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Abstract

Most macroeconomic models for monetary policy analysis are approximated around a zero-inflation steady state, but most central banks target inflation at a rate of about 2 percent. Many economists have recently proposed even higher inflation targets to reduce the incidence of the zero lower bound (ZLB) constraint on monetary policy. In this survey, we show the importance of appropriately accounting for a low, positive trend inflation rate for the conduct of monetary policy. We first review empirical research on the evolution and dynamics of U.S. trend inflation, as well as some proposed new measures to assess the volatility and persistence of trend-based inflation gaps. Then we construct a generalized New Keynesian model that accounts for a positive trend inflation rate. We find that, in this model, higher trend inflation is associated with a more volatile and unstable economy and tends to destabilize inflation expectations. This analysis offers a note of caution in evaluating recent proposals to address the existing ZLB situation by raising the underlying rate of inflation.

Key words: trend inflation, inflation target, inflation persistence, monetary policy

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1 Introduction

The notion of price stability, as defined in modern monetary theory and central banking practice today, is typically associated with a moderate rate of price inflation: during the most recent period of relatively stable inflation, 1990 to 2008, average inflation (as measured by the consumer price index) was about 2.8 percent in the US, 2.2 percent in Germany and 4 percent in OECD countries, for example. Countries whose central banks officially adopt an 'inflation targeting' regime typically target inflation at about 2 percent. Recent communication by the FOMC states 2 percent change in the personal consumer expenditure price deflator as the annualized rate of inflation "most consistent over the longer run with the Federal Reserve's statutory mandate."[1] The recent crisis has led some researchers to advocate a higher target inflation even in normal times, to increase the room for monetary policy to react to deflationary shocks, e.g., Blanchard et al. (2010), Williams (2009) and Ball (2013). In particular, the proposal in Blanchard et al. (2010) of adopting a 4 percent inflation target in the U.S. to address the current crisis situation, where monetary policy is constrained by the zero lower bound, stimulated a lively debate in both policy and academic circles.[2] In various speeches, however, Fed Chairman Bernanke argued against such proposals. In his 2010 Jackson Hole speech, for example, he observed that "Inflation expectations appear reasonably well-anchored, and both inflation expectations and actual inflation remain within a range consistent with price stability. In this context, raising the inflation objective would likely entail much greater costs than benefits. Inflation would be higher and probably more volatile under such a policy, undermining confidence and the ability of firms and households to make longer-term plans, while squandering the Fed's hard-won inflation credibility. Inflation expectations would also likely become significantly less stable, and risk premiums in asset markets—including inflation risk premiums—would rise."[3]

In light of this debate it is worth asking what are the implications of a higher rate of inflation for the conduct of monetary policy in normal times. In particular: what are the implications of different underlying rates of inflation - we call these inflation 'trends' - for the volatility and persistence of inflation and for the dynamics and the volatility of the economy as a whole? What are the implications for anchoring inflation expectations?[4]

While a large time series literature has developed on estimating the rate of trend inflation, there is no comprehensive discussion in the literature of the implications of different trend inflation rates for the conduct of monetary policy. Workhorse monetary models most often assume zero inflation in steady state, or otherwise mute the theoretical

[1] FOMC statement of longer-run goals and policy strategy, released the first time by the Federal Reserve in January 2012, and revised in January 2013: http://www.federalreserve.gov/monetarypolicy/files/FOMC_LongerRunGoals.pdf

[2] E.g., see the recent policy commentaries by Paul Krugman, 29 April 2012 in the New York Times (http://www.nytimes.com/2012/04/29/magazine/chairman-bernanke-should-listen-to-professor-bernanke.html?pagewanted=all&_moc.semityn.www) and B. DeLong in The Economist Debate on Inflation Target: http://www.economist.com/debate/overview/203.

[3] Chairman Ben S. Bernanke, "The Economic Outlook and Monetary Policy," Remarks at the Federal Reserve Bank of Kansas City Economic Symposium, Jackson Hole, Wyoming, August 27, 2010, http://www.federalreserve.gov/newsevents/speech/bernanke20100827a.pdf

[4] Throughout this survey, we use the terms trend inflation, steady state inflation and inflation target interchangeably.

relevance of a positive average inflation rate by imposing appropriate indexation to trend inflation. In this survey we address the above questions by examining a recent literature which started to investigate the implications of modeling trend inflation and its evolution from an empirical, theoretical and policy perspective.

Trend inflation is tied to the behavior of monetary policy, while short-run fluctuations of inflation around trend reflect price setting dynamics, external shocks, and monetary policy. Trend inflation itself may have interesting dynamics: policymakers may have targets that change over time, for example when they implement gradual disinflations (or reflations), or learn about the structure of the economy over time; or policymakers may have implicit inflation targets, which agents have to learn over time. Understanding the dynamics of the short- and long-run components of inflation helps estimating the persistence of inflation and has significant implications for the design of monetary policy: for example, monetary policy rules that are optimal in forward-looking models deliver bad outcomes in models that feature intrinsic persistence. Furthermore, in standard models, the dynamics of the economy are affected by the rate of inflation that characterizes the long-run equilibrium. Hence, the assumptions one makes about trend inflation have important implications for the cyclical properties of the economy, affecting the policy trade-offs and the conduct of monetary policy.

To review these implications, we first discuss empirical research on the evolution of the dynamics and persistence of inflation. Next, we lay down what an appropriate account of a positive rate of trend inflation implies for the theory and practice of monetary policy. Finally, we address the current policy debate by evaluating under which circumstances it may be desirable to have a higher trend inflation in the economy.

The road map of this survey is as follows. We start by discussing the dynamic properties of US inflation, addressing the issue of whether its volatility and persistence have evolved over time. We review reduced form methods to characterize the evolving inflation dynamics and then discuss the consequences of allowing for a time-varying trend inflation on the assessment of inflation persistence in a structural Phillips curve relationship [Section 2].

We then turn to the implications of a positive inflation target for the dynamics and the volatility of the economy as a whole and for anchoring inflation expectations [Section 3]. The framework we use for our analysis is the New Keynesian model. Although this is not the only model that could be used for the analysis, it is the workhorse framework of modern monetary policy literature. It therefore allows both a unified treatment of the issues we intend to address and a direct comparison with other results in the literature that do not take trend inflation into account. As in standard versions of this model, we build the supply side on the Calvo price-setting framework. We start by reviewing the implications of a *low* trend inflation rate for the dynamic properties of the model economy. In particular, we discuss the implications of a moderate rate of trend inflation both for the dynamic response to shocks and for the determinacy and E-stability of the rational expectations equilibrium. We discuss results in the literature that show that, everything else equal, higher trend inflation increases the cost of price dispersion, the volatility of the economy as well as the likelihood of sunspots fluctuations and of unstable expectations dynamics under learning.

We also briefly consider whether these implications hold under other time-dependent models of price adjustment. We do not cover the literature on state-dependent prices.

While important, this literature has played a less central role in the monetary policy debate. Indeed our focus on the implications of a *low* rate of trend inflation makes reasonable to assume that trend inflation does not affect the frequency of price adjustments.[5] Where appropriate, however, we point out to results that obtain in our framework but may not hold under state-dependent pricing.

Finally, we also cover normative issues, addressing research both on optimal stabilization policy conditional on a chosen target, illustrating the sensitivity of optimal policy analysis to accounting for positive trend inflation in the model, and, briefly, on the optimal inflation rate.

In the last section [Section 4] we bring the survey to bear on the current debate, by addressing the issue of whether it may be desirable for policymakers to target a higher long-run rate of inflation in the current situation of continued binding of the zero lower bound (*ZLB*) on the nominal interest rate. Here we address two somewhat separate issues.

The first one concerns whether the presence of a *ZLB* alters traditional considerations about the optimality of keeping long-run inflation at a low level. The rationale for a higher target is to allow more room to policymakers for lowering the real rate of interest in case of deflationary shocks, making the *ZLB* less likely to be reached. The analysis of monetary policy that we discuss in the course of the survey shows, however, that a higher inflation target increases the cost of price dispersion and it makes more difficult for monetary policy to stabilize inflation around target. Hence to answer the question of whether a higher inflation target can mitigate the zero bound constraint, one needs to balance the costs of higher inflation in normal times with the benefit of reducing the likelihood of hitting the *ZLB* constraint.

The second issue concerns whether, even if a higher inflation target can in principle mitigate the zero bound constraint, there are alternative monetary policy strategies that can help exiting the *ZLB* without the drawbacks of a permanently higher target.

2 Trend inflation, inflation persistence and the Phillips curve

The aim of the section is to discuss empirical evidence on the low frequency component of inflation and show how a distinction between inflation and inflation gap (the difference between inflation and its trend) matters for estimating the dynamics of inflation and inflation persistence in structural models.

The extent of inflation persistence is in turn important for the design of appropriate monetary policy: knowing the length of time it takes for inflation to approach a new equilibrium after a shock is crucial for a central bank in determining how to adjust its tools to reach its desired objectives. But shifts in monetary policy also affect the fundamentals that drive inflation and therefore alter its dynamic properties: one should therefore distinguish, for example, whether persistence is intrinsic to the inflation process, in which case it should be taken as given by policymakers, or it is instead

[5] Recent contributions also suggest that the degree of state-dependence is not very high for changes in the rate of inflation when average inflation is moderate, as currently observed in advanced economies. Hence the Calvo model provides a relatively good approximation in this case (see Alvarez et al., 2011, Costain and Nakov 2011a,b, Woodford, 2009b, and Kehoe and Midrigan, 2012).

the result of how monetary policy is conducted. This would be the case if persistence were primarily detected in the trend component of inflation, which is typically associated with the long-run objective of the policymakers.

The debate on inflation persistence is part of the more general debate on whether the more stable inflation that characterized the so-called Great Moderation period (the period post-1984 and until the Great Recession) was due to lower volatility of the shocks (better 'luck') or less persistence in the effects of the shocks, which could be partly attributed to better policy.

Evidence on the topic remains controversial. In a recent, comprehensive, survey (to which we refer the interested reader) Fuhrer (2011) discusses a large body of research on inflation persistence, conducted with an array of statistical methods on the most common measures of US inflation. These analyses have investigated the extent to which there were structural breaks, and whether these breaks were likely associated with a change in the systematic behavior of monetary policy, or should rather be attributed to changes in the persistence of the shocks.

In the context of this survey, the most relevant analyses are those that examine the evolution of persistence over time in connection with the evolution of the trend component of inflation. We discuss how changes in inflation persistence can manifest as a change in the volatility of the innovations to the trend component (e.g. Stock and Watson, 2007) or as a decline in the inflation gap persistence (Cogley et al., 2010).

We then turn to investigate the presence of intrinsic persistence in a structural model of inflation dynamics that allows for positive trend inflation. To do so we introduce a formulation of the New Keynesian Phillips Curve (*NKPC*) which is obtained by approximating firms' optimal pricing conditions around a steady state with positive (and possibly time varying) inflation (we call this a Generalized New Keynesian Phillips curve, or *GNKPC*). Relative to standard formulations of the *NKPC*, in this generalized model the dynamics of inflation is more forward-looking, and changes in trend inflation change the inflation/real activity trade-off. The *GNKPC* is the backbone of the general equilibrium model that we use for policy analysis in section 3.

In the last part of this section we present a way to estimate the *GNKPC*, and discuss its implications for the assessment of the intrinsic persistence in the inflation process (Cogley and Sbordone, 2008; Barnes et al. 2009). A caveat to the empirical literature on the *NKPC*, however, is the weak identification of the parameters (Kleibergen and Mavroedis, and Mavroedis et al., 2012).

2.1 Inflation persistence in reduced form frameworks

Most measures of inflation in the US and other advanced economies were significantly higher in the mid- '70s and early '80s relatively to the previous and the following years, which raises the issue of accounting for possible structural changes when estimating inflation dynamics over those periods. Several research contributions show indeed that the analysis of persistence is sensitive to whether one accounts for variation in the autoregression coefficients when fitting univariate processes to inflation time series. Levin and Piger (2004), for example, analyze inflation dynamics in 12 industrial countries for the period 1984-2003, allowing for possible structural breaks at unknown dates. They find strong evidence of a structural break in the intercept of the autoregressive equations, but little evidence of a break in any of the autoregressive coefficients. Allowing for a break in the mean, the sum of the autoregressive coefficients reveals very little persistence in most of the inflation series, leading the authors to conclude that high inflation persistence is not an intrinsic feature of industrial economies.[6] Pivetta and Reis (2007) analyze US inflation persistence and find no evidence of significant changes over time. They allow for a unit root in inflation and consider several measures of persistence trying to distinguish between changes in volatility and changes in persistence. They conclude that there has been no change in persistence over the last three decades, and attribute their findings to a proper account of structural breaks; a break is a way of accounting for a shifting trend inflation.

Evidence that the dynamics of inflation have been largely dominated by the trend component is provided by Stock and Watson (2007). Their analysis focuses on the forecastability of inflation: based on a split-sample analysis, where the split is set around the beginning of the second term of Volcker's chairmanship at the Federal Reserve, they observe that inflation has become *more* predictable, because its innovation variance is smaller, but *less* predictable because future inflation is less closely correlated with current inflation and other predictors. Changes in the forecastability are affected by changes in the volatility and persistence of inflation. To identify the nature of the changes in inflation dynamics as well as the timing of these changes, Stock and Watson (2007) estimate a univariate time-varying trend-cycle model with stochastic volatility.[7] They found large variations in the standard deviations of the permanent innovations $\sigma_{\varepsilon,t}$ (reported in Figure 1). The period from the 1970 through 1983 was a period of high volatility; the previous period, from mid-'50s through late '60s, as well the '84-'90 period show moderate volatility, and since mid-'90 the volatility of the permanent innovation fell to very low levels. By contrast, the variance of the transitory component ($\sigma_{\eta,t}$, in Figure 2) remained largely unchanged.

Stock and Watson's analysis suggests that inference on inflation persistence may be quite different when conducted on an 'inflation gap' measured as deviation of inflation from a time-varying trend. However in their setting inflation innovations are serially uncorrelated, which makes the model unsuitable to investigate persistence in the inflation gap.

A number of recent contributions have analyzed inflation dynamics in a multivariate framework, aiming at relating shifts in the dynamics of inflation to the evolution of monetary policy. Particularly important has been the seminal contribution of Cogley and Sargent (2001) who introduced Bayesian Vector Autoregression models with drifting coefficients for the study of the joint dynamics of inflation, unemployment and the short-term nominal interest rate. They estimate the trend component of inflation and find that this component appears mostly responsible both for the rise and fall of US inflation

[6] They report the sum of the autoregressive coefficients to be less than 0.7, and they can reject the null hypothesis of a unit root at the 95 percent confidence level.

[7] In this UC-SV model, inflation is the sum of two components, a permanent stochastic trend component τ_t and a serially uncorrelated transitory component η_t . Specifically: $\pi_t = \tau_t + \eta_t$, and $\tau_t = \tau_{t-1} + \varepsilon_t$, where the processes η_t and ε_t are respectively $\eta_t = \sigma_{\eta,t}\zeta_{\eta,t}$, and $\varepsilon_t = \sigma_{\varepsilon,t}\zeta_{\varepsilon,t}$. The stochastic volatilities evolve as driftless geometric random walks: $ln\ \sigma_{i,t} = ln\ \sigma_{i,t-1} + \nu_{i,t}$ for $i = \eta, \varepsilon$. Furthermore, $\zeta_t = (\zeta_{\eta,t}, \zeta_{\varepsilon,t})$ is iid $N(0, \gamma I_2)$, $\nu_t = (\nu_{\eta,t}, \nu_{\varepsilon,t})$ is iid $N(0, \gamma I_2)$, and ζ_t and ν_t are independently distributed. γ is a scale parameter which controls the smoothness of the stochastic volatility process. The figures reported here show the smoothed estimates of the standard deviation of the permanent and transitory component ($\sigma_{\varepsilon,t}$ and $\sigma_{\eta,t}$, respectively) of inflation, measured by the GDP deflator, computed by Markov Chain Monte Carlo (MCMC).

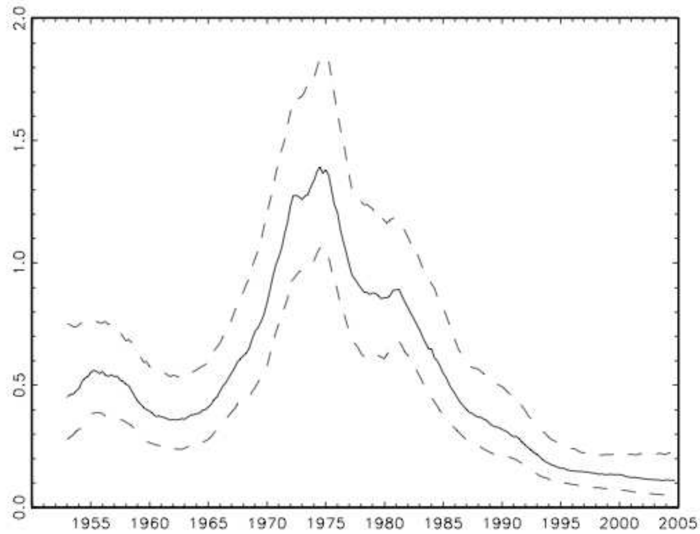

Figure 1: Stock and Watson (2007, p.18) – Figure 2(a): Estimated $\sigma_{\varepsilon,t}$

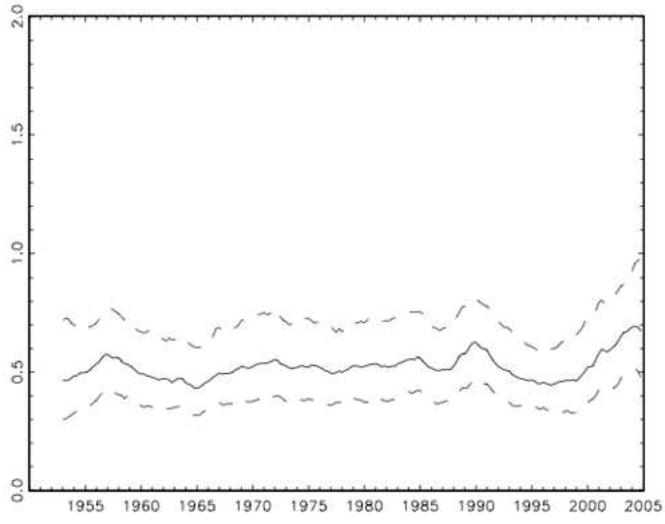

Figure 2: Stock and Watson (2007, p.18) – Figure 2(b): Estimated $\sigma_{\eta,t}$

in the post World War II period, and for movements in inflation persistence. The results of this paper and its companion (Cogley and Sargent, 2005), where the authors also account for stochastic volatility, contrast with other results in the structural *VAR* literature. Modeling discrete shifts in regimes, for example, Sims and Zha (2006) found that the historical patterns emphasized by Cogley and Sargent can be generated by "stable monetary policy reactions to a changing array of major disturbances." (p. 77). Primiceri (2006) also tends towards this interpretation, arguing that the high volatility of the shocks that characterized the 1970's and early 1980's seems a more likely candidate to explain the peaks in inflation that occurred in those periods.

While the significance of the decline in persistence and its attribution to monetary policy remain debated, undoubtedly more attention is now paid to modeling the dynamics of trend inflation, hence of the inflation gap. Indeed Cogley et al. (2010) extend further the work of Cogley and Sargent (2001, 2005), focusing explicitly on the inflation gap. They provide a new measure of inflation persistence based on predictability, and detect a significant decline in inflation gap persistence in the post-Volcker era. They also find that inflation innovations account for a small fraction of the unconditional variance of inflation, implying that most of the volatility is in the trend component of inflation. This result is consistent with the univariate analysis of Stock and Watson (2007) we discussed above.

To illustrate these results we now formally introduce the notion of trend inflation and its estimation in the context of a Bayesian *VAR* model of the kind introduced by Cogley and Sargent (2001). This allows to evaluate the persistence measures just described, and set the stage for the discussion of the dynamics of the inflation gap derived from a structural model of price setting, which we discuss in the second half of this section.

