# Low Competition Traps*

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**Abstract**

We develop a multi-industry growth model with oligopolistic competition and variable markups. Our model features a complementarity between capital accumulation and competition, which can give rise to multiple competitive regimes – regimes characterized by a large capital stock and strong competition and regimes featuring low capital and weak competition (*low competition traps*). Negative transitory shocks can trigger a transition from a high to a low competition regime. We also show that, as the firm size/markup distribution becomes more dispersed, the economy is increasingly likely to enter a *low competition trap*. In a calibrated version of our model, a transition from a high to a low competition regime rationalizes important features of the US great recession and its aftermath, such as the persistent drop in output and aggregate TFP, the decline of the labor share, the increase in the profit share, and the decline in the number of firms.

**JEL Classification:** E22, E24, E25, E32, L16

**Key words:** competition, market power, multiple equilibria, poverty traps, great recession

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# 1 Introduction

The US economy appears to have experienced a fundamental change over the past four decades. Several studies and indicators have highlighted different secular trends concerning the structure of product markets, the distribution of income across factors and the distribution of activity across firms. Some of the trends attracting prominent attention in the recent debate include[1]

1. the decline in the labor share
2. the increase in aggregate markups
3. the increase in the profit share
4. the increase in concentration
5. the decline in the firm entry rate
6. the persistent deviation of aggregate output and TFP from trend after 2008

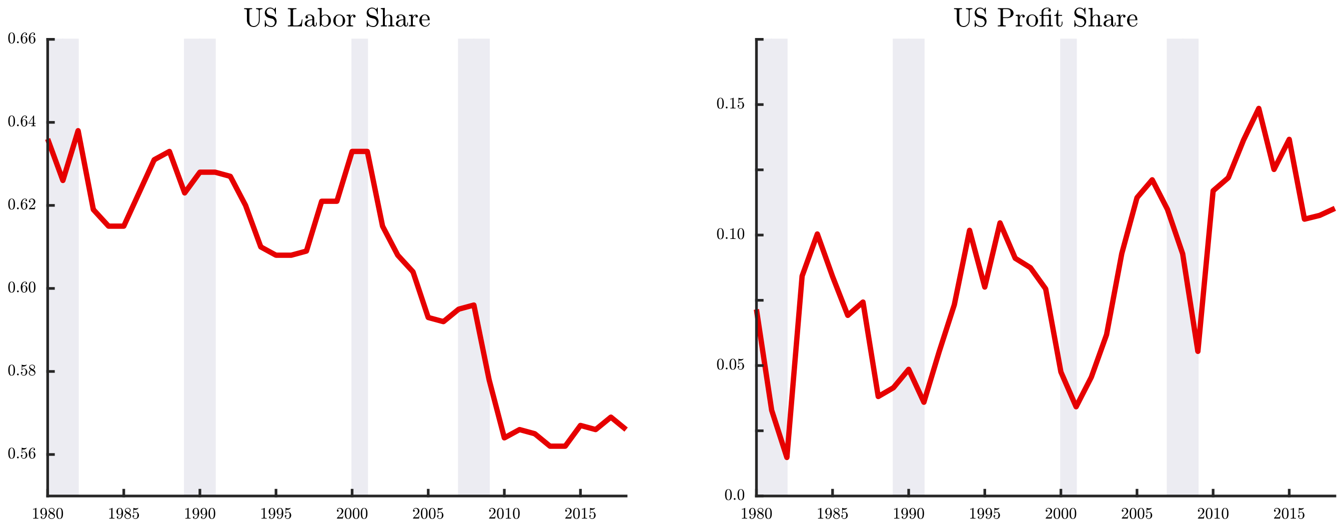
Although these trends are still the object of discussion in the literature, two aspects are starting to gain consensus. First, the first four trends in the list above appear to be driven mostly by a reallocation of activity from small, low markup firms towards large, high markup firms. For example, Autor et al. (2017) and Kehrig and Vincent (2018) find that the decrease in the US labor share has not been driven by a change in the labor share of the median establishment, but rather by a shift in activity towards large establishments (with high markups and low labor shares). Similarly, De Loecker and Eeckhout (2017) show that the rise in aggregate markups is not explained by a change in the markup of the median firm, but rather by an increasing share of large firms and growing dispersion in the markup distribution. More broadly, these facts relate to the empirical observation that firm differences have become increasingly pronounced in recent decades – with several studies documenting rising dispersion in size, revenue TFP, markups and profit margins within industries.[2]

Second, these trends have become especially pronounced in the aftermath of the last two recessions – the 2001 crisis and, in particular, the great recession of 2008. For example, as Figure 1(a) shows, the decline of the US labor share between 1980 and 2018 (-7.0 pp) is concentrated in two relatively short periods: 2001-2003 (-2.5 pp) and 2008-2010 (-3.1 pp). The rise in the aggregate profit share appears to be concentrated in the period after 2008 (Figure 1(b)). Similarly, a large part of the decline in the firm entry

[1] See for example Eggertsson et al. (2018), Aghion et al. (2019) and Akcigit and Ates (2019a).

[2] Several independent studies have recently documented (i) widening revenue productivity gaps between firms in the same industry (Andrews et al. (2015), Kehrig (2015), Decker et al. (2018)), (ii) growing dispersion in size (Bonfiglioli et al. (2018), Autor et al. (2017)), (iii) in price-cost markups (De Loecker and Eeckhout (2017), Calligaris et al. (2018), Díez et al. (2018)), and also (iv) in profit margins (Gao et al. (2013)). See Van Reenen (2018) for a summary of the recent findings.

rate takes place in the aftermath of the 2008 recession; this has been associated with a persistent decline in the number of active firms after 2008 (see Appendix A.1). The 2008 recession has, however, had a broader impact on the macroeconomy, being associated with a persistent deviation of real GDP and aggregate TFP from trend – the so called *great deviation* (Figure 2).



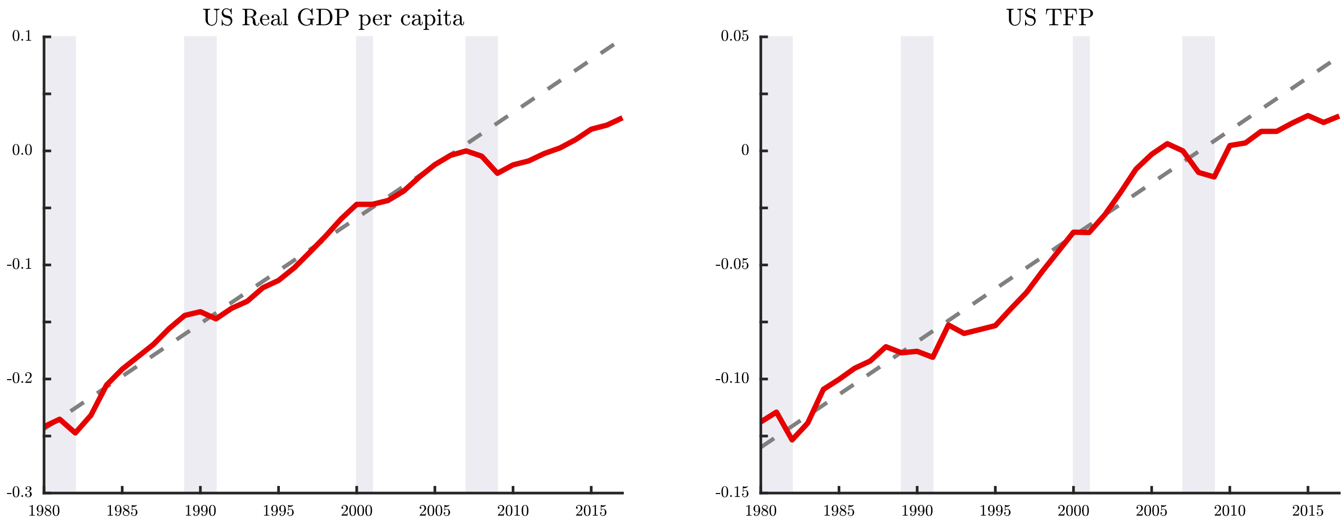

(a) **Labor Share of the Corporate Business Sector** (b) **Profit Share of the Corporate Business Sector**

Figure 1: US Labor and Profit Shares (1980-2018)

The left panel shows the labor share of the corporate business sector (from the BLS). The right panel shows the aggregate profit share, constructed as the ratio of aggregate profits to gross value added (for the US non-financial corporate sector). Aggregate profits are the difference between gross value added and the sum of total labor compensation, the user cost of capital and taxes to production less subsidies (see Appendix A.1.2 for details).

These trends suggest that the US economy may have changed both because of a long-run process of reallocation towards large firms, and because of aggregate fluctuations that may have had a persistent impact on the macroeconomy. In this paper, we investigate a natural, yet unexplored, connection between these two sets of observations. We provide a framework to study the interactions between business cycles, firm heterogeneity and the product market structure. We have two main questions. First, we ask whether a temporary recession can have a persistent impact on the product market structure and on the macroeconomy. Second, we investigate whether the firm size/markup distribution can affect the response of the economy to aggregate shocks. In particular, we ask whether the long-run increase in size/markup dispersion, which the literature has documented, may have contributed to the severity and persistence of the 2008 recession.

Our theory builds on the neoclassical growth model, with a representative household and the standard accumulation of capital. We depart, however, from the canonical model by introducing an endogenous market structure as in Atkeson and Burstein (2008). We model a multi-industry economy, with endogenous entry and oligopolistic competition. Firms face fixed production costs and make their entry decisions based on their idiosyncratic productivity draws and on the aggregate level of capital/output. The endogenous number of players in every market, together with the distribution of productivities, determine the overall distribution of markups and outputs.[3] In this environment, there is a complementarity between capital accumulation and the degree of competition in product markets. On the one hand, a larger stock of capital allows more firms to break even and hence results in a more competitive market structure. Consequently, profit shares decline and factor shares increase. Higher competition, on the other hand, increases the incentives for capital accumulation. Larger factor shares result in higher factor prices (wages and rental rates) and hence a joint increase in the supply of labor and capital.




(a) **US Real GDP per capita** (b) **US Real TFP**

Figure 2: The *Great Deviation*

The left panel shows real GDP per capita (from the BEA). The right panel shows Fernald (2012) aggregate TFP series. The two series are in logs, undetrended and centered around 2007. The linear trends are computed for the 1980-2007 period.

Two main insights emerge from our theory. The first is that the complementarity between capital accumulation and competition may give rise to multiple competitive regimes or stochastic steady-states. In particular, there can be regimes featuring a large stock of capital, a large number of firms and hence intense competition (low profit shares and high factor shares); and regimes featuring a low stock of capital, a small number of firms and weak competition (*low competition traps*).[4] Small temporary shocks, which have a reduced impact on the aggregate stock of capital, will typically make the economy fluctuate within one particular regime. However, whenever the economy is hit by a sufficiently large (temporary) shock, it

[3]This setup nests the canonical neoclassical model with monopolistic competition as a special case in which all industries have a single player.

[4]The existence of multiple regimes or stochastic steady-states does not rely on the existence of multiple equilibria. In other words, our economy can feature multiple steady-states in spite of the existence of a unique equilibrium path. The steady-state the economy will reach depends on the initial condition and the history of aggregate shocks.

can experience a transition across regimes. In particular, when it starts in a high competition regime and is hit by a negative shock that significantly depresses capital accumulation, the economy can experience a persistent transition to a low competition regime. In such a case, the economy follows a path that, in many aspects, resembles the 2008 recession and subsequent *great deviation*. There is a persistent decline in the labor share, an increase in the profit share, as well as a persistent drop in the number of active firms.

The second insight to emerge from our theory concerns the role of firm heterogeneity. We analyze the consequences of larger firm differences by considering a shock that increases the right tail of the distribution of idiosyncratic productivity draws.[5] This shock has two main consequences. First, from a static point of view, it generates a reallocation of activity from small unproductive firms towards large productive firms, which is capable of explaining the first four facts we listed earlier. In particular, it is consistent with a fall in the labor share, a rise in aggregate markups and in concentration. Second, as firm heterogeneity increases, the economy becomes more likely to fall in a low competition regime. In other words, a high competition regime becomes increasingly difficult to sustain, so that even a relatively small temporary shock can trigger a persistent transition to a *low competition trap*.

To exemplify the mechanism, think of an economy in which two firms can operate in any industry: a productive/large firm and an unproductive/small firm. Since our model features a tight connection between productivity/size and markups, the large firm will be charging a high markup, and the small firm a low one. Due to the existence of fixed costs, the small firm can enter only when it makes sufficiently large profits (i.e. if it breaks even). This fact translates into a threshold of aggregate capital below which it remains out of the market. We show that such a threshold is likely to increase if the large firm increases its productivity advantage over the small one. As the large/productive firm becomes even more productive, its markup will increase, whereas the markup of the small firm will decrease. As a result, it becomes more difficult for the small firm to survive, even when the aggregate stock of capital is relatively large. This translates into a lower likelihood that a high competition regime can be maintained (i.e. the basin of attraction of a high competition regime shrinks). Our model therefore suggests that larger firm differences may have increased the likelihood of a recession like the 2008 crisis, with output experiencing a persistent deviation from trend.

We calibrate our model to match some moments of the markup distribution of public firms in 2007 (in particular, its average and its standard deviation). We ensure that the economy features at least two competitive regimes, and adopt the interpretation that it starts at the highest one. We simulate our model and compute business cycle moments and correlations. We show that, relative to a standard business cycle model with a fixed number of firms and market structure, our model features considerably larger amplification and persistence. We also ask whether our model can generate a pattern similar to that of Figure 2(a). We feed the model with a sequence of temporary negative TFP shocks (to replicate the

[5]In our model, firm heterogeneity is exclusively driven differences in idiosyncratic productivities. However, as we discuss below, this is a parsimonious way of modeling firm heterogeneity and other sources of heterogeneity (such as firm-specific demand shifters) would yield identical results.

observed behavior of aggregate TFP in 2008/2009) and show that they can trigger a transition from the high to the low regime. Quantitatively, our model can replicate well the persistent drop in output, employment, investment and aggregate TFP that we observe in the data. It also matches the persistent drop in the labor share observed after 2008.

To evaluate the role of larger average markups and larger markup dispersion, we calibrate our model to match the same two moments of the markup distribution in 1985. We show that, relative to the 2007 model, the 1985 economy exhibits lower amplification and persistence. Furthermore, the size of the shock needed to trigger a transition from the high to the low regime is larger in the 1985 economy. In particular, the sequence of negative TFP shocks that we feed in the 2007 economy is not sufficient to generate a transition from the high to the low regime in 1985. Overall, these results indicate that the increase in average markups and in markup dispersion may have rendered the US economy more vulnerable to aggregate shocks. Our theory also suggests that this increased fragility may have been difficult to identify, as it manifests itself only in reaction to sufficiently large shocks.

Finally, we present cross-industry empirical evidence on our mechanism. Our model offers predictions on how industries should respond to a transition from a high to a low regime. In particular, industries featuring initially a larger concentration should experience a larger contraction as the economy enters a low competition trap. This happens because, in the model, a larger concentration reflects greater productivity differences. We test this prediction using data from the US census and focusing again on the 2008 crisis. Consistent with the predictions of the model, we find that industries featuring a larger concentration in 2007 experienced a greater cumulative contraction over the 2007-2016 period.

The rest of the paper is organized as follows. Section 3 presents the model. Section 4 discusses the calibration and presents the quantitative results. Section 5 focuses on the US great recession and its aftermath. Section 6 presents the cross-industry empirical evidence. Section 7 concludes.

## 2 Related Literature

Our paper is related to three different strands of the literature. In the first place, our paper belongs to the macroeconomic literature studying the cyclical properties of competition and markups, which includes, among others, the contributions of Rotemberg and Saloner (1986), Chatterjee et al. (1993), and Gilchrist et al. (2017) and Bils et al. (2018). Close to our approach, Chatterjee et al. (1993) build a multi-industry model with Cournot competition and an endogenous number of firms. Their model features a complementarity between aggregate output and firm entry decisions, which is capable of generating multiple equilibria and multiple steady-states. Cooper and John (2000) show that the combination of oligopolistic competition with variable markups can generate significant amplification of shocks. With respect to this literature, we make two main contributions. First, we embed an endogenous market structure in the neoclassical growth model and provide a quantification of our mechanism. Second, we discuss the role of firm heterogeneity in generating multiplicity and shaping the response of the economy to aggregate shocks. Second, this paper relates to a large and growing literature documenting long-run trends in firm heterogeneity and market power. There are several signs indicating rising market power in the US and other advanced economies. For example, Autor et al. (2017) use data from the US census and document rising sales and employment concentration at the industry level; they also show that industries with a faster increase in concentration experienced a greater decline in the labor share. Akcigit and Ates (2019b) also document a rise in patenting concentration. Other studies have documented a secular rise in price-cost markups. Using data from national accounts, Hall (2018) finds that the average sectoral markup increased from 1.12 in 1988 to 1.38 in 2015. De Loecker and Eeckhout (2017) document a steady increase in sales-weighted average markups for US public firms between 1980 and 2016.[6] This was driven by both an increasing share of large firms and by rising dispersion in the markup distribution. Identical findings are obtained by Díez et al. (2019) and by Calligaris et al. (2018), who use data from ORBIS (Bureau van Dijk) and include different countries in their analysis.[7] In our model, rising dispersion in size and markups is driven by increasing productivity differences. Several studies do indeed appear to indicate secular increase in productivity differences across firms (Andrews et al. (2015), Kehrig (2015), Decker et al. (2018)). We contribute to this literature by investigating the business cycle consequences of rising market power and discussing its potential impact on the 2008 crisis.

Lastly, this paper relates to the literature focusing on the persistent impact of the 2008 crisis. Schaal and Taschereau-Dumouchel (2015) study a model with endogenous capacity utilization. Their model features a complementarity between firms' capacity utilization and aggregate output, which gives rise to multiple steady-states. Like us, they interpret the post-2008 deviation as a transition to a low steady-state. Other authors have proposed explanations based on the complementarity between firms' innovation decisions and aggregate output (Benigno and Fornaro (2017), Anzoategui et al. (2019)). Finally, Clementi et al. (2017) argue that the persistent decline in firm entry, observed after 2008, is crucial if we want to understand the post-crisis growth dynamics. The extensive margin also plays a central role in our framework; we argue that rising firm differences may be important to understand such decline in the number of firms. All in all, while we view our theory as complementary to the above-mentioned articles, we believe we are the first to link the *great deviation* to the long-run increase in firm level heterogeneity, and to propose an explanation based on the interactions between market size and market structure.

[6] Edmond et al. (2018) show that a cost-weighted average markup (as opposed to sales-weighted) displays a less pronounced upward trend. They show that a cost-weighted average markup is the one that is relevant for welfare analysis, as it accounts for the fact that high markup firms are also more productive. See also Karabarbounis and Neiman (2019) and Traina (2018) for a critique on the De Loecker and Eeckhout (2017) methodology.

[7] There are other signs suggesting an increase in market power. Decker et al. (2014) document a secular decline in measures of business dynamism, while Decker et al. (2018) and Kehrig and Vincent (2018) use data from the US census to show that firms have become increasingly less reactive to changes in idiosyncratic productivity.

## 3 A Growth Model with Oligopolistic Competition and Variable Markups

This section presents our theoretical framework. We start by describing the demand side and the technology structure. We then analyze the equilibrium of a particular industry (taking aggregate variables as given). Finally, we characterize the general equilibrium.

### 3.1 Preferences

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. There is a representative, infinitely-lived household with lifetime utility

$$U_t = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, L_t\right),$$

where $0 < \beta < 1$ is the discount factor, $C_t \geq 0$ is consumption of the final good and $L_t \geq 0$ is labor. We adopt a period utility function as in Greenwood et al. (1988)

$$U\left(C_t, L_t\right) = \frac{1}{1-\gamma} \left( C_t - \frac{L_t^{1+\nu}}{1+\nu} \right)^{1-\gamma},$$

where $0 \leq \gamma \leq 1$ and $\nu > 0$.

The representative household contains many individual members, which will be denoted by $j$. Each individual member can run a firm in the corporate sector. We will be assuming that if two or more individuals run a firm in the same industry, they will behave in a non-cooperative way – i.e. they will compete against each other and will not collude. Nevertheless, all individuals will pool together the profits they make. There is, hence, a single dynamic budget constraint

$$K_{t+1} = \left[R_t + (1-\delta)\right] K_t + W_t L_t + \Pi_t^N - C_t,$$

where $K_t$ is capital, $R_t$ is the rental rate, $W_t$ is the wage rate and $\Pi_t^N = \sum_j \Pi_j^N$ are the profits accruing from all the firms in the economy net of fixed production costs. Capital depreciates at rate $0 \leq \delta \leq 1$ and factor prices $R_t$ and $W_t$ are taken as given. The representative household therefore solves the dynamic optimization problem

$$\begin{aligned} \max_{(C_t, L_t, K_{t+1})} \quad & \mathbb{E} \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, L_t\right) \\ \text{s.t.} \quad & K_{t+1} = \left[R_t + (1-\delta)\right] K_t + W_t L_t + \Pi_t^N - C_t. \end{aligned} \tag{1}$$

Note that, from the perspective of the household, labor and entrepreneurial decisions are separable; indeed, individuals can simultaneously run a firm and supply labor. We decide to abstract from an occupational decision problem (becoming an entrepreneur *versus* working) just for the sake of simplicity. This assumption does not however imply that there is a fixed supply of entrepreneurs, since the equilibrium number of entrepreneurs (and hence the number of active firms) will be endogenous.

Our choice of GHH preferences implies that the aggregate labor supply is a simple function of the wage rate

$$L_t = W_t^{1/\nu},$$

$\nu$ is hence the inverse of the wage elasticity of labor supply.

## 3.2 Technology

There is a final good (the *numeraire*), which is a CES aggregate of $I$ different industries

$$Y_t = \left( \sum_{i=1}^{I} y_{it}^{\rho} \right)^{\frac{1}{\rho}},$$

where $y_{it}$ is the quantity of industry $i \in [0,1]$, $0 < \rho < 1$ and $\sigma_I = \dfrac{1}{1-\rho}$ is the elasticity of substitution across industries.[8] $I$ is assumed to be large, so that each individual industry has a negligible size in the economy. The output of each industry $i$ is itself a CES composite of differentiated goods or varieties

$$y_{it} = \left( \sum_{j=1}^{n_{it}} y_{jit}^{\eta} \right)^{\frac{1}{\eta}},$$

where $n_{it}$ is the number of active firms in industry $i$ at time $t$ (to be determined endogenously), $0 < \eta \leq 1$ and $\sigma_G = \dfrac{1}{1-\eta}$ is the within-industry elasticity of substitution. Following Atkeson and Burstein (2008), we assume that the within-industry elasticity of substitution is larger than the across-industry elasticity of substitution.

**Assumption.** $0 < \rho < \eta \leq 1$

The inverse demand for each industry $i$ is given by

$$p_{it} = \left( \frac{Y_t}{y_{it}} \right)^{1-\rho}. \tag{2}$$

Firms within a certain industry will be producing differentiated varieties and will hence face different demands. The inverse demand function for variety $j$ in industry $i$ is equal to

$$p_{ijt} = \left( \frac{Y_t}{y_{it}} \right)^{1-\rho} \left( \frac{y_{it}}{y_{ijt}} \right)^{1-\eta}. \tag{3}$$

[8] As $\rho \to 0$, final output becomes a Cobb-Douglas aggregate of intermediate industries; in such a case, production requires a strictly positive amount of each industry and the degree of differentiation is high. As $\rho \to 1$, industries become perfect substitutes and the degree of differentiation is zero.

