# Product Design in Selection Markets*

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February 2014

**Abstract**

Insurers choose plan characteristics to sort for profitable consumers. In a model with multidimensional types, this sorting incentive is proportional to the covariance, among marginal consumers, between marginal willingness-to-pay and cost to the insurer. Standard forms of cost-sharing successfully repel costly consumers, but reducing plan comprehensiveness instead alienates the risk-averse. In a perfectly competitive equilibrium, the sorting incentive must vanish. Market power increases insurance quality and welfare. Nonetheless, a competitive equilibrium with positive insurance is possible when insurance value is sufficiently negatively correlated with cost. However, in a calibration to Handel, Hendel and Whinston (2013)'s data, equilibrium still fails to exist.

**Keywords**: selection markets, cream-skimming, insurance markets, multidimensional heterogeneity, product design

**JEL Classification Codes**: D41, D42, D43, D86, I13

*This paper replaces earlier working papers "Multidimensional Heterogeneity and Platform Design" and "Multidimensional Product Design". We thank Eduardo Azevedo, Bruno Jullien, Renato Gomes, Jacques Crémer, Matthew Gentzkow, Jon Levin, Jesse Shapiro, Mike Whinston and many workshop and conference participants for helpful comments as well as Jonathan Baker, Liran Einav, Jean Tirole and Preston McAfee for excellent formal discussions of the paper. We acknowledge the financial support of the NET Institute, the research program on intellectual property at the Institut d'Economie Industrielle (IDEI), the Marion Ewing Kauffman Foundation and the Fundação para a Ciência e a Tecnologia which funded the research assistance of Kevin Qian. We are especially grateful to Ben Handel, Igal Hendel and Mike Whinston for providing us the summary statistics that allow our calibration.

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# 1 Introduction

Insurers choose non-price characteristics of their products, such as co-insurance rates and deductibles, to selectively attract the most valuable customers. For instance, in Rothschild and Stiglitz (1976) (henceforth RS), lowering insurance quality disproportionately hurts high risk consumers, so insurers "skim the cream" from rivals by raising cost-sharing and lowering price, which causes a "death spiral" towards zero insurance. However, such a strategy can be ineffective if the risk-averse are also repelled, and if these consumers are also typically less costly, as found by Finkelstein and McGarry (2006) and Fang, Keane and Silverman (2008). Despite this, existing analyses of product design in selection markets assume individuals are only heterogeneous along a single dimension, usually risk-type. This assumption rules out sorting incentives in monopoly models (Stiglitz, 1977; Veiga and Weyl, 2013) and eliminates competitive equilibria (RS, Riley, 1979). In this paper we propose a simple characterization of sorting incentives in the presence of continuous multidimensional types that implies the possibility of a competitive positive insurance pooling equilibrium, quantifies how market power affects equilibrium product design, and can be calibrated based on simple reduced-form statistics.

We allow for flexible forms of consumer heterogeneity and preferences and gain tractability by assuming that the set of contracts offered can be described by a vector, as it is in RS, rather than a menu rich enough to match the richness of heterogeneity. Thus we do not allow for non-linear pricing of a continuum of products, described by a function as in Stiglitz (1977) and Rochet and Choné (1998). Our analysis is based on the characterization of the marginal incentive of an insurer to use a non-price product characteristic, such as an actuarial rate, to sort in favor of low-cost consumers. This is the product of two components. The first component is the density of marginal consumers and captures how *many* buyers change their purchase decision when a product characteristic changes. The second component captures *which* buyers adopt the product. This second term equals the covariance, within the set of marginal buyers, between the marginal willingness-to-pay (WTP) for the non-price product characteristic and the cost of consumers to the firm. This covariance vanishes in one-dimensional models because the set of marginal types is a singleton.

Although this covariance is endogenous, it can be characterized to yield economically important results. We begin by showing that the covariance is signed when marginal WTP and cost can be written as monotonic functions of an (endogenous) common index. This occurs naturally in environments with non-linear pricing or when types are bidimensional. We illustrate this result in a simple model where individuals differ in expected loss and risk aversion. Increasing the generosity of a linear actuarial rate disproportionately attracts the

worse risks, which we refer to "adverse sorting." We also show (computationally) that more generous deductibles and indemnity caps also sort adversely, but increasing the probability that a shock is covered ex-post (a measure of plan comprehensiveness) sorts advantageously. These results help explain why contracts typically share costs along dimensions such as deductibles and coinsurance rates, but usually offer plans covering all catastrophic outcomes.

We then extend the model to a simple competitive setting à la Hotelling (1929). We assume firms are symmetrically differentiated, that the market is covered and that Hotelling preferences over insurers are independent of risk and risk aversion. Moreover, we focus on marginal incentives at symmetric (pooling) contracts, which we call a "local deviations pooling equilibrium" or LDPE. In a simple bidimensional context, the LDPE is unique whenever it exists. However, full insurance is never an LDPE even thought it is social optimal in the absence of moral hazard. The incentive to increase cost-sharing near full insurance is always positive because marginal insurance value vanishes near perfect insurance. Therefore, insurance at a competitive LDPE is always insufficient. By contrast, a monopolist has a socially optimal incentive to provide insurance quality (albeit at an excessive price) because it internalizes the socially harmful effects of cream-skimming. Therefore, as market power increases and reduces the first element of the sorting incentive, insurance quality rises. In the absence of moral hazard, this implies that market power increases welfare.

We then consider the case of perfect competition. When consumers differ only in their risk type (RS; Riley, 1979) the incentive to cream-skim undermines any positive level of insurance under perfect competition. Multidimensional types re-introduce the possibilty of a pooling equilibrium by mitigating the second component of the sorting incentive if and only if value for insurance is sufficiently negatively correlated with mean risk, as found empirically by Finkelstein and McGarry (2006). In the limit of undifferentiated Bertrand competition, LDPE insurance has a simple structure: the covariance term must vanish because the density of switching consumers becomes arbitrarily large. If $\beta$ is the OLS regression coefficient of insurance value on mean risk in the entire population, a competitive LDPE with positive coverage requires $\beta < -1$. When this is the case, sorting is sufficiently advantageous that insurers gain from raising cost-sharing above zero. When $\beta$ is sufficiently positive, the unique LDPE features zero insurance. For an intermediate range of $\beta$, no LDPE exists for the same reasons as in RS: there are first order losses from raising risk-sharing above zero, but there are also second order gains from doing so.

We calibrate our analysis with summary statistics from Handel, Hendel and Whinston (2013) (henceforth HHW), who adopt a CARA-Normal framework to measure the joint distribution of types.[1] In their data, $\beta \approx 13$, implying that only zero insurance is a candidate

[1] We thank these authors for generously sharing with us these statistics.

LDPE. However, second-order conditions show that zero insurance is not an LDPE either and thus the RS non-existence result holds in this setting. Our approach thus uses a simple empirical summary statistic to explain the extreme adverse selection and market collapse found in HHW's more detailed structural analysis. While market power restores positive insurance in LDPE and approaches the first-best as it grows large, a relative-to-cost mark-up of almost 170% is necessary for an LDPE to even exist, and a markup of twice cost is necessary to achieve an actuarial rate of 80%.

We then perform three additional calibration exercises as robustness checks to our analysis of competition. First, we allow for moral hazard. This implies that full insurance is not socially optimal, but it is still the case that LDPE insurance (zero) is dramatically below the socially optimal level (86%). Second, we discuss why the welfare enhancing effect of market power appears to contradict existing results (Armstrong and Vickers, 2001; Rochet and Stole, 2002), where competition drives out distortions from second-degree price discrimination, leading to efficiency. The reason is that these models consider ex-post contracting where *only moral hazard* (ex-post efficient consumption) is relevant while we consider an ex-ante situation where *only insurance* is relevant. We give an example of a model where consumers first incur health shocks and then purchase healthcare ex-post. Ex-post welfare is maximized when consumers internalize their costs, which occurs under perfect competition. A calibrated version of this model is (positively) quantiatively similar to our benchmark model while yielding opposite welfare conclusions. Third, we consider market expansion. When the market is not covered, market power redcued the quantity of individuals covered while increasing the quality of insurance, so the effect on welfare is ambiguous. However, at the interior optimal degree of market power (a relative mark-up of around 100%), welfare is remarkably 98% of its level at the social optimum, thus reinforcing the conclusion that market power is may be beneficial to solve the cream-skimming problem.

The remainder of the paper is organized as follows. Section 2 presents our results in a simple CARA-normal setting where users differ in mean risk and risk aversion. The emphasis is on the discussion of our substantive assumptions, the economic content of the model and sketches of the proofs. Section 3 contains the empirical calibration to the HHW data. Section 4 contains proofs and generalizations. Section 5 contains robustness checks. We conclude in Section 6, where we discuss the applicability of our approach beyond insurance markets and policy implications of our analysis. Less instructive proofs, detailed calcuations and details of our calibration exercises are collected into the appendices following the main text.

# 2 A Simple Model of Sorting

This section develops the main results of the paper in a simple CARA-Normal setting where insurance is characterized by a linear actuarial rate and consumers differ in risk and risk aversion. We describe the ingredients of the model, consider a monopoly insurer, then extend the model to a competitive setting.

## 2.1 Setup

A unit mass of individuals face an uncertain verifiable wealth shock. A monopoly commits to absorbing a share $x \in [0,1]$ of the shock for a price $p \in \mathbb{R}_+$. The actuarial rate $x$ captures the quality of insurance, or $1-$ the amount of cost sharing. Consumers have constant absolute risk aversion (CARA) preferences, so maximize the expected value (over realizations of the shock) of $-e^{-aw}$, where $w$ is final wealth and $a > 0$ is absolute risk aversion. Wealth shocks are normally distributed with mean $\mu > 0$ and variance $\sigma^2 > 0$. The expected cost to the risk-neutral insurer of providing quality $x$ to a type $\mu$ is $c(x,\mu) \equiv x\mu$.

Initial wealth is zero. Expected utility without insurance is $-\exp\left\{a\mu + \frac{a^2\sigma^2}{2}\right\}$, while with insurance it is $-\exp\left\{a(1-x)\mu + ap + \frac{(1-x)^2a^2\sigma^2}{2}\right\}$. Willingness-to-pay (WTP) for $x$ is the price at a given individual has these two expected utilities be equal. This WTP is

$$u(x,\mu,v) \equiv x\mu + \frac{1-(1-x)^2}{2}v.$$

where $v \equiv a\sigma^2 > 0$ is an individuals value for insurance, the amount by which she values insurance beyond the off-loading of mean risk to the insurer ($x\mu$). The vector $(\mu, v)$ summarizes the relevant bidimensional individual heterogeneity and it is not contractible.

Buyers are those for whom WTP exceeds price: $u(x,\mu,v) > p$. For $x > 0$, we equivalently define the set of buyers by $\mu > \mu^\star(p,x,v) \equiv \frac{1}{x}\left(p - \frac{1-(1-x)^2}{2}v\right)$. The set of marginal buyers are those for whom $\mu = \mu^\star(p,x,v)$. The set of of buyers is a two-dimensional surface, but the set of marginal consumers is a one-dimensional curve in $\mathbb{R}^2$, as in Figure 1.

The insurer knows $(\mu, v)$ is distributed according to the atomless and full support probability density function $f(\mu,v) : \left[\underline{\mu},\overline{\mu}\right] \times [\underline{v},\overline{v}] \to \mathbb{R}_{++}$. Defining $\mu^\star(p,x,v)$ allows us to express the quantity of buyers as a standard iterated integral: $Q(p,x) \equiv \int_{\underline{v}}^{\overline{v}} \int_{\mu^\star(p,x,v)}^{\overline{\mu}} f(\mu,v)\, d\mu dv$. Intuitively $\frac{\partial Q(p,x)}{\partial p} < 0$, so there exists a price $P(x,q)$ which solves $Q(P(x,q),x) \equiv q$ for any $q$. This allows us to follow Spence (1975) in defining the firm's profits as a function of quantity $q$ and quality $x$. As we explain below, it is useful to consider changes in $x$ holding

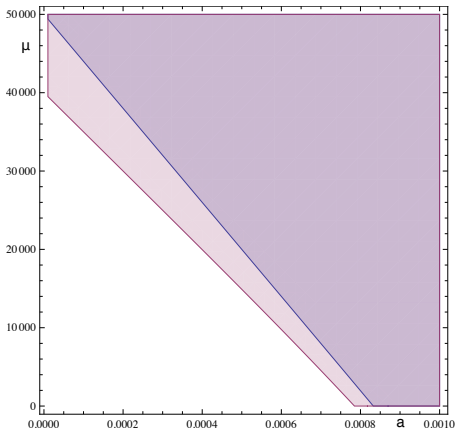


Figure 1: Set of buyers for $\mu \in [0, 10^4]$, $a \in [10^{-5}, 10^{-3}]$, $\sigma^2 = 10^4$ and $p = 40000$. Quality is $x = 0.8$ (blue) and $x = 1$ (red).

fixed the total number of buyers, which is straightforward in this setup. Total cost is

$$C = \int_{\underline{v}}^{\overline{v}} \int_{\mu^\star(P(q,x),x,v)}^{\overline{\mu}} c\left(x,\mu\right) f\left(v,\mu\right) d\mu dv.$$

and profit is $\Pi = qP\left(q,x\right) - C$.

As is typical, $\mathbb{E}_u\left[\zeta\left(x,\mu,v\right) \mid \mu > \mu^\star\right] \equiv \frac{1}{q}\int_{\underline{v}}^{\overline{v}} \int_{\mu^\star(P(q,x),x,v)}^{\overline{\mu}} \zeta\left(x,\mu,v\right) f\left(v,\mu\right) d\mu dv$ for an integrable function $\zeta\left(x,\mu,v\right)$. The density of marginal users is $M \equiv -\frac{\partial Q(p,x)}{\partial p}$. Let $\mu^\star = \mu^\star\left(p,x,v\right)$. Then $\mathbb{E}_u\left[\zeta\left(x,\mu,v\right) \mid \mu = \mu^\star\right] = \frac{\int_{\underline{v}}^{\overline{v}} \zeta(x,\mu^\star,v) f(\mu^\star,v) dv}{\int_{\underline{v}}^{\overline{v}} f(\mu^\star,v) dv}$ is the expectation conditional on the margin, and similarly for the covariance along the margin $\mathrm{Cov}_u\left[\cdot \mid \mu = \mu^\star\right]$.[2]

## 2.2 Monopoly

Given this setup, a profit-maximizing monopoly's First Order Condition (FOC) is captured by the following result, where functional arguments are omitted for simplicity. We denote $\frac{\partial u(x,\mu,v)}{\partial x} \equiv u'$ and $\frac{\partial c(x,\mu)}{\partial x} \equiv c'$.

**Proposition 1.** *A necessary FOC for a monopoly's profit maximizing choice of $x$ is*

$$\underbrace{-q\mathbb{E}\left[c' \mid \mu > \mu^\star\right]}_{Cost} + \underbrace{q\mathbb{E}_u\left[u' \mid \mu = \mu^\star\right]}_{Spence\ term} = \underbrace{M\mathrm{Cov}_u\left[u', c \mid \mu = \mu^\star\right]}_{Sorting}, \tag{1}$$

*Proof.* By Leibniz's rule, $M = -\frac{\partial Q(p,x)}{\partial p} = \frac{1}{x}\int_{\underline{v}}^{\overline{v}} f\left(\mu^\star, v\right) dv$ .

[2]Typically, defining $\mathbb{E}_u\left[\cdot \mid \mu = \mu^\star\right]$ requires some care to define an economically useful measure on this set. However, in this simple example, such complications do not arise and therefore we delay discussing them until Section 4.