A *VAR* model with time-varying coefficients can be written as follows:[8]

$$\mathbf{x}_t = \mathbf{X}_t' \boldsymbol{\vartheta}_t + \boldsymbol{\varepsilon}_{\mathbf{x}t}, \tag{1}$$

where $\mathbf{x}_t$ is a $N \times 1$ vector of endogenous variables, $\mathbf{X}_t' = \mathbf{I}_N \otimes \begin{bmatrix} 1 & \mathbf{x}_{t-l}' \end{bmatrix}$, where $\mathbf{x}_{t-l}'$ represents lagged values of $\mathbf{x}_t$ and $\boldsymbol{\vartheta}_t$ denotes a vector of time-varying conditional mean parameters. $\boldsymbol{\vartheta}_t$ is assumed to evolve as a driftless random walk:[9]

$$\boldsymbol{\vartheta}_t = \boldsymbol{\vartheta}_{t-1} + \mathbf{v}_t \tag{2}$$

where the innovation $\mathbf{v}_t$ is normally distributed, with mean $\mathbf{0}$ and variance $\boldsymbol{\Omega}$. In addition to time varying coefficients, stochastic volatility in the *VAR* innovations $\boldsymbol{\varepsilon}_{\mathbf{x}t}$ adds further dynamics to the model.[10] We assume that $\boldsymbol{\varepsilon}_{\mathbf{x}t}$ can be expressed as:

$$\boldsymbol{\varepsilon}_{\mathbf{x}t} = \mathbf{V}_t^{1/2} \boldsymbol{\xi}_t,$$

where $\boldsymbol{\xi}_t$ is a standard normal vector, which we assume to be independent of parameters innovation $\mathbf{v}_t$ ($E(\boldsymbol{\xi}_t \mathbf{v}_s) = 0$, for all t, s). $\mathbf{V}_t$ is modeled as a multivariate stochastic

[8] This description is based on Cogey and Sargent (2005). The model specification estimated below follows Cogley and Sbordone (2008). More detail on the models and their estimation can be found in the original papers.

[9] This is also subject to a reflecting barrier that guarantees non-explosive roots for the *VAR* at every date.

[10] Cogley et al. (2010) extend further this framework to include stochastic volatility also in the parameter innovations v_t.

volatility process:

$$\mathbf{V}_t = \mathbf{B}^{-1}\mathbf{H}_t\mathbf{B}^{-1\prime}, \tag{3}$$

where $\mathbf{H}_t$ is diagonal and $\mathbf{B}$ is lower triangular. To represent permanent shifts in innovation variance, the diagonal elements of $\mathbf{H}_t$ are assumed to be independent, univariate stochastic volatilities that evolve as driftless geometric random walks:

$$\ln h_{it} = \ln h_{it-1} + \sigma_i \eta_{it}. \tag{4}$$

The innovations η_{it} have a standard normal distribution, are independently distributed, and are assumed independent of innovations $\mathbf{v}_t$ and $\boldsymbol{\xi}_t$.[11] The vector $\mathbf{x}_t$ includes a measure of inflation, and typically a measure of real activity and a policy variable. We report below estimates on a vector $\mathbf{x}_t$ which includes inflation, output growth, a measure of marginal costs and the short term interest rate, variables that comove with inflation in the structural model that we analyze later.[12]

Trend inflation is defined in terms of infinite horizon forecast: $\bar{\pi}_t = \lim_{j\to\infty} E_t \pi_{t+j}$, following Beveridge and Nelson (1981). It is therefore the level to which inflation is expected to settle after short-run fluctuations die out. To compute such measure, we use the companion-form notation of (1):

$$\mathbf{z}_t = \boldsymbol{\mu}_t + \mathbf{A}_t \mathbf{z}_{t-1} + \varepsilon_{\mathbf{z}t},$$

where the vector $\mathbf{z}_t = (\mathbf{x}_t, \mathbf{x}_{t-1}, ..., \mathbf{x}_{t-p+1})'$, the matrix $\mathbf{A}_t$ refers to the autoregressive parameters in $\boldsymbol{\vartheta}_t$, and the vector $\boldsymbol{\mu}_t$ includes the intercepts; we approximate $\bar{\pi}_t$ by calculating a local-to-date t estimate of mean inflation from the *VAR*:

$$\bar{\pi}_t = \mathbf{e}_\pi'(\mathbf{I} - \mathbf{A}_t)^{-1}\boldsymbol{\mu}_t, \tag{5}$$

where $\mathbf{e}_\pi'$ is a selection vector that picks up inflation from the vector $\mathbf{z}_t$. This definition implies that, to a first-order approximation, inflation evolves as a driftless random walk.

Figure 3 reports the estimate of trend inflation obtained from this model with quarterly US data from 1960Q1 through 2012Q4. The thin black line in the Figure is actual inflation and the thick line is the median estimate of trend inflation at each date (all expressed at annual rates). The estimates are conditioned on data through the end of the sample. As the Figure shows, trend inflation rose from slightly above 2 percent in the early 1960s to around 5 percent in the 1970s, then fell to just about 2 percent at the end of the sample. This path is also largely consistent with long-term inflation expectations derived from survey data. For example, the correlation between our estimate of trend inflation and the 10-year inflation expectations from the Survey of Professional Forecasters (computed from 1981) is 0.96. A time-varying inflation trend implies that the inflation gap measured as deviation of inflation from the time-varying trend is quite

[11] The factorization in (3) and the log specification in (4) guarantee that $\mathbf{V}_t$ is positive definite, while the free parameters in $\mathbf{B}$ allow for time-varying correlation among the *VAR* innovations $\boldsymbol{\varepsilon}_{\mathbf{x}t}$.

[12] This is indeed the specification in Cogley and Sbordone (2008). Inflation is computed from the implicit deflator of GDP for the nonfarm business sector, output growth is the rate of growth of GDP of the non farm business sector, marginal costs is approximated by unit labor costs for the nonfarm business sector, and the short term interest rate is the effective federal funds rate. The calibration of the priors for the *VAR* parameters and the estimation procedure follow the original paper, to which we refer for details.

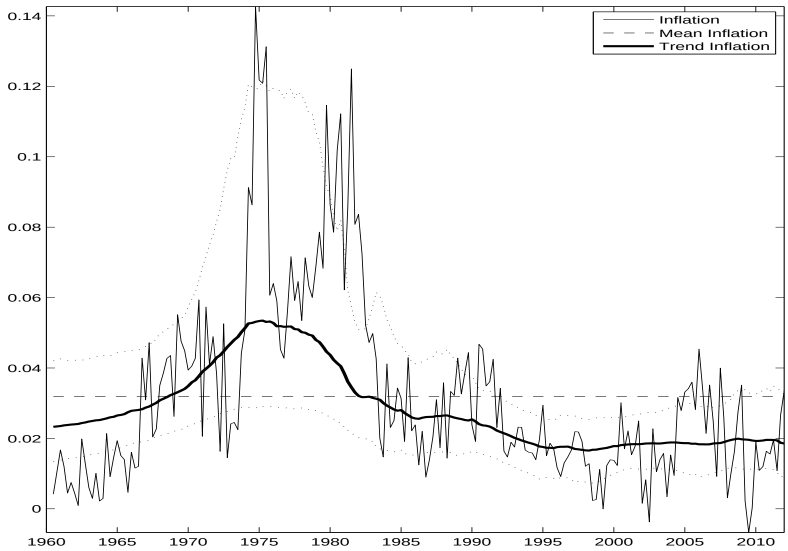

Figure 3: Inflation, Mean Inflation and Trend Inflation

different from deviations of inflation from a constant mean.[13] In particular, the persistence of these two series is different: in the Figure there are long runs at the beginning, middle, and end of the sample when inflation does not cross the mean line, while it crosses the trend line more often, especially after the Volcker disinflation.

The Figure also shows, however, that there is a lot of uncertainty about the level of trend inflation at any given date, as the marginal 90 percent credible sets at each date, displayed as dotted lines in the Figure, show.[14]

Table 1 summarizes the autocorrelation of the inflation gap. The first row refers to the inflation gap computed as deviation from the mean, the second to the inflation gap measured by subtracting the median estimate of trend inflation from actual inflation. The autocorrelation of the mean-based gap hovers around 0.7-0.8 both for the whole sample and for the two subsamples. The persistence of the trend-based gap instead, while still elevated for the period before the Volcker disinflation, drops substantially afterwards, to around 0.35.[15]

[13] As we we will discuss later in this survey, deviation from a constant mean, reflecting the assumption of a constant steady-state rate of inflation, are those typically analyzed in conventional versions of the *NKPC*.

[14] A credible set is a Bayesian analog to a confidence interval. The marginal credible sets portray uncertainty about the location of $\bar{\pi}_t$ at a given date. The estimates of the structural parameters we present later take this uncertainty into account because they are based on the entire posterior sample for trend inflation, not just on the mean or median path.

[15] These results are consistent with Kozicki and Tinsley (2002), among the first to point out the importance of shifts in trend inflation to assess the persistence of inflation. From the sum of the autoregressive coefficients of an AR(4) model fit to inflation data for the 1962-2001 period and for various subsamples they found that persistence in the trend-based gap is generally lower, and declined in the recent subsamples.

Table 1: Autocorrelation of Inflation Gaps			
	1960-2012	1960-1983	1984-2012
Mean-Based Gap	0.875	0.855	0.718
Trend-Based Gap	0.727	0.785	0.352

To analyze further the decline in the persistence of the trend-based gap we use the statistical measure introduced by Cogley et al. (2010). They compute the fraction of the total variation in the inflation gap that is due to shocks inherited from the past, relative to those that will occur in the future. Specifically, defining the following forecasting model for the vector of gap variables:

$$(\mathbf{z}_{t+1} - \overline{\mathbf{z}}_t) = \mathbf{A}_t(\mathbf{z}_t - \overline{\mathbf{z}}_t) + \varepsilon_{z,t+1},$$

(where $\overline{\mathbf{z}}_t$ is approximated by $\overline{\mathbf{z}}_t \approx (\mathbf{I} - \mathbf{A}_t)^{-1}\boldsymbol{\mu}_t$), the j-period ahead forecasts of the gap variables are approximated by $\mathbf{A}_t^j\ \widehat{\mathbf{z}}_t$ where $\widehat{\mathbf{z}}_t = \mathbf{z}_t - \overline{\mathbf{z}}_t$.[16] The forecast-error variance of $\widehat{\mathbf{z}}_{t+j}$ is then approximated by:

$$var_t(\widehat{\mathbf{z}}_{t+j}) \approx \Sigma_{h=0}^{j-1} \left(\mathbf{A}_t^h\right) var\left(\varepsilon_{z,t+1}\right) \left(\mathbf{A}_t^h\right)'. \tag{6}$$

The unconditional variance of $\widehat{\mathbf{z}}_{t+1}$, is approximated by taking the limit of the conditional variance as the forecast horizon j increases:

$$var(\widehat{\mathbf{z}}_{t+1}) \approx \Sigma_{h=0}^{\infty} \left(\mathbf{A}_t^h\right) var\left(\varepsilon_{z,t+1}\right) \left(\mathbf{A}_t^h\right)',$$

and by virtue of the anticipated utility approximation, this is also the unconditional variance of $\widehat{\mathbf{z}}_{t+s}$, for $s > 1$. Inflation persistence at any given date t is then defined as the fraction of the total variance of the inflation gap due to shocks inherited from the past. The rationale is that, if past shocks die out fast, persistence is weak, and this measure will converges to 0 rapidly. If past shocks explain instead a high proportion of the variation of future inflation gaps, then inflation gap persistence is high. Cogley et al. (2010) call this measure of persistence R_{jt}^2 (being akin to an R-square statistic for j-step ahead forecasts) and compute it as 1 minus the fraction of total variation due to future shocks, where the latter can be expressed as the ratio of conditional to unconditional variance (since future shocks account for the forecast error):

$$R_{jt}^2 = 1 - \frac{var_t(\mathbf{e}_\pi'\widehat{\mathbf{z}}_{t+j})}{var(\mathbf{e}_\pi'\widehat{\mathbf{z}}_{t+j})} \approx 1 - \frac{\mathbf{e}_\pi'\Sigma_{h=0}^{j-1}\left(\mathbf{A}_t^h\right) var\left(\varepsilon_{z,t+1}\right)\left(\mathbf{A}_t^h\right)'\mathbf{e}_\pi}{\mathbf{e}_\pi'\Sigma_{h=0}^{\infty}\left(\mathbf{A}_t^h\right) var\left(\varepsilon_{z,t+1}\right)\left(\mathbf{A}_t^h\right)'\mathbf{e}_\pi}. \tag{7}$$

The R_{jt}^2 statistic (by definition between 0 and 1) converges to 0 as the forecast horizon j lengthens, and its degree of convergence indicates the degree of persistence. Cogley et al. (2010) underline how this statistic is more informative than typical statistics used to summarize persistence in VARs, such as the largest autoregressive root in A_t, because it retains all information in A_t, but at the same time it is sensitive to changes in the conditional variance V_t. For example, they note, this measure of persistence would decline if the composition of the structural shocks would change towards a predominance of those shocks for which the impulse response declines more rapidly.

[16] Note the use of the anticipated utility approximation to have $E_t\overline{z}_{t+j} = \overline{z}_t$.

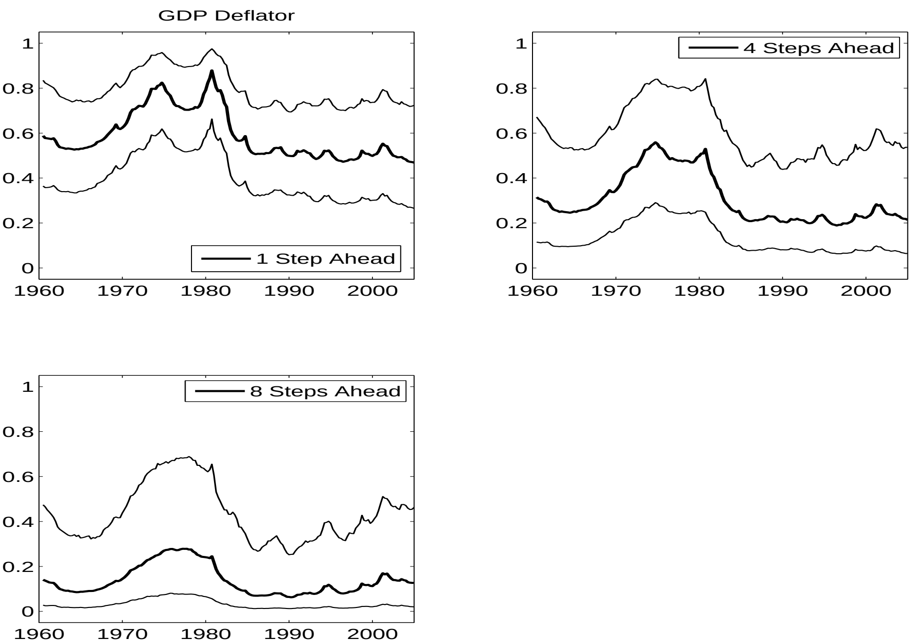

Figure 4: R^2_{jt} statistics

Computing such measure on our estimated VAR, we find, as in Cogley et al. (2010), that inflation innovations account for a small fraction of the unconditional variance of inflation, implying that most of the volatility is in the trend component of inflation. Furthermore, the R^2_{jt} statistics for one, two, and eight quarter ahead, reported in Figure 4, overall suggest that inflation gap persistence was higher in the Great Inflation period and lower after mid-'80s.[17]

To illustrate whether the decline in persistence is statistically significant, we follow again Cogley et al. (2010) by considering the joint posterior distribution of the R^2_{1t} statistic across two pairs of time periods: 1960 and 1980 and 1980 and 2012.[18] We find that for both pairs the draws from the posterior distribution fall mostly on one side of the 45-degree line; in the upper graph of Figure 5, which compares 1980 and 2012, the draws lie below the line ($R^2_{1,1980} > R^2_{1,2012}$) indicating that with high probability inflation persistence went down from 1980 to 2012. By converse, the draws from the pairs 1960 and 1980 lay mostly above the 45-degree line ($R^2_{1,1980} > R^2_{1,1960}$), implying that inflation persistence likely went up from 1960 to 1980. The evidence we obtain is not as sharp as that reported in Cogley et al. (2010), where almost the entire distribution of the pairs collapses away from the 45 degree line, but nonetheless provides clear evidence of a

[17] The figure reports the posterior median and the interquartile range for R^2_{jt} at each date t, for $j = 1, 4, 8$ quarters ahead. By construction the statistic is lower the longer is the forecast period.

[18] Using the beginning and the end of the sample period (in our case respectively 1960 and 2012) as well as the middle of the period, which represents the high point of the statistic and coincides with the eve of the Volcker disinflation, is a way of capturing the points where persistence has likely changed.

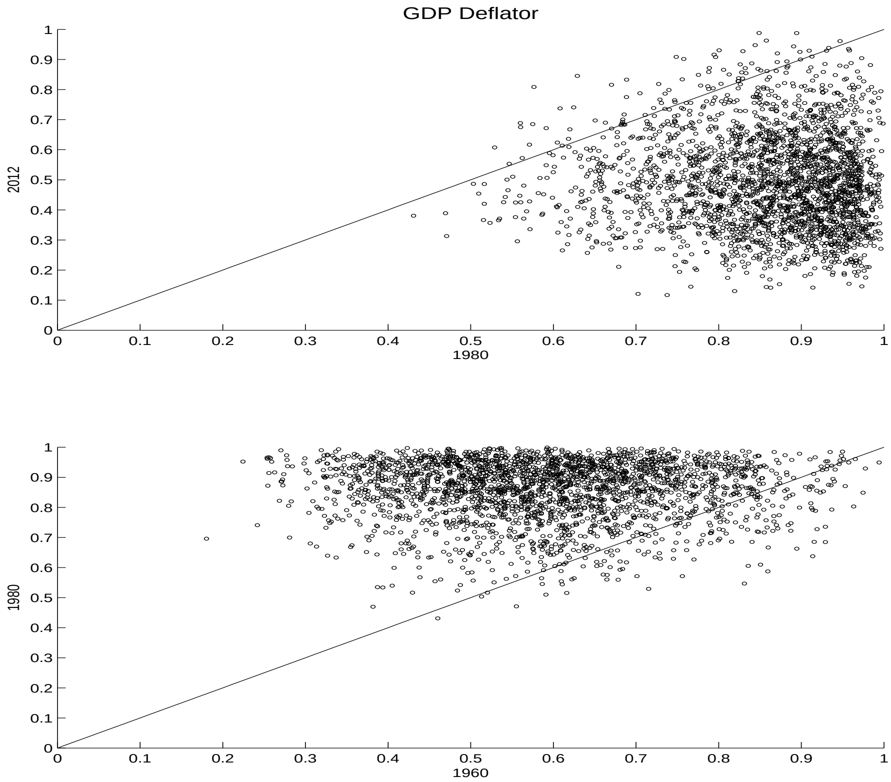

Figure 5: Joint distribution of R_1^2 statistics: 1960-80 and 1980-2012

change in persistence across periods, validating the inference from the serial correlation statistics reported in the table.

2.2 Persistence in structural models

The debate on inflation persistence affects the formulation of structural models in at least two distinct ways. The first is the form of the aggregate supply or Phillips curve relationship. Those who argue that textbook forward-looking new Keynesian models fail to account for inflation persistence typically include backward components to better fit the data. The literature offers many examples of hybrid curves, for example, based on some ad hoc indexation assumption. Alternatively, lagged inflation terms are justified by modifying the standard assumptions underlying the new Keynesian model by introducing state-contingent pricing (e.g. Dotsey et al., 1999, Wolman, 1999) or departing from the rational expectations assumption (e.g. Erceg and Levin, 2003, Milani, 2005, 2007). The previous discussion suggests, however, that a forward looking model may indeed account well for inflation dynamics, once one correctly specifies the object of study to be the dynamics of the *trend-based* inflation gap, rather than the mean-based gap; this is because the persistence of the inflation gap, as we discussed, is significantly lower than that of overall inflation, and has also likely declined over time.

To investigate inflation persistence in a structural model, we now introduce a *NKPC* which accounts for trend inflation. We build this relationship from firms' optimal price-setting behavior, discuss briefly the important features of the model, and show an assessment of its empirical fit. The microfoundations of our *GNKPC* are those of the Calvo-Yun model,[19] where price setting firms face random intervals between price adjustments. This model is the central feature of the small general equilibrium model that we build in the section 3 to analyze the implications of positive trend inflation for the evaluation of monetary policy.