Each variety $i \in [0,1]$ can be produced by up to $N \in \mathbb{N}$ different entrepreneurs, so that $n_{it} \leq N$. Entrepreneur $j$ can produce a variety in industry $i$ by combining capital $k_{ijt}$ and labor $l_{ijt}$ through a Cobb-Douglas technology

$$y_{it} = \underbrace{e^{z_t}\ \pi_{ij}}_{\tau_{ijt}} (k_{ijt})^{\alpha} (l_{ijt})^{1-\alpha}. \tag{4}$$

According to this specification, the productivity of each entrepreneur depends on two terms: (i) a time-varying aggregate component $e^{z_t}$ (common to all industries and types) and (ii) a time-invariant idiosyncratic term $\pi_{ij}$. We refer to $z_t$ as aggregate productivity and to $\pi_{ij}$ as $j$'s idiosyncratic productivity in industry $i$. Aggregate productivity $z_t$ follows an auto-regressive process

$$z_t = \phi_z z_{t-1} + \varepsilon_t, \tag{5}$$

where $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$ is a white-noise process. We will let $\mathcal{F}_i \coloneqq \{\ \pi_{i1}\ ,\ \pi_{i2}\ ,\ \pi_{i3}\ , \ldots\}$ denote the distribution of idiosyncratic productivity terms in industry $i$. With no loss of generality, we will let type $j = 1$ have the highest productivity, type $j = 2$ have the second highest productivity, and so on

$$\pi_{i1} \geq \pi_{i2} \geq \pi_{i3} > \cdots.$$

Most of the general equilibrium analytical results will be derived for the case in which the distribution $\mathcal{F}_i$ is common across all industries. However, for the time being, $\mathcal{F}_i$ can be taken as industry-specific. In what follows, it will be convenient to define the total productivity term

$$\tau_{ijt} = e^{z_t}\ \pi_{ij}.$$

Labor is hired at the competitive wage $W_t$ and capital at the competitive rental rate $R_t$. Our choice of a Cobb-Douglas technology implies that entrepreneur $j$ can produce good $i$ with constant marginal cost $\dfrac{\Theta_t}{\tau_{ijt}}$, where

$$\Theta_t \equiv \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$$

is the marginal cost function for a Cobb-Douglas technology with unit productivity. We will often refer to $\Theta_t$ as the *factor cost index*. In addition to all variable costs, the production of each variety entails a fixed production cost $c_f \geq 0$ per period. Such a cost is in units of the *numeraire*.

### 3.3 Market Structure

Firms play a static Cournot game: all firms that decide to enter (thus incurring the fixed cost $c_f$) will simultaneously announce quantities, taking the output of the other competitors as given. We follow Atkeson and Burstein (2008) and assume that firms make sequential entry decisions in reverse order of productivity.[9] Therefore, if entrepreneur $j$ decides to produce in industry $i$, he chooses the amount of output $y_{ijt}$ that maximizes his profits given the output of his competitors. Specifically, he solves

$$\max_{y_{ijt}} \quad \left(p_{ijt} - \frac{\Theta_t}{\tau_{ijt}}\right) y_{ijt} \qquad \text{s.t.} \quad \begin{aligned} p_{ijt} &= \left(\frac{Y_t}{y_{it}}\right)^{1-\rho} \left(\frac{y_{it}}{y_{ijt}}\right)^{1-\eta} \\ y_{it} &= \left(\sum_{j=1}^{n_{it}} y_{jit}^{\eta}\right)^{\frac{1}{\eta}}. \end{aligned} \tag{6}$$

The solution to this static problem yields a system of $n_{it}$ first order conditions (one for each active firm)

$$p_{ijt}\left[\eta - (\eta - \rho)\, s_{ijt}\right] = \frac{\Theta_t}{\tau_{ijt}}, \tag{7}$$

where $s_{ijt} = \dfrac{p_{it}\, y_{it}}{p_{ijt}\, y_{ijt}}$ is the market share of firm $j = 1, \ldots, n_{it}$. As we can see from the previous equation, entrepreneur $j$ will charge a markup

$$\mu_{ijt} = \frac{1}{\eta - (\eta - \rho)\, s_{ijt}} \tag{8}$$

over his marginal cost $\dfrac{\Theta_t}{\tau_{ijt}}$. There are two observations that are worth mentioning. First, equation (8) establishes a positive relationship between market shares and markups. To understand such a relationship, note that firms are not price-takers and that they internalize the impact of size on the industry price $p_{it}$. Furthermore, since large firms have a greater influence on $p_{it}$, they restrict output disproportionately more, thereby charging a high markup. Second, market shares are themselves a positive function of revenue TFP $p_{ijt}\, \tau_{ijt}$, as equation (7) also highlights. Our model thus features a positive association between revenue productivity, size and markups. This sheds light on the empirical evidence mentioned in Section 1. Recall that that there is growing evidence that (within an industry) firms are becoming more heterogeneous in terms of revenue productivity, size and markups. Through the lens of our model, these facts are hence closely related − any shock that increases revenue productivity dispersion in our model will also be translated into larger size and markup dispersion.

The set of first order conditions in (7) defines a system of $n_{it}$ non-linear equations in the prices $\{p_{ijt}\}_{j=1}^{n_{it}}$. Such a system admits only a close-form solution in the limit case in which there is no differentiation within an industry ($\eta = 1$), as shown in the next example.

**Example.** *(Industry Equilibrium with $\eta = 1$) Suppose that $\eta = 1$, so that varieties are perfect substitutes*

[9] When the fixed cost $c_f$ is non-negligible, there can be multiple equilibria. Suppose, for instance, that there are two types of firms: low productivity types $\pi_L$ and high productivity types $\pi_H$. Suppose, further, that aggregate variables are such that any firm can profitably produce alone, but not if there is another competitor. In such a case, there are two possible equilibria: a monopoly with a high type or a monopoly with a low type. Sequential entry in reverse order of productivity is a way to select a particular equilibrium (in this case, the monopoly with a high type).

*within an industry. In such a case, firms charge a common price $p_{ijt} = p_{it}$, which is given by*

$$p_{it} = \frac{1}{n_{it} - (1-\rho)} \sum_{j=1}^{n_{it}} \frac{\Theta_t}{\tau_{ijt}}. \tag{9}$$

*Furthermore, firm $j$ has a market share*

$$s_{ijt} = \frac{1}{1-\rho} \left[ 1 - \frac{n_{it} - (1-\rho)}{\sum\limits_{k=1}^{n_{it}} \frac{1}{\tau_{ikt}}} \frac{1}{\tau_{ijt}} \right]. \tag{10}$$

To conclude the description of the industry equilibrium, we need to determine the number of active firms $n_{it}$. To this end, let

$$\Pi\left(j, n_{it}, \mathcal{F}_{it}, X_t\right) := \left(p_{it} - \frac{\Theta_t}{\tau_{ijt}}\right) y_{ijt}$$

denote the equilibrium profits of firm $j \leq n_{it}$ in industry $i$ (gross of the fixed production cost), when there are $n_{it}$ active firms, given a productivity distribution $\mathcal{F}_{it}$ and a vector of aggregate variables $X_t := [z_t, Y_t, \Theta_t]$.

The equilibrium number of firms must be such that (i) the profits of each active firm are not lower than the fixed cost $c_f$ and (ii) if an additional firm were to enter, its profits would be lower than the fixed cost. Mathematically, an interior solution $n_{it}^* < N$ to the equilibrium number of firms must satisfy

$$\left[\Pi\left(n_{it}^*, n_{it}^*, \mathcal{F}_i, X_t\right) - c_f\right]\left[\Pi\left(n_{it}^*+1, n_{it}^*+1, \mathcal{F}_i, X_t\right) - c_f\right] \leq 0. \tag{11}$$

Although we cannot provide an analytical characterization of the profit function under $\eta < 1$, we can obtain a closed form solution in the limit case where $\eta = 1$. Proposition 1 provides four important results.

**Proposition 1.** *(Profit Function Under $\eta = 1$) When $\eta = 1$, the profit function $\Pi\left(j, n_{it}, \mathcal{F}_i, X_t\right)$ satisfies*

1. $\dfrac{\partial \Pi\left(j, n_{it}, \mathcal{F}_i, X_t\right)}{\partial Y_t} > 0$
2. $\dfrac{\partial \Pi\left(j, n_{it}, \mathcal{F}_i, X_t\right)}{\partial n_{it}} < 0 \qquad , n_{it} > j$
3. $\dfrac{\partial \Pi\left(j, n_{it}, \mathcal{F}_i, X_t\right)}{\partial \pi_{ij}} > 0$
4. $\dfrac{\partial \Pi\left(j, n_{it}, \mathcal{F}_i, X_t\right)}{\partial \pi_{ik}} < 0 \qquad , \forall k \neq j.$

*Proof.* See Appendix B.1.3 ■

Besides stating that profits are strictly decreasing in the number of active firms $n_{it}$, Proposition 1 says the profits of any firm $j$ are increasing in its own idiosyncratic productivity $\pi_{ij}$ and decreasing in the idiosyncratic productivity of all the other firms $\pi_{ik}$ (*ceteris paribus*). The fact that the profits of $j$ decrease in the productivity of any competitor is crucial to understand the mechanism at the heart of the model. Suppose, for example, that the most productive firm becomes even more productive, while the productivity of all the other firms remains constant. An implication of Proposition 1 is that the profits of the least productive firm will necessarily decrease. But if this firm experiences a sufficiently large decrease in profits, those profits may become lower than the fixed production cost $c_f$, and the firm may be driven out of the market. We next consider a simple example to illustrate this mechanism in more detail.

**One leader *versus* multiple followers** Take the limit case in which $\eta = 1$, so that goods are homogeneous within any industry, and assume that aggregate productivity is constant and equal to $e^z = 1$. Suppose further that there is a high productivity firm with productivity $\pi_{i1} = \pi > 1$ (the *leader*), while all the other firms $j = 2, 3, \ldots$ have productivity $\pi_{ij} = 1$ (the *followers*). The parameter $\pi \geq 1$ measures the relative productivity of the leader with respect to the followers. In such a case, when there are $n_{it}$ active firms (one leader and $n_{it} - 1$ followers) the price of the undifferentiated variety is equal to

$$p_{it} = \frac{1 + (n_{it} - 1)\,\pi}{n_{it} - (1 - \rho)} \frac{\Theta_t}{\pi},$$

so that the markup $\mu_{ijt} := \dfrac{p_{it}}{(\Theta_t / \pi_{ij})}$ charged by each type of firm is equal to

$$\begin{aligned}
\mu_{iLt} &= \frac{1 + (n_{it} - 1)\,\pi}{n_{it} - (1 - \rho)}, \\
\mu_{iFt} &= \frac{(1/\pi) + (n_{it} - 1)}{n_{it} - (1 - \rho)}.
\end{aligned}$$

We can therefore see that, as $\pi$ increases, the markup of the leader increases, while the markup of each follower decreases (when there is at least one follower producing, i.e $n_{it} \geq 2$). We can also obtain an expression for the market share of each type of firm

$$\begin{aligned}
s_{iLt} &= \frac{1}{1 - \rho} \frac{2 - \pi - \rho + n_{it}\,(\pi - 1)}{1 + (n_{it} - 1)\,\pi}, \\
s_{iFt} &= \frac{1}{1 - \rho} \frac{1 - \rho\pi}{1 + (n_{it} - 1)\,\pi}.
\end{aligned}$$

As we can see from the set of equations above, the followers produce only when $\rho\pi < 1$.[10] Finally, when there are $n_{it}$ firms, the profits that each follower makes are given by

$$\Pi^F(n_{it}, \Theta_t, Y_t) := \begin{cases} 0 & \text{if } n_{it} \leq 1 \\ \left[\dfrac{1-\rho\pi}{1+(n_{it}-1)\pi}\right]^2 \left[\dfrac{n_{it}-(1-\rho)}{1+(n_{it}-1)\pi}\pi\right]^{\frac{\rho}{1-\rho}} \dfrac{\Theta_t^{-\frac{\rho}{1-\rho}} Y_t}{1-\rho} & \text{if } n_{it} \geq 2. \end{cases} \tag{12}$$

Not surprisingly, profits are increasing in aggregate output $Y_t$. Recall that Proposition 1 also tells us that $\Pi^F(\cdot)$ decrease in $\pi$ (*ceteris paribus*). This result is illustrated in Figure 3, which shows how the profits of a follower change with $\pi$, for a fixed number of firms $n_{it} = 2$ (i.e. a duopoly with the leader and only one follower). Each curve represents $\Pi^F(\cdot)$ for a different value of aggregate output: a high value $Y_0$ and a low value $Y_1 < Y_0$ (the factor cost index $\Theta_t$ is kept constant).

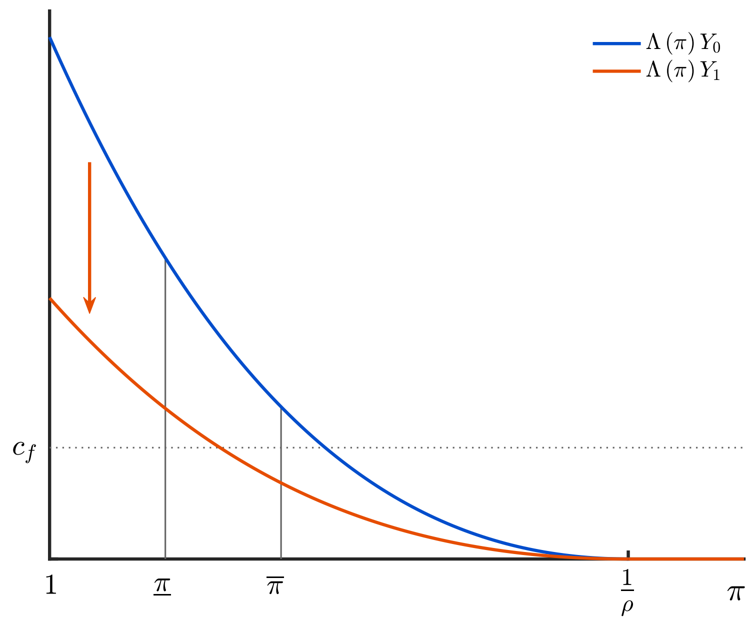



Figure 3: Profits of the follower

This example provides a partial equilibrium illustration of the mechanism that is at the heart of our model. Suppose that aggregate output decreases from $Y_0$ to $Y_1$.[11] If the leader has a small productivity advantage (for instance $\pi = \underline{\pi}$), the follower makes lower profits, but remains in operation (as the profits are still above $c_f$). However, if the leader has a large productivity advantage (for instance $\pi = \overline{\pi}$), the follower will be forced to leave the market following the fall in aggregate output (as the profits are now below $c_f$). Given the infinitesimal size of industry $i$, the exit of one follower will have a negligible effect on aggregate variables. However, if a positive mass of industries experiences a similar dynamics, there can be

[10] Otherwise, the leader's monopoly price would be below the followers' marginal cost, i.e. $p_{it} = \dfrac{1}{\rho}\dfrac{\Theta_t}{\pi} < \Theta_t$

[11] The results would be qualitatively identical for a drop in aggregate productivity.

non-negligible aggregate effects, which can be a source of amplification of business cycle fluctuations. We will se such an effect below, once we solve for the general equilibrium of this economy.

## 3.4 General Equilibrium

### Equilibrium Definition

We start by defining an equilibrium for this economy. Denoting the history of aggregate productivity shocks by $Z^t = \{z_t, z_{t-1}, ...\}$ we have the following definition.

**Definition 1.** *A general equilibrium consists of a sequence of household policies* $\left\{C_t\left(Z^t\right), K_{t+1}\left(Z^t\right), L_t\left(Z^t\right),\right\}$, *firm policies* $\left\{y_{ijt}\left(Z^t\right), k_{ijt}\left(Z^t\right), l_{ijt}\left(Z^t\right)\right\}$, *and a set of active firms* $\{n_{it}\}_{i=1}^{I}$ *such that*

*(i) households optimize*

*(ii) all active firms optimize*

*(iii) all active firms break even*

*(iv) no additional firm is willing to enter*

*(v) capital and labor markets clear*

### 3.4.1 Static Equilibrium

We now describe the general equilibrium of this economy. We start by focusing on a static equilibrium, in which we describe production and labor supply decisions, taking the aggregate level of capital $K_t$ as given. Later on, we describe the equilibrium dynamics.

**Aggregate Production Function** Given a $(I \times N)$ matrix of productivity draws $\mathbb{Z}_{\tau t}$ and when the $n_{it}$ most productive firms of industry $i$ are active, aggregate output can be written as

$$Y_t = \Phi\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right) \; L_t^{1-\alpha} K_t^{\alpha}. \tag{13}$$

The term $\Phi\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right)$ represents aggregate TFP and is equal to

$$\Phi\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right) = \left[\sum_{i=1}^{I}\left(\sum_{j=1}^{n_{it}}\omega_{ijt}^{\eta}\right)^{\frac{\rho}{\eta}}\right]^{\frac{1}{\rho}}\left(\sum_{i=1}^{I}\sum_{j=1}^{n_{it}}\frac{\omega_{ijt}}{\tau_{ijt}}\right)^{-1}, \tag{14}$$

where

$$\omega_{ijt} \coloneqq \left[\sum_{k=1}^{n_{it}}\left(\frac{\mu_{ikt}}{\tau_{ikt}}\right)^{\frac{\eta}{1-\eta}}\right]^{\frac{\eta-\rho}{\eta}\frac{1}{1-\rho}}\left(\frac{\tau_{ijt}}{\mu_{ijt}}\right)^{\frac{1}{1-\eta}}.$$

Although we cannot provide a general analytical characterization of $\Phi(\cdot)$, we can however analyze one particular case, which will help us understand some of the quantitative results of this paper. It focuses on the role played by the number of active firms.

**Example.** *(Identical Industries and Identical Firms) Suppose that all firms are identical and have constant productivity $\tau_{ijt} = \tau$. Suppose further that the aggregate equilibrium is such that all industries have the same number of firms $n_{it} = n$. In such a case, aggregate productivity is equal to*

$$\Phi = I^{\frac{1-\rho}{\rho}} n^{\frac{1-\eta}{\eta}} \tau.$$

As one can see, in the particular case highlighted above, aggregate TFP increases in both the number of industries $I$ (which is always fixed on our model) and in the number of firms per industry $n$, provided that $\eta < 1$. Such a fact simply reflects a *love for variety*, which is embedded in our production structure.

**Factor Prices and Factor Shares** We can aggregate firms' best responses, given by equation (7), to find an expression for the aggregate factor cost index. Given again a $(I \times N)$ matrix of productivity draws $\mathbb{Z}_{\tau t}$ and when there the $n_{it}$ most productive firms are active in industry $i$, the equilibrium factor cost index is equal to

$$\Theta\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right) = \left\{\sum_{i=1}^{I}\left[\sum_{j=1}^{n_{it}}\left(\frac{\tau_{ijt}}{\mu_{ijt}}\right)^{\frac{\eta}{1-\eta}}\right]^{\frac{1-\eta}{\eta}\frac{\rho}{1-\rho}}\right\}^{\frac{1-\rho}{\rho}}. \tag{15}$$

There are two facts that are worth highlighting. First, for a given distribution of markups $\mu_{ijt}$, economies with larger productivity levels $\tau_{ijt}$ should have on average larger factor prices. Second, for a given distribution of productivities, economies with larger markups should have on average larger profit shares, and hence lower factor shares and factor prices. From (15), we can also see how (15) varies with changes in the number of firms. Suppose that the number of firms in each industry $i$ increases from $n_{it}$ to $n_{it} + 1$. In such a case, $\Theta(\cdot)$ changes for two reasons. First, there is one additional firm in each industry, which necessarily increases factor demand even when all the remaining players do change their markups (*entry effect*). Second, the entry of one additional firm increases the level of competition in the industry, i.e. the preexisting firms respond by cutting their markups. $n_{it}$ firms respond to a more competitive market structure by increasing factor demand and cutting their markups. As it is clear from (15), a reduction in the markup of preexisting players will result in a larger factor cost index (*competition effect*).

$$\left\{\sum_{i=1}^{I}\left[\underbrace{\sum_{j=1}^{n_{it}}\left(\frac{\tau_{ijt}}{\tilde{\mu}_{ijt}}\right)^{\frac{\eta}{1-\eta}}}_{\uparrow\text{(competition effect)}} + \underbrace{\left(\frac{\tau_{ikt}}{\tilde{\mu}_{ikt}}\right)^{\frac{\eta}{1-\eta}}}_{\uparrow\text{(entry effect)}}\right]^{\frac{1-\eta}{\eta}\frac{\rho}{1-\rho}}\right\}^{\frac{1-\rho}{\rho}} > \left\{\sum_{i=1}^{I}\left[\sum_{j=1}^{n_{it}}\left(\frac{\tau_{ijt}}{\mu_{ijt}}\right)^{\frac{\eta}{1-\eta}}\right]^{\frac{1-\eta}{\eta}\frac{\rho}{1-\rho}}\right\}^{\frac{1-\rho}{\rho}}.$$

We can write $\Theta\left(\mathbb{Z}_{\tau t},\{n_{it}\}_{i=1}^{I}\right)$ as an explicit function of individual productivities when goods are undifferentiated within an industry ($\eta=1$) and industries are identical (with the same number of firms $n_i=n$ and identical distribution of productivities $\mathcal{F}_i=\mathcal{F}$).

**Example.** *($\eta=1$ and Identical Industries) Suppose that goods are undifferentiated within an industry ($\eta=1$) and that all industries are identical (with the same number of firms $n_i=n$ and identical distribution of productivities $\mathcal{F}_i=\mathcal{F}$). In such a case, the equilibrium factor cost index is equal to*

$$\Theta\left(\mathcal{F},n\right)=\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\tau_k}}.$$

*which is an increasing function of $n$ (see Appendix C.2 for a proof).*

We can also characterize the factor and profit shares. Let $\Omega\left(\cdot\right)$ denote the aggregate factor share, i.e. $\Omega=\left(W_t\,L_t+R_t\,K_t\right)/Y_t$. Note that the aggregate profit share (exclusive of fixed production costs) is given by $1-\Omega$. As shown in Appendix C.2, we have that

$$\Omega\left(\mathbb{Z}_{\tau t},\{n_{it}\}_{i=1}^{I}\right)=\frac{\Theta\left(\mathbb{Z}_{\tau t},\{n_{it}\}_{i=1}^{I}\right)}{\Phi\left(\mathbb{Z}_{\tau t},\{n_{it}\}_{i=1}^{I}\right)}. \tag{16}$$

$\Omega\left(\mathbb{Z}_{\tau t},\{n_{it}\}_{i=1}^{I}\right)$ takes a complicated expression, for which we cannot provide a general characterization. However, we can again characterize the aggregate factor share in the particular case in which (i) $\eta=1$ and (ii) all industries are identical (have the same number of firms $n$ and the same productivity distribution $\mathcal{F}$). Proposition 2 characterizes $\Omega\left(\mathcal{F},n\right)$ in such a case.

**Proposition 2.** *($\eta=1$ and Identical Industries) Let $\Omega\left(\mathcal{F},n\right)$ be the aggregate factor share in a symmetric equilibrium in which all industries are identical (have the same number of firms $n$ and the same productivity distribution $\mathcal{F}$). We have that*

1. *$\Omega\left(\mathcal{F},n\right)$ increases in the average number of firms per industry, i.e.*

$$\Omega\left(\mathcal{F},n+1\right)>\Omega\left(\mathcal{F},n\right).$$

2. *Let $\pi_j$ be an idiosyncratic productivity type such that $\pi_j\geq\frac{1}{n}\sum_{k=1}^{n}\pi_k$. Suppose that $\pi_j$ increases to $\pi_j'>\pi_j$ in all industries but all other types remain unchanged. Then, the new distribution $\mathcal{F}'$ is such that*

$$\Omega\left(\mathcal{F}',n\right)<\Omega\left(\mathcal{F},n\right).$$

*Proof.* See Appendix C.2. ■

The previous proposition states an important result of the model: it characterizes how the productivity distribution $\mathcal{F}$ affects the aggregate factor share $\Omega(\cdot)$. To understand this proposition, pick some productivity type $\pi_j$ and let it increase identically in all industries. Then, if $\pi_j$ is already above the average $\frac{1}{n}\sum_{k=1}^{n}\pi_k$, the aggregate factor share will decrease. In other words, when the right tail of the productivity distribution increases, the aggregate factor share decreases (and the profit share increases). The intuition is simple. In every industry, high productivity firms are larger and charge higher markups. As large firms are able to increase their markups even further, the aggregate profit share increases and the aggregate factor share decreases. This proposition therefore explains how rising productivity/size differences across firms may generate

**Factor Market Clearing** Having obtained an expression for the aggregate factor cost index, we can determine the factor demand schedules for labor $L_t$ and capital $K_t$

$$\begin{aligned} W_t = &\quad (1-\alpha)\ \Theta\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right)\ L_t^{-\alpha}K_t^{\alpha}, \\ R_t = &\quad \alpha\ \Theta\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right)\ L_t^{1-\alpha}K_t^{\alpha-1}. \end{aligned} \tag{17}$$

These two demand schedules can be combined with the labor and capital supply equations

$$\begin{aligned} L_t^S = &\quad W_t^{1/\nu}, \\ K_t^S = &\quad K_t \end{aligned} \tag{18}$$

to determine the factor market equilibrium. Figure 4 shows how the factor market equilibrium is determined (in an example in which all industries are identical and have $n$ firms). Note that because $\Theta(\mathcal{F}, n)$ is increasing in $n$, an increase in the number of active firms necessarily results in a larger equilibrium employment and wage rate. Given that the capital supply is vertical in the short run, an increase in the number of active firms results in a larger rental rate.