By the Implicit Function Theorem (IFT), $u\left(x, v, \mu^{\star}\left(x, v, p\right)\right) \equiv p \Rightarrow \frac{\partial \mu^{\star}}{\partial x} = -\frac{1}{x}u'$. By the IFT, $Q\left(P\left(x, q\right), x\right) = q \Rightarrow \frac{\partial P}{\partial x} = -\frac{\partial Q/\partial x}{\partial Q/\partial p}$. By Leibniz's rule $\frac{\partial P}{\partial x} = -\frac{\partial Q/\partial x}{\partial Q/\partial p} = -\frac{M\mathbb{E}_u[u'|\mu=\mu^{\star}]}{-M} = \mathbb{E}_u\left[u' \mid \mu = \mu^{\star}\right]$. Similarly by Leibniz's rule, $\frac{\partial C}{\partial x} = -M\mathbb{E}_u\left[u'c \mid \mu = \mu^{\star}\right] + q\mathbb{E}\left[c' \mid \mu = \mu^{\star}\right]$. Moreover, $\frac{\partial C}{\partial p} = M\mathbb{E}_u\left[c \mid \mu = \mu^{\star}\right]$. The FOC is $\frac{\partial \Pi}{\partial x} = q\frac{\partial P(x,q)}{\partial x} - \frac{\partial C}{\partial x} - \frac{\partial C}{\partial p}\frac{\partial P(x,q)}{\partial x} = 0$. Replacing each of these elements of the FOC yields the result. For details, see Theorem 1. □

This characterization decomposes the insurer's marginal incentive to raise quality ($x$) into three components. The first two are familiar. First, the monopolist loses the average increase in the cost of buyers ($\mathbb{E}\left[c' \mid \mu > \mu^{\star}\right]$) multiplied by the number of buyers ($q$), which follows mechanically from the additional share of the shock absorbed by the insurer. Second, increasing $x$ causes the number of buyers to increase. In order to keep $q$ fixed, price implicitly increases to all buyers ($q$) by the average marginal WTP of marginal consumers ($\mathbb{E}_u\left[u' \mid \mu = \mu^{\star}\right]$), as in Spence (1975).[3]

The third effect is the focus of our analysis. Increasing $x$ not only changes the *number* of buyers, but also their *composition* since $x$ attracts some individuals (those with large $u'$) more than others. This alters the composition of buyers, sorting in favor of marginal buyers with high $u'$. All buyers pay the same price $p$, so the impact that buyer composition has on profit depends on whether the marginal consumers most strongly attracted by $x$ tend to be also those with particularly high cost ($c$). If this is the case, sorting increases the insurer's costs. It is therefore natural that the effect of sorting on profit is captured by the covariance, among marginal consumers, between the cost of providing them with the product and their WTP for an increase in $x$: $\mathrm{Cov}_u\left[u', c \mid \mu = \mu^{\star}\right]$. If $\mathrm{Cov}_u\left[u', c \mid \mu = \mu^{\star}\right] > 0$, additional quality ($x$) increases average cost, so there is "adverse sorting." If $\mathrm{Cov}_u\left[u', c \mid \mu = \mu^{\star}\right] < 0$, there is "advantageous sorting."

It is useful to distinguish sorting from what is commonly referred to as selection, which captures the response of average cost to a change in the quantity of buyers. "Adverse selection" occurs when, among all buyers, those with higher *level of WTP* are most costly. Then, as the number of buyers increases (price falls), average cost falls as in Akerlof (1970) and Einav, Finkelstein and Cullen (2010). "Adverse sorting" occurs when, among marginal buyers, those with higher *marginal WTP* are most costly. Then, holding fixed the number of buyers, as quality $x$ increases, the effect this has on the composition of buyers causes average cost to increase.

Importantly, the sorting term vanishes when consumers are homogeneous in their marginal WTP for $x$ or in their cost. It also vanishes when there is a unique type of marginal consumer, as in all unidimensional type models we are aware of. Finally, notice that this effect

[3]The effect was independently and nearly simultaneously identified by Sheshinski (1976), although we follow the convention of associating it with Spence.

is scaled by $M = -\frac{\partial Q}{\partial p}$ which captures the responsiveness of demand. Loosely speaking, in Figure 1, $M$ captures translations of the line that defines the marginal set (changes in the number of buyers), while $\text{Cov}_u\left[u', c \mid \mu = \mu^\star\right]$ captures rotations of that line (changes in the composition of buyers).

## 2.3 Signing the sorting incentive

The sorting term may seem an unlikely object to focus on since it is endogenously determined by $q$ and $x$. However, its sign can be ascertained directly and exogenously in a variety of contexts to yield results of economic importance. We use these results to determine the sign of sorting for canonical insurance design dimensions such as a deductible or an indemnity cap. In this section, we do not use the notation $v \equiv a\sigma^2$.[4]

An advantage of the simple two-dimensional setting described above is that, in this case, the set of marginal consumers is a one-dimensional curve. Then, conditional on this set, the functions $u'(x, \mu, a)$ and $c(x, \mu)$ become $u(x, a, \mu^\star(p, x, a))$ and $c(x, \mu^\star(p, x, a))$ and are therefore univariate in $a$ for a given $(p, x)$. Then, if two univariate functions are co- or anti-monotone, the covariance between them is signed.[5] This is stated in the following result which, importantly, applies for instruments other than a linear actuarial rate. Let $u = u(x, \mu, a)$.

**Proposition 2.** *Let* $S = S(p, x, a, \mu^\star(p, x, a)) \equiv \frac{\partial^2 u/\partial x \partial a}{\partial u/\partial a} - \frac{\partial^2 u/\partial x \partial \mu}{\partial u/\partial \mu} \mid_{\mu=\mu^\star(p,x,a)}$. *If* $S$ *is signed, it has the opposite sign of* $\text{Cov}_u\left[u', c \mid \mu = \mu^\star\right]$.

*Proof.* Marginal buyers have WTP equal to price. Since $\frac{\partial u}{\partial \mu} > 0$ and $\frac{\partial u}{\partial a} > 0$, for a fixed level of WTP, $\frac{d\mu^\star}{da} < 0$, as in Figure 1. Thus $\frac{dc(x,\mu^\star(p,x,a))}{da} = x\frac{\partial \mu^\star}{\partial a} < 0$ is monotonic. Then, the sorting incentive depends only on $\frac{d}{da}\left(\frac{\partial u(x,a,\mu^\star(p,x,a))}{\partial x}\right) = \frac{\partial^2 u}{\partial x \partial a} + \frac{\partial^2 u}{\partial x \partial \mu}\frac{\partial \mu^\star}{\partial a}$. Since $\frac{\partial \mu^\star}{\partial a} = -\frac{\partial u/\partial a}{\partial u/\partial \mu}$ and $\frac{\partial u}{\partial a} > 0$, this has the same sign as $S$. If $S < 0$, then $u'$ is co-monotonic with $c$ within the margin, so sorting is adverse, and vice-versa. For an extension, see Theorem 2 □

We are concerned with the set of marginal users, where the level of WTP is constant (and equal to price). Therefore, sorting is adverse when risk ($\mu$) raises marginal WTP relative to level of WTP more than risk aversion ($a$) raises marginal WTP relative to the level WTP. Theorem 2 below shows that this logic holds for more general preferences and cost functions, namely if types are bidimensional. Then, if $\theta_i$ is the type dimension that increases marginal WTP of $x$ most rapidly relative to the rate it increases WTP, and $\theta_i$ also increases cost most

[4] This notation is not useful when considering instruments other than a linear actuarial rate. However, in the case of the linear actuarial rate, all results stated in this section remain true if $a$ is replaced by $v = a\sigma^2$ even if $\sigma^2$ is allowed to be heterogeneous across individuals.

[5] For a simple proof, see Schmidt (2003).

rapidly relative to the rate at which it increases WTP, then $x$ sorts adversely. Since the curvature of a function captures the difference between its margin and its average, sorting favors the dimension of type which generates a more convex demand for $x$. The following corollary is immediate.

**Corollary 1.** *An actuarial rate sorts adversely (*$\text{Cov}_u\left[u', c \mid \mu = \mu^\star\right] \geq 0$*).*

*Proof.* $\frac{\partial^2 u/\partial x \partial a}{\partial u/\partial a} - \frac{\partial^2 u/\partial x \partial \mu}{\partial u/\partial \mu} = -\sigma^2 \frac{x}{2} \leq 0.$ □

WTP is $u = x\mu + \frac{1-(1-x)^2}{2} a\sigma^2$. Demand induced by $\mu$ is linear, while that induced by $a$ is concave in $x$. There is a constant incentive to transfer mean risk, but insurance value has decreasing returns. For profitable marginal individuals (high $a$, low $\mu$) additional $x$ has rapidly decreasing marginal benefit, but for unprofitable marginal individuals (high $\mu$, low $a$) each unit of $x$ has only slowly decreasing marginal benefit. This is why raising $x$ attracts bad risks more intensely than good risks, as suggested by Figure 1.

It is also useful to consider sorting by instruments other than an actuarial rate. Since these cases are less analytically tractable, we proceed computationally, providing details in Appendix A. For realism, we consider cases where reimbursements occur only for negative shocks and study three instruments: a standard actuarial rate (a share $x$ of the shock are covered), a deductible (value of shock above a threshold $x$ are covered), and an indemnity cap (shock is covered up to a value $x$). Finally, we consider a measure of comprehensiveness, where there is a probability $x$ that a shock is covered. We assume shocks $l$ have a Gaussian density $f$ with heterogeneous mean $\mu$ and homogeneous variance $\sigma^2$. The insurer makes a payment $G(l, x) \geq 0$ when the individual incurs a loss $l$. Consumers have CARA utility. The homogeneous initial wealth is $w_0$. Without insurance, final wealth is $w_N = w_0 - l$. With insurance, it is $w_I = w_0 - l + G(l, x) - p$. Expected surplus from insurance is $U = \int \left[e^{-aw_N} - e^{-aw_I}\right] f dl$. Appendix A shows how to express WTP (and thus $S$) in terms of $U$. Computationally, we generate draws of $(p, x, a)$, compute the value of $\mu = \mu^\star(p, x, a)$ that makes an individual marginal ($U = 0$), then evaluate $S\,|_{\mu=\mu^\star(p,x,a)}$. We calibrate our analysis following HHW: we set $\sigma^2 = 10^8$, which is the mean value of $\sigma^2$ is that data. We consider $a \in [10^{-5}, 10^{-3}]$ and $\mu \in [0, 5 \cdot 10^4]$, which are approximately the ranges of risk aversion and risk in that data. We focus on the relatively high levels of insurance and assume insurance is perfect along every dimension other than the one we consider. The computational output justifying the following claims is in Appendix A.

*Claim* 1. Additional insurance sorts adversely ($\text{Cov}_u\left[u', c \mid \mu = \mu^\star\right] > 0$) when:

- $x$ is an actuarial rate, so $G(l, x) = \max\{0, xl\}$.

- $x$ is a deductible, so $G(l, x) = \max\{0, l - x\}$.
- $x$ is an indemnity cap, so $G(x, l) = \min\{\max\{0, l\}, x\}$.

These canonical dimensions of insurance quality sort adversely (for a deductible, increasing $x$ implies less generous insurance). This suggests an explanation for why insurers have an incentive to share costs with consumers relative to full insurance, since this would improve the composition of its buyers. Notice that this does not imply that insurance would be reduced to zero: a monopoly must balance the incentive to sort on the right hand side (RHS) of Equation 1, with the incentive to exploit the gains from trade from more insurance on the left hand side (LHS).

Finally, we consider an instrument that sorts advantageously even at full insurance. We assume an insurer covers the full loss $l$ with probability $x$, which we refer to as the plan's "comprehensiveness." This can be thought of as the share of conditions that are covered by insurance, or the completeness of the insurance contract.[6] Final wealth without insurance is $w_N$. With insurance, with probability $x$, final wealth is $w_{I+} = w_0 - p$; with probability $1 - x$ it is $w_{I-} = w_0 - l - p$. Surplus from insurance $U$ is as above.

*Claim* 2. Comprehensiveness sorts advantageously ($\text{Cov}_u[u', c \mid \mu = \mu^\star] < 0$).

In this case, additional insurance (increasing $x$) sorts advantageously even at full insurance ($x = 1$). Given that gains from trade also tend to raise coverage towards full insurance, this suggests that insurers have a incentive to offer contracts that are as complete as possible, since this disproportionately attracts the risk-averse (although doing so may entail additional costs that are not modeled above). In particular, no insurance contract that we are aware of explicitly makes the probability of reimbursement random.

## 2.4 Competition

The general message of RS and of a large subsequent literature is that competing insurers have incentives to offer low levels of insurance, and that, as a result, equilibrium may not exist. In their conclusion, RS suggest that frictions, such as multidimensional types or imperfect competition, might mitigate these extreme results. Consistent with this hypothesis, Einav, Finkelstein and Cullen (2010) and others have shown how multidimensional types can mitigate or even reverse the welfare effects of adverse selection (that is, selection on the number of individuals à la Akerlof (1970), without endogenous product quality).[7] Despite

[6] This is equivalent to: 1) a condition is drawn from a $\mathcal{U}[0, 1]$ distribution; 2) an associated loss drawn from a $\mathcal{N}(\mu, \sigma^2)$. Then $x$ captures the share of conditions covered. Notice that all conditions have the same likelihood and loss distribution. The term $x$ can also be thought of as capturing the completeness of the contract offered by the insurer.

[7] See Einav and Finkelstein (2011) for an excellent summary of the literature.

this, to our knowledge, no paper has quantified the effect of such frictions on outcomes in the RS environment, where non-price product characteristics are endogenous.[8]

Our characterization of the sorting incentive allows us to shed light on the interaction of both these frictions with product design. We confirm RS's intuition that these frictions may restore the existence of a perfectly competitive pooling equilibria with positive insurance. However, our calibration in the next section shows that market power is more likely than multidimensional heterogeneity to actually do so. Moreover, our characterization allows us to describe the level of insurance under perfect competition and it implies that market power tends to increases insurance quality.

We extend the framework above to a simple Hotelling environment as in Villas-Boas and Schmidt-Mohr (1999) and Bénabou and Tirole (2013). We consider two insurers, indexed by $i \in \{0, 1\}$, where $i$ captures location on the Hotelling unit interval. Insurer $i$ chooses a linear actuarial rate $x_i$ and a price $p_i$. We assume the two insurers are identical apart from their Hotelling location, so cost is $c(x, \mu) = x\mu$ for either insurer. Consumers have an additional dimension of type, $b \in [0, 1]$, which captures preferences over insurers. An individual with type $b$ incurs a cost $tb$ by purchasing from firm 0 and a cost $t(1 - b)$ from purchasing from firm 1, and this cost is fungible with price. Thus, $t$ captures market power. WTP is as in Subsection 2.2. We return to using $v \equiv a\sigma^2$.

We assume that every consumer purchases from one of the insurers. Doing so allows us to focus on the effect of competition on quality ($x$), abstracting from its effect on the number of consumers covered. Moreover, we are particularly interested in the competitive limit where $t \to 0$, in which case the results of a covered and uncovered market are qualitatively similar, as we argue in Subsection 5.3. Finally, it is often the case that a government mandate implies

[8] In fact, few papers have studied models of competitive product design that include either of these features. While two recent papers have considered purchasers who are heterogeneous in two dimensions (Wambach, 2000; Smart, 2000), both assume four discrete types, providing partial characterizations of perfectly competitive equilibrium. Two other recent papers that have considered equilibrium product design with imperfectly competitive firms (Villas-Boas and Schmidt-Mohr, 1999; Bénabou and Tirole, 2013) in environments with binary (unidimensional) types and conclude that market power may enhance social welfare, though neither paper focuses on insurance markets. All of this work has focused on theoretical possibilities rather than calibration.

This contrasts sharply with extensive empirical evidence that multidimensional types and market power are crucial to understanding the functioning of insurance markets. Dozens of recent papers surveyed by Einav, Finkelstein and Levin (2010) find that multidimensional heterogeneity of individuals is crucial to explain observed behavior in insurance markets.Chiappori et al. (2006) provide empirical evidence that, they argue, is hard to explain unless firms have market power, writing "(O)ur findings...suggest that more attention should be devoted to the interaction between imperfect competition and adverse selection on risk aversion...there is a crying need for such models." On a similar note, Einav and Finkelstein (2011) write, "On the theoretical front, we currently lack clear characterizations of the equilibrium in a market in which firms compete over contract dimensions as well as price, and in which consumers may have multiple dimensions of private information (like expected cost and risk preferences)." A major goal of this paper it to provide such a characterization.

a covered market, and indeed this is a common assumption in the literature.[9]

Individuals purchase from the insurer that offers the highest utility net of transportation costs. We define $b^\star \equiv \frac{1}{2t}\left(u(x_0,\mu,v) - p_0 - (u(x_1,\mu,v) - p_1)\right) + \frac{1}{2}$. Then buyers of insurer 0 are those for whom $b < b^\star$, buyers of insurer 1 have $b > b^\star$, and marginal individuals have $b = b^\star$. We assume, following Rochet and Stole (2002), that $b$ is distributed uniformly on the interval $[0,1]$, independent of $(\mu, v)$, so the joint distribution of types is $f(\mu, v)$.[10] This is a natural benchmark, since there is no obvious relationship between these variables in a market with symmetrically differentiated firms.

We will focus on local deviations pooling equilibria (LDPE). Intuitively, we will assume both insurers choose $x_0 = x_1 = x^\star$ and $p_0 = p_1 = p^\star$, and we consider as an LDPE a point $(x^\star, p^\star)$ where both firms' First and Second Order Conditions (SOC) for profit maximization are satisfied.

