Income Mobility as an Equalizer of Permanent Income

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Abstract: Do market-orientated economies with relatively large cross-sectional levels of inequality have higher income mobility and therefore less permanent inequality? To answer this question, we introduce a formal representation of income mobility as an equalizer of permanent income. The proposed representation is called a mobility curve and forms the basis for comparison of income distributions according to income mobility. The mobility curve captures the extent to which the distribution of permanent income is equalized because of changes in individuals' relative income over time. From the derivative of the mobility curve, we can assess the equalizing effect of income mobility in the lower, middle and upper part of the distribution of permanent income. The mobility curve allows us to develop dominance criteria that provide partial orderings of income distributions according to income mobility. We obtain complete orderings through an axiomatically justified family of rank-dependent measures of income mobility, which summarizes the informational content of the mobility curve. We illustrate the usefulness of these methods by re-examining previous findings of income mobility across countries. In contrast to the conclusions in previous studies, we find that changes in relative income over time contribute more (as much) to equality in permanent income in the US as in the Nordic countries and Germany.

Keywords: Inequality, mobility, permanent income, social welfare, rank-dependent measures

JEL classification: D31, D63

Acknowledgement: The project received financial support from the Norwegian Research Council (project number 194339). While carrying out this research, we have been associated with the Centre of Equality, Social Organization, and Performance (ESOP) at the Department of Economics at the University of Oslo. ESOP is supported by the Research Council of Norway through its Centres of Excellence funding scheme, project number 179552.

Sammendrag

Formålet med denne artikkelen er å utvikle metoder for måling og sammenligning av inntektsmobilitet.

1. Introduction

"If income mobility were very high, the degree of inequality in any given year would be unimportant, because the distribution of lifetime income would be very even" (Krugman, 1992).

It was long claimed that the US economy generates much income inequality in any given year in exchange for greater income mobility and therefore less permanent inequality. But several researchers have recently reached conclusions that appear to turn conventional wisdom on its head: Despite higher cross-sectional levels of inequality, Americans enjoy no more income mobility than their peers in the Nordic countries (e.g. Aaberge et al., 2002) and in Germany (e.g. Burkhauser and Poupure, 1997).

When interpreting these findings, however, caution is in order: Following Shorrocks (1978), the above studies employ measures of income mobility that capture the share of cross-sectional inequality that is transitory. [1] This means that the estimated mobility is not necessarily higher in a society where changes in the relative incomes of individuals occur more frequently or are greater in magnitude. In particular, if cross-sectional inequality is low then even minor changes in relative income over time may translate into high income mobility. This raises the concern that traditional measures of income mobility do not adequately distinguish between changes in the income structure that equalize the cross-sectional income distribution, and those that affect individuals' relative incomes over time. This concern needs to be put in context: The traditional mobility measures capture the concept they were designed to measure, namely the share of cross-sectional inequality that is transitory. What they do not capture is the widespread notion of income mobility as an equalizer of permanent income, as proposed by Friedman (1962) and emphasized by Krugman (1992).

In this paper, we introduce a formal representation of income mobility as an equalizer of permanent income. The proposed representation is called a mobility curve and forms the basis for comparison of income distributions according to income mobility. The mobility curve captures the extent to which the distribution of permanent income is equalized because of changes in individuals' relative income over time. The state of no mobility is defined to occur when the individuals' positions in the cross-sectional income distributions are constant over time. The derivative of the mobility curve allows us to directly assess the equalizing effect of income mobility in the lower, middle and upper part of the distribution of permanent income.

The mobility curve plays a similar role in our analysis of income mobility as the Lorenz curve plays in analysis of income inequality. By displaying the deviation of each individual share in the distribution of permanent income from the share that corresponds to no income mobility, the mobility

[1] We refer to Chakravarty et al. (1985), Atkinson et al. (1992), Dardanoni (1993), Fields (2009), Gottschalk and Spolare (2002), Ruiz-Castillo (2004), Tsui (2009) and D'Agostino and Dardanoni (2009) for discussions of alternative approaches to measuring intra-generational income mobility. A number of empirical studies have employed Shorrock's approach to measure income mobility, including Bjørklund, (1993), Burkhauser and Puopore (1997), Maasoumi and Trede (2001), Aaberge et al. (2002), Ayala and Sastre (2004), Chen (2009), and Kopczuk et al. (2009). We refer to Burkhauser and Couch (2009) for a recent review of the empirical literature on intra-generational income mobility.

curve captures how changes in relative incomes over time equalize the distribution of permanent income. Ranking income distributions in accordance with first-degree mobility dominance means the higher of non-intersecting mobility curves unambiguously show more income mobility. The normative justification of this criterion follows from the fact that the higher of two non-intersecting mobility curves can be obtained from the lower mobility curve through income transfers that increase the frequency or magnitude of changes in relative incomes of individuals over time, while preserving the cross-sectional distributions of income. [2]

In practice, however, mobility curves may intersect, in which case weaker criteria than first-degree mobility dominance are required. To address this challenge, we introduce two alternative generalizations of first-degree mobility dominance; one that integrates the mobility curve from below (second-degree upward mobility dominance) and the other that integrates the mobility curve from above (second-degree downward mobility dominance). Since first-degree mobility dominance implies upward and downward mobility dominance of second degree, it follows that both criteria preserve first-degree mobility dominance. However, the transfer sensitivity of these second-degree dominance criteria differs. While upward mobility dominance places more emphasis on inequality in the lower part of the permanent income distribution, second-degree downward mobility dominance emphasizes on inequality in the upper part of the permanent income distribution. As a result, they complement each other: Downward dominance allows one to assess whether the rising share of top incomes in many countries is accompanied by changes in the composition of the top income classes; upward dominance focuses attention on whether income mobility attenuates the persistence of low income in a society.

In situations where neither upward nor downward mobility dominance of second-degree provides unambiguous rankings of income distributions, it is useful to employ summary measures of income mobility. Summary measures of income mobility also allow us to quantify the equalizing effect of income mobility. We use an axiomatic approach to derive a general family of rank-dependent measures of income mobility, which summarizes the informational content of the mobility curve. The members of this family measure the extent to which the distribution of permanent income is equalized because of changes in relative income over time. The family is completely axiomatized, and has an intuitive social welfare interpretation. We also characterize the relationship between the upward and downward mobility dominance criteria and two parametric subfamilies of mobility measures in the ranking of income distributions by mobility. The subfamily associated with upward dominance is characterized by the principle of downside positional transfer sensitivity (Zoli, 1999; Aaberge, 2000; 2009), while the subfamily associated with downward dominance is characterized by the principle of

[2] Analogously, the Lorenz curve captures the descriptive features of income inequality by displaying the deviation of each individual income share from the income share that corresponds to perfect equality. As shown by Atkinson (1970), ranking income distributions in accordance with first-degree Lorenz dominance means that the higher of non-intersecting Lorenz curves is preferred; the normative significance of this criterion follows from the fact that the higher of two non-intersecting Lorenz curves can be obtained from the lower Lorenz curve by rank-preserving income transfers from richer to poorer individuals.

upside positional transfer sensitivity (Aaberge, 2009). The two principles differ in the sensitivity to inequality in the lower versus the upper part of the permanent income distribution.

To illustrate the usefulness of our methods for measuring income mobility as an equalizer of permanent income, we exploit a population panel data set from Norway with information on individuals' incomes over their working life span. We also apply the methods to re-examine the pattern of income mobility across countries. In contrast to the conclusions reached in previous studies, we find that changes in relative income over time contribute more (as much) to equality in permanent income in the US as in the Nordic countries (Germany).

Our paper complements the literature on intra-generational income mobility in several ways. The introduction of a mobility curve allows us to develop dominance criteria that provide partial orderings of income distributions according to income mobility. The mobility curve also allows us to assess the equalizing impact of income mobility across the entire distribution of permanent income. The axiomatically justified family of rank-dependent measures of income mobility provides complete orderings by summarizing the informational content of the mobility curve. Our representation of income mobility is also fundamentally different, in that we accommodate the widespread notion of income mobility as an equalizer of permanent income. This representation has important implications for the interpretation of income mobility estimates: In contrast to the traditional measures, the mobility curve approach ensures that high mobility will equalize permanent income and raise social welfare more than low mobility. Our empirical results highlight these differences: Due to low cross-sectional inequality in the Nordic countries, even small changes in relative incomes over time – which matter little for social welfare and equality in permanent income – translate into high estimates of income mobility when applying traditional mobility measures.