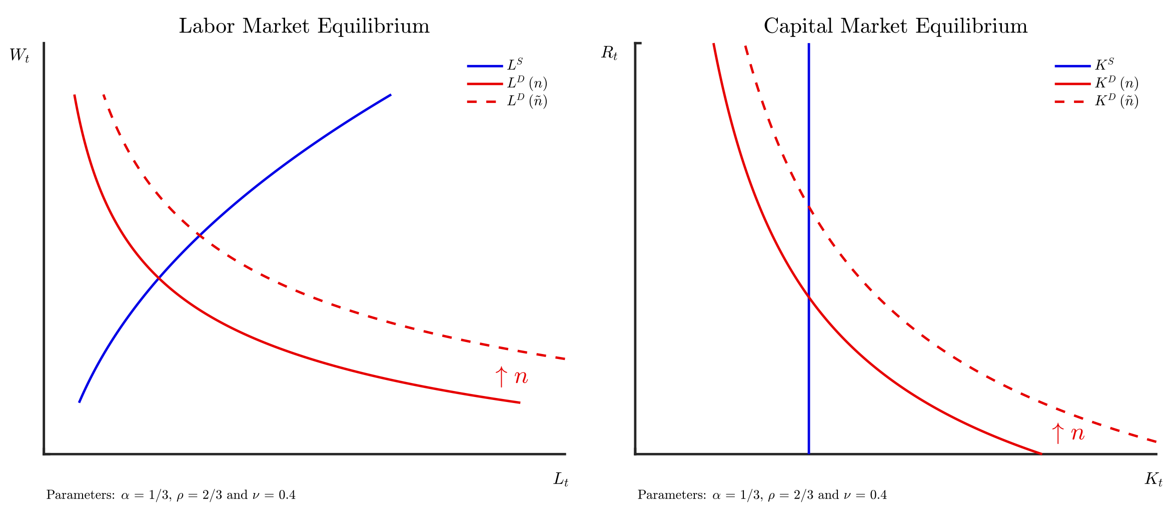



Figure 4: Factor Market Equilibrium

We can combine equations (17) and (18) to find an expression for equilibrium employment as a function of aggregate capital $K_t$, the productivity distribution $\mathbb{Z}_{\tau t}$ and the set of active firms $\{n_{it}\}_{i=1}^{I}$

$$L_t = \left[(1-\alpha)\,\Theta\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right)\right]^{\frac{1}{\nu+\alpha}} K_t^{\frac{\alpha}{\nu+\alpha}}. \tag{19}$$

Finally, we can combine equations (13) and (19) to write aggregate output as a function of the aggregate capital stock $K_t$, the productivity distribution $\mathbb{Z}_{\tau t}$ and the set of active firms $\{n_{it}\}_{i=1}^{I}$

$$Y_t = \Phi\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right)\left[(1-\alpha)\,\Theta\left(\mathbb{Z}_{\tau t}, \{n_{it}\}_{i=1}^{I}\right)\right]^{\frac{1-\alpha}{\nu+\alpha}} K_t^{\alpha\frac{1+\nu}{\nu+\alpha}}. \tag{20}$$

To conclude the characterization of the static equilibrium, we need to determine the set of active firms $\{n_{it}\}_{i=1}^{I}$.

**Equilibrium Set of Firms** All the results derived so far took the set of active firms $\{n_{it}\}_{i=1}^{I}$ as given. The number of active firms in each industry $i$ shall be jointly determined by equations (15), (20) and the set of inequalities defined in (11). Such a joint system does not admit however a general analytical characterization. We can nevertheless analyze the particular case in which all industries are ex-ante identical and have the same distribution of productivities $\mathcal{F}$. Proposition 3 states the conditions under which it is possible to have a symmetric equilibrium where all industries have the same number of firms $n$.

**Proposition 3.** *(Symmetric Equilibrium) Suppose that there is a symmetric equilibrium with $n$ firms per industry at time $t$. Then $K_t$ must be such that*

$$\underline{K}(\mathcal{F}, n) \leq K_t \leq \overline{K}(\mathcal{F}, n),$$

*where*

$$\underline{K}(\mathcal{F}, n) \coloneqq \left\{\frac{c_f}{\Lambda(\mathcal{F}, n, n)}(1-\alpha)^{-\frac{1-\alpha}{\nu+\alpha}}\left[\Phi(\mathcal{F}, n)\right]^{-1}\left[\Theta(\mathcal{F}, n)\right]^{\frac{\rho}{1-\rho}-\frac{1-\alpha}{\nu+\alpha}}\right\}^{\frac{\nu+\alpha}{\alpha(1+\nu)}}$$
$$\overline{K}(\mathcal{F}, n) \coloneqq \left\{\frac{c_f}{\Lambda(\mathcal{F}, n+1, n+1)}(1-\alpha)^{-\frac{1-\alpha}{\nu+\alpha}}\left[\Phi(\mathcal{F}, n)\right]^{-1}\left[\Theta(\mathcal{F}, n)\right]^{\frac{\rho}{1-\rho}-\frac{1-\alpha}{\nu+\alpha}}\right\}^{\frac{\nu+\alpha}{\alpha(1+\nu)}}.$$

*The bounds $\overline{K}(\cdot)$ and $\underline{K}(\cdot)$ are both increasing in $n$.*

*Proof.* See Appendix C.3. ∎

Intuitively, $K_t$ must be (i) sufficiently large so that all existing $n$ firms can break even, (ii) but cannot be too high, for otherwise an additional firm could profitably enter in at least one industry.

**Unique Static Equilibrium** Whenever the bounds $\overline{K}(\cdot)$ and $\underline{K}(\cdot)$ satisfy

$$\overline{K}(\mathcal{F}, n) < \underline{K}(\mathcal{F}, n+1) \quad , \qquad \forall n$$

the equilibrium is unique.[12] As we have just seen, when $K_t \in \left[\underline{K}(\mathcal{F}, n), \overline{K}(\mathcal{F}, n)\right]$, the economy will feature a symmetric equilibrium with $n$ firms in all industries. When however $K_t \in \left[\overline{K}(\mathcal{F}, n), \underline{K}(\mathcal{F}, n+1)\right]$, a symmetric equilibrium is not possible. In such a case, the economy will have some industries with $n$ players, and others with $n+1$ players. The fraction of industries with $n+1$ firms will be such that, in these industries, the last firm exactly breaks even, i.e.

$$\Pi(n+1, n+1, \mathcal{F}, \Theta_t, Y_t) = c_f.$$

A detailed characterization of this non-symmetric equilibrium is provided in Appendix C.3.[13]

We now discuss how a larger capital stock $K_t$, by boosting firm entry, can be associated with a more competitive market structure. Figure 5 shows aggregate output, the profit share, and the equilibrium wage and rental rate as a function of aggregate capital $K_t$. In the regions represented by the full line, the economy features a symmetric equilibrium. In the regions represented by the dashed line, the economy exhibits an asymmetric equilibrium. When the capital stock is low, the economy can accommodate only one firm per industry and will thus consist of a collection of identical monopolies. As capital increases and surpasses $\overline{K}(1)$, then at least some industries will have a second player. When it achieves $\underline{K}(2)$, all industries will have two players. As one can see, output $Y_t$ is not globally concave in the capital stock $K_t$. To understand this note that, as the average number of firms increases (say from $n = 1$ to $n = 2$), competition becomes more intense and the profit share decreases (top right panel). This translates into a larger factor share and a disproportionately larger wage (bottom left panel). However, a disproportionately larger wage results in a disproportionately larger labor supply (through equation (18)), which explains the break in the concavity of $Y_t$. The fact that factor shares increase as the economy transitions into a more competitive regime also explains why, as in the case represented in Figure 5, the rental rate $R_t$ is not strictly decreasing in the aggregate capital stock $K_t$. Such a result helps us understand how multiple steady-states can occur. Note that the steady-state rental rate is pinned down by the household's discount factor $\beta$. If the the steady-state rental rate is such that it crosses the map represented in Figure 5 more than once, then steady-state multiplicity occurs.

[12] We discuss next the case in which static multiplicity occurs.

[13] Since industries are symmetric ex-ante, our model cannot determine which industries will have $n$ firms and which industries will have $n+1$ firms. Such indeterminacy would, however, disappear in the presence of ex-ante heterogeneity, either in terms of productivities or fixed costs.

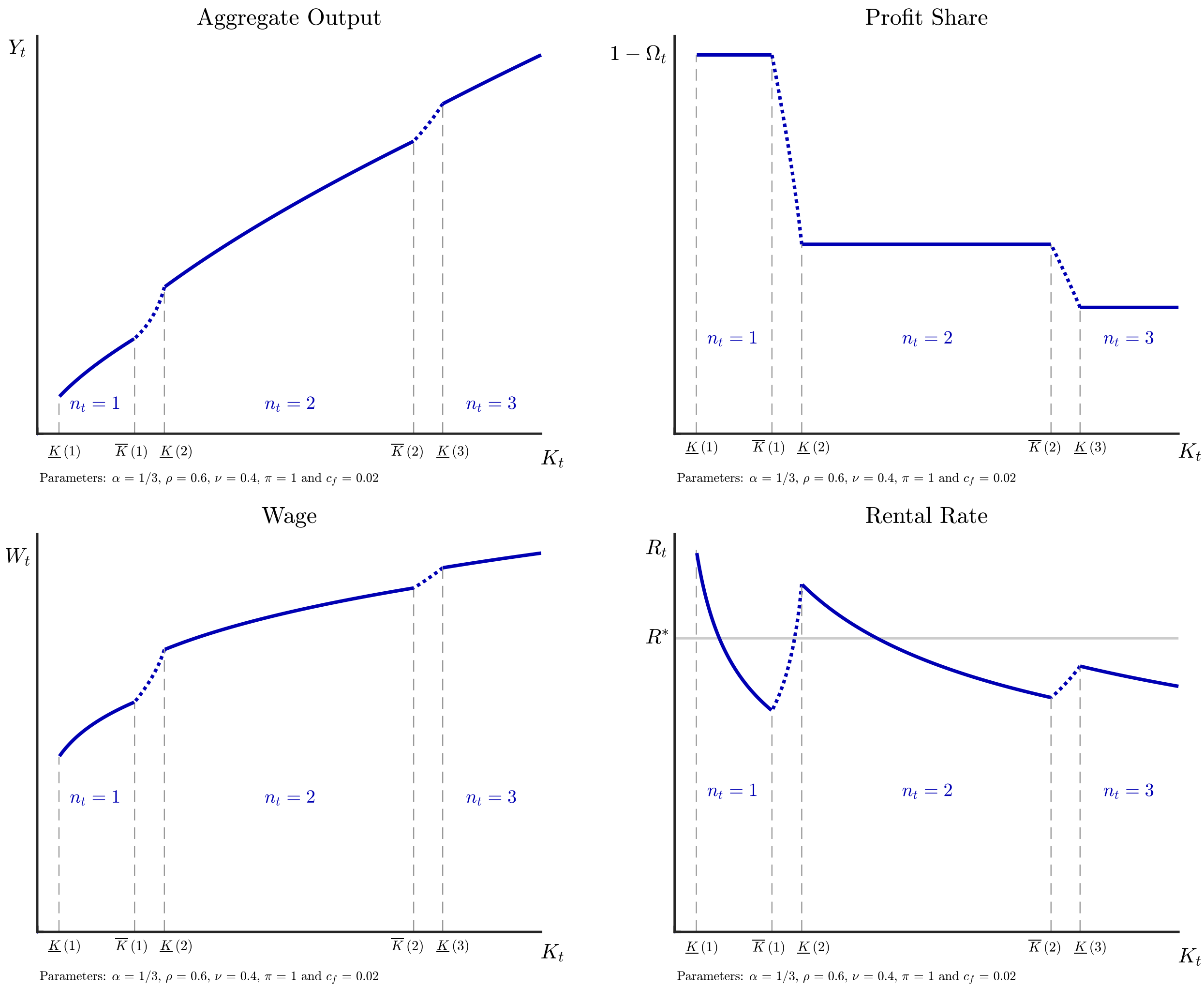



Figure 5: Static Equilibrium

**Multiple Static Equilibria** Whenever the bounds $\overline{K}(\cdot)$ and $\underline{K}(\cdot)$ satisfy

$$\underline{K}(\mathcal{F}, n+1) < \overline{K}(\mathcal{F}, n) \quad , \qquad \text{for some } n$$

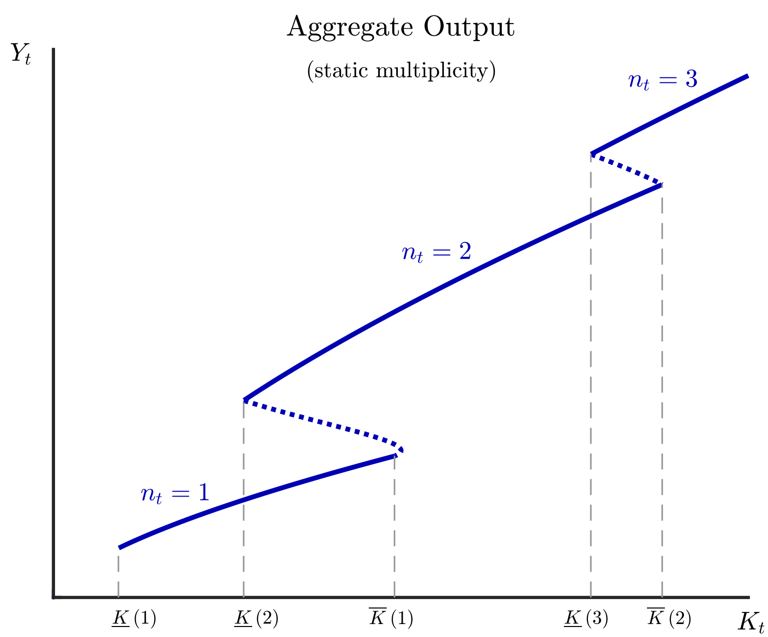
the equilibrium may not be unique. When $K_t \in \left[\underline{K}(\mathcal{F}, n+1), \overline{K}(\mathcal{F}, n)\right]$, the economy can feature multiple equilibria: it can feature a symmetric equilibrium with $n$ firms per industry, a symmetric equilibrium with $n+1$ per industry, and also an asymmetric equilibrium with $n$ firms in some industries and $n+1$ in some others. Figure 6 shows aggregate output $Y_t$ as a function of the aggregate capital stock $K_t$ for an economy in which static multiplicity can occur. As before, the full lines represent symmetric equilibria in which all industries are identical, whereas the dashed lines represent asymmetric equilibria. Note that static multiplicity arises because of a positive complementarity between competition and labor supply. For a given capital stock level $K_t$ there can be an equilibrium featuring a large number of active firms, and hence high factor shares, high wages and high labor supply; and another possible equilibrium with a lower number of firms, and hence low factor shares, low wages and a depressed labor supply. Appendix C.4 characterizes the conditions for the existence of static multiplicity. Proposition 4 below states the conditions under which static multiplicity can occur under full symmetry (identical industries and firms).

**Proposition 4.** *(Static Multiplicity with No Productivity Differences) When all firms are equally productive there can be equilibrium multiplicity if and only if*

$$\frac{\rho}{1-\rho} < \frac{1-\alpha}{\nu+\alpha}.$$

*Proof.* See Appendix C.4. ■

As Proposition 4 makes it clear, in the limit case in which firms are equally productive, static multiplicity depends on three main parameters: $\rho$, $\nu$ and $\alpha$. In particular, when there are no productivity differences, static multiplicity arises whenever (i) $\rho$ is low (so that differentiation across varieties is large and markups/factor shares display a high responsiveness to changes in the number of firms), (ii) the wage elasticity of labor supply $\frac{1}{\nu}$ is large or (iii) when the labor elasticity of output $1-\alpha$ is large. Note that in the limit case of perfect competition ($\rho = 1$, $c_f = 0$ and no productivity differences), static multiplicity can never arise.




Parameters: $\alpha = 1/3$, $\rho = 0.3$, $\nu = 0.4$, $\pi = 1$ and $c_f = 0.02$

Figure 6: Static Multiplicity

### 3.4.2 Equilibrium Dynamics

We next explore the dynamic properties of our economy. The positive interaction between aggregate output and competition, already discussed in the characterization of the static equilibrium, will also have important implications for the equilibrium dynamics. Even though we cannot derive a general law of motion for our economy in closed form, we can nevertheless characterize the steady-state aggregate savings rate.

**Proposition 5.** *(Steady-State Savings Rate) In a steady-state with a fixed distribution of firm productivities $\mathbb{Z}_\tau$ and a fixed set of active firms $\{n_i\}_{i=1}^I$, the aggregate savings rate is equal to*

$$s^* = \frac{\beta\delta}{1-(1-\delta)\beta}\,\alpha\,\Omega\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right).$$

As Proposition 5 makes it clear, the steady-state savings rate will depend on the competitive structure of the economy. A more competitive market structure will be associated with a larger factor share $\Omega\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)$, and hence a larger equilibrium rental rate and savings rate. In the discussion of the static general equilibrium, we have emphasized a (static) complementarity between aggregate output and competition, operating through the labor market – an increase in aggregate output stimulates firm entry, thereby resulting in a larger labor share and larger aggregate labor supply. The relationship defined in Proposition 5 highlights a similar complementarity, but this one in a dynamic context – an increase in aggregate output stimulates firm entry, thereby resulting in a larger capital share and larger aggregate capital supply. Indeed, note Proposition 5 does not depend on $\nu$ – it holds even if labor supply is totally inelastic ($\nu \to \infty$). To better understand the dynamic properties of our economy, we will again analyze the special case in which all industries are ex-ante identical.

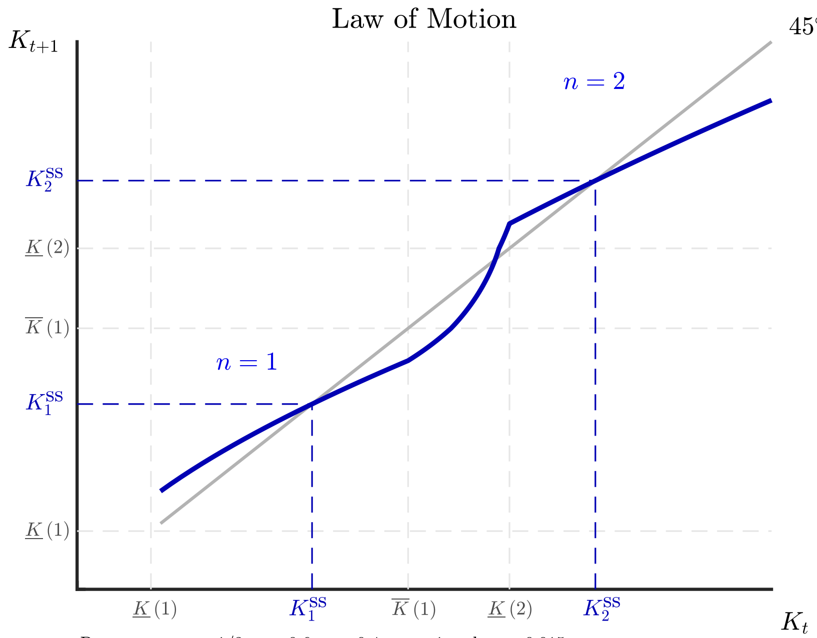
**No productivity differences** Let us start by assuming that aggregate productivity is constant and equal to $z_t = 1$ and that firms are equally productive, so that $\pi_{ij} = 1$. In such a case, aggregate TFP is constant and equal to one (and thus independent of the number of firms):

$$\Phi(\mathcal{F}, n_t) = 1.$$

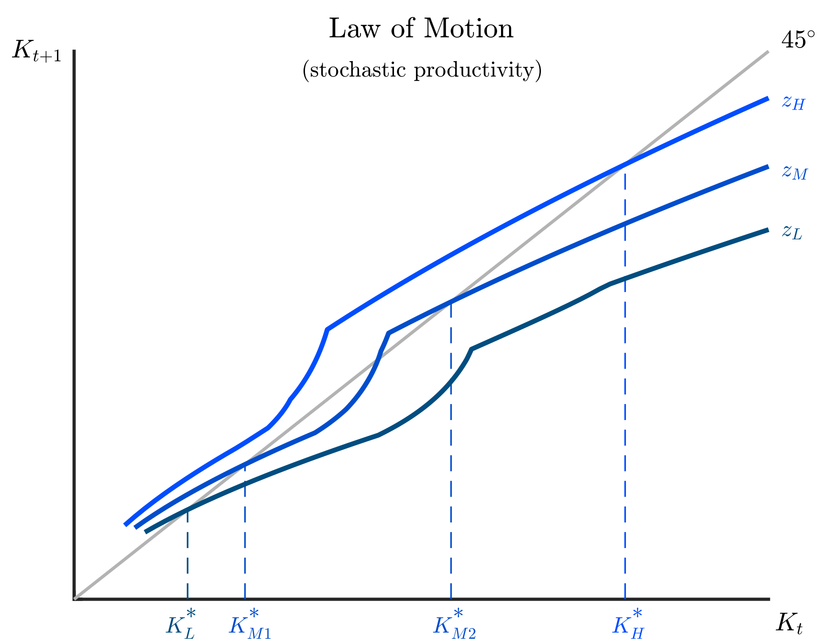
The aggregate factor share depends on the number of active firms per industry $n$

$$\Omega(\mathcal{F}, n_t) = \frac{n_t - (1-\rho)}{n_t}.$$

The aggregate factor share thus goes from $\Omega(n_t = 0) = \rho$ (monopoly) to $\Omega(n_t \to \infty) = 1$ (perfect competition). From equation (16) we have that the factor cost index coincides with the aggregate factor share, $\Theta(\mathcal{F}, n_t) = \Omega(\mathcal{F}, n_t)$. Figure 7 below shows the law of motion of this economy. Note that the law of motion is not globally concave and exhibits a convex region for $K_t \in \left[\overline{K}(1), \underline{K}(2)\right]$. Such a convexity occurs for the mechanism highlighted earlier – as $K_t$ increases, the economy moves towards a more competitive regime, with a larger factor share $\Omega(\mathcal{F}, 2) > \Omega(\mathcal{F}, 1)$. A larger factor share results in a disproportionately larger wage rate and labor supply (resulting in larger $Y_t$ for a given $K_t$), but also in a larger savings rate (resulting in larger $K_{t+1}$ for a given $Y_t$). Because of this complementarity between capital accumulation and competition, the law of motion exhibits two steady-states: one where all industries are a monopoly ($K_1^{\text{ss}}$), and another where all industries are a duopoly ($K_2^{\text{ss}}$).[14] Note that despite the existence of two steady-states, there is a unique equilibrium: there is a unique value of $K_{t+1}$ for each value of $K_t$ (the state variable).




Parameters: $\alpha = 1/3$, $\rho = 0.6$, $\nu = 0.4$, $\pi_L = 1$ and $c_f = 0.015$




Parameters: $\alpha = 1/3$, $\rho = 0.6$, $\nu = 0.4$ and $c_f = 0.015$

Figure 7: Law of motion: no productivity differences

Figure 7 represents the law of motion for a fixed value of aggregate productivity $z_t = 1$. However,

[14]The existence of exactly two steady-states is obviously not guaranteed. Figures 24 and 25 in Appendix C.5 show examples of economies with one or three steady-states.