2.2.1 The Calvo-Yun price setting model

In each period t, a final good Y_t is produced by perfectly competitive firms, which combine a continuum of intermediate inputs $Y_{i,t}$, $i \in [0,1]$, via the technology:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{8}$$

where $\varepsilon > 1$ is the elasticity of substitution among intermediate inputs. Profit maximization and the zero profit condition imply that the price index associated with the final good Y_t is a CES aggregate of the prices of the intermediate inputs $P_{i,t}$:

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}, \tag{9}$$

and the demand schedule for intermediate input $Y_{i,t}$ is:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t. \tag{10}$$

Intermediate inputs are produced by a continuum of firms with a simple linear technology in labor, which is the only input of production:

$$Y_{i,t} = A_t N_{i,t}, \tag{11}$$

where A_t is a stationary process for aggregate technology.[20] Due to the assumption of constant return to scale technology and assuming that nominal wages are set in perfectly competitive markets, real marginal costs of firm i, $MC_{i,t}$, depend only on aggregate variables and thus are the same across firms:

$$MC_{i,t} = MC_t = \frac{W_t}{A_t P_t}. \tag{12}$$

Intermediate firms' price setting problem - Imperfect substitutability generates market power for intermediate goods producers, that are thus price-setters. We assume random intervals between price resets: in each period a firm can re-optimize

[19] The model was introduced by Calvo (1983) and the discrete time version we use here was first introduced by Yun (1996).

[20] The on line Appendix presents the more general case of decreasing returns to labor.

its nominal price with fixed probability $1-\theta$, while with probability θ it maintains the price charged in the previous period.[21] The problem of firm i which sets price at time t is to choose price $P_{i,t}^{*}$ to maximize expected profits:

$$E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} \left[\frac{P_{i,t}^{*}}{P_{t+j}} Y_{i,t+j} - TC_{t+j}\left(Y_{i,t+j}\right) \right], \tag{13}$$

subject to the demand function (10). $\mathcal{D}_{t,t+j}$ is a stochastic discount factor and $TC_{t+j}\left(Y_{i,t+j}\right) = \frac{W_{t+j}}{P_{t+j}} \frac{Y_{i,t+j}}{A_{t+j}}$ is the total cost function. Letting $p_{i,t}^{*}$ denote the relative price of the optimizing firm at t ($p_{i,t}^{*} = P_{i,t}^{*}/P_t$), the first order condition of this problem can be written as:

$$p_{i,t}^{*} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} Y_{t+j} \Pi_{t,t+j}^{\varepsilon} MC_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} Y_{t+j} \Pi_{t,t+j}^{\varepsilon-1}} \tag{14}$$

where $\Pi_{t,t+j}$ represents the *cumulative gross inflation rate* over j periods:

$$\Pi_{t,t+j} = \begin{cases} 1 & \text{for } j = 0 \\ \left(\frac{P_{t+1}}{P_t}\right) \times \cdots \times \left(\frac{P_{t+j}}{P_{t+j-1}}\right) = \Pi & \text{for } j = 1, 2, \cdots \end{cases}.$$

In what follows we denote the gross inflation rate by $\pi_t = \frac{P_t}{P_{t-1}}$.

Note that future expected inflation rates enter both the numerator and the denominator in (14), affecting the relative weights on future variables. The numerator is the present discounted value of future marginal costs. Forward-looking firms know that they may be stuck with the price set at t while inflation will erode over time their markup, hence they discount future marginal costs taking into account future expected inflation rates. Higher these future expected inflation rates, higher the relative weights on expected future marginal costs; firms become effectively more forward-looking, weighting the future more relatively to present economic conditions.

Note also that equation (14) in steady state is:

$$p_i^{*} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \left(\beta\theta\bar{\pi}^{\varepsilon}\right)^j MC}{E_t \sum_{j=0}^{\infty} \left(\beta\theta\bar{\pi}^{\varepsilon-1}\right)^j}, \tag{15}$$

where p_i^{*} is the steady state value of the relative price $p_{i,t}^{*}$, $\bar{\pi}$ is steady state (trend) inflation, β is the steady state value of the stochastic discount factor $\mathcal{D}_{t,t+j}$ and MC is the steady state value of real marginal cost. Hence, the model constrains the feasible inflation rate in steady state: if steady state inflation is positive (i.e., $\bar{\pi} > 1$), for the sums in (15) to converge, it must be that $\beta\theta\bar{\pi}^{\varepsilon-1} < 1$ and $\beta\theta\bar{\pi}^{\varepsilon} < 1$. This implies upper bounds on trend inflation: $\bar{\pi} < (1/\theta\beta)^{1/(\varepsilon-1)}$ and $\bar{\pi} < (1/\theta\beta)^{\frac{1}{\varepsilon}}$.[22]

The aggregate price level evolves according to:

$$P_t = \left[\int_0^1 P_{i,t}{}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_{i,t}^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{16}$$

[21] In the baseline model we do not assume indexation of prices to past inflation and/or to trend inflation, an assumption we find counterfactual. We discuss indexation in Section 3.6.1.

[22] For somewhat standard calibration values, $\theta = 0.75$, $\beta = 0.99$, $\varepsilon = 10$, those bounds would be 14.1% and 12.6% annual rates, respectively.

2.2.2 The *GNKPC*

Most studies at this point take a log-linear approximation of the firms' equilibrium conditions and the aggregate price relation around a steady state characterized by zero inflation, obtaining an expression of the type:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \, \widehat{mc}_t \tag{17}$$

where hatted variables denote log-deviations from steady state values (for any variable $x_t : \hat{x}_t = \ln(x_t/\bar{x})$) and the coefficient of the marginal cost κ is a combination of the parameters governing the price setting problem: $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}$. Here we depart from this practice and instead log-linearize the equilibrium conditions around a steady state characterized by a shifting trend inflation and, with usual manipulations, derive a version of the *NKPC* which can be written as:[23]

$$\begin{aligned}\hat{\pi}_t \;=\;& \kappa(\boldsymbol{\psi}, \bar{\pi}_t)\;\widehat{mc}_t + b_1(\boldsymbol{\psi}, \bar{\pi}_t) E_t \hat{\pi}_{t+1} + b_2(\boldsymbol{\psi}, \bar{\pi}_t) E_t \sum_{j=2}^{\infty} \varphi(\boldsymbol{\psi}, \bar{\pi}_t)^{j-1} \hat{\pi}_{t+j} \qquad (18)\\ & + b_3(\boldsymbol{\psi}, \bar{\pi}_t) E_t \sum_{j=0}^{\infty} \varphi(\boldsymbol{\psi}, \bar{\pi}_t)^{j} \widehat{\mathcal{D}}_{t+j,t+j+1} + u_t.\end{aligned}$$

This equation differs from conventional versions of the *NKPC* in two respects. First, all the coefficients are non-linear functions of trend inflation $\bar{\pi}_t$ and the parameters of the pricing model, which we collected in a vector denoted by $\boldsymbol{\psi}$; even though these parameters are assumed to be constant, the coefficients of equation (18) drift when trend inflation drifts. In particular, higher trend inflation implies a lower weight on current marginal cost (in the plane $(\hat{\pi}_t, \widehat{mc}_t)$ the short-run *NKPC* flattens) and a greater weight on expected future inflation. Second, a number of additional variables appear on the right-hand side of (18).[24] These include higher-order leads of expected inflation, and terms involving the discount factor $\widehat{\mathcal{D}}_t$. Excluding these variables when estimating traditional Calvo equations would result in omitted-variable bias on the estimated coefficients if the omitted terms are correlated with those variables. The standard $NKPC$ emerges as a special case when steady-state inflation is zero ($\bar{\pi}_t = 1$ for all t). In those cases, $b_2(\boldsymbol{\psi},\bar{\pi}_t) = b_3(\boldsymbol{\psi},\bar{\pi}_t) = 0$, while the other coefficients collapse to those of the standard model (17).

Furthermore, from (15) and (30) evaluated in steady state one derives a restriction between trend inflation and steady-state marginal cost:

$$\left[\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \left[\frac{1-\theta\;\beta\bar{\pi}_t^{\varepsilon}}{1-\theta\;\beta\bar{\pi}_t^{\varepsilon-1}}\right] = \frac{\varepsilon}{\varepsilon-1}\overline{mc}_t. \tag{19}$$

[23] Following the derivation in Cogley and Sbordone (2008), to obtain all future values of inflation in the equation we have evaluated multi-step expectations when parameters drift under the 'anticipated utility' approximation (i.e. agents treat drifting parameters as if they would remain constant at the current level going forward in time). The online Appendix contains a derivation of the equation.

[24] We also included the error term u_t to account for the fact that this equation is an approximation and to allow for other possible mis-specifications. In the estimation we assume that u_t is a white noise process.

2.2.3 Estimation of the *GNKPC*

To investigate the importance of trend inflation for assessing inflation persistence, Cogley and Sbordone (2008) formulate and take to the data a slightly more general form of the *GNKPC* (18). Specifically, they assume that firms that do not optimally reset prices nonetheless change their price by indexing it to past inflation.[25] Prices that are *not* set optimally thus evolve according to $P_{i,t} = \pi_{t-1}^{\varrho} P_{i,t-1}$,where $\varrho \epsilon [0,1]$ measures the degree of indexation. In this more general formulation the coefficients of (18) depend also on the parameter ϱ, which is included in the vector $\boldsymbol{\psi}$.[26] Importantly, the more general formulation introduces a backward-looking term in the *GNKPC* and therefore allows testing for the presence of 'intrinsic persistence' in inflation dynamics.

The estimation method exploits a set of cross-equation restrictions between the parameters of the Calvo model and those of a reduced-form vector autoregression with drifting parameters, where the latter takes the form discussed in 2.1. The intuition is that if inflation is determined according to the *GNKPC*, the *VAR* should also satisfy a collection of nonlinear cross-equation restrictions, embedded in (18) which relates the cyclical components of inflation and marginal cost, and (19) which constrains the evolution of their steady-state values. These relations involve non-linear combinations of the underlying parameters of the Calvo model, collected in the vector $\boldsymbol{\psi}$, which is now defined as $\boldsymbol{\psi} = [\theta,\varepsilon,\ \varrho]'$, where ϱ indicates the degree of indexation.

Following Cogley and Sbordone (2008), we perform the estimation of $\boldsymbol{\psi}$ with a two-step procedure. First, we fit to the data an unrestricted reduced-form *VAR* of the type described in 2.1 in order to estimate trend inflation. Then, conditional on those estimates, we estimate the vector of parameters $\boldsymbol{\psi}$ as:

$$\widehat{\boldsymbol{\psi}} = \arg\min \mathcal{F}\left(\widehat{\boldsymbol{\mu}}_t, \hat{\mathbf{A}}_t, \boldsymbol{\psi}\right)' \mathcal{F}\left(\widehat{\boldsymbol{\mu}}_t, \hat{\mathbf{A}}_t, \boldsymbol{\psi}\right). \tag{20}$$

where the function $\mathcal{F}\left(\widehat{\boldsymbol{\mu}}_t, \hat{\mathbf{A}}_t, \boldsymbol{\psi}\right)$ embeds the cross equation restrictions.[27]

Table 2 summarizes the second-stage estimates. Because the distributions are non-normal, we report in the table the median and 90 percent confidence intervals. As in Cogley and Sbordone (2008) (the only difference here is the use of an extended sample), the estimates accord well with microeconomic evidence and are reasonably precise.

Table 2: Estimates of the Structural Parameters

	θ	ϱ	ε
Median	0.593	0	9.32
90 percent Confidence Interval	(0.49,0.66)	(0,0.17)	(7.66,11.00)

[25] A 'hybrid' New Keynesian Phillips curve, including lagged inflation, was formulated by Galí and Gertler (1999) assuming the presence of rule-of-thumb firms. A similar hybrid curve is more frequently obtained in the literature assuming indexation, as in Christiano et al. (2005).

[26] The model estimated in Cogley and Sbordone is more general than (18) in few more respects: it assumes a concave production function, allows for output growth, and allows for the presence of strategic complementarity in price setting. The estimates we report below are obtained for this more general formulation.

[27] Note that here hats on vectors $\widehat{\psi}$, $\widehat{\boldsymbol{\mu}}_t$ and matrices $\hat{\mathbf{A}}_t$ indicate that they are estimated values. For a full description of the estimation method we defer to Cogley and Sbordone (2008).

In particular, there is no evidence of a significant backward component, as discussed in Cogley and Sbordone (2008). We interpret this result as due to the proper account, in this equation, of the dynamics of trend inflation, and as a cautionary note against relying on *NKPC* formulations with important backward-looking components for monetary policy analyses. We return to this issue in section 3.4.[28]

A further dimension in which trend inflation matters is in its effect on the coefficients of the *GNKPC,* which become time varying. We can evaluate these coefficients by combining the estimates in table 2 with the estimated trend inflation, illustrating the extent to which, as we noted above, the relationship is more forward-looking than standard *NKPCs*. Figure 6 portrays as solid lines the coefficients κ_t, b_{1t}, b_{2t} and b_{3t}, computed as in (18): their time variations reflect time variation in $\overline{\pi}_t$. By comparison, the dashed lines report the value of the same coefficients that would obtain under a standard approximation of the Phillips curve, i.e. assuming zero trend inflation. The figure shows that the weight on current marginal cost (upper left graph) varies inversely with trend inflation, while the forward-looking coefficients b_{it} $(i = 1, 2, 3)$ are all positively related to trend inflation and larger than in the standard approximation case; in particular, the weight on next period inflation expectations is always bigger than 1, and higher order expectations matter. Relative to the conventional approximation, current costs matter less while anticipations about the future matter more.

3 Trend inflation and monetary policy

We now embed the *GNKPC* in a simple general equilibrium model in order to illustrate the implications of a positive trend inflation for the effects of monetary policy. We assume in this Section that trend inflation is positive, but constant, and exogenously set at a value $\bar{\pi}$. We analyze how modifying the standard framework by allowing for a positive steady state inflation alters the dynamics of the model and the trade-offs confronted by policymakers. We show in particular that in this Generalized New Keynesian model (*GNK*) higher trend inflation is associated with a more volatile and unstable economy and tends to destabilize inflation expectations.

3.1 The baseline generalized New Keynesian (*GNK*) model

To build our baseline *GNK* model we first introduce the aggregate demand side and a monetary policy rule. We then complete the description of the supply side of the model by introducing a measure of price dispersion and showing how price dispersion increases marginal costs and reduces the output produced per unit of inputs. The cost of price dispersion in turn carries implications for the long-run relationship between inflation and output. Finally we present a log-linear approximation of the model around a steady state characterized by positive inflation, comparing it with a model approximated around a

[28] A recent paper by Barnes et al (2009) challenges this conclusion, though, arguing that it results from considering too simple a model of indexation. Under the assumption that non optimally reset prices are indexed to a weighted average of aggregate inflation of the past two periods, they obtain a *GNKPC* with two lags of inflation whose coefficients are estimated to be statistically significant, albeit quite small.

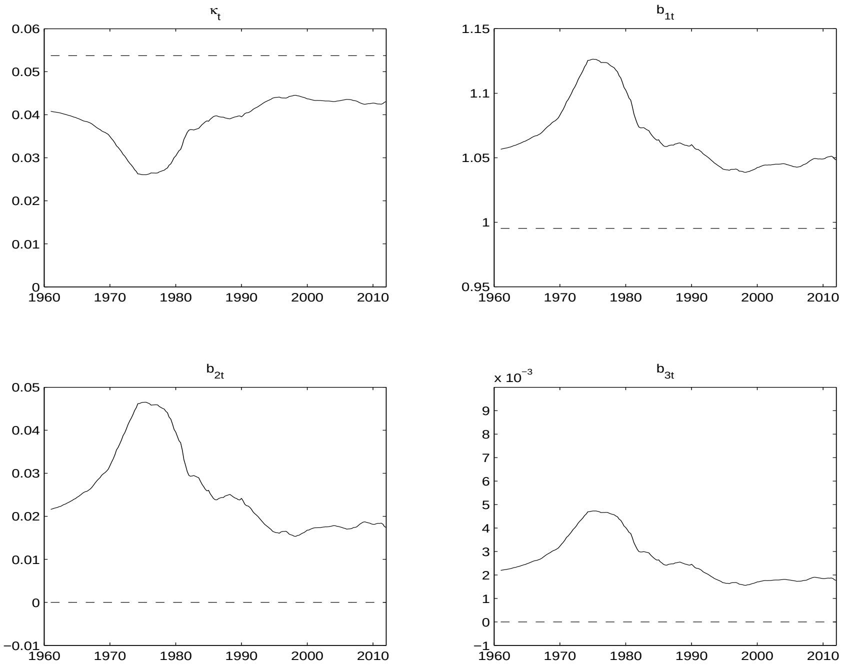

Figure 6: *GNKPC* coefficients

zero steady state inflation.[29]

The aggregate demand side. The demand side of the model features a representative household, who maximizes an intertemporal utility function, separable in consumption (C) and labor (N):

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - d_n e^{\varsigma_t} \frac{N_{t+j}^{1+\varphi}}{1+\varphi} \right], \tag{21}$$

subject to the period by period budget constraint:

$$P_t C_t + (1+i_t)^{-1} B_t = W_t N_t + D_t + B_{t-1}. \tag{22}$$

Here i_t is the nominal interest rate, B_t is holdings of a one-period bond, W_t is the nominal wage rate, and D_t is profits (distributed firms dividends). ς_t is a labor supply shock, σ is the intertemporal elasticity of substitution in consumption, and ϕ is the Frish elasticity of labor supply. The household optimization problem yields the following first-order conditions:

$$\text{Euler equation}: \quad \frac{1}{C_t^{\sigma}} = \beta E_t \left[\left(\frac{P_t}{P_{t+1}} \right) (1+i_t) \left(\frac{1}{C_{t+1}^{\sigma}} \right) \right], \tag{23}$$

[29] Our baseline *GNK* model with positive trend inflation is essentially the same as in Ascari and Ropele (2009), augmented to include the technology and the labour supply shocks and abstracting from indexation. See on line Appendix for the derivation.

$$\text{Labor supply equation} \quad \frac{W_t}{P_t} = d_n e^{\varsigma_t} N_t^{\varphi} C_t^{\sigma}. \tag{24}$$

The policy rule. We assume a very simple Taylor rule of the form:

$$\left(\frac{1+i_t}{1+\bar{\imath}}\right) = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_Y} e^{v_t}, \tag{25}$$

where $\bar{Y}$ is steady state output and v_t is a monetary policy shock.

Recursive formulation of the optimal price setting equation. The joint dynamics of the optimal reset price and inflation can be compactly described by rewriting the first order condition for the optimal price (14) as follows:

$$p_{i,t}^{*} = \frac{\varepsilon}{\varepsilon - 1}\frac{\psi_t}{\phi_t}, \tag{26}$$

where ψ_t and ϕ_t are auxiliary variables that allow to rewrite in recursive formulation the infinite sums that appear on the numerator and denominator of (14):[30]

$$\psi_t \equiv MC_t Y_t^{1-\sigma} + \theta\beta E_t\left[\pi_{t+1}^{\varepsilon}\psi_{t+1}\right], \tag{27}$$

and

$$\phi_t \equiv Y_t^{1-\sigma} + \theta\beta E_t\left[\pi_{t+1}^{\varepsilon-1}\phi_{t+1}\right]. \tag{28}$$

These variables can be interpreted as the present discounted value of marginal costs and marginal revenues (for a unit change in the optimal reset price), respectively. It is important to note the different exponents on future expected inflation rates in (27) and (28) that reflect the different elasticity of marginal costs and of marginal revenues to a change in the relative price in (13).[31]

We should also note that equation (16) implies:

$$p_{i,t}^{*} = \left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}}. \tag{30}$$

The role of price dispersion. Price dispersion affects the relationship between aggregate employment and aggregate output. From individual firms' production functions, aggregating over i, aggregate labor demand is derived as:

$$N_t = \int_0^1 N_{i,t}di = \int_0^1 \frac{Y_{i,t}}{A_t}di = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di. \tag{31}$$

[30] Here we substituted the stochastic discount factor from the Euler equation, i.e., $\mathcal{D}_{t,t+1} = \beta C_{t+1}^{-\sigma}/C_t^{-\sigma}$ and $C_t = Y_t$.

[31] This can be easily seen by rewriting (14) as

$$E_t\sum_{j=0}^{\infty}\theta^j \mathcal{D}_{t,t+j}Y_{t+j}\left(\frac{P_{i,t}^{*}}{P_t}\Pi_{t,t+j}^{\varepsilon-1} - \frac{\varepsilon}{\varepsilon-1}\Pi_{t,t+j}^{\varepsilon}MC_{t+j}\right) = 0 \tag{29}$$

Future inflation changes the relative price of the firms that can not modify their price. This change affects future marginal costs and marginal revenues in a different way, and relatively more the former (ε) than the latter ($\varepsilon - 1$).