This concept is defined rigorously in Subsection 4.4 and builds on a recent literature in the design of mechanisms for environments with rich multidimensional types. In some such settings, LDPE may be a more plausible solution concept than is standard Nash Equilibrium that allows for global deviations. While an insurer could likely identify a non-local deviation in the simpler environment of RSs, this may be less plausible in a setting with continuous multidimensional types. For example, Erdil and Klemperer (2010) write that "in a complex environment, bidders are unlikely to understand the full space of alternatives, but may have a clearer view of where and how to gain from smaller deviations" and Carroll (2012) writes that "one might consider that agents are not capable of contemplating every possible misreport of their preferences... (but believe) that agents are at least rational enough to be capable of imitating any nearby type."

Moreover, considering LDPEs greatly increases the model's tractability without eliminating the main effects of interest in the literature following on RS. In particular, in the continuous type version of that paper, there is a local first-order local deviation from all candidate positive insurance symmetric competitive equilibria, and there is a second-order local deviation from the candidate equilibrium with zero insurance (Riley, 1979). Thus considering local deviations is sufficient to rule out the existence of equilibrium in the RS model. As a result, we show below that many of the intuitions present in the literature (including non-existence) still arise using the more tractable concept of LDPE.[11] Furthermore note that

[9] For instance, a covered market is mandated under the recent Affordable Care and Patient Protection Act in the United States. The assumption is mirrored in several recent papers such as Rochet and Stole (2002) and Handel, Hendel and Whinston (2013).

[10] This independence assumption is relaxed in an earlier version of this paper, Veiga and Weyl (2012) and uniformity can easily be dispensed with.

[11] Following on the non-existence results of RS, several papers have introduced modified concepts of equilibrium. Riley (1979) showed that non-existence is generally the case with continuous types although, from

the set of LDPE is strictly larger than the set of pooling (symmetric) equilibria, as LDPE imposes strictly fewer conditions.

Turning to this analysis, let $Q_i$ be the number of buyers from insurer $i$. At an LDPE, the market is split evenly so $b^\star = Q_0 = Q_1 = \frac{1}{2}$ and $M = -\frac{\partial Q_0}{\partial p_0} = -\frac{\partial Q_1}{\partial p_1} = \frac{1}{2t}$. The set of marginal consumers has the same composition as the set of all consumers, so all expectations and covariances are unconditional. Moreover, a profit maximizer catering to its marginal consumers is simultaneously catering to its average consumers, so we abstract from the Spence distortion mentioned in Section 4, thereby focusing on the effect of the sorting incentive on $x$. Then $\Pi_0 = \int_{\underline{v}}^{\overline{v}} \int_{\underline{\mu}}^{\overline{\mu}} \int_0^{b^\star} (p_0 - x_0\mu) f(\mu, v)\, db d\mu dv$ is the profit of insurer 0, and similarly for insurer 1.

It is convenient to define $\phi = \phi(x, v) \equiv u(x, \mu, v) - c(x, \mu) = \frac{1-(1-x)^2}{2} v$ as the social surplus from insurance or the value a consumer places on insurance beyond the off-loading of mean risk to the insurer.

**Proposition 3.** *There exists a unique $x^\star \in (0, 1)$ that satisfies the competitive FOC*

$$\mathbb{E}[\phi'] = \frac{1}{t} \mathrm{Cov}[u', c].$$

*Thus, sorting is adverse at an LDPE and full insurance is never an LDPE.*

*Proof.* The FOC can be derived from Proposition 1, using $Q = \frac{1}{2}$, $M = \frac{1}{2t}$, making the expectation and covariance unconditional (for details, see Theorem 3) Then, the FOC becomes $(1 - x^\star)\mathbb{E}[v]\, t = \mathrm{Cov}[\mu + v(1 - x^\star), x^\star \mu]$, which can be written as $\frac{t}{x^\star}\frac{\mathbb{E}[v]}{\mathbb{V}[\mu]} - \frac{1}{1-x^\star} = \frac{\mathrm{Cov}[v,\mu]}{\mathbb{V}[\mu]}$. As a function of $x^\star \in (0, 1)$, $\frac{t}{x^\star}\frac{\mathbb{E}[v]}{\mathbb{V}[\mu]} - \frac{1}{1-x^\star}$ has range $\mathbb{R}$ and is continuous strictly decreasing. Therefore, for any $\frac{\mathrm{Cov}[v,\mu]}{\mathbb{V}[\mu]}$, there exists a unique $x^\star \in (0, 1)$ where the FOC holds. Sorting must be adverse at an LDPE because $0 \leq (1 - x^\star)\mathbb{E}[v] = \mathbb{E}[\phi']$. Moreover, at full insurance $\mathbb{E}[\phi'] = 0$ so $\frac{\partial \Pi}{\partial x} \mid_{x=1} < 0$. □

On the one hand, increasing $x$ generates gains from insurance $\mathbb{E}[\phi']$, which the firm can perfectly capture because of the absence of a Spence distortion. On the other hand, firms use $x$ to sort for the most valuable consumers thereby skimming the cream from its rival. The relative weight on these two forces is determined by market power, since the sorting term is multiplied by $\frac{1}{t}$. When competition is intense (in the sense of undifferentiated Bertrand, $t$ low), a large weight is placed on sorting, as in the perfectly competitive world of RS where a small change in $x$ creates a large amount of cream-skimming, as we discuss below.

Dasgupta and Maskin (1986), equilibrium still exists in mixed strategies. Miyazaki (1977) and Wilson (1977) suggest notions of equilibrium which allow for richer reactions on the part of other firms. We use the concept of LDPE for tractability, not existence, purposes. Broadly speaking, we find that RS's non-existence conclusions hold also using the concept of LDPE for the circumstances considered in that paper.

Proposition 3 implies that full insurance is never an LDPE, independently of $f(\mu, v)$. The reason is that, at full insurance, the marginal utility for additional insurance ($u' = \mu + (1-x)v$) stems entirely from off-loading mean risk to the insurer ($\mu$) rather than from insurance per se ($(1-x)v \mid_{x=1} = 0$). This has two implications. First, there are no gains from trade at full insurance, so even a small incentive to sort reduces coverage away from $x = 1$. Second, at full insurance, sorting must be adverse ($\text{Cov}[\mu + v(1-x), x\mu] \mid_{x=1} = \mathbb{V}[\mu] > 0$). Thus, near full insurance, costly marginal individuals are more intensely attracted than individuals with high insurance value. When coverage is low, additional coverage may lead to advantageous sorting by attracting individuals with high insurance value and low risk types, as found by Finkelstein and McGarry (2006), but sorting always puts downward pressure on insurance quality near full insurance.

Proposition 3 also shows that sorting is also adverse at an LDPE, again independently of $f(\mu, v)$. Intuitively, if sorting were advantageous, raising $x$ would increase the number of buyers and improves their composition. This intuition will constitute the basis of the next result.

**Proposition 4.** *Market power increases insurance quality ($\frac{\partial x^\star}{\partial t} \geq 0$).*

*Proof.* See Appendix C. □

As we saw, sorting is adverse at an LDPE, thus placing downward pressure on $x^\star$. Market power reduces the incentive to sort, thus leading to higher insurance quality at an LDPE. As far as we are aware, this result establishes formally for the first time the intuition of RS that market power improves the quality of insurance, though related results have been derived in simpler credit (Villas-Boas and Schmidt-Mohr, 1999) and compensation (Bénabou and Tirole, 2013) contracting environments.[12]

We derive the SOC for an imperfectly competitive market in Appendix B. In Section 4 we argue that, generically, the SOC is satisfied for sufficiently large $t$. Therefore, we focus below on a discussion of the competitive limit ($t \to 0$), where the SOC is least likely to hold. Notice that, in this limit, firms make zero profit (as in RS) since $t$ captures the markup in this setting.

**Proposition 5.** *Let $\beta \equiv \frac{\text{Cov}[v,\mu]}{\mathbb{V}[\mu]}$ and $\gamma \equiv \frac{1}{4}\frac{\mathbb{E}[v]^2}{\mathbb{V}[\mu]} \geq 0$. In the limit where $t \to 0$:*

[12] In our particular setting, it is easy to see that the social optimum is full insurance, since there is no moral hazard. Thus market power brings $x^\star$ closer to its welfare-maximizing level. However, we emphasize that this is an artifact of our assumption (no moral hazard, a covered market and no Spence distortion), although the positive effect of $t$ on $x^\star$ is more robust. In Subsection 5.3 we discuss further the issue of welfare and present and scenario that, while similar to the one presented above, it is competition that brings $x^\star$ closer from its welfare maximizing level. In Subsection 5.3 we argue that an interior optimal degree of market power is typically optimal because increased prices reduce the quantity of individuals covered.

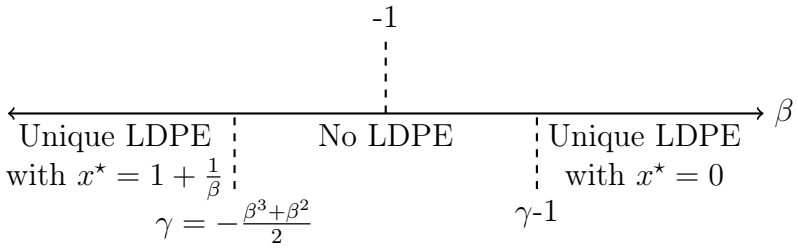


Figure 2: LDPE for various values of $\beta$ (increasing to the right).

- *There is at most a single LDPE, which must satisfy the first-order condition* $\text{Cov}\left[u', c\right] = x^\star \mathbb{V}\left[\mu\right]\left(1 + \left(1 - x^\star\right)\beta\right) = 0$ *and the second-order condition* $\left(1 - x^\star\right)^2 \gamma - 1 < \left(1 - \frac{3}{2}x^\star\right)\beta$.
- *If* $-1 < \gamma - 1 < \beta$, *the unique LDPE has* $x^\star = 0$. *If* $2\gamma < -\left(\beta^3 + \beta^2\right)$, *then* $\beta < -1$ *and the unique LDPE has* $x^\star = 1 + \frac{1}{\beta}$ .
- *For any* $\gamma > 0$, *there is a range of* $\beta$ *for which there is no LDPE, and this range increases in* $\gamma$. *However, for* $\gamma > 0$, *there is a unique* $\tilde{\beta} < -1$ *such that there exists an LDPE if* $\beta < \tilde{\beta} < -1$.

*Proof.* See Appendix C. □

These results are expressed in Figure 2. A competitive LDPE must have zero sorting incentive, which is a generalization of the logic of Rothschild and Stiglitz (1976) to a context of multidimensional types. Cream skimming incentives are proportional to $M$, so in an LDPE where $t \to 0$ demand becomes infinitely responsive ($M = \frac{1}{2t} \to \infty$) since, loosely speaking, all consumers are marginal. Thus, any incentive to sort will cause firms to deviate from the LDPE. This implies that the unique actuarial rate possible at an LDPE can be computed solely on the basis of a simple moment of the distribution of types in the population. This rate is $x^\star = 1 + \frac{1}{\beta}$ where $\beta = \frac{\text{Cov}[\mu, v]}{\mathbb{V}[\mu]}$ is the uncontrolled OLS regression coefficient of $\mu$ on $v$ in the entire population. This provides a simple condition to take to data in order to calibrate how much insurance would be provided at a competitive LDPE which we do in Section 3.

Positive insurance at an LDPE requires $\beta < -1$. This occurs when risk aversion is sufficiently negatively correlated with risk so that sorting is advantageous at zero insurance, as in the data of Finkelstein and McGarry (2006). Recalling the results above, it is useful to think of the sign of sorting as $x$ decreases from full insurance. Sorting is always adverse at full insurance and at the LDPE, independently of the distribution of types. However, the level of insurance quality at the LDPE (and specifically whether it is positive) does depend on the distribution of types through $\beta = \frac{\text{Cov}[\mu, v]}{\mathbb{V}[\mu]}$. Importantly, this results requires

multidimensional types. Otherwise, we would have $\text{Cov}[v, \mu] = \beta = 0$ so the only candidate LDPE is $x^\star = 0$, as in RS.

However, LDPE also requires that SOCs are satisfied. If $\gamma = 0$, then $x^\star = 1 + \frac{1}{\beta}$ is the LDPE when $\beta < -1$, and $x^\star = 0$ is the LDPE if $\beta > -1$. In this case, an LDPE always exists although there is no demand for insurance since $\mathbb{E}[v] = 0 \Rightarrow v \equiv 0$. The intuition again mirrors that of RS, extended to a multidimensional context. Here, $\gamma$ is a force pushing for second-order deviations away from a low-insurance LDPE, arising from the gains from trade from insurance, $\mathbb{E}[v]$. The smaller is $\gamma$, the smaller is the range over which no LDPE exists. If $\beta > \gamma - 1$, a no-insurance LDPE exists because sorting is so adverse at zero insurance that it overcomes the pressure of $\gamma$ to deviate from that LDPE. If $\beta$ is negative enough, sorting is sufficiently advantageous that there is a first-order gain from raising insurance quality above zero. In the intermediate region, adverse sorting local to zero-insurance eliminates the possibility of a positive insurance LDPE, but insurance demand makes a zero-insurance LDPE vulnerable to second-order deviations, so no LDPE exists.

# 3 Empirical Calibration

This section uses summary statistics from Handel, Hendel and Whinston (2013) to perform a calibration of the competitive model described in Section 2.4 above.[13] These authors use proprietary claims data from a large employer. The data does not arise from a competitive market, instead stemming from a firm which uses cross-subsidies to achieve a variety of objectives other than single-plan profit maximization, such as employee retention and productivity. Assuming that a widely-used proprietary risk estimation package represents the information set of individuals, the authors are able to recover the joint distribution of $\mu$ and $v$ for the entire population from the joint distribution of claims, risk estimates based on the package and plan choice by individuals.[14] We use moments of this distribution to determine the properties of the LDPE if individuals in this data bough insurance from a competition market such as the one described in Section 2.4. For the purposes of the calibration, we follow HHW in allowing three dimensions of type: $a$, $\mu$ and $\sigma^2$. However, all the formulae from Section 2 remain valid.

To the first two significant digits, HHW find $\mathbb{E}[\mu] = 6.6 \cdot 10^3$, $\mathbb{E}[v] = 68 \cdot 10^3$, $\mathbb{V}(\mu) = 50 \cdot 10^6$ and $\text{Cov}[\mu, v] = 630 \cdot 10^6$ (units are dollars and dollars squared). This implies $\beta \approx 13 > 0$ and thus the unique candidate LDPE is $x = 0$. However $\gamma \approx 23 > 13 + 1$, thus the zero-

[13] We thanks these authors for their generosity.

[14] They follow an approach analogous to that of Cohen and Einav (2007), using individuals' choices among available plans coupled with an assumption that $a$ is distributed normally conditional on $\mu, \sigma^2$and other covariates to estimate the joint distribution of $\mu, \sigma^2$ and $a$.

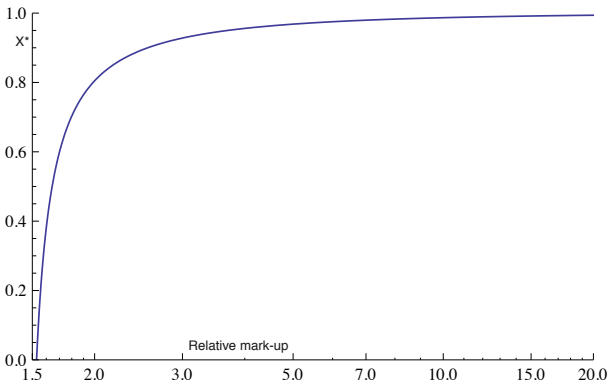


Figure 3: Unique candidate LDPE actuarial rate $x^\star$ as a function of the relative markup ($R$), shown on a logarithmic scale.

insurance candidate is not actually an LDPE because the average value of insurance is too great. While these statistics are taken for the full population, the same qualitative features apply to the market in every five-year age bucket. Thus, in their data, no LDPE exist.

Multidimensionality of private information exacerbates rather than mitigates adverse selection in this case. Insurance value is positively correlated with mean risk, as in Cohen and Einav (2007) but unlike in Finkelstein and McGarry (2006). Interestingly, HHW find that $a$ is negatively correlated with $\mu$, as in Finkelstein and McGarry, which would suggest $\beta < 0$. However, there is an even stronger positive correlation between $\sigma^2$ and $\mu$ because sick individuals have both high and highly variable expenditures, which makes $\beta > 0$. This illustrates how seemingly innocuous simplifications of the nature of consumer heterogeneity can have important effects on a model's predictions.[15]

These results are consistent in spirit with those reported by HHW. They analyze two-plan equilibria between plans with exogenously specified (and approximately linear) actuarial rates. They find severe adverse selection leading almost always to complete collapse of the plan with the higher actuarial rate. Thus the value of $\beta$ is a reduced-form statistic that helps provide intuition for their structural results.