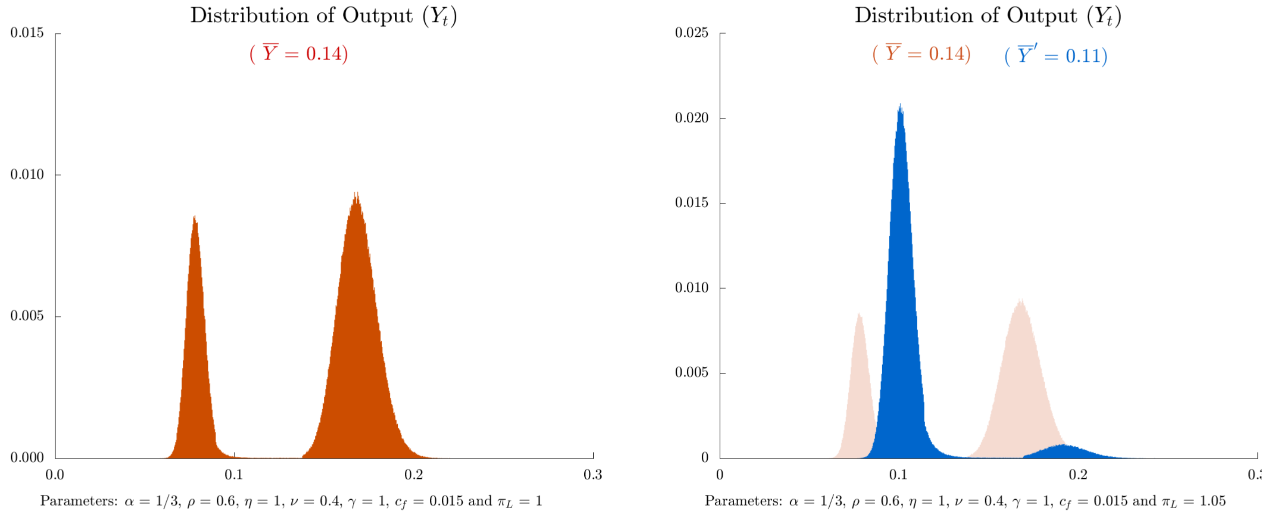
the law of motion will necessarily vary with aggregate productivity $z_t$. Therefore, we also consider an example with stochastic productivity. Suppose for simplicity that $z_t$ can take three values: a low value $z_L$, an intermediate value $z_M$ and a high value $z_H$. The bottom panel of Figure 7 represents the law of motion under each value of aggregate productivity.

As one can see, when aggregate productivity is low and equal to $z_L$, the economy exhibits a unique steady-state where all industries are a monopoly ($K_L^*$). Under $z_H$, on the other hand, there is only a unique steady-state where all industries are a duopoly ($K_H^*$). Finally, when aggregate productivity takes the intermediate value $z_M$, the economy exhibits two steady-states: a low one where all industries are a monopoly ($K_{M1}^*$) and a high one where all industries are a duopoly ($K_{M2}^*$).

To exemplify the dynamics of the model, suppose that aggregate productivity starts at $z_H$ and that the economy is at the steady-state $K_H^*$. Suppose now that there is a negative aggregate productivity shock, which reduces aggregate productivity permanently to $z_M$. After this shock, the economy will converge to the new steady-state $K_{M2}^*$. Output is lower than before, but the market structure is identical – all sectors are still a duopoly.

Now suppose that, instead of falling permanently to $z_M$, aggregate productivity falls first to $z_L$, and later increases to $z_M$. Suppose further that aggregate productivity remains at $z_L$ for sufficiently large period, so that the economy approaches the low steady-state $K_L^*$. Then, as aggregate productivity increases to $z_M$, the economy will approach $K_{M1}^*$. Note now that in the new steady-state all sectors are a monopoly. The economy therefore experiences a persistent transition to a regime featuring a more concentrated market structure.

We now let aggregate productivity fluctuate according to equation (5). Figure 8(a) below shows the distribution of output. There are two stochastic steady-states: a low one where all sectors are a monopoly, and a high one where all sectors are a duopoly. Given the particular parameters chosen, the economy is mostly around the high steady-state.




(a) Output Distribution: no productivity differences

(b) Output Distribution: Leader *versus* Follower

Figure 8: Output Distributions

**One leader *versus* multiple followers** Let us now revisit the leader-follower example we have seen in partial equilibrium. Suppose now that in each industry there is a productive firm with productivity $\pi_L = \pi > 1$ (the leader), while all the other firms $j = 2, 3, \ldots$ have productivity $\pi_F = 1$ (the followers). Figure 9 represents the effects of an increase in $\pi$. The law of motion under the initial value of $\pi$ is represented in light blue. Two facts stand out. First, the two concave segments of the law of motion (representing the symmetric equilibria with $n = 1$ and with $n = 2$ firms) move up. This fact simply represents an expansion in the economy's production possibility frontier – because of the larger productivity advantage of the leaders, aggregate output will increase for any fixed number of active firms $n$. Second, part of the convex segment lying between $\overline{K}(1)$ and $\underline{K}(2)$ lies below the initial law of motion. This change reflects the fact that the leaders increase their productivity over the followers. Because of such a larger advantage, the followers can only enter at increasingly larger levels of aggregate capital, which results in a simultaneous increase in $\overline{K}(1)$ and $\underline{K}(2)$. In other words, the increase in $\pi$ may have ambiguous effects: (i) whenever the number of firms remains unchanged, it will necessarily result in larger output (because of an expansion of the economy's production possibility frontier), (ii) but since it makes it increasingly harder for the followers to profitably enter the market, it may result in a lower equilibrium number of firms, with a potentially lower output.

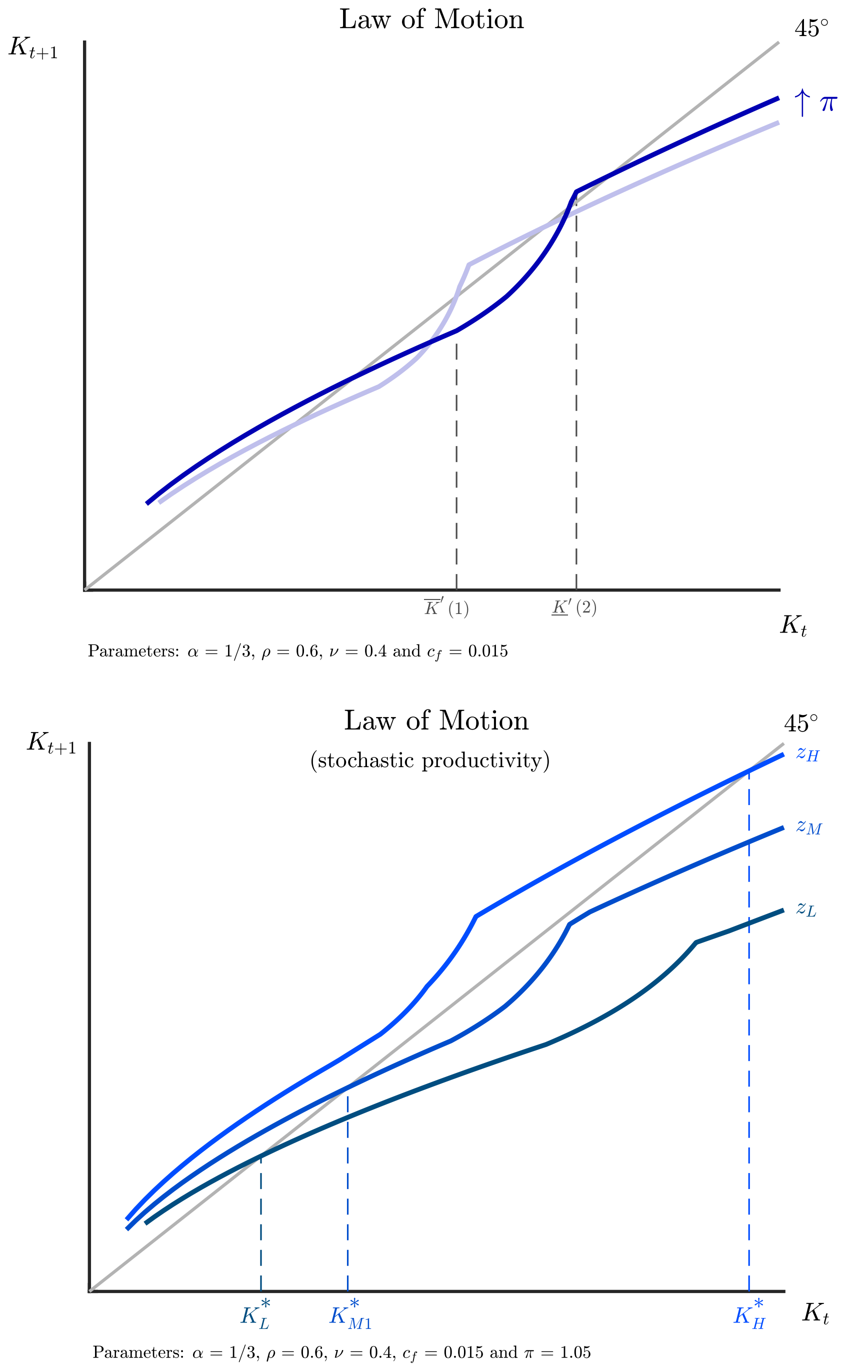



Figure 9: Law of motion: no productivity differences

Note that the increase in $\pi$ results in a reduction of the basin of attraction of the high steady-state. Although we cannot provide a full analytical characterization of the impact of $\pi$ on the basin of attraction (i.e. whether it always shrinks or can actually increase), we can nevertheless characterize its impact on the subset of the basin of attraction that falls under a symmetric equilibrium. Proposition 6 states the conditions under which an increase in the productivity of the leader $\pi$ may reduce the basin of attraction of the high steady-state.

**Proposition 6.** *(Basin of Attraction Falling Under a Symmetric Equilibrium, $\eta = 1$) Let $K^*(n)$ be a steady-state with $n$ firms. Suppose that some type $\pi_k \geq \pi_n$ experiences a productivity increase, while the productivity of all other types remains constant, i.e. $\uparrow \pi_k$ for some $\pi_k \geq \pi_n$.*
*The basin of attraction falling under $\left[\underline{K}(n), K^*(n)\right]$ shrinks provided that*

$$\frac{\partial}{\partial \pi_k}\left[\frac{K^*(n)}{\underline{K}(n)}\right] < 0.$$

*When there are no productivity differences, this condition becomes*

$$\frac{1/\nu + \alpha}{1-\alpha} < \frac{(4 + 1/n)\left[n - (1-\rho)\right] - n}{1-\rho}.$$

*Proof.* See Appendix C.6. ■

Proposition 6 says that, as productivity differences increase, the basin of attraction of a symmetric steady-state shrinks provided that (i) $1/\nu$ is not too large and that (ii) $\rho$ is not too low. On the one hand, the wage elasticity of labor supply $1/\nu$ cannot be too high, so that labor is relatively inelastic and types $\pi_k \geq \pi_n$ effectively crowd out types $\pi_n$. On the other hand, $\rho$ must be sufficiently large, so that average markups are low and there is a significant pass-through from productivity to profits.[15]

The bottom panel of Figure 9 replicates the example with three values for aggregate productivity. The main difference with respect to Figure 7 is that now, for the intermediate value of aggregate productivity $z_M$, there is only one steady-state. Note that the two steady-states $K_L^*$ and $K_H^*$ are larger after the increase in the leaders' productivity. The same happens with the low steady-state when aggregate productivity is equal to $z_M$ ($K_{M1}^*$). This result is not surprising. Keeping the market structure constant (for example, a monopoly in every industry), the higher the productivity of the leader, the higher is aggregate output. Figure 9 shows however that it becomes increasingly more difficult to sustain a duopoly and, as a consequence, the steady-state $K_{M2}^*$ disappears.

Finally, we let aggregate productivity fluctuate again according to equation (5). Figure 8(b) shows the distribution of output. There are again two steady-states, which are higher than the ones before the increase in $\pi$. However, the economy is now more likely to be around the low steady-state. As a consequence, average output may decrease (in this example, it actually decreases from $\overline{Y} = 0.14$ to $\overline{Y} = 0.11$).

[15] When this condition is not satisfied, an increase in the productivity of the leader may increase the basin of attraction of the high steady-state. In this case, a larger production possibility frontier makes entry easier for the followers. See Figure 26 in Appendix C.6 for an example.

# 4 Quantitative Results

In this section, we provide a quantitative evaluation of our model. There are two objects we need to parametrize – the distribution from which firms make their idiosyncratic productivity draws and the distribution of fixed production costs. We will assume that firms draw their idiosyncratic productivities from a Pareto distribution with tail parameter $\lambda$

$$\pi_{ij} \sim \mathrm{Pa}(\lambda).$$

Recall that each industry $i$ will be characterized by $N$ such draws. Since $N$ is a finite number, industries will be characterized by different ex-post distributions of idiosyncratic productivities $\{\pi_{ij}\}_{j=1}^{N}$.

With respect to the distribution of fixed costs, we assume that there are two types of industry – a fraction $f_{\mathrm{comp}}$ of all industries have a zero fixed cost $c_i = 0$, whereas the remaining fraction $1 - f_{\mathrm{comp}}$ faces a positive fixed production cost $c_i = c > 0$. We hence have that

$$c_i = \begin{cases} 0 & \text{if } i \leq f_{\mathrm{comp}} \cdot I \\ c & \text{if } i > f_{\mathrm{comp}} \cdot I \end{cases}.$$

There are two aspects about this assumption that should be explained. First, by imposing a zero fixed cost $c_i = 0$ in some industries, we are somehow introducing a *competitive* sector in the economy.[16] Note that the extensive margin will be shut down in these industries as all potential $N$ entrants will always be active. In the absence of this assumption, industries would exhibit an identical degree of concentration and would tend to move together in response to a shock, which could imply an excessive degree of amplification/persistence.[17] The parameter $f_{\mathrm{comp}}$, which measures the relative importance of the *competitive* sector, will be calibrated to match the share of aggregate employment allocated to non-concentrated industries, as explained below.

Second, we assume that there is a common fixed cost $c > 0$ among all *noncompetitive* industries. Although we make this assumption mostly for simplicity, we should highlight that it is not completely innocuous. In particular, when there are differences in fixed costs within these industries, and if these differences are large, multiplicity may disappear – since for multiplicity to arise, we need a sufficiently large number of industries that move together. Recall however that, even if they share the same fixed cost $c > 0$, *noncompetitive* industries will still be heterogeneous, as they will have different ex-post distributions

[16] Note however these industries will not necessarily always operate close to perfect competition. First, because $N$ is finite. Second, even when $N$ is large, there can be large productivity differences across firms (for example, when the first firm has a clear advantage over all the others), which results in high concentration and high markups for the largest firms.

[17] There are reasons to think that some industries should not be significantly affected by a contraction of aggregate output. One of them is trade. Although we have a closed-economy where all industry output is sold domestically, in reality there are tradable industries whose output can be exported. If external demand does not experience a significant contraction for some of these industries, they should not be affected by the transition to the low steady-state.

of idiosyncratic productivity draws $\{\pi_{ij}\}_{j=1}^{N}$. These industries may display in fact a different number of players, as we will see below.

### 4.1 Calibration

We next describe the calibration of all the parameters. The model is calibrated at a quarterly frequency. Under the parameters we use, the economy will feature two steady-states. Our calibration strategy relies on the interpretation that the economy starts in the high steady-state. Some parameters are standard and taken from the literature. For the preference parameters, we work with an annualized discount factor of 0.96 and set $\gamma = 1$ to have log utility. The inverse of the elasticity of labor supply is set to $\nu = 0.4$ as in Jaimovich and Rebelo (2009). We set the capital elasticity to $\alpha = 0.3$ and assume a 10% depreciation rate. For the two parameters governing the elasticities of substitution, we follow Mongey (2019) and use $\sigma_I = 1.5$ and $\sigma_G = 10$. These two parameters are important for the results, as they determine the degree of complementarity between capital accumulation and competition. Edmond et al. (2015) estimate $\sigma_I = 1.24$ and $\sigma_G = 10.5$ in a static trade model with oligopolistic competition. Atkeson and Burstein (2008) use $\sigma_I \approx 1$ and $\sigma_G = 10$. Several other studies also use $\sigma_I \approx 1$ so that the final good is a Cobb-Douglas aggregate of the different industries.[18] In general, increasing the elasticity of substitution across industries $\sigma_I$ depresses markups and weakens the complementarity between capital accumulation and competition, making multiplicity less likely to arise.[19] Therefore, we see $\sigma_I = 1.5$ as a conservative choice. However, in Appendix D.8, we provide two alternative calibrations where we use different values for the cross-industry elasticity of substitution. We set the number of industries to $I = 5,000$ and the maximum number of firms per industry to $N = 100$. The maximum number of firms per industry will play a role in the *competitive* industries (i.e. those with zero fixed cost, and where all potential firms always produce). We perform robustness exercises with $N = 50$ and $N = 200$ and obtain similar results.

There are three important parameters that we need to calibrate – the fraction of *competitive* industries $f_{\text{comp}}$, the fixed cost for the *noncompetitive* sector ($c$) and the Pareto shape of the productivity distribution of the pool of potential entrants ($\lambda$). These three parameters are jointly calibrated to target three data moments observed in 2007 (i.e. before the 2008 crisis). To calibrate $f_{\text{comp}}$, we target the fraction of aggregate employment that is allocated to highly concentrated industries. In our model, *noncompetitive* industries will be highly concentrated and will not have more than 4 firms. We hence define an industry as concentrated if the 4 largest firms represent at least 90% of the output of the 8 largest firms.[20] Using data from the US

[18]See, for example, Hsieh and Klenow (2009) and Hottman et al. (2016).

[19]To see this, note that markups will be comprised between $\mu_{\text{PC}} = 1$ (perfect competition) and $\mu_{\text{M}} = \dfrac{\sigma_I}{\sigma_I - 1} \geq 1$ (monopoly). A larger $\sigma_I$ will be hence associated with a lower monopoly markup. In other words, as $\sigma_I$ increases markups become less sensitive to changes in the number of firms.

[20]We would like to think of an industry at the highest possible level of disaggregation (e.g. 10-digit NAICS). However, the US census provides concentration metrics only at the 6-digit NAICS level. This is why we do not look directly at the share of

Census, we find that 7.62% of aggregate employment is allocated to such 6-digit industries.

We calibrate the other two parameters by targeting two moments of the markup distribution of public firms in 2007: the average sales-weighted markup (as computed by De Loecker and Eeckhout (2017)) and its standard deviation.[21] Intuitively, the average level of markups pins down the fixed cost, $c$ – a lower fixed cost will be associated with larger entry and hence lower average markups, for a given level of productivity dispersion. Dispersion in markups will, on the other hand, pin down the Pareto tail of entrants' productivity, $\lambda$ – given the positive link between productivity and markups, larger dispersion in productivities will be associated with larger dispersion in markups (and vice-versa) for a given number of firms. We obtain a fraction of *competitive* industries of $f_{\text{comp}} = 0.785$, a Pareto tail of $\lambda = 5.43$ and a fixed cost of $c = 0.0375$.

Finally, we need to calibrate the two parameters governing the dynamics of aggregate productivity: the autocorrelation parameter $\phi_z$ and the standard deviation of the innovations $\sigma_\varepsilon$. We do so by targeting the first order autocorrelation and the standard deviation of aggregate TFP (between 1985 and 2018).[22]

To assess the business cycle implications of larger firm level heterogeneity, we also provide an alternative calibration of the model. In particular, we calibrate the Pareto tail $\lambda$ and the fixed cost $c$ to target the same two moments of the markup distribution in 1985. Note that both the observed sales-weighted average markup and its standard deviation are lower in 1985 than in 2007 (Table 1). All other parameters are kept the same.[23] In this alternative calibration, we obtain a Pareto tail of $\lambda = 7.69$ and a fixed cost of $c = 0.0152$.

Tables 1 and 2 below report our targeted moments, with their model counterparts. We match the average markup in both the 1985 and the 2007 economies extremely well. We match its standard deviation reasonably well in 2007, but less so in 1985. The employment share of highly concentrated industries is slightly overestimated.

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the top 4 firms, but instead scale it by the share of the top 8. We have checked the robustness of our criterion. In particular, we considered alternative thresholds for the ratio of the top 4 to the top 8 (85% and 95%). The results were identical.

[21] See Appendix D.1 for details.

[22] We use the series by Fernald (2012) and remove a linear trend, computed for the 1985-2007 period.

[23] We want to compare the 1985 and 2007 economies over a limited set of dimensions – in particular, the degree of firm heterogeneity and of fixed costs. For this reason, we keep $f_{\text{comp}}$ constant. In Appendix D.7, we provide a robustness exercise where we also recalibrate $f_{\text{comp}}$.

Parameter Values

| Description | Parameter | Value | Source/Target |
|---|---|---|---|
| Between-Industry ES | $\sigma_I$ | 1.5 | Mongey (2019) |
| Within-Industry ES | $\sigma_G$ | 10 | Mongey (2019) |
| Elasticity of Labor Supply | $\nu$ | 0.4 | Jaimovich and Rebelo (2009) |
| Capital Elasticity | $\alpha$ | 1/3 | Standard value |
| Depreciation Rate | $\delta$ | $1-0.9^{1/4}$ | Standard value |
| Discount Factor | $\beta$ | $0.96^{1/4}$ | Standard value |
| Coefficient of Risk Aversion | $\gamma$ | 1 | log utility |
| Persistence of $z_t$ | $\rho_z$ | 0.90 | Autocorrelation of log output |
| Standard Deviation of $\varepsilon_t$ | $\sigma_\varepsilon$ | 0.004 | Standard deviation of log output |
| Number of Industries | $I$ | 5,000 | See text |
| Maximum Number of Firms per Industry | $N$ | 100 | See text |
| Fraction of Competitive Industries | $f_{\text{comp}}$ | 0.785 | Employment Share in Concentrated Industries |
| Pareto Tail 1985 | $\lambda_{85}$ | 8.19 | Markup Dispersion 1985 |
| Fixed Cost 1985 | $c_{85}$ | $3.58 \times 10^{-3}$ | Average Markup 1985 |
| Pareto Tail 2007 | $\lambda_{07}$ | 5.43 | Markup Dispersion 2007 |
| Fixed Cost 2007 | $c_{07}$ | $10.1 \times 10^{-3}$ | Average Markup 2007 |

| | Markups: Average | | Markups: Std. Deviation | | Emp. Share Concent. Ind. | |
|---|---|---|---|---|---|---|
| | Data | Model | Data | Model | Data | Model |
| 1985 | 1.27 | 1.30 | 1.44 | 1.12 | - | 11.7% |
| 2007 | 1.46 | 1.45 | 1.74 | 1.69 | 7.62% | 9.48% |

Table 1: Targeted Moments and Model Counterparts

| | Autocorrelation | Standard Deviation |
|---|---|---|
| Data: 1985-2018 | 0.934 | 0.025 |
| Model: 1985 calibration | 0.983 | 0.027 |
| Model: 2007 calibration | 0.936 | 0.017 |

Table 2: Targeted Moments and Model Counterparts: Aggregate TFP

| | $\text{share}_4/\text{share}_8$ | Labor Share | Profit Share | I/Y |
|---|---|---|---|---|
| 1985 | 0.798 | 0.564 | 0.149 | 0.174 |
| 2007 | 0.845 | 0.525 | 0.211 | 0.161 |

Table 3: Model: Summary Statistics

### 4.2 Quantitative Results

We now describe the quantitative results. We start by comparing the steady-states of the 1985 and 2007 economies. We also include a parameterization for the years 1990, 1995 and 2000, assuming that average markups and markup dispersion follow a linear trend between 1985 and 2007. Figure 10 shows the steady-state values of output per hour, aggregate TFP, the labor share and aggregate markups for the five different parameterizations.[24] Our model predicts an overall increase in aggregate output per hour between 1985 and 2007 of roughly 30%.[25] Aggregate TFP increases by 26%. Note that the increase in both output per hour and aggregate TFP are driven by the increase in the tail of the Pareto distribution – which results in a larger production possibility frontier. When looking at the data counterparts, we observe that real output per worker increases by 50%, while aggregate TFP increases by 26%. Therefore, though the lens of our model, the increase in the Pareto tail of the distribution of idiosyncratic draws can explain 60% of the increase in real output per hour. Our model replicates, however, the evolution of aggregate TFP, which is a non-targeted moment. Regarding the labor share, our model predicts a 3.9 percentage point decline in the aggregate labor share (from 0.564 to 0.525). In the data, it falls by only 2 percentage points (from 0.615 to 0.595). Note that the labor share in our model is about 5 to 6 percentage points lower than the one observed in the data. Therefore, our model underestimates the level of the labor share, but overestimates its decline. Such a discrepancy can be explained by the fact that we target average markups for public firms, which tend to display larger profit shares (and hence lower labor shares) than the average firm in the economy.