The remainder of this paper proceeds as follows. Section 2 describes the Norwegian panel data that we use to illustrate the mobility curve. Section 3 presents the mobility curve and shows how it can be used to compare income distributions according to income mobility. Section 4 compares our methods to the traditional measures of income mobility, and demonstrates empirically how they reach different conclusions about the pattern of income mobility across countries. The final section offers some concluding remarks.

2. Data

Our empirical analysis uses a longitudinal dataset containing records for every Norwegian from 1967 to 2010. The variables captured in this dataset include demographic information (sex, year of birth, municipality of birth) and socio-economic information (education and income). We focus on the 1947 cohort, which ensures data on income from age 20 to 63.[3] We exclude a small number of individuals

[3] Although the formal retirement age is 67 years, many individuals are eligible for early retirement schemes in their early 60s.

whose information on annual income is missing. The final sample used in the analysis consists of 51,804 individuals.

Our measure of income is the sum of pre-tax market income from wages and self-employment. We use the consumer price index to make incomes from different years comparable (with 1960 as the base year). Our measure of *permanent income* is the annuity value of the discounted sum of real income

$$Z = \frac{Y_T + \sum_{t=1}^{T-1} Y_t \prod_{j=1+t}^{T} (1 + r_j)}{1 + \sum_{t=1}^{T-1} \prod_{j=1+t}^{T} (1 + r_j)}, \tag{2.1}$$

where r_t denotes the real interest rates on income-transfers from year *t-1* to *t*.[4]

It should be noted that the Norwegian income data have several advantages over those available in many other countries. First, there is no attrition from the original sample because it is not necessary to ask permission from individuals to access their tax records. In Norway, these records are in the public domain. Second, our income data pertain to all individuals, and not only to jobs covered by social security. Third, we have nearly career-long income histories for certain cohorts, and do not need to extrapolate the income profiles to ages not observed in the data.

Table 1. Descriptive statistics for annual and permanent income (1960 NOK)

	Mean	St. Dev.	St. Dev/Mean.
Annual income:			
Age 20	58,448	61,518	1.05
Age 30	182,139	145,757	0.80
Age 40	252,444	172,332	0.68
Age 50	288,587	231,357	0.80
Age 60	306,076	299,883	0.98
Permanent income:	206,697	122,234	0.59
Observations		51,804	

Notes: The sample consists of individuals born 1947. Permanent income is defined as the annuitized value of real income from age 20 to 63.

Table 1 displays the mean and standard deviation in annual and permanent income. Average annual income increases over the life cycle, and is most similar to average permanent income when individuals are in their mid 30s. The growth in average annual income over the life-cycle is

[4] The annual real interest rates is set equal to 2.3 percent. This corresponds to the average interest rate on borrowing and savings over the period 1967–2006. The average income is another much used measure of permanent income. This is of course a special case of the annuity income where the real interest rates are set equal to zero.

accompanied by an increase in the variance of annual income. The last column shows that there is much less relative variability in the distribution of permanent income than in the cross-sectional distribution of income at any given age. This indicates that changes in relative incomes over time could be important as an equalizer of permanent income. In the next section, we introduce a framework that allows us to rigorously assess this conjecture.

3. The mobility curve approach

This section presents the mobility curve and shows how it can be used to compare income distributions according to income mobility.

3.1. Mobility Curve

To represent mobility as an equalizer of permanent income, we introduce the concept of a mobility curve. The mobility curve is defined as

(3.1)
$$M(u) = L_Z(u) - L_{Z_R}(u), \; u \in [0,1]$$

where L_Z denote the Lorenz curve for the distribution F_Z of the observed permanent income Z defined by (2.1); and L_{Z_R} denotes the Lorenz curve for the distribution F_{Z_R} of the reference permanent income Z_R in the case of no mobility. In the distribution F_{Z_R}, the rank of each individual is the same in every period; this distribution can be formed by assigning the lowest income in every period to the poorest individual in period the first period, the second lowest to the second poorest, and so on.[5]

Since L_Z can be attained from L_{Z_R} by a sequence of Pigou-Dalton transfers in permanent income that keep the period-specific distributions unchanged, we have that $L_Z(u) \geq L_{Z_R}(u)$ for all $u \in [0,1]$, and that $L_Z(u) = L_{Z_R}(u)$ for all u if and only if Z is equal to Z_R. The mobility curve captures the extent to which permanent income is equalized because of changes in relative incomes over time. An equal distribution of permanent income can either be due to equality in the cross-sectional distributions of income or high income mobility.

Inserting (2.1) for Z and Z_R in (3.1) yields the following convenient expression,

(3.2)
$$M(u) = L_Z(u) - \sum_{t=1}^{T} \frac{\mu_t}{\mu_Z} b_t L_t(u)$$

where

[5] Note that reference distribution that corresponds to no mobility is unique. For example, the reference permanent income does not depend on whether we assign incomes to individuals according to their rank in, say, the first or the last period.

(3.3) $$b_t = \frac{\prod_{j=t+1}^{T}(1+r_j)}{1+\sum_{s=1}^{T-1}\prod_{j=s+1}^{T}(1+r_j)},\ t=1,2,...,T-1,\ b_T = 1,$$

and $\mu_t = EY_t$ while $\mu_Z = \sum_{t=1}^{T} b_t \mu_t$. Expression (3.2) highlights that an unequal distribution of permanent income (L_Z) can be due to high inequality in annual income (L_t) or low mobility *(M*).

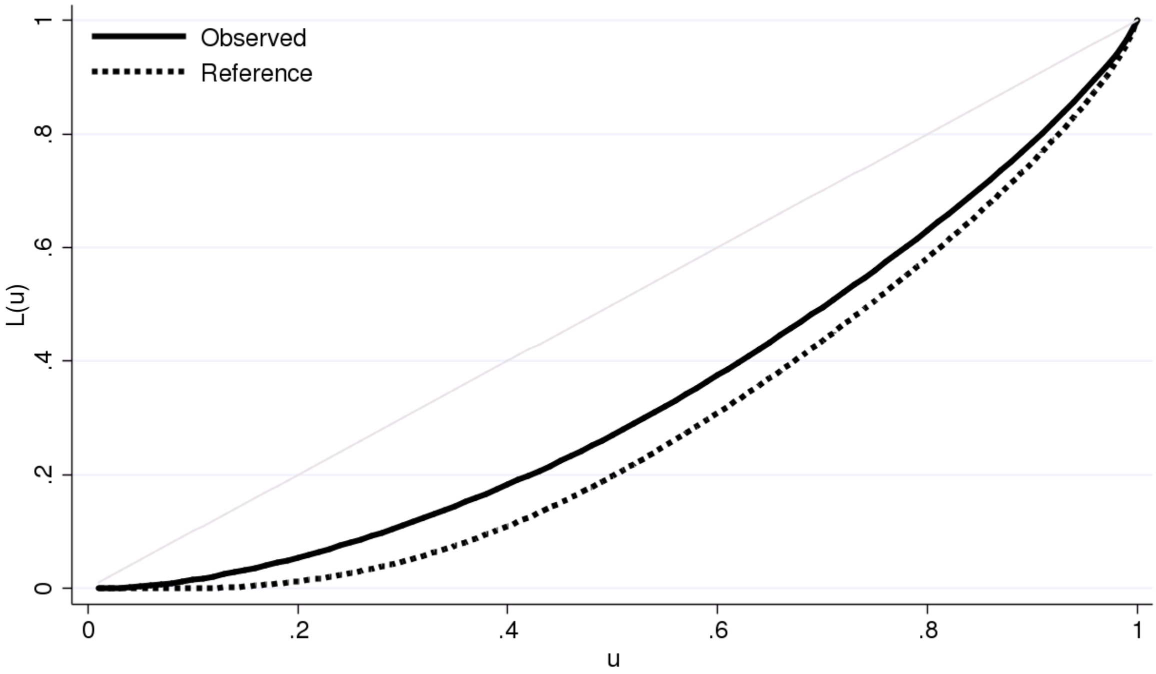
In Figure 1, we use the income data for the 1947 cohort to graph the Lorenz curves in the distribution of observed annuity income and the distribution of the reference annuity income. By construction, the former always lies weakly above the latter, reflecting that income mobility will unambiguously equalize the distribution of permanent income.