We then use the same moments to calibrate the tendency of market power to increase insurance provision (Proposition 4). Markup in this Hotelling framework is $t$. Since insurer cost is per consumer is $x\mathbb{E}[\mu]$, we measure market power using the proportional-to-cost mark-up $R = \frac{t}{x\mathbb{E}[\mu]}$, which monotonically increases with $t$. We can therefore solve the FOC of Proposition 3 for $R = \frac{t}{x^\star\mathbb{E}[\mu]} = \frac{1}{1-x^\star}\frac{\mathbb{V}[\mu]+\mathrm{Cov}[\mu,v]}{\mathbb{E}[v]\mathbb{E}[\mu]}$. Figure 3 shows the unique candidate LDPE actuarial rate $x^\star$ as a function of this relative-to-cost mark-up.

At a mark-up of 161%, the actuarial rate at the candidate LDPE is 40%. A relative

[15] Even if $\sigma^2$ is treated as homogeneously equal to its average value, $\beta$ is negative but always above $-1$ in their data so that no positive insurance candidate LDPE exists even in this case.

markup of 198% is necessary for an actuarial rate of 80% to emerge as an LDPE. Calibration of the second-order condition shows that a relative mark-up of 170% ($R = 1.7$) is necessary for an LDPE to exist.[16] While such large mark-ups would likely generate political resistence given their distributive consequences, they clearly increase welfare in our model.[17]

# 4 A General Model of Sorting

In this section we show how the techniques and results of Section 2 generalize to richer settings and provides rigorous proofs of the results above. We will focus on relating these more general concepts to those of the previous sections, rather than discussion of the results or assumptions.

## 4.1 Setup

A monopoly offers an insurance contract characterized by a price $p \in \mathbb{R}$ and a non-price characteristic $x \in \mathbb{R}$. The scalar $x$ can trivially be extended to be a vector (see Veiga and Weyl (2013)). There is a unit mass of consumers characterized by their type, a $T$-dimensional vector $\theta = (\theta_1, ..., \theta_T) \in (\underline{\theta_1}, \overline{\theta_1}) \times \cdots \times (\underline{\theta_T}, \overline{\theta_T}) \subseteq \mathbb{R}^T$, which is not contractible. It is common knowledge that $\theta$ is distributed according to the atomless and full support probability density function $f(\theta)$.

Consumers face a wealth shock $l \in \mathbb{R}$, distributed according to the atomless density $g(l, \theta) > 0$. Consumers have a utility function $U = U(w, \theta)$, where $w$ is final wealth and $\frac{\partial U}{\partial w} \equiv U' > 0$ and $\frac{\partial^2 U}{\partial w^2} < 0$. An insurer reimburses $G = G(l, x)$ if the consumers incurs a loss $l$. Then, $G = l$ is full insurance and $G = 0$ is no insurance. Initial wealth is $w_0$ (potentially heterogeneous). Let $\mathbb{E}_l[\cdot]$ denote expectation over $l$. The WTP of a consumer of type $\theta$ is $u(x, \theta)$, the level of $p$ that equates expected utility with insurance to that without insurance:

$$\mathbb{E}_l[U(w_0 - l + G(l, x) - u(x, \theta), \theta) \mid \theta] = \mathbb{E}_l[U(w_0 - l, \theta) \mid \theta]. \tag{2}$$

We assume that $u(x, \theta)$ is continuously differentiable in all arguments and strictly increasing in all arguments. We define $\frac{\partial u(x,\theta)}{\partial x} \equiv u' > 0$. Expected cost of type $\theta$ to the risk-neutral insurer is $c = c(x, \theta) = \mathbb{E}_l[G \mid \theta]$, so $c' \equiv \mathbb{E}_l\left[\frac{\partial G}{\partial x} \mid \theta\right]$. We assume $c(x, \theta)$ is twice continuously

[16] This result is also consistent with the very low actuarial rates for non-group markets prior to the Affordable Care Act in the US.

[17] Of course in a more realistic model where population coverage is endogenous, such prices are likely to significantly undermine the sustainability of an individual mandate and thus likely reduce the fraction of the population covered. In Subsection 5.3 we show that allowing for such uncovered market puts additional downward pressure on price, implying that more reasonable rates of mark-ups can achieve these levels of insurance quality and again lead to nearly first-best welfare.

differentiable in all arguments. Differentiating Equation 2 yields $u' - c' = \frac{\text{Cov}_l\left[U', \frac{\partial G}{\partial x} | \theta\right]}{\mathbb{E}_l[U'|\theta]} \equiv \phi'$.[18] Increasing $x$ captures more generous insurance if $\phi' > 0$.

The set of consumers purchasing the product is $\Theta \equiv \{\theta : u(x,\theta) \geq p\}$, while the set of marginal consumers is $\partial\Theta \equiv \{\theta : u(x,\theta) = p\}$. We define $\theta_{-T} = (\theta_1, ..., \theta_{T-1})$ and follow Veiga and Weyl (2013) in making the following assumption

**Assumption 1.** *There exists a function* $\theta_T^\star(p, x, \theta_{-T})$ *such that* $u(x,\theta) \geq p \Leftrightarrow \theta_T \geq \theta_T^\star(p, x, \theta_{-T})$.

This function therefore defines the margin.[19] Defining $t \equiv (\theta_{-T}, \theta_T^\star(p, x, \theta_{-T}))$, an integral over $\partial\Theta$ can be expressed as an iterated integral over the $\theta_{-T}$. That is, for an integrable function $\zeta(x,\theta)$, we define

$$\int_\Theta \zeta(x,\theta)\, d\theta \equiv \int_{\underline{\theta_1}}^{\overline{\theta_1}} ... \int_{\theta_T^\star(p,x,\theta_{-T})}^{\overline{\theta_T}} \zeta(x,\theta)\, d\theta,$$

$$\int_{\partial\Theta} \zeta(x,t)\, d\theta_{-T} \equiv \int_{\underline{\theta_1}}^{\overline{\theta_1}} ... \int_{\underline{\theta_{T-1}}}^{\overline{\theta_{T-1}}} \zeta(x, \theta_{-T}, \theta_T^\star(p,x,\theta_{-T}))\, d\theta_{-T}$$

The quantity of buyers is $Q = Q(x,p) \equiv \int_\Theta f(\theta) d\theta$. It is shown in the proof of Proposition 1 below that $\frac{\partial Q}{\partial p} < 0$ for all $Q > 0$. We can therefore invert $Q(x,p)$ with respect to $p$ to recover the function $P(x,q)$, which solves $Q(P(x,q),x) = q$. Profit is $\Pi \equiv qP(x,q) - C$, where $C \equiv \int_\Theta c(x,\theta)\, d\theta$ is total cost.

We define the density of the margin and expectation conditional on the margin as

$$M \equiv -\frac{\partial Q}{\partial p} = \int_{\partial\Theta} \frac{f(t)}{\partial u(x,t)/\partial\theta_T} d\theta_{-T}$$

For details on $\frac{\partial Q}{\partial p}$, see the proof of Proposition 1 below. For integrable functions $\zeta_1 = \zeta_1(x,\theta)$ and $\zeta_2 = \zeta_2(x,\theta)$, expectation conditional on the margin is

$$\mathbb{E}_u[\zeta_1(x,\theta) \mid \partial\Theta] \equiv \frac{1}{M} \int_{\partial\Theta} \frac{\zeta_1(x,t) f(t)}{\partial u(x,t)/\partial\theta_T} d\theta_{-T}.$$

Similarly, $\text{Cov}_u[\zeta_1, \zeta_2 \mid \partial\Theta] \equiv \mathbb{E}_u[\zeta_1\zeta_2 \mid \partial\Theta] - \mathbb{E}_u[\zeta_1 \mid \partial\Theta]\,\mathbb{E}_u[\zeta_2 \mid \partial\Theta]$.[20]

[18] The amount by which type $\theta$ values additional insurance ($u'$), is the transfer of mean risk to the insurer $c' = \mathbb{E}_l\left[\frac{\partial G}{\partial x} \mid \theta\right]$ in addition to the marginal social surplus insurance $\phi'$. This marginal surplus captures whether insurance quality increases for those states with highest marginal utility (i.e., high realizations of $l$). Under risk neutrality, $U'$ is constant so $\phi' = 0$. In Section 2, we had $\phi' = (1-x)v$.

[19] In Section 2, this was the function $\mu^\star(p,x,v)$.

[20] In Section 2, we had $\frac{\partial u}{\partial\theta_T} = \frac{\partial u}{\partial\mu} = \frac{1}{x}$, so this term vanishes in $\mathbb{E}_u[\zeta(x,\theta) \mid \partial\Theta]$ since it is present in the numerator and denominator. This occurs whenever the margin is a straight line in the space of types.

## 4.2 Monopoly

The firm chooses $q$ and $x$ to maximize profit. The FOC with respect to $q$ is familiar ($P - \frac{q}{M} = \mathbb{E}_u[c \mid \partial\Theta]$, or marginal revenue equals marginal cost) so we do not discuss it more extensively.[21] Instead, our analysis will focus on the the non-price product dimension $x$.

**Theorem 1.** *A necessary FOC for a monopoly's profit maximizing choice of $x$ is*

$$-\underbrace{q\mathbb{E}[c' \mid \Theta]}_{cost} + \underbrace{q\mathbb{E}_u[u' \mid \partial\Theta]}_{Spence\ term} = \underbrace{M\mathrm{Cov}_u[u', c \mid \partial\Theta]}_{sorting\ effect}.$$

*Proof.* See Appendix C. □

## 4.3 Signing the sorting incentive

The following Theorem presents commonly-satisfied conditions under which the sorting incentive can be signed. Essentially, this is the case when $u'$ and $c$, conditional on $\theta \in \partial\Theta$, can be expressed as monotone univariate functions.

**Theorem 2.** *Suppose there exists a function $g(x,\theta) : \mathbb{R} \times (\underline{\theta_1}, \overline{\theta_1}) \times \cdots \times (\underline{\theta_T}, \overline{\theta_T}) \mapsto \mathbb{R}$ such that $c \equiv \hat{c}(x, g(x,\theta))$ and $u' \equiv \hat{u}(x, g(x,\theta))$. Suppose $\hat{c}(x, g(x,\theta))$ and $\hat{u}(x, g(x,\theta))$ are monotone in $g(x,\theta)$. Then, $\mathrm{Cov}_u[u', c \mid \partial\Theta]$ has the same sign as*

$$\frac{\partial\hat{c}(x, g(x,\theta))}{\partial g}\frac{\partial\hat{u}(x, g(x,\theta))}{\partial g}.$$

*Alternatively, suppose that $\theta = (\theta_1, \theta_2) \in \mathbb{R}^2$. If $\forall\theta \in \partial\Theta$, we have*

$$\left(\frac{\partial u'/\partial\theta_1}{\partial u/\partial\theta_1} - \frac{\partial u'/\partial\theta_2}{\partial u/\partial\theta_2}\right)\left(\frac{\partial c/\partial\theta_1}{\partial u/\partial\theta_1} - \frac{\partial c/\partial\theta_2}{\partial u/\partial\theta_2}\right) > 0,$$

*then $\mathrm{Cov}_u[u', c \mid \partial\Theta] > 0$. The statement remains true if both inequalities are reversed.*

*Proof.* See Appendix C. □

The assumptions of Theorem 2, while seemingly special, apply naturally in contexts such as non-linear pricing, where the scalar amount purchased by each individual determines both her cost to the firm and her marginal utility from a change in the per-unit price, as in the model of Subsection 5.2. The logic of this theorem can also be generically applied when types are bi-dimensional, say $(\theta_1, \theta_2)$, so that the margin ($\partial\Theta$) is a one-dimensional curve. Since

[21] Its more sophisticated component, marginal cost $\mathbb{E}_u[c \mid \partial\Theta]$, is discussed in Veiga and Weyl (2013), Einav, Finkelstein and Cullen (2010) and Einav and Finkelstein (2011).

WTP is is increasing in all arguments, if $(\theta_1, \theta_2)$ and $(\hat{\theta}_1, \hat{\theta}_2)$ are both indifferent, $\hat{\theta}_1 > \theta_1$ must imply $\hat{\theta}_2 < \theta_2$. Then, within the margin, $\theta_2$ is a decreasing function of $\theta_1$. Thus an index can be constructed and the logic of Theorem 2 can be applied. Whether the relevant terms in the statement of Theorem 2 can be signed depends on the details of the specific model, although we have yet to find a case where this approach cannot be usefully applied, as in the computational exercise of Subsection 2.3.

## 4.4 Competition

We extend our analysis to a simple Hotelling (1929) competitive environment. We consider two insurers, indexed by $i \in \{0, 1\}$, where $i$ captures the insurer's location on a Hotelling line and $-i$ refers $i$'s competitor. Each insurer $i$ chooses a non-price characteristic $x_i$ and a price $p_i$. The two insurers are identical apart from their Hotelling location. Consumer types $\theta \in \mathbb{R}^T$ is augmented by $b \in [0, 1]$. An individual with type $b$ incurs a cost of $tb$ by purchasing from firm 0 and by $t(1-b)$ from purchasing from firm 1, where $t$ captures market power. This cost is fungible with price. Individuals have a WTP $u$ as above. We make the two following simplifying assumptions.

**Assumption 2.** *The market is covered.*

**Assumption 3.** *The joint distribution of heterogeneity is $f(\theta)$.*

Given these assumptions, individuals purchasing product 0 are those for whom $b \leq b^\star \equiv \frac{1}{2t}\left(u(x_0, \theta) - p_0 - u(x_1, \theta) + p_1\right) + \frac{1}{2}$ and those purchasing from insurer 1 are those for whom $b > b^\star$. The set of consumer for whom $b = b^\star$ is marginal to both insurers.[22]

We will focus on *local deviation pooling equilibri*a (LDPE), which we define as follows.

**Definition 1.** A *local deviation pooling equilibrium* (LDPE) is a pair $(x^\star, p^\star)$ such that, if firm $i$ plays $(x_i, p_i) = (x^\star, p^\star)$ then there exists an $\epsilon > 0$ such that $(x^\star, p^\star)$ maximizes the profit of firm $-i$ in the set $(x^\star - \epsilon, x^\star + \epsilon) \times (p^\star - \epsilon, p^\star + \epsilon)$.

Then, if the FOC and SOC are satisfied at $(x^\star, p^\star)$, then $(x^\star, p^\star)$ is an LDPE. At an LDPE, the market is split evenly, so $b^\star = \frac{1}{2}$. Defining $Q_0 \equiv \int_\theta \int_0^{b^\star} f(\theta)\, dbd\theta$ and similarly for $Q_1$, at an LDPE $Q_0 = Q_1 = \frac{1}{2}$. Moreover, $M \equiv -\frac{\partial Q_0}{dp_0} = \frac{1}{2t}$ (for details, see Appendix B). At an LDPE, the set of marginal consumers is $\partial\Theta = \left\{(\theta, b) : b = \frac{1}{2}\right\}$ and it has the same composition as the set of buyers overall. Otherwise the model is as in Subsection 4.1, with subscripts denoting the relevant firm.

[22] Assumption 3 is relaxed in an earlier version of this paper, Veiga and Weyl (2013).

Pricing at an LDPE in this Hotelling context is well-known: $p+t = \mathbb{E}[c]$, where marginal cost $\mathbb{E}[c]$ is the average cost in the full population (by Assumptions 3 and 2). Recall that $\phi' \equiv u' - c'$. We define $\frac{\partial \phi'}{\partial x} \equiv \phi''$.

**Theorem 3.** *For $t > 0$, a necessary FOC for the profit maximizing value of $x$ is*

$$\mathbb{E}[\phi'] = \frac{1}{t}\text{Cov}[u', c].$$

*Sorting is adverse at an LDPE. If $\text{Cov}[c', c] > 0$ at full insurance, then sorting is adverse at full insurance and there exists an interior value of $x$ at which the FOC holds. The SOC is*

$$t\mathbb{E}[\phi''] + \frac{1}{2}\mathbb{E}[\phi']^2 < \text{Cov}[u'', c] + 2\text{Cov}[u', c'].$$

*Proof.* See Appendix C. □

**Theorem 4.** *Market power increases insurance quality ($\frac{\partial x^\star}{\partial t} \geq 0$).*

*Proof.* Applying the IFT to the FOC yields $\frac{\partial x^\star}{\partial t} = -\frac{\mathbb{E}[\phi']}{\partial^2 \Pi / \partial x^2}$. If the SOC is satisfied, then $\partial^2\Pi/\partial x^2 < 0$. If $x$ captures higher insurance quality, $\phi' > 0$. Then $\frac{\partial x^\star}{\partial t} \geq 0$. □

We can now consider the competitive limit as $t \to 0$. Recall $G = l$ corresponds to full insurance while $G = 0$ corresponds to zero insurance. Notice that, in this limit, firms make zero profit as in RS.