[24] In all the different five calibrations, we ensure that the economy features two steady-states and is at the highest one.

[25] Given that GHH preferences are not consistent with a balanced growth path, we report output per hour (and not output) and compare it to its data counterpart.

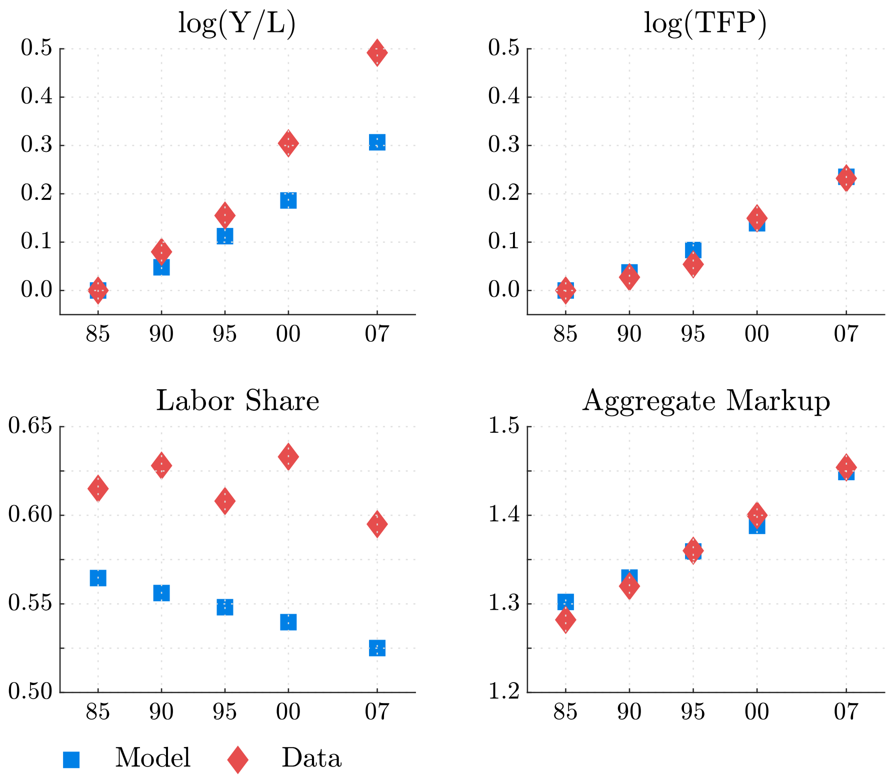
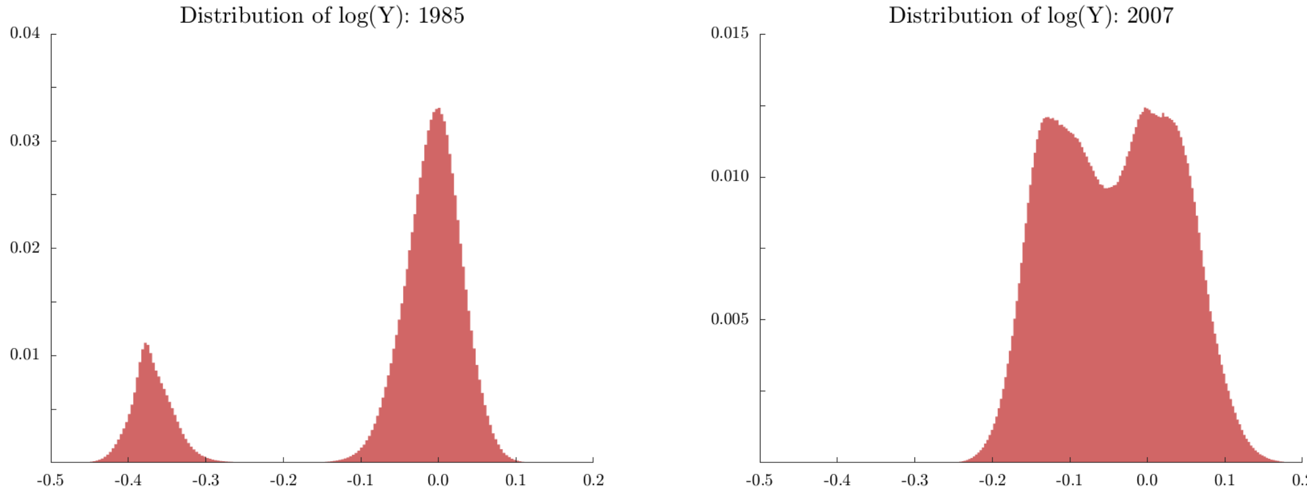



Figure 10: Model *versus* data: 1985-2007. The data series are respectively (i) Business Sector: Real Output Per Hour of All Persons (from BLS), (ii) Aggregate TFP from Fernald (2012), (iii) Business Sector: Labor Share (from BLS), and (iv) Aggregate Markup from De Loecker and Eeckhout (2017).

We then compare the dynamic properties of the 1985 and the 2007 economies. We start by simulating each economy over 10,000,000 periods. Figure 11 shows the ergodic distribution of log output; the distributions are centered around the high steady-state, so that the horizontal axis represents output in percentage deviation from the high steady-state. Two facts stand out. First, compared to the 2007 distribution, the 1985 distribution exhibits a larger mass around the high regime or steady-state. Second, in the 1985 distribution, the two steady-states are further apart. Taken together, these facts imply that in the 2007 economy (i) the low steady-state will be reached more frequently and (ii) transitions across regimes are going to be more likely.[26] Recall that the 1985 and the 2007 economies only differ in the Pareto tail parameter $\lambda$ and the fixed production cost in the *noncompetitive* sector. In particular, the 2007 economy exhibits a more dispersed Pareto distribution and larger fixed costs. These facts mean that in 2007, small firms in the *noncompetitive* sector will have a lower share of the market and their entry/exit decisions will be more sensitive to aggregate fluctuations.

[26]Figure 28 in Appendix D.1 shows the long-run demand and supply of capital under the 1985 and the 2007 parameters. It illustrates how multiple steady-state can arise.




(a) Output Distribution: 1985

(b) Output Distribution: 2007

Figure 11: Output Distributions

| | Output | Consumption | Investment | Hours |
|---|---|---|---|---|
| | | Correlation with Output | | |
| Data: 1985-2018 | 1.00 | 0.99 | 0.87 | 0.89 |
| Model: 1985 calibration | 1.00 | 0.99 | 0.97 | 1.00 |
| Model: 2007 calibration | 1.00 | 0.99 | 0.83 | 1.00 |
| | | Standard Deviation Relative Output | | |
| Data: 1985-2018 | 1.00 | 1.00 | 2.53 | 1.07 |
| Model: 1985 calibration | 1.00 | 0.98 | 1.12 | 0.76 |
| Model: 2007 calibration | 1.00 | 0.96 | 1.56 | 0.77 |

Table 4: Business Cycle Moments

We next study the impulse response functions to a negative TFP shocks in the two economies.

**Impulse Response Functions: Small Negative Shock** We start by characterizing the reaction of the economy to a small negative shock. We consider a shock to the innovation of the exogenous TFP process that is equal to $\varepsilon_t = -\sigma_\varepsilon$ and lasts for two quarters. Such a shock will, however, have a persistent impact on exogenous TFP $z_t$ through equation (5). Figure 12 shows the impulse responses for both the 1985 and the 2007 economy. The simulation of the transition dynamics covers 100 quarters. This shock generates different responses for the two economies, as evidenced by the middle top panel of 12 and Table 5. The 2007 economy exhibits both greater amplification and greater persistence. First, the 1985 economy experiences a 1.6% reduction in aggregate output after 5 quarters, against a 2.2% reduction in the 2007 economy. Second, the 1985 economy is back at steady state levels of output after approximately 90 quarters, while the 2007 economy has a much more prolonged downturn, being still 1% below steady-state after 100 quarters.

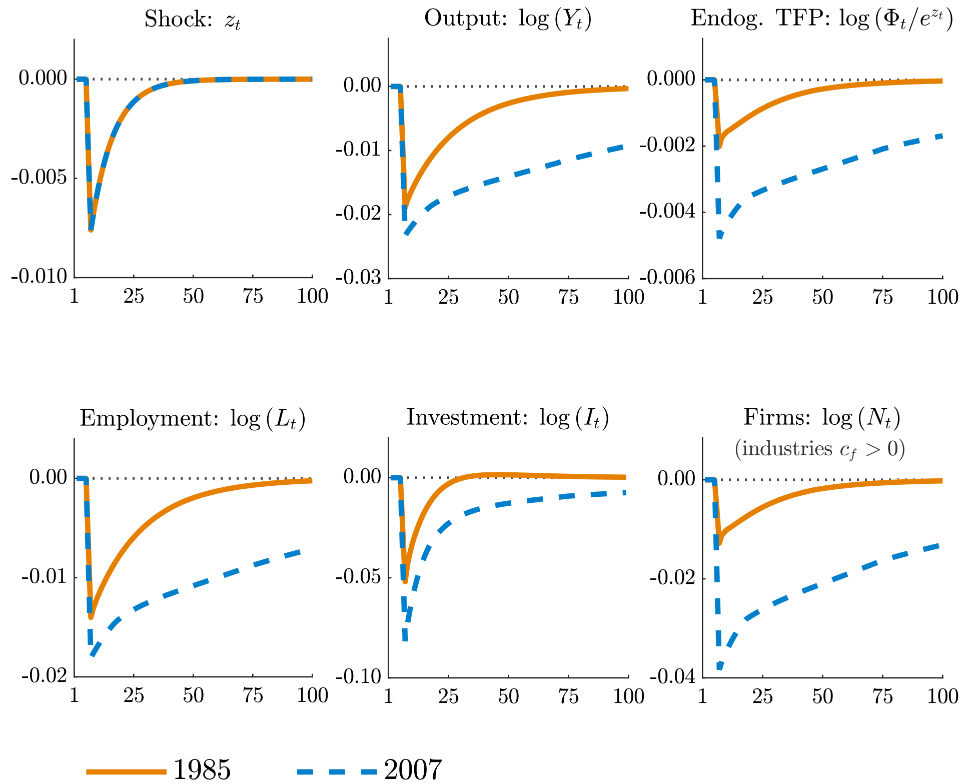



Figure 12: Impulse Responses: Small Shock

% Deviation of $\log(Y_t)$ from High Steady-State

| Calibration | $T = 5$ | $T = 10$ | $T = 20$ | $T = 100$ |
|---|---|---|---|---|
| 1985 | -0.016 | -0.013 | -0.008 | 0.000 |
| 2007 | -0.022 | -0.020 | -0.017 | -0.009 |

Table 5: Impulse Response Functions of $\log(Y_t)$. This table shows the values $\log(Y_t)$ in deviation from its steady-state, after a negative shock to $\varepsilon_t$. This shock is equal to $\varepsilon_t = -\sigma_\varepsilon$ and lasts for two quarters.

The mechanism underlying such increased amplification and persistence can be better understood by looking at the right bottom panel, which plots the transition dynamics of the number of firms in the *noncompetitive* industries (which in our calibration represent 21.5% of all industries). In 2007, there is a much more significant reduction in the number of firms, due to the mechanisms outlined above: increased productivity dispersion and larger fixed costs make small, unproductive firms more sensitive to aggregate shocks. Such additional action in the extensive margins generates both additional amplification and persistence. Note that greater amplification and persistence can be observed also in employment, investment and in the endogenous component of aggregate TFP. The drop in the endogenous component of aggregate TFP is due both to a reduction in the number of firms (as our economy features a love for variety) and to an increase in industry misallocation (as industries featuring positive fixed costs display a disproportionately larger contraction). The reaction of the endogenous component of aggregate TFP will however be explained in more detail in Section 5.

**Impulse Response Functions: Large Negative Shock** After the temporary negative shock just considered, both economies transition back to their steady-states. We now describe the dynamics after a larger negative shock. To this end, we repeat the same exercise for the two economies, but now introduce a negative shock equal to $\varepsilon_t = -3\sigma_\varepsilon$, which lasts for three quarters.

The dynamics are shown in Figure 13 and in Table 6. As before, there is greater amplification and persistence in the 2007 economy. However, the 2007 economy now experiences a permanent drop in aggregate output, i.e. it transitions to a lower steady-state (a low competition trap). In the example we consider, there is a permanent 11.1% loss in output. In this setup, employment drops permanently by 8.6%, while investment decreases by 72% on impact and 11.8% in the long run.

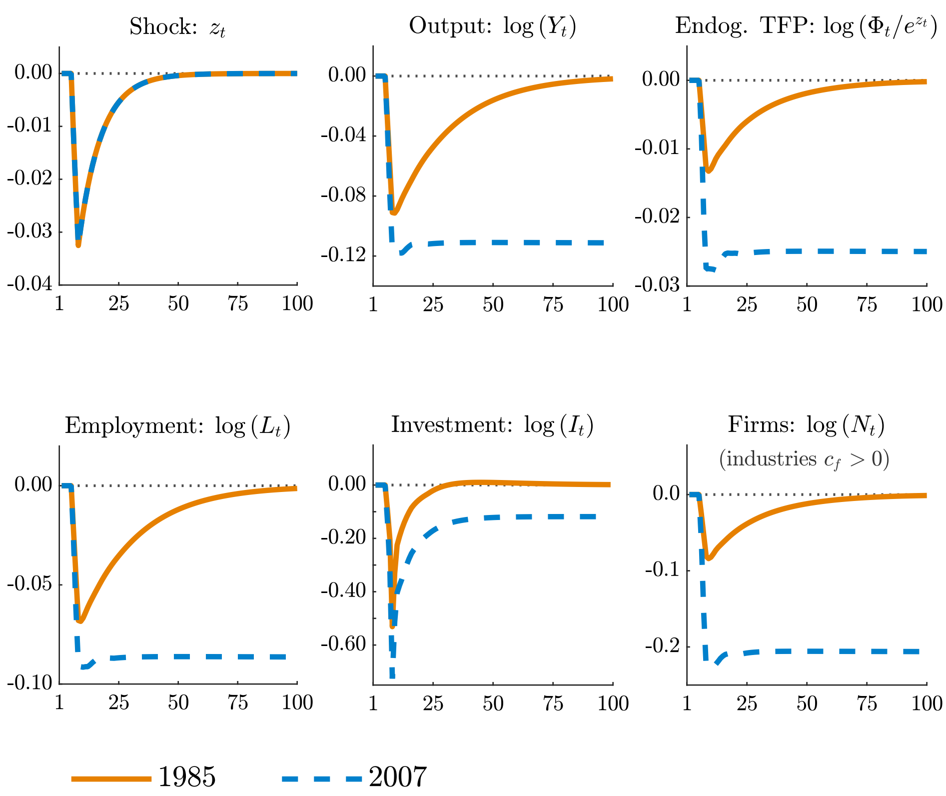
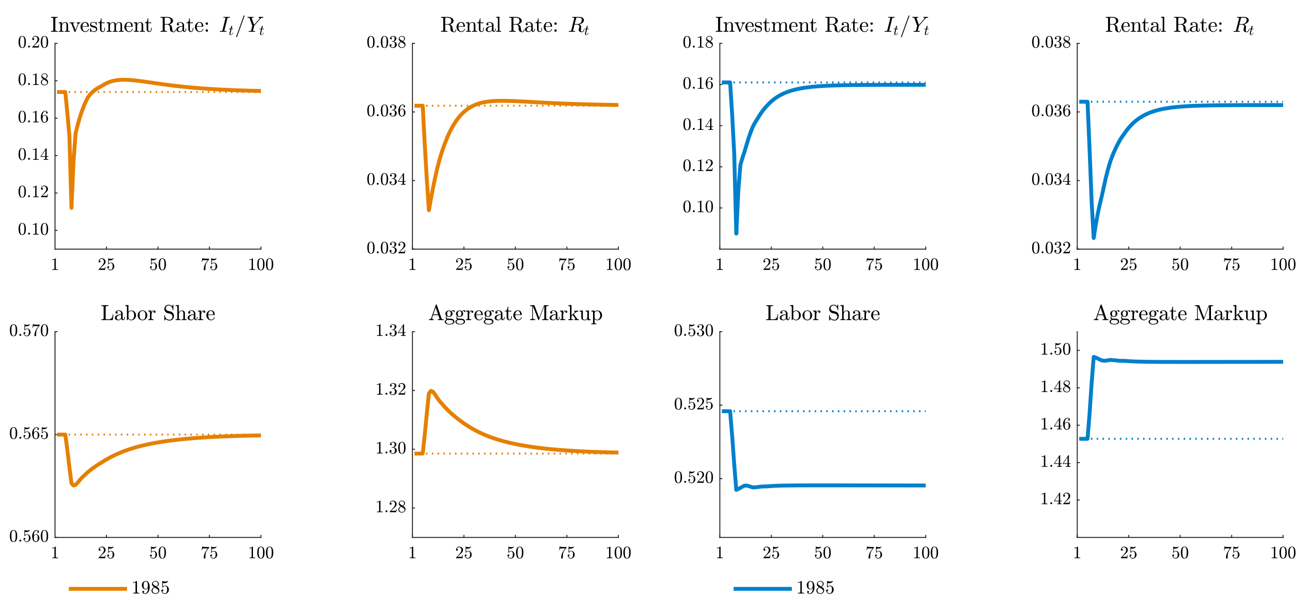



Figure 13: Impulse Responses: Large Shock

| % Deviation of $\log(Y_t)$ from High Steady-State | | | | |
|---|---|---|---|---|
| Calibration | $T = 5$ | $T = 10$ | $T = 20$ | $T = 100$ |
| 1985 | -0.089 | -0.072 | -0.047 | 0.002 |
| 2007 | -0.118 | -0.114 | -0.112 | -0.111 |

Table 6: Impulse Response Functions of $\log(Y_t)$. This table shows the values $\log(Y_t)$ in deviation from its steady-state, after a negative shock to $\varepsilon_t$. This shock is equal to $\varepsilon_t = -3\sigma_\varepsilon$ and lasts for three quarters.

In figures 14(a) and 14(b) we plot the responses of the gross investment rate $I_t/Y_t$, the rental rate, the labor share and the (sales-weighted) average markup.[27] With respect to the behavior of the investment rate, there is one difference that is worth highlighting. In the 1985 economy, the investment rate suffers a significant drop on impact, but ultimately recovers and overshoots its steady-state level – so that the capital stock can recover to its long-run value. This is not true for the 2007 economy. As the economy is converging to a lower steady state, during the transition there is "too much" capital in the system, which yields a gradual reduction of the stock through a depressed investment rate over the transition. Aggregate markups increase on impact as there is firm exit. In the 1985 economy, this increase is reabsorbed as the economy transitions back to its steady state and firms enter the market. In the 2007 economy, such absorption does not take place since the number of firms never goes back to the previous level. Not surprisingly, the labor share exhibits the oppposite behavior. In the 1985 economy, the reduction quickly reverts, while the 2007 economy experiences a 0.5 pp permanent drop in the labor share.




(a) 1985 Economy

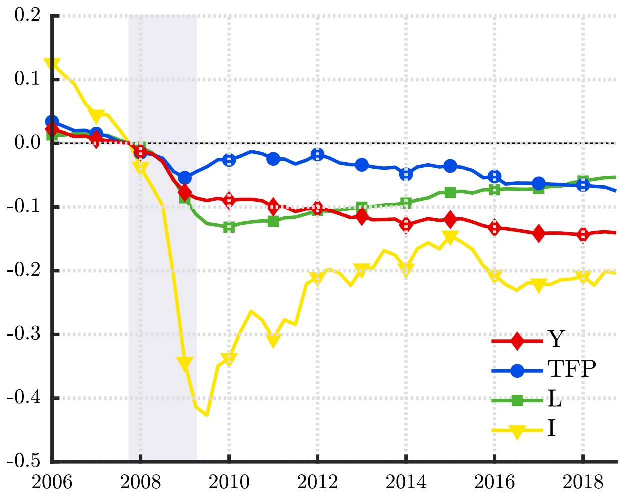
(b) 2007 Economy

Figure 14: IRF (Large Shock)

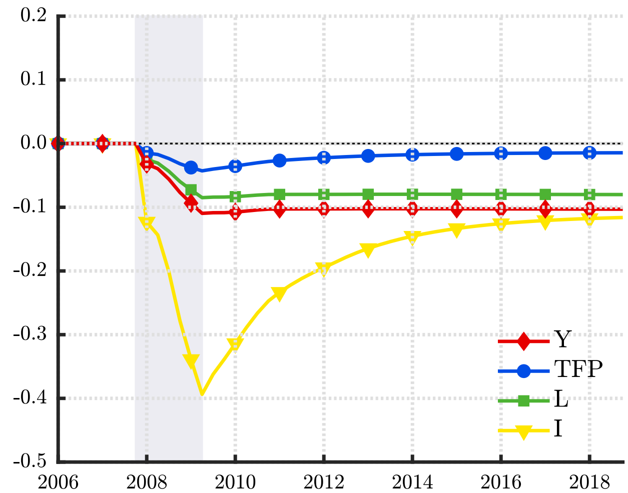
[27] In our setup, the steady-state rental rate is independent of the regime of the economy, as it is pinned down by the discount factor of the representative household.

# 5 The 2008 Recession and Its Aftermath

In this section, we take a deeper look at the 2008 recession and its aftermath. The left panel of Figure 15 shows the behavior of some aggregate variables from 2006 to 2018 – real GDP, real gross private investment and total hours (all in per capita terms), as well as aggregate TFP.[28] All variables are in logs, detrended (with a linear trend computed over 1985-2007) and centered around 2007Q4. The four variables decline on impact and do not seem to rebound to their pre-recession trends. For example, in the first quarter of 2018, real GDP per capita is 13.3% below trend (Table 7). Aggregate TFP has experienced a 6.8% negative deviation from trend. Investment declines by more than 40% on impact, and then seems to stabilize at approximately 20% below the pre-crisis trend.




