditional distribution of returns exhibits heavy tails: this leads to the notion of tail risk and to the application of EVT for risk measurement as in Embrechts *et al.* (1997), Longin and Solnik (2001) and Poon *et al.* (2004). In order to allow for time-varying conditional moments, Engle (1982) and Bollerslev (1986) follow the observation-driven approach to build econometric models under the maintained assumption of conditional Gaussian distribution; and Bollerslev (1987) allows for a more flexible student-t conditional distribution. We do not impose any distributional assumption and provide an approximation for the lower and upper tails of the conditional distribution of returns. We further study how tails and tail connectedness relate to the macroeconomic environment following similar steps as in Andersen *et al.* (2005) and Andersen *et al.* (2013): we focus on business cycle indicators (see Stock and Watson (2014)) and macroeconomic uncertainty (see Bloom (2014)) and perform a comprehensive analysis that, to the very best of our knowledge, is novel to the literature. We report results for the U.S. and international equity markets in Sections 4 and 5, respectively.

3 Modelling the Conditional Distribution of Extreme Returns

In this section we propose a new framework to study the conditional distribution of extreme returns. We discuss the components of the model in Sections 3.1 and 3.2: the dynamic POT advanced in Massacci (2014); and the score-based updating mechanism of Creal *et al.* (2013) and Harvey (2013). In Section 3.3 we address the issues of estimation and inference. We introduce tail connectedness in Section 3.4.

3.1 Conditional Distribution of Extreme Returns

Let $\{R_t\}_{t=1}^{T}$ be the time sequence of random returns on a portfolio of risky assets, where T denotes the size of the available sample. Following common practice in the literature, we define extreme returns in terms of exceedances with respect to a fixed threshold value γ (see Longin and Solnik (2001) and Chavez-Demoulin *et al.* (2014)): we then refer to the events $R_t < \gamma$ and $R_t > \gamma$ as to negative and positive exceedances, respectively. For expositional purposes, we focus on positive exceedances: analogous results hold for negative exceedances by an argument of symmetry. Let $F_t\left(r_t \left| \mathfrak{F}_{t-1}\right.\right)$ be the conditional cumulative distribution function of R_t, where $\mathfrak{F}_t$ denotes the information set available at period t: we let $F_t(r_t|\mathfrak{F}_{t-1})$ be time-varying; we also assume it is absolutely continuous and positive everywhere on the real line for all t, so that an underlying probability density function for R_t exists at each point in time. For a given quantile q_t of the conditional distribution of R_t, it follows that $\Pr(R_t \leq q_t|\mathfrak{F}_{t-1}) = F_t(q_t|\mathfrak{F}_{t-1})$; the event $R_t > \gamma$ is then assigned a conditional probability $p_t = \Pr(R_t > \gamma|\mathfrak{F}_{t-1}) = 1 - F_t(\gamma|\mathfrak{F}_{t-1})$. We focus on positive exceedances and work under the assumption $\gamma > 0$.

Let the conditional cumulative distribution function of positive exceedances over γ be

$$F_t^{\gamma}(r_t|\mathfrak{F}_{t-1}) = \Pr(\gamma \leq R_t \leq r_t|R_t > \gamma; \mathfrak{F}_{t-1}) = \frac{F_t(r_t|\mathfrak{F}_{t-1}) - F_t(\gamma|\mathfrak{F}_{t-1})}{1 - F_t(\gamma|\mathfrak{F}_{t-1})}, \quad 0 < \gamma \leq r_t,$$

which is unknown without distributional assumptions on the sequence of returns $\{R_t\}_{t=1}^{T}$, namely without assumptions about the analytical expression for $F_t(r_t|\mathfrak{F}_{t-1})$. An approximation for $F_t^{\gamma}(r_t|\mathfrak{F}_{t-1})$ is available under the POT of Picklands (1975) and Davison and Smith (1990): following Balkema and De Haan (1974) and Picklands (1975), the GPD is the only nondegenerate distribution that approximates that of exceedances as the threshold γ approaches the upper bound of the conditional distribution of R_t. Formally, let the conditional cumulative distribution function of the GPD as applied to R_t be

$$G_t^{\gamma}(r_t|\mathfrak{F}_{t-1}) = \begin{cases} 1 - \left(1 + k_t \dfrac{r_t - \gamma}{\eta_t}\right)_+^{-1/k_t}, & r_t \geq \gamma, \quad \gamma > 0, \quad k_t \neq 0 \quad \eta_t > 0, \\ 1 - \exp\left(-\dfrac{r_t - \gamma}{\eta_t}\right), & r_t \geq \gamma, \quad \gamma > 0, \quad k_t = 0 \quad \eta_t > 0, \end{cases} \tag{1}$$

where $(\cdot)_+$ denotes the positive part of the argument between brackets: we allow both the shape parameter k_t and the scale parameter η_t to be time-varying, so that $G_t^{\gamma}(r_t|\mathfrak{F}_{t-1})$ is time-dependent. The following uniform convergence result as applied to $F_t^{\gamma}(r_t|\mathfrak{F}_{t-1})$ holds (see Balkema and De Haan (1974) and Picklands (1975)):

$$\lim_{\gamma \to +\infty} \sup_{\gamma \leq r_t < +\infty} |F_t^{\gamma}(r_t|\mathfrak{F}_{t-1}) - G_t^{\gamma}(r_t|\mathfrak{F}_{t-1})| = 0. \tag{2}$$

Given $\eta_t > 0$, the support of $G_t^{\gamma}(r_t|\mathfrak{F}_{t-1})$ depends on k_t: the support is $r_t \geq \gamma$ and $\gamma \leq r_t \leq \gamma - \eta_t/k_t$ for $k_t \geq 0$ and $k_t < 0$, respectively. The shape of $G_t^{\gamma}(r_t|\mathfrak{F}_{t-1})$ depends on η_t and k_t (see Smith (1985), Davison and Smith (1990) and Ledford and Tawn (1996) for technical details). The scale parameter $\eta_t > 0$ depends on the threshold γ. The parameter k_t determines the shape of the upper tail of the conditional distribution of returns and it is independent of γ: the case $k_t < 0$ corresponds to distributions with finite support (e.g., uniform distribution); the case $k_t = 0$ to an exponentially declining tail (e.g., Gaussian distribution); and the case $k_t > 0$ to a heavy-tailed distribution obeying the power law (e.g., student-t distribution). The conditional distribution of financial returns exhibit heavy tails (see Bollerslev (1987)) and we assume $k_t > 0$ for all t: the higher k_t, the higher the value of $p_t = \Pr(R_t > \gamma \,|\mathfrak{F}_{t-1})$. In what follows, we interchangeably refer to k_t as to the shape parameter or the tail index. Since k_t plays a key role in measuring tail risk, we aim at characterizing its dynamic properties.

We treat realizations of R_t below γ as censored at γ (see Ledford and Tawn (1996)). We define $Y_t = \max(R_t - \gamma, 0)$, with corresponding conditional cumulative distribution function $\Pr(Y_t \leq y_t \,|\mathfrak{F}_{t-1}) = H_t(y_t \,|\mathfrak{F}_{t-1})$: for $y_t = 0$ it follows that $H_t(y_t \,|\mathfrak{F}_{t-1}) = 1 - p_t$; and for $y_t > 0$ we obtain the approximation

$$H_t(y_t \,|\mathfrak{F}_{t-1}) = 1 - p_t + p_t G_t^{\gamma}(r_t \,|\mathfrak{F}_{t-1}) = 1 - p_t \left(1 + k_t \frac{y_t}{\eta_t}\right)_+^{-1/k_t}, \quad k_t > 0, \quad \eta_t > 0.$$

Let $\mathbb{I}(\cdot)$ denote the indicator function; the conditional distribution function of positive exceedances is

$$H_t(y_t \,|\mathfrak{F}_{t-1}) = \mathbb{I}(y_t = 0)(1 - p_t) + \mathbb{I}(y_t > 0)\left[1 - p_t \left(1 + k_t \frac{y_t}{\eta_t}\right)_+^{-1/k_t}\right], \quad k_t > 0, \quad \eta_t > 0. \tag{3}$$

The function $H_t(y_t \,|\mathfrak{F}_{t-1})$ in (3) requires information about $p_t = \Pr(R_t > \gamma \,|\mathfrak{F}_{t-1})$. Under $k_t > 0$, the conditional distribution of returns belongs to the maximum domain of attraction of the Fréchet distribution (see Embrechts *et al.* (1997)). Following Massacci (2014), we approximate p_t as a power law multiplied by a time-varying function $L_t(q_t)$ slowly varying at infinity: formally,

$$\Pr(R_t > q_t \,|\mathfrak{F}_{t-1}) = L_t(q_t)\, q_t^{-1/k_t}, \quad \lim_{q_t \to +\infty} \frac{L_t(c q_t)}{L_t(q_t)} = 1, \quad q_t > 0, \quad k_t > 0, \quad c > 0. \tag{4}$$

We parameterize the function $L_t(q_t)$ as

$$L_t(q_t) = \left(\frac{q_t}{1 + q_t}\right)^{1/k_t}, \quad q_t > 0, \quad k_t > 0. \tag{5}$$

From (4) and (5) it follows that

$$p_t = \Pr(R_t > \gamma \,|\mathfrak{F}_{t-1}) = \left(\frac{1}{1 + \gamma}\right)^{1/k_t}, \quad \gamma > 0, \quad k_t > 0: \tag{6}$$

the parameterization of $L_t(q_t)$ in (5) ensures that p_t lies within the unit interval and it is a probability measure for all $\gamma > 0$; in addition, p_t is monotonically decreasing in γ and increasing in k_t and satisfies $\lim_{\gamma\to 0} p_t = 1$, $\lim_{\gamma\to+\infty} p_t = 0$, $\lim_{k_t\to 0} p_t = 0$ and $\lim_{k_t\to+\infty} p_t = 1$.

From (3) and (6), we obtain the following analytical expression for $H_t(y_t\,|\mathfrak{F}_{t-1})$:

$$\begin{aligned} H_t(y_t\,|\mathfrak{F}_{t-1}) &= \mathbb{I}(y_t=0)\left[1-\left(\frac{1}{1+\gamma}\right)^{1/k_t}\right] \\ &\quad +\mathbb{I}(y_t>0)\left[1-\left(\frac{1}{1+\gamma}\right)^{1/k_t}\left(1+k_t\frac{y_t}{\eta_t}\right)_+^{-1/k_t}\right], \quad \gamma>0, \quad k_t>0, \quad \eta_t>0. \end{aligned} \tag{7}$$

The analytical formulation in (7) combines two models from EVT: the POT for the conditional cumulative distribution function of exceedances; and the power law for the conditional probability of an exceedance. The model is an example of a dynamic censored regression model, the dynamic Tobit is a textbook example of (see Hahn and Kuersteiner (2010) and references therein): compared to the dynamic Tobit, we replace the distributional assumption imposed on the underlying continuous dependent variable with an approximation for the conditional distribution of positive exceedances dictated by EVT.

3.2 Time-Varying Parameters

We now parameterize the laws of motion for the time-varying parameters k_t and η_t in $H_t(y_t\,|\mathfrak{F}_{t-1})$ in (7). We follow the observation-driven approach and apply the score-based mechanism of Creal *et al.* (2013) and Harvey (2013). We resort to Harvey (2013) to build a GPD with time-varying parameters[2]. We define $\varsigma_t = 1/k_t$ and $\alpha_t = \eta_t/k_t$ and reparameterize (1) as

$$G_t^{\gamma}(r_t\,|\mathfrak{F}_{t-1}) = 1-\left[1+\frac{(r_t-\gamma)}{\alpha_t}\right]_+^{-\varsigma_t}, \quad \gamma>0, \quad \varsigma_t>0, \quad \alpha_t>0,$$

where ς_t and α_t are shape and scale parameters, respectively: $G_t^{\gamma}(r_t\,|\mathfrak{F}_{t-1})$ then comes from the Burr distribution by setting to unity a suitable shape parameter[3]. It follows that $G_t^{\gamma}(r_t\,|\mathfrak{F}_{t-1})$ with $\varsigma_t > 0$ is heavy-tailed and the limiting case $\varsigma_t \to \infty$ leads to an exponentially declining tail. In the rest of the paper, we interchangeably refer to ς_t as to the shape parameter or the tail index: the dynamics of ς_t are

[2] See Harvey (2013), Section 5.3.5.
[3] On the relationship between the GPD and the Burr distribution see Harvey (2013), Sections 5.3.4 and 5.3.5.

the main focus of this paper. By reparameterizing (6), we have

$$p_t = \Pr\left(R_t > \gamma \left|\mathfrak{F}_{t-1}\right.\right) = \left(\frac{1}{1+\gamma}\right)^{\varsigma_t}, \quad \gamma > 0, \quad \varsigma_t > 0, \tag{8}$$

which satisfies $\lim_{\gamma \to 0} p_t = 1$, $\lim_{\gamma \to +\infty} p_t = 0$, $\lim_{\varsigma_t \to 0} p_t = 1$ and $\lim_{\varsigma_t \to +\infty} p_t = 0$: from (3) and (8), the cumulative distribution function $H_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right)$ becomes

$$\begin{aligned} H_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right) &= \mathbb{I}\left(y_t = 0\right)\left[1 - \left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right] \\ &\quad + \mathbb{I}\left(y_t > 0\right)\left[1 - \left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left(1 + \frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right] \end{aligned}, \quad \gamma > 0, \quad \varsigma_t > 0, \quad \alpha_t > 0. \tag{9}$$

The dynamics of ς_t are central to our work: it is then important to understand how the model in (9) relates to other specifications that allow to estimate the sequence of shape parameters $\{\varsigma_t\}_{t=1}^{T}$. We obtain a less informative set up by replacing (9) with

$$\tilde{H}_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right) = \mathbb{I}\left(y_t = 0\right)\left[1 - \left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right] + \mathbb{I}\left(y_t > 0\right)\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}, \quad \gamma > 0, \quad \varsigma_t > 0:$$

$\tilde{H}_t\left(\tilde{y}_t \left|\mathfrak{F}_{t-1}\right.\right)$ does not use the information about the magnitude of the exceedance coming from the POT and provides a less efficient estimator for ς_t than $H_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right)$. As in the Hill estimator (see Hill (1975)), one could use information stemming from exceedances only and define

$$\dot{H}_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right) = \mathbb{I}\left(y_t > 0\right)\frac{\left[1 - \left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left(1 + \frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right]}{\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}, \quad \gamma > 0, \quad \varsigma_t > 0, \quad \alpha_t > 0:$$

$\dot{H}_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right)$ ignores information from the probability mass at $y_t = 0$ and efficiency issues arise.