Figure 2 shows the mobility curve associated with the Lorenz curves in the observed and the reference distribution of permanent income. The derivative of the mobility curve allows us to directly assess the equalizing impact of income mobility across the entire distribution of permanent income. The derivative of M is given by

(3.4) $$M'(u) = \frac{F_Z^{-1}(u)}{\mu_Z} - \frac{F_{Z_R}^{-1}(u)}{\mu_{Z_R}},\ u \in [0,1].$$

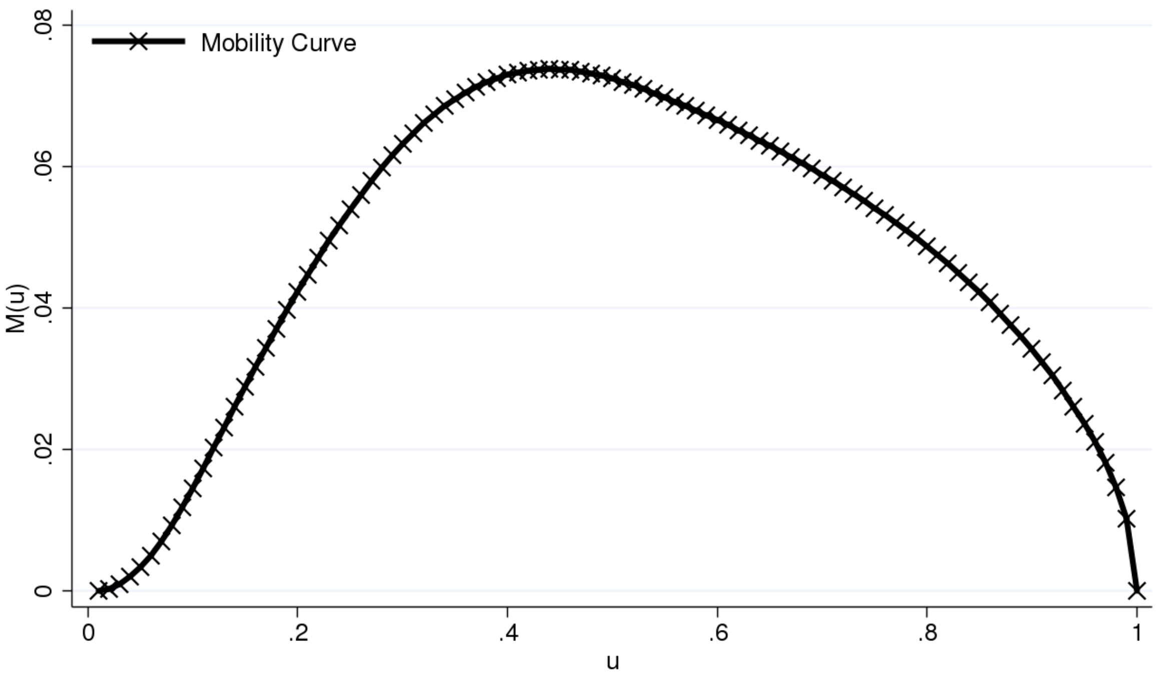
Individuals for which $M'(u)$ is positive (negative) become better (worse) off because of income mobility: Their shares of total income are higher (lower) than what they would have been in the absence of changes in relative incomes over time. Figure 3 displays the derivatives of the mobility curve for the 1947 cohort, where we represent the derivatives as the difference in income shares with and without mobility at every percentile. The poorest 44 percent of the population benefits from income mobility, at the cost of the richest 56 percent. The gains peak at the 13th percentile where mobility increases the share of total income by 0.29 percentage points (from .07 percent with A_R to 0.36 percent with A). There is considerable income mobility in the uppermost part of the permanent income distribution, reducing the share of top incomes considerably. By way of comparison, $M'(u)$ would be zero for high values of u if there were no mobility in top incomes.

Figure 1. Lorenz curves in the distributions of observed and reference annuity income



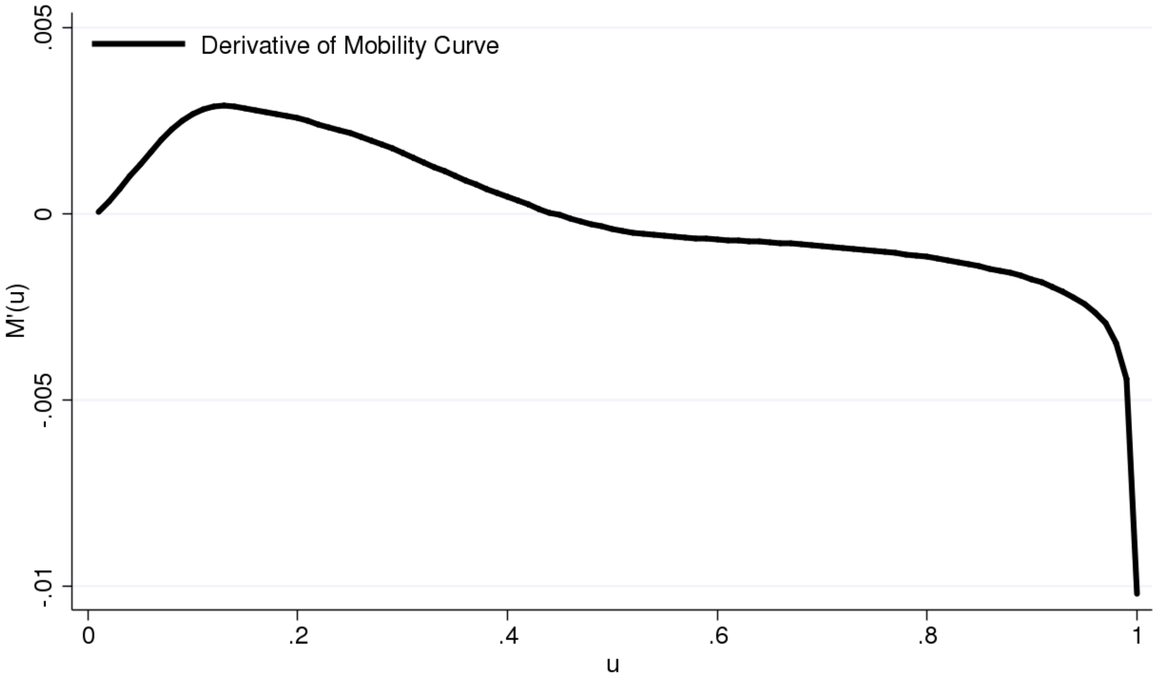
Notes: The sample consists of individuals born 1947. Permanent income is defined as the annuitized value of real income from age 20 to 63. The reference annuity income represents the distribution of permanent income with no mobility.

Figure 2. Mobility curve from the distributions of observed and reference annuity income



Notes: The sample consists of individuals born 1947. Permanent income is defined as the annuitized value of real income from age 20 to 63. The reference annuity income represents the distribution of permanent income with no mobility.
The mobility curve is defined in equation (3.2).

Figure 3. Derivatives of the mobility curve



Notes: The sample consists of individuals born 1947. Permanent income is defined as the annuitized value of real income from age 20 to 63. The derivative of the mobility curve is defined in equation (3.4). We represent the derivatives as the difference in income shares with and without mobility at every percentile.

3.2. Partial rankings

Assume that M_1 and M_2 are two mobility curves, where $M_1(u) \geq M_2(u)$ *for all* $u \in [0,1]$ and the inequality is strict for at least one $u \in \langle 0,1 \rangle$. Then we say that M_1 exhibits more mobility than M_2. Thus, ranking income distributions in accordance with first-degree mobility dominance means the higher of non-intersecting mobility curves unambiguously shows more income mobility.