(a) Data




(b) 2007 Model

Figure 15: The great recession and its aftermath

| | Data | | | Model | | |
|---|---|---|---|---|---|---|
| | 2009Q4 | 2015Q1 | 2018Q1 | 2009Q4 | 2015Q1 | 2018Q1 |
| Output | -0.084 | -0.119 | -0.133 | -0.109 | -0.102 | -0.103 |
| Aggregate TFP | -0.037 | -0.026 | -0.068 | -0.038 | -0.016 | -0.015 |
| Hours | -0.124 | -0.062 | -0.037 | -0.084 | -0.080 | -0.080 |
| Investment | -0.352 | -0.153 | -0.220 | -0.340 | -0.133 | -0.117 |

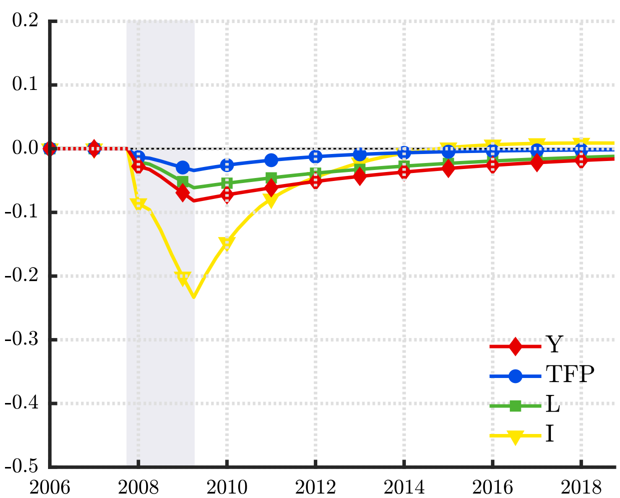
Table 7: The great recession and its aftermath

We then ask whether our model can replicate the behavior of these four variables. We feed our model with a sequence of shocks to the innovation of TFP ($\varepsilon_t$) that lasts for six quarters (2008Q1:2009Q2), so that

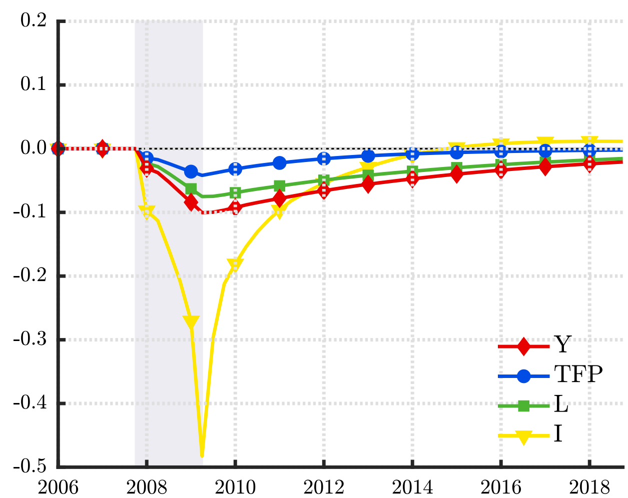
[28] See Appendix A.1.1 for the data sources.

endogenous aggregate TFP in our model ($\Phi_t$) matches the observed aggregate TFP series over the same period. The economy starts at the high steady-state (with $z_t = 0$). We set the innovations to productivity to zero after 2008Q1 and let the economy recover afterwards. The right panel of Figure 15 shows the implied responses of output, aggregate TFP, employment and investment, generated by our model. The series of shocks that we feed happen to be sufficient to trigger a transition to the low steady-state. Our model provides a reasonable description of the evolution of the four variables. Output experiences a 10.3% decline in the long-run, whereas employment drops by 8.0% (Table 7). Both reactions are of the same order of magnitude as observed in the data (with our model underpredicting the drop in output and overpredicting the drop in total hours). The same happens for investment, which declines by 34.0% on impact, and then stabilizes at 11.7% below its high steady-state value. Finally, our model generates a 1.5% permanent drop in aggregate TFP – we can hence explain approximately 1/5 of the decline in aggregate TFP observed in the data. In the next subsection we study more closely the dynamics of aggregate TFP and other variables.

We next ask whether the sequence of aggregate TFP shocks $z_t$ that we feed in the 2007 economy can also trigger a transition to the low steady-state in the 1985 economy. Figure 16(a) shows the transition dynamics. Not only does the economy exhibit substantially less amplification, but it also reverts back to the high steady-state. These shocks would however imply a decline in aggregate TFP lower than the one observed in the data. Therefore, we also recalibrate the sequence of shocks to match the observed behavior of aggregate TFP between 2008Q1:2009Q2. The results are shown in Figure 16(b). The new shocks are substantially greater in magnitude, so that investment experiences a large collapse on impact. However, all aggregates end up converging again to the initial steady-state.




(a) 1985 Model: shocks used in Figure 15(b)




(b) 1985 Model: recalibrated shocks

Figure 16: The *great recession* in the 1985 Model

These results suggest that, in the 1985 economy, a downturn of the magnitude of the 2008-2009 recession would not be large enough to generate a persistent deviation from trend. The economy would have experienced a relatively fast reversal to trend, due to a lower endogenous amplification and persistence. We then conclude that the structural differences between the 1985 and the 2007 economies (namely larger productivity differences and larger fixed costs) are key to understand the 2008 crisis and the subsequent *great deviation*.

### 5.1 Aggregate Productivity

As we have seen before, as the economy enters the low competition trap, it experiences a persistent decline in aggregate TFP $\Phi(\cdot)$. Note that this happens in spite of the exit of low productivity firms. To understand such a result, note first that aggregate TFP is not a weighted average of firm level TFP. Indeed, the latter exhibits a permanent increase, as shown in the right panel of Figure 17.[29]

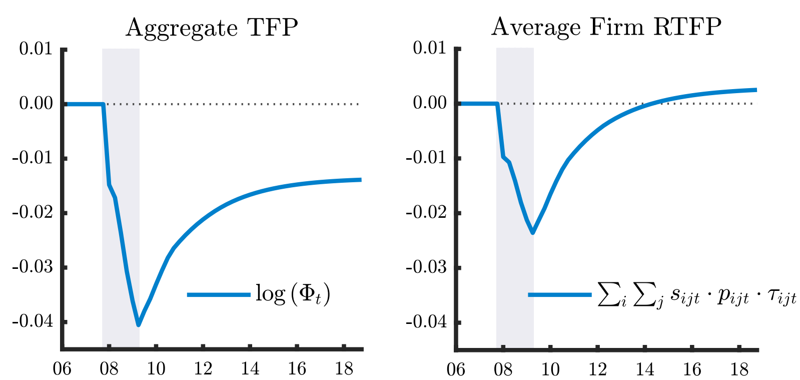



Figure 17: Aggregate TFP versus Average Firm Level TFP

The left panel shows aggregate TFP, as defined in equation (14). The right panel shows a sales-weighted average of firm level revenue TFP $p_{ijt} \cdot \tau_{ijt}$.

But why is there a decrease in aggregate TFP in the model, despite the exit of low productivity firms? There are two reasons: (i) a reduction in the number of firms and (ii) an increase in cross-industry misallocation.

To understand the first effect, note that our model embeds a *love for variety* effect. Recall that in the limit case in which there is no heterogeneity across firms or industries (all industries have the same number of firms $n$ and all firms have the same productivity $\tau$), aggregate TFP is equal to

$$\Phi = I^{\frac{1-\rho}{\rho}} n^{\frac{1-\eta}{\eta}} \tau.$$

As we can see, aggregate TFP increases in both the number of industries $I$ (which is always fixed on our model) and in the number of firms per industry, $n$.

[29]Figure 17 reports a sales-weighted average of firm level revenue TFP. A similar pattern emerges if one uses physical TFP instead.

Second, as the economy moves to a low competition trap, the economy is likely to experience an increase in cross industry misallocation. This fact happens because industries experiencing a larger contraction are the industries with positive fixed costs $c > 0$, i.e. industries whose output is already restricted. Figure 18 shows the evolution of the standard deviation of (log) industry outputs

$$\text{std}_i \left[\log\left(y_{it}\right)\right].$$

As we can see, once the economy enters the low steady-state, there is an increase in cross-industry dispersion of outputs. This also contributes to the decline in aggregate TFP.

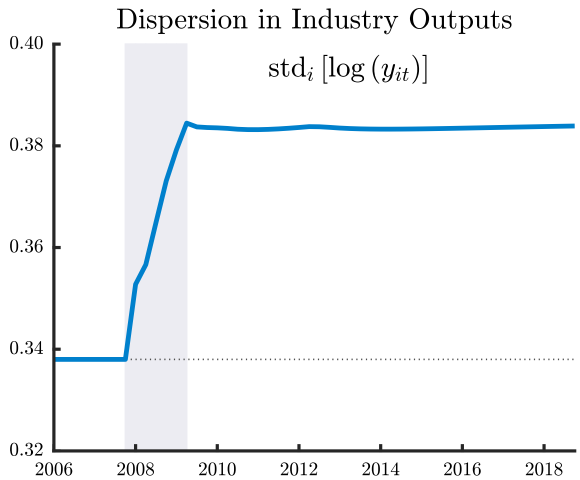



Figure 18: Dispersion in $\log\left(y_{it}\right)$

All in all, our model provides two possible reasons why aggregate TFP may have experienced a permanent drop after 2008. Consistent with the model, such a drop in aggregate TFP may have occurred in spite of the exit of low productivity firms.[30]

### 5.2 Aggregate Markups and the Labor Share

Our model suggests that a transition from the high to the low steady-state should be accompanied by a change in the competitive structure of the economy. In particular, we should observe signs of declining competition, such as larger markups and a lower labor share. Figure 19 shows the evolution of the labor share (left panel) and the De Loecker and Eeckhout (2017) aggregate markup series for publicly listed firms (right panel). The grey dashed line represents a linear trend computed for the 1985-period.

[30] Foster et al. (2016) show that, as in previous recessions, manufacturing firms exiting during the great recession were on average less productive.

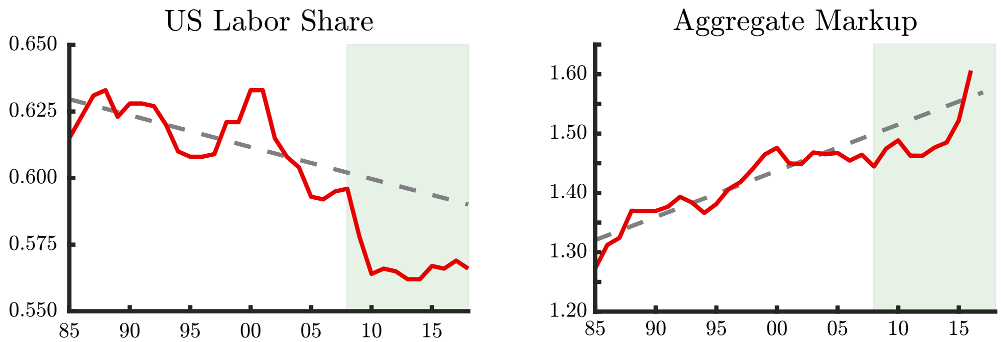



Figure 19: **US Labor Share and Aggregate Markup: 1985-2017**

(i) Labor Share of the Corporate Business Sector (from the BLS) and (ii) De Loecker and Eeckhout (2017) aggregate markup series. For each series, the dashed grey line shows the corresponding average for the 1985-2007 period.

Table 8 compares the evolution of the aggregate labor share and the aggregate markup series between 2007 and 2016 observed in the data and obtained in our model. Overall, our model predicts a 0.6 pp decline in the aggregate labor share, which is approximately 1/5 of the observed decline between 2007 and 2016. Markups increase by 4.1 points in our model, which represents 29% of the observed increase (14.2 points).

| | Data | | | Model | | |
|---|---|---|---|---|---|---|
| | 2007 | 2016 | $\Delta 2007-2016$ | 2007 | 2016 | $\Delta 2007-2016$ |
| Labor Share | 0.595 | 0.567 | -0.028 | 0.525 | 0.519 | -0.006 |
| Aggregate Markup | 1.46 | 1.61 | 14.2 | 1.45 | 1.49 | 4.1 |

Table 8: Labor share and aggregate markups

### 5.3 The Decline in Corporate Investment and Real Interest Rates

Gutiérrez and Philippon (2017) and Jones and Philippon (2016) have pointed out that the US investment rate has been low in recent years, in spite of historically low interest rates. Figure 20 illustrates these facts. It shows the corporate investment rate (gross capital formation divided by value added) and interest rate between 1980 and 2017.

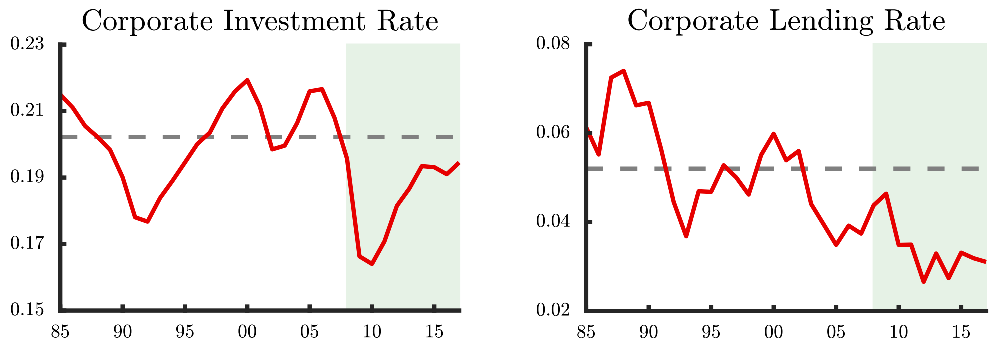



Figure 20: **US Corporate Investment and Lending Rates: 1985-2017**

(i) The corporate investment rate is the ratio of gross capital formation to gross value added, for the US private business sector (from the BEA) and (ii) the corporate lending rate is Moody's Seasoned BAA Corporate Bond Yield (from Moody's) minus a 5-year moving average of past CPI inflation (from the BLS). For each series, the dashed grey line shows the corresponding average for the 1985-2007 period.

For instance, the investment rate experienced a significant decline between 2008 and 2010 (of more than 4 percentage points). It then increased between 2011 and 2014, and seems to have stabilized at a lower level: the average investment rate between 2015 and 2017 was 19.3%, against an average of 20.4% between 1980 and 2007. The second panel suggests that the real corporate lending rate has been below its pre-crisis average (the average interest rate between 2015 and 2017 was 3.20%, while the 1980-2007 average was 5.28%).

The impulse responses shown in Figure 14(b) seem to be consistent with this pattern. Note, however, that our model cannot explain a persistent drop in the interest rate, as its steady-state value is always pinned down by the discount factor of the representative household – and is hence independent of the competitive regime of the economy.[31]

## 6 Empirical Evidence

The main purpose of this section is to provide empirical evidence on the model's mechanism. The results presented in Section 3.3 offer cross-industry predictions that can be tested in the data. Recall that according to Proposition 1 if we take two industries with the same number of firms, the one featuring a more uneven distribution of productivities should be more sensitive to aggregate fluctuations. This result comes from the higher likelihood of small firms exiting the market when the economy is hit by a negative shock. Therefore, industries featuring larger firm heterogeneity, should be the ones in which the post-2007 drop in aggregate output had a more persistent impact.

Ideally, we should have data on within-industry productivity dispersion. However, with a lack of such information, we use concentration ratios as a proxy for within-industry productivity dispersion. The

[31]We could obtain a variable steady-state interest rate under an heterogeneous agent framework (e.g. in OLG model).

intuition is that, given the positive association between productivity and market shares highlighted in equation (10) (for a given number of firms $n_{it}$), industries with a larger productivity dispersion should feature larger dispersion in market shares and, hence, on average be more concentrated.

We build a dataset combining the 2002 and 2007 US Census data on industry concentration to the Statistics of US Businesses (SUSB) and the Bureau of Labor Statistics (BLS) to obtain outcomes as output, employment, wage bill and the number of firms at the industry level (6-digits NAICS). The final dataset includes 791 6-digit industries. In 2016, the average (median) industry had 5,631 (1,316) firms, 121,621 (36,910) workers and a total payroll of \$6,394 million (\$1,880 million).

We study the correlations between the response to the 2007 recession and the degree of concentration across industries. In particular, we are interested in whether industries with a larger concentration before the crisis experienced a larger post-crisis decline. To investigate this relation we estimate the following model

$$\frac{\Delta y_{i,07-16}}{\mathrm{y}_{i,07}} = \beta_0 + \beta_1 \text{ concent}_{i,07} + \beta_2 \log\left(\text{firms}_{i,07}\right) + a_s \mathbb{1}\{i \in s\} + u_i.$$

Where $y_i$ is an outcome for industry $i$, $concent_i$ is sector $i$ largest 4 firms' combined market share and we control for the number of firms before the crisis. The outcomes always take the form of the annualized growth rate between 2007 and 2016 in a specific industry. In all regressions we include macro sector fixed effects, denoted by $a_s$, $s$ being the more aggregate industry. The unit of observation is a 6-digit industry.

We start by studying the change in employment. The results, showed in Table 9, suggest that more concentrated industries experienced lower employment growth in the aftermath of the great recession. This pattern is robust to the inclusion of the pre-crisis number of firms and the previous 5 years cumulative employment growth.

Table 9: Change in Employment: 2007-2016

| VARIABLES | $\Delta \ln Empl_{07-16}$ | $\Delta \ln Empl_{07-16}$ | $\Delta \ln Empl_{07-16}$ |
|---|---|---|---|
| $\text{concent}_{07}$ | -0.0220*** | -0.0185** | -0.0194*** |
| | (0.00567) | (0.00744) | (0.00738) |
| $\log(\text{Firms}_{07})$ | | 0.000857 | 0.000683 |
| | | (0.00116) | (0.00115) |
| $\Delta \ln Empl_{03-07}$ | | | 0.0933*** |
| | | | (0.0245) |
| Observations | 769 | 769 | 768 |
| R-squared | 0.050 | 0.051 | 0.070 |
| Sector FE | YES | YES | YES |

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Quantitatively, the estimation suggests that a 1pp higher pre-crisis concentration correlates with a 2pp lower employment growth rate between 2007 and 2016. A similar pattern is found for the total payroll. The correlation suggests that a one point increase in the concentration measure is associated with a 2 percentage points reduction in the post crisis growth of payroll.

Table 10: Change in Total Payroll: 2007-2016

| VARIABLES | $\Delta \ln Payroll_{07-16}$ | $\Delta \ln Payroll_{07-16}$ | $\Delta \ln Payroll_{07-16}$ |
|---|---|---|---|
| $\text{concent}_{07}$ | -0.0213*** | -0.0206*** | -0.0215*** |
| | (0.00580) | (0.00761) | (0.00756) |
| $\log(\text{Firms}_{07})$ | | 0.000162 | 1.67e-05 |
| | | (0.00119) | (0.00118) |
| $\Delta \ln Payroll_{03-07}$ | | | 0.0750*** |
| | | | (0.0224) |
| Observations | 773 | 773 | 772 |
| R-squared | 0.045 | 0.045 | 0.060 |
| Sector FE | YES | YES | YES |

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Finally, we study the correlation between the measure of concentration and net entry after the crisis.

Our finding suggests that a 1 percentage point increase in the concentration measure is associated with a 2 to 3 percentage points decrease in the post crisis net entry.

Table 11: Change in Number of Firms: 2007-2016

| VARIABLES | $\Delta \ln Firms_{07-16}$ | $\Delta \ln Firms_{07-16}$ | $\Delta \ln Firms_{07-16}$ |
|---|---|---|---|
| $\text{concent}_{07}$ | -0.0336*** | -0.0203*** | -0.0219*** |
| | (0.00511) | (0.00685) | (0.00683) |
| $\log(\text{Firms}_{07})$ | | 0.00313*** | 0.00298*** |
| | | (0.00108) | (0.00108) |
| $\Delta \ln Firms_{03-07}$ | | | 0.0912*** |
| | | | (0.0274) |
| Observations | 790 | 790 | 790 |
| R-squared | 0.119 | 0.128 | 0.140 |
| Sector FE | YES | YES | YES |

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The main takeaway from this empirical analysis is that, consistent with our model, more concentrated industries show larger post crisis decline both in terms of employment and total payroll, and on the extensive margin.

## 7 Conclusion

The US economy appears to have experienced a fundamental change over the past decades, with several studies and data sources indicating a reallocation of activity towards large, high markup firms. This observation has raised concerns in academic and policy circles about increasing market power, and it has been proposed as an explanation for recent macroeconomic *puzzles* – such as low aggregate investment, low wage growth or declining labor shares. Besides their impact on factor shares and factor prices, our model suggests that rising firm differences and greater market power can also have an impact on business cycles and provide an amplification and persistence mechanism to aggregate fluctuations. In particular, larger firm heterogeneity and greater market power may have rendered the US economy more vulnerable to aggregate shocks and more likely to experience quasi-permanent recessions. Through the lens of our theory, such increased fragility may have been difficult to identify, as it manifests itself only in reaction to large shocks.

In broader terms, our theory indicates that the firm size/markup distribution can be an important determinant of the response of the economy to aggregate shocks. This observation suggests that product market considerations should gain relevance within macroeconomic research and policy analysis. In particular, the standard toolkits used by macroeconomists should increasingly incorporate a realistic characterization of product market frictions.

We conclude by mentioning two extensions we are considering in our future work. First, we are planning to introduce endogenous growth to research the dynamic interplay between market power and innovation in a context of multiple competitive regimes. As documented in Figure 2, both real GDP per capita and aggregate TFP have experienced a widening deviation from trend after 2008, which indicates that growth rates have become persistently lower. We think that an extended version of our model with endogenous R&D has the potential to account for this. In a world where firms conduct R&D because of an *escape-from-competition* effect, a decrease in product market competition will likely reduce firms' incentives to innovate.

Second, we also plan to consider a setup with nominal rigidities to think about the monetary policy implications of increasing firm differences and of rising market power. Our theory suggests at least two relevant insights for the design of monetary policy. First, as industries become more concentrated, firms' pricing decisions are likely to become increasingly rigid and less sensitive to aggregate fluctuations. This suggests that the degree of price rigidity may endogenously respond to changes in the product market structure, which has obvious implications for the effects of monetary policy. Second, market power can have a negative impact on interest rates and hence be associated with the greater likelihood of a binding zero lower bound. The examination of these two hypotheses is an important avenue for future research.