**Theorem 5.** *Assume $\mathbb{E}[\phi']$ is bounded. In the limit as $t \to 0$, an LDPE must satisfy $\text{Cov}[u', c] = 0$, no insurance is an LDPE if $\frac{1}{4}\frac{\mathbb{E}[\phi']^2}{\mathbb{V}[c']}\,|_{G=0} - 1 < \frac{\text{Cov}[\phi', c']}{\mathbb{V}[c']}\,|_{G=0}$ and there is an interior candidate LDPE if $\frac{Cov(\phi', c')}{\mathbb{V}[c']}\,|_{G=0} < -1$.*

*Proof.* See Appendix C. □

It is useful to relate these results to those of Section 2. There, we had $\frac{Cov[\phi, c']}{\mathbb{V}[c']}\,|_{G=0} = \frac{(1-x)Cov[v,\mu]}{\mathbb{V}[\mu]}\,|_{x=0} = \beta$, so the condition for existence of an interior candidate LDPE is a generalization of Proposition 5. Similarly, the SOC at no insurance presented above implies that, in Section 2, the SOC holds for $x^\star = 0$ if $\gamma - 1 < \beta$, as in Proposition 5.

# 5 Robustness Checks & Welfare Analysis

In this section we discuss the robustness of our conclusions to the inclusion of moral hazard and market expansion. We also consider the welfare implications of our model in a third robustness check.

## 5.1 Moral hazard

Our calibration in Section 3 relied on the assumption that mean risk $\mu$ was invariant to the generosity of insurance coverage $x$, which implies that full insurance was optimal. In reality, there is substantial evidence of moral hazard in insurance markets, which would imply that the social optimum prescribes less than full insurance (Aron-Dine, Einav and Finkelstein, 2013). In this subsection, we consider a simple form of moral hazard.

In the model of Subsection 2.4, suppose that mean health expenditures $\mu$ respond to out-of-pocket expenses according to a constant elasticity function of the kind used in Saez (2001). Then, mean risk at an actuarial rate of $x$ is $\mu(x) = \mu(\hat{x})\left(\frac{1-\hat{x}}{1-x}\right)^{\epsilon}$, where $\epsilon$ is the constant elasticity of utilization and $\hat{x}$ is a reference level of insurance against. We calibrate this model using the canonical estimate of $\epsilon = 0.2$ for the elasticity of medical expenditures from the RAND experiment.[23] We use as common reference level for all individuals $\hat{x} = 0.85$, which is the mean actuarial rate in the HHW data.

Then, the socially optimal actuarial rate (see the next subsection) is approximately 87%. The reason why even high expenditure elasticities ($\epsilon$) do not significantly reduce optimal insurance is that, with a high elasticity, even a small decrease of generosity below full insurance discourages much of the wasteful mean expenditure. Even an unrealistically high elasticity of $\epsilon = 1$ yields a socially optimal insurance of about 78%. Therefore moral hazard reduces our estimate of socially optimal insurance rates, though not significantly. Given that in our calibration of Section 3 insurance is driven entirely out of the market, our conclusions that LDPE insurance is vastly below the social optimum seem robust to plausible degrees of moral hazard.

## 5.2 Welfare

A natural question following our analysis above is the socially optimal pooling contract: constrained to choosing $x_0 = x_1 = x$ and $p_0 = p_1 = p$, what is the welfare maximizing value of $(x, p)$? To proceed with this analysis, we equate WTP with utility as in the standard Kaldor (1939)-Hicks (1939) analysis. If the market is covered, there is no optimal value of $p$ because prices are purely distributive and do not affect behavior. Thus, we focus on the optimal level of $x$ and on the effect of market power on welfare through $x^\star$ at the LDPE.

As in analysis above, competition increases the importance of sorting (attracting the right kind of consumers) relative to the importance of gains from trade (attracting more consumers). However, this can increase or decrease welfare. To emphasize this point, we consider both the model of ex-ante insurance contracting without moral hazard and a variant

[23] See Manning et al. (1987) and Aron-Dine, Einav and Finkelstein (2013).

with ex-post contracting and moral hazard that yields a sharply different result.[24]

First, consider the model of Subsection 4.4, and thus define welfare as $W = \mathbb{E}[\phi]$.

**Proposition 6.** *In the competitive model of Subsection 4.4, full insurance* ($x = 1$) *is socially optimal. If* $\text{Cov}[u', c]$ *is bounded, then* $\lim_{t\to\infty} x^\star = 1$.

*Proof.* See Appendix C. □

Since there is no moral hazard (unlike in the previous subsection), full insurance is the social optimum. However, even with some moral hazard, LDPE always has insufficient insurance (Theorem 3). Since market power increases $x^\star$ (Theorem 4) and the level of $p$ has no effect on welfare, market power increases welfare in this model.

We now present a model that, while using the same basic framework, focuses on moral hazard and ex-post efficient consumption, rather than ex-ante efficient insurance. We introduce a Spence distortion driven by ex-post expenditure heterogeneity that incentivizes firms to create moral hazard that is in turn mitigated by sorting.

Individuals incur a (heterogeneous) health shock of value $L$ and then purchase healthcare of value $D = D(L, x)$, as a function of $L$ and a cost-sharing rate $x$, described below. The realized distribution of losses in the population has density $f(L) > 0$. An individual with loss $L$ has demand $D = L(1-x)^{-\epsilon}$, where $\epsilon > 0$ captures the degree of moral hazard. Of this amount, consumers pay a share $1-x$, so the cost to a hospital of serving an individual with loss $L$ is $xL(1-x)^{-\epsilon}$. Thus $x$ is "quasi-insurance" since a larger $x$ allows consumers to better mitigate their health shock through ex-post healthcare purchases. By the envelope theorem, the change in consumer surplus from an increase in $x$ is equal to $-D$. Thus, consumer surplus is $\int_0^x D(L, \hat{x})d\hat{x} = L\frac{\left[1-(1-x)^{1-\epsilon}\right]}{1-\epsilon}$ and wealth loss by the consumer is $L\left(1 - \frac{\left[1-(1-x)^{1-\epsilon}\right]}{1-\epsilon}\right) = L\lambda(x)$. Let $\lambda = \lambda(x)$.[25] Individuals have CARA utility, $-e^{-aw}$, where $w$ is final wealth and $a$ is absolute risk aversion. The market is covered. There are two competing hospitals. Individuals are uniformly distributed on $[0, 1]$ with respect to $b$, which is independent of $L$. There is a wealth cost $tb$ from using hospital 0 and $t(1-b)$ from using hospital 0. We consider an LDPE. Hospitals choose cost-sharing rates $x$ and prices $p$.

Social welfare generated by an individual $w = L(1-\lambda) - xD$ is maximized for $x = 0$, the point at which each individual fully internalizes her costs and purchases $L$ units of healthcare.

[24] This builds on a general insight of Rochet and Stole (2002) and Yin (2004): competition in non-linear pricing leads to efficiency when average and marginal consumers have the same preferences.

[25] For instance, when $x = \lambda(0) = 0$ there is no surplus. It is also possible that $\lambda(x) < 0$ for $x$ large enough, in which case those with highest $L$ are the best-off (for example, if $\epsilon = .2$ then $\lambda(0.82) = 0$, then for smaller $x$ it is positive, and for larger it is negative). However, we will see below that in equilibrium $x$ will always be below this level.

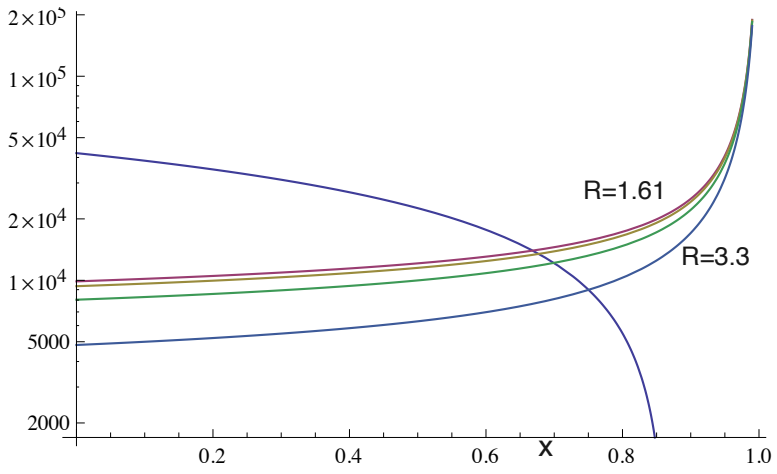


Figure 4: The LHS (downward-sloping) and RHS (upward sloping) of Equation 3 as a function of $x$, for different values of the relative mark-up ($R$). Intersections represent LDPE.

The FOC for the profit maximizing choice of $x$ is

$$\frac{1}{2}\left(\mathbb{E}_u\left[D\right]-\mathbb{E}\left[D\right]\right)=x^\star\left(\frac{1}{2}\frac{\epsilon}{1-x^\star}\mathbb{E}\left[D\right]+M\mathbb{V}_u\left[D\right]\right) \tag{3}$$

where $\mathbb{E}_u\left[\zeta \mid b=b^\star\right]=\frac{\int_L \zeta e^{aL\lambda}f(L)dL}{\int_L e^{aL\lambda}f(L)dL}=\mathbb{E}_u\left[\zeta\right]$. It is clear that $\frac{Q\epsilon}{1-x}\mathbb{E}\left[D\right]+M\mathbb{V}_u\left[D\right]>0$. Since $e^{aL\lambda}$ places greater weight on larger values of $L$, we have $\mathbb{E}_u\left[D\right]>\mathbb{E}\left[D\right]$, so for $t>0$, we must have $x^\star>0$ for the FOC to hold at an LDPE. At a competitive LDPE the sorting term $(x^\star M\mathbb{V}_u\left[D\right])$ must vanish, so we must have $\lim_{t\to\infty}x^\star=0$ . Therefore, the social optimum obtains at the competitive limit.

We then calibrate this model to match the distribution of losses and the mean risk-aversion in the Handel, Hendel and Whinston (2013) data, as detailed in Appendix D, to compare it quantitatively to that of Section 3. The FOC with respect to price is $p^\star - x^\star\mathbb{E}_u\left[D\right]=\frac{1}{2M}$. Absolute markup is $\frac{1}{2M}$ and markup as a ratio of the cost absorbed by the firm is $R=\frac{1}{2Mx\mathbb{E}[D]}$. One can solve for $M$ as a function of the the markup $R$, then plug that expression into the FOC for $x$. Letting $\epsilon=0.2$, we now plot the RHS and the LHS of the FOC for $x$, as a function of $x$, for different values of $R$. The results are shown in Figure 4.

We observe a levels of insurance similar to that we saw in the basic model (approximately .65 to .76). Furthermore, the mark-up increases, LDPE quasi-insurance rises, as in Section 3, where we also considered mark-ups as a fraction of the expenditure absorbed by the firm. However $x^\star$ is now much less responsive to $R$; as $R$ ranges from 1.61 to 3.3, $x^\star$ ranges from .65 to .76 rather than from .4 to .9 as in Section 3. This is likely because moral hazard makes large $x$ values very unattractive. Nonetheless both models generate fairly high

levels of insurance (here quasi-insurance) for reasonably large mark-ups and both have this insurance increasing in market power, despite radically different normative implications of these results.

## 5.3 Market expansion

The model of competition in Section 2.4 assumes a covered market. We now relax that assumption and, with some additional structure, calibrate this model with market expansion. We outline our analysis here and provide details in Appendix E. We consider the CARA-Normal linear co-insurance set-up of Section 2.4, maintain that $b$ is distributed independently of $(\mu, v)$ and focus on LDPE. We then assume, following White and Weyl (2012) that the Hotelling transportation cost to the nearest insurer is zero, while to the furthest insurer is $t(1-2b)$. This preserves the difference in transportation costs between the insurers and therefore the marginal incentives of consumers to choose between insurers. However, at an LDPE every consumer purchases from the closer of the two firms and thus no consumer actually incurs any transportation cost. As a result, at each $b \in [0,1]$, the set of types $(\mu, v)$ purchasing insurance is the same. Under these assumptions, consumers on the "switching" margin between the two firms, $\left\{(\mu, v, b) : b = \frac{1}{2}, \mu > \mu^\star(p, x, v)\right\}$, are representative of all buyers in terms of $(\mu, v)$. Consumers on the exiting margin of either firm, $\{(\mu, v, b) : \mu = \mu^\star(p, x, v)\}$, are not. These two margins are pictured in Figure 5, with $(\mu, v)$ collapsed to a single dimension to allow 2-dimensional representation. The threshold for purchasing $\mu^\star(p, x, v) = \mu^\star = \frac{p}{x} - \frac{2-x}{2}v$.

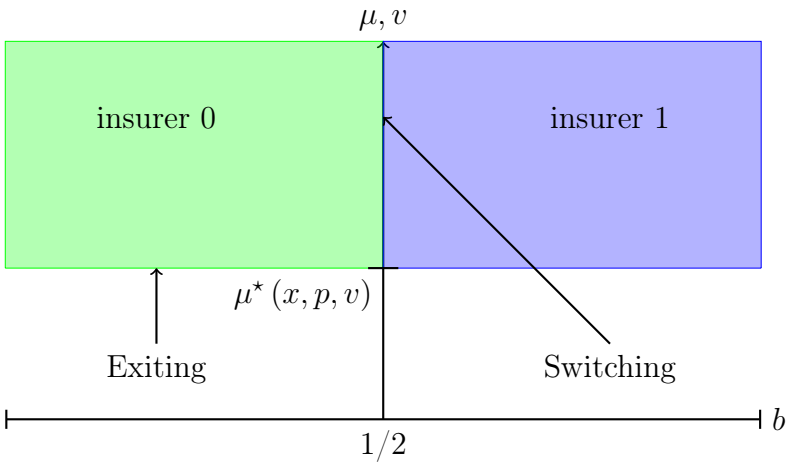


Figure 5: The $b - \mu$ plane of a competitive market with an expansion margin.

Market expansion changes LDPE conditions in several ways. All expressions evaluated

for marginal consumers are now a mixture of the switching and exiting margins, combining the logic of Subsection 2.2 with that of Subsection 2.4. We denote the density of individuals along the exiting margin of either firm when they are pooled at $(p, x)$ as $M^X(p,x) \equiv \frac{1}{2x}\int_{\underline{v}}^{\overline{v}} f\left(v, \frac{p}{x} - \frac{2-x}{2}v\right) dv$ and the density of individuals along the switching margin as $M^S(p,x) = \frac{Q(p,x)}{2t}$. Note that as $t \to 0$, $M^S \to \infty$ so that $x\text{Cov}[\mu, \mu + (1-v)x \mid \mu > \mu^\star] = 0$ is still necessary for a competitive LDPE. Thus the necessity of zero sorting incentive at a perfectly competitive limit carries over.

The FOC for maxmizing welfare with respect to $x$ is

$$\underbrace{q(1-x^\star)\mathbb{E}\left[v \mid \mu \geq \mu^\star\right]}_{\text{marginal social surplus}} = \underbrace{M^X \text{Cov}_u[\mu, \mu + (1-v)x^\star \mid \mu = \mu^\star]}_{\text{monopolistic/socially-optimal sorting}}. \tag{4}$$

For any fixed $x$, it is always socially optimal to include as many consumers as possible as all have strictly positive insurance value. Once all individuals are thus covered, $M^X = 0$ and thus $x^\star = 1$ solves Equation 4. Thus our basic conclusion is robust to the possibility that the market is not fully covered: perfect competition often drives out the possibility of a positive insurance pooling LDPE while pooling on full insurance is socially optimal.

To confirm our quantitative conclusions, we calibrate a model with market expansion using the HHW data. We assume that $(\mu, v)$ have a joint log-normal distribution with parameters matched to the means and variance-covariance matrix of the HHW data.[26] We then compute, as described in Appendix E, the first-order equilibrium conditions using numerical integration. Finally, we approximately solve for a candidate LDPE for a given ratio of markup to average cost, by hand using a Tâttonement process.