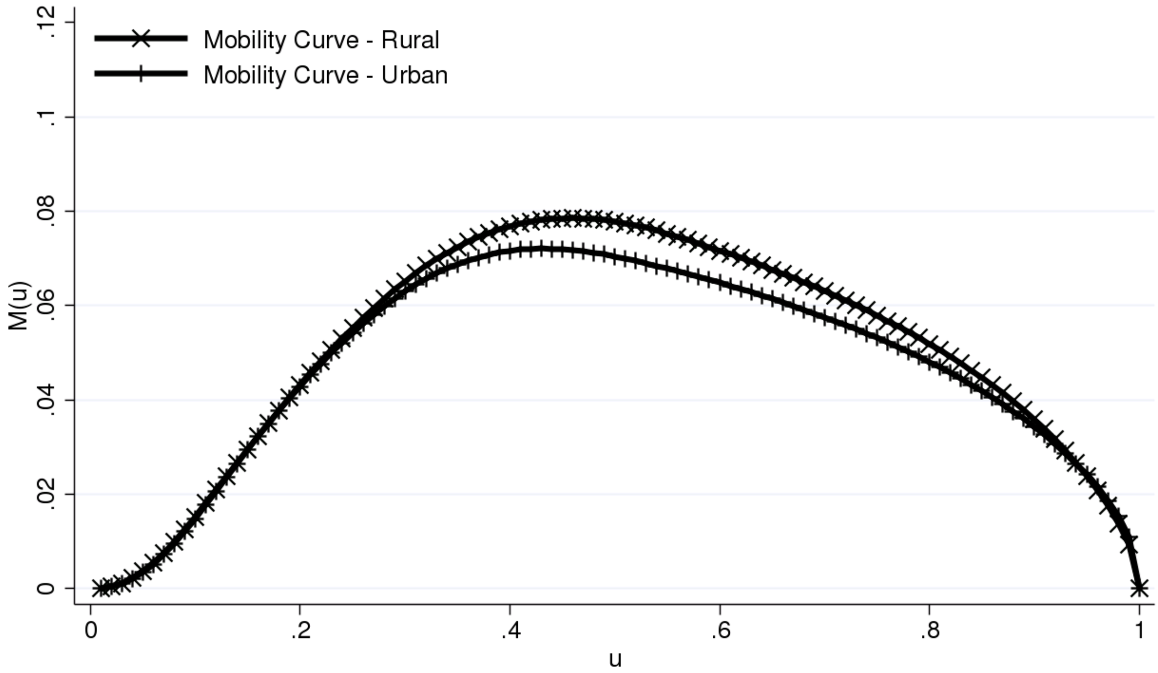
Definition 3.1. *A mobility curve M_1 is said to* ***first-degree dominate*** *a mobility curve M_2 if*

$$M_1(u) \geq M_2(u) \quad \textit{for all} \quad u \in [0,1]$$

and the inequality holds strictly for some $u \in \langle 0,1 \rangle$.

Figure 4 shows an example of first-degree mobility dominance. In this figure, we have divided the 1947 cohort into two subgroups according to whether the individuals were born in a rural or an urban municipality. We can see that the mobility curve of the individuals born in rural areas always lies (weakly) above that of individuals born in urban areas. Therefore, we can unambiguously conclude that income mobility equalizes permanent income the most in the former group.

Figure 4. Mobility curves for individuals born in urban and rural municipalities



Notes: The sample consists of individuals born 1947. Permanent income is defined as the annuitized value of real income from age 20 to 63. The mobility curve is defined in equation (3.2).

To provide a normative justification for first-degree mobility dominance, we introduce a permanent income version of the Pigou-Dalton principle of transfers.

Definition 3.2. ***A Pigou-Dalton permanent income transfer*** *is a transfer in the permanent income distribution F from a person of rank t with income* $F^{-1}(t)$ *to a person of rank s with income* $F^{-1}(s)$, *where* $0 < s < t \leq 1$, *such that the period-specific income distributions are kept unchanged.*

The higher of two non-intersecting mobility curves can be obtained from the lower mobility curve by Pigou-Dalton permanent income transfers. Since such income transfers preserve the period-specific income distributions, L_{Z_R} is unchanged. As a result, the dominating mobility curve M_1 can be attained from the dominated mobility curve M_2 by Pigou-Dalton permanent income transfers that equalizes the permanent income distribution $F_{Z,1}$.[6]

Theorem 3.1. *Let* M_1 and M_2 *be members of* ***M***. *Then the following statements are equivalent,*

(i) $$M_1(u) \geq M_2(u) \quad \text{for all} \quad u \in [0,1]$$

(ii) M_1 *can be attained from* M_2 *by Pigou-Dalton permanent income transfers*.

[6] In practice, a Pigou-Dalton permanent income transfer is achieved by a transfer of period-specific income from a poor to a rich person in permanent income that increases the changes in relative incomes over time, while preserving the marginal distributions of period-specific incomes.

The proof of Theorem 3.1 is omitted because it is analogue to the proof of the equivalence between the criterion of first-degree Lorenz curve dominance and the standard Pigou-Dalton principle of transfers, which means that the dominating Lorenz curve can be attained from the dominated Lorenz curve by transferring income from richer to poorer persons (Fields and Fei, 1978).

In practice, however, mobility curves may intersect, in which case weaker criteria than first-degree mobility dominance are required. We use the mobility curve to introduce two alternative generalizations of first-degree mobility dominance. By integrating the mobility curve from below we get the criterion of second-degree upward dominance:

Definition 3.3A. *A mobility curve M_1 is said to* ***second-degree upward dominate*** *a mobility curve M_2 if*

$$\int_0^u M_1(\mathrm{t})\,dt \geq \int_0^u M_2(\mathrm{t})\,dt \quad \textit{for all} \;\; u \in [0,1]$$

and the inequality holds strictly for some $u \in \langle 0,1 \rangle$.

If we instead integrate the mobility curve from above we get the criterion of second-degree downward dominance:

Definition 3.3B. *A mobility curve M_1 is said to* ***second-degree downward dominate*** *a mobility curve M_2 if*

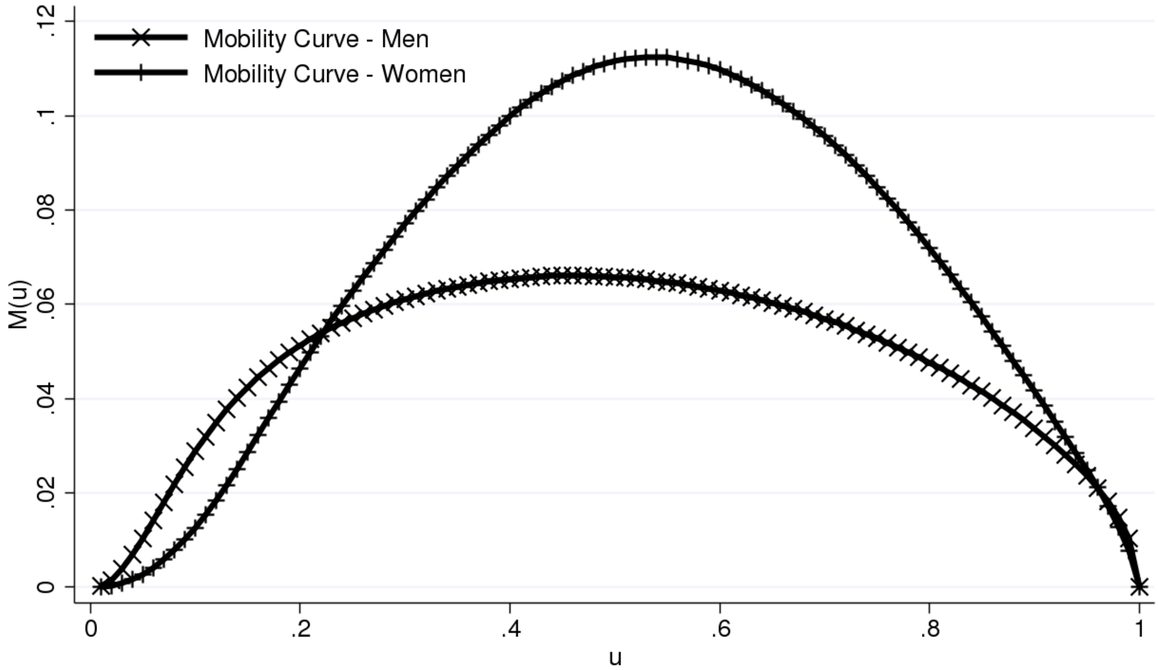
$$\int_u^1 M_1(\mathrm{t})\,dt \geq \int_u^1 M_2(\mathrm{t})\,dt \quad \textit{for all} \;\; u \in [0,1]$$

and the inequality holds strictly for some $u \in \langle 0,1 \rangle$.