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# A Data Appendix

## A.1 Number of Firms

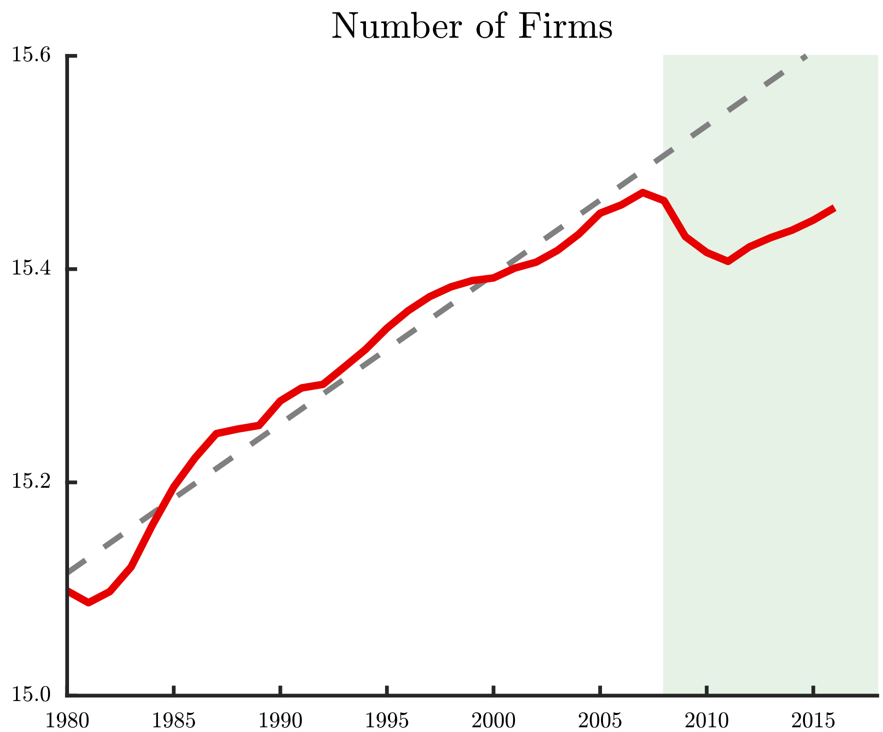



Figure 21: **Number of Firms per Sector: 1980-2016**

The red line shows the number of firms with at least one employee (in logs). The dashed grey line shows a linear trend computed over the 1980-2007 period. Data is from the US Business Dynamics Statistics

**Number of Firms per Sector**

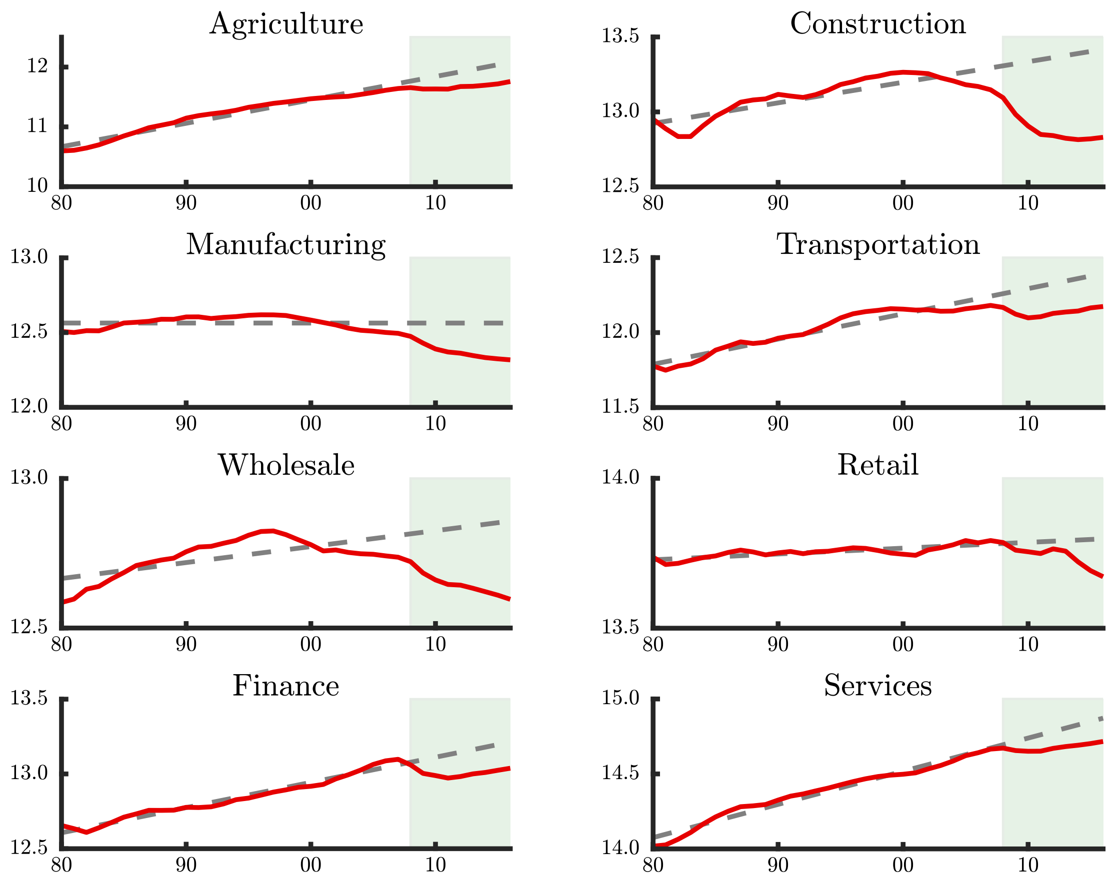



Figure 22: **Number of Firms per Sector: 1980-2016**

Each panel shows the number of firms with at least one employee in each sector (in logs). For each series, the dashed grey line shows a linear trend computed over the 1980-2007 period. Data is from the US Business Dynamics Statistics

**Entry and Exit Rates**

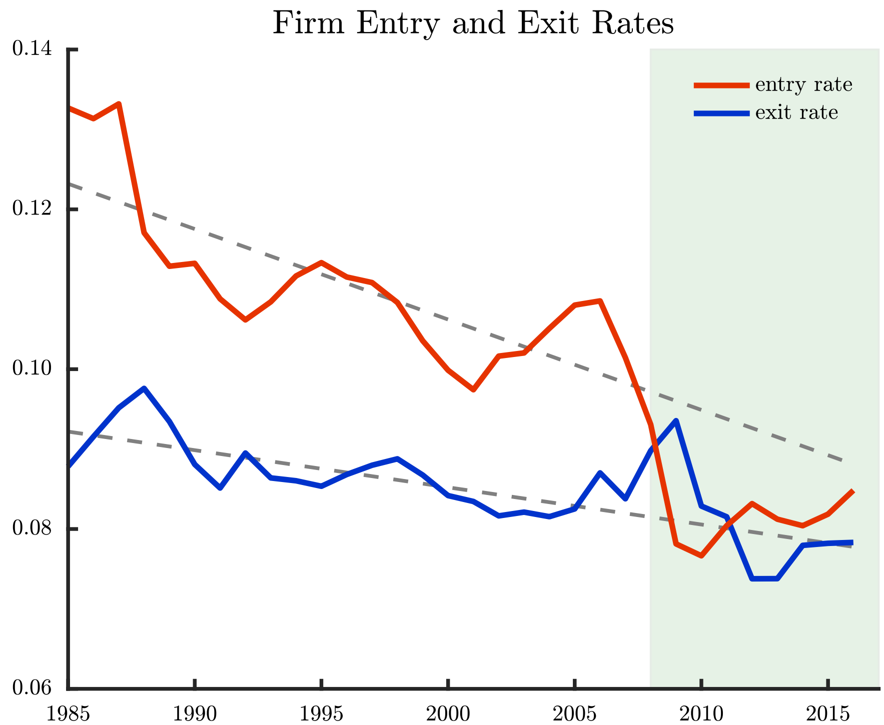



Figure 23: **US Firm Entry and Exit Rates: 1980-2017**

The entry (exit) rate is ratio of the number of startups (exiting firms) to the number of active firms in the previous year (data is from the US Business Dynamic Statistics). The dashed grey line shows a linear trend computed for the 1985-2007 period.

### A.1.1 Data Definition

Table 12 provides information on all the data sources used in Section 5.

Table 12: Data sources

| Variable | Source |
|---|---|
| Real GDP | BEA – NIPA Table 1.1.3 (line 1) |
| Real Gross Private Domestic Investment | BEA – NIPA Table 1.1.3 (line 7) |
| Total Hours | BLS – Nonfarm Business sector: Hours of all persons |
| Aggregate TFP | Fernald (2012): Raw Business Sector TFP |
| Population | BEA – NIPA Table 2.1 (line 40) |

### A.1.2 Aggregate Profit Share

The aggregate profit share is computed as

$$\text{profit\_share}_t = \frac{\text{VA}_t - \text{W}_t - \text{T}_t - r_t \cdot \text{K}_t - \text{DEP}_t}{\text{VA}_t}$$

Following Barkai and Benzell (2018), $\text{VA}_t$ is the total value added of the corporate non-financial sector (NIPA Table 1.14, line 17), $\text{W}_t$ is total labor compensation (line 20) and $\text{T}_t$ is the value of taxes on production minus subsidies (line 23).

$\text{K}_t$ is the value of non-residential capital (including intangibles) of the corporate non-financial sector (NIPA Table 4.1, line 37) and $\text{DEP}_t$ is depreciation (NIPA Table 4.4, line 37). Finally, $\text{r}_t$ is the required rate of return. We follow Eggertsson et al. (2018) and compute it as the difference between Moody's Seasoned BAA Corporate Bond Yield and a 5-year moving average of past CPI inflation (used as a proxy for expected inflation).

# B Model Derivation and Proofs: Industry Equilibrium

## B.1 Static Cournot Game

### B.1.1 Equilibrium Price and Output

When $n$ firms produce, we have a system of $n$ first order conditions

$$p\left[1-(1-\rho)\, s_j\right] = \frac{\Theta}{\pi_j}$$

Dividing the first order condition of firm $j$ by that of firm 1 we obtain

$$\begin{aligned}
& \frac{1-(1-\rho)\, s_j}{1-(1-\rho)\, s_1} = \frac{\pi_1}{\pi_j} \\
\Leftrightarrow \quad & 1-(1-\rho)\, s_j = \frac{\pi_1}{\pi_j}\left[1-(1-\rho)\, s_1\right] \\
\Leftrightarrow \quad & s_j = \frac{1}{(1-\rho)}\left\{1-\frac{\pi_1}{\pi_j}\left[1-(1-\rho)\, s_1\right]\right\}
\end{aligned}$$

Note that

$$\begin{aligned}
& \sum_{k=1}^{n} s_k = 1 \\
\Leftrightarrow \quad & \sum_{k=1}^{n} \frac{1}{(1-\rho)}\left\{1-\frac{\pi_1}{\pi_k}\left[1-(1-\rho)\, s_1\right]\right\} = 1 \\
\Leftrightarrow \quad & n-\pi_1\left[1-(1-\rho)\, s_1\right]\sum_{k=1}^{n}\frac{1}{\pi_k} = 1-\rho \\
\Leftrightarrow \quad & \frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}} = \pi_1\left[1-(1-\rho)\, s_1\right]
\end{aligned}$$

Plugging the last equation into the first order condition of firm 1 we obtain

$$\begin{aligned}
& p\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}} = \Theta \\
\Leftrightarrow \quad & p = \frac{\sum_{k=1}^{n}\frac{1}{\pi_k}}{n-(1-\rho)}\Theta
\end{aligned}$$

Total output is hence equal to

$$\begin{aligned}
& y = p^{-\frac{1}{1-\rho}} Y \\
\Leftrightarrow \quad & y = \left[\frac{\sum_{k=1}^{n}\frac{1}{\pi_k}}{n-(1-\rho)}\Theta\right]^{-\frac{1}{1-\rho}} Y
\end{aligned}$$

#### B.1.2 Market Shares

Plugging the previous equation into the first order condition of firm $j$ we have

$$
\begin{aligned}
1-(1-\rho)\, s_j &= \frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j} \\
\Leftrightarrow \quad s_j &= \frac{1}{1-\rho}\left[1-\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right]
\end{aligned}
$$

It is easy to verify that each firm's market share decreases in the total number of active firms. To see this, suppose that the number of firms increases from $n$ to $n+1$. The new entrant will have a market share

$$
s_{n+1} = \frac{1}{1-\rho}\left[1-\frac{n+1-(1-\rho)}{\sum_{k=1}^{n+1}\frac{1}{\pi_k}}\frac{1}{\pi_{n+1}}\right]
$$

which is non-negative provided that

$$
\pi_{n+1}\sum_{k=1}^{n+1}\frac{1}{\pi_k} > n+1-(1-\rho) \tag{21}
$$

and below one given that

$$
\pi_{n+1}\sum_{k=1}^{n+1}\frac{1}{\pi_k} < \frac{1}{\rho}\left[n+1-(1-\rho)\right] \tag{22}
$$

If we compare the market share of firm $j$ when there $n$ and $n+1$ firms in the market, we have

$$
\begin{aligned}
& s_j\,|n+1 < s_j\,|n \\
\Leftrightarrow \quad & \frac{1}{1-\rho}\left[1-\frac{n+1-(1-\rho)}{\sum_{k=1}^{n+1}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right] < \frac{1}{1-\rho}\left[1-\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right] \\
\Leftrightarrow \quad & \frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}} < \frac{n+1-(1-\rho)}{\sum_{k=1}^{n+1}\frac{1}{\pi_k}} \\
\Leftrightarrow \quad & [n-(1-\rho)]\left(\frac{1}{\pi_{n+1}}+\sum_{k=1}^{n}\frac{1}{\pi_k}\right) < [n+1-(1-\rho)]\sum_{k=1}^{n}\frac{1}{\pi_k} \\
\Leftrightarrow \quad & [n-(1-\rho)]\frac{1}{\pi_{n+1}} < \sum_{k=1}^{n}\frac{1}{\pi_k} \\
\Leftrightarrow \quad & \pi_{n+1}\sum_{k=1}^{n+1}\frac{1}{\pi_k} > n-(1-\rho)
\end{aligned}
$$

Note that the last condition is implied by (21).

### B.1.3 Profits

When there are $n$ active firms, type $\pi_j$ makes production profits

$$\begin{aligned}\Pi\left(\pi_j, n, \mathcal{F}, \Theta, Y\right) &= \left(p - \frac{\Theta}{\pi_j}\right) s_j\, y_j \\ &= \frac{1}{1-\rho}\left[1 - \frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right]^2 \left[\frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\right]^{\frac{\rho}{1-\rho}} \Theta^{-\frac{\rho}{1-\rho}} Y\end{aligned}$$

We now prove Proposition 1. We start by showing that $\Pi\left(\cdot\right)$ increases in $\pi_j$

$$\begin{aligned}
&2\left[1 - \frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right]^{-1}\left\{-\frac{-\left[n-(1-\rho)\right]\left[-\left(\frac{1}{\pi_j}\right)^2\right]}{\left(\sum\limits_{k=1}^{n}\frac{1}{\pi_k}\right)^2}\frac{1}{\pi_j} + \frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\left(\frac{1}{\pi_j}\right)^2\right\} + \\
&\frac{\rho}{1-\rho}\left[\frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\right]^{-1}\frac{-\left[n-(1-\rho)\right]\left[-\left(\frac{1}{\pi_j}\right)^2\right]}{\left(\sum\limits_{k=1}^{n}\frac{1}{\pi_k}\right)^2} > 0 \\
\Leftrightarrow\quad &2\left[1 - \frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right]^{-1}\left\{-\frac{1}{\left(\sum\limits_{k=1}^{n}\frac{1}{\pi_k}\right)^2}\frac{1}{\pi_j} + \frac{1}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\right\} + \\
&\frac{\rho}{1-\rho}\left[\frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\right]^{-1}\frac{1}{\left(\sum\limits_{k=1}^{n}\frac{1}{\pi_k}\right)^2} > 0 \\
\Leftrightarrow\quad &2\left[1 - \frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right]^{-1}\left\{-\frac{1}{\pi_j} + \sum_{k=1}^{n}\frac{1}{\pi_k}\right\} + \frac{\rho}{1-\rho}\left[\frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\right]^{-1}\frac{1}{\left(\sum\limits_{k=1}^{n}\frac{1}{\pi_k}\right)^2} > 0 \\
\Leftrightarrow\quad &2\left[1 - \frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right]^{-1}\left(\sum_{k\neq j}^{n}\frac{1}{\pi_k}\right) + \frac{\rho}{1-\rho}\left[\frac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\frac{1}{\pi_k}}\right]^{-1} > 0
\end{aligned}$$

To prove points (ii) and (iii) it suffices to show that $\Lambda(\cdot)$ is decreasing in $\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}$.[32] We have that

$$
\begin{aligned}
& 2\left[1-\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right]^{-1}\left(-\frac{1}{\pi_j}\right)+\frac{\rho}{1-\rho}\left[\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\right]^{-1}<0 \\
\Leftrightarrow\quad & \frac{\rho}{1-\rho}\left[1-\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\frac{1}{\pi_j}\right]<2\left[\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\right]\frac{1}{\pi_j} \\
\Leftrightarrow\quad & \frac{\rho}{1-\rho}<\left(2+\tfrac{\rho}{1-\rho}\right)\left[\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\right]\frac{1}{\pi_j} \\
\Leftrightarrow\quad & \rho<(2-\rho)\left[\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}\right]\frac{1}{\pi_j} \\
\Leftrightarrow\quad & \pi_j\sum_{k=1}^{n}\frac{1}{\pi_k}<\frac{2-\rho}{\rho}\left[n-(1-\rho)\right]
\end{aligned}
$$

The last condition is implied by (22).

[32] We know that $\frac{n-(1-\rho)}{\sum_{k=1}^{n}\frac{1}{\pi_k}}$ increases in $n$.

# C Model Derivation and Proofs: General Equilibrium

## C.1 Aggregate TFP

Suppose all industries are identical and that $\eta = 1$. Let $s_j$ denote the market share of firm $j$ and let $v_j$ denote its input share within the industry. Note that firm $j$ produces $\frac{s_j}{s_1}$ as much output as firm 1 and uses $\frac{s_j}{s_1}\frac{\pi_1}{\pi_j}$ as many inputs. We have that

$$
\begin{aligned}
& \sum_{k=1}^{n} v_k = 1 \\
\Leftrightarrow \quad & v_1 \frac{\pi_1}{s_1} \sum_{k=1}^{n} \frac{s_k}{\pi_k} = 1 \\
\Leftrightarrow \quad & v_1 = \frac{s_1}{\pi_1} \left( \sum_{k=1}^{n} \frac{s_k}{\pi_k} \right)^{-1}
\end{aligned}
$$

Note that we can write aggregate output as

$$
\begin{aligned}
& Y_t = \left[ \sum_{i=1}^{I} \left( \sum_{k=1}^{n} y_k \right)^{\rho} \right]^{\frac{1}{\rho}} \\
\Leftrightarrow \quad & Y_t = I^{\frac{1}{\rho}} \sum_{k=1}^{n} y_k \\
\Leftrightarrow \quad & Y_t = I^{\frac{1}{\rho}} \sum_{k=1}^{n} \pi_k \left( v_k I^{-1} L \right)^{1-\alpha} \left( v_k I^{-1} K \right)^{\alpha} \\
\Leftrightarrow \quad & Y_t = I^{-\frac{1-\rho}{\rho}} \left( \sum_{k=1}^{n} \pi_k v_k \right) L^{1-\alpha} K^{\alpha} \\
\Leftrightarrow \quad & Y_t = I^{-\frac{1-\rho}{\rho}} \left( \sum_{k=1}^{n} \pi_k \frac{s_k}{s_1} \frac{\pi_1}{\pi_k} v_1 \right) L^{1-\alpha} K^{\alpha} \\
\Leftrightarrow \quad & Y_t = I^{-\frac{1-\rho}{\rho}} \Bigg( \frac{\pi_1}{s_1} v_1 \underbrace{\sum_{k=1}^{n} s_k}_{=1} \Bigg) L^{1-\alpha} K^{\alpha} \\
\Leftrightarrow \quad & Y_t = I^{-\frac{1-\rho}{\rho}} \left( \sum_{k=1}^{n} \frac{s_k}{\pi_k} \right)^{-1} L^{1-\alpha} K^{\alpha} \\
\Leftrightarrow \quad & Y_t = \underbrace{I^{-\frac{1-\rho}{\rho}} \left\{ \sum_{k=1}^{n} \frac{1}{\pi_k} \frac{1}{1-\rho} \left[ 1 - \frac{n-(1-\rho)}{\sum_{h=1}^{n} \frac{1}{\pi_h}} \frac{1}{\pi_k} \right] \right\}^{-1}}_{:=\Phi(n)} L^{1-\alpha} K^{\alpha}
\end{aligned}
$$

In this particular case, in which all industries are identical and $\eta = 1$, aggregate productivity decreases in the number of active firms. This result simply reflects the fact that firms enter in reverse order of productivity. To prove it, note the each firm's market share decreases in the number of active firms in its industry. We hence have that $s_k < \tilde{s}_k$ , $\forall k$ and that $\pi_{n+1} < \pi_k$ , $\forall k \leq n$ . These facts imply that

$$
\sum_{k=1}^{n} s_k \frac{1}{\pi_k} < \left( \sum_{k=1}^{n} \tilde{s}_k \frac{1}{\pi_k} \right) + \tilde{s}_{n+1} \frac{1}{\pi_{n+1}}
$$

## C.2 Factor Costs and Factor Shares

### Aggregate Factor Costs

Suppose all industries are identical and that $\eta = 1$. When there are $n$ firms in every industry we have

$$\Theta = \frac{\sum_{k=1}^{n} \frac{1}{\pi_k}}{n - (1-\rho)}$$

We can show that

$$\begin{aligned}
& \frac{n+1-(1-\rho)}{\sum_{k=1}^{n} \frac{1}{\pi_k} + \frac{1}{\pi_{n+1}}} > \frac{n-(1-\rho)}{\sum_{k=1}^{n} \frac{1}{\pi_k}} \\
\Leftrightarrow \quad & \frac{n+1-(1-\rho)}{n-(1-\rho)} > \frac{\sum_{k=1}^{n} \frac{1}{\pi_k} + \frac{1}{\pi_{n+1}}}{\sum_{k=1}^{n} \frac{1}{\pi_k}} \\
\Leftrightarrow \quad & \frac{1}{n-(1-\rho)} > \frac{\frac{1}{\pi_{n+1}}}{\sum_{k=1}^{n} \frac{1}{\pi_k}} \\
\Leftrightarrow \quad & \pi_{n+1} \sum_{k=1}^{n} \frac{1}{\pi_k} > n - (1-\rho)
\end{aligned}$$

The last condition is implied by (21).

### Aggregate Factor Share: Proof of Proposition 2

Recall that the aggregate factor share is equal to

$$\Omega(\mathcal{F}, n) = \frac{\Theta(\mathcal{F}, n)}{\Phi(\mathcal{F}, n)}$$

As we have seen before, the aggregate factor cost index $\Theta(\mathcal{F}, n)$ is increasing in $n$, and aggregate TFP $\Phi(\mathcal{F}, n)$ is decreasing in $n$. To prove the second part of the proposition, note that the aggregate factor can be written as

$$\Omega(\mathcal{F}, n) = \left(\frac{n}{1-\rho} - 1\right) \left\{ \sum_{k=1}^{n} \frac{\frac{1}{\pi_k}}{\sum_{h=1}^{n} \frac{1}{\pi_h}} \left[ 1 - [n - (1-\rho)] \frac{\frac{1}{\pi_k}}{\sum_{h=1}^{n} \frac{1}{\pi_h}} \right] \right\}$$

Take some $j < n$ such that $\pi_j \geq \frac{1}{n} \sum_{h=1}^{n} \pi_h$. First note that we must have that

$$\frac{\frac{1}{\pi_j}}{\frac{1}{n} \sum_{h=1}^{n} \frac{1}{\pi_h}} \leq 1$$

To see this, note that

$$
\begin{aligned}
& \frac{1}{\pi_j} \leq \frac{1}{n}\sum_{h=1}^{n}\frac{1}{\pi_h} \\
\Leftrightarrow \quad & 1 \leq \frac{1}{n}\sum_{h=1}^{n}\frac{\pi_j}{\pi_h} \\
\Leftrightarrow \quad & 1 \leq \frac{1}{n}\sum_{h=1}^{n}\frac{\pi_j}{\frac{1}{n}\sum_{k=1}^{n}\pi_k}\frac{\frac{1}{n}\sum_{k=1}^{n}\pi_k}{\pi_h} \\
\Leftrightarrow \quad & 1 \leq \underbrace{\frac{\pi_j}{\frac{1}{n}\sum_{k=1}^{n}\pi_k}}_{\geq 1}\underbrace{\frac{1}{n}\sum_{h=1}^{n}\frac{\frac{1}{n}\sum_{k=1}^{n}\pi_k}{\pi_h}}_{\geq 1}
\end{aligned}
$$