Figure 5.3 pictures LDPE values of $x^\star$, $p^\star$ as a function the ratio of markup to average cost $R = \frac{P(q^\star, x^\star) - x^\star \mathbb{E}[\mu | \mu \geq \mu^\star]}{x^\star \mathbb{E}[\mu | \mu \geq \mu^\star]}$.[27] As $t \to 0$, $x^\star \to 0$, as does welfare. Welfare peaks when the relative markup is 99%, when 92% actuarial rate is achieved with 84% population coverage. Welfare is then 97.5% of first-best welfare ($W/W_* = .975$). Thus, surprisingly to us, nearly all of first-best welfare can be achieved with through appropriate degree of market power with an expanding market. After this point welfare then declines as population coverage shrinks despite the actuarial rate continuing to increase. However, welfare is a large fraction of the first-best even for very large market power (70% or above), while it approaches zero under perfect competition.

The findings are consistent with those of Section 3. However, here both the possibility of exit and competition discipline pricing. Therefore, significant insurance can be sustained

[26] We do not know $\mathbb{V}[v]$ from HHW, so we assumed that $a$ was homogeneous and equal to $\mathbb{E}[a]$, and constructed $\mathbb{V}[v] = (\mathbb{E}[a])^2 \mathbb{V}[\sigma^2]$.

[27] This grows steadily with $t$, except at very extreme values of $t$ where effects are weak and our numerical integration procedures introduce some error.

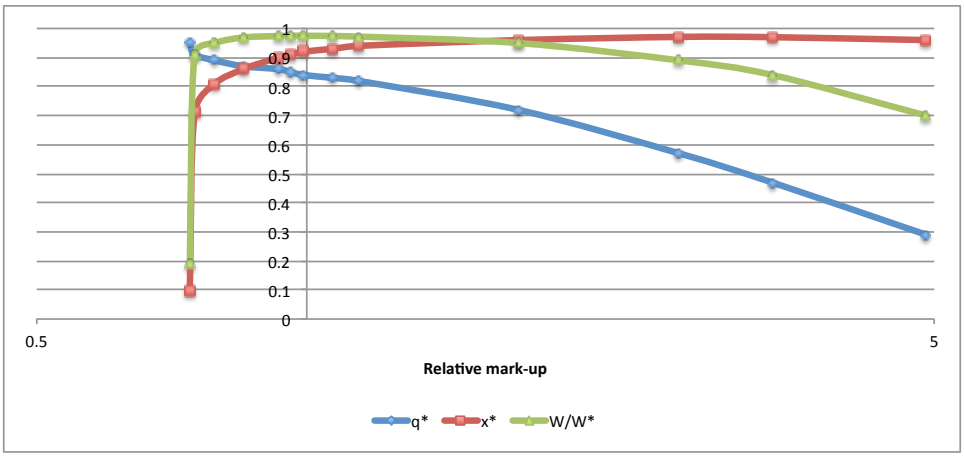


Figure 6: As a function of markup relative to average cost ($R$), share of users who purchase insurance ($q^\star$), insurance quality ($x^\star$) and welfare as a share of the socially optimal value ($W/W^\star$).

without the exorbitant mark-ups required in Section 3 where, for example, a 94% actuarial rate required a relative markup of more than 330%. Market power distorts population coverage downward, as in Mahoney and Weyl (2013), so very high market power is also suboptimal and instead there is an interior optimal degree of market power, but this level still manages to acheive welfare very close to the first-best.

# 6 Conclusion

This paper makes three primary contributions. First, we provide a general characterization of a firm's incentive to use non-price instruments in a selection market with multidimensional heterogeneity to attract the most profitable consumers. Second, we apply this characterization to a simple model of imperfect competition, deriving reduced-form statistics that characterize the LDPE level of insurance. Third, we calibrate these characterizations to show the direction and size of sorting effects, their impact on social welfare and their interaction with market power.

From a policy perspective, our results suggest there may be a case for restricting harmful competition over non-price product characteristics. Blanket structural attempts to limit competition are unlikely to be truly optimal policy because competition on price is often beneficial to population coverage, particularly under adverse selection (Mahoney and Weyl, 2013). However such remedies may be superior to allowing cream-skimming to destroy a market altogether, if better regulation is infeasible. In such a second-best case, estimating

reduced-form statistics such as the one we develop may help determine whether it is better to allow competition with its other benefits (efficient quantities, cost reductions or innovations) or whether competition is likely to be so destructive that it should curtailed. Such estimation may also help determine ways in which markets can be re-organized through various forms of pooling so that dimensions of type offset one another within each pooled group thereby preventing harmful cream-skimming. In particular, employment relationships typically bundle together safety measures, multiple tasks, several types of insurance, etc. These packages may help to mitigate cream-skimming in sub-elements of the employment package, thereby allowing valued services to be provided at competitive equilibria.

While we focused throughout the paper on applications to insurance markets, selection is of interest in many other contexts, including employment relationships, matching markets with non-transferable utility, platform markets, etc.[28] Research on these markets has, in recent years, increasingly focused on the possibility multidimensional types to which our analysis is adapted.

Additionally, a number of theoretical issues remain unaddressed in our analysis. We do not consider the possibility of asymmetric "separating" equilibria, of more than two firms, or non-local deviations by firms. It would also be interesting to study alternatives to our assumption that switching consumers are representative of the full population of consumers, as Bonatti (2011) does for the Rochet and Stole (2002) context.

[28] An earlier version of this paper, Veiga and Weyl (2012) contains several detailed analyses of such settings.

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# A Computational signing of monopoly sorting incentive (Subsection 2.3)

Each consumer faces a normally distributed wealth loss $l \sim \mathcal{N}(\mu, \sigma^2)$, where $\mu$ is heterogeneous between consumers. Let $f = f(l, \mu, \sigma^2)$ denote the Gaussian density of shocks of an individual with type $\mu$. The insurer's policy prescribes a payment $G = G(l, x)$ when the insurer incurs a loss $l$ and the insurer's instrument is $x$. Consumers have CARA utility $-e^{-aw}$, where $w$ is final wealth and $a$ is the (heterogeneous) absolute risk aversion. Let the (homogeneous) initial wealth be $w_0$.

Without insurance, final wealth is $w_N = w_0 - l$. With insurance, it is $w_I = w_0 - l + G - p$. Then, expected surplus from insurance in

$$U = U(x, a, \mu, p) = \int \left[-e^{-aw_I} - \left(-e^{-aw_N}\right)\right] f dl.$$

Then, we express the elements of $S = S(p, x, a, \mu) = \frac{\partial^2 u/\partial x \partial a}{\partial u/\partial a} - \frac{\partial^2 u/\partial x \partial \mu}{\partial u/\partial \mu}$ in terms of the function $U$ by applying the IFT to $U(x, a, \mu, u(x, a, \mu)) \equiv 0$. We obtain $\frac{\partial u}{\partial x} = -\frac{\partial U/\partial x}{\partial U/\partial p}$, $\frac{\partial u}{\partial v} = -\frac{\partial U/\partial v}{\partial U/\partial p}$ and $\frac{\partial u}{\partial \mu} = -\frac{\partial U/\partial \mu}{\partial U/\partial p}$. Moreover,

$$\frac{\partial^2 u}{\partial x \partial a} = \frac{\partial}{\partial a}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial a}\left(-\frac{\partial U/\partial x}{\partial U/\partial p}\right) = -\frac{\left(\frac{\partial U}{\partial x \partial a} + \frac{\partial U}{\partial x \partial p}\frac{\partial u}{\partial a}\right)\frac{\partial U}{\partial p} - \left(\frac{\partial U}{\partial p \partial a} + \frac{\partial U}{\partial p \partial p}\frac{\partial u}{\partial a}\right)\frac{\partial U}{\partial x}}{(\partial U/\partial p)^2}$$

$$\frac{\partial^2 u}{\partial x d\mu} = \frac{\partial}{\partial \mu}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial \mu}\left(-\frac{\partial U/\partial x}{\partial U/\partial p}\right) = -\frac{\left(\frac{\partial U}{\partial x \partial \mu} + \frac{\partial U}{\partial x \partial p}\frac{\partial u}{\partial \mu}\right)\frac{\partial U}{\partial p} - \left(\frac{\partial U}{\partial p \partial \mu} + \frac{\partial U}{\partial p \partial p}\frac{\partial u}{\partial \mu}\right)\frac{\partial U}{\partial x}}{(\partial U/\partial p)^2}.$$

Following Handel, Hendel and Whinston (2013), we set $w_0 = 10^6$ and $\sigma^2 = 10^8$. We consider $a \in [10^{-5}, 10^{-3}]$ and $\mu \in [0, 5 \cdot 10^4]$. We evaluate $S$ as a function of $a$, $p$ and $x$, knowing that there exists some function $\mu^\star(x, p, a)$ that determines the boundary, which can be characterized by $U = 0$. In all cases below properties of the Normal distribution imply that $S$ can be computed analytically using mathematical software.

We consider $p \in \{16000, 20000, 24000\}$. We focus on relatively high levels of insurance, since these are the ones with greater economic relevance. We assume insurance is complete in dimensions other than $x$. In each case, we consider 3 values of $x$ equally spaced within a given range, described for each of the instruments below. In the graphs presented, the top row corresponds to $p = 16000$, while the bottom row has $p = 24000$. Each column corresponds to a level of $x$, with the left column capturing the lowest value of $x$.

Then, we make take 100 evenly-spaced draws of $a$ from the range on which where is some type $\mu^\star(p, x, a) > 0$ that is marginal. For each value of $a$, we compute the $\mu^\star(p, x, a)$ that makes a consumer indifferent and then compute $S(p, x, a, \mu^\star(p, x, a))$. We then plot $S$ as a function of $\mu^\star$, with the draws of $a$ acting as parametric variables for the plot. At times, the estimated value of $\mu^\star$ converges to a negative value, which should be ignored. Notice that these computations require no assumptions on the joint distribution of $(\mu, v)$.

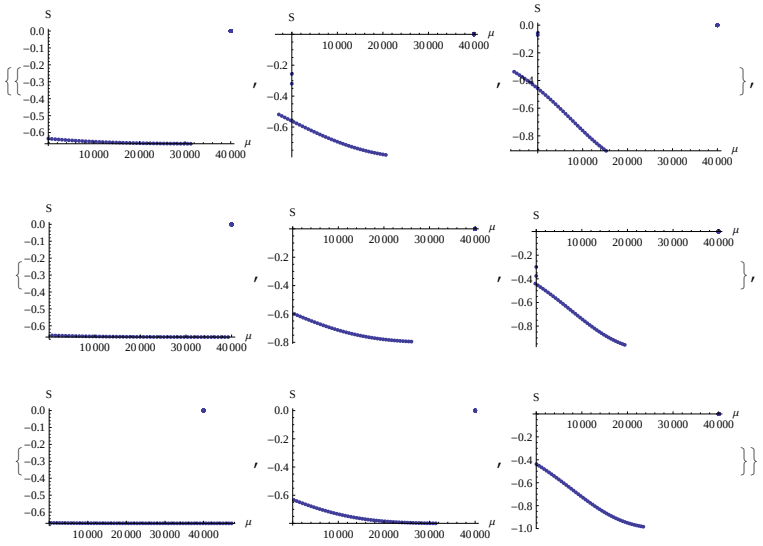

Figure 7: The sorting sign $S$ with an actuarial rate on losses only for various values of that rate (corresponding to columns) and various prices (corresponding to rows) as described in the text. Each graph is $S$ as a function of $\mu$, where $a$ responds to keep the individual marginal to purchasing.

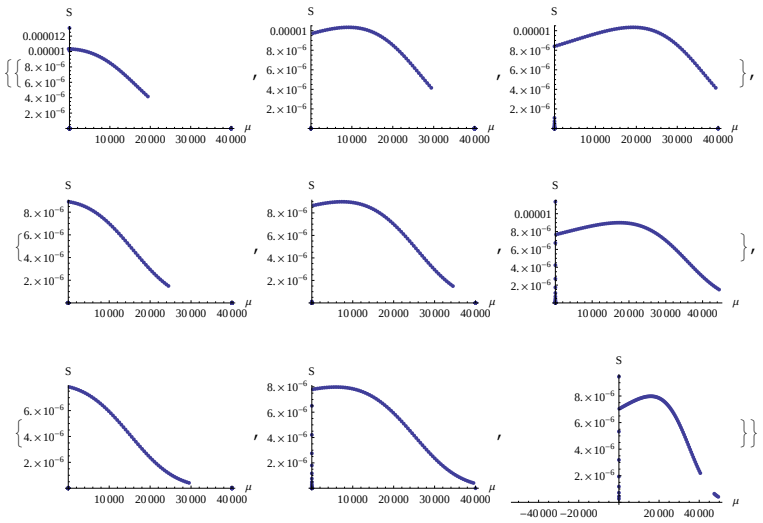

Figure 8: The sorting sign with a deductible rate. Because increasing $x$ *decreases* the amount of insurance, the sign is reversed relative to the other cases.

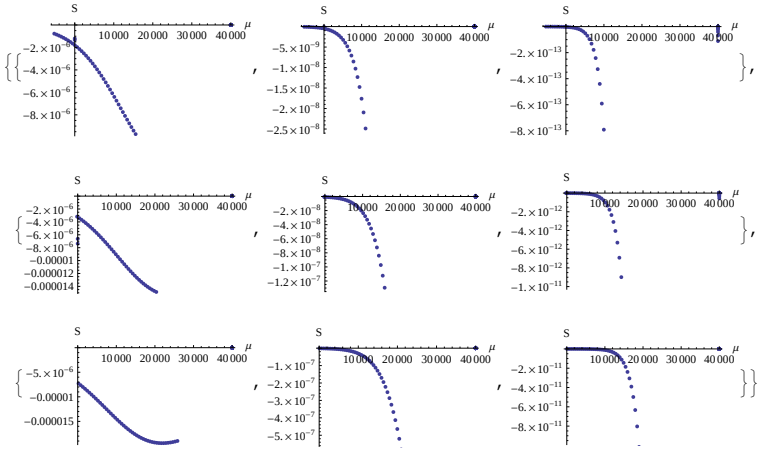

Figure 9: The sorting sign $S$ with an indemnity cap.

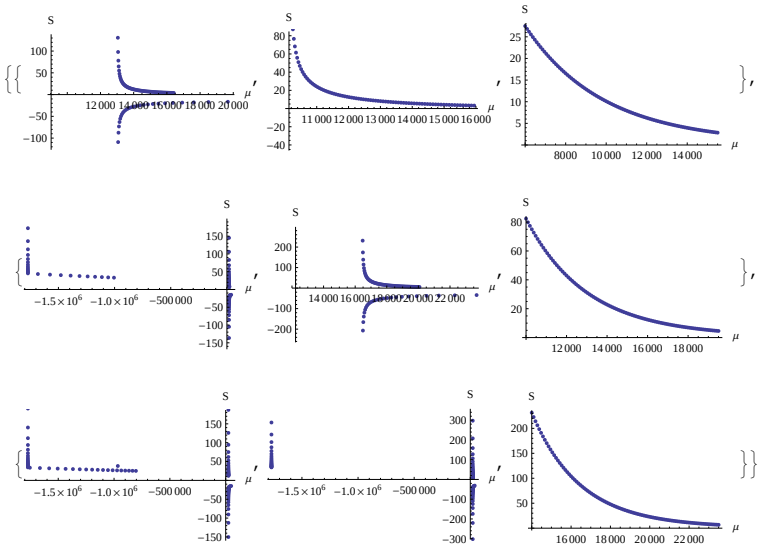

Figure 10: The sorting sign $S$ with comprehensiveness.

First, we consider the case where $x$ is a regular actuarial rate, then $G = \max\{0, xl\}$. The range of the instrument is $x \in \left[\frac{1}{2}, 1\right]$. It is clear from the graphs that $S < 0$. Since higher values of $x$ correspond to better insurance, insurance sorts adversely.

Second, we consider the case where $x$ is a deductible, then $G = \max\{0, l - x\}$. The considered range is $x \in [0, 2000]$. It is clear from the graphs that $S > 0$. Notice that, in the case of a deductible, a low $x$ captures more generous insurance. Thus more insurance sorts adversely in this case also.

Third, we consider the case of an indemnity cap, so $G = \min\{\max\{0, l\}, x\}$. The considered range is $x \in [30000, 70000]$. It is clear from the graphs that $S < 0$. In this case, higher values of $x$ correspond to better insurance, so more generous insurance sorts adversely also in this case.

Finally, we consider that the insurer covers the loss $l$ with probability $x$, but with probability $1 - x$, the consumer incurs the full wealth shock and pays the insurer the insurance

premium. Final wealth without insurance is $w_N = w_0 - l$. With insurance, with probability $x$ final wealth is $w_{I+} = w_0 - p$, and with probability $1 - x$ final wealth is $w_{I-} = w_0 - l - p$. Then, surplus from insurance in

$$U = \int \left[ x \left( -e^{-aw_{I+}} \right) + (1 - x) \left( -e^{-aw_{I-}} \right) - \left( -e^{-aw_N} \right) \right] f dl.$$

The range of $x$ considered is $x \in [0.9, 1]$.