Figures 5 and 6 illustrate situations in which first-degree dominance is insufficient to rank income distributions by income mobility. Figure 5 shows mobility curves for men and women, while Figure 6 displays mobility curves for individuals with and without a college degree. In both cases, second-degree downward dominance is sufficient to rank these income distributions by income mobility.

Since first-degree mobility dominance implies upward and downward mobility dominance of second degree, it follows that both criteria preserve first-degree mobility dominance and thus are consistent with the Pigou-Dalton principle of permanent income transfers. To judge the normative significance of the criteria of second-degree upward and downward mobility dominance, the next section introduces permanent income versions of the principles of downside and upside position transfer sensitivity.[7]

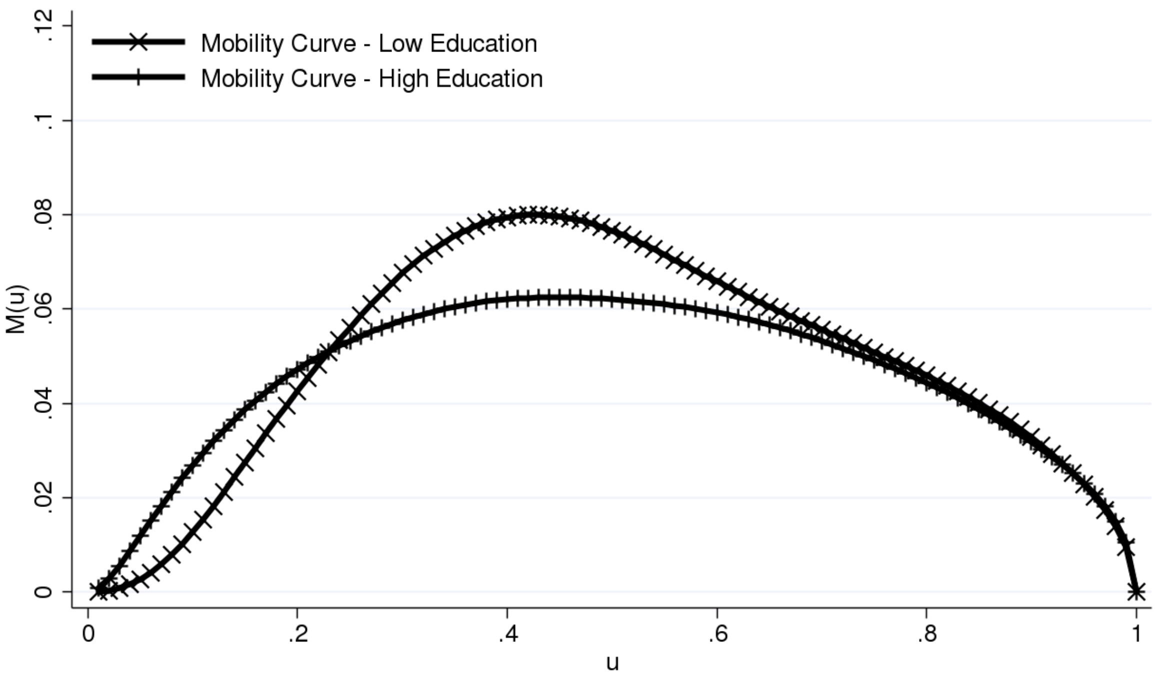
Figure 5. Mobility curves for men and women



Notes: The sample consists of individuals born 1947. Permanent income is defined as the annuitized value of real income from age 20 to 63. The mobility curve is defined in equation (3.2).

[7] Similar principles have been used by Kolm (1976a, 1976b), Zoli (1999) and Aaberge (2000, 2009) to characterize second degree upward Lorenz dominance, while Aaberge (2009) introduced and characterized second-degree downward Lorenz dominance.

Figure 6. Mobility curves for individuals with low and high education



Notes: The sample consists of individuals born 1947. Permanent income is defined as the annuitized value of real income from age 20 to 63. The mobility curve is defined in equation (3.2). High (low) education is defined as (not) having a college degree.

3.3. Complete rankings

In situations where neither upward nor downward mobility dominance of second-degree provides unambiguous rankings of income distributions, it is useful to employ summary measures of income mobility. Summary measures of income mobility also allow us to quantify the equalizing effect of income mobility. In this section, we will use an axiomatic approach to derive a general family of rank-dependent measures of income mobility, which summarizes the informational content of the mobility curve.

Consider the ordering $\succeq$ defined on the family ***M*** of mobility curves. Since the mobility curve *M* is uniquely determined by two Lorenz curves, we can impose similar conditions on the ordering $\succeq$ as Aaberge (2001) used for an ordering defined on the family of Lorenz curves. That is, the ordering $\succeq$ is assumed to be transitive, continuous, complete and rank $M_1 \succeq M_2$ if $M_1(u) \geq M_2(u)$ *for all* $u \in [0,1]$. More importantly, to give the order relation $\succeq$ an empirical content we introduce the following independence condition[8]

[8] These four conditions are analogue to the axioms underlying the expected utility theory for choice under uncertainty. For a proof of the characterization result, we refer to Fishburn (1982).

Independence condition: *Let M_1, M_2 and M_3 be members of **M** and let* $\alpha \in [0,1]$. *Then* $M_1 \succeq M_2$ *implies* $\alpha M_1 + (1-\alpha)M_3 \succeq \alpha M_2 + (1-\alpha)M_3$.

It can be proved that the ordering $\succeq$ which satisfies these conditions can be represented by the following family of mobility measures

$$\Lambda_p(M) = \int_0^1 p(u)dM(u), \tag{3.5}$$

where M is the mobility curve associated with the Lorenz curves $L_Z(u)$ and $L_{Z_R}(u)$, and the weighting function p is a positive non-increasing function defined on the unit interval where $\int p(t)dt = 1$. Note that the condition of non-increasing p follows from the axiom of first-degree mobility dominance. To ensure that Λ_p has the unit interval as its range, the normalization $p(1) = 0$ is imposed.

The preference function p assigns weights to the incomes of the individuals in accordance with their rank in the distribution of permanent income. Therefore, the functional form of p reveals the attitude towards permanent income inequality of a policymaker or researcher who employs Λ_p to judge between mobility curves. Inserting for (3.1) in (3.5) yields

$$\Lambda_p(M) = \int_0^1 p(u)dL_Z(u) - \int_0^1 p(u)dL_{Z_R}(u) = J_p(L_{Z_R}) - J_p(L_Z), \tag{3.6}$$

where the inequality measure $J_p(L)$ for the Lorenz curve L of distribution F with mean μ is defined by

$$J_p(L) = 1 - \int_0^1 p(u)\,d\,L(u) = 1 - \frac{1}{\mu}\int_0^1 p(u)F^{-1}(u)\,du\,. \tag{3.7}$$

Thus, the mobility measure Λ_p shows the extent to which income mobility equalizes the distribution of permanent income, when inequality is measured by the rank-dependent inequality measure J_p.

It is straightforward to verify that $0 \le \Lambda_p(M) \le 1$, with $M=0$ if and only if the distribution of permanent income Z is equal to the distribution of the reference permanent income Z_R. Thus, the state of no mobility occurs when each individual's position in the period-specific income distributions is constant over time. Mobility takes the maximum value of one when there is complete inequality in each period (i.e. $J_p(L_{Z_R}) = 1$) and complete equality in the distribution of permanent incomes (i.e. $J_p(L_Z) = 0$).