We specify the laws of motion for ς_t and α_t by following similar steps as in Massacci (2014). We write $\varsigma_t > 0$ in exponential form as

$$\ln \varsigma_t = \phi_0 + \phi_1 \ln \varsigma_{t-1} + \phi_2 u_{t-1}: \tag{10}$$

ϕ_0, ϕ_1 and ϕ_2 are scalar parameters; $\ln \varsigma_{t-1}$ introduces an autoregressive component; and the update u_{t-1} requires $\phi_2 \neq 0$ for identification purposes (unless ϕ_1 is *a priori* known to be zero). The scale parameter $\alpha_t > 0$ enters the conditional distribution function in (9) only when a positive exceedance occurs (i.e., when $y_t > 0$) and we do not observe it over the entire sample period $t = 1, \ldots, T$: we model α_t in exponential form as

$$\ln \alpha_t = \varphi_0 + \varphi_1 \ln \varsigma_{t-1} + \varphi_2 u_{t-1}, \tag{11}$$

where φ_0, φ_1 and φ_2 are scalar parameters. In writing (11) we assume that the components that determine the dynamics in ς_t also drive the law of motion for α_t. The model in (11) is more general than what suggested in Chavez-Demoulin *et al.* (2005), who assume the realized exceedance is the only driver of the scale parameter[4] (i.e., $\varphi_1 = 0$ and $u_t = y_t$): Massacci (2014) empirically shows that the tail index provides valuable information for understanding the time-varying properties of the scale parameter.

The key component in the laws of motion for ς_t and α_t in (10) and (11) is the common updating mechanism u_t: this is known given $\mathfrak{F}_t$ under the observation-driven approach. A natural choice would be $u_t = y_t$, as in Chavez-Demoulin *et al.* (2005) and Massacci (2014): this easily implementable solution is however an *ad hoc* and restrictive choice. We then opt for the data-driven score-based mechanism of Creal *et al.* (2013) and Harvey (2013): this innovative methodology relates the innovation u_t to the score of the underlying likelihood function, which is a known quantity given $\mathfrak{F}_t$. Formally, from (9) let

$$\begin{aligned} h_t\left(Y_t \left| \mathfrak{F}_{t-1}\right.\right) &= \mathbb{I}\left(Y_t = 0\right)\left[1 - \left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right] \\ &\quad + \mathbb{I}\left(Y_t > 0\right)\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\frac{\varsigma_t}{\alpha_t}\left(1 + \frac{Y_t}{\alpha_t}\right)_+^{-\varsigma_t - 1}\right], \quad \gamma > 0, \quad \varsigma_t > 0, \quad \alpha_t > 0: \end{aligned} \tag{12}$$

according to the score-based mechanism, u_t is known given $\mathfrak{F}_t$ and defined as

$$u_t = -\left\{\mathrm{E}\left\{\frac{\partial^2 \ln\left[h_t\left(Y_t \left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln \varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}\right\}^{-1}\frac{\partial \ln\left[h_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \ln \varsigma_t},$$

where (see Appendix A for details) the realized score with respect to $\ln \varsigma_t$ is

$$\frac{\partial \ln\left[h_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \ln \varsigma_t} = \mathbb{I}\left(y_t > 0\right)\left\{1 + \ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left(1 + \frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right]\right\} - \mathbb{I}\left(y_t = 0\right)\left[\frac{\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}{1 - \left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}\right]\ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right]$$

[4] See Chavez-Demoulin *et al.* (2005), p. 231.

and the information quantity is

$$\mathrm{E}\left\{\frac{\partial^2 \ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}=-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left\{1+\frac{1}{1-\left(\dfrac{1}{1+\gamma}\right)^{\varsigma_t}}\left\{\ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right]\right\}^2\right\}.$$

At first sight the dynamic score-based updating mechanism may seem to be an *ad hoc* choice. Creal *et al.* (2013) show that it nests a variety of popular models such as the GARCH (see Bollerslev (1986)) and the ACD (see Engle and Russell (1998)). Blasques *et al.* (2014) prove that this solution is information theoretic optimal, in the sense that the parameter updates always reduce the local Kullback-Leibler divergence between the true conditional density and the model implied conditional density. Harvey (2013) relates the dynamic score-based update to the unobserved components described in Harvey (1989): this last interpretation makes our model particularly suitable to study the connections between tail risk and the macroeconomic environment.

3.3 Estimation and Inference

3.3.1 Threshold Value

As explained in Section 3.1, we keep γ in (9) fixed over the entire sample and set it equal to a prespecified quantile of the empirical distribution of returns. The choice of γ is a delicate issue. On the one hand, a low γ makes many observations with $y_t > 0$ available to estimate the parameters of $H_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)$; at the same time, the GPD may provide a poor approximation to the true unknown underlying distribution. On the other hand, a high γ is consistent with the theoretical limiting result in (2); however, only few exceedances become available. The choice of γ creates a trade-off between model misspecification and estimation noise, which leads to a trade-off between bias and efficiency. The literature has proposed rigorous methods to select γ, as recently surveyed in Scarrot and MacDonald (2012). In this paper we follow Chavez-Demoulin and Embrechts (2004) and Chavez-Demoulin *et al.* (2014) and set γ such that 10% of the realized returns are classified as exceedances: Chavez-Demoulin and Embrechts (2004) show that small variations of γ lead to little variation on the estimated values of the parameters.

3.3.2 Maximum Likelihood Estimation

Let $\boldsymbol{\theta} = (\phi_0, \phi_1, \phi_2, \varphi_0, \varphi_1, \varphi_2)'$ denote the vector of parameters that characterize the laws of motion for ς_t and α_t in (10) and (11), respectively: from the observation-driven approach, the conditional distribution of Y_t given the information set $\mathfrak{F}_{t-1}$ is known up to $\boldsymbol{\theta}$. Score-based models can be estimated by maximum likelihood (see Creal *et al.* (2013) and Harvey (2013)): we then suitably extend the maximum likelihood estimator discussed in Smith (1985) and Davison and Smith (1990). From (9), the likelihood contribution from the realization of Y_t is

$$\begin{aligned} h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right) &= \mathbb{I}\left(y_t = 0\right)\left[1 - \left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right] \\ &\quad + \mathbb{I}\left(y_t > 0\right)\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\frac{\varsigma_t}{\alpha_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t - 1}\right], \quad \gamma > 0, \quad \varsigma_t > 0, \quad \alpha_t > 0: \end{aligned}$$

the maximum likelihood estimator $\hat{\boldsymbol{\theta}}$ for the true vector of parameters $\boldsymbol{\theta}^*$ solves

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} L\left(\boldsymbol{\theta}\right),$$

where $L\left(\boldsymbol{\theta}\right) = \prod_{t=1}^{T} h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)$ is the likelihood function. In the static case, Smith (1985) provides sufficient conditions for the maximum likelihood estimator of the GPD to be consistent, asymptotically normally distributed and efficient. In the dynamic set up we consider, we proceed as in Creal *et al.* (2013) and conjecture that under appropriate regularity conditions $\hat{\boldsymbol{\theta}}$ is consistent and satisfies

$$T^{1/2}\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^*\right) \xrightarrow{d} \mathrm{N}\left(\boldsymbol{\theta}, \boldsymbol{\Omega}^*\right), \quad \boldsymbol{\Omega}^* = -\mathrm{E}\left[\left.\frac{\partial^2 \log L\left(\boldsymbol{\theta}\right)}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^*}\right].$$

3.4 Tail Connectedness

Given a set of portfolios of risky assets, we identify connectedness measures with respect to the underlying metric and risk drivers: Billio *et al.* (2012) apply principal components analysis to financial returns; Diebold and Yilmaz (2014) build several measures from variance decompositions to study realized volatilities. Let $\boldsymbol{\varsigma}_t$ be the $N \times 1$ vector of shape parameters from N portfolios of assets: we track the sequence $\{\boldsymbol{\varsigma}_t\}_{t=1}^{T}$ and quantify connectedness using the risk fraction as in Billio *et al.* (2012). Let

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{\varsigma}_t - \bar{\boldsymbol{\varsigma}}\right)\left(\boldsymbol{\varsigma}_t - \bar{\boldsymbol{\varsigma}}\right)'$$

be the $N \times N$ sample covariance matrix of $\boldsymbol{\varsigma}_t$, where $\bar{\boldsymbol{\varsigma}} = T^{-1} \sum_{t=1}^{T} \boldsymbol{\varsigma}_t$ is the sample mean of $\boldsymbol{\varsigma}_t$. The risk fraction is the ratio between the maximum eigenvalue of $\hat{\boldsymbol{\Sigma}}$ and the sum of all eigenvalues of $\hat{\boldsymbol{\Sigma}}$: it lies within the unit interval by construction; and the higher the risk fraction, the higher the degree of connectedness among the elements of $\boldsymbol{\varsigma}_t$, as the first principal component is able to explain a higher portion of the variance of $\boldsymbol{\varsigma}_t$.

4 Results from the U.S. Stock Market

In this section we utilize the theory presented in Section 3 in relation to returns from U.S. stock portfolios: we build all empirical models as described in Sections 3.1 and 3.2; we classify realized returns as exceedances according to the criterion discussed in Section 3.3.1; we run maximum likelihood estimation in line with the methodology presented in Section 3.3.2; and we measure tail connectedness as in Section 3.4. We perform the analysis in Ox 7.1 (see Doornik (2012)); and we implement the maximization algorithm with starting values ς_1 and α_1 equal to the estimates from the static model obtained by setting $\phi_1 = \phi_2 = 0$ and $\varphi_1 = \varphi_2 = 0$ in (10) and (11), respectively. We provide details about data, estimation results, connectedness and links to the macroeconomy in Sections 4.1, 4.2, 4.3 and 4.4, respectively.

4.1 Data

We use daily observations from the Center for Research in Security Prices (CRSP) and consider two sets of data: the value-weighted price index for NYSE, AMEX, and NASDAQ; and the price indices for size-sorted decile portfolios. From each index value, we construct the return r_t as the percentage continuously compounded return. The sample period begins in January 1954 and ends in December 2012, a total of 14851 daily observations: the long time series of available data allows to conduct inference on extreme returns, which we define by setting the threshold γ as discussed in Section 3.3.1. We use daily observations from datasets previously employed in important empirical contributions on risk measurement in equity markets. Given a sample of quarterly observations for the value-weighted price index for NYSE, AMEX, and NASDAQ, Ludvigson and Ng (2007) follow a factor approach to analyze the risk-return relation. Perez-Quiros and Timmermann (2000) use monthly data for size-sorted decile portfolios and show that firms of different size exhibit heterogeneous responses to business cycle fluctuations. We gain information about market wide tail risk from the returns on the value-weighted portfolio. Decile-sorted portfolios let us study tail risk dynamics across firm size (see Gabaix *et al.* (2003)) and measure tail connectedness: Decile 1 and Decile 10 correspond to the smallest (i.e., small caps) and the largest firms (i.e., large caps), respectively, and firm size monotonically increases from the former to the latter.

We show descriptive statistics and correlation matrix for daily portfolio returns in Table 1.

Table 1 about here

As expected, returns from the value-weighted index and from the portfolio of largest firms have similar features. The portfolio mean decreases in market size, and the first four decile portfolios have lower standard deviation than the remaining six: the portfolios from the two sets of smaller firms are optimal in a mean-variance sense. Unlike in standard asset pricing models, mean and standard deviation do not decline as one moves from smallest firms to largest firms portfolios: the decline in mean and standard deviation as a function of firm size is restored with monthly data (see Perez-Quiros and Timmermann (2000) and the difference is due to the sampling frequency. All portfolio returns exhibit negative skewness and excess kurtosis: the null hypothesis of Gaussianity is always rejected at any conventional level. Finally, the correlation between decile portfolios is higher the closer they are ranked in terms of degree of market capitalization.

4.2 Estimation Results

We now estimate the model in (9), (10) and (11). We collect results for negative and positive exceedances in Tables 2 and 3, respectively.

Table 2 about here

Table 3 about here

The results for negative exceedances (see Table 2) show that ϕ_1 and ϕ_2 in (10) are positive in all empirical models: ϕ_1 is very close to unity and tail risk is highly persistent both at market level and across firms; ϕ_1 is also increasing in firm size. Analogous results apply to positive exceedances (see Table 3), where the persistence of the shape parameter is more pronounced than it is for negative exceedances.

We provide further insights on the unconditional distribution of estimated daily tail indices in Tables 4 and 5, where we report descriptive statistics and correlation matrix for both negative and positive exceedances, respectively. The sample period is $1955 - 2012$, a total of 14600 observations: we exclude estimates from 1954 to 1955 to minimize the effects induced by the starting values chosen in the estimation algorithm previously discussed.

Table 4 about here

Table 5 about here

In the case of negative exceedances (see Table 4), the descriptive statistics (see Panel A) show that sample mean, standard deviation and median of ς_t decrease with firm size: on average and median terms, tail risk is higher for larger firms than it is for smaller ones; and the volatility of tail risk decreases in firm size. The average value of the shape parameter falls approximately between 3.549 (Decile 10) and 4.447 (Decile 2): the former value relates to the portfolio of largest firms and resembles the unconditional point estimate discussed in Gabaix *et al.* (2003). The empirical distribution of shape parameters is negatively skewed across all portfolios and the degree of skewness diminishes with firm size. The pairwise correlations between tail indices are seizable and tend to be higher the closer portfolios are ranked in terms of firm size (see Panel B). Similar results hold in the case of positive exceedances (see Table 5).

We provide a graphical representation of the results collected in Tables 4 and 5 by plotting the sequences of daily shape parameters estimated over the sample period $1955 - 2012$ for negative and positive exceedances in Figure 1: to highlight the main features, we concentrate on value-weighted, small caps (i.e., Decile 1) and large caps (i.e., Decile 10) portfolios.

Figure 1 about here

Consistently with the results in Table 2, the sequence of tail indices for negative exceedances from large caps is more persistent than the one from small caps and it is similar to that from the value-weighted portfolio; it also shows a more pronounced cyclical behavior associated to stronger countercyclical tail risk. Analogous considerations hold for positive exceedances, where the series are more persistent than the counterparts from negative exceedances. The graphical analysis suggests that the business cycle may affect large firms more than small ones: we provide stronger evidence of this result in Section 4.4.