Denoting by s_t the following measure of price dispersion:

$$s_t = \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di, \tag{32}$$

aggregate output is expressed as:

$$Y_t = \frac{A_t}{s_t} N_t. \tag{33}$$

Schmitt-Grohé and Uribe (2007) show that $s_t \geq 1$, being equal to one only if all the prices are the same, that is, if there is no price dispersion. Eq. (33) shows that s_t is a convenient variable to express the resource *cost* of price dispersion in this model:[32] higher the dispersion of relative prices, higher is s_t, hence higher is the labor input needed to produce a given amount of aggregate output. Thus it immediately follows that, for any given level of output, price dispersion increases the equilibrium real wage (see (24)) and hence the marginal cost of the firms (see (12)). Furthermore, one can derive that price dispersion is an inertial variable, which evolves as:[33]

$$s_t = (1-\theta) \left(p_{i,t}^* \right)^{-\varepsilon} + \theta \pi_t^{\varepsilon} s_{t-1}. \tag{34}$$

The complete non-linear model. In this simple baseline model there is no capital, and no fiscal spending, hence the aggregate resource constraint is simply given by $Y_t = C_t$. Using the latter to substitute for consumption, the non-linear model is described by the following equations: (12), (26), (27), (28), (30), (23), (24), (25), (33) and (34), which determine the evolution of the following variables: MC_t, p_{it}^*, ϕ_t, ψ_t, π_t, Y_t, N_t, s_t, i_t, w_t.

3.1.1 The cost of price dispersion and long-run implications of trend inflation

Price dispersion is probably the pivotal characteristic of price staggering models as it determines the costs of inflation. In an economy with moderate trend inflation, price dispersion is therefore an important first-order variable that matters when studying the long-run behavior, the short-run dynamic and the welfare properties of the class of *NK* models we are analyzing.

To illustrate how costly price dispersion is in our benchmark model we first derive the steady state price dispersion from (34), using expression (30) for the optimal price

[32] For brevity, we will also refer to s as price dispersion, because it is a model consistent index of price dispersion.

[33] The expression is obtained by observing that s_t can be written as

$$\begin{aligned} s_t = (1-\theta) \left[\frac{P_{i,t}^*}{P_t} \right]^{-\varepsilon} + \theta \pi_t^{\varepsilon} \times \\ \times \left\{ (1-\theta) \left[\frac{P_{i,t-1}^*}{P_{t-1}} \right]^{-\varepsilon} + \theta^2 (1-\theta) \left[\frac{P_{i,t-2}^*}{P_{t-1}} \right]^{-\varepsilon} + \cdots \right\}, \end{aligned}$$

where the expression in the curly brackets is exactly the definition of s_{t-1}.

$p^*_{i,t}$, evaluated in steady state:

$$s = \frac{1-\theta}{1-\theta\bar{\pi}^{\varepsilon}}\left(\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{35}$$

This expression shows that in a steady state with positive inflation price dispersion increases in θ, ε, and $\bar{\pi}$. Quite intuitively, higher is the average duration of prices, higher is price dispersion: if prices were completely flexible ($\theta = 0$), there would be no price dispersion ($s = 1$). Price dispersion is also increasing in ε, because the inefficient allocation among firms due to the distortion in relative prices is larger, the larger the elasticity of substitution among goods. Finally, trend inflation increases price dispersion by causing a greater difference between the price set by the resetting firms and the average price level.

As (33) shows, price dispersion is akin to a negative aggregate productivity shock (recall that $s_t \geq 1$), since it increases the amount of labor that must be employed to produce a given level of output. Hence we follow Damjanovic and Nolan (2010) and measure the cost of price dispersion by defining the variable $\tilde{A} = \frac{A_t}{s_t}$ as a measure of 'effective' aggregate productivity, and map a percentage *increase* in s into an equivalent percentage *decrease* in aggregate productivity $\tilde{A}$. Figure 7 shows this decrease in aggregate productivity as a function of θ, ε and $\bar{\pi}$.[34] Aggregate productivity is rapidly decreasing with trend inflation and with values of the price stickiness parameter above 0.75. Moreover, higher trend inflation makes the cost of price dispersion more sensitive to the value of the structural parameters θ and ε.[35]

Given such high aggregate costs, it would be desirable to measure the degree of price dispersion in the data.[36] Damjanovic and Nolan (2010) provide an indirect estimate of the productivity loss by mapping the price dispersion measure s_t into the coefficient of variation of prices ($cvar$).[37] From the approximation:

$$s \approx 1 + \frac{1}{2}\frac{\varepsilon}{\varepsilon+1}\frac{cvar^2}{1-\frac{1}{2}\frac{\varepsilon}{\varepsilon+1}\left(cvar^2+1\right)}, \tag{36}$$

given a value of ε, it is possible to calculate the cost of price dispersion and hence the percentage loss in aggregate productivity for a range of the coefficient of variation estimated in the data. For example, if the coefficient of variation is between 10 percent to 30 percent, as suggested by some empirical evidence (see reference therein), then the productivity loss ranges from 0.8 percent to 3.3 percent.

[34] Unless otherwise stated, in numerical computations we use these rather standard parameter values: $\beta = 0.99$, $\sigma = 1$, $\varepsilon = 10$, $\theta = 0.75$, $\varphi = 1$.

[35] Decreasing returns to labor (i.e. a firm technology $Y_{i,t} = A_t N_{i,t}^{1-\alpha}$) would make price dispersion more sensitive to the rate of trend inflation, hence would deliver a larger equivalent change in aggregate productivity (see on line Appendix). For example, assuming $\alpha = 0.3$, then if annual trend inflation goes from 0 to 4%, the cost of price dispersion increases by almost 9%, corresponding to an equivalent negative shock to aggregate productivity of 5.8%.

[36] Alvarez et al. (2012) is an important step in this direction.

[37] The coefficient of variation $cvar$ is defined as

$$cvar = \frac{\left[\int p^2(i)di - \left(\int p(i)di\right)^2\right]^{\frac{1}{2}}}{\int p(i)di}.$$

See Damjanovic and Nolan (2010) for details.

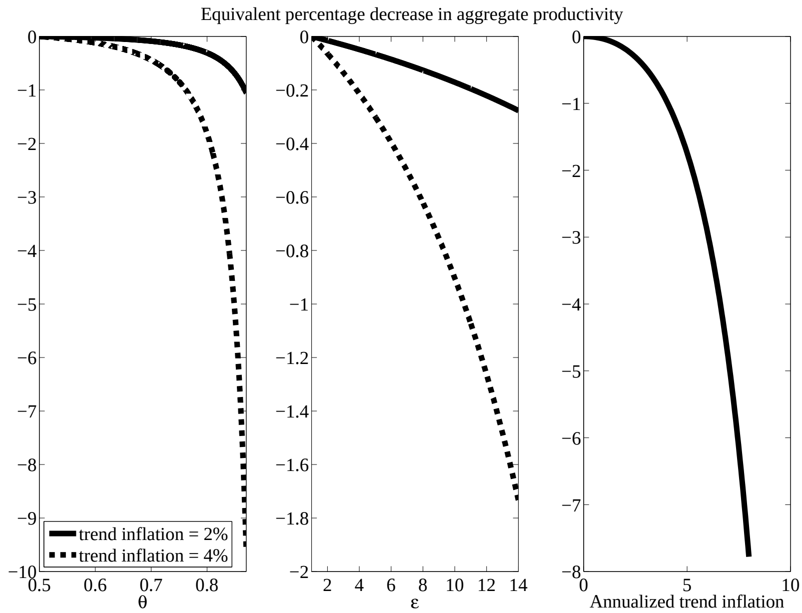

Figure 7: The cost of price dispersion

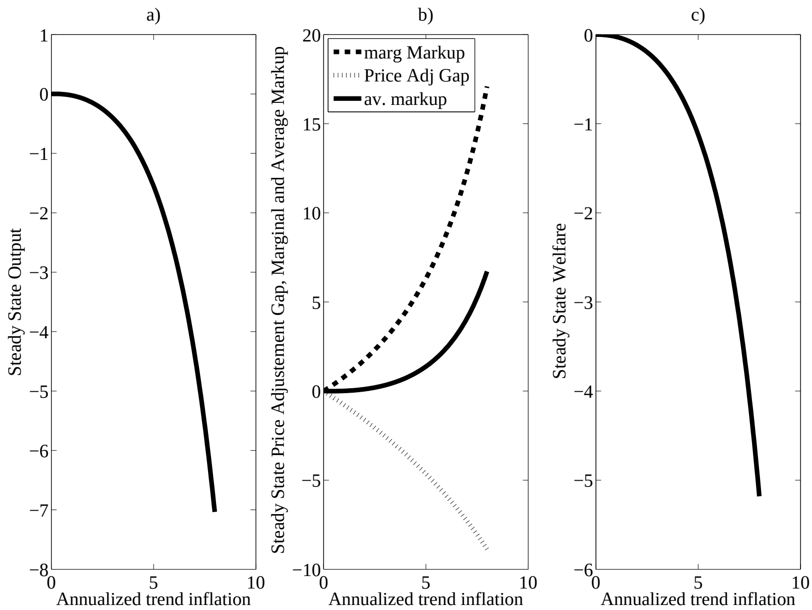

Figure 8: Trend inflation and steady state variables. Variables expressed as percentage deviation from the zero inflation steady state.

A loss in aggregate productivity in turn translates into an output loss, as shown in Figure 8. Panel a) in the figure displays steady state output as a function of trend inflation. Our stylized New Keynesian model implies that long-run superneutrality breaks down, and a negative long-run relationship emerges between inflation and output (King and Wolman, 1996, Ascari, 2004, Yun, 2005).[38] To grasp the intuition for this result, it is useful to use expressions (12) and (24), evaluated in steady state, to write the steady state output level as:

$$Y = \left(\frac{A^{\varphi+1}}{d_n} \frac{1}{s^{\varphi} \frac{1}{MC}} \right)^{\frac{1}{\varphi+\sigma}} = \left(\frac{A^{\varphi+1}}{d_n} \frac{1}{s^{\varphi} \mu} \right)^{\frac{1}{\varphi+\sigma}}, \tag{37}$$

where A is the steady state level of technology and $\mu \equiv \frac{1}{MC}$ is the steady state (real) markup. As the equation shows, the steady state level of output depends on trend inflation via price dispersion and the average markup. First, panel a) in Figure 8 mostly reflects the right panel of Figure 7, showing that the negative effect of trend inflation on output through price dispersion is quite powerful. Second, the average markup also increases with trend inflation (see panel b) in Figure 8). The intuition for this result can be best seen using the decomposition by King and Wolman (1996):

$$\mu = \left(\frac{P}{P_i^*} \right) \left(\frac{p_i^*}{MC} \right),$$

where the mark up is composed of a *price adjustment gap* $\left(\frac{P}{P_i^*} \right)$ and a *marginal markup* $\left(\frac{p_i^*}{MC} \right)$, which is the ratio of the newly adjusted price to marginal cost. The first term decreases with current inflation (see (30)) because inflation erodes the relative prices of firms that reset in past periods. Hence, higher the current inflation rate, more the newly set price is (mechanically) away from the average price level. The *marginal markup* increases with trend inflation in steady state, since from (15) it can be written as:

$$\frac{p_i^*}{MC} = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \beta\theta\bar{\pi}^{\varepsilon-1}}{1 - \beta\theta\bar{\pi}^{\varepsilon}}. \tag{38}$$

This is due to the forward-looking behavior of firms and the interactions of future expected inflation rates with the monopolistic competition framework. As we discussed in Section 3.1, future inflation rates affect relatively more marginal costs (ε) than marginal revenues ($\varepsilon - 1$) (see (14)). Thus the overall effect of trend inflation on the average markup depends on the elasticity of demand ε, a key parameter of the model. The *marginal markup* term in general dominates the *price adjustment gap* one,[39] as it can be seen in Figure 2(b), so that higher trend inflation yields a larger average markup, hence a larger economy's monopolistic distortion and a larger negative effect on steady-state output.

[38] See Karanassou et al., 2005, for an empirical appraisal of the long-run Phillips Curve , or Beyer and Farmer (2007) for a positive empirical relationship between inflation and unemployment. Vaona (2012) finds empirical support for a negative relationship between inflation and output growth.

[39] Actually, the price adjustment gap is stronger for extremely low values of trend inflation, such that average markup first slightly decreases and then increases with trend inflation (see King and Wolman, 1996).

It is worth noting another element that affects the marginal markup, and partially offsets the effects of trend inflation: discounting. The more firms discount the future, the less they are going to worry about the erosion of future markups: this implies a lower *marginal markup*[40], a lower aggregate markup and a higher steady state output.

Finally Panel c) in Figure 8 shows that welfare is also strongly decreasing with trend inflation in this simple *NK* model, both because trend inflation reduces steady state output and hence consumption, and because it increases the amount of labor needed to produce a given amount of output. We will analyze further these implications, when we discuss normative issues in Section 3.4.

3.1.2 The log-linearized *GNK* model

Much of the success of the standard *NK* model is due to the elegance of condensing a quite sophisticated microfounded model into three simple linear equations. The first two equations derive from the log-linear approximation of the Euler equation (23), imposing the resource constraint $Y = C$:

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma^{-1} \left[\hat{\imath}_t - E_t \hat{\pi}_{t+1} \right], \tag{39}$$

and the Taylor rule (25):

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + v_t. \tag{40}$$

The third is an *NKPC* that links inflation and output obtained by log-linearizing the supply side equations of the model around a zero inflation steady state. The *NKPC* (17) (and the *GNKPC* (18)) that we have discussed so far in Section 2.2.2 is however a relationship between inflation and marginal costs. It is straightforward to transform (17) into a relationship between inflation and output to get:[41]

$$\hat{\pi}_t = \lambda \hat{Y}_t + \beta E_t \hat{\pi}_{t+1} + \kappa \left(\varsigma_t - (\varphi + 1) \hat{A}_t \right), \tag{41}$$

where the slope is given by $\lambda \equiv \kappa (\sigma + \varphi)$.

While the first two equations still hold, to get a corresponding expression to (41) for the case of positive trend inflation is unfortunately more cumbersome. Under the assumption of constant trend inflation the *GNKPC* can be written in a recursive form, exploiting the recursive formulation (26)-(28) of Section 3.1. To do so, we combine (26) and (30) to obtain an expression for ϕ_t, which we substitute in (28). This gives a Phillips curve which comprises two equations describing respectively the dynamics of inflation and the evolution of the present discounted value of future marginal costs ψ_t. Log-linearizing these two equations around a constant trend inflation $\overline{\pi}$, we obtain (see the online Appendix):

$$\begin{aligned} \hat{\pi}_t = {} & \kappa (\overline{\pi}) \widehat{mc}_t + b_1 (\overline{\pi}) E_t \hat{\pi}_{t+1} \\ & + b_2 (\overline{\pi}) \left((1 - \sigma) \hat{Y}_t - E_t \hat{\psi}_{t+1} \right), \end{aligned} \tag{42}$$

[40] It is easy to show that the marginal markup increases with β if $\overline{\pi} > 1$ (see on line Appendix).

[41] The derivation is obtained substituting in (17) a log-linear approximation of the marginal cost expression (12), the labor supply equation (24), and the aggregate production function (33), with $\hat{s}_t = 0$.

and

$$\hat{\psi}_t = (1-\theta\beta\bar{\pi}^{\varepsilon})\left(\widehat{mc}_t + (1-\sigma)\hat{Y}_t\right) + (\theta\beta\bar{\pi}^{\varepsilon}) E_t\left(\hat{\psi}_{t+1} + \varepsilon\hat{\pi}_{t+1}\right). \tag{43}$$

where $\kappa(\bar{\pi}) \equiv \frac{(1-\theta\bar{\pi}^{\varepsilon-1})(1-\theta\beta\bar{\pi}^{\varepsilon})}{\theta\bar{\pi}^{\varepsilon-1}}$, $b_1(\bar{\pi}) \equiv \beta\left[1+\varepsilon\left(1-\theta\bar{\pi}^{\varepsilon-1}\right)(\bar{\pi}-1)\right]$ and $b_2(\bar{\pi}) \equiv \beta\left(1-\theta\bar{\pi}^{\varepsilon-1}\right)(1-\bar{\pi})$. Finally, to express the Phillips curve as an inflation-output relationship we substitute out the marginal cost, accounting for the price dispersion term in the aggregate production function: $\hat{Y}_t = \hat{A}_t + \hat{N}_t - \hat{s}_t$. Equations (42) and (43) then become, respectively:

$$\begin{aligned}\hat{\pi}_t &= \lambda(\bar{\pi})\hat{Y}_t + b_1(\bar{\pi})E_t\hat{\pi}_{t+1} + \kappa(\bar{\pi})\left(\varphi\hat{s}_t + \varsigma_t - (\varphi+1)\hat{A}_t\right)\\ &+ b_2(\bar{\pi})\left(\hat{Y}_t(1-\sigma) - E_t\hat{\psi}_{t+1}\right),\end{aligned} \tag{44}$$

and

$$\hat{\psi}_t = (1-\theta\beta\bar{\pi}^{\varepsilon})\left(\varphi\hat{s}_t + (\varphi+1)\left(\hat{Y}_t - \hat{A}_t\right) + \varsigma_t\right) + \theta\beta\bar{\pi}^{\varepsilon}E_t\left(\hat{\psi}_{t+1} + \varepsilon\hat{\pi}_{t+1}\right), \tag{45}$$

where $\lambda(\bar{\pi}) \equiv \kappa(\bar{\pi})(\sigma+\varphi)$, and the dynamic behavior of $\hat{s}_t$ is derived by log-linearizing (34):[42]

$$\hat{s}_t = \left[\frac{\varepsilon\theta\bar{\pi}^{\varepsilon-1}}{1-\theta\bar{\pi}^{\varepsilon-1}}(\bar{\pi}-1)\right]\hat{\pi}_t + [\theta\bar{\pi}^{\varepsilon}]\,\hat{s}_{t-1}. \tag{46}$$

The log-linearized *GNK* model is thus composed of the Euler equation (39), the policy rule (40), and three equations for the supply side, (44), (45), (46), which together describe the evolution of the variables $\hat{\pi}_t$, $\hat{\psi}_t$, $\hat{Y}_t$, $\hat{s}_t$, and $\hat{\imath}_t$.

Two things should be noted here. First, the supply side block of the *GNK* model includes now three dynamic equations (44), (45) and (46) rather than one, (41), as in the approximation around a zero steady state inflation. Setting $\bar{\pi} = 1$ in these three equations yields back the standard *NKPC* (41). Second, $\hat{s}_t$ is a backward-looking variable, so its inclusion adds inertia to the adjustment of inflation; moreover, trend inflation increases the weight on $\hat{s}_{t-1}$ yielding, *ceteris paribus,* a more persistent adjustment path for $\hat{s}_t$. The dynamics of the *GNK* model is thus richer, and, as we stressed in Section 2.2.2, its coefficients depend on the level of trend inflation.

3.1.3 The zero inflation steady state approximation

Despite the high output and welfare costs discussed in Section 3.1.1, price dispersion is generally considered a term of second order importance in linearized models, because the vast majority of papers in the literature log-linearize the model around a zero inflation steady state. There are two main reasons behind this choice: the first is that zero is the optimal long-run value for inflation for many specifications of the *NK* framework (see Goodfriend and King, 2001, Woodford, 2003, and the discussion in Section 3.5). The second, as shown above, is analytical convenience: the resulting model is very simple and tractable.

Zero steady state inflation, however, does not coincide with the concept of 'price stability' held by central bankers, who generally target a positive inflation rate. From

[42] We use (30) to substitute for $p^*_{i,t}$ in (34).