As demonstrated by Yaari (1988) and Aaberge (2001), the J_p-family represents a preference relation defined either on the class of distribution functions or on the class of Lorenz curves, where *p* can be interpreted as a preference function of a social planner. We consider both convex and concave preference functions. To choose between them, more powerful principles than the Pigou-Dalton principle of permanent income transfers are needed.

In order to provide a formal definition of the necessary principles, it is useful to consider a discrete permanent income distribution. We also introduce the notation $\Delta\Lambda_p(\delta,h,s)$, denoting the change in Λ_p of a Pigou-Dalton permanent income transfer δ from an individual with rank $s+h$ to an individual with rank s in the distribution of permanent income.[9] Further, let

$$\Delta\Lambda_p(\delta,h,r,s) = \Delta\Lambda_p(\delta,h,r) - \Delta\Lambda_p(\delta,h,s).$$

We can then define the mobility principles of downside and upside positional transfer sensitivity:

Definition 3.4 A. *Λ satisfies the mobility principle of downside positional transfer sensitivity (DPTS) if and only if* $\Delta\Lambda_p(\delta,h,r,s) > 0$ when $r < s$.

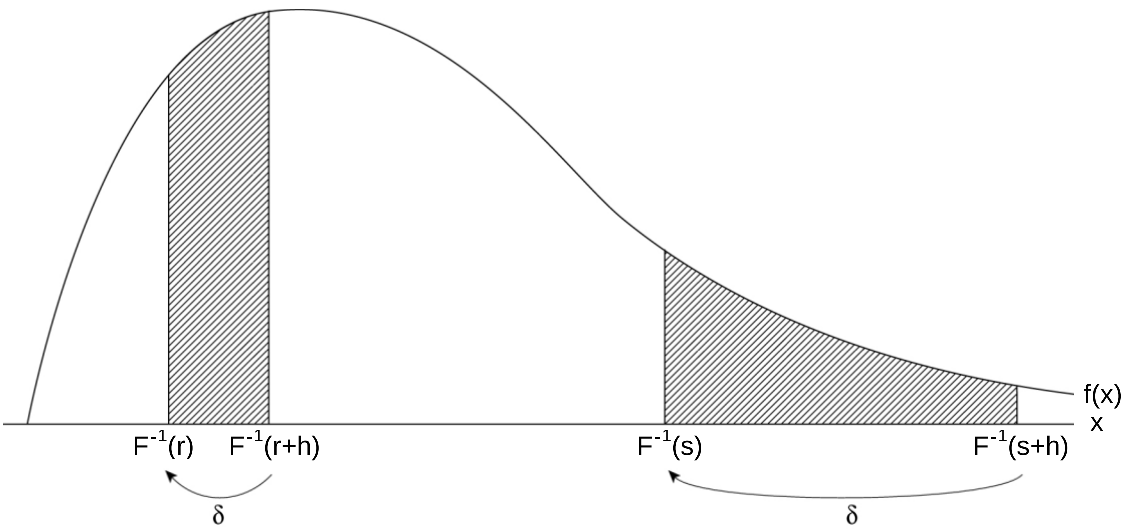
Definition 3.4 B. *Λ satisfies the mobility principle of upside positional transfer sensitivity (UPTS) if and only if* $\Delta\Lambda_p(\delta,h,r,s) < 0$ when $r < s$.

To better understand these transfer principles and how they relate to the Pigou-Dalton principle of permanent income transfers, consider Figure 7 where we draw the probability density *f* of a right-skewed permanent income distribution *F*. We have also drawn two alternative Pigou-Dalton permanent income transfers: One from an individual at rank $r+h$ to an individual at rank *r*, and another from rank $s+h$ to rank *s*; the equal difference in rank *h* is reflected in the equal size of the shaded areas.

According to the Pigou-Dalton principle of permanent income transfers, both transfers should reduce permanent income inequality. According to UPTS (DPTS), the transfer at lower ranks has a weaker (stronger) equalizing effect than the transfer at higher ranks. An inequality averse social planner that supports the principle of UPTS (DPTS) is therefore said to exhibit upside (downside) positional inequality aversion. The choice between DPTS and UPTS clarifies, therefore, whether equalizing transfers between poorer individuals should be considered more or less important for equality in permanent income as compared to equalizing transfers between richer individuals.

[9] For convenience, the dependence of Λ on *F* is suppressed in the notation for Λ.

Figure 7. Illustration of transfer principles



Armed with these transfer principles, we are able to characterize and interpret the relationship between upward and downward dominance of second degree and the general family of mobility measures Λ_p.

Theorem 3.2A. *Let M_1 and M_2 be members of **M**. Then the following statements are equivalent,*

(i) *M_1 second-degree upward dominates M_2*

(ii) $\Lambda_p(M_1) > \Lambda_p(M_2)$ for all non-increasing convex *p such that* $p'(1) = 0$

(iii) $\Lambda_p(M_1) > \Lambda_p(M_2)$ *for all p being such that Λ_p obeys the principle of DPTS*

(Proof in Appendix).

Theorem 3.2B. *Let M_1 and M_2 be members of **M**. Then the following statements are equivalent,*

(i) *M_1 second-degree downward dominates M_2*

(ii) $\Lambda_p(M_1) > \Lambda_p(M_2)$ *for all non-increasing concave p such that* $p'(0) = 0$

(iii) $\Lambda_p(M_1) > \Lambda_p(M_2)$ *for all p being such that Λ_p obeys the principle of UPTS*

(Proof in Appendix).

The equivalence between (i) and (ii) in Theorem 3.2A reveals the least-restrictive set of mobility measures that allows an unambiguous ranking of income distributions in accordance with second-degree upward mobility dominance. This is ensured by imposing the requirement of a convex preference function *p*. Further, the equivalence with (iii) provides a normative justification for ranking distribution functions according to second-degree upward mobility dominance. Theorem 3.2B provides analogous results for second-degree downward dominance. By comparing Theorems 3.2A and 3.2B, it is clear that the choice between second degree upward mobility dominance and second degree downward mobility dominance depends on the weight assigned to the equalizing effect of income mobility in the lower versus the upper part of the permanent income distribution.

The transfer principles allow us to interpret the dominance results displayed in Figures 5 and 6. In both cases, second-degree downward dominance is sufficient to rank these income distributions by income mobility. We can therefore conclude that changes in relative incomes over time equalize the distribution of permanent income more for women and low educated individuals, provided that more attention is paid to inequality reduction in the upper than in the lower part of the permanent income distribution. If one is more concerned with inequality reduction in the lower part of the permanent income distribution, weaker criteria than second-degree mobility dominance is required to rank these distributions by income mobility.

3.4. Social welfare interpretation

Analogous to the expected utility type of social welfare functions proposed by Atkinson (1970), Yaari (1988) introduced the so-called dual family of social welfare functions defined by

(3.8) $$W_p(F)=\int_0^1 p(u)F^{-1}(u)du,$$

where F is an income distribution with mean μ and associated Lorenz curve L. As was recognized by Ebert (1987), the social welfare function in (3.8) can alternatively be expressed as

(3.9) $$W_p(F)=\mu\left(1-J_p(L)\right),$$

where the product $\mu J_p(L)$ can be interpreted as a measure of the loss in social welfare due to inequality in the distribution F. A mean-independent ordering of income distributions in terms of inequality (i.e. an ordering of Lorenz curves) forms the basis of Ebert's approach.[10]

To obtain a welfare interpretation of the income mobility measures, we rewrite expression (3.6) by inserting (3.9) into $J_p(L_Z)$ and $J_p(L_{Z_R})$. This yields

(3.10) $$A_p(M_Z)=\frac{1}{\mu_Z}\left(W_p\left(F_Z\right)-W_p\left(F_{Z_R}\right)\right),$$

[10] See Aaberge (2001) for a theory for ranking Lorenz curves.

where μ_Z and μ_{Z_R} are the means of F_Z and F_{Z_R} and $\mu_{Z_R} = \mu_Z$.