4.3 Connectedness Analysis

We now measure connectedness among decile-sorted CRSP portfolios using the strategy detailed in Section 3.4. We treat the estimated sequences of shape parameters as the object of interest (see Andersen *et al.* (2003)): this allows us to overcome empirical and theoretical issues related to estimation noise. We allow for time-varying connectedness to account for dynamic effects induced by events such as the business cycle or financial crises. We estimate the covariance matrix on a daily frequency using a rolling window of 100 daily observations, which provides a good balance between estimation error and flexibility: the sequences of risk fractions for negative and positive exceedances run over the period $1955-2012$, a total of 14600 observations. If we assume an underlying linear factor specification, the risk fraction is a function of the parameters from the underlying model: allowing for time-varying risk fraction is equivalent to allowing for time-varying parameters in the factor model for the shape parameters; and linear models with time-varying parameters are general forms of nonlinear models, as emphasized in White's Theorem reported in Granger (2008). Allowing for time-varying risk fraction then means allowing for an underlying nonlinear factor model. We provide a measure of connectedness: unlike Billio *et al.* (2012) and Diebold and Yilmaz (2014), we do not attempt to uncover the underlying network structure.

We show the sequences of risk fractions for negative and positive exceedances in Figure 2.

Figure 2 about here

Both sequences exhibit high degree of time dependence and countercyclical pattern; when connectedness peaks, the maximum eigenvalue captures almost the entire risk fraction. We collect results from analytical analysis of the series of maximum eigenvalues in Table 6, which reports descriptive statistics, correlation matrix and autoregressive coefficient obtained from fitting an autoregressive process of order one.

Table 6 about here

The results clearly show that tail connectedness is generally higher and less volatile for negative exceedances than it is for positive ones (see Panel A); it is also highly persistent (see Panel C).

4.4 Macroeconomic Determinants

We now study the links between tail risk and the macroeconomy. We proceed in two steps: we first run a "non-causal" analysis in Section 4.4.1 to place our work within a macroeconomic framework (see Andersen *et al.* (2013)); we then perform a structural investigation in Section 4.4.2 to study the effects of uncertainty shocks on tail risk.

4.4.1 Non-Causal Analysis

We relate the sequences of shape parameters for negative and positive exceedances and of maximum eigenvalues to key macroeconomic indicators tracking business cycle dynamics and macroeconomic uncertainty: the business cycle is a key driver of market risk (see Andersen *et al.* (2013)) and asymmetrically affects the distribution of stock returns across different firm sizes (see Perez-Quiros and Timmermann (2000)); macroeconomic uncertainty interacts with tail risk in producing effects on real activity (see Kelly and Jiang (2014)); and macroeconomic uncertainty is countercyclical (see Bloom (2014)). We employ macroeconomic indicators available at monthly frequency and aggregate daily sequences into monthly values by computing monthly medians: Kelly (2014) calculates averages within the month; however, unlike moments, quantiles of distributions are robust to outliers and the median is likely to provide more accurate information about the central tendency of the monthly distribution of tail indices (see Kim and White (2004)); quantiles are also equivariant under monotone transformations (see Koenker (2005)) and we can map monthly medians of tail indices into monthly medians of conditional probabilities of exceedances.

We consider several macroeconomic indicators. The first one is the recession dummy (RD), which takes unit value during recession periods defined according to the NBER classification and it is otherwise equal to zero: the binary RD indicator models the business cycle as a discrete process, a view implicitly embedded in Perez-Quiros and Timmermann (2000). We then look at three sets of continuous macroeconomic indicators. Returns volatility is countercyclical (see Schwert (1989)) and for each individual index we consider two measures of volatility: monthly realized volatility (RV), computed as the sum of squared daily returns within the month (see Schwert (1989)); and the monthly long-run market volatility measure (LRV) proposed in Mele (2007). For each portfolio, we compute RV and LRV from the corresponding sequence of returns. Following Stock and Watson (2014) we consider five coincident indicators: industrial production (IP), nonfarm employment (EMP), real manufacturing and wholesale-retail trade sales (MT), real personal income less transfers (PIX) and the index published monthly by The Conference Board (TCB)[5]; we compute log-differences for IP (ΔIP), EMP (ΔEMP), MT (ΔMT), PIX (ΔPIX) and TCB (ΔTCB); other conditions being equal, an increase in ΔIP, ΔEMP, ΔMT, ΔPIX or ΔTCB signals an improvement in the underlying macroeconomic conditions. Finally, we analyze the linkages with macroeconomic uncertainty through the measures U(h) proposed in Jurado *et al.* (2014) for horizons $h = 1, 3, 6, 12$[6]. The indicators RD, RV, LRV, U(1), U(3), U(6) and U(12) are countercyclical; ΔIP, ΔEMP, ΔMT, ΔPIX or ΔTCB are cyclical.

We perform a correlation analysis as suggested in Andersen *et al.* (2013). We collect the results for negative and positive exceedances and for the risk fractions in Tables 7, 8 and 9, respectively.

Table 7 about here

Table 8 about here

Table 9 about here

Each table reports descriptive statistics, correlation matrix, least squares estimates for the autoregressive coefficient from fitting an autoregressive process of order one, and correlations with the macroeconomic indicators discussed above. Due to data availability, we compute correlations with macroeconomic indicators using different sample periods: $1955:01-2012:12$ for RD, RV and LRV; $1960:01-2010:06$ for ΔIP, ΔEMP, ΔMT, ΔPIX and ΔTCB; and $1961:01-2011:11$ for U(1), U(3), U(6) and U(12).

Starting from negative exceedances (see Table 7), the persistence in the sequence of monthly medians increases in firm size (see Panel C): this is consistent with the pattern in ϕ_1 shown in Table 3. Tail risk is clearly countercyclical (see Panel D). Negative correlations arise across all portfolios with RD, the volatility measures RV and LRV, and the uncertainty measures U(1), U(3), U(6) and U(12): higher tail risk in equity markets is associated with higher volatility and higher macroeconomic uncertainty. On the other hand, we observe positive correlations with ΔIP, ΔEMP, ΔMT, ΔPIX and ΔTCB: higher tail risk is associated with a deterioration of macroeconomic fundamentals. Correlations with macroeconomic

[5] We thank Mark Watson for making the dataset available online at http://www.princeton.edu/~mwatson/publi.html .
[6] We thank Sydney Ludvigson for making the dataset available online at http://www.econ.nyu.edu/user/ludvigsons/jlndata.zip

indicators clearly increase with firm size. This result creates tension with Perez-Quiros and Timmermann (2000), who show that *monthly* returns from small caps are more affected by the business cycle than returns from large caps: our findings complement those in Cenesizoglu (2011), who shows that at *daily* frequency large firms react more than small ones to macroeconomic news; and suggest that further insights on tail risk may be gained by looking at the scaling properties of the distribution of returns (see Mantegna and Stanley (1995) and Timmermann (1995)). Over all investment portfolios, tail risk is most highly correlated with the volatility measures RV and LRV; it is also correlated with the macroeconomic uncertainty measures proposed in Jurado *et al.* (2014), which we deal with in greater details in Section 4.4.2. Analogous qualitative results hold for positive exceedances (see Table 8). As for the sequences of risk fractions (see Table 9), tail connectedness is countercyclical over both sides of the conditional distribution of returns and it increases with macroeconomic uncertainty.

4.4.2 Causal Analysis: the Role of Macroeconomic Uncertainty

We now run a causal analysis and investigate the impact of macroeconomic uncertainty on tail risk by estimating impulse response functions from a vector autoregressive (VAR) similar to those constructed in Bloom (2009), Kelly and Jiang (2014) and Jurado *et al.* (2014). We opt for the uncertainty measures constructed in Jurado *et al.* (2014): these are defined as the common variation in the unforecastable component of a large number of economic indicators; they measure macroeconomic uncertainty, as opposed to microeconomic metrics available in the literature (see Bloom (2014)); unlike measures of dispersion such as market volatility, they are not contaminated by factors related to time-varying risk aversion and therefore provide a more accurate measure of the true economic uncertainty (see Bekaert *et al.* (2013) and Jurado *et al.* (2014)). As in Kelly and Jiang (2014), in our VAR specification U(h) is first, followed by the monthly median values of shape parameters (i.e., the measure of tail risk), the Federal Funds Rate, log average hourly earnings, log consumer price index, hours, log employment and log industrial production[7]: uncertainty is countercyclical (see Bloom (2014)) and the inclusion of coincident indicators allows to measure more accurately the impact of uncertainty on tail risk. Unlike Kelly and Jiang (2014), we focus on the financial implications of macroeconomic uncertainty, rather than on the effect of tail risk on macroeconomic aggregates: in this respect, our work relates to Andersen *et al.* (2005). We estimate

[7] Average hourly earnings, hours and employment are calculated for the manufacturing sector as in Bloom (2009).

the VAR over the period $1961:03-2011:12$ for $h=1,3,6,12$ and in each model we select two lags according to the BIC criterion.

Based on the results presented in Section 4.4.1, we estimate impulse responses for negative and positive exceedances, and for value-weighted, small caps (i.e., Decile 1) and large caps (i.e., Decile 10) portfolios. In Figure 3 we plot percentage changes in the conditional probabilities of exceedances induced by a one-standard deviation shock to uncertainty: we obtain the responses from those of the shape parameters through the equivariance property of quantiles.

Figure 3 about here

In the case of negative exceedances and depending on the uncertainty horizon, the value-weighted portfolio experiences an increase in tail risk between 6 and 12 months after the shock, with magnitude related to h and comprised between 2% and 2.5%; the recovery is completed after 38 to 45 months, and a slow convergence to the equilibrium follows. More visible results hold for the Decile 10 portfolio. The response of small caps to macroeconomic uncertainty shocks is less pronounced and exhibit an earlier timing. As for positive exceedances, all portfolios have impulse responses with lower peaks and slower recovery than the homologous from negative exceedances. The results from impulse response analysis show that uncertainty shocks impact large firms more than small ones.

5 Evidence from International Stock Markets

We now turn the attention to international equity markets (see Gabaix *et al.* (2003)). We use daily observations from the Morgan Stanley Capital International (MSCI) indices. We consider the following eleven developed markets: Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom and the U.S.. We take the perspective of an unhedged U.S. investor and express all indices in U.S. dollars. From each index value, we construct the return r_t as the percentage continuously compounded return. The sample period begins in January 1975 and finishes in December 2012. We account for holidays by deleting observations from trading days for which at least one market has a return identically equal to zero: we end up with 9300 daily data points; and we define negative and positive exceedances as in Section 4. International equity markets have different opening times and a synchronization issue arises (see Martens and Poon (2001)): we however do not run a causality analysis between markets and synchronization is not of first order importance. Table 10 shows that all international portfolio returns exhibit negative skewness and excess kurtosis, and the null hypothesis of Gaussianity is always rejected at any conventional level.

Table 10 about here

We collect results from model estimation for negative and positive exceedances in Tables 11 and 12, respectively.

Table 11 about here

Table 12 about here

The results for negative exceedances (see Table 11) show that ϕ_1 and ϕ_2 in (10) are positive in all markets: ϕ_1 is very close to unity and it falls between 0.972 (Australia) and 0.991 (Sweden). Analogous results apply to positive exceedances (see Table 12): ϕ_1 ranges between 0.987 (Switzerland) and 0.997 (Canada). Results from international markets confirm that tail risk is highly persistent over both sides of the conditional distribution of returns; and that persistency is more pronounced along the right tail. We provide further insights about the empirical distribution of the estimated sequences of shape parameters for negative and positive exceedances in Tables 13 and 14, respectively.

Table 13 about here

Table 14 about here

The sample period is $1976 - 2012$, a total of 9077 observations: we exclude estimates from 1975 to 1976 to minimize the effects induced by the starting values in the estimation algorithm discussed in Section 4. In the case of negative exceedances (see Table 13), the descriptive statistics (see Panel A) show that the sample mean falls between 2.483 (Italy) and 3.229 (the U.S.) and that the empirical distribution is negatively skewed across all international portfolios; pairwise correlations are sizeable (see Panel B), ranging from 0.317 (Japan and the U.K.) to 0.796 (Germany and the Netherlands). Analogous results hold in the case of positive exceedances (see Table 14): exceptions are the empirical distributions for Germany, Japan and the United Kingdom, which are positively rather than negatively skewed (see Panel A). In Figure 4 we plot the sequences of daily shape parameters over the sample period $1976 - 2012$ for negative and positive exceedances for Germany, Japan and the United Kingdom: the series confirm that tail-heaviness is highly time-varying.

Figure 4 about here

Table 15 collects results from analytical analysis of the series of risk fractions (estimated with a rolling window of 100 observations) by reporting descriptive statistics, correlation matrix and autoregressive coefficient obtained from fitting an autoregressive process of order one: it is important to notice that means and medians are substantially lower than those reported in Table 6 in relation to the U.S. market.

Table 15 about here

We shed light on this result in Figure 5, which shows the sequences of risk fractions for negative and positive exceedances: both sequences exhibit a clear upward trend within the second half of the sample, with a corresponding increase in connectedness[8].

Figure 5 about here

We present this finding from a different angle by computing the mean values for the sequences of risk fractions over the four overlapping sub-samples $1976-2012$, $1986-2012$, $1996-2012$ and $2006-2012$: the figures (in percentage terms) for negative exceedances are 59.46, 61.97, 67.79 and 77.12, respectively; and those for positive exceedances are 58.55, 60.88, 65.89 and 74.16, respectively. These values confirm the upward trend in tail connectedness graphically shown in Figure 5. Christoffersen *et al.* (2012) document an analogous behavior in returns correlations in developed countries: our results are complementary as we are interested in the tail indices as risk drivers.

We finally conduct a non-causal analysis. We consider several macroeconomic indicators sampled at monthly frequency. The first one is realized volatility (RV) (see Schwert (1989)): for each tail index,

[8] We run a formal statistical analysis to strengthen the visual inspection we make in the paper: the results are available upon request.

we compute it as the sum of squared daily returns within the month from the corresponding country portfolio; as for the risk fraction, we proceed as in Gourio *et al.* (2013) and construct it as the average country-level realized volatility, and it then proxies global volatility. As coincident indicators, we include log-differences for industrial production at country level (ΔIP) and for the G7 countries (ΔIP_G7), the latter being a proxy for global economic activity[9]. Uncertainty measures such as those constructed in Jurado *et al.* (2014) are not available for international economies. We define macroeconomic uncertainty in terms of volatility of industrial production growth (see Segal *et al.* (2014)): we measure country-specific and global macroeconomic uncertainty as Mele's (2007) monthly long-run volatility of ΔIP (LRV_IP) and ΔIP_G7 (LRV_IP_G7), respectively. Finally, we study the potential connections to U.S. specific macroeconomic uncertainty by including U(h), for $h = 1, 3, 6, 12$. Due to data availability, we compute summary correlations over the period $1976 - 2011$ and collect the results in Table 16.