While the numerical optimization algorithm struggles to converge consistently for the smaller values of $x$, it is clear that for $x = 1$ we have $S > 0$. Thus, this instrument sorts advantageously even at full insurance this case.

# B LDPE FOC and SOC (Proposition 3 and Theorem 3)

We have $b^\star \equiv \frac{u(x_0,\theta)-p_0-(u(x_1,\theta)-p_1)}{2t} + \frac{1}{2}$. Then $\frac{\partial b^\star}{\partial p} = -\frac{1}{2t}$ and $\frac{\partial b^\star}{\partial x} = \frac{u'}{2t}$.

Profit is $\pi = \int_\theta \int_0^{b^\star} (p - c(x,\theta)) f(\theta) \, dbd\theta$. We obtain

$$\frac{\partial \pi}{\partial p} = \int_\theta \int_0^{b^\star} f(\theta) \, dbd\theta + \int_\theta -\frac{1}{2t} \left[ p - c(x,\theta) \right] f(\theta) \, d\theta = Q - M\mathbb{E}\left[ p - c \right].$$

$$\frac{\partial \pi}{\partial p^2} = \int_\theta -\frac{1}{2t} f(\theta) \, d\theta + \int_\theta -\frac{1}{2t} f(\theta) \, d\theta = -2M < 0.$$

$$\frac{\partial \pi}{\partial p \partial x} = \int_\theta \frac{u'}{2t} f(\theta) \, d\theta + \int_\theta -\frac{1}{2t} \left( -c' \right) f(\theta) \, d\theta = M\mathbb{E}\left[ u' + c' \right] > 0.$$

$$\frac{\partial \pi}{\partial x} = \int_\theta \int_0^{b^\star} -c' f(\theta) \, dbd\theta + \int_\theta \frac{u'}{2t} (p - c) f(\theta) \, d\theta.$$

$$\begin{aligned} \frac{\partial \pi}{\partial x^2} &= \int_\theta \frac{u'}{2t} \left( -c' \right) f(\theta) \, d\theta + \int_\theta \int_0^{b^\star} \left( -c'' \right) f(\theta) \, dbd\theta + \frac{1}{2t} \int_\theta \left\{ u''(p - c) + u' \left( -c' \right) \right\} f(\theta) \, d\theta \\ &= M \left\{ t\mathbb{E}\left[ u'' - c'' \right] - \mathrm{Cov}\left[ u'', c \right] - 2\mathbb{E}\left[ u' \right] \mathbb{E}\left[ c' \right] - 2\mathrm{Cov}\left[ u', c' \right] \right\}. \end{aligned}$$

Then the Hessian determinant is

$$\begin{aligned} H &= \frac{\partial \pi}{\partial p^2} \frac{\partial \pi}{\partial x^2} - \left( \frac{\partial \pi}{\partial p \partial x} \right)^2 = \\ &= -2MM \left\{ t\mathbb{E}\left[ u'' - c'' \right] - \mathrm{Cov}\left[ u'', c \right] - 2\mathbb{E}\left[ u' \right] \mathbb{E}\left[ c' \right] - 2\mathrm{Cov}\left[ u', c' \right] \right\} - \left( M\mathbb{E}\left[ u' + c' \right] \right)^2 \\ &= -M^2 \left\{ 2t\mathbb{E}\left[ u'' - c'' \right] - 2\mathrm{Cov}\left[ u'', c \right] - 4\mathrm{Cov}\left[ u', c' \right] + \mathbb{E}\left[ u' - c' \right]^2 \right\}. \end{aligned}$$

Then $H$ is positive if

$$t\mathbb{E}\left[\phi''\right]+\frac{1}{2}\mathbb{E}\left[\phi'\right]^2<\mathrm{Cov}\left[u'',c\right]+2\mathrm{Cov}\left[u',c'\right].$$

Evaluating this SOC at $t=G=0$ yields $\frac{1}{4}\frac{\mathbb{E}[\phi']^2}{\mathbb{V}[c']}-1<\frac{\mathrm{Cov}[\phi',c']}{\mathbb{V}[c']}$.

# C Omitted Proofs

*Proof of Proposition 4.* Applying the IFT to the FOC from From Proposition 3 yields $\frac{\partial x^\star}{\partial t}=\frac{(1-x^\star)\mathbb{E}[v]}{\mathbb{E}[v]t+\mathbb{V}[\mu]+(1-2x^\star)\mathrm{Cov}[\mu,v]}$. The numerator is positive. Using the FOC to substitute for $\mathbb{E}[v]t$ in the denominator yields the leftmost term of

$$\frac{\mathbb{V}[\mu]}{1-x^\star}+(1-x^\star)\mathrm{Cov}[\mu,v]>\mathbb{V}[\mu]+(1-x^\star)\mathrm{Cov}[\mu,v]=\frac{1-x^\star}{x^\star}\mathbb{E}[v]t>0$$

where $\frac{\mathbb{V}[\mu]}{1-x^\star}>\mathbb{V}[\mu]$ because $x^\star\in(0,1)$ and the equality follows from the FOC.

□

*Proof of Proposition 5.* From Proposition 3, as $t\to 0$, $\mathrm{Cov}\left[u',c\right]\to 0$ since $\mathbb{E}\left[\phi'\right]=(1-x)\,\mathbb{E}\left[v\right]$ is bounded. In this setting, $\mathrm{Cov}\left[u',c\right]=x\mathbb{V}\left[\mu\right](1+(1-x)\,\beta)$, so LDPE requires $x^\star=0$ or $x^\star=1+\frac{1}{\beta}$. The SOC is derived in Appendix B. When $t\to 0$, it is $(1-x)^2\,\gamma-1<\left(1-\frac{3}{2}x\right)\beta$. The FOC always holds at $x^\star=0$, but the SOC requires $-1<\gamma-1<\beta$. The FOC holds at $x^\star=1+\frac{1}{\beta}$ if $\beta<-1$, in which case the SOC requires $2\gamma<-\beta^3-\beta^2$. The threshold defined by $\beta=\gamma-1$ increases with $\gamma$ ($\frac{\partial\beta}{\partial\gamma}=1$), while that defined by $2\gamma=-\left(\beta^3+\beta^2\right)$ decreases with $\gamma$ ($\frac{\partial\beta}{\partial\gamma}=-\frac{1}{2}\left(3\beta^2+2\beta\right)<0$ if $\beta<-1$). For $\beta<-1$, we have $-\beta^3-\beta^2$ positive, with range $\mathbb{R}_+$ and strictly decreasing.

□

*Proof of Theorem 1.* We follow Veiga and Weyl (2013). We apply the IFT to $u\left(x,t\right)-p=0$ to obtain

$$\frac{\partial\theta_T^\star}{\partial p}=-\frac{1}{\partial u(x,t)/\partial\theta_T}$$
$$\frac{\partial\theta_T^\star}{\partial x}=-\frac{\partial u(x,t)/\partial x}{\partial u(x,t)/\partial\theta_T}$$

Using the Leibniz Rule to differentiate $Q$ and $C$ yields

$$\frac{\partial Q}{\partial p}=\int_{\partial\Theta}\frac{\partial\theta_T^\star}{\partial p}f(t)d\theta_{-T}=\int_{\partial\Theta}-\frac{f(t)}{\partial u(x,t)/\partial\theta_T}d\theta_{-T}=-M$$
$$\frac{\partial Q}{\partial x}=\int_{\partial\Theta}\frac{\partial\theta_T^\star}{\partial x}f(t)d\theta_{-T}=\frac{M}{M}\int_{\partial\Theta}\frac{u'(x,t)f(t)}{\partial u(x,t)/\partial\theta_T}d\theta_{-T}=M\mathbb{E}_u\left[u'\mid\partial\Theta\right]$$
$$\frac{\partial C}{\partial p}=\int_{\partial\Theta}-\frac{c(x,t)f(t)}{\partial u(x,t)/\partial\theta_T}d\theta_{-T}=-M\mathbb{E}_u\left[c\mid\partial\Theta\right]$$

$$\frac{\partial C}{\partial x} = \int_{\partial\Theta} \frac{\partial \theta_T^\star}{\partial x} c(x,t) f(t) d\theta + \int_{\Theta} c' f(\theta) d\theta = M\mathbb{E}_u \left[u'c \mid \partial\Theta\right] + q\mathbb{E}\left[c' \mid \Theta\right]$$

Applying the IFT to $Q\left(P\left(x,q\right),x\right) = q$, and using the results above for $\frac{\partial Q}{\partial x}$ and $\frac{\partial Q}{\partial p}$ we obtain

$$\frac{\partial P(x,q)}{\partial x} = -\frac{\partial D/\partial x}{\partial D/\partial p} = -\frac{M\mathbb{E}_u\left[u' \mid \partial\Theta\right]}{-M} = \mathbb{E}_u\left[u' \mid \partial\Theta\right].$$

Differentiating $\Pi\left(x,q\right)$ with respect to $x$ yields

$$\begin{aligned}\frac{\partial \Pi}{\partial x} &= q\frac{\partial P}{\partial x} - \frac{\partial C}{\partial x} - \frac{\partial C}{\partial p}\frac{\partial P}{\partial x} \\ &= q\mathbb{E}_u\left[u' \mid \partial\Theta\right] - M\mathbb{E}_u\left[u'c \mid \partial\Theta\right] + q\mathbb{E}\left[c' \mid \Theta\right] - M\mathbb{E}_u\left[c \mid \partial\Theta\right]\mathbb{E}_u\left[u' \mid \partial\Theta\right]\end{aligned}$$

The definition of $\mathrm{Cov}\left[u', c\right]$ yields the result.

□

*Proof of Theorem 2.* We have $\mathrm{Cov}_u\left(u', c \mid \theta \in X\right) = \mathrm{Cov}_u\left(\hat{c}(x; g\left(x,\theta\right)), \hat{u}(x; g\left(x,\theta\right)) \mid \theta \in X\right)$for any sub-set of consumers $X$. If $\hat{c}(x; g\left(x,\theta\right))$ and $\hat{u}(x; g\left(x,\theta\right))$ are monotone increasing in their second argument, from Schmidt (2003), the covariance of two monotone increasing functions of a single variable is positive. The other cases are obtained by changing the direction of monotonicity of $\hat{c}(x, g\left(x,\theta\right))$ and $\hat{u}(x, g\left(x,\theta\right))$.

Given Assumption 1, $\partial\Theta$ is defined by a function $\theta_2^\star\left(p,x,\theta_1\right)$ such that $u\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right) = p$. Thus, conditional on $\partial\Theta$, $u' = u'\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right) = \hat{u}(x,\theta_1)$ and $c = c\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right) = \hat{c}(x,\theta_1)$. Thus $\theta_1$ is a unidimensional index. It thus remains only to determine conditions for the monotonicity of $\hat{u}$ and $\hat{c}$ in $\theta_1$. First, we apply the IFT to $u\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right) = p$, yielding

$$\frac{\partial \theta_2^\star(x,p,\theta_1)}{\partial\theta_1} = -\frac{\partial u\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right)/\partial\theta_1}{\partial u\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right)/\partial\theta_2}.$$

Then, we compute

$$\frac{\partial \hat{u}(x,\theta_1)}{\partial\theta_1} = \frac{\partial u'\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right)}{\partial\theta_1} - \frac{\partial u'\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right)}{\partial\theta_2}\frac{\partial u\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right)/\partial\theta_1}{\partial u\left(x,\theta_1,\theta_2^\star\left(p,x,\theta_1\right)\right)/\partial\theta_2}$$

Since $u$ increasing in all arguments, $\frac{\partial\hat{u}(x;\theta_1)}{\partial\theta_1}$ has the sign of $\frac{\partial^2 u/\partial x\partial\theta_1}{\partial u/\partial\theta_1} - \frac{\partial^2 u/\partial x\partial\theta_2}{\partial u/\partial\theta_2}$. The analogous formula applies for $\frac{\partial\hat{c}}{\partial\theta_1}$ by the same logic.

□

*Proof of Theorem 3.* The competitive FOC and SOC are derived in Appendix B. If $\phi' > 0$, the FOC implies $\mathrm{Cov}\left[u', c\right] > 0$ at an LDPE. The derivative of profit evaluated at no insurance is $\frac{\partial\Pi}{\partial x} = \frac{1}{2}\mathbb{E}[\phi'] + 0 > 0$, but evaluated at full insurance it is $\frac{\partial\Pi}{\partial x} = -\frac{1}{t}\mathrm{Cov}\left[c', c\right] < 0$. By continuity, there is some interior point at which $\frac{\partial\Pi}{\partial x} = 0$.

We emphasize that $\mathrm{Cov}\left[c', c\right] > 0$ is a natural condition: it implies only that reducing

insurance quality implies larger cost reductions for the costliest consumers.[29] As in Section 2.4, sorting is adverse at an LDPE and at full insurance. Thus sorting places a downward pressure on insurance quality $x$ at an LDPE.

□

*Proof of Theorem 5.* If $\mathbb{E}[\phi']$ is bounded, satisfying Equation 3 requires $\lim_{t\to 0} \text{Cov}[u', c] = 0$.

At zero insurance, $c$ is constant, so $\text{Cov}[u', c] = 0$ holds. Evaluating the SOC at $t = G = 0$, and noticing $\text{Cov}[u'', c] \mid_{g=0} = 0$, yields the first inequality result.

Theorem 3 shows $\text{Cov}[u', c] \mid_{G=l} > 0$. Then, by continuity, the sorting term vanishes at some interior point if $\frac{\partial \text{Cov}[u',c]}{\partial x} \mid_{G=0} < 0$. To show this, first we compute $\frac{\partial Cov[u',c]}{\partial x} = Cov[u'', c] + Cov[u', c']$. Then, $Cov[u'', c] \mid_{G=0} = 0$ and therefore we require $\frac{\partial Cov[u',c]}{\partial x} \mid_{G=0} = \mathbb{V}[c'] + Cov[\phi', c'] < 0$.

□

*Proof of Proposition 6.* We have $u' - c' = \phi' = \frac{\text{Cov}_l\left[U', \frac{\partial G}{\partial x}|\theta\right]}{\mathbb{E}_l[U'|\theta]}$. At full insurance, $U'$ does not vary, so the FOC for the maximization of welfare is satisfied ($\mathbb{E}[\phi'] = 0$). The derivative of profit at full insurance is $\frac{\partial \Pi}{\partial x} = \mathbb{E}[\phi'] - \text{Cov}[c' + \phi', c]$. At full insurance, $\phi' = 0$, so this derivative is negative since we have assumed $\text{Cov}[c', c] \mid_{x=1} > 0$. From Proposition 3, an LDPE satisfies $\mathbb{E}[\phi']\, t = \text{Cov}[u', c]$. If the SOC holds, then $\frac{\partial \Pi}{\partial x^2} < 0$. Applying the IFT to the FOC yields $\frac{\partial x}{\partial t} = -\frac{\mathbb{E}[\phi']}{\partial \Pi / \partial x^2} > 0$, since $\phi' > 0$.

□

# D Welfare (Subsection 5.2)

First we obtain $\frac{\partial \lambda}{\partial x} = -(1-x)^{-\epsilon} = -\frac{1}{L} D$ and $\frac{\partial D}{\partial x} = -\epsilon L (1-x)^{-\epsilon - 1} (-1) = \frac{\epsilon}{1-x} D$. Intuitively, the share of the burden faced by consumers ($\lambda$) decreases with $x$, whereas the demand for healthcare ($D$) increases. For an individual with type $(L, b)$, the surplus of purchasing from hospital 0 over hospital 1 is $-e^{-a(-L\lambda(x_0) - p_0)} - bt - \left(-e^{-a(-L\lambda(x_1) - p_1)} - (1-b)\, t\right)$. Letting $b^\star$ be the level of $b$ that makes this surplus vanish, we obtain $b^\star = \frac{1}{2} + \frac{e^{a(L\lambda(x_1) + p_1)} - e^{a(L\lambda(x_0) + p_0)}}{2t}$.

We compute

$$\frac{\partial b^\star}{\partial p_0} = -\frac{1}{2t} a e^{aL\lambda(x_0)} e^{ap_0}$$

$$\frac{\partial b^\star}{\partial x_0} = -\frac{1}{2t} a e^{aL\lambda(x_0)} e^{ap_0} L \frac{\partial \lambda}{\partial x} = \frac{1}{2t} a e^{aL\lambda(x_0)} e^{ap_0} D$$

Intuitively, an increase in price decreases demand, whereas an increase in cost-sharing increases demand.