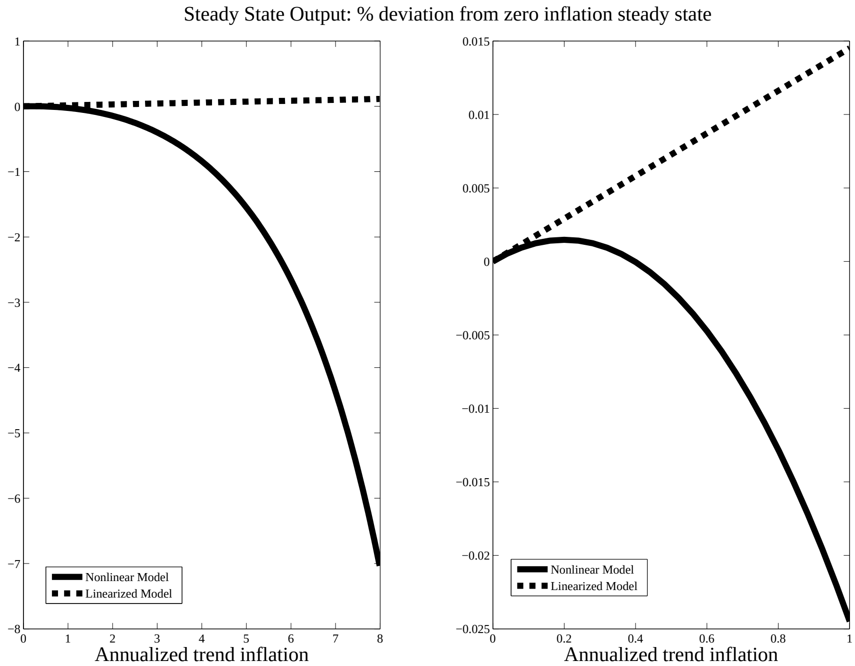

Figure 9: Relation between steady state inflation and output in the model linearized around a zero steady state and in the nonlinear model (the right panel plots the same curves as the left panel, but it zooms the Figure for trend inflation rates lower than 1%).

a theoretical perspective, assuming zero inflation in steady state eliminates some interesting effects that derive from the interaction of trend inflation, relative prices, and the monopolistic competition framework.

In Section 3.1.1, we distinguished three such effects: price dispersion, marginal markup, and discounting. When the model is log-linearized around a zero inflation steady state, the price dispersion effect is completely nullified at first order: price dispersion does not matter not only in steady state, but also in the dynamics of the model. Setting $\bar{\pi} = 1$ into (46), one gets:

$$\hat{s}_t = \theta \hat{s}_{t-1}. \tag{47}$$

Small perturbations have a zero first order impact on $\hat{s}_t$, because the model is log-linearized exactly around the point in which price dispersion is at its minimum. The marginal markup effect, by which price-setting firms take into account that future inflation will erode their relative prices and markup, is also muted, and would appear only in a second order approximation. Discounting interacts with trend inflation in opposite direction relative to the other two effects: with positive discounting output and inflation are positively linked in the long-run. The *NKPC* (41) in the long-run implies:

$$\hat{Y} = \frac{1-\beta}{\lambda} \hat{\pi}, \tag{48}$$

so that discounting makes the long-run Phillips curve positively sloped. Figure 9 shows, however, that this conclusion is an artifact due to the log-linearization around zero inflation. The Figure plots the relation between steady state output and inflation from (37) and the long-run Phillips Curve from the model log-linearized around a zero inflation steady state, (48):[43] except at extremely low rates of inflation, in the non-linear model the discounting effect is weak and easily dominated by the effects of price dispersion and marginal markup. At zero inflation, however, these two effects are absent at first order and the time-discounting generates a positive inflation-output relationship, represented by the tangent at zero to the non linear curve, whose slope at that point is obviously equal to $\frac{1-\beta}{\lambda}$.

Three main implications follows from this analysis. First, the fact that marginal markup and price dispersion effects cancel out under the zero inflation steady state assumption makes the model behave as if there is only one representative firm, hence price dispersion is no more an issue. Indeed, the model is first order observationally equivalent to the Rotemberg (1982) model, which assumes convex costs of price adjustment, with no staggering nor price dispersion.[44] The zero inflation steady state assumption is therefore not just a convenient simplification, but washes out some implications of the microfoundations of the Calvo model.

Second, a notable thing about the standard *NKPC* (17) or (41) is the absence of a role for the parameter ε. This stands in contrast with our emphasis in Section 3.1 on the effects of the interaction between trend inflation and the monopolistic competition framework, which naturally depends on the demand elasticity ε.[45]

Third, Figure 9 makes clear that the zero inflation steady state approximation is increasingly problematic for policy inference, the higher is the steady state rate of inflation. Of course, the extent by which a linear approximation worsens as one moves away from the approximation point depends on how nonlinear the original model is near that approximation point. The standard *NK* model is unfortunately especially nonlinear around the zero inflation steady state: moving the approximation point slightly away from zero inflation delivers a log-linearized model whose long-run and dynamic properties are quite different from those of the model log-linearized around zero inflation. In particular, the explicit account of a positive steady state inflation in the *GNK* model changes several results of the standard *NK* model about the determinacy of the rational expectation equilibrium, the dynamic response to shocks and the design of optimal policy. This is what we turn to next.

3.2 Trend inflation and the anchoring of expectations

Many economist have advocated an increase in central banks' inflation targets to respond to the current economic recession, while others, as Fed's Chairman Bernanke, counterargued that such a move could destabilize inflation expectations. Within our framework

[43] The graph on the right is a magnified version of the one of the left, to emphasize the behavior at very low rates of inflation. A similar figure appears in Yun (2005), Levin and Yun (2007) and Ascari and Merkl (2009).

[44] In this model firms change prices in every period and in the same way (see the discussion in Section 3.6.2).

[45] This can be seen by comparing κ and $\kappa(\bar{\pi})$. Note however that there are other modifications of the standard model, such as firm specific factors, or a different demand aggregator, under which the elasticity ε enters the slope of the standard model independently of the effect of price dispersion.

we can address to what extent a higher inflation target makes it more difficult for a central bank to anchor inflation expectations.

3.2.1 Rational expectations and determinacy

In the standard *NK* model, log-linearized around a zero inflation steady state, the economy can be simply described by the Euler equation (39), the *NKPC* (41), and the policy rule (40).[46] With this specification, one can show that a necessary and sufficient condition for the Rational Expectations equilibrium (REE) to be unique is:

$$\phi_\pi + \frac{(1-\beta)}{\lambda}\phi_Y > 1, \tag{49}$$

with ϕ_π, ϕ_Y both positive. As stressed by Bullard and Mitra (2002) and Woodford (2001, 2003, Ch. 4.2.2) among others, this condition is a generalization of the Taylor (1993) principle: the nominal interest rate should rise more than proportionally with respect to inflation, i.e., $\phi_\pi > 1$. Condition (49) states that in a *NK* model the Taylor principle should be amended according to the long-run properties of the model: the nominal interest rate should rise by more than the long run increase in inflation (as we saw in (48), $(1-\beta)/\lambda$ represents the long run multiplier of inflation on output in a standard *NKPC* log-linearized around the zero inflation steady state).[47] In other words, the Taylor principle should be understood as saying that:

$$\left.\frac{\partial \hat{\imath}}{\partial \hat{\pi}}\right|_{LR} = \phi_\pi + \phi_Y \left.\frac{\partial \hat{Y}}{\partial \hat{\pi}}\right|_{LR} > 1, \tag{50}$$

where LR stands for long run. The intuition is straightforward: a sunspot increase in inflation requires a more than proportional increase in the nominal interest rate in order to drive up the real interest rate. This in turns reduces output via the Euler equation, curbing the initial rise in inflation via the *NKPC*.[48] Figure 10 shows the determinacy region in the space of the policy parameters (ϕ_π, ϕ_Y). Panel (a), on the left of the Figure, portrays this region in the zero inflation steady state case, as a shaded area.[49]

The case of positive trend inflation is portrayed in panel (b). In this case, as we saw in Section 3.1.2, the log-linearized model comprises additional equations describing the dynamics of price dispersion $\hat{s}$, and of the variable $\hat{\psi}$; most importantly, the parameters of these equations are generally function of trend inflation. As a consequence,

[46] Since this section analyzes the indeterminacy properties of the model, we can abstract from the shocks.

[47] Hence, the left-hand side of (49) "represents the long-run increase in the nominal interest rate prescribed [...] for each unit of permanent increase in the inflation rate" (Woodford, 2003, p. 254). Therefore, "The Taylor principle continues to be a crucial condition for determinacy, once understood to refer to the *cumulative* response to a *permanent* inflation increase" (Woodford, 2003, p. 256, italics in the original).

[48] An intuitive analytical explanation is provided again by Woodford (2003). Indeed, (39), (40) and the standard *NKPC* continue to be satisfied if inflation, output and interest rates are increased at all dates by constant factors satisfying (50) with equality. This means that a real eigenvalue of value one corresponds to that equality.

[49] It is worth noticing that another condition needs to be satisfied: $(\phi_Y + \lambda\phi_\pi > \beta - 1)$. This condition is often neglected in the literature because it is always satisfied in the positive orthant of the space (ϕ_π, ϕ_Y).

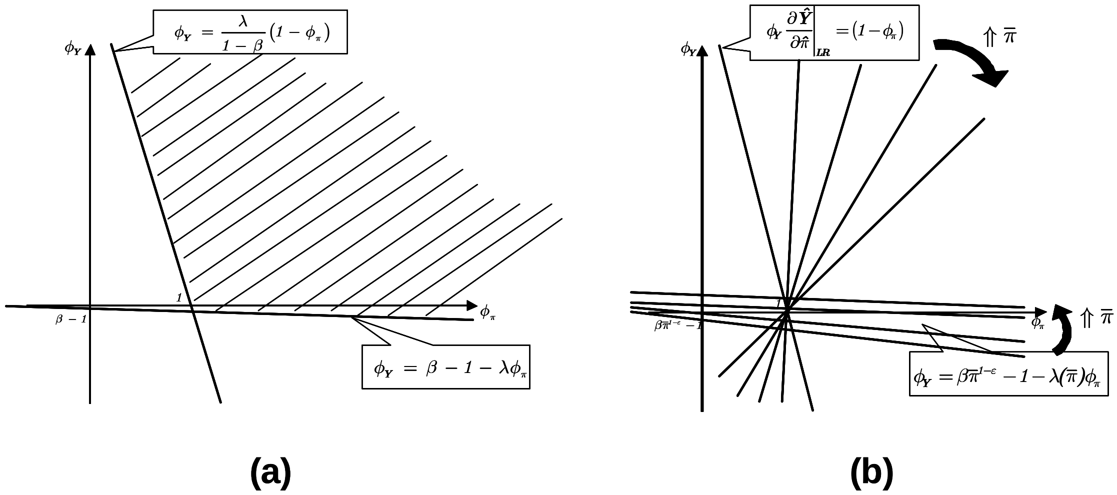

Figure 10: The determinacy region: (a) the zero inflation steady state case; (b) the positive trend inflation case

the determinacy region is affected by the particular value of trend inflation. Despite the higher order dynamics, under some simplifying assumptions it is possible to derive analytically the determinacy conditions, and show that the generalized Taylor principle (50) still holds.[50] The long run multiplier of inflation $\left.\frac{\partial \hat{Y}}{\partial \hat{\pi}}\right|_{LR}$, however, now depends on the trend inflation rate because it evidently coincides with the slope of the tangent to the non-linear model long-run Phillips Curve, shown in Figure 9. Hence, while positive and equal to $(1-\beta)/\lambda$ at zero inflation, it decreases as trend inflation increases, becoming negative for low values of trend inflation and increasingly so. The line that defines the generalized Taylor principle (50) therefore rotates clockwise, shrinking the determinacy region, as Figure 10(b) shows. Note also that the second condition moves the bottom line upward in the diagram, so that eventually it enters the positive orthant. As a consequence of the joint movement of the two lines, the minimum level of ϕ_π necessary to induce a unique REE increases with trend inflation. Again the intuition is straightforward. Since trend inflation makes price-setting firms more forward-looking, the inflation rate becomes less sensitive to variations in the current economic conditions (the short-run *GNKPC* flattens), hence monetary policy should respond more strongly in order to induce a reduction in output that achieves a given change in inflation.

Simulating the determinacy region for the benchmark calibration of the model (see footnote 34) yields Figure 11. In the baseline *GNK* model the determinacy region shrinks very rapidly with trend inflation, requiring a weaker policy response to output and a stronger response to inflation to guarantee determinacy as trend inflation increases. In the Figure, a cross is drawn at the classical values of the Taylor rule parameters, i.e., $\phi_\pi = 1.5$ and $\phi_Y = 0.5/4$, as in Taylor (1993). These values would result in an indeterminate REE for trend inflation bigger than 4 percent.

[50] The simplifying assumptions we use are log preference in consumption ($\sigma = 1$) and indivisible labor ($\varphi = 0$). In this case the second condition becomes $\phi_Y + \lambda(\bar{\pi})\phi_\pi > \beta\bar{\pi}^{1-\varepsilon} - 1$ (see Figure 10). See Ascari and Ropele (2009) for details.

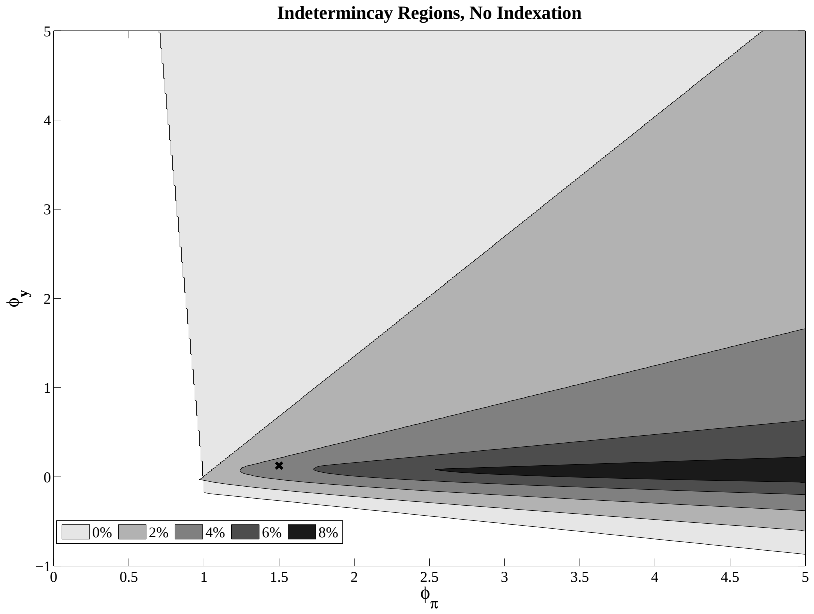

Figure 11: The determinacy region and trend inflation

Two general policy implications emerge from this analysis. First, a higher trend inflation tends to destabilize inflation expectations. Second, in a higher inflation targeting regime, monetary policy should respond more to inflation deviations from target and less to output gaps. Ascari and Ropele (2009) show that these two messages are qualitatively robust across different parameterizations and different types of Taylor rules (backward-looking, forward-looking and inertial policy).[51] As in the zero inflation steady state case, inertial policies tend to stabilize expectations also in the case of positive steady state inflation. Similarly, Taylor-type rules that respond to output growth rather than to the output gap widen the determinacy region, thus making easier for policy to induce a unique REE (Coibion and Gorodnichenko, 2011).

Coibion and Gorodnichenko (2011) provide an interesting empirical application of these results: they offer a reinterpretation of the economic instability of the period before the U.S. Great Moderation as due to a combination of high trend inflation and a mild policy response. A mainstream interpretation of the volatility of the Great Inflation period is that high inflation was due to the failure of monetary policy to satisfy the Taylor principle (Clarida et al., 2000). By contrast, in the post-Volker period monetary policy responded strongly to inflation (the policy parameter ϕ_π is estimated to be substantially bigger than one) bringing expectations back in line with a unique stable REE. In this interpretation, the 'conquest of American inflation' after the '80s is due to a switch from 'bad' to 'good' policy. Coibion and Gorodnichenko (2011) identify, however, another possible dimension of the 'bad vs good' policy argument: the substantial reduction in the inflation target. To show that, they first estimate a Taylor rule using real-time US

[51] The case of a backward-looking rule is somewhat different. See Ascari and Ropele (2009) for details.

data based on Greenbook forecast for two sub-samples: 1969–1978 and 1983–2002.[52] They then conduct some counterfactual experiments on the estimated policy coefficients of each of the two sub-samples and on the level of trend inflation, using two values that correspond to the average inflation in the two periods (6 percent and 3 percent, respectively).[53] In line with Clarida et al. (2000), they find that the economy was very likely to be in an indeterminate region according to the pre-'79 estimates, while the opposite is true according to the post-'82 estimates. However, contrary to Clarida et al. (2000), they also show that changing the policy rule coefficients from the pre-'79 estimates to the post-'82 ones, without at the same time reducing the target inflation rate, would not have been sufficient to rule out indeterminacy.[54] To dig further into this result, Coibion and Gorodnichenko (2011) estimate a time-varying parameters version of their Taylor rule, and recover a time-varying measure of trend inflation from the time-varying intercept of the rule. Moreover, from the estimated time series of the Taylor rule parameters and the estimated measure of trend inflation, they construct time series of the probability of determinacy for the US economy given the estimated distribution of the Taylor rule parameters. Figure 12 shows the implied evolution of these probabilities and nicely visualizes the two main messages of their work.[55] First, the US economy was very likely to be subject to indeterminacy from the early '70s until the start of the Volcker disinflation. Second, the reduction in trend inflation was an important component of the change in the probability of a unique REE. The two dashed lines are far apart, meaning that a level of average inflation of 3 percent rather than 6 percent matters in determining the likelihood of a determinate REE. Had policy switched to the 'good' post-'82 reaction coefficient without reducing average inflation to 3 percent, the US economy would have still been at the edge of the indeterminacy region.

3.2.2 Learning and expectations

Learning provides a natural framework to study the concern that a higher inflation target could unanchor inflation expectations.[56] Kobayashi and Muto (2011) and Ascari and Florio (2012) extend the analysis in Ascari and Ropele (2009) by introducing adaptive

[52] They drop the period from 1979 to 1982 in which the Fed officially abandoned interest rate targeting for monetary aggregate targeting. In addition to a response to inflation and output gap, their estimated Taylor rule features interest rate inertia and a response to output growth.

[53] For each sample period, they draw 10,000 times from the distribution of the estimated parameters of the Taylor rule and assess the fraction of draws that yield a determinate REE at 3% and at 6% trend inflation.

[54] In particular, it would have increased the likelihood of a determinate REE, but it would have still left a large uncertainty, because only 60 percent of draws from the distribution of estimated parameters from the Taylor rule predicted determinacy.

[55] Figure 12 reproduces the Figure 4 p. 363 in Coibion and Gorodnichenko (2011). The dashed (dotted) black line assumes a constant rate of trend inflation of 3 percent (6 percent), while the solid line uses the time-varying estimated measure of trend inflation. See Coibion and Gorodnichenko (2011) for details.

[56] A suggestion in that sense came from Chairman Bernanke: "What is the right conceptual framework for thinking about inflation expectations in the current context? [...] Although variations in the extent to which inflation expectations are anchored are not easily handled in a traditional rational expectations framework, they seem to fit quite naturally into the burgeoning literature on learning in macroeconomics. [...] In a learning context, the concept of anchored expectations is easily formalized." Fed Chairman Ben S. Bernanke, speech at the NBER Monetary Economics Workshop, July 2007.

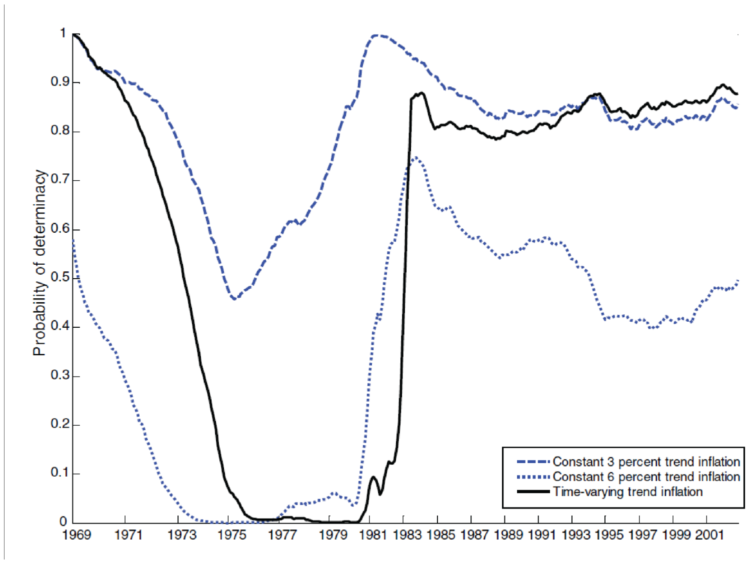

Figure 12: Coibion and Gorodnichenko (2011) Figure 4, p. 363: Probability of Determinacy Using Time-Varying Response Function by the Federal Reserve

learning à la Evans and Honkapohja (2001)[57] in the baseline *GNK* model with trend inflation. As it is known in the learning literature, when the perceived law of motion of the agents converges to the actual law of motion of the economy, the REE is said to be learnable or E-stable, in that expectations under learning converge asymptotically to the REE. Along the lines of Bullard and Mitra (2002), Kobayashi and Muto (2011) and Ascari and Florio (2012) investigate the E-stability regions - the regions of the parameter space (ϕ_π, ϕ_Y) of a given Taylor rule, where the REE is E-stable. Their main message is that higher trend inflation generally makes the REE more difficult to learn, reduces the E-stability region, and requires a stronger policy response to inflation and weaker response to output in order to guarantee E-stability.[58]

Trend inflation therefore affects the E-stability region under adaptive learning in a similar way in which it affects the determinacy region under rational expectations,[59] supporting the claim that a higher inflation target could unanchor expectations. Moreover, Ascari and Florio (2012) show that, conditional on E-stability, a higher trend inflation reduces the speed of convergence of expectations to the REE. Hence, a higher inflation target destabilizes expectations both asymptotically (E-stability) and in the transition phase.