It follows from (3.10) and (3.9) that the welfare produced by the permanent income distribution F_Z admits the following decomposition,

(3.11) $$W_p(F_Z) = W_p(F_{Z_R}) + \mu_Z \Lambda_p(M) = \mu_Z \left(1 - J_p(L_{Z_R}) + \Lambda_p(M)\right),$$

where $W_p(F_{Z_R})$ gives the level of social welfare attained when there is no mobility and $\mu_Z \Lambda_p(M)$ expresses the gains in social welfare due to income mobility. The last equality highlights an important point: If income mobility is very high, the degree of inequality in any given year will be unimportant for social welfare because the distribution of permanent income will be very even.[11] Note that $W_p(F_Z) \le \mu_Z$ and that $W_p(F) = \mu_Z$ if and only if the permanent incomes are equally distributed. Thus, $W_p(F_Z)$ can be given a money-metric interpretation as the equally distributed equivalent permanent income; this represents the level of permanent income per capita which, if shared equally, would generate the same social welfare as the observed distribution of permanent income.

3.5. Parametric sub-families of mobility measures

Until now, the results and discussion have centered on characterizing the relationship between dominance criteria and Λ_p in the ranking of income distributions by income mobility. This section extends our framework to not only answer whether one distribution has higher income mobility than another distribution, but also get an estimate of by how much. To this end, we employ two parametric sub-families of mobility measures.

Consider the following parametric classes of convex and concave weighting functions,

(3.12) $$p_k(u) = (k+1)(1-u)^k, \; k \ge 1,$$

and

(3.13) $$p_k(u) = (k+1)(1-u^k), \; k \ge 1,$$

where $p'_{1k}(1) = 0$ *and* $p'_{2k}(0) = 0$. The weighting classes (3.12) and (3.13) define two alternative families of mobility measures,

[11] Following the literature on income mobility, we abstract from risk due to income fluctuations over time. Incorporating the welfare loss from income risk would require a certainty equivalent (i.e. risk adjusted) measure of permanent income.

$$(3.14)\quad \Lambda_p(M) = \Lambda_{1,k}(M) \equiv (k+1)\int_0^1 (1-u)^k \, d\left(L_Z(u) - L_{Z_R}(u)\right) = G_k(L_{Z_R}) - G_k(L_Z), k \geq 1$$

where $G_k(L) = 1-(k+1)\int_0^1 (1-u)^k \, dL(u)$ is equal to the extended Gini family of inequality measures introduced by Donaldon and Weymark (1980), and

$$(3.15)\quad \Lambda_p(M) = \Lambda_{2,k}(M) \equiv (k+1)\int_0^1 \left(1-u^k\right) d\left(L_Z(u) - L_{Z_R}(u)\right) = D_k(L_{Z_R}) - D_k(L_Z),\ k \geq 1$$

where $D_k(L) = 1-(k+1)\int_0^1 \left(1-u^k\right) dL(u),\ k \geq 1$ is equal to the Lorenz family of inequality measures introduced by Aaberge (2000, 2007).[12]

Inserting for *k=1* in (3.14) and (3.15), we find that both weighting functions form the following mobility measure,

$$(3.16)\qquad \Lambda_p(M) = \Lambda_G(M) \equiv \int_0^1 (1-u)\, d\left(L_Z(u) - L_{Z_R}(u)\right) = G(L_{Z_R}) - G(L_Z),$$

where G is the Gini coefficients. Note that the *p*-function that corresponds to the Gini coefficient, $p(u) = 2(1-u)$, is neither strictly concave nor strictly convex. Since $p''(u) = 0$ for all *u*, the Gini coefficient is the only member of Λ_p that neither preserves second-degree upward mobility dominance nor second-degree downward mobility dominance.

For *k*>1, however, the members of the extended Gini and Lorenz families differ in their sensitivity to whether changes take place in the lower or upper part of the permanent income distributions. As *k* increases, the extended Gini measures G_k assign more weight to inequality in the lower part of the permanent income distribution, whereas the Lorenz measures D_k emphasises on inequality in the upper part of the permanent income distribution. As $k \to \infty$ we get that

$$(3.17)\qquad \Lambda_{1,k}(M) \to \frac{F_Z^{-1}(0+)}{\mu_Z} - \frac{F_{Z_R}^{-1}(0+)}{\mu_{Z_R}}$$

and

$$(3.18)\qquad \Lambda_{2,k}(M) \to 0$$

[12] Aaberge (2001) provided an axiomatic justification of these two families of inequality measures based on a theory for ranking Lorenz curves.

Equation (3.17) shows that the highest degree of aversion to inequality in the lower part of the permanent income distribution is achieved when focus is exclusively turned to the situation of the poorest in the population. In this case, the social welfare function corresponds to the Rawlsian maximin criterion, and income mobility matters for social welfare insofar it increases the income share of the poorest individual. Equation (3.18) shows the other extreme situation, when focus is exclusively turned to the mean permanent income. In this case, any equalizing effect of income mobility does not matter for social welfare.

In Table 2, we use the income data for the 1947 cohort to illustrate the parametric measures of income mobility. For simplicity, we focus on the case where k is equal to 1. The first column reports the Gini coefficients in the distribution of permanent income with no income mobility. The second column shows how income mobility reduces the Gini coefficients in permanent income. In the population as a whole, income mobility reduces the Gini-coefficient by 9.6 percentage points (or 23 percent). Put into perspective, this reduction corresponds to introducing a 23 percent proportional tax on permanent incomes and then redistributing the derived tax as equal sized amounts to the individuals (Aaberge, 1997). This suggests that income mobility as an equalizer of permanent income can be economically important. The last column supports this conjecture, showing that income mobility increased social welfare by 12.4 percent. Table 2 also looks at income mobility within different subgroups. Consistent with the dominance results, we find that income mobility is relatively high among males, individuals with low education levels, and people born in rural areas. As a consequence, these groups experience the largest relative increase in social welfare.

Table 2. Inequality and mobility estimates

Groups:	$G(L_{A_R})$	$G(L_{A_R}) - G(L_A)$	Increase in welfare due to mobility
Males	0.312	0.096	+ 12.2 %
Females	0.457	0.135	+ 19.8 %
Rural	0.412	0.101	+ 14.7 %
Urban	0.417	0.095	+ 14.0 %
Low Education	0.431	0.097	+ 14.6 %
High Education	0.334	0.091	+ 12.0 %
Full sample	0.417	0.096	+ 12.4 %

Notes: The sample consists of individuals born 1947. High (low) education is defined as (not) having a college degree.

4. Re-examining the pattern of income mobility

This section compares our method to traditional measures of income mobility, and demonstrates empirically how they reach different conclusions about the pattern of income mobility across countries.

4.1. Traditional measures of income mobility

Following Shorrocks (1978), a large number of studies employ measures of income mobility capturing the *share* of cross-sectional inequality that is transitory. These income mobility measures are derived from a factor decomposition of inequality measures and can be written as

(4.1) $$\tilde{\Lambda}_p(L_Z) = \frac{J_p(L_{Z_R}) - J_p(L_Z)}{J_p(L_{Z_R})},$$

when the rank-dependent family of inequality measures form the basis for the measurement of inequality. Equation (4.1) shows that $\tilde{\Lambda}_p(L_Z)$ is not necessarily higher in a society where changes in the relative incomes of individuals occur more frequently or are greater in magnitude. In particular, if $J_p(L_{Z_R})$ is low then even minor changes in relative income over time may translate into high $\tilde{\Lambda}_p(L_Z)$. This raises the concern that the traditional measures of mobility does not adequately distinguish between changes in the income structure that equalize the cross-sectional income distributions, and those that affect individuals' relative incomes over time.