Table 16 about here

The results confirm that country-level tail risk and tail connectedness (as measured by the risk fraction) are countercyclical along both tails of the conditional distribution of returns, as evidenced by the summary correlations calculated with respect to RV, ΔIP and ΔIP_G7; they also increase in the level of uncertainty as measured by LRV_IP, LRV_IP_G7 and U(h).

6 Concluding Remarks

Understanding the connections between tail risk and the macroeconomy is crucial for risk measurement and management. We build a novel univariate econometric model to describe the conditional distribution of extreme returns: the model extends the peaks over threshold from extreme value theory to allow for time-varying parameters, which we parameterize according to the recently proposed score-based approach. We further introduce the concept of tail connectedness to study the comovement in the shape of the tails of the conditional distributions of univariate extreme returns.

We analyze the dynamics of tail risk on a daily basis as a function of firm size and of the underlying market. We provide empirical evidence that tail risk in the U.S. stock market is countercyclical across all

[9] We collected the series for industrial production from the OECD website. Data for Australia and Switzerland are not available and are then omitted.

levels of market capitalizations: smaller firms display higher time variation in tail risk than large ones; the latter respond more to a deterioration in economic fundamentals or to an increase in macroeconomic uncertainty (or both) than small firms do; and tail connectedness is countercyclical and increases with macroeconomic uncertainty. The analysis of international developed markets confirms that tail risk is countercyclical and correlated with macroeconomic uncertainty and shows that tail connectedness has experienced an upward sloping trend.

Our results have important implications for investors and for policy makers. Several extensions are possible. The most immediate is the multivariate setting to measure extreme dynamic correlation: this will be the topic of future research.

A Updating Mechanism of Shape Parameter

Given the low of motion for ς_t in (10), let $\lambda_t = \ln \varsigma_t \Leftrightarrow \varsigma_t = \exp(\lambda_t)$: from (12), we can write

$$\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right] = \mathbb{I}\left(y_t = 0\right)\ln\left[h_{0t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right] + \mathbb{I}\left(y_t > 0\right)\ln\left[h_{1t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right],$$

where the contributions $h_{0t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)$ and $h_{1t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)$ are equal to

$$h_{0t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right) = 1 - \left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}, \qquad h_{1t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right) = \left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\frac{\exp(\lambda_t)}{\alpha_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\exp(\lambda_t)-1},$$

respectively. It follows that the score can be written as

$$\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\lambda_t} = \mathbb{I}\left(y_t = 0\right)\frac{\partial\ln\left[h_{0t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\lambda_t} + \mathbb{I}\left(y_t > 0\right)\frac{\partial\ln\left[h_{1t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\lambda_t}.$$

We have

$$\frac{\partial\ln\left[h_{0t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\lambda_t} = -\frac{\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}{1-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right]$$

and

$$\frac{\partial\ln\left[h_{1t}\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\lambda_t} = 1 + \ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\exp(\lambda_t)}\right]:$$

therefore, the score is equal to

$$\begin{aligned}\frac{\partial \ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \lambda_t} &= \mathbb{I}\left(y_t>0\right)\left\{1+\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\exp(\lambda_t)}\right]\right\} \\ &\quad -\mathbb{I}\left(y_t=0\right)\left[\frac{\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}{1-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}\right]\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right].\end{aligned}$$

As for the information quantity,

$$\frac{\partial^2 \ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \lambda_t^2}=\mathbb{I}\left(Y_t=0\right)\frac{\partial^2 \ln\left[h_{0t}\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \lambda_t^2}+\mathbb{I}\left(Y_t>0\right)\frac{\partial^2 \ln\left[h_{1t}\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \lambda_t^2}.$$

We then have

$$\begin{aligned}\frac{\partial^2 \ln\left[h_{0t}\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \lambda_t^2} &= -\left[\frac{\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}{1-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}\right]\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right] \\ &\quad \times\left\{1+\frac{1}{1-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right]\right\}\end{aligned}$$

and

$$\frac{\partial^2 \ln\left[h_{1t}\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \lambda_t^2}=\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\left(1+\frac{Y_t}{\alpha_t}\right)_+^{-\exp(\lambda_t)}\right]$$

so that

$$\begin{aligned}\frac{\partial^2 \ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial \lambda_t^2} &= \mathbb{I}\left(Y_t>0\right)\left\{\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\left(1+\frac{Y_t}{\alpha_t}\right)_+^{-\exp(\lambda_t)}\right]\right\} \\ &\quad -\mathbb{I}\left(Y_t=0\right)\left[\frac{\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}{1-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}\right]\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right] \\ &\quad \times\left\{1+\frac{1}{1-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right]\right\}:\end{aligned}$$

this implies that

$$
\begin{aligned}
\mathrm{E}\left\{\frac{\partial^2 \ln\left[h_t\left(Y_t\,|\mathfrak{F}_{t-1}\right)\right]}{\partial\lambda_t^2}\,|\mathfrak{F}_{t-1}\right\} \quad = \quad & \mathrm{E}\left\{\mathbb{I}\left(Y_t>0\right)\left\{\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\left(1+\frac{Y_t}{\alpha_t}\right)_+^{-\exp(\lambda_t)}\right]\right\}|\mathfrak{F}_{t-1}\right\} \\
& -\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right] \\
& \times\left\{1+\frac{1}{1-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right]\right\}.
\end{aligned}
$$

Notice that

$$
\begin{aligned}
& \mathrm{E}\left\{\frac{\partial \ln\left[h_t\left(Y_t\,|\mathfrak{F}_{t-1}\right)\right]}{\partial\lambda_t}\,|\mathfrak{F}_{t-1}\right\} \quad = \quad 0 \\
\Leftrightarrow \quad & \mathrm{E}\left\{\mathbb{I}\left(Y_t>0\right)\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\left(1+\frac{Y_t}{\alpha_t}\right)_+^{-\exp(\lambda_t)}\right]|\mathfrak{F}_{t-1}\right\} \quad = \quad -\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\left\{1-\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right]\right\}
\end{aligned}
$$

and the analytical expression for the information quantity

$$
\mathrm{E}\left\{\frac{\partial^2 \ln\left[h_t\left(Y_t\,|\mathfrak{F}_{t-1}\right)\right]}{\partial\lambda_t^2}\,|\mathfrak{F}_{t-1}\right\}=-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\left\{1+\frac{1}{1-\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}}\left\{\ln\left[\left(\frac{1}{1+\gamma}\right)^{\exp(\lambda_t)}\right]\right\}^2\right\}
$$

easily follows.

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Figure 1: Daily Shape Parameters, CRSP Portfolios, 1955-2012

This figure displays daily sequences of tail indices for negative and positive exceedances for Value-Weighted, Decile 1 (smallest firms) and Decile 10 (largest firms) portfolios. The sample period is $1955-2012$, a total of 14600 daily observations.

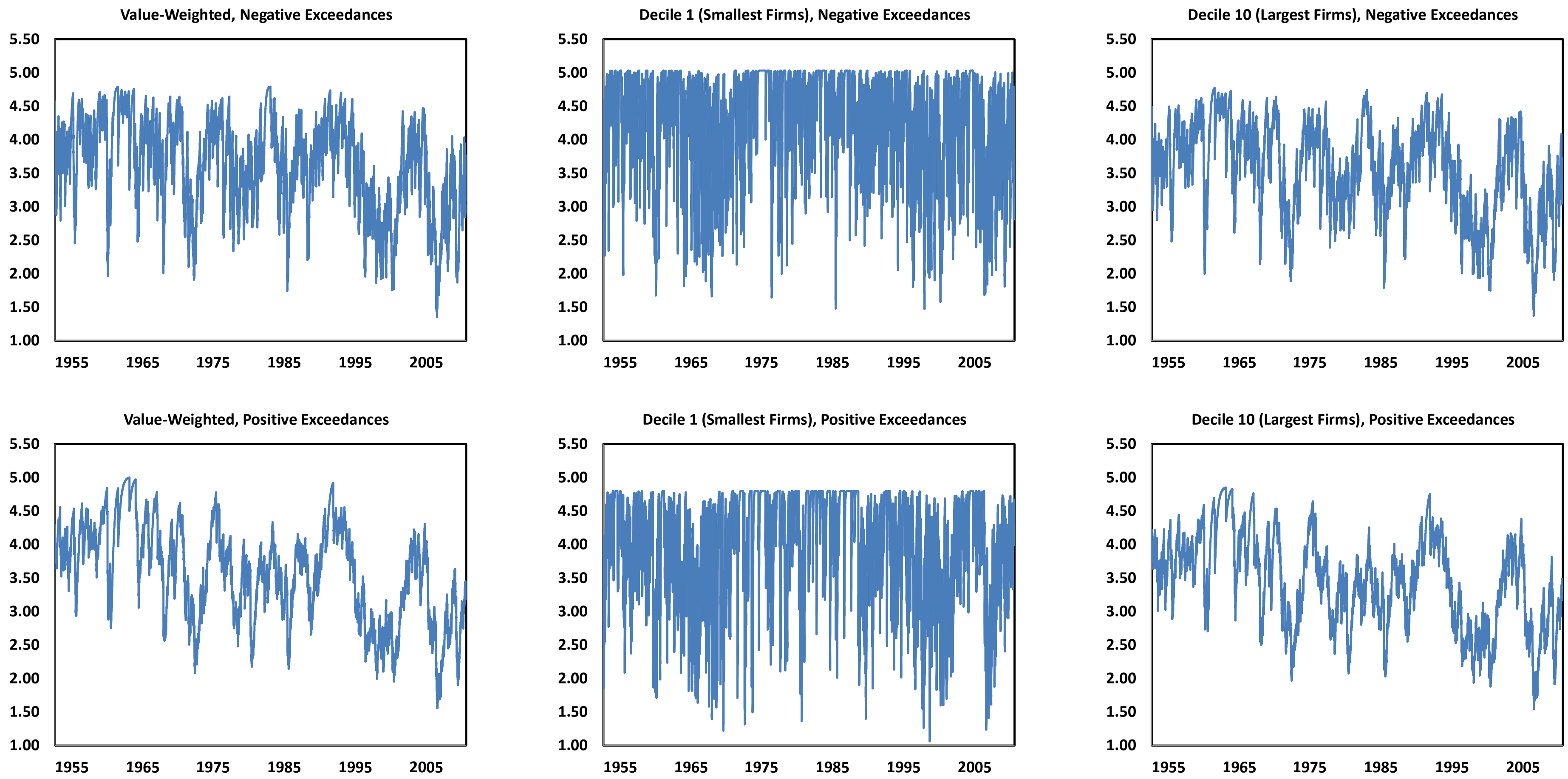


Figure 2: Daily Risk Fractions, Covariance Matrix of Daily Shape Parameters, CRSP Portfolios, 1955-2012

This figure displays daily sequences of risk fractions (in percentage terms) of the correlation matrix of daily tail indices for negative and positive exceedances over the period 1955 − 2012, a total of 14600 daily observations. Estimation of the correlation matrix is carried out over a rolling window of 100 daily observations.

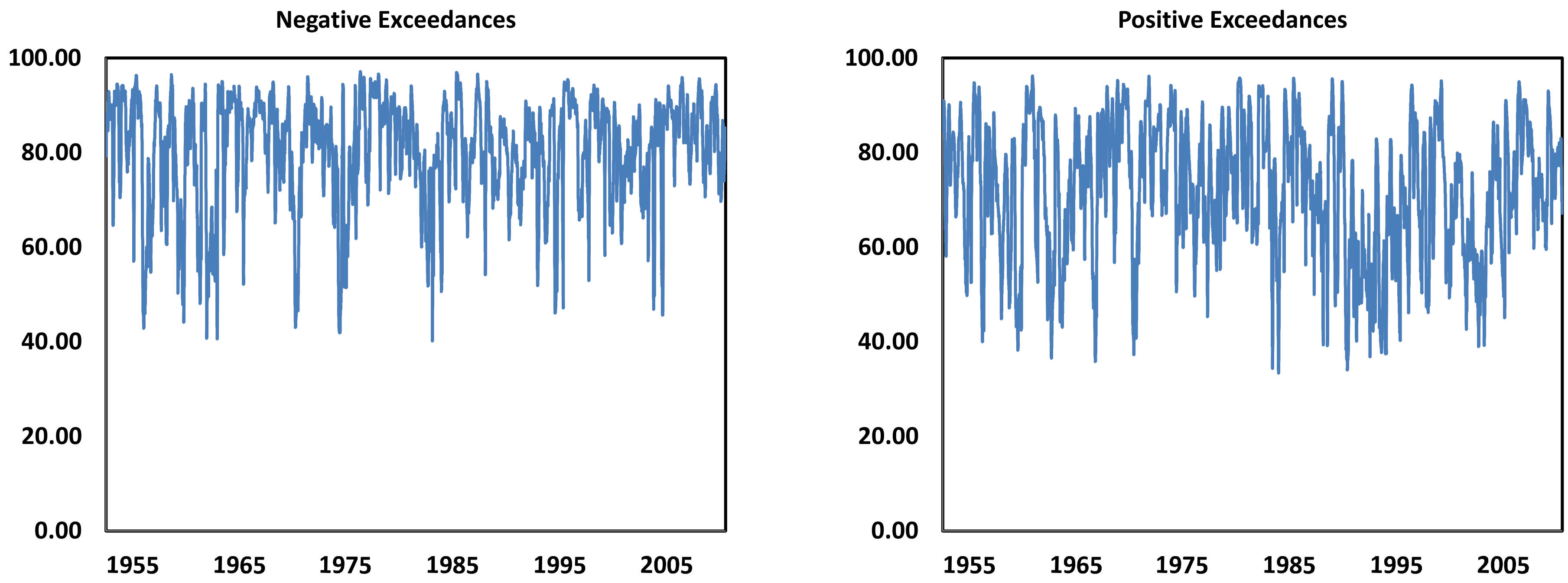


Figure 3: Macroeconomic Uncertainty Impulse Response Functions, Exceedance Probabilities, CRSP Portfolios

This figure plots impulse responses for a one standard deviation shock to macroeconomic uncertainty on exceedance probabilities (in percentage terms). In the underlying VAR specification uncertainty is first, followed by the shape parameter, the Federal Funds Rate, log average hourly earnings, log consumer price index, hours, log employment and log industrial production. Uncertainty is measured as in Jurado *et al.* (2014) for horizon $h = 1, 3, 6, 12$.