Branch and Evans (2011) and Cogley et al. (2010) follow a different approach to investigate the relationship between learning and the inflation target. They both study

[57] Agents form expectations through recursive least squares with decreasing gain and their perceived laws of motion usually takes the minimum state variable (MSV) form.

[58] Despite some differences in the assumptions regarding the learning process across the two papers, the main result seems robust across different specifications of the Taylor rule (backward, forward and/or inertial).

[59] See Ellison and Pearlman (2011) for a formal analysis of the link betwen the determinacy and the E-stability region.

the dynamics of the model when there is imperfect information about a change in the long-run inflation target. Branch and Evans (2011) show how an increase in the inflation target could generate near-random walk beliefs and temporary unstable dynamics due to self-fulfilling paths. The assumption that agents form expectations using adaptive learning with constant gain (that is, that agents discount past data) is key for the result, and it can be justified by a prior on possible structural change of parameters. In this way, the model can generate significant and persistent departures from RE. The mechanics of this result is the following. After an increase in the inflation target, inflation expectations increase less than proportionally because of the adaptive learning process. As a result inflation would be below target and the central bank would reduce the nominal interest rates. Given the specification of the learning process, agents will detect a structural change in the persistence of inflation, leading to higher inflation expectations that feed back into higher inflation rates. Eventually agents come to believe that inflation follows a random walk. Inflation could then largely overshoot the target before returning to its new long-run value. Imperfect information on the inflation target can thus generate instability in the inflation rate and in inflation expectations, which temporarily follow a non-stationary dynamics. To prevent that, monetary policy should be perfectly credible and transparent, informing agents about the size and the timing of the change in the target.

A related work by Cogley et al. (2010) studies optimal disinflation policy with Taylor-type rules under learning. They analyze the case of a central banker that inherits high inflation: under full information the optimal disinflation policy implies more aggressive response coefficients, while under learning optimal disinflation occurs more gradually, and implies a larger sacrifice ratio. The optimal disinflation under learning is gradual because the equilibrium law of motion in that case is potentially explosive. However, the authors find that the crucial source of potential explosiveness is the imperfect information about the policy feedback parameters, rather than about the long-run inflation target. Further analysis by the authors also shows that the form of the Taylor rule, in particular whether it is backward, contemporaneous, or forward-looking, matters in generating potentially unstable dynamics.

3.3 Trend inflation, macroeconomic dynamics and monetary policy trade-offs

Trend inflation also affects the transmission mechanism of monetary policy and the dynamics of the macroeconomy in response to shocks, because it flattens the Phillips curve, as we have shown. To analyze these effects, we specify the following autoregressive processes for the three shocks of our baseline model:

(i) the technology shock:

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + u_{At}; \tag{51}$$

(ii) the labor supply shock in (24) (which is equivalent to a cost push shocks in the marginal cost equation (12)):

$$\varsigma_t = \rho_\varsigma \varsigma_{t-1} + u_{\varsigma t}; \tag{52}$$

(iii) the monetary policy shock in the Taylor rule (40):

$$v_t = \rho_v v_{t-1} + u_{vt}. \tag{53}$$

The innovations u_{At}, $u_{\varsigma t}$, and $u_{\nu t}$ are assumed to be *i.i.d.* standard normal processes.

To obtain analytical expressions for the responses of output and inflation to the shocks, we use the following simplifying assumptions: log preference in consumption ($\sigma = 1$), indivisible labor ($\varphi = 0$)[60] and no persistence in the shocks ($\rho_i = 0$ for $i = A, \varsigma, v$). With these assumptions the log-linearized *GNK* model can be described by the following equations:

$$\hat{\pi}_t = \lambda(\bar{\pi})\left(\hat{Y}_t - \hat{A}_t + \varsigma_t\right) + b_1(\bar{\pi}) E_t\hat{\pi}_{t+1} - b_2(\bar{\pi}) E_t\hat{\psi}_{t+1}, \tag{54}$$

$$\hat{\psi}_t \equiv (1 - \theta\beta\bar{\pi}^{\varepsilon})\left(\hat{Y}_t - \hat{A}_t + \varsigma_t\right) + (\theta\beta\bar{\pi}^{\varepsilon}) E_t\left(\hat{\psi}_{t+1} + \varepsilon\hat{\pi}_{t+1}\right), \tag{55}$$

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - (\hat{\imath}_t - E_t\hat{\pi}_{t+1}), \tag{56}$$

$$\hat{\imath}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{Y}_t + v_t. \tag{57}$$

The model is completely forward-looking as in the case of zero steady state inflation, because the assumption of indivisible labour makes price dispersion irrelevant for the dynamics of output and inflation (but not for the one of hours), as evident from (44) and (45). Moreover, given that the shocks are i.i.d., there is no transitional dynamics and the economy returns to steady state in the period after the shock. The impact effect of the different shocks can be derived as:[61]

$$\hat{\pi}_t = \frac{\lambda(\bar{\pi})}{1 + \lambda(\bar{\pi})\frac{\phi_\pi}{1+\phi_y}}\left(\varsigma_t - \hat{A}_t - \frac{1}{1+\phi_y}v_t\right), \tag{58}$$

$$\hat{y}_t = \frac{1}{\frac{1+\phi_y}{\phi_\pi} + \lambda(\bar{\pi})}\left(\lambda(\bar{\pi})\left(\hat{A}_t - \varsigma_t\right) - \frac{1}{\phi_\pi}v_t\right). \tag{59}$$

As in the standard *NK* model, a technology (labor supply) shock reduces (increases) inflation and increases (lowers) output, while a monetary policy shock contracts both output and inflation.[62] However, because trend inflation flattens the Phillips curve by reducing $\lambda(\bar{\pi})$, an increase in $\bar{\pi}$ reduces the impact effect of a technology (labor supply) shock on output and inflation.[63] The impact effect of a monetary shock on output is larger for higher trend inflation, while the one on inflation is smaller. Thus the effect of trend inflation on the volatility of aggregate variables depends on the type of shock.

If we remove the assumption of $\varphi = 0$ used to obtain the analytical derivations above, the dynamic response to shocks is affected also by the dynamics of price dispersion. As we discussed, when trend inflation increases, deviations of price dispersion from steady

[60] Note that $\sigma = 1$ and $\varphi = 0$ imply $\lambda(\bar{\pi}) = \kappa(\bar{\pi})$.

[61] Under our simplifying assumptions, the labour supply shock is equivalent to a negative technology shock. We will thus comment only the effects of technology shocks, the opposite being valid for labor supply shocks.

[62] Moreover, the higher $\phi_\pi(\phi_y)$, the lower (higher) the response of inflation to shocks and the higher (lower) the one of output.

[63] It is worth noting that very often in the literature a cost-push shock is simply added to the *NKPC* equation. If we had done that, adding a shock to (54), then the analytical response of output and inflation to such a shock would have implied that the impact effect both on output and inflation increases with trend inflation. This explain the result in Ascari and Ropele (2007) that the policy frontier, i.e., the locus of feasible combinations of output and inflation variances attainable by monetary policy, shifts outward with trend inflation (see proposition 3 therein).

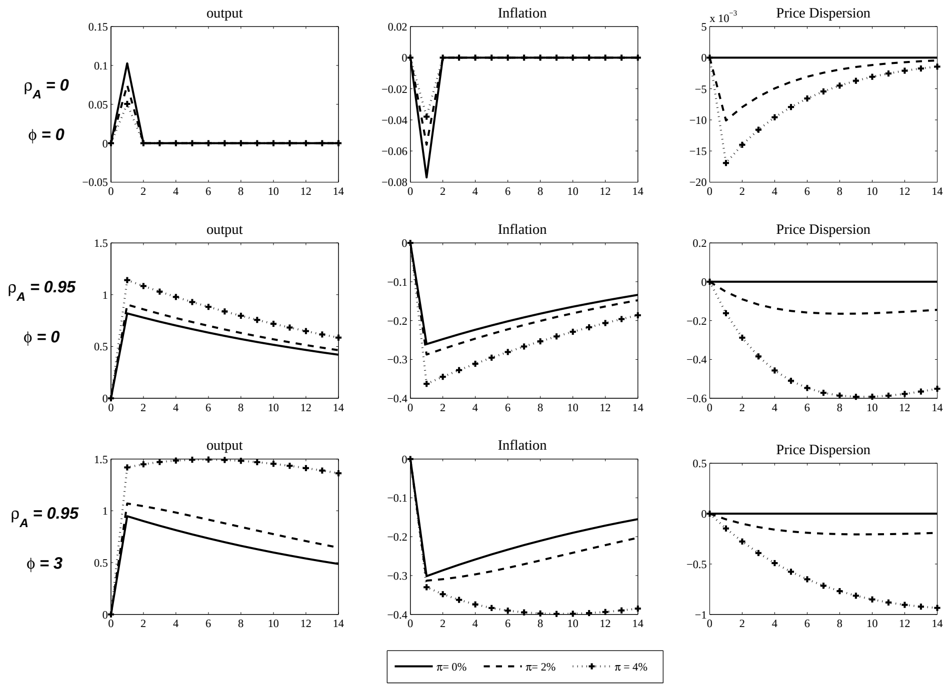

Figure 13: Impulse response functions to a 1% positive technological shock

state are more persistent. This feeds back into the marginal costs - and hence the *NKPC* - so that trend inflation tends to increase the persistence of macroeconomic variables, particularly of inflation. This in turn also affects the unconditional variance of most economic variables. We display these effects in the following figures.

Technology Shock. Figure 13 shows the impulse response functions of output, inflation and price dispersion to a positive 1 percent technology shock for different values of trend inflation: 0, 2 and 4 percent. Each row of the Figure corresponds to a different parametrization of the shock persistence ρ_A and the Frish elasticity φ.[64] The first row assumes the same parameters as in our analytical investigation; the second line assumes $\rho_A = 0.95$ and the third sets $\rho_A = 0.95$ and $\varphi = 3$. The first line displays the analytical result we discussed above: trend inflation dampens the impact effect of a technology shock on output and inflation, and amplifies that on price dispersion. Output and inflation return immediately to steady state after the impact, because they do not inherit any persistence from the shock (since $\rho_A = 0$), nor through price dispersion (because s does not enter the *GNKPC* when $\varphi = 0$). Price dispersion does not move much, because the change in inflation is small and short-lived.[65] However, when the technology shock displays high persistence, as often assumed in calibration studies, trend inflation *amplifies* the impact of a technology shock on output and inflation, as price-setting firms are in this case more forward-looking. The price dispersion response is also

[64] Calibration for the Taylor rule parameters is: $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$.

[65] Note that price dispersion does not move in the case of zero steady state inflation, because it does not matter to first order in that case, as we explained.

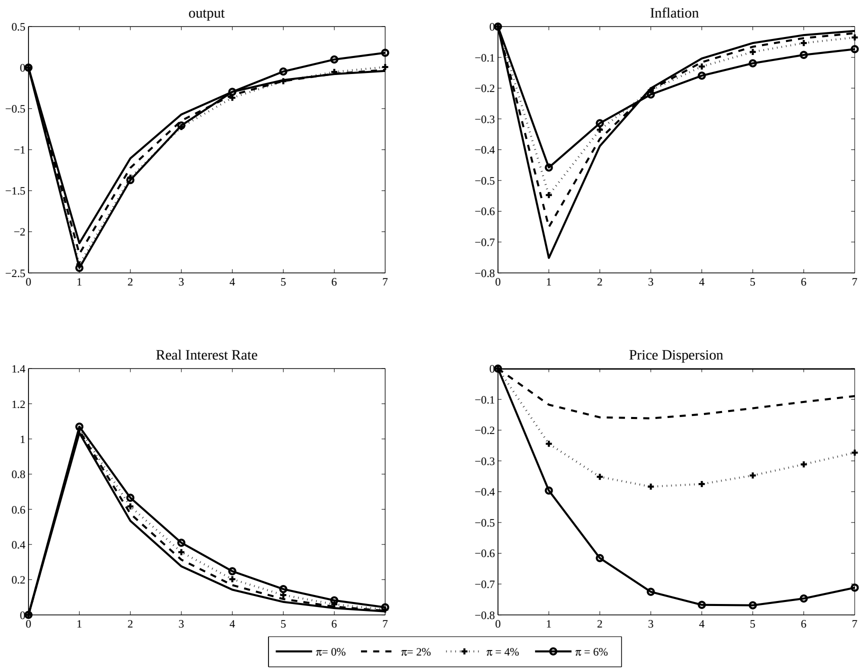

Figure 14: Impulse response functions to a 1% positive monetary policy shock

quite sizeable and persistent in this case, as expected from (46). Finally, when $\varphi > 0$ (bottom row of the Figure) price dispersion increases the persistence of output and inflation, as in this case there is mutual feedback between inflation and price dispersion, whose strength is governed by the parameter φ.

Monetary Policy Shock. Figure 14, constructed as Figure 13**,** shows the impulse responses of output, inflation and price dispersion to a 1 percent shock to the Taylor rule.[66] Price resetting firms react less to the current drop in output, so that inflation reacts less, and the interest rate induces a larger reaction of output according to the Euler Equation. Trend inflation matters for these impulse responses through the *GNKPC* slope $\lambda(\bar{\pi})$: because the effect of a change in current output on inflation decreases with trend inflation, higher trend inflation reduces the impact of monetary policy shocks on inflation.[67] Note also that through price dispersion, higher trend inflation increases inflation persistence.

The main message from these simulations is that trend inflation tends to increase the

[66] Here we set $\phi_{\pi}= 2$ (rather than 1.5, with the other parameters as in the benchmark calibration of footnote 34) to allow determinacy for the 6% trend inflation case. The change in parameters has no effect on the shape of the impulse responses. For realism, we also add inertia to the Taylor rule, specified as:

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)\left(\phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t\right) + v_t, \tag{60}$$

with $\rho_i = 0.8$.

[67] Ascari and Ropele (2007) shows that this result is valid also for shocks to the rate of growth of the money supply when monetary policy is described by an exogenous autoregressive process for the money supply.

persistence of macroeconomics variables. Its effect on volatility, however, depends on the type of shocks, the variable and the calibration. In particular, when supply shocks are very persistent, as commonly calibrated in the literature, trend inflation strongly increases the volatility of macroeconomic variables. For example, in our benchmark model an increase in the inflation target from 0 to 4 percent more than doubles the variance of output (from 1.3 to 2.85) and that of inflation (from 0.41 to 0.85), assuming only technology shocks with persistence and variance equal to 0.95 and 0.45, respectively (as in Smets and Wouters, 2007).

These results are in line with those of Amano et al. (2007), one of the first papers in the literature to analyze the macroeconomic effects of trend inflation. Amano et al. (2007) also uncover the effects of trend inflation on the stochastic means of variables by simulating the model dynamics using a second-order approximation. They show that the spread between deterministic steady states values of the variables and their stochastic means increases with trend inflation. In the case of zero trend inflation the stochastic mean of inflation is slightly higher than the deterministic trend inflation rate, while the stochastic means of output, consumption and employment are slightly lower than their steady state counterparts. With positive trend inflation these spreads dramatically increase. The effect is particularly large for the stochastic mean of inflation which, in the case of a 4 percent inflation target, is equal to 7.8 percent! The mechanism again works through price dispersion, whose effects are nonlinear: persistent increases in the deviations of price dispersion from steady state have a greater impact on the model endogenous variables than do decreases, leading to a spread between deterministic steady state and the stochastic mean. This result has an important policy implication: higher the inflation target, more often inflation would exceeds its target (unless the monetary authority is willing to compensate these effects by running a policy where output, on average, is forced to fall short of its deterministic steady-state value).

3.4 Trend inflation and optimal stabilization policy

As we discussed in Section 2, including trend inflation in the aggregate supply relationship leads to attribute most of the inflation persistence to the persistence of its underlying trend. Under this interpretation inflation persistence is an effect of monetary policy, rather than an intrinsic features of the economy: as a consequence, when accounting for a time-varying trend inflation the optimal response to exogenous shocks may be different from the optimal response in an economy which features intrinsic inflation persistence. Sbordone (2007) illustrates this point for the case of cost push shocks. She considers a small new Keynesian model with two different specifications of the Phillips curve, one with a backward-looking component as in the Galí and Gertler (1999) model, the other purely forward-looking with a time-varying trend inflation as in Cogley and Sbordone (2008), and considers the consequences of implementing stabilization policy under assumptions about intrinsic persistence that differ from what the true model implies. Her simulations suggest that the cost of implementing stabilization policy overestimating the degree of intrinsic inflation persistence is in general higher than the cost of responding to shocks ignoring possible structural persistence. The results are sharper when the inflation stabilization has more weight in the policymaker's loss function, as it is the case when the loss function is welfare-based.

Another way of evaluating the effect of trend inflation on stabilization policy is by revisiting the well-known analysis of optimal stabilization policy under discretion and commitment by Clarida et al. (2000), in the context of our *GNK* model. Ascari and Ropele (2007) conduct such exercise in a model like our baseline, adding a cost-push shock to the *GNKPC* (54*)* as in Clarida et al. (2000), i.e. without deriving it from microfoundations, and assuming an ad-hoc quadratic loss function of the form:

$$\mathcal{W} = \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left(\hat{\pi}_{t+j}^2 + \chi \hat{Y}_{t+j}^2 \right), \tag{61}$$

where χ represents the relative weight between output and inflation stabilization around the target.

Optimal policy under discretion has the same prescription in the *GNK* model as Clarida et al. (2000) obtained for the standard case of zero inflation steady state: optimal policy has to 'lean against the wind', distributing the effects of the cost-push shock between output and inflation according to:

$$\hat{Y}_t = -\frac{\lambda(\bar{\pi})}{\chi} \hat{\pi}_t. \tag{62}$$

However, since the slope of the Phillips Curve $\lambda(\bar{\pi})$ decreases with trend inflation $\bar{\pi}$, inflation is less sensitive to a change in output engineered by monetary policy. As a consequence, the degree by which optimal policy is aggressive in response to a cost-push shock depends on trend inflation: higher is trend inflation, more the shock is passed into inflation and less into output, because higher trend inflation lowers the gain in terms of reduced inflation for each unit of output loss. This result carries two implications. First, the optimal policy is at first more aggressive as $\bar{\pi}$ increases, but becomes less aggressive for further increases in $\bar{\pi}$. The reason for this non monotone response is that the interest rate is a weaker policy instrument in the presence of trend inflation: at some point the output costs of controlling inflation becomes too high so the optimal response is increasingly cautious. Second, the variability of inflation relative to output increases with trend inflation.

Turning to optimal policy under commitment, a positive inflation target strengthens the policy features of the optimal commitment in Clarida et al. (2000), such as a lower impact effects on endogenous variables and greater persistence in impulse responses. Inflation dynamics is in fact more forward-looking with positive trend inflation, so that, according to the model, there are greater incentives for the monetary authority to use forward guidance to manage expectations. The nominal interest rate reacts less aggressively also under commitment, because monetary policy becomes less effective with trend inflation. Hence, also in this case the relative volatility of inflation and output increases. Finally, the gains from commitment are increasing up to certain level of trend inflation, because trend inflation increases the importance of influencing future expectations, but then they decrease, as the second effect of positive trend inflation takes over, namely the reduction of the effectiveness of monetary policy. At high levels of trend inflation, the effectiveness of policy is reduced to such an extent that inflation becomes very costly to control. Policymakers find it optimal to keep the output gap almost constant and give up on inflation stabilization both under commitment and discretion, so there are not much gains from commitment.