The number of buyers from hospital 0 is $Q_0 = \int_L \int_0^{b^\star} f(L)\, dbdL$. Then,

$$M = -\frac{\partial Q_0}{\partial p_0} = -\int_L \frac{\partial b^\star}{\partial p_0} f(L)\, dL = \int_L \frac{1}{2t} a e^{aL\lambda(x_0)} e^{ap_0} f(L)\, dL$$

[29] For instance, in Section 2, this condition becomes $\text{Cov}[c', c] = x\mathbb{V}[\mu] > 0$.

$$\mathbb{E}_u\left[\zeta \mid b=b^\star\right] = \frac{\int_L \frac{\partial b^\star}{\partial p_0}\zeta f(L)dL}{\int_L \frac{\partial b^\star}{\partial p_0}f(L)dL} = \frac{\int_L \zeta e^{aL\lambda(x_0)} f\left(L\right)dL}{\int_L e^{aL\lambda(x_0)} f\left(L\right)dL} = \mathbb{E}_u\left[\zeta\right].$$

Notice that expectations conditional on the margin are different from unconditional expectations, but are independent of $b$.

Profit is $\pi_0 = \int_L \int_0^{b^\star} \left[p_0 - x_0 D\left(L, x_0\right)\right] f\left(L\right) dbdL$. We now compute the two FOCs. At an LDPE we have $x_1 = x_0 = x$ and $p_1 = p_0 = p$. The FOC for price is

$$\frac{\partial \pi_0}{\partial p_0} = \int_L \int_0^{b^\star} f\left(L\right) dbdL + \int_L \frac{\partial b^\star}{\partial p_0}\left[p - xD\left(L, x\right)\right] f\left(L\right) dL = 0 \Rightarrow \frac{Q}{M} = \mathbb{E}_u\left[p - xD\right]$$

while the FOC for $x$ is

$$\begin{aligned}
\frac{\partial \pi_0}{\partial x_0} &= -\int_L \int_0^{b^\star} \left[D + x_0 \frac{\partial D}{\partial x}\right] f\left(L\right) dbdL + \int_L \frac{\partial b^\star}{\partial x_0}\left[p_0 - x_0 D\right] f\left(L\right) dL \\
&= -\int_L \int_0^{b^\star} \left[D + x_0 \frac{\partial D}{\partial x}\right] f\left(L\right) dbdL + \int_L \frac{1}{2t} a e^{aL\lambda(x_0)} e^{ap_0} f\left(L\right) dL \left(\frac{\int_L e^{aL\lambda(x_0)} D\left[p_0 - x_0 D\right] f\left(L\right) dL}{\int_L e^{aL\lambda(x_0)} f\left(L\right) dL}\right) \\
&= -Q\mathbb{E}\left[D\right] - \frac{Qx\epsilon}{1-x}\mathbb{E}\left[D\right] + Q\mathbb{E}_u\left[D\right] - xM\mathbb{V}_u\left[D\right] = 0
\end{aligned}$$

$$Q\left(\mathbb{E}_u\left[D\right] - \mathbb{E}\left[D\right]\right) = x\left(Q\frac{\epsilon}{1-x}\mathbb{E}\left[D\right] + M\mathbb{V}_u\left[D\right]\right)$$

At a competitive LDPE the sorting term must vanish, so we must have $xM\mathbb{V}_u\left[D\right] = 0 \Rightarrow x = 0$ . Therefore, the social optimum obtains at the competitive limit. Moreover, since $e^{aL\lambda(x)}$ is increasing in $L$, we have

$$\mathbb{E}_u\left[D\right] = \frac{\int_L \frac{L}{(1-x)^\epsilon} e^{aL\lambda(x_0)} f(L)dL}{\int_L e^{aL\lambda(x_0)} f(L)dL} = \frac{1}{(1-x)^\epsilon}\frac{\int_L L e^{aL\lambda(x)} f(L)dL}{\int_L e^{aL\lambda(x)} f(L)dL} > \frac{1}{(1-x)^\epsilon}\int_L L f(L)dL = \mathbb{E}\left[D\right]$$

It is clear that $\frac{Qx\epsilon}{1-x}\mathbb{E}\left[D\right] + xM\mathbb{V}_u\left[D\right] > 0$. Since $\mathbb{E}_u\left[D\right] - \mathbb{E}\left[D\right] > 0$, for $t > 0$, we must have $x > 0$ for the FOC to hold at an LDPE.

We now calibrate these results to show that they are empirically similar to those of Section 3. Recall that, in that section, we had $\mathbb{E}\left[\mu\right] = 6.6 \cdot 10^3$ and $\mathbb{V}\left[\mu\right] = 5 \cdot 10^7$. We assume individuals have homogeneous constant absolutely risk-aversion equal to $4 \cdot 10^{-4}$. $L$ is drawn from an individual-specific normal distribution with mean $\mu$ and variance $\sigma^2 = 1.6 \cdot 10^8$, so $\mu \sim \mathcal{N}\left(6.5 \cdot 10^3, 5 \cdot 10^7\right)$ and $L \mid \mu \sim \mathcal{N}\left(\mu, 1.6 \cdot 10^8\right)$. Notice $\lambda = \lambda\left(x\right)$ is not a function of $L$. Then, $\mathbb{E}\left[D\right] = \frac{1}{(1-x)^\epsilon} 6.6 \cdot 10^3$. We then obtain

$$f\left(L\right) = f_{L|\mu}\left(L \mid \mu\right) f_\mu\left(\mu\right) = \frac{e^{-\frac{(L-\mu)^2}{2(1.6\cdot 10^8)}}}{\sqrt{2\pi\left(1.6 \cdot 10^8\right)}} \frac{e^{-\frac{\left(\mu - 6.5\cdot 10^3\right)^2}{2(5.0\cdot 10^7)}}}{\sqrt{2\pi\left(5.0 \cdot 10^7\right)}}$$

$$M = \frac{1}{2t} a e^{ap} \int_L e^{aL\lambda} f(L)\, dL = \frac{1}{2t} a e^{ap_0} e^{\lambda(16.8\lambda+2.64)}$$

$$\mathbb{E}_u[D] = \frac{\int_\mu \int_L \frac{L}{(1-x)^\epsilon} e^{aL\lambda} f(L) dL d\mu}{\int_\mu \int_L e^{aL\lambda} f(L) dL d\mu} = \frac{8.4 \cdot 10^4 \lambda + 6.6 \cdot 10^3}{(1-x)^\epsilon}$$

$$\mathbb{V}_u[D] = \mathbb{E}_u\left[D^2\right] - \mathbb{E}_u[D]^2 = \frac{1}{(1-x)^{2\epsilon}} \frac{\int_\mu \int_L L^2 e^{aL\lambda} f(L) dL d\mu}{\int_\mu \int_L e^{aL\lambda} f(L) dL d\mu} - \mathbb{E}_u[D]^2 = \frac{5 \cdot 10^7 + 1.6 \cdot 10^8}{(1-x)^{2\epsilon}} = \frac{2.1 \cdot 10^8}{(1-x)^{2\epsilon}}$$

Notice that the weighting $e^{aL\lambda}$ will enter into the normal density $f(L)$ leaving the denominator unchanged, where the denominator captures the variance of the normal distribution. Therefore we should not expect $\mathbb{V}_u[D]$ to depend on $\lambda$.

Now, consider the FOC with respect to price, using $Q = \frac{1}{2}$ at the LDPE: $p - \mathbb{E}_u[xD] = \frac{1}{2M}$. Absolute markup is $\frac{1}{2M} = \frac{t}{a e^{ap_0} e^{\lambda(16.8\lambda+2.64)}}$. Markup as a ratio of the cost absorbed by the firm is $R = \frac{1}{2Mx\mathbb{E}[D]}$. One can solve for $M = \frac{1}{2Rx\mathbb{E}[D]}$. We can then plug this value of $M$ into the FOC for $x$:

$$Q\left(\mathbb{E}_u[D] - \mathbb{E}[D]\right) = x\left(Q\frac{\epsilon}{1-x}\mathbb{E}[D] + \frac{\mathbb{V}_u[D]}{2Rx\mathbb{E}[D]}\right)$$

Note that all other quantities have been computed above as functions of $x$ and $\epsilon$. Letting $\epsilon = 0.2$, we now plot the RHS and the LHS of the FOC for $x$, as a function of $x$, for different values of $R$. The resulting graphs and discussion are in Subsection 5.2.

# E Market Expansion (Subsection 5.3)

A derivation of the LDPE and social optimality conditions appears in Veiga and Weyl (2012). At a LDPE, individuals purchase insurance from one firm if $\mu \geq \mu^\star = \frac{p}{x^\star} - \frac{2-x^\star}{2}v$. Thus the fraction of individuals who buy is $Q(p,x) \equiv \int_{\underline{v}}^{\overline{v}} \int_{\mu^\star}^{\overline{\mu}} f(v,\mu) d\mu dv$ and the average cost of these individuals is now endogenous. Inverting $Q$ with respect to $p$ for a given $x$ yields $P(q,x)$ as in Subsection 2.2. Recall that, at an LDPE, we have $\mu \geq \mu^\star$. Specializing the conditions from Veiga and Weyl to the expectations here among exiters and switchers yields LDPE conditions

$$\underbrace{P}_{\text{price}} - \underbrace{x^\star \frac{M^X \mathbb{E}_u[\mu \mid \mu = \mu^\star] + M^S \mathbb{E}[\mu \mid \mu \geq \mu^\star]}{M^X + M^S}}_{\text{average cost of marginal consumers}} = \underbrace{\frac{q}{M^X + M^S}}_{\text{mark-up}}. \tag{5}$$

$$\underbrace{q^\star \frac{M^X \mathbb{E}_u[\mu + (1-x^\star)v \mid \mu = \mu^\star] + M^S \mathbb{E}[\mu + (1-x^\star)v \mid \mu \geq \mu^\star]}{M^X + M^S}}_{\text{marginal utility of average marginal consumer}} - q^\star \underbrace{\mathbb{E}[\mu \mid \mu \geq \mu^\star]}_{\text{marginal cost of average consumer}} =$$

$$x\left[\underbrace{M^X \text{Cov}_u[\mu, \mu + (1-x^\star)v \mid \mu = \mu^\star]}_{\text{monopolistic sorting}} + \underbrace{M^S \text{Cov}[\mu, \mu + (1-x^\star)v \mid \mu > \mu^\star]}_{\text{cream-skimming}}\right]. \tag{6}$$

From Equations 6 one can derive the analog of Proposition 5, which is mentioned in Subsection 5.3 above.

Now we turn to the calibration. To two significant digits, in the HHW data, we have $\mathbb{E}[\mu] \approx 6.6 \cdot 10^3$, $\mathbb{E}[v] \approx 6.7 \cdot 10^4$, $\mathbb{V}[\mu] = 5.0 \cdot 10^7$ and $\text{Cov}[\mu, v] \approx 6.3 \cdot 10^7$. We do not have$\mathbb{V}[v]$. However, the variation in $a$ seems to be quite small and only weakly correlated with that in $\sigma^2$, so we simply set $\mathbb{V}[v] = \mathbb{E}[a]^2 \mathbb{V}[\sigma^2] \approx 9.8 \cdot 10^9$. We assume that $(\mu, v)$ has a joint log-normal distribution:

$$\begin{pmatrix} \log(\mu) \\ \log(v) \end{pmatrix} \sim \mathcal{N}\left( \begin{matrix} m_\mu \\ m_v \end{matrix}, \begin{matrix} V_\mu & C \\ C & V_v \end{matrix} \right).$$

Then $\mathbb{E}[\mu] = e^{m_\mu + \frac{V_\mu}{2}}$ and similarly for $\mathbb{E}[v]$, $\mathbb{V}[\mu] = (\mathbb{E}[\mu])^2 \left(e^{V_\mu} - 1\right)$ and similarly for $\mathbb{V}[v]$ and $\text{Cov}[\mu, v] = \mathbb{E}[v]\mathbb{E}[\mu] \left(e^C - 1\right)$. This system of equations of 5 equations with 5 unknowns can be solved analytically and uniquely (calculations omitted here) to yield $m_\mu = 8.4$, $m_v = 11$, $V_\mu = .76$, $V_v = 1.2$ and $C = .63$.

All quantities in Equations 5-6 can then be computed, though not analytically. Instead we employ numerical quadrature, sampling 100 points uniformly spaced from the inverse CDF of $v$ and $\mu$ given $v$. Note that, by Normal updating and given $v$, $\log(\mu) \sim \mathcal{N}\left(8.4 + .53(\log(v) - 11), .43\right)$. Let $f(v), F(v)$ be the marginal PDF and CDF of $v$, respectively. Let $g(\mu \mid v)$ and $G(\mu \mid v)$ represent the conditional PDF and CDF of $\mu$ given $v$, respectively.

Rather than discussing all of the calculations, we now just take two sample equations and discuss how they were computed. For instance,

$$\mathbb{E}\left[\mu \mid \mu > \frac{p}{x} - \frac{2-x}{2}v\right] = \frac{\int_{v=0}^{\infty} \int_{\mu=\max\{0, \frac{p}{x} - \frac{2-x}{2}v\}}^{\infty} \mu g(\mu \mid v) f(v) d\mu dv}{\int_{v=0}^{\infty} \int_{\mu=\max\{0, \frac{p}{x} - \frac{2-x}{2}v\}}^{\infty} g(\mu \mid v) f(v) d\mu dv} \approx \frac{\frac{1}{100}\sum_{i=1}^{100} I\left(F^{-1}\left(.005 + \frac{i}{100}\right)\right)}{\frac{1}{100}\sum_{i=1}^{100} J\left(F^{-1}\left(.005 + \frac{i}{100}\right)\right)},$$

where $I$ and $J$ are functions, described shortly, that represent the value of the interior of the two integrals. We have used the Riemann sum approximation of the outer integral $\left(\int_{v=0}^{\infty}(\cdot) f(v) dv\right)$ on both top and bottom based on 100 points evenly spaced in probability space, as is standard in numerical quadrature. The inner integral $\left(\int_{\mu=\max\{0, \frac{p}{x} - \frac{2-x}{2}v\}}^{\infty}(\cdot) g(\mu \mid v) d\mu\right)$ is then evaluated in one of two ways in order to form the functions $I$ and $J$ that enter into this sum. If $\frac{p}{x} < \frac{2-x}{2}v$, the integral is taken over the full space of values and has the analytic expression for the mean of a log-normal distribution in the case of $I$ $\left(e^{8.4 + .53(\log(v) - 11) + .5 \cdot .43}\right)$ and a value of 1 in the case of $J$. If $\frac{p}{x} \geq \frac{2-x}{2}v$ then for $J$ the expression is analytic, simply $1 - G\left(\frac{p}{x} - \frac{2-x}{2}v \mid v\right)$ and in the case of $I$ we again compute it by quadrature as $\frac{1}{100}\sum_{j=1}^{100} 1_{G^{-1}\left(.005 + \frac{j}{100} \mid v\right) > \frac{p}{x} - \frac{2-x}{2}v} G^{-1}\left(.005 + \frac{j}{100} \mid v\right)$. As our other example

$$M^X(p, x) \equiv \frac{1}{2x}\int_0^{\infty} f\left(v, \frac{p}{x} - \frac{2-x}{2}v\right) dv \approx \frac{1}{200x}\sum_{i=1}^{100} f\left(F^{-1}\left(.005 + \frac{i}{100}\right), \frac{p}{x} - \frac{2-x}{2}F^{-1}\left(.005 + \frac{i}{100}\right)\right).$$

These procedures allowed us to calculate the FOCs at an LDPE. Then, for each $t$ of interest, we manually searched for an LDPE, beginning at the covered-market LDPE $p^\star$ and $x^\star$ computed analytically in Section 3. To do so we followed an informal single-equation Tâttonement process using $p$ and $x$ as our search variables: if the LHS>RHS of Equation 5, we incremented price, and if LHS<RHS we decremented it until we had

reached satisfied the first-order equilibrium conditions to 3 significant digits. In particular we ensured that no change less fine than this number of digits avoided overshooting. Then we applied the same process to Equation 6 to find an approximately equilibrating value of $x$. We then took this value back to Equation 5 and iterated until both had converged to 4 significant digits. In all cases convergence obtained within 5 minutes of trial. We then computed welfare by a similar quadrature method. The results, for $t = 5 \cdot 10^2, 3 \cdot 10^3, 5 \cdot 10^3, 6 \cdot 10^3, 7 \cdot 10^3, 7.5 \cdot 10^3, 8 \cdot 10^3, 9 \cdot 10^3, 1 \cdot 10^4, 2 \cdot 10^4, 5 \cdot 10^4, 1 \cdot 10^5$ and $1 \cdot 10^8$, are presented in the chart in the main text.