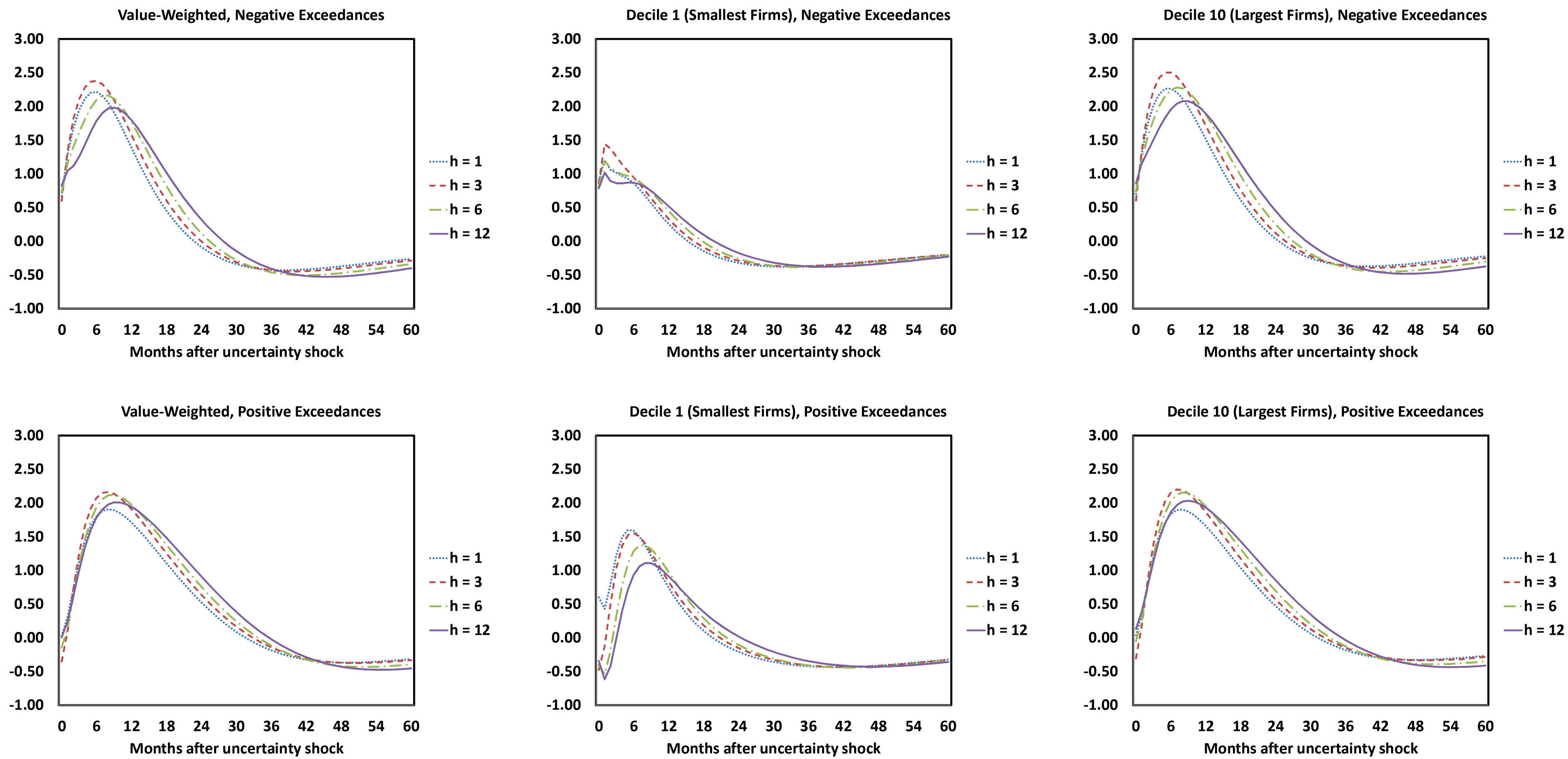


Figure 4: Daily Shape Parameters, International Portfolios, 1976-2012

This figure displays daily sequences of tail indices for negative and positive exceedances for Germany, Japan and the United Kingdom. The sample period is 1976 – 2012, a total of 9077 daily observations.

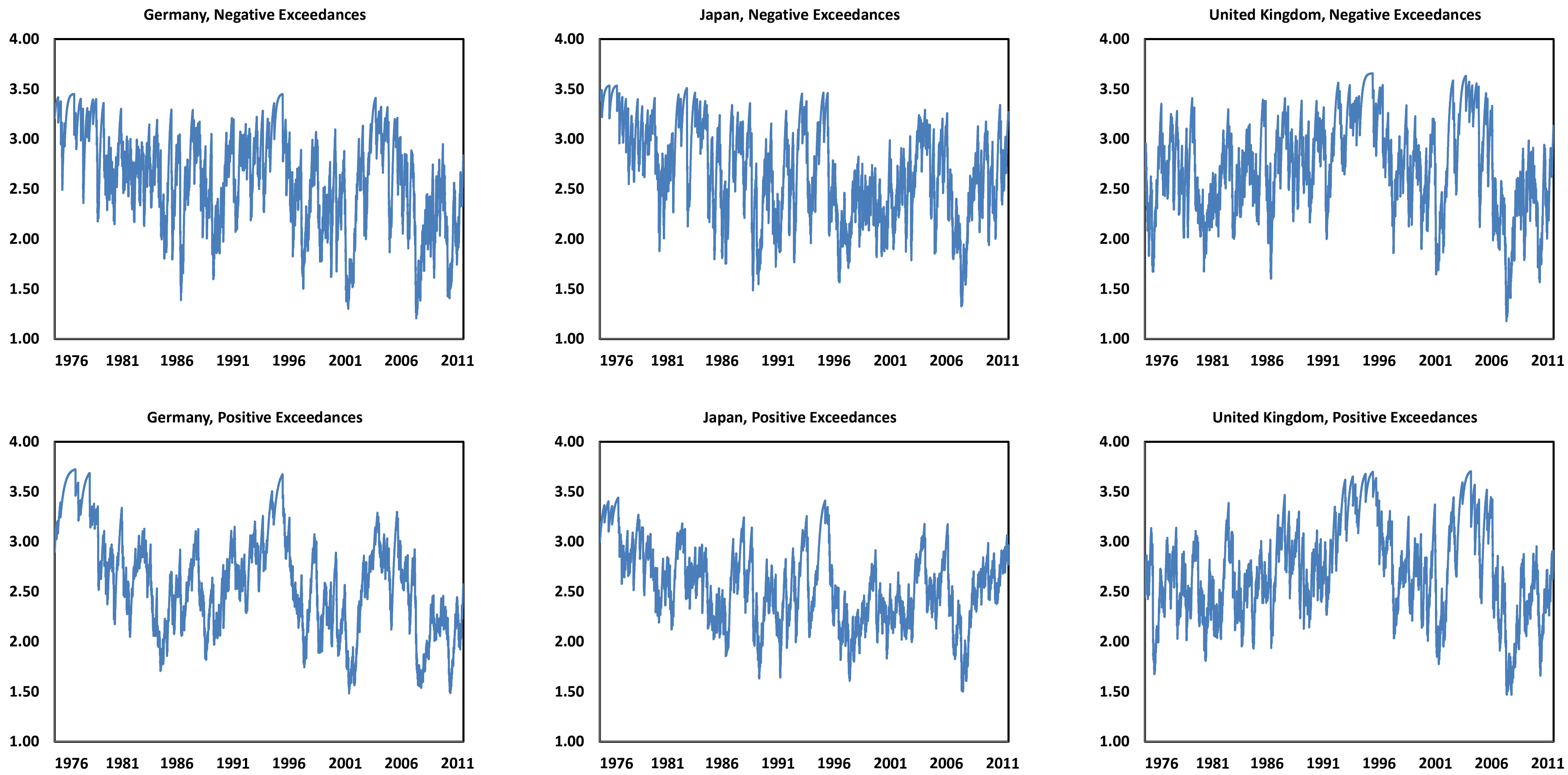
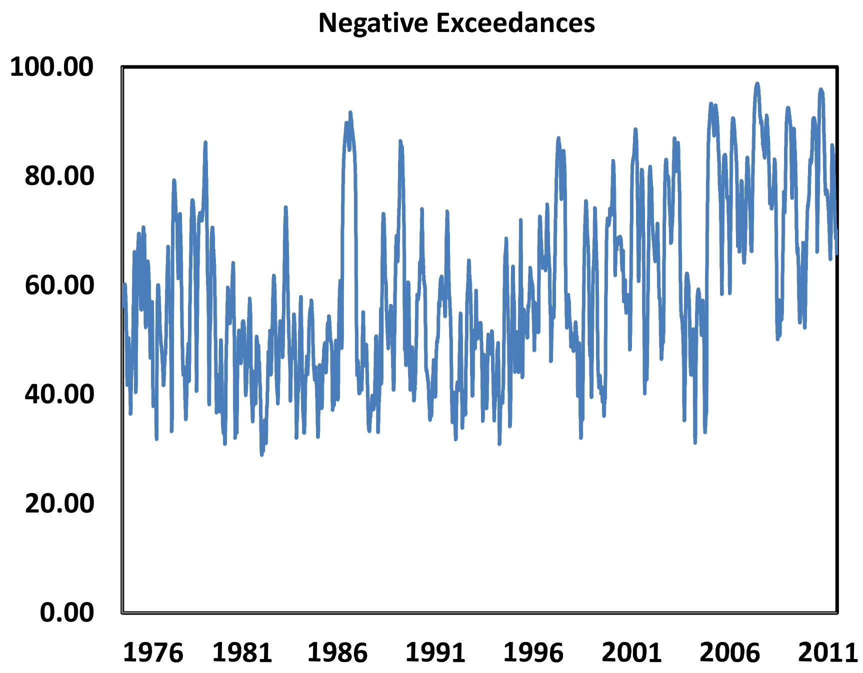


Figure 5: Daily Risk Fractions, Covariance Matrix of Daily Shape Parameters, International Portfolios, 1976-2012

This figure displays daily sequences of risk fractions (in percentage terms) of the correlation matrix of daily tail indices for negative and positive exceedances over the period $1976-2012$, a total of 9077 daily observations. Estimation of the correlation matrix is carried out over a rolling window of 100 daily observations.



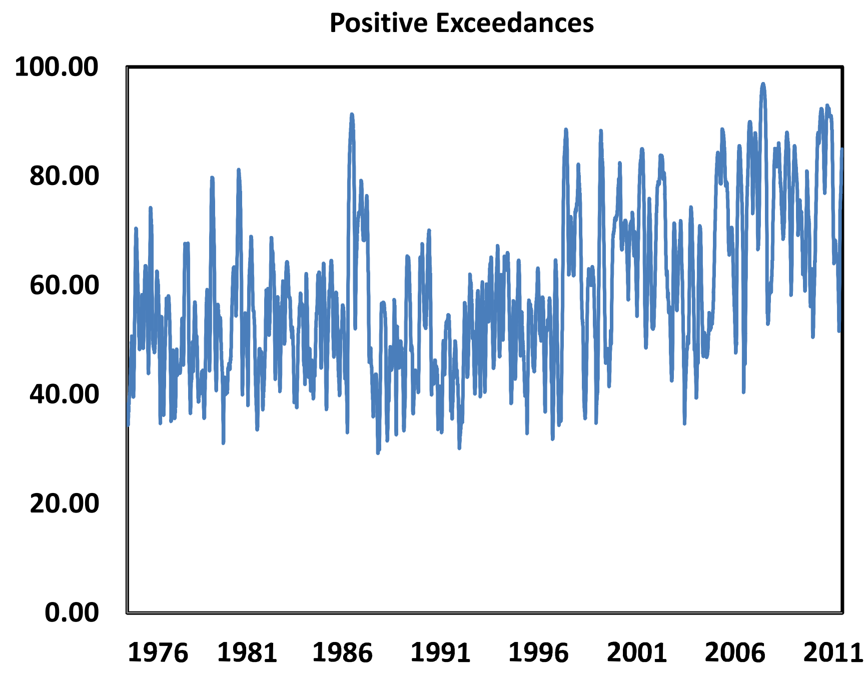


Table 1: Descriptive Statistics and Correlation Matrix, Daily Returns, CRSP Portfolios, 1954-2012

This table presents descriptive statistics and correlation matrix for the empirical distribution of daily returns from U.S. stock portfolios over the period $1954:01-2012:12$, a total of 14851 observations. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level.

Panel A: Descriptive Statistics

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Mean	0.040	0.066	0.053	0.047	0.048	0.045	0.045	0.045	0.045	0.045	0.038
Std. Dev.	0.954	0.806	0.745	0.774	0.832	0.918	0.949	0.967	0.970	0.955	0.979
Median	0.074	0.089	0.089	0.098	0.104	0.102	0.108	0.105	0.105	0.098	0.063
Maximum	10.902	7.713	6.058	6.535	8.065	9.735	10.245	9.237	9.231	10.204	11.173
Minimum	-18.796	-8.535	-9.468	-11.151	-11.110	-12.150	-11.525	-12.202	-12.491	-14.214	-20.186
Skewness	-0.852	-0.356	-0.873	-1.153	-1.048	-0.949	-0.872	-0.825	-0.784	-0.836	-0.832
Kurtosis	22.848	12.219	13.544	16.429	15.734	17.531	16.570	15.438	16.244	17.527	24.357
Jarque-Bera ($\times 10^5$)	2.456*	0.529*	0.707*	1.149*	1.031*	1.329*	1.158*	0.974*	1.101*	1.323*	0.284*

Panel B: Correlation Matrix

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Value-Weighted	1.000	0.537	0.646	0.723	0.784	0.834	0.866	0.888	0.911	0.942	0.996
Decile 1 (Smallest Firms)		1.000	0.799	0.793	0.752	0.704	0.675	0.652	0.627	0.609	0.499
Decile 2			1.000	0.888	0.859	0.816	0.786	0.765	0.738	0.719	0.605
Decile 3				1.000	0.920	0.889	0.864	0.844	0.816	0.800	0.680
Decile 4					1.000	0.942	0.924	0.908	0.886	0.862	0.740
Decile 5						1.000	0.966	0.953	0.937	0.909	0.791
Decile 6							1.000	0.973	0.963	0.938	0.824
Decile 7								1.000	0.977	0.958	0.848
Decile 8									1.000	0.974	0.875
Decile 9										1.000	0.911
Decile 10 (Largest Firms)											1.000

Table 2: Estimation Results, Daily Returns, CRSP Portfolios, 1954-2012, Negative Exceedances

This table reports maximum likelihood estimation results for the model

$$H_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right) = \mathbb{I}\left(y_t=0\right)\left[1-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\right] + \mathbb{I}\left(y_t>0\right)\left[1-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right], \quad \gamma<0, \quad \varsigma_t>0, \quad \alpha_t>0,$$

$$y_t=\max\left(\gamma-r_t,0\right), \qquad \ln\varsigma_t=\phi_0+\phi_1\ln\varsigma_{t-1}+\phi_2 u_{t-1}, \qquad \ln\alpha_t=\varphi_0+\varphi_1\ln\varsigma_{t-1}+\varphi_2 u_{t-1},$$

$$u_t=-\left\{\mathrm{E}\left\{\frac{\partial^2\ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}\right\}^{-1}\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\ln\varsigma_t},$$

$$\mathrm{E}\left\{\frac{\partial^2\ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}=-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\left\{1+\frac{1}{1-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}}\left\{\ln\left[\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\right]\right\}^2\right\},$$

$$\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\ln\varsigma_t}=\mathbb{I}\left(\rho_t>0\right)\left\{1+\ln\left[\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right]\right\}-\mathbb{I}\left(\rho_t=0\right)\left[\frac{\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}}{1-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}}\right]\ln\left[\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\right].$$

where r_t is the daily return on a U.S. portfolio. The sample period is $1954:01-2012:12$, a total of 14851 observations. Standard errors appear in parentheses below parameter estimates.