In the standard optimal policy literature the *NKPC* is usually expressed in terms of output gap $\tilde{Y}_t$, defined as the log-deviation of current output from the output that would arise in a flexible price environment, namely the natural level of output Y_t^n (Woodford, 2003 and Galí, 2008): $\tilde{Y}_t = \log Y_t - \log Y_t^n$. $\tilde{Y}_t$ is a measure of the nominal distortion implied by sticky prices. In our benchmark model under zero steady state inflation, the output gap can be written as $\tilde{Y}_t = (\log Y_t - \log \bar{Y}) - (\log Y_t^n - \log \bar{Y}) = \hat{Y}_t - \frac{\varphi+1}{\sigma+\varphi}\hat{A}_t + \frac{1}{\sigma+\varphi}\varsigma_t$, where $\bar{Y}$ is the flexible price steady state output. Thus, (44) can be written in the familiar form:

$$\hat{\pi}_t = \lambda \tilde{Y}_t + \beta E_t \hat{\pi}_{t+1}. \tag{63}$$

An optimal policy that is able to close the output gap also stabilizes inflation: this is what Blanchard and Galí (2007) called the 'divine coincidence' in the *NK* model.

To write an expression analogous to (63) under trend inflation we introduce the following decomposition:

$$\begin{aligned} \tilde{Y}_t &= (\log Y_t - \log \bar{Y}(\bar{\pi})) - (\log Y_t^n - \log \bar{Y}) - (\log \bar{Y} - \log \bar{Y}(\bar{\pi})) \\ &= \hat{Y}_t - \frac{\varphi+1}{\sigma+\varphi}\hat{A}_t + \frac{1}{\sigma+\varphi}\varsigma_t + \tilde{Y} \end{aligned} \tag{64}$$

where we defined $\bar{Y}(\bar{\pi})$ as the steady state output level for a given trend inflation. Expression (64) includes the extra term $\tilde{Y}$ to take into account the fact that the model is log-linearized around a positive $\bar{\pi}$. The term $\tilde{Y} = (\log \bar{Y}(\bar{\pi}) - \log \bar{Y})$ is the deviation of the level of output associated with steady state inflation $\bar{\pi}$ from the level of long-run output under flexible prices (or when steady state inflation is zero). As such, $\tilde{Y}$ represents a *long-run output gap* (recall Figure 9). Using (64) we can write the *GNKPC* (44) and (45) as:

$$\begin{aligned} \hat{\pi}_t &= \lambda(\bar{\pi})\left(\tilde{Y}_t - \tilde{Y}\right) + \kappa(\bar{\pi})\varphi\hat{s}_t + b_1(\bar{\pi}) E_t \hat{\pi}_{t+1} \\ &+ b_2(\bar{\pi})\left(\left(\tilde{Y}_t - \tilde{Y} + \frac{\varphi+1}{\sigma+\varphi}\hat{A}_t - \frac{1}{\sigma+\varphi}\varsigma_t\right)(1-\sigma) - E_t\hat{\psi}_{t+1}\right), \end{aligned} \tag{65}$$

$$\hat{\psi}_t \equiv (1-\theta\beta\bar{\pi}^{\varepsilon})\left[\varphi\hat{s}_t + (1+\varphi)\left(\tilde{Y}_t - \tilde{Y} + \frac{1-\sigma}{\sigma+\varphi}\hat{A}_t - \frac{1}{\sigma+\varphi}\varsigma_t\right)\right] + \theta\beta\bar{\pi}^{\varepsilon} E_t\left(\hat{\psi}_{t+1} + \varepsilon\hat{\pi}_{t+1}\right). \tag{66}$$

It is then immediately evident that it not sufficient for policy to close the output gap in order to stabilize inflation, so that the 'divine coincidence' does not hold when trend inflation is positive.[68]

Using this output gap definition, Coibion et al. (2012) and Lago-Alves (2012) derive a second-order approximation to the utility function of the representative agent under trend inflation, following the approach in Woodford (2003).[69]

Woodford (2003) shows that under the zero inflation steady state assumption the steady state inefficiency Φ_y is simply due to the monopolistic distortion: $\Phi_y = 1 - \frac{1}{\mu} =$

[68] In other words, the shocks in our benchmark *GNK* model would not generate any policy trade-off under the zero inflation steady state assumption, but they do under positive trend inflation. See Lago-Alves (2012).

[69] This provides an extension of the Ascari and Ropele (2007) analysis to the case of a microfounded loss function. This extension is important because the parameters of the microfounded loss function provide another channel for the effects of trend inflation.

$\frac{1}{\varepsilon}$. The second-order approximation of the utility function around the efficient level of output (Rotemberg and Woodfod, 1999) is then derived under the assumption that the inefficiency Φ_y is 'small enough'.[70] When trend inflation is positive, however, Lago-Alves (2012) shows that the monopolistic distortion is potentially larger, since the steady state markup is now (see (38) and (30)):

$$\overline{\mu} = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \beta\theta\bar{\pi}^{\varepsilon-1}}{1 - \beta\theta\bar{\pi}^{\varepsilon}} \left(\frac{1 - \theta\bar{\pi}^{\varepsilon-1}}{1 - \theta} \right)^{\frac{1}{\varepsilon-1}}. \tag{67}$$

Lago-Alves (2012) also illustrates two results from the analysis of the welfare function. First, the non-linear welfare function decreases fast and becomes increasingly concave as the inflation rate rises. Hence the microfounded loss function derived under the zero inflation steady state assumption can be a poor approximation to the true welfare function when average inflation is positive. At 2 percent inflation, for example, the curvature of the true loss function is two and a half times as large as the curvature at the steady state with zero inflation, and at 4 percent inflation it is ten times as large. An approximation around positive trend inflation would be more precise. The second result pertains to the relative weight on output in the welfare function, which is now:[71]

$$\chi(\bar{\pi}) = \frac{1 - \theta\bar{\pi}^{\varepsilon-1}}{1 - \theta\bar{\pi}^{\varepsilon}} \frac{\lambda(\bar{\pi})}{\varepsilon} \tag{68}$$

rather than $\chi = \frac{\lambda}{\varepsilon}$ as under the zero steady state inflation assumption. $\chi(\bar{\pi})$ is rapidly decreasing with trend inflation: with trend inflation at 3 percent, for example, $\chi(\bar{\pi})$ is about 75 percent of the corresponding value under the zero steady state inflation assumption. Thus optimal policy has a lower weight on output gap volatility (or a higher one on inflation volatility) when the target level for inflation is higher. This is very intuitive. Inflation fluctuations are costlier when trend inflation is higher, because they have a larger and more prolonged impact on price dispersion: hence the weight on inflation stabilization should be higher.

Finally, the analysis in Lago-Alves shows that the features of optimal policy under a positive inflation target that we discuss for the case of an ad hoc loss function are robust to the fact that the optimal weight χ depends on trend inflation. The welfare effects of trend inflation are obviously critical to the issue of the optimal inflation target to which we turn next.

3.5 Trend inflation and the optimal inflation rate

This survey discusses the consequence of a positive inflation target. We showed so far that incorporating a positive long-run rate of inflation in the benchmark *NK* model tends to unanchor inflation expectations, enhance macroeconomic volatility and lower welfare. These results are consistent with the vast literature on the optimal long-run rate of inflation, recently covered in a quite exhaustive survey by Schmitt-Grohè and Uribe (2011). In most New Keynesian models the optimal long-run rate of inflation is

[70] Alternatively, one may assume another policy tool (e.g., a subsidy) to take care of this inefficiency.
[71] See Proposition 2 in Lago-Alves (2012) and the subsequent discussion therein.

indeed zero[72] (Goodfriend and King, 1998, Yun, 2005, Benigno and Woodford, 2005, Khan et al., 2003, and Amano et al., 2009), a conclusion shown to be quite robust in cashless models with staggered price setting. Moreover, Schmitt-Grohè and Uribe (2011) showed the Ramsey optimal policy call for price stability (i.e., zero inflation) at all times in such a model. Even in the presence of uncertainty, for plausible calibrations, the inflation rate is always very close to zero (i.e., has very low variability). This result is perhaps not surprising, given the emphasis we have put on the cost of price dispersion: zero inflation is optimal simply because it eliminates this cost. Nor it is surprising that the optimality of zero inflation disappears only in the extreme and unrealistic case of full indexation. When prices are fully indexed, there is no price dispersion in steady state, so the optimal steady state rate in this case is just equal to the one that existsted initially (Schmitt-Grohè and Uribe, 2011).

The pivotal role of price dispersion in this class of models is even more evident by considering how the optimal policy is affected by a positive inherited level of price dispersion. In this case the optimality of zero inflation at all times is no longer valid. While the optimal policy does converge to the steady state with zero inflation, the transitional dynamics call for a quicker reduction of the inherited level of price dispersion than under a zero inflation policy. This result is easy to show in our framework, following Yun (2005). The Ramsey problem is to maximize the utility function of the representative consumer under just the resource constraint and the evolution of price dispersion as described by (34).[73] The optimal policy should satisfy the following two conditions (see the online Appendix):

$$\pi_t = \frac{s_t}{s_{t-1}} \tag{69}$$

and

$$s_t = s_{t-1} \left[\theta + (1-\theta)\, s_{t-1}^{\varepsilon-1}\right]^{\frac{1}{1-\varepsilon}} . \tag{70}$$

From these, it immediately follows that: (i) the optimal policy implies zero inflation in steady state ($\bar{\pi} = 1$); (ii) if there is no inherited price dispersion, i.e., $s_{t-1} = 1$, then a policy setting $\pi_t = 1$ for all t is optimal; (iii) however, if there is inherited persistence, i.e., $s_{t-1} > 1$, then optimal policy would prescribe a gradual reduction in price dispersion to satisfy (70) toward a steady state with no price dispersion.[74] In this latter case, then, the inflation rate is negative ($\pi_t < 1$) along the adjustment path exactly to engineer a faster reduction in price dispersion than the one implied by a zero inflation policy. Furthermore, along the optimal path, as long as there is price dispersion the optimal reset price is below the average price level:

$$p_{i,t}^* = \frac{1}{s_t}. \tag{71}$$

To sum up, the *NK* model would predict an optimal inflation rate either equal to zero (absent money demand distortions) or negative during the transition whenever the

[72] Or negative if there is a demand for money. In this case the optimal rate will trade off the opportunity cost of holding money due to a positive nominal interest rate and the cost of price dispersion due to a positive inflation rate. Since we only considered a cashless framework, we did not address the money demand case, which is comprehensively treated in Schmitt-Grohé and Uribe (2011).

[73] Using (30) to substitute for $p_{i,t}^*$ in (34).

[74] Since $\varepsilon > 1$, if $s_{t-1} > 1$ then (70) implies $s_t < s_{t-1}$.

initial level of price dispersion is not zero. The goal of the policy is to minimize the distortions due to price dispersion. These results show once again how the costs of inflation arise naturally in a *NK* model with trend inflation from the key feature of price dispersion.

3.6 Trend inflation, alternative models of price setting and generalization of the basic *NK* model

Before heading to the implications of the macroeconomics of trend inflation for the current policy debate, we discuss in this section some generalizations of the framework we used throughout the survey. These include considering other features within the Calvo price-setting model as well as alternative price setting models.[75]

3.6.1 Microeconomic features

Firm specific inputs. When factors of production are firm specific, the effects of trend inflation described in the previous sections are amplified. In our baseline model we assumed a common-factor market, which implies strategic substitutability in pricing decisions, while specific-factor markets (either in capital or in labor) generate strategic complementarity in price setting. Bakhshi et al. (2007) show that firm specific factors increase the long-run negative effects of trend inflation and the elasticity of the slope of the Phillips Curve with respect to trend inflation. The presence of firm-specific factors also strengthens the effects of trend inflation on the determinacy and E-learnability regions we analyzed in Section 3.2 (Hornstein and Wolman, 2005 and Kurozumi and Van Zandweghe, 2012).

The role of different microeconomic features is often muted when models are approximated around a steady state with zero inflation. This is what Levin et al. (2008) call *macroeconomic equivalence* and *microeconomic dissonance*: when two different models are log-linearized around the flexible-price equilibrium, their different microfoundations may become irrelevant. They show this phenomenon by considering two different types of strategic complementarity (or real rigidity): one generated by a kinked demand curve, the other by the presence of firm-specific inputs. Up to first order these two models yield the same standard *NKPC*, which is flatter than the standard case because of the real rigidities, but this is an effect of their approximation: if they had taken the approximation around a generic trend inflation level (or even better, worked with the non-linear model) the model differences would have been preserved.[76]

In Levin et al. (2008) the different microfoundations of the two models carry only implications for welfare, since welfare is approximated to second order (around the efficient allocation). Price dispersion is not very costly in the kinked demand model, where relative-demand reactions (and so equilibrium allocations of resources) are quite price-insensitive, while it is very costly in the model with firm-specific factors. This in turn leads to very different normative policy prescriptions, since the two models

[75] We do not discuss models with growth. However, Amano et al. (2009), Vaona and Snower (2008) and Vaona (2012) show that growth generally strengthens the effects of trend inflation in an environment with nominal rigidities.

[76] Taking second order approximations around the zero inflation steady state will generally make the difference in microfoundations to emerge, but this would matter only at second-order.

imply different optimal relative weights on output and inflation volatility in the welfare function.

Discounting. We saw above that another microeconomic feature that interacts with trend inflation is discounting. Graham and Snower (2008, 2011) demonstrate that hyperbolic discounting could generate a positive relationship between inflation and output in steady state - so that the optimal rate of inflation could be positive in this case. Hyperbolic discounting strengthens the discounting effect described above, which eventually dominates on the relative price and price dispersion effect.[77]

Medium-scale models. Two recent papers investigate the effects of trend inflation in medium-scale *NK* models. Arias (2013) carries out a similar exercise as in Section 3.2.1 in a *GNK* model with capital accumulation, including investment adjustment costs and variable capital utilization, two real frictions typically embedded in *NK* medium-scale models. He finds that real frictions that smooth aggregate demand make the determinacy region less sensitive to variations in trend inflation. Ascari et al. (2012) reexamine the results of Coibion and Gorodnichenko (2011), described in Section 3.2.1, in the context of a medium-scale model a là Christiano et al. (2005). They show that in this model the likelihood of falling in the indeterminacy region is less affected by trend inflation, relative to the baseline model. In the US economy indeterminacy appears confined to the second half of the 1970s, when trend inflation reached its historical peak in the post-WWII sample. It follows that in this model changes in the Taylor rule parameters appear the key drivers of the conquest of U.S. inflation, in line with the message popularized by Clarida et al. (2000).

Indexation. Very often in the *NK* models literature non optimized prices are assumed to be somewhat indexed either to steady state inflation (Yun, 1996) or to past inflation rates (Christiano et al., 2005) or to both (Smets and Wouters, 2007). The purpose of backward-looking indexation is mainly to capture persistence in inflation data, as discussed in Section 2. Indexation counteracts the effects of trend inflation (Ascari, 2004): the theoretical results discussed in previous sections are mitigated when assuming partial indexation of non-resetted prices (either to trend or to past inflation), and are actually muted if indexation is full. In fact, under full indexation the steady state of the model is exactly the flexible-price one: there is no price dispersion and all the firms charge the same price. The assumption of indexation, however, does not seem justified on theoretical nor on empirical grounds. Theoretically, indexation is not typically derived from optimal behavior of price setters, and empirically, analyses based on price microdata show that prices do not change all the time, as automatic indexation would imply. Furthermore, as we discussed in Section 2.2.3, indexation doesn't appear to be needed to fit the *NKPC*, once trend inflation is taken into account.

3.6.2 Other time-dependent pricing models

The New Keynesian literature often employs two other time-dependent pricing models: the Taylor (1979, 1980) overlapping contracts model and the Rotemberg (1982) cost-of-adjustment model.

[77] They use a model with overlapping contract a là Taylor. We conjecture this result will be harder to get in a Calvo setup, where the price dispersion effect is stronger.

The effects of trend inflation we discussed for the Calvo model still hold qualitatively for the Taylor model, but are quantitatively far less important. This occurs because there is a much lower degree of price dispersion in this model, since price resetting occurs a fixed number of times (typically four, so there are only four different prices in each period). Ascari (2004) shows that in the Taylor model the steady state and the dynamics of the model are much less sensitive to changes in trend inflation than in the Calvo model.[78] In line with Ascari and Ropele's (2009) results, Kiley (2007) shows that in a Taylor model with an interest rate rule trend inflation makes the determinacy of the rational expectation equilibrium more difficult to achieve, and it increases inflation volatility following cost-push shocks. Finally, Amano et al. (2007) also show that the features that limit price dispersion relative to the Calvo model dampen the effects of trend inflation on the model dynamics.[79]

The Rotemberg price-setting model implies instead qualitatively different results from the Calvo model. The two models are indeed an example of *macroeconomic equivalence* and *microeconomic dissonance* in the *NK* literature: to a first order approximation around a zero inflation steady state, they have the same reduced-form dynamics (Rotemberg, 1987, Roberts, 1995) and equivalent welfare implications (Nisticò, 2007). As a consequence, these two models are treated in the literature as equivalent from a macroeconomic perspective, despite their different microfoundation and their different welfare implications. Under positive trend inflation the two models differ because there is no price dispersion in the Rotemberg model - all firms change their price every period, subject to an adjustment cost, hence none of the effects of relative prices on firms price setting that we described in Section 3.1.1 are present. There is still however the discounting effect: in changing their price firms weight today's adjustment cost of moving away from yesterday's price relatively more than tomorrow's adjustment cost of setting a new price away from today's one. It follows that in the Rotemberg model the steady state average markup decreases and the steady state output increases with trend inflation.[80] Ascari and Rossi (2009) show in particular that in the Rotemberg model, unlike in Calvo's: (i) the long-run *NKPC* is positively sloped; (ii) the log-linear *GNK* model implies a different dynamics; (iii) when monetary policy follows a Taylor rule positive trend inflation enlarges the determinacy region and dampens the dynamic response of output and inflation to a persistent technology shock. A comparison of the empirical fit on U.S. data of the log-linearized Calvo's and Rotemberg's price setting models under trend inflation supports the view that trend inflation is an empirically relevant feature and points to evidence in favor of the Calvo price setting model (Ascari et al., 2011).

3.6.3 Endogenous frequency of price adjustment

Finally, throughout our analysis, we treated the frequency of price adjustment (the Calvo parameter) as fixed under different trend inflation rates. It may however be the case that when inflation is higher the frequency of price resetting is also higher. In the standard *NK* model higher price rigidity implies higher price dispersion, for any given

[78] See also Ascari (2000) and Graham and Snower (2004).

[79] They consider both the Taylor model and an hybrid version of the Calvo model with a maximum price duration.

[80] Note that, however, consumption and welfare decrease with trend inflation, as in the Calvo model, because the higher trend inflation, the larger the adjustment cost that firms have to pay.

level of trend inflation. As a consequence, a lower Calvo parameter (i.e., higher price flexibility induced by higher inflation) would dampen the effects of trend inflation.

In models where the frequency of price adjustment is endogenous the coefficients of the *GNKPC* (which exhibits in this case more inflation inertia) are still function of steady-state inflation, and are also function of the steady-state distribution of price vintages and the number of price vintages, which evolve endogenously in the model.[81] If trend inflation rises, firms adjust their prices more frequently, and the number of price vintages declines while the number of firms in recent vintages increases, so price dispersion decreases. Monetary policy shocks in this framework (under high policy inertia), have a larger impact effect on inflation the higher is trend inflation, while the persistence in inflation is lower. The opposite, instead, occurs for output. These effects are however quantitatively minor. Levin and Yun (2007) look at the steady state properties of a model where firms are subject to a cost of price adjustment but are allowed to choose the frequency of price changes once and for all, given the constant average steady state inflation. The equilibrium average frequency of price adjustment is then endogenous and it is an increasing function of steady state inflation. In this model the steady state relationship between output and inflation is different from the one we showed in Figure 9. For very high rate of inflation, firms adjust prices every period so that the equilibrium is the same as the flexible price one and the model exhibits superneutrality. For low and intermediate rates of trend inflation, the relationship is first positive and then negative as in Figure 9, but the effects are far less dramatic. How much less, obviously depends on the calibration, in particular on how much the price stickiness parameter responds to changes in trend inflation. Only a very high elasticity of response could substantially dampen the results surveyed above .