Inserting (3.9) for $J_p(F_Z)$ and $J_p(F_{Z_R})$ in (4.1) yields the following alternative expression for $\tilde{\Lambda}_p$,

(4.2) $$\tilde{\Lambda}_p(L_Z) = \frac{W_p(F_Z) - W_p(F_{Z_R})}{\mu_Z - W_p(F_{Z_R})}$$

where the numerator of (4.2) can be considered as a measure of the gain in social welfare due to income mobility, and the denominator as a measure of maximum attainable gain in social welfare due to income mobility when $W_p(F)$ is used as a measure of social welfare. By rearranging equation (4.2), we find that $W_p(F_Z)$ admits the following decomposition

(4.3) $$W_p(F_Z) = W_p(F_{Z_R}) + \tilde{\Lambda}_p(L_Z)(\mu_Z - W_p(F_{Z_R})).$$

where the first term gives the level of social welfare attained when there is no mobility. The second term, however, is more difficult to interpret as it depends on the interaction between the cross-sectional inequality and the income mobility. Put differently, social welfare in permanent income is not additively decomposable with respect to the contributions from the cross-sectional distributions and the income mobility. Equation (4.3) shows that even if $\tilde{\Lambda}_p(L_Z)$ is very high, the degree of inequality in any given year is important for social welfare. Therefore, $\tilde{\Lambda}_p(L_Z)$ is not a suitable measure of income mobility as an equalizer of permanent income.

4.2. Income mobility across countries

Consider first Table 3, which shows estimates of income mobility for the 1947 cohort. The first column reports the Gini coefficients in the distribution of permanent income with no income mobility. The second column shows the estimates of income mobility from the mobility curve approach, while the third displays income mobility estimates based on the traditional measures. The results suggest that the traditional measures of income mobility do not adequately distinguish between changes in the income structure that equalize the cross-sectional income distribution, and those that affect individuals' relative incomes over time. As shown in the third column, the groups that have the lowest cross-sectional levels of inequality are always recorded with the highest income mobility when applying the traditional measures. This does not mean, however, that income mobility is more important for the distribution of permanent income for these groups. As shown in the second column, changes in relative incomes over time equalize permanent income the most among females, who have relatively high levels of cross-sectional inequality.

In Table 4, we re-examine the pattern of income mobility across countries. In each panel, we use the estimates of inequality and mobility reported in previous studies to compute our measure of income mobility as an equalizer of permanent income. In Panel A, we use the results reported in Aaberge et al. (2002) to compare income mobility between the US and the Nordic countries. We find that changes in relative incomes over time contribute more to equality in long-run incomes in the US than in the Nordic countries. However, due to low cross-sectional inequality in the Nordic countries, even small changes in relative incomes over time translate into high estimates of income mobility when applying traditional measures.

In Panel B, we shift attention to the between the US and Germany. In this case, we use the results reported in Burkhauser and Poupure (1997). As pointed out in their study, the traditional measures suggest that Germany has somewhat higher income mobility than the US. This result, however, is due low cross-sectional levels of inequality. Changes in relative incomes over time contribute as much to equality in long-run incomes in the US as in Germany.

Table 3. Inequality and mobility estimates

Groups:	$G(L_{A_R})$	$G(L_{A_R})-G(L_A)$	$\left[G(L_{A_R})-G(L_A)\right]/G(L_{A_R})$
Males	0.312	0.096	0.308
Females	0.457	0.135	0.294
Rural	0.412	0.101	0.246
Urban	0.417	0.095	0.227
Low Education	0.431	0.097	0.225
High Education	0.334	0.091	0.270
Full sample	0.417	0.096	0.230

Notes: The sample consists of individuals born 1947. High (low) education is defined as (not) having a college degree.

Table 4. Estimates of mobility and inequality in permanent income

Country and period:	$G(Z_R)$	$G(Z_R)$- $G(Z)$	$[G(Z_R)$- $G(Z)]/ G(Z_R)$
Panel A:			
Denmark, 80-90	0.239	0.019	0.080
Norway, 80-90	0.275	0.019	0.069
Sweden, 80-90	0.252	0.018	0.073
U.S., 80-90	0.404	0.026	0.065
Panel B:			
Germany, 83-88	0.240	0.015	0.065
U.S., 83-88	0.340	0.016	0.048

Notes: In Panel A, the estimates of columns 1 and 3 are from Aaberge et al. (2002). In Panel B, the estimates of columns 1 and 3 are from Burkhauser and Poupure (1997).

5. Concluding remarks

Do market-orientated economies with relatively large cross-sectional levels of inequality have higher income mobility and therefore less permanent inequality? To answer this question, we have introduced a formal representation of the notion of income mobility as an equalizer of permanent income. The proposed representation is called a mobility curve and forms the basis for comparison of income distributions according to income mobility. The mobility curve captures the extent to which the distribution of permanent income is equalized because of changes in individuals' relative income over time. We applied our method to re-examine the conclusions in recent studies about the pattern of income mobility across countries. We find that changes in relative income over time contribute more (as much) to equality in permanent income in the US as in the Nordic countries and Germany.

Our paper complements the literature on intra-generational income mobility in several ways. The introduction of a mobility curve allows us to develop dominance criteria providing partial orderings of income distributions according to income mobility. The mobility curve also allows us to assess the equalizing impact of income mobility across the entire distribution of permanent income. An axiomatically justified family of rank-dependent measures of income mobility provides complete orderings by summarizing the informational content of the mobility curve. Our representation of income mobility is also fundamentally different, in that we accommodate the widespread notion of income mobility as an equalizer of permanent income. This representation has important implications for the interpretation of our income mobility estimates: High mobility will equalize permanent income and raise social welfare more than low mobility. Our empirical results highlight these differences: Due to low cross-sectional inequality in the Nordic countries, even small changes in relative incomes over time – which matter little for social welfare and equality in permanent income – translate into high estimates of income mobility when applying traditional mobility measures.

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Appendix: Proofs

Proof of Theorem 3.2A. Using integration by parts we have that

$$\Lambda_p(M_1)-\Lambda_p(M_2)=\int_0^1 p(u)d\left(M_1(u)-M_2(u)\right)=\int_0^1 p'(u)\left(M_1(u)-M_2(u)\right)du$$

$$=-p'(1)\int_0^1\left(M_1(t)-M_2(t)\right)dt+\int_0^1 p''(u)\int_0^u\left(M_1(t)-M_2(t)\right)dtdu$$

Thus, if (i) holds then $\Lambda_p(M_1)>\Lambda_p(M_2)$ for all non-increasing convex *p such that* $p'(1)=0$.

To prove the converse statement we restrict to non-increasing convex *p such that* $p'(1)=0$ .. Hence,

$$\Lambda_p(M_1)-\Lambda_p(M_2)=\int_0^1 p''(u)\int_0^u\left(M_1(t)-M_2(t)\right)dtdu$$

and the desired result it obtained by applying Lemma 1 (see below).

To prove the equivalence between (ii) and (iii) consider a case where we transfer a small amount γ from persons with permanent incomes $F^{-1}\left(s+h_1\right)$ and $F^{-1}\left(t+h_1\right)$ to persons with permanent incomes $F^{-1}(s)$ and $F^{-1}(t)$, respectively, where *t* is assumed to be larger than *s*. Then Λ_p defined by (3.5) obeys DPTS if and only if

$$p(r)-p\left(r+h_1\right)>p(s)-p\left(s+h_1\right)$$

which for small h_1 is equivalent to

$$p'(s)-p'(r)>0\,.$$

Next, inserting for $s=r+h_2$, we find, for small h_2, that this is equivalent to $p''(s)>0$.

Proof of Theorem 3.2B.

The proof of Theorem 3.2B is analogue to the proof of Theorem 2.2A and is based on the expression

$$\Lambda_p(M_1)-\Lambda_p(M_2)=-p'(0)\int_0^1\left(M_1(t)-M_2(t)\right)dt-\int_0^1 p''(u)\int_u^1\left(M_1(t)-M_2(t)\right)dtdu\,,$$

which is obtained by using integration by parts. Thus, by arguments like those in the proof of Theorem 3.2A the results of Theorem 3.2B are obtained.

Lemma 1. *Let H be the family of bounded, continuous and non-negative functions on [0,1] which are positive on* $\langle 0,1 \rangle$ *and let g be an arbitrary bounded and continuous function on [0,1]. Then*

$$\int g(t)\,h(t)\,dt > 0 \quad \textit{for all} \quad h \in H$$

implies

$$g(t) \geq 0 \quad \textit{for all} \quad t \in [0,1]$$

and the inequality holds strictly for at least one $t \in \langle 0,1 \rangle$.

The proof of Lemma 1 is known from mathematical textbooks.