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Shape Parameter											
ϕ_0	0.017	0.099	0.126	0.122	0.092	0.071	0.050	0.042	0.036	0.030	0.013
	(0.003)	(0.013)	(0.017)	(0.016)	(0.013)	(0.011)	(0.008)	(0.006)	(0.006)	(0.005)	(0.003)
ϕ_1	0.986	0.930	0.914	0.915	0.933	0.946	0.961	0.967	0.972	0.977	0.989
	(0.003)	(0.009)	(0.011)	(0.011)	(0.009)	(0.008)	(0.006)	(0.005)	(0.004)	(0.004)	(0.002)
ϕ_2	0.032	0.076	0.085	0.086	0.075	0.068	0.057	0.053	0.049	0.044	0.027
	(0.003)	(0.006)	(0.006)	(0.007)	(0.006)	(0.006)	(0.005)	(0.005)	(0.004)	(0.004)	(0.003)
Scale Parameter											
φ_0	0.833	0.655	0.455	0.545	0.530	0.674	0.605	0.611	0.710	0.663	0.950
	(0.161)	(0.157)	(0.157)	(0.152)	(0.147)	(0.146)	(0.143)	(0.140)	(0.142)	(0.144)	(0.166)
φ_1	-0.366	-0.184	-0.006	-0.055	-0.038	-0.124	-0.081	-0.078	-0.162	-0.124	-0.499
	(0.136)	(0.116)	(0.113)	(0.112)	(0.115)	(0.119)	(0.120)	(0.119)	(0.120)	(0.120)	(0.142)
φ_2	-0.039	-0.021	-0.015	0.002	-0.016	0.012	-0.010	-0.007	-0.008	-0.020	-0.037
	(0.020)	(0.019)	(0.019)	(0.019)	(0.020)	(0.020)	(0.021)	(0.020)	(0.020)	(0.021)	(0.021)

Table 3: Estimation Results, Daily Returns, CRSP Portfolios, 1954-2012, Positive Exceedances

This table reports maximum likelihood estimation results for the model

$$\begin{aligned} H_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right) &= \mathbb{I}\left(y_t=0\right)\left[1-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right] \\ &+\mathbb{I}\left(y_t>0\right)\left[1-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right], \quad \gamma>0, \quad \varsigma_t>0, \quad \alpha_t>0, \end{aligned}$$

$$y_t=\max\left(r_t-\gamma,0\right), \qquad \ln\varsigma_t=\phi_0+\phi_1\ln\varsigma_{t-1}+\phi_2 u_{t-1}, \qquad \ln\alpha_t=\varphi_0+\varphi_1\ln\varsigma_{t-1}+\varphi_2 u_{t-1},$$

$$u_t=-\left\{\mathrm{E}\left\{\frac{\partial^2\ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}\right\}^{-1}\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\ln\varsigma_t},$$

$$\mathrm{E}\left\{\frac{\partial^2\ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}=-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left\{1+\frac{1}{1-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}\left\{\ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right]\right\}^2\right\},$$

$$\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\ln\varsigma_t}=\mathbb{I}\left(y_t>0\right)\left\{1+\ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right]\right\}-\mathbb{I}\left(y_t=0\right)\left[\frac{\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}{1-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}\right]\ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right].$$

where r_t is the daily return on a U.S. portfolio. The sample period is $1954:01-2012:12$, a total of 14851 observations. Standard errors appear in parentheses below parameter estimates.

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Shape Parameter											
ϕ_0	0.007	0.067	0.066	0.061	0.032	0.024	0.017	0.010	0.012	0.009	0.007
	(0.001)	(0.008)	(0.009)	(0.009)	(0.006)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.001)
ϕ_1	0.994	0.950	0.953	0.957	0.977	0.982	0.987	0.992	0.990	0.993	0.994
	(0.001)	(0.006)	(0.006)	(0.006)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)
ϕ_2	0.019	0.075	0.073	0.067	0.047	0.041	0.034	0.027	0.029	0.024	0.020
	(0.002)	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)
Scale Parameter											
φ_0	0.425	-0.487	-0.729	-0.492	-0.083	-0.119	0.236	0.281	0.346	0.487	0.570
	(0.163)	(0.111)	(0.115)	(0.125)	(0.134)	(0.130)	(0.138)	(0.133)	(0.137)	(0.146)	(0.168)
φ_1	-0.061	0.478	0.650	0.493	0.186	0.293	0.028	0.003	-0.056	-0.174	-0.212
	(0.140)	(0.092)	(0.092)	(0.098)	(0.108)	(0.109)	(0.118)	(0.115)	(0.118)	(0.125)	(0.148)
φ_2	-0.029	-0.039	-0.013	-0.009	-0.018	0.014	0.008	-0.021	-0.004	-0.029	-0.038
	(0.020)	(0.017)	(0.018)	(0.018)	(0.018)	(0.017)	(0.016)	(0.016)	(0.018)	(0.017)	(0.021)

Table 4: Descriptive Statistics and Correlation Matrix, Daily Shape Parameters, CRSP Portfolios, 1955-2012, Negative Exceedances

This table reports descriptive statistics and correlation matrix for the estimated sequence of daily shape parameters for negative exceedances from U.S. stock portfolios over the period $1955:01-2012:12$, a total of 14600 observations. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level.

Panel A: Descriptive Statistics

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Mean	3.623	4.266	4.447	4.265	4.043	3.801	3.661	3.618	3.654	3.732	3.549
Std. Dev.	0.699	0.762	0.782	0.761	0.733	0.699	0.687	0.704	0.721	0.748	0.688
Median	3.720	4.482	4.702	4.524	4.256	3.976	3.818	3.750	3.791	3.869	3.622
Maximum	4.793	5.034	5.193	4.982	4.789	4.560	4.479	4.486	4.581	4.753	4.775
Minimum	1.353	1.475	1.431	1.350	1.362	1.257	1.236	1.254	1.251	1.284	1.372
Skewness	-0.526	-1.016	-1.094	-1.069	-0.962	-0.890	-0.795	-0.701	-0.679	-0.635	-0.454
Kurtosis	2.612	3.237	3.444	3.324	3.078	2.976	2.833	2.675	2.636	2.635	2.517
Jarque-Bera	765.42*	2546.40*	3030.10*	2844.60*	2255.50*	1927.70*	1554.70*	1259.50*	1201.50*	1063.20*	644.52*

Panel B: Correlation Matrix

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Value-Weighted	1.000	0.553	0.550	0.587	0.665	0.715	0.790	0.837	0.878	0.928	0.986
Decile 1 (Smallest Firms)		1.000	0.842	0.838	0.810	0.798	0.770	0.740	0.697	0.669	0.482
Decile 2			1.000	0.909	0.884	0.846	0.789	0.745	0.696	0.663	0.483
Decile 3				1.000	0.923	0.892	0.830	0.790	0.742	0.708	0.516
Decile 4					1.000	0.940	0.892	0.859	0.814	0.783	0.594
Decile 5						1.000	0.948	0.924	0.883	0.840	0.640
Decile 6							1.000	0.969	0.944	0.906	0.718
Decile 7								1.000	0.970	0.941	0.769
Decile 8									1.000	0.968	0.820
Decile 9										1.000	0.883
Decile 10 (Largest Firms)											1.000

Table 5: Descriptive Statistics and Correlation Matrix, Daily Shape Parameters, CRSP Portfolios, 1955-2012, Positive Exceedances

This table reports descriptive statistics and correlation matrix for the estimated sequence of daily shape parameters for positive exceedances from U.S. stock portfolios over the period $1955:01-2012:12$, a total of 14600 observations. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level

Panel A: Descriptive Statistics

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Mean	3.567	3.948	4.233	4.243	4.026	3.865	3.802	3.746	3.718	3.748	3.444
Std. Dev.	0.722	0.833	0.900	0.885	0.854	0.845	0.858	0.889	0.859	0.849	0.686
Median	3.641	4.177	4.460	4.464	4.197	3.983	3.918	3.856	3.843	3.855	3.487
Maximum	5.001	4.802	5.176	5.206	5.147	5.026	5.091	5.159	5.032	5.196	4.852
Minimum	1.557	1.062	1.174	1.335	1.509	1.297	1.325	1.349	1.304	1.393	1.541
Skewness	-0.207	-0.924	-0.866	-0.807	-0.630	-0.508	-0.393	-0.346	-0.375	-0.350	-0.128
Kurtosis	2.344	2.973	2.863	2.692	2.481	2.386	2.202	2.109	2.134	2.238	2.366
Jarque-Bera	365.75*	2077.80*	1837.80*	1640.40*	1130.60*	857.33*	763.53*	774.88*	797.61*	651.31*	284.23*

Panel B: Correlation Matrix

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Value-Weighted	1.000	0.282	0.345	0.449	0.571	0.651	0.740	0.809	0.853	0.921	0.988
Decile 1 (Smallest Firms)		1.000	0.820	0.751	0.657	0.596	0.535	0.453	0.432	0.391	0.258
Decile 2			1.000	0.876	0.788	0.692	0.607	0.518	0.488	0.445	0.331
Decile 3				1.000	0.878	0.785	0.701	0.617	0.581	0.546	0.427
Decile 4					1.000	0.905	0.845	0.775	0.735	0.693	0.537
Decile 5						1.000	0.947	0.897	0.866	0.802	0.603
Decile 6							1.000	0.959	0.937	0.886	0.687
Decile 7								1.000	0.969	0.936	0.754
Decile 8									1.000	0.960	0.805
Decile 9										1.000	0.882
Decile 10 (Largest Firms)											1.000

Table 6: Descriptive Statistics, Correlation Matrix and AR(1) Coefficient, Daily Risk Fractions, Daily Shape Parameters, CRSP Portfolios, 1955-2012

For the sequences of risk fractions (in percentage terms) of the covariance matrix of estimated daily tail indices, this table reports descriptive statistics, correlation matrix and least squares estimates for the autoregressive coefficient obtained from fitting an autoregressive process of order one. The sample period is $1955:01-2012:12$, a total of 14600 observations. The covariance matrix is estimated using a rolling window of 100 observations. * indicates significance at the 1% level.

Panel A: Descriptive Statistics

	Negative Exceedances	Positive Exceedances
Mean	80.318	72.018
Std. Dev.	11.285	14.010
Median	82.924	73.727
Maximum	97.119	96.239
Minimum	40.1371	33.372
Skewness	-1.064	-0.421
Kurtosis	3.752	2.357
Jarque-Bera	3101.00*	681.74*

Panel B: Correlation Matrix

	Negative Exceedances	Positive Exceedances
Negative Exceedances	1.000	0.249
Positive Exceedances		1.000

Panel C: AR(1) Coefficient

	Negative Exceedances	Positive Exceedances
	0.998	0.998

Table 7: Descriptive Statistics, Correlation Matrix, AR(1) Coefficient and Correlations with Recession Dummy, Volatility Measures, Coincident Economic Indicators and Macroeconomic Uncertainty Measures, Monthly Median Values of Daily Shape Parameters, CRSP Portfolios, 1955-2012, Negative Exceedances

For monthly medians of estimated daily tail indices for negative exceedances, this table reports descriptive statistics, correlation matrix, least squares estimates for the autoregressive coefficient obtained from fitting an autoregressive process of order one, and summary correlations between the series and the following macroeconomic indicators: recession dummy (RD); monthly realized volatility (RV); monthly long-run volatility (LRV); log-difference in industrial production (ΔIP), nonfarm employment (ΔEMP), real manufacturing and wholesale-retail trade sales (ΔMT), real personal income less transfers (ΔPIX) and the index published monthly by The Conference Board (ΔTCB); and macroeconomic uncertainty through U(h) for horizons $h = 1, 3, 6, 12$. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level. Correlations with RD, RV and LRV are computed over the period $1955:01-2012:12$; correlations with ΔIP, ΔEMP, ΔMT, ΔPIX and ΔTCB are computed over the period $1960:01-2010:06$; correlations with U(1), U(3) and U(12) are computed over the period $1961:01-2011:11$.