While the sensitivity of the frequency of price adjustment to the average inflation rate is ultimately an empirical question, we should note that our interest is on the effects of low-to-moderate trend inflation rates, hence treating the Calvo parameter as constant is, we believe, a reasonable approximation. Many macroeconomic studies which estimated various versions of the *NK* model in the pre- and post-Volcker periods report no significant changes in the Calvo parameter.[82] Microeconomic data also find no empirical evidence of sizeable changes in the frequency of price adjustment. Alvarez et al. (2013), for example, using a uniquely rich dataset from Argentina, conclude that "the steady state frequency of price changes is unresponsive to inflation for low inflation rates" (below 10 percent).[83] Surveying comprehensively various studies on the subject, they show that their results are similar to those obtained for other countries. A similar result holds for price dispersion, though. From the perspective of having an operational price staggering framework useful for macroeconomic models, this may suggest that a more problematic feature of the Calvo model could be the existence of very long tails, that is the existence of very old, and then relatively very low, prices leading to a large cost of price dispersion which is very sensitive to degree of trend inflation.

[81] Bakhshi et al. (2007) obtain the *GNKPC* in the state-dependent model of Dotsey et al. (1999).

[82] E.g., see Fernández-Villaverde and Rubio-Ramirez (2008) and Cogley, Primiceri and Sargent (2010). In the working paper version of their 2008 AER article, Cogley and Sbordone (2005) show that the estimate of the Calvo probability on US data appears quite stable in the post-WWII period.

[83] See also Gagnon (2009).

4 Trend inflation and the Zero Lower Bound (*ZLB*)

This Section brings the results surveyed so far to bear on the debate about the opportunity of raising the inflation target. This issue, which gained particular attention after Japan's experience in the 1990's and was much debated in U.S. academic and policy circles during the low interest rate period of 2003-2004, has reemerged since the recent financial crisis brought the Federal Reserve's policy rate effectively to zero.

The debate, however, involves two somewhat separate issues. On the one hand, whether the presence of a zero lower bound alters traditional considerations about the optimal long-run inflation rate (see Section 3.5). On the other, even if a higher inflation target can in principle mitigate the zero bound constraint, whether alternative policies should be preferred. We discuss these issues in turn.

4.1 Is low long-run inflation optimal in a *ZLB* environment?

The first question regards whether the central bank should adopt a higher inflation target in normal times to reduce the probability of hitting the *ZLB*. In the course of the survey we discussed how alternative targets for long-run inflation impact economic stability and overall welfare. The general lesson we drew is that a higher inflation target tends to decrease average output, unanchor inflation expectations, increase the volatility of the economy, worsen the policy trade-offs, and lower welfare. Hence, according to our model framework, a higher inflation target seems to be a bad policy prescription.

So how to rationalize these results with the fact that central banks typically target a positive level of inflation, most often around 2 percent per year? The literature has long pointed out some reasons why policymakers may want to target a higher rate of inflation. There are measurement errors in inflation: inflation indexes likely overstate 'true' inflation by failing to completely adjust for quality improvements. Nominal wages are arguably downwardly rigid: a moderate inflation can then 'grease the wheels' of labor market facilitating a reduction in real wages in response to negative shocks. Schmitt-Grohè and Uribe (2011) discuss both these reasons, as well as the role that uncertainty about parameter values and alternative price staggering models, both time- and state-dependent, may have in delivering a higher optimal inflation rate. But none of these modifications, they argue, generates a significant departure from a very low optimal inflation rate, typically much lower than 2 percent.

The most prominent argument in the recent debate against targeting too low a rate of inflation is, however, the risk of hitting the *ZLB* constraint. As argued by Summers (1991), since stabilizing the economy may occasionally require real negative interest rates, and nominal rates cannot fall below zero, too low an average inflation may limit the ability of central banks to conduct effective stabilization policy. A positive inflation target could provide instead more room for manoeuver to policymakers. Summers (1991) and Fischer (1996) suggested that an inflation target in the 1 to 3 percent range would allow to accommodate the need for a negative real rate of interest when a zero bound constraint is binding. Krugman (1998) made a similar argument and suggested indeed that, in its then current economic situation, Japan needed a positive inflation target of 4 percent for several years in order to generate the needed negative real rates and to curb deflation.

Raising the inflation target, though, implies raising the distortions associated with a positive rate of inflation that we discussed at length, and these distortions would remain when the natural real rate ceases to be negative. Evaluating whether a higher inflation target is appropriate would then depend crucially upon the assessment of how likely it is for the *ZLB* constraint to be hit. This in turn depends critically upon the size of the exogenous shocks that affect the economy and on the transmission mechanism of these shocks; hence it must be conducted in the context of theoretical models.

Schmitt-Grohè and Uribe (2011) analyze the issue in an estimated medium-scale macroeconomic model with both nominal and real frictions, without directly imposing the *ZLB*: they evaluate its relevance by assessing how often the optimal interest rate violates the zero bound. For a reasonable calibration of the model's exogenous shocks, they find that the optimal inflation rate remains mildly negative.[84] They also find that under optimal policy the standard deviation of the interest rate is only 0.9 percentage points at an annual rate: with a mean of the Ramsey optimal nominal interest rate estimated at 4.4 percent, that implies that "for the nominal interest rate to violate the zero bound, it must fall by more than 4 standard deviations below its target level." (p. 703) They conclude that the probability that the Ramsey optimal nominal rate violates the *ZLB* is practically zero. One important assumption underlying these results is the value of the subjective discount rate, which is set at 3 percent per year. Combined with an average growth rate of per capita output of 1.8 percent, that implies a real rate of interest in the deterministic steady state of 4.8 percent. Reducing the discount rate mitigates the above result; however, the authors note that a discount rate of 1 percent per year will still require the nominal rate to fall by almost three standard deviations below its mean for the *ZLB* to be violated. The conclusion that the *ZLB* constraint would rarely bind under an optimal monetary policy regime implies that considerations of the constraint do not essentially alter the optimal rate of inflation.[85]

Coibion et al. (2012) compute the optimal inflation rate in a setting very similar to our baseline model, where they account for a positive steady state inflation and monetary policy follows a Taylor rule subject to the zero bound constraint.[86] They find that for plausible calibrations, with costly but infrequent instances of the *ZLB* constraint, the optimal inflation is above 0, but typically lower than 2 percent. The intuition for this result is straightforward: "the unconditional cost of the *ZLB* is small even though each individual *ZLB* event is quite costly." (p. 1373) This occurs because in their model *ZLB* events, defined as 8-quarter episodes, are rare, occurring about once every 20 years. So while it is true that conditional on being in a *ZLB* situation raising the inflation target could significantly reduce the costs associated with it, from a long-run perspective the expected gain of such a policy is rather small.[87] Conversely, the cost of higher trend inflation is much more modest, but must be paid every period. That is why the optimal

[84] These shocks are: a permanent neutral labor-augmenting technology shock, a permanent investment-specific technology shock, and a temporary shock to government spending. The model also features money demand distortions in the form of a transaction technology.

[85] This is in line with the conclusion of Adam and Billi (2006) and Billi (2011), who explicitly impose the zero bound constraint on the interest rate, though in a model without trend inflation.

[86] They follow the approach of Bodenstein et al. (2009) by solving for the duration of the zero bound endogenously. The duration of the zero bound episodes as well as their frequency affect the volatility of inflation and output, which are in turn important determinants of the welfare cost of inflation.

[87] They calculate that a ZLB situation lasting for 8 quarters at 2% trend inflation is equivalent to a 6.2% permanent reduction in consumption, but the unconditional cost of that situation drops to a 0.08 percent permanent reduction in consumption, because the probability of this event is very low.

inflation rate remains below 2 percent even in presence of a *ZLB* concern.

The frequency and the cost of *ZLB* episodes depend very much upon the structure of the model and the calibration of its parameters, particularly those driving the distribution of the shocks. Coibion et al. (2012), for example, show that their results are sensitive to the calibration of the risk premium shock, because an increase in the persistence and volatility of that shock has a large effect on the frequency and duration of being at the *ZLB*, hence raising the benefit of a higher steady-state inflation. In their baseline calibration they choose a combination of parameter values that closely matches the historical incidence of the *ZLB* in the US; they show, however, that even considering "parameter values that double or even triple the frequency of hitting the *ZLB* at the historical average rate of inflation for the U.S., the optimal inflation rate rises only to about 3 percent. This suggests that the evidence for an inflation target in the neighbourhood of 2 percent is robust to a wide range of plausible calibrations of hitting the *ZLB*." (p. 1394)

4.2 Incidence of the *ZLB* constraint in more complex models

The implications we reviewed so far are normative, and obtained in the context of a single framework. The likelihood and the consequences of hitting the *ZLB*, however, have also been examined in classes of structural and statistical models different from the one we use in this survey, and through a variety of policy rules; this literature is more supportive of the need to set a higher inflation target in normal times.

An early estimate of the effects of *ZLB* for macroeconomic stability under different rates of trend inflation is provided by Reifschneider and Williams (2000). They use the FRB/US model where monetary policy follows a standard Taylor rule. Simulating the model under historical disturbances (whose variance is estimated from the equation residuals over for the 1966-95 period) they study how the steady state distribution of output, inflation and interest rate vary as policymakers change the target rate of inflation. The zero bound, in their simulations, gives rise to a trade-off between the average rate of inflation and the variability of output, but no significant trade-off between the average rate of inflation and inflation variability. They estimate that if the inflation target in the standard Taylor rule were set at 2 percent, the federal funds rate would be near zero only 5 percent of the time, and the typical *ZLB* episode would last about 4 quarters. Raising target inflation to 4 percent would reduce the incidence of the zero bound to less than 1 percent of the time, and the length of the episode to two quarters. Overall, their results suggest that, under the assumptions that fiscal policy is characterized by a historically average degree of activism, and the equilibrium real rate of interest is around 2-1/2 percent, "macroeconomic stability would likely deteriorate somewhat if the target rate of inflation were to fall below 1 or 2 percent." (p. 956)[88] The authors also discuss how the negative impact of the *ZLB* in a low-inflation environment can be mitigated by simple modifications of the Taylor rule: specifically, if the Taylor rule were to respond to the cumulative deviations of inflation and output from their target levels (which is the form of efficient simple rules in FRB/US) that would lead to a small

[88] Of course for this paper too the results depend on the features of the model. In his comments to to the authors Sims (2000), for example, is skeptical about their conclusion on the ground that the FRB/US lacks a careful treatment of real balances and of fiscal policy, which may either mitigate or aggravate the effects of the ZLB constraint.

efficiency loss from the *ZLB*.[89] In other words, policies that are efficient in the absence of the zero bound implicitly incorporate the response to past constraints that provide the best set of policy trade-offs between output and inflation variability.[90]

Williams (2009) revisits the constraint imposed by the *ZLB* using again the FRB/US model. His objective is to estimate how much the *ZLB* has limited monetary policy during the recent recession and evaluate the optimal inflation rate in light of this experience. He comes to two main conclusions. First, the *ZLB* has been a significant factor in slowing the recovery both in the US and other economies: the inability of reducing further the nominal interest rate, he computes, has imposed a cost in term of lost output of about \$1.8 trillion. Second, if the current conditions imply a permanently lower equilibrium real rate, then a 2 percent steady state inflation may indeed be an inadequate buffer against the *ZLB* and it may be prudent to aim at a moderately higher rate.[91] Williams' analysis is further expanded by Chung et al. (2012) to assess whether estimates of the frequency of the *ZLB* in the literature were too benign. They use an array of statistical and structural models to investigate what we can learn from recent events about the expected frequency, duration and magnitude of *ZLB* episodes, and how severe the zero bound constraint has been in the current crisis. The authors consider both structural models - FRB/US, EDO (a medium-size DSGE model developed at the Board), and the Smets and Wouters (2007) model - and statistical models, which impose less tight constraints and allow the possibility of structural changes.[92] For each model they conduct stochastic simulation exercises.

Comparing the actual course of events with what each model would have predicted prior to the crisis, from the standpoint of 2007 (the authors base the forecast distribution on historic shocks, starting from 1960), they observe that the drop in output and the federal funds rate, as well as the increase in unemployment, were big surprises, while there was no much surprise in inflation, given that only a mild disinflation occurred. Furthermore, while the structural models deliver very low probability of hitting the *ZLB*, the statistical models give a 2 to 9 percent probability. The probability of hitting the *ZLB*, as well as that of being stuck at the *ZLB* for 4 or more quarters, in general increase in the structural models when the estimation sample is extended to 2010 (which gives three more years of information about the shocks), and when the simulations account for the uncertainty about model parameters and latent variables; by converse, these probabilities decrease when only observations for the Great Moderation period are used.

The broad message of Chung et al. (2012) is that uncertainty about model parameters significantly increases the probability of hitting the zero bound; that parametrizations based on the Great Moderation period tend to understate the incidence and severity of the *ZLB*; and that the propagation mechanism of typical DSGE models is

[89] Efficient simple rules are those delivering the best combinations of standard deviations of inflation and output gap known as the policy frontier (see Williams 2009).

[90] Price-level targeting rules are a particular case of the modified Taylor rules considered, hence they can overcome the effect of the zero bound, as shown in Wolman (1998). We discuss this point later.

[91] This conclusion is based on a calibration of the equilibrium real rate at 1 percent, and a distribution of shocks estimated from data covering the pre-Great Moderation period only. Both assumptions are arguably quite extreme (see Woodford, 2009a).

[92] The statistical models considered are a time-varying parameter *VAR*, the Laubach and Williams (2003) model, which allows a unit root in output growth and the equilibrium interest rate, and a univariate model with GARCH error processes.

unable to generate sustained periods of policy stuck at the *ZLB*.

The question, however, is whether pinning down the probability of hitting the zero bound can resolve the issue of whether the long-run target for inflation should be raised, or whether one should consider alternative policies that could help respond to the kind of negative shocks that brought about the Great Recession. We turn to this issue next.

4.3 History-dependent policy as an alternative to a higher inflation target

Interpreting the conclusion of simulations performed under a 'simple' Taylor rule for monetary policy requires some caution. As noted by Reischneifer and Williams (2000), and demonstrated by Eggertsson and Woodford (2003), such rule is a poor form of policy in the case in which the zero bound binds, because it is a commitment to a purely forward-looking policy, that is a policy that takes into account at each point in time only of the evolution of the economy from that point on; as soon as the constraint is no longer binding, policy is again conducted as it would be if the constraint had never hit. As we discussed above, Reischneifer and Williams (2000) note indeed that the economy would perform better under a modified Taylor rule that responds "to the *cumulative* deviations of output and inflation from their respective target levels." (p. 961) Eggertsson and Woodford (2003) address this issue and show how rules that incorporate a form of history dependence, as those do, can approximate optimal policy in the presence of the zero bound constraint. They recast Krugman's (1998) analysis in a simple dynamic general equilibrium model, where policymakers objective function trades off inflation and output stabilization, and characterize optimal policy subject to the zero bound constraint. They do find that the zero bound constrains stabilization policy in a low inflation environment when a real shocks lowers the natural rate of interest. They show, however, that it is possible to mitigate the effects of the zero bound constraint by creating "the right kind of expectations regarding how monetary policy will be used after the constraint is no longer binding, and the central bank again has room to maneuver." (p. 143).

In the sort of dynamic models that we discussed the effectiveness of monetary policy rests on its ability to affect private sector's expectations about the future path of the short term rates, because these determine equilibrium long-term rates which in turn affect most spending decisions. Hence "a commitment to create subsequent inflation involves the commitment to *keep interest rates low for some time* in the future", (Eggertsson and Woodford, p. 144) and this commitment can help stimulate aggregate demand today, even if current nominal rates cannot be lowered any further.

The commitment to a history-dependent policy is key to generate higher inflation expectations and help exiting the zero lower bound. This commitment to future policy can achieve what a higher inflation target is supposed to achieve, according to its advocates: generating expectations of inflation in the future (when the constraint would no longer bind) in order to obtain the desired decline in the current real rate. Rather than permanently raise the inflation target, policymakers can generate these expectations by being explicit about how they will conduct policy *after* the zero bound constraint is no more binding.

Eggertsson and Woodford (2003) also show that the optimal commitment takes the form of a price level targeting: people should expect inflation to be only temporarily high following periods in which the price level has fallen below its target path: the price level target implements a sort of history-dependent inflation target.[93]

The “forward guidance” adopted by the Federal Reserve in the recent crisis is consistent with the history dependence feature of Eggertsson and Woodford’s optimal policy. Indeed, their recommendation of compensating for the current zero bound constraint by providing more accommodation in the future is echoed in the FOMC’s post-meeting statement language adopted since September 2012: “... the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens.”[94] It has also found expression in a concrete policy proposal advanced by the Chicago Fed President Evans (2012). He argued for the adoption of a state-contingent price level targeting regime, and illustrated the way it would work in the specific case of a desired price path which would grow at 2 percent per year (the FOMC’s long-run inflation objective, see fn. 1). Every time the annual rate of inflation would run below 2 percent, as it had been the case on average since the recession started, he argued, the price level would have to grow at a faster rate afterwards, in order to close the gap between the actual and the desired price paths. The expected path of inflation would be therefore for some time above the long-run inflation target of 2 per cent, which continues to be the long-run objective.[95]

Similar recommendation is made by Coibion et al (2012), the contribution that we discussed before. They derive optimal policy rules under commitment and simulate the model under baseline parameters. They obtain that when the central bank can commit to the Ramsey policy the cost of *ZLB* can be negligible. This is because in the event of a large shock, the credible commitment to keep the interest rate low for an extended period reduces the impact of the shock and helps exiting the *ZLB* sooner. The impact of a negative shock is therefore smaller and the *ZLB* events are shorter, despite the fact that the optimal inflation rate is very close to zero, hence the zero bound binds more frequently. Commitment is the key for this result: the optimal policy under discretion would entail much higher costs of a *ZLB* situation, and a higher inflation rate of about 2.7 percent. Evaluating price level targeting rules, Coibion et al (2012) obtain that such policies stabilize expectations, reducing substantially output and inflation volatility and increasing welfare. Under those rules the zero bound binds less frequently even at a low level of steady state inflation.[96]

Nominal GDP targeting rules contain as well the history dependence feature of price-level targeting, and have therefore been advocated by some, both in the academic and business communities, as another promising alternative to an increase in the long-run inflation objective for addressing the *ZLB* constraint (e.g. Woodford, 2012; Hatzius and

[93] A similar argument holds if instead of targeting the price level, monetary policy is going to use the exchange rate as an instrument (Svensson, 2003; McCallum, 2000).

[94] FOMC post-meeting statement, September 13, 2012. From October 2012 on, that sentence of the statement was amended to include a reference to the purchase program: “.. the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the purchase program ends and the economic recovery strengthens.”

[95] Price level targeting has been for some time under the attention of Bank of Canada researchers: e.g. Cote’, 2007, Ambler, 2009 and Crawford et al., 2009.

[96] Analyzing the Japan "lost decade", Leigh (2010) shows that simply increasing the inflation target would have had only limited and short-lived effects on output, while a price-level targeting rule would have proved a much better stabilization tool.

Stehn, 2011).[97]

5 Conclusion

The literature lacks a comprehensive discussion about the implications of different rates of trend inflation, and the macroeconomic effects of conducting policy with a higher inflation target. In this survey we contribute to fill this gap by reviewing a number of empirical, theoretical, and policy issues related to setting a positive rate of trend inflation in an otherwise standard macro model that represents the workhorse for the analysis of monetary policy.

We start the survey by stressing the importance of accounting for the evolution of trend inflation in empirical models of inflation dynamics. In Section 2 we discuss the impact of a time-varying inflation trend on measuring the persistence of inflation and accounting for inflation dynamics. In this respect, the theoretical underpinnings of time variation in trend inflation and its relation to the policymakers' inflation target remain an active area of research.

In Sections 3 we review the theoretical implications of moderate levels of trend inflation for the conduct and the effects of monetary policy, both from a positive and a normative point of view. We conclude, in section 4, by evaluating the proposal of rising the inflation target to address the need for a more stimulative policy in a *ZLB* situation, a proposal that generated a lively policy debate. The results surveyed in Section 3 suggest that rising the inflation target can carry significant costs. On the other hand, these costs need to be evaluated against the importance of staying away from the *ZLB*, and the uncertainty about the efficacy of alternative monetary policy tools in such environment.

[97] Discussion of other policies to stimulate the economy when at the ZLB are beyond the scope of this Survey.

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