Panel A: Descriptive Statistics

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Mean	3.627	4.308	4.503	4.322	4.088	3.834	3.684	3.637	3.673	3.745	3.551
Std. Dev.	0.689	0.692	0.692	0.675	0.672	0.654	0.658	0.680	0.702	0.732	0.681
Median	3.723	4.498	4.704	4.535	4.270	4.002	3.818	3.745	3.812	3.882	3.614
Maximum	4.790	5.034	5.193	4.982	4.789	4.560	4.479	4.486	4.581	4.753	4.770
Minimum	1.530	1.959	2.158	2.023	1.859	1.562	1.440	1.432	1.466	1.479	1.520
Skewness	-0.535	-0.983	-1.058	-1.051	-0.966	-0.894	-0.800	-0.700	-0.681	-0.634	-0.459
Kurtosis	2.637	3.175	3.374	3.303	3.115	3.011	2.875	2.689	2.641	2.642	2.526
Jarque-Bera	36.99*	112.95*	133.93*	130.69*	108.54*	92.77*	74.74*	59.57*	57.51*	50.32*	30.99*

Panel B: Correlation Matrix

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Value-Weighted	1.000	0.576	0.578	0.616	0.688	0.730	0.800	0.845	0.885	0.933	0.986
Decile 1 (Smallest Firms)		1.000	0.867	0.866	0.824	0.816	0.789	0.759	0.714	0.690	0.503
Decile 2			1.000	0.936	0.899	0.868	0.810	0.768	0.713	0.688	0.508
Decile 3				1.000	0.933	0.904	0.844	0.802	0.756	0.728	0.545
Decile 4					1.000	0.947	0.901	0.868	0.824	0.797	0.617
Decile 5						1.000	0.952	0.930	0.889	0.849	0.655
Decile 6							1.000	0.971	0.949	0.914	0.728
Decile 7								1.000	0.972	0.944	0.776
Decile 8									1.000	0.971	0.827
Decile 9										1.000	0.888
Decile 10 (Largest Firms)											1.000

Panel C: AR(1) Coefficient

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
	0.871	0.527	0.445	0.499	0.571	0.620	0.700	0.750	0.779	0.807	0.901

Table 7-*Continued*: Descriptive Statistics, Correlation Matrix, AR(1) Coefficient and Correlations with Recession Dummy, Volatility Measures, Coincident Economic Indicators and Macroeconomic Uncertainty Measures, Monthly Median Values of Daily Shape Parameters, CRSP Portfolios, 1955-2012, Negative Exceedances

Panel D: Correlations with Recession Dummy, Volatility Measures, Coincident Economic Indicators and Macroeconomic Uncertainty Measures											
	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
RD	-0.355	-0.160	-0.231	-0.227	-0.256	-0.271	-0.264	-0.270	-0.267	-0.289	-0.371
RV	-0.753	-0.632	-0.668	-0.717	-0.726	-0.748	-0.755	-0.779	-0.770	-0.770	-0.742
LRV	-0.551	-0.277	-0.295	-0.313	-0.301	-0.309	-0.334	-0.396	-0.425	-0.511	-0.599
ΔIP	0.273	0.086	0.108	0.096	0.159	0.178	0.195	0.216	0.219	0.224	0.278
ΔEMP	0.411	0.143	0.157	0.156	0.232	0.264	0.299	0.326	0.347	0.352	0.427
ΔMT	0.205	0.082	0.106	0.112	0.158	0.167	0.176	0.188	0.190	0.191	0.204
ΔPIX	0.270	0.104	0.121	0.122	0.182	0.202	0.224	0.241	0.247	0.251	0.277
ΔTCB	0.364	0.120	0.144	0.147	0.224	0.249	0.278	0.302	0.313	0.319	0.374
U(1)	-0.488	-0.133	-0.259	-0.275	-0.341	-0.374	-0.367	-0.393	-0.411	-0.436	-0.507
U(3)	-0.517	-0.146	-0.269	-0.285	-0.351	-0.386	-0.384	-0.416	-0.436	-0.462	-0.536
U(6)	-0.521	-0.128	-0.246	-0.261	-0.332	-0.369	-0.372	-0.409	-0.434	-0.461	-0.543
U(12)	-0.519	-0.099	-0.209	-0.222	-0.298	-0.337	-0.346	-0.391	-0.423	-0.453	-0.546

Table 8: Descriptive Statistics, Correlation Matrix, AR(1) Coefficient and Correlations with Recession Dummy, Volatility Measures, Coincident Economic Indicators and Macroeconomic Uncertainty Measures, Monthly Median Values of Daily Shape Parameters, CRSP Portfolios, 1955-2012, Positive Exceedances

For monthly medians of estimated daily tail indices for positive exceedances, this table reports descriptive statistics, correlation matrix, least squares estimates for the autoregressive coefficient obtained from fitting an autoregressive process of order one, and summary correlations between the series and the following macroeconomic indicators: recession dummy (RD); monthly realized volatility (RV); monthly long-run volatility (LRV); log-difference in industrial production (ΔIP), nonfarm employment (ΔEMP), real manufacturing and wholesale-retail trade sales (ΔMT), real personal income less transfers (ΔPIX) and the index published monthly by The Conference Board (ΔTCB); and macroeconomic uncertainty through U(h) for horizons $h = 1, 3, 6, 12$. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level. Correlations with RD, RV and LRV are computed over the period $1955:01 - 2012:12$; correlations with ΔIP, ΔEMP, ΔMT, ΔPIX and ΔTCB are computed over the period $1960:01 - 2010:06$; correlations with U(1), U(3) and U(12) are computed over the period $1961:01 - 2011:11$.

Panel A: Descriptive Statistics

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Mean	3.568	3.964	4.249	4.259	4.029	3.870	3.803	3.746	3.718	3.747	3.446
Std. Dev.	0.721	0.798	0.864	0.856	0.839	0.836	0.851	0.886	0.855	0.846	0.684
Median	3.637	4.191	4.488	4.491	4.182	3.999	3.915	3.860	3.851	3.847	3.494
Maximum	4.999	4.802	5.176	5.206	5.147	5.025	5.091	5.155	5.027	5.191	4.850
Minimum	1.629	1.327	1.574	1.717	1.641	1.410	1.419	1.429	1.378	1.477	1.637
Skewness	-0.211	-0.901	-0.854	-0.788	-0.634	-0.503	-0.386	-0.339	-0.370	-0.350	-0.130
Kurtosis	2.347	2.921	2.864	2.641	2.476	2.377	2.199	2.104	2.126	2.237	2.374
Jarque-Bera	17.50*	94.25*	85.20*	75.80*	54.53*	40.65*	35.87*	36.67*	37.98*	31.06*	13.33*

Panel B: Correlation Matrix

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
Value-Weighted	1.000	0.289	0.356	0.464	0.579	0.655	0.745	0.811	0.856	0.923	0.988
Decile 1 (Smallest Firms)		1.000	0.847	0.773	0.673	0.605	0.544	0.463	0.438	0.397	0.263
Decile 2			1.000	0.895	0.804	0.702	0.618	0.531	0.500	0.456	0.340
Decile 3				1.000	0.894	0.796	0.714	0.635	0.594	0.560	0.439
Decile 4					1.000	0.909	0.849	0.781	0.740	0.699	0.544
Decile 5						1.000	0.950	0.901	0.870	0.804	0.605
Decile 6							1.000	0.962	0.940	0.889	0.690
Decile 7								1.000	0.970	0.937	0.756
Decile 8									1.000	0.961	0.807
Decile 9										1.000	0.884
Decile 10 (Largest Firms)											1.000

Panel C: AR(1) Coefficient

	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
	0.960	0.634	0.665	0.703	0.811	0.847	0.890	0.930	0.926	0.939	0.953

Table 8-*Continued*: Descriptive Statistics, Correlation Matrix, AR(1) Coefficient and Correlations with Recession Dummy, Volatility Measures, Coincident Economic Indicators and Macroeconomic Uncertainty Measures, Monthly Median Values of Daily Shape Parameters, CRSP Portfolios, 1955-2012, Positive Exceedances

Panel D: Correlations with Recession Dummy, Volatility Measures, Coincident Economic Indicators and Macroeconomic Uncertainty Measures											
	Value-Weighted	Decile 1 (Smallest Firms)	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10 (Largest Firms)
RD	-0.298	-0.017	-0.110	-0.139	-0.204	-0.220	-0.239	-0.263	-0.261	-0.277	-0.309
RV	-0.593	-0.639	-0.567	-0.551	-0.568	-0.563	-0.584	-0.588	-0.605	-0.596	-0.599
LRV	-0.745	-0.470	-0.536	-0.539	-0.573	-0.540	-0.560	-0.625	-0.638	-0.712	-0.743
ΔIP	0.277	-0.003	0.048	0.095	0.143	0.202	0.233	0.248	0.254	0.261	0.279
ΔEMP	0.483	0.131	0.162	0.203	0.292	0.352	0.411	0.449	0.453	0.466	0.476
ΔMT	0.146	-0.042	-0.012	0.030	0.060	0.087	0.110	0.125	0.119	0.127	0.150
ΔPIX	0.276	0.064	0.087	0.125	0.169	0.186	0.226	0.239	0.249	0.250	0.272
ΔTCB	0.375	0.049	0.093	0.142	0.212	0.261	0.309	0.338	0.342	0.350	0.374
U(1)	-0.507	-0.044	-0.208	-0.273	-0.382	-0.418	-0.435	-0.462	-0.460	-0.479	-0.507
U(3)	-0.534	-0.035	-0.197	-0.266	-0.373	-0.415	-0.441	-0.473	-0.475	-0.499	-0.535
U(6)	-0.551	-0.008	-0.166	-0.238	-0.345	-0.392	-0.426	-0.466	-0.475	-0.504	-0.554
U(12)	-0.570	0.029	-0.119	-0.194	-0.298	-0.349	-0.394	-0.444	-0.465	-0.501	-0.578

Table 9: Descriptive Statistics, Correlation Matrix, AR(1) Coefficient and Correlations with Recession Dummy, Volatility Measures, Coincident Economic Indicators and Macroeconomic Uncertainty Measures, Monthly Median Values of Daily Risk Fractions, CRSP Portfolios, 1955-2012

For monthly medians of risk fractions (in percentage terms) estimated with a rolling window of daily 100 observations, this table reports descriptive statistics, correlation matrix, least squares estimates for the autoregressive coefficient obtained from fitting an autoregressive process of order one, and summary correlations between the series and the following macroeconomic indicators: recession dummy (RD); monthly realized volatility (RV); monthly long-run volatility (LRV); log-difference in industrial production (ΔIP), nonfarm employment (ΔEMP), real manufacturing and wholesale-retail trade sales (ΔMT), real personal income less transfers (ΔPIX) and the index published monthly by The Conference Board (ΔTCB); and macroeconomic uncertainty through U(h) for horizons $h = 1, 3, 6, 12$. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level. Correlations with RD, RV and LRV are computed over the period $1955:01-2012:12$; correlations with ΔIP, ΔEMP, ΔMT, ΔPIX and ΔTCB are computed over the period $1960:01-2010:06$; correlations with U(1), U(3) and U(12) are computed over the period $1961:01-2011:11$.

Panel A: Descriptive Statistics

	Negative Exceedances	Positive Exceedances
Mean	72.111	0.804
Std. Dev.	13.936	0.111
Median	73.918	0.831
Maximum	96.002	96.806
Minimum	0.338	41.950
Skewness	-0.421	-1.074
Kurtosis	2.383	3.787
Jarque-Bera	31.56*	151.79*

Panel B: Correlation Matrix

	Negative Exceedances	Positive Exceedances
Negative Exceedances	1.000	0.300
Positive Exceedances		1.000

Panel C: AR(1) Coefficient

	Negative Exceedances	Positive Exceedances
	0.772	0.734

Panel D: Correlations with Recession Dummy, Volatility Measures, Coincident Economic Indicators and Macroeconomic Uncertainty Measures

	Negative Exceedances	Positive Exceedances
RD	0.191	0.207
RV	0.166	0.224
LRV	0.367	0.280
ΔIP	-0.118	-0.176
ΔEMP	-0.154	-0.200
ΔMT	-0.042	-0.132
ΔPIX	-0.088	-0.117
ΔTCB	-0.131	-0.190
U(1)	0.228	0.289
U(3)	0.220	0.296
U(6)	0.207	0.295
U(12)	0.184	0.287

Table 10: Descriptive Statistics and Correlation Matrix, Daily Returns, International Portfolios, 1975-2012

This table presents descriptive statistics and correlation matrix for the empirical distribution of daily returns from international stock portfolios over the period $1975:01-2012:12$, a total of 9300 observations. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level.

Panel A: Descriptive Statistics

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA
Mean	0.027	0.023	0.028	0.025	0.013	0.024	0.031	0.039	0.035	0.034	0.031
Std. Dev.	1.397	1.192	1.415	1.428	1.568	1.357	1.321	1.616	1.170	1.328	1.114
Median	0.053	0.054	0.058	0.046	0.024	0.034	0.056	0.052	0.048	0.055	0.051
Maximum	8.809	10.278	11.849	11.594	12.470	12.272	10.527	14.053	9.735	12.161	11.043
Minimum	-26.806	-14.245	-11.572	-13.739	-12.388	-18.300	-11.515	-17.138	-11.741	-14.065	-22.827
Skewness	-1.469	-0.799	-0.122	-0.203	-0.210	-0.097	-0.162	-0.150	-0.259	-0.173	-1.1299
Kurtosis	26.626	15.345	9.127	9.167	8.230	11.324	9.872	10.011	8.742	10.396	29.599
Jarque-Bera ($\times 10^5$)	2.196*	0.600*	0.145*	0.148*	0.107*	0.269*	0.183*	0.191*	0.129*	0.212*	2.761*

Panel B: Correlation Matrix

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA
AUS	1.000	0.334	0.365	0.378	0.329	0.390	0.381	0.384	0.371	0.378	0.122
CAN		1.000	0.473	0.461	0.380	0.182	0.491	0.440	0.411	0.472	0.654
FRA			1.000	0.734	0.609	0.257	0.754	0.642	0.697	0.648	0.379
DEU				1.000	0.597	0.271	0.747	0.641	0.725	0.596	0.386
ITA					1.000	0.204	0.588	0.548	0.550	0.511	0.291
JPN						1.000	0.258	0.258	0.305	0.250	0.058
NLD							1.000	0.621	0.713	0.701	0.402
SWE								1.000	0.591	0.543	0.330
CHE									1.000	0.591	0.321
GBR										1.000	0.367
USA											1.000

Table 11: Estimation Results, Daily Returns, International Portfolios, 1975-2012, Negative Exceedances

This table reports maximum likelihood estimation results for the model

$$\begin{aligned} H_t\left(y_t \left|\mathfrak{F}_{t-1}\right.\right) &= \mathbb{I}\left(y_t=0\right)\left[1-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\right] \\ &+\mathbb{I}\left(y_t>0\right)\left[1-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right], \quad \gamma<0, \quad \varsigma_t>0, \quad \alpha_t>0, \end{aligned}$$

$$y_t=\max\left(\gamma-r_t,0\right), \qquad \ln\varsigma_t=\phi_0+\phi_1\ln\varsigma_{t-1}+\phi_2 u_{t-1}, \qquad \ln\alpha_t=\varphi_0+\varphi_1\ln\varsigma_{t-1}+\varphi_2 u_{t-1},$$

$$u_t=-\left\{\mathrm{E}\left\{\frac{\partial^2\ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}\right\}^{-1}\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\ln\varsigma_t},$$

$$\mathrm{E}\left\{\frac{\partial^2\ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}=-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\left\{1+\frac{1}{1-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}}\left\{\ln\left[\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\right]\right\}^2\right\},$$

$$\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\ln\varsigma_t}=\mathbb{I}\left(\rho_t>0\right)\left\{1+\ln\left[\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right]\right\}-\mathbb{I}\left(\rho_t=0\right)\left[\frac{\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}}{1-\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}}\right]\ln\left[\left(\frac{1}{1-\gamma}\right)^{\varsigma_t}\right].$$

where r_t is the daily return on an international portfolio. The sample period is $1975:01-2012:12$, a total of 9300 observations. Standard errors appear in parentheses below parameter estimates.

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA
Shape Parameter											
ϕ_0	0.027	0.011	0.018	0.013	0.012	0.014	0.013	0.008	0.024	0.012	0.011
	(0.008)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.005)	(0.003)	(0.003)
ϕ_1	0.972	0.990	0.981	0.986	0.987	0.986	0.987	0.991	0.978	0.988	0.990
	(0.008)	(0.003)	(0.004)	(0.003)	(0.004)	(0.004)	(0.003)	(0.002)	(0.005)	(0.003)	(0.003)
ϕ_2	0.037	0.027	0.034	0.031	0.029	0.030	0.029	0.028	0.032	0.029	0.025
	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)
Scale Parameter											
φ_0	0.721	0.903	0.962	0.785	0.673	0.626	0.973	0.629	1.198	0.530	0.876
	(0.210)	(0.187)	(0.197)	(0.173)	(0.175)	(0.195)	(0.196)	(0.142)	(0.263)	(0.188)	(0.207)
φ_1	-0.274	-0.401	-0.665	-0.429	-0.130	-0.256	-0.569	-0.207	-0.821	-0.209	-0.432
	(0.222)	(0.177)	(0.220)	(0.196)	(0.208)	(0.215)	(0.203)	(0.173)	(0.255)	(0.202)	(0.191)
φ_2	-0.030	-0.056	-0.051	-0.039	-0.080	-0.065	-0.063	-0.046	-0.094	-0.068	-0.020
	(0.029)	(0.024)	(0.030)	(0.032)	(0.033)	(0.035)	(0.034)	(0.031)	(0.029)	(0.027)	(0.031)

Table 12: Estimation Results, Daily Returns, International Portfolios, 1975-2012, Positive Exceedances

This table reports maximum likelihood estimation results for the model

$$\begin{aligned} H_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right) &= \mathbb{I}\left(y_t=0\right)\left[1-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right] \\ &+\mathbb{I}\left(y_t>0\right)\left[1-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right], \quad \gamma>0, \quad \varsigma_t>0, \quad \alpha_t>0, \end{aligned}$$

$$y_t=\max\left(r_t-\gamma,0\right), \qquad \ln\varsigma_t=\phi_0+\phi_1\ln\varsigma_{t-1}+\phi_2 u_{t-1}, \qquad \ln\alpha_t=\varphi_0+\varphi_1\ln\varsigma_{t-1}+\varphi_2 u_{t-1},$$

$$u_t=-\left\{\mathrm{E}\left\{\frac{\partial^2\ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}\right\}^{-1}\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\ln\varsigma_t},$$

$$\mathrm{E}\left\{\frac{\partial^2\ln\left[h_t\left(Y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\left(\ln\varsigma_t\right)^2}\left|\mathfrak{F}_{t-1}\right.\right\}=-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left\{1+\frac{1}{1-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}\left\{\ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right]\right\}^2\right\},$$

$$\frac{\partial\ln\left[h_t\left(y_t\left|\mathfrak{F}_{t-1}\right.\right)\right]}{\partial\ln\varsigma_t}=\mathbb{I}\left(y_t>0\right)\left\{1+\ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\left(1+\frac{y_t}{\alpha_t}\right)_+^{-\varsigma_t}\right]\right\}-\mathbb{I}\left(y_t=0\right)\left[\frac{\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}{1-\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}}\right]\ln\left[\left(\frac{1}{1+\gamma}\right)^{\varsigma_t}\right].$$

where r_t is the daily return on an international portfolio. The sample period is $1975:01-2012:12$, a total of 9300 observations. Standard errors appear in parentheses below parameter estimates.

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA
Shape Parameter											
ϕ_0	0.006	0.004	0.009	0.005	0.006	0.007	0.008	0.005	0.013	0.009	0.006
	(0.002)	(0.001)	(0.003)	(0.001)	(0.002)	(0.003)	(0.002)	(0.002)	(0.004)	(0.002)	(0.002)
ϕ_1	0.994	0.997	0.990	0.994	0.993	0.992	0.992	0.994	0.987	0.991	0.995
	(0.002)	(0.001)	(0.003)	(0.001)	(0.002)	(0.003)	(0.002)	(0.002)	(0.004)	(0.002)	(0.002)
ϕ_2	0.017	0.019	0.023	0.019	0.020	0.020	0.022	0.020	0.024	0.025	0.017
	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.004)	(0.003)	(0.002)
Scale Parameter											
φ_0	0.659	0.614	0.684	0.827	0.741	0.003	0.819	0.921	0.108	0.421	0.747
	(0.219)	(0.177)	(0.215)	(0.199)	(0.193)	(0.228)	(0.204)	(0.178)	(0.242)	(0.193)	(0.225)
φ_1	-0.363	-0.309	-0.426	-0.518	-0.474	-0.045	-0.474	-0.647	0.217	-0.202	-0.362
	(0.237)	(0.172)	(0.242)	(0.227)	(0.237)	(0.257)	(0.215)	(0.217)	(0.245)	(0.210)	(0.211)
φ_2	-0.035	-0.043	-0.033	-0.011	-0.044	-0.078	-0.021	-0.052	-0.028	-0.056	-0.019
	(0.033)	(0.024)	(0.032)	(0.032)	(0.026)	(0.030)	(0.036)	(0.032)	(0.029)	(0.030)	(0.030)

Table 13: Descriptive Statistics and Correlation Matrix, Daily Shape Parameters, International Portfolios, 1976-2012, Negative Exceedances

This table reports descriptive statistics and correlation matrix for the estimated sequence of daily shape parameters for negative exceedances from international portfolios over the period $1976:01-2012:12$, a total of 9077 observations. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level.

Panel A: Descriptive Statistics

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA
Mean	2.688	3.145	2.597	2.618	2.483	2.679	2.818	2.527	2.919	2.733	3.229
Std. Dev.	0.380	0.627	0.428	0.483	0.426	0.474	0.511	0.546	0.398	0.480	0.595
Median	2.746	3.229	2.649	2.662	2.512	2.677	2.853	2.583	2.950	2.736	3.293
Maximum	3.299	4.216	3.330	3.451	3.264	3.535	3.689	3.506	3.637	3.658	4.265
Minimum	1.182	1.242	1.243	1.205	1.174	1.326	1.286	1.094	1.512	1.178	1.338
Skewness	-0.752	-0.428	-0.502	-0.401	-0.381	-0.152	-0.393	-0.329	-0.540	-0.181	-0.494
Kurtosis	3.432	2.326	2.673	2.567	2.516	2.186	2.442	2.254	3.050	2.586	2.602
Jarque-Bera	925.22*	449.42*	421.07*	313.72*	308.07*	285.27*	351.73*	374.57*	442.34*	114.43*	428.53*

Panel B: Correlation Matrix

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA
AUS	1.000	0.585	0.571	0.609	0.493	0.418	0.597	0.520	0.592	0.582	0.525
CAN		1.000	0.620	0.608	0.470	0.457	0.716	0.681	0.533	0.567	0.727
FRA			1.000	0.757	0.692	0.436	0.766	0.695	0.743	0.731	0.661
DEU				1.000	0.613	0.578	0.796	0.762	0.774	0.593	0.686
ITA					1.000	0.332	0.567	0.590	0.596	0.614	0.478
JPN						1.000	0.448	0.521	0.494	0.317	0.519
NLD							1.000	0.700	0.714	0.748	0.731
SWE								1.000	0.623	0.567	0.673
CHE									1.000	0.625	0.630
GBR										1.000	0.599
USA											1.000

Table 14: Descriptive Statistics and Correlation Matrix, Daily Shape Parameters, International Portfolios, 1976-2012, Positive Exceedances

This table reports descriptive statistics and correlation matrix for the estimated sequence of daily shape parameters for positive exceedances from international portfolios over the period $1976:01-2012:12$, a total of 9077 observations. Jarque-Bera denotes the Jarque-Bera goodness of fit test for normality: the joint null hypothesis is that skewness and kurtosis are equal to 0 and 3, respectively. * indicates significance at the 1% level.

Panel A: Descriptive Statistics

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA
Mean	2.644	3.164	2.562	2.614	2.417	2.558	2.771	2.443	2.825	2.722	3.139
Std. Dev.	0.392	0.744	0.392	0.508	0.425	0.384	0.496	0.460	0.389	0.474	0.566
Median	2.679	3.122	2.592	2.603	2.415	2.548	2.798	2.487	2.831	2.689	3.167
Maximum	3.484	4.620	3.365	3.724	3.343	3.441	3.767	3.416	3.671	3.706	4.346
Minimum	1.354	1.399	1.518	1.482	1.335	1.500	1.552	1.241	1.686	1.469	1.552
Skewness	-0.535	-0.082	-0.327	0.170	-0.045	0.088	-0.205	-0.231	-0.158	0.105	-0.166
Kurtosis	3.135	2.110	2.551	2.473	2.364	2.554	2.140	2.315	2.711	2.480	2.368
Jarque-Bera	440.69*	309.52*	237.51*	148.89*	155.96*	86.98*	343.54*	258.33*	69.14*	118.89*	192.42*

Panel B: Correlation Matrix

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA
AUS	1.000	0.535	0.570	0.601	0.476	0.396	0.540	0.546	0.527	0.546	0.572
CAN		1.000	0.495	0.532	0.303	0.357	0.693	0.624	0.368	0.438	0.717
FRA			1.000	0.696	0.651	0.416	0.738	0.682	0.738	0.707	0.593
DEU				1.000	0.508	0.583	0.704	0.712	0.741	0.470	0.677
ITA					1.000	0.275	0.554	0.518	0.579	0.467	0.398
JPN						1.000	0.390	0.514	0.565	0.305	0.509
NLD							1.000	0.644	0.638	0.684	0.765
SWE								1.000	0.551	0.472	0.573
CHE									1.000	0.565	0.569
GBR										1.000	0.606
USA											1.000

Table 15: Descriptive Statistics, Correlation Matrix and AR(1) Coefficient, Monthly Maximum Eigenvalues, Daily Risk Fractions, International Portfolios, 1976-2012

For the sequences of risk fractions (in percentage terms) of the covariance matrix of estimated daily tail indices, this table reports descriptive statistics, correlation matrix and least squares estimates for the autoregressive coefficient obtained from fitting an autoregressive process of order one. The sample period is $1976:01-2012:12$, a total of 9077. The covariance matrix is estimated using a rolling window of 100 observations. * indicates significance at the 1% level.

Panel A: Descriptive Statistics

	Negative Exceedances	Positive Exceedances
Mean	59.465	58.548
Std. Dev.	16.194	14.565
Median	56.484	56.304
Maximum	97.011	96.9334
Minimum	28.886	29.234
Skewness	0.400	48.626
Kurtosis	2.161	2.443
Jarque-Bera ($\times 10^5$)	508.29*	474.87*

Panel B: Correlation Matrix

	Negative Exceedances	Positive Exceedances
Negative Exceedances	1.000	0.535
Positive Exceedances		1.000

Panel C: AR(1) Coefficient

	Negative Exceedances	Positive Exceedances
	0.999	0.999

Table 16: Correlations with Realized Volatility, Coincident Economic Indicators and Macroeconomic Uncertainty Measures, Monthly Median Values of Daily Tail Indices and Risk Fractions, International Portfolios, 1976-2011

For monthly medians of estimated tail indices for international markets, this table reports summary correlations between the series and the following macroeconomic indicators: realized volatility (RV); log-difference in industrial production at country level (ΔIP) and for the G7 countries (ΔIP_G7); monthly long run volatility of log-difference in industrial production at country level (LRV_IP) and for the G7 countries (LRV_IP_G7); and U.S. macroeconomic uncertainty through U(h) for horizons $h = 1, 3, 6, 12$.

Panel A: Negative Exceedances

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA	Risk Fraction
RV	-0.671	-0.727	-0.758	-0.779	-0.708	-0.712	-0.752	-0.728	-0.701	-0.692	-0.717	0.285
ΔIP	-	0.123	0.112	0.078	0.057	0.185	0.061	0.075	-	0.030	0.212	-
ΔIP_G7	0.243	0.233	0.195	0.199	0.154	0.288	0.208	0.247	0.209	0.215	0.220	-0.171
LRV_IP	-	-0.242	-0.206	-0.151	-0.039	-0.191	-0.136	0.076	-	-0.224	-0.220	-
LRV_IP_G7	-0.231	-0.234	-0.254	-0.180	-0.253	-0.030	-0.233	-0.171	-0.141	-0.340	-0.267	0.194
U(1)	-0.398	-0.565	-0.382	-0.227	-0.256	-0.149	-0.445	-0.236	-0.305	-0.499	-0.426	0.199
U(3)	-0.398	-0.581	-0.385	-0.236	-0.258	-0.165	-0.450	-0.237	-0.313	-0.496	-0.444	0.200
U(6)	-0.384	-0.577	-0.367	-0.223	-0.240	-0.154	-0.441	-0.218	-0.301	-0.480	-0.431	0.187
U(12)	-0.369	-0.563	-0.344	-0.208	-0.218	-0.140	-0.428	-0.190	-0.288	-0.459	-0.411	0.162

Panel B: Positive Exceedances

	AUS	CAN	FRA	DEU	ITA	JPN	NLD	SWE	CHE	GBR	USA	Risk Fraction
RV	-0.521	-0.624	-0.612	-0.641	-0.615	-0.583	-0.641	-0.666	-0.561	-0.574	-0.583	0.367
ΔIP	-	0.089	0.110	0.043	0.042	0.112	0.030	0.065	-	0.043	0.196	-
ΔIP_G7	0.196	0.187	0.208	0.165	0.092	0.221	0.161	0.209	0.215	0.219	0.188	-0.165
LRV_IP	-	-0.279	-0.168	-0.172	-0.110	-0.171	-0.196	0.032	-	-0.348	-0.262	-
LRV_IP_G7	-0.333	-0.270	-0.271	-0.156	-0.266	-0.003	-0.304	-0.182	-0.230	-0.342	-0.309	0.236
U(1)	-0.360	-0.542	-0.373	-0.191	-0.180	-0.145	-0.441	-0.224	-0.280	-0.493	-0.384	0.263
U(3)	-0.355	-0.560	-0.374	-0.202	-0.180	-0.162	-0.446	-0.226	-0.289	-0.491	-0.405	0.267
U(6)	-0.336	-0.563	-0.359	-0.200	-0.172	-0.157	-0.441	-0.211	-0.281	-0.477	-0.399	0.250
U(12)	-0.314	-0.554	-0.338	-0.198	-0.166	-0.153	-0.431	-0.187	-0.273	-0.458	-0.388	0.221