Now suppose that $\pi_j$ increases to $\tilde{\pi}_j > \pi_j$. We want to show that

$$
\begin{aligned}
&\Omega\left(\tilde{\mathcal{F}},n\right)<\Omega\left(\mathcal{F},n\right)\\
\Leftrightarrow\quad&\sum_{k=1}^{n}\frac{\frac{1}{\tilde{\pi}_k}}{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}}\left[1-\left[n-(1-\rho)\right]\frac{\frac{1}{\tilde{\pi}_k}}{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}}\right]<\sum_{k=1}^{n}\frac{\frac{1}{\pi_k}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\left[1-\left[n-(1-\rho)\right]\frac{\frac{1}{\pi_k}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\\
\Leftrightarrow\quad&\sum_{k=1}^{n}\frac{1}{\tilde{\pi}_k}\left[\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\frac{1}{\tilde{\pi}_k}\right]<\frac{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\sum_{k=1}^{n}\frac{1}{\pi_k}\left[\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\frac{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\frac{1}{\pi_k}\right]\\
\Leftrightarrow\quad&\sum_{k=1}^{n}\frac{1}{\tilde{\pi}_k}\left[\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\frac{1}{\tilde{\pi}_k}\right]<\left[1+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\cdot\\
&\cdot\sum_{k=1}^{n}\frac{1}{\pi_k}\left\{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\left[1+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\frac{1}{\pi_k}\right\}\\
\Leftrightarrow\quad&\sum_{k=1}^{n}\frac{1}{\tilde{\pi}_k}\left[\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\frac{1}{\tilde{\pi}_k}\right]<\sum_{k=1}^{n}\frac{1}{\pi_k}\left[\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\frac{1}{\pi_k}\right]+\\
&+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\sum_{k=1}^{n}\frac{1}{\pi_k}\left\{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\left[1+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\frac{1}{\pi_k}\right\}-\sum_{k=1}^{n}\frac{1}{\pi_k}\left\{\left[n-(1-\rho)\right]\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\frac{1}{\pi_k}\right\}\\
\Leftrightarrow\quad&\frac{1}{\tilde{\pi}_j}\left[\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\frac{1}{\tilde{\pi}_j}\right]<\frac{1}{\pi_j}\left[\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\frac{1}{\pi_j}\right]+\\
&+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\sum_{k=1}^{n}\frac{1}{\pi_k}\left\{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\left[1+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\frac{1}{\pi_k}\right\}-\left[n-(1-\rho)\right]\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\sum_{k=1}^{n}\left(\frac{1}{\pi_k}\right)^2\\
\Leftrightarrow\quad&\left(\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}\right)\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\left[\left(\frac{1}{\tilde{\pi}_j}\right)^2-\left(\frac{1}{\pi_j}\right)^2\right]<\\
&<\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\sum_{k=1}^{n}\frac{1}{\pi_k}\left\{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\left[1+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\frac{1}{\pi_k}\right\}-\left[n-(1-\rho)\right]\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\sum_{k=1}^{n}\left(\frac{1}{\pi_k}\right)^2\\
\Leftrightarrow\quad&\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\left(\frac{1}{\pi_j}+\frac{1}{\tilde{\pi}_j}\right)>\\
&>\frac{1}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\sum_{k=1}^{n}\frac{1}{\pi_k}\left\{\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h}-\left[n-(1-\rho)\right]\left[1+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\frac{1}{\pi_k}\right\}-\left[n-(1-\rho)\right]\frac{\sum_{k=1}^{n}\left(\frac{1}{\pi_k}\right)^2}{\sum_{h=1}^{n}\frac{1}{\pi_h}}
\end{aligned}
$$

$$\Leftrightarrow \quad \sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h} - [n-(1-\rho)]\left(\frac{1}{\pi_j}+\frac{1}{\tilde{\pi}_j}\right) > \underbrace{\frac{\sum_{k=1}^{n}\frac{1}{\pi_k}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}}_{=1}\sum_{h=1}^{n}\frac{1}{\tilde{\pi}_h} - \frac{n-(1-\rho)}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\left[2+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\sum_{k=1}^{n}\left(\frac{1}{\pi_k}\right)^2$$

$$\Leftrightarrow \quad -[n-(1-\rho)]\left(\frac{1}{\pi_j}+\frac{1}{\tilde{\pi}_j}\right) > -\frac{n-(1-\rho)}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\left[2+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\sum_{k=1}^{n}\left(\frac{1}{\pi_k}\right)^2$$

$$\Leftrightarrow \quad \frac{1}{\pi_j}+\frac{1}{\tilde{\pi}_j} < \frac{1}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\left[2+\frac{\frac{1}{\tilde{\pi}_j}-\frac{1}{\pi_j}}{\sum_{h=1}^{n}\frac{1}{\pi_h}}\right]\sum_{k=1}^{n}\left(\frac{1}{\pi_k}\right)^2$$

$$\Leftrightarrow \quad 1+\frac{\pi_j}{\tilde{\pi}_j} < \frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}}\left(2+\frac{\frac{\pi_j}{\tilde{\pi}_j}-1}{\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}}\right)$$

$$\Leftrightarrow \quad \tilde{\pi}_j+\pi_j < 2\tilde{\pi}_j\frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}} - (\tilde{\pi}_j-\pi_j)\frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2}$$

$$\Leftrightarrow \quad \pi_j\left[1-\frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2}\right] < \tilde{\pi}_j\left[2\frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}} - \frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2} - 1\right]$$

$$\Leftrightarrow \quad \frac{\pi_j}{\tilde{\pi}_j} < \frac{2\frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}} - \frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2} - 1}{1-\frac{\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2}{\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2}}$$

It only suffices to show that the right hand side of the above inequality is greater than one

$$\begin{aligned}
& 2\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right) - \sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2 - \left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2 > \left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2 - \sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2 \\
\Leftrightarrow \quad & 2\sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right) - \left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2 > \left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2 \\
\Leftrightarrow \quad & \sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right) > \left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)^2 \\
\Leftrightarrow \quad & \sum_{k=1}^{n}\left(\frac{\pi_j}{\pi_k}\right)^2\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right) > \left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right)\left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right) \\
\Leftrightarrow \quad & \sum_{h=1}^{n}\left(\frac{\pi_j}{\pi_h}\right)^2 > \sum_{h=1}^{n}\frac{\pi_j}{\pi_h}
\end{aligned}$$

It is easy to prove that the last inequality is satisfied provided that $\pi_j \geq \frac{1}{n}\sum_{k=1}^{n}\pi_k$. Note that

$$
\begin{aligned}
& \sum_{h=1}^{n} \frac{\frac{1}{n}\sum_{k=1}^{n}\pi_k}{\pi_h}\frac{\pi_j}{\pi_h} > \sum_{h=1}^{n}\frac{\pi_j}{\pi_h} \\
\Leftrightarrow \quad & \left(\sum_{h=1}^{n}\frac{\pi_j}{\pi_h}\right) + \left(\sum_{h=1}^{n}\frac{\frac{1}{n}\sum_{k\neq h}^{n}\pi_k}{\pi_h}\frac{\pi_j}{\pi_h}\right) > \sum_{h=1}^{n}\frac{\pi_j}{\pi_h}
\end{aligned}
$$

### C.3 Asymmetric Equilibrium

Suppose that

$$\overline{K}(\mathcal{F},n) < K < \underline{K}(\mathcal{F},n+1)$$

In such a case, there will be an asymmetric equilibrium at time $t+1$: some industries will contain $n$ firms, whereas some industries will contain $n+1$ firms. The fraction of industries with $n+1$ will be pinned down by a zero profit condition for the marginal entrant in an industry with $n+1$ firms

$$\Lambda(\mathcal{F},\pi_{n+1},n+1)\,\Theta^{-\frac{\rho}{1-\rho}}Y = c_f$$

The equilibrium is characterized by 4 variables: the fraction of the industries with $n+1$ firms ($\eta$), aggregate output ($Y$), aggregate productivity ($\Phi$) and the aggregate cost index ($\Theta$). These 4 variables are pinned down by the following 4 equations

$$Y = \Phi\left[(1-\alpha)\Theta\right]^{\frac{1-\alpha}{\nu+\alpha}} K^{\alpha\frac{1+\nu}{\nu+\alpha}}$$

$$\Phi = \frac{\left\{(1-\eta)\left[\dfrac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\dfrac{1}{\pi_{1k}}}\right]^{\frac{\rho}{1-\rho}} + \eta\left[\dfrac{n+1-(1-\rho)}{\sum\limits_{k=1}^{n+1}\dfrac{1}{\pi_{2k}}}\right]^{\frac{\rho}{1-\rho}}\right\}^{\frac{1}{\rho}}}{(1-\eta)\left[\dfrac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\dfrac{1}{\pi_{1k}}}\right]^{\frac{1}{1-\rho}}\left(\sum\limits_{k=1}^{n}\dfrac{s_{1k}}{\pi_{1k}}\right) + \eta\left[\dfrac{n+1-(1-\rho)}{\sum\limits_{k=1}^{n+1}\dfrac{1}{\pi_{2k}}}\right]^{\frac{1}{1-\rho}}\left(\sum\limits_{k=1}^{n+1}\dfrac{s_{2k}}{\pi_{2k}}\right)}$$

$$\Theta = \left\{(1-\eta)\left[\dfrac{n-(1-\rho)}{\sum\limits_{k=1}^{n}\dfrac{1}{\pi_{k}}}\right]^{\frac{\rho}{1-\rho}} + \eta\left[\dfrac{n+1-(1-\rho)}{\sum\limits_{k=1}^{n+1}\dfrac{1}{\pi_{k}}}\right]^{\frac{\rho}{1-\rho}}\right\}^{\frac{1-\rho}{\rho}}$$

$$\Lambda(\mathcal{F},\pi_{n+1},n+1)\,\Theta^{-\frac{\rho}{1-\rho}}Y = c_f$$

The market share $s_{1k}$ refers to a certain firm $k$ in an industry with $n$ firms, whereas $s_{2k}$ refers to firm $k$ in an industry with $n+1$ firms. They are defined in Appendix B.1.2.

## C.4 Static Multiplicity

The following proposition provides the conditions for static multiplicity.

**Proposition 7.** *(Static Multiplicity) Suppose that an equilibrium with $n$ firms per industry is possible at time $t$. An equilibrium with $n+1$ firms is also possible provided that*

$$\frac{\Phi\left(\mathcal{F},n\right)}{\Phi\left(\mathcal{F},n+1\right)}<\left[\frac{\Theta\left(\mathcal{F},n\right)}{\Theta\left(\mathcal{F},n+1\right)}\right]^{\frac{\rho}{1-\rho}-\frac{1-\alpha}{\nu+\alpha}}$$

*Proof.* Suppose that

$$\underline{K}\left(\mathcal{F},n\right)\leq K_t\leq\overline{K}\left(\mathcal{F},n\right)$$

so that a symmetric equilibrium with $n$ firms in every industry is possible. A symmetric equilibrium with $n+1$ firms will also be possible provided that

$$\begin{aligned}
& \underline{K}\left(\mathcal{F},n+1\right)<\overline{K}\left(\mathcal{F},n\right)\\
\Leftrightarrow\quad & \frac{c_f}{\Lambda\left(n+1,\pi_{n+1}\right)}\left[\Phi\left(\mathcal{F},n+1\right)\right]^{-1}\left[\Theta\left(\mathcal{F},n+1\right)\right]^{\frac{\rho}{1-\rho}-\frac{1-\alpha}{\nu+\alpha}}<\frac{c_f}{\Lambda\left(n+1,\pi_{n+1}\right)}\left[\Phi\left(\mathcal{F},n\right)\right]^{-1}\left[\Theta\left(\mathcal{F},n\right)\right]^{\frac{\rho}{1-\rho}-\frac{1-\alpha}{\nu+\alpha}}\\
\Leftrightarrow\quad & \frac{\Phi\left(\mathcal{F},n\right)}{\Phi\left(\mathcal{F},n+1\right)}<\left[\frac{\Theta\left(\mathcal{F},n\right)}{\Theta\left(\mathcal{F},n+1\right)}\right]^{\frac{\rho}{1-\rho}-\frac{1-\alpha}{\nu+\alpha}}
\end{aligned}$$

■

**Corollary 8.** *(Static Multiplicity with No Productivity Differences) When all firms are equally productive there can be equilibrium multiplicity if and only if*

$$\frac{\rho}{1-\rho}<\frac{1-\alpha}{\nu+\alpha}$$

*Proof.* when there are no productivity differences, the condition becomes

$$\begin{aligned}
& \left[\frac{\Theta\left(\mathcal{F},n\right)}{\Theta\left(\mathcal{F},n+1\right)}\right]^{\frac{\rho}{1-\rho}-\frac{1-\alpha}{\nu+\alpha}}>1\\
\Leftrightarrow\quad & \frac{\rho}{1-\rho}-\frac{1-\alpha}{\nu+\alpha}<0\\
\Leftrightarrow\quad & \frac{\rho}{1-\rho}<\frac{1-\alpha}{\nu+\alpha}\\
\Leftrightarrow\quad & \rho<\frac{1-\alpha}{\nu+\alpha}\left(1-\rho\right)\\
\Leftrightarrow\quad & \frac{\rho}{1-\rho}<\frac{1-\alpha}{\nu+\alpha}
\end{aligned}$$

■

### C.5 Steady-State

In a steady-state with a constant productivity distribution $\mathbb{Z}_\tau$ and a set of active firms $\{n_i\}_{i=1}^I$, the aggregate savings rate is equal to

$$s = \frac{\beta\delta}{1-(1-\delta)\,\beta}\,\alpha\,\Omega\left(\mathbb{Z}_{\tau t}, \{n_i\}_{i=1}^I\right)$$

Recall that we also have

$$Y = \Phi\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\left[(1-\alpha)\,\Theta\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\frac{1-\alpha}{\nu+\alpha}} K^{\alpha\frac{1+\nu}{\nu+\alpha}}$$

We can combine the above two equations with

$$\delta K = sY$$

to write

$$\begin{aligned}
& Y = \Phi\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\left[(1-\alpha)\,\Theta\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\frac{1-\alpha}{\nu+\alpha}}\left[\frac{\beta}{1-(1-\delta)\,\beta}\,\alpha\,\Omega\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)Y\right]^{\alpha\frac{1+\nu}{\nu+\alpha}} \\
\Leftrightarrow\ & Y^{\frac{\nu-\alpha\nu}{\nu+\alpha}} = \Phi\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\left[(1-\alpha)\,\Theta\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\frac{1-\alpha}{\nu+\alpha}}\left[\frac{\beta}{1-(1-\delta)\,\beta}\,\alpha\,\Omega\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\alpha\frac{1+\nu}{\nu+\alpha}} \\
\Leftrightarrow\ & Y = \left[\Phi\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\frac{\nu+\alpha}{\nu-\alpha\nu}}\left[(1-\alpha)\,\Theta\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\frac{1-\alpha}{\nu-\alpha\nu}}\left[\frac{\beta}{1-(1-\delta)\,\beta}\,\alpha\,\Omega\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\alpha\frac{1+\nu}{\nu-\alpha\nu}} \\
\Leftrightarrow\ & Y = \left[\Phi\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\frac{\nu+\alpha}{\nu-\alpha\nu}-\alpha\frac{1+\nu}{\nu-\alpha\nu}}\left[\Theta\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\frac{1-\alpha}{\nu-\alpha\nu}+\alpha\frac{1+\nu}{\nu-\alpha\nu}}(1-\alpha)^{\frac{1-\alpha}{\nu-\alpha\nu}}\left[\frac{\beta\alpha}{1-(1-\delta)\,\beta}\right]^{\alpha\frac{1+\nu}{\nu-\alpha\nu}} \\
\Leftrightarrow\ & Y = \Phi\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\left[\Theta\left(\mathbb{Z}_\tau, \{n_i\}_{i=1}^I\right)\right]^{\frac{1+\alpha\nu}{\nu(1-\alpha)}}(1-\alpha)^{\frac{1-\alpha}{\nu-\alpha\nu}}\left[\frac{\beta\alpha}{1-(1-\delta)\,\beta}\right]^{\alpha\frac{1+\nu}{\nu-\alpha\nu}}
\end{aligned}$$

**Example with Unique Steady-State**

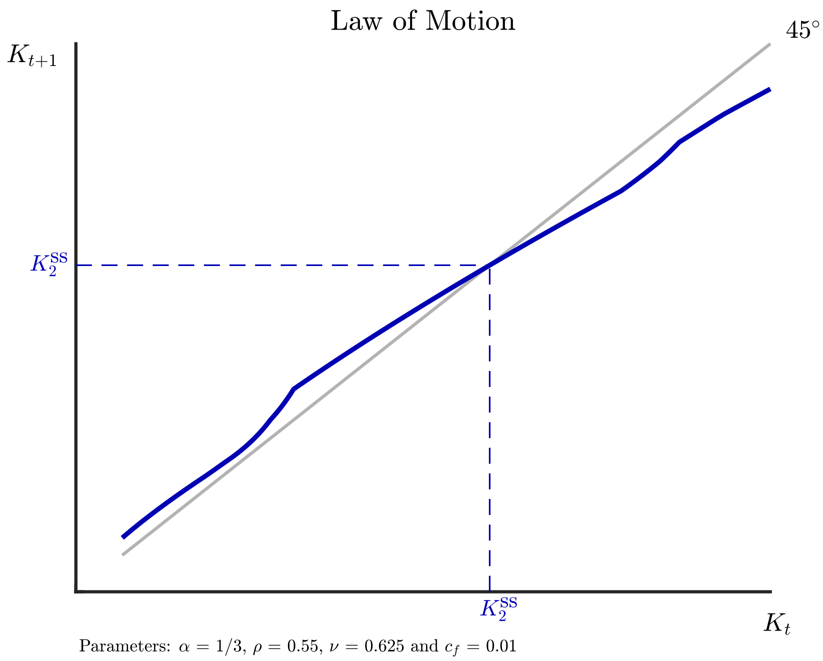



Figure 24: Economy with Unique Steady-State

**Example with Three Steady-State**

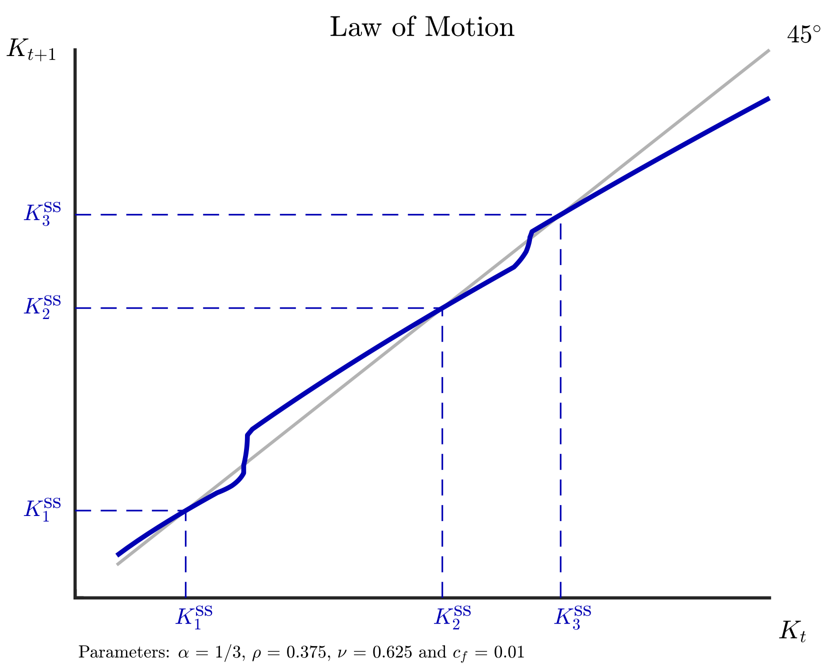



Figure 25: Economy with Three Steady-States

## C.6 Steady-State Multiplicity and Basins of Attraction

A symmetric steady-state with $n$ firms per industry is characterized by

$$Y^*(\mathcal{F},n)=\Phi(\mathcal{F},n)\left[\Theta(\mathcal{F},n)\right]^{\frac{1+\alpha\nu}{\nu(1-\alpha)}}(1-\alpha)^{\frac{1-\alpha}{\nu-\alpha\nu}}\left[\frac{\beta\alpha}{1-(1-\delta)\beta}\right]^{\alpha\frac{1+\nu}{\nu-\alpha\nu}}$$

and the minimum level of output consistent with $n$ firms per industry is given by

$$\underline{Y}(\mathcal{F},n)=c_f(1-\rho)\left[1-\frac{\Theta(\mathcal{F},n)}{\pi_n}\right]^{-2}$$

We therefore have that

$$\frac{Y^*(\mathcal{F},n)}{\underline{Y}(\mathcal{F},n)}=\propto\ \Phi\ \left[\Theta(\mathcal{F},n)\right]^{\frac{1+\alpha\nu}{\nu(1-\alpha)}}\ \left[1-\frac{\Theta(\mathcal{F},n)}{\pi_n}\right]^2$$

For any $\pi_k>\pi_n$, we have that

$$\begin{aligned}\frac{\partial\left(\frac{Y^*}{\underline{Y}}\right)}{\partial\pi_k}<0= &\ \frac{\partial\Phi}{\partial\pi_k}+\\ &\Phi\left\{\frac{1+\alpha\nu}{\nu(1-\alpha)}\Theta^{-1}\frac{\partial\Theta}{\partial\pi_k}+2\left(-\frac{1}{\pi_j}\frac{\partial\Theta}{\partial\pi_k}\right)\left[1-\frac{\Theta}{\pi_n}\right]^{-1}\right\}<0\end{aligned}$$

In the special case in which $\pi_k=1\ \forall k$

$$\begin{aligned}\frac{\partial\Phi(\mathcal{F},n)}{\partial\pi_k} &= \frac{1}{1-\rho}\left[1-\frac{2n+1}{n}\Theta(\mathcal{F},n)\right]\\ \frac{\partial\Theta(\mathcal{F},n)}{\partial\pi_k} &= \frac{\Theta(\mathcal{F},n)}{n}\end{aligned}$$

The above condition hence becomes

$$\begin{aligned}&\frac{1}{1-\rho}\left(1-\frac{2n+1}{n}\Theta\right)\left[\frac{1+\alpha\nu}{\nu(1-\alpha)}\frac{1}{n}-2\frac{\Theta}{n}(1-\Theta)^{-1}\right]<0\\ \Leftrightarrow\quad&\frac{1}{1-\rho}\left[n-(2n+1)\Theta\right]+\left[\frac{1+\alpha\nu}{\nu(1-\alpha)}-2\frac{\Theta}{1-\Theta}\right]<0\end{aligned}$$

Recall that $\Theta = \dfrac{n-(1-\rho)}{n}$ when $\pi_k = 1\ \forall k$, we can write

$$
\begin{aligned}
& \frac{1}{1-\rho}\left[n-(2n+1)\frac{n-(1-\rho)}{n}\right]+\left[\frac{1+\alpha\nu}{\nu(1-\alpha)}-2\frac{n-(1-\rho)}{1-\rho}\right]<0 \\
\Leftrightarrow\quad & \frac{1}{1-\rho}\left\{n-\left(2+\frac{1}{n}\right)[n-(1-\rho)]-2[n-(1-\rho)]\right\}+\frac{1+\alpha\nu}{\nu(1-\alpha)}<0 \\
\Leftrightarrow\quad & \frac{1}{1-\rho}\left\{n-\left(2+\frac{1}{n}\right)[n-(1-\rho)]-2[n-(1-\rho)]\right\}+\frac{1+\alpha\nu}{\nu(1-\alpha)}<0 \\
\Leftrightarrow\quad & \frac{1}{1-\rho}\left\{n-\left(4+\frac{1}{n}\right)[n-(1-\rho)]\right\}+\frac{1+\alpha\nu}{\nu(1-\alpha)}<0 \\
\Leftrightarrow\quad & \frac{\frac{1}{\nu}+\alpha}{1-\alpha}<\frac{\left(4+\frac{1}{n}\right)[n-(1-\rho)]-n}{1-\rho}
\end{aligned}
$$

**Counterexample**

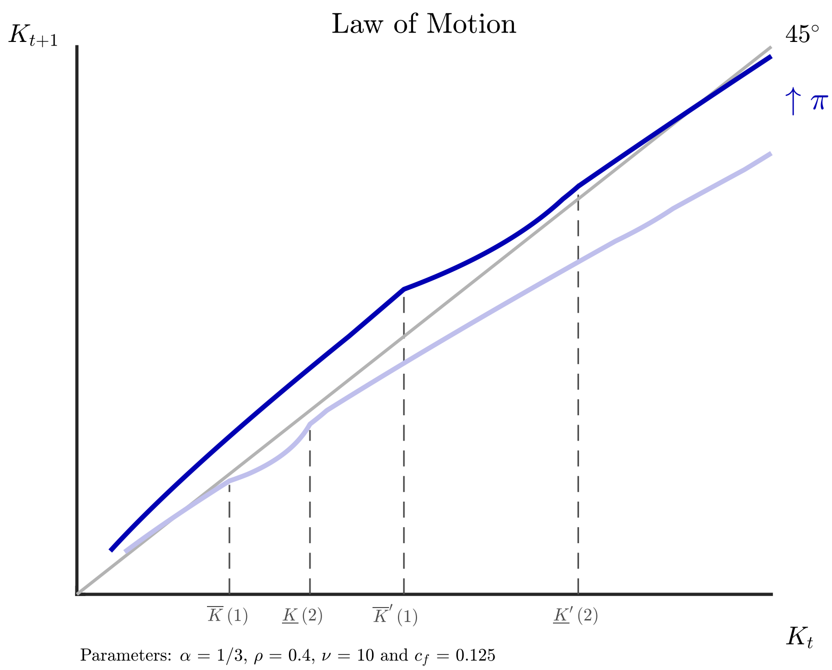



Figure 26: In this economy, an increase in the productivity of the leader, makes possible a steady-state with a symmetric duopoly.

# D The Quantitative Model

## D.1 Calibration

### D.1.1 Steady-State

We perform two different calibrations of our model – to match the average level of markups and its dispersion in 1985 and in 2007. The two parameters we need to calibrate are the Pareto tail $\lambda$ and the fixed production cost $c$.

We specify a grid of possible candidates for $\lambda$ and for $c$. We also specify a grid with values for the aggregate capital stock $K$. we compute the aggregate equilibrium for each parameter combination $(\lambda, c)$ and for each value $K$.[33] We start by assuming that all firms are active, so that there are $N$ firms in each of the $I$ industries. We compute the aggregate equilibrium using equations (14) and (15). We then compute the profits net of the fixed cost that each firm makes

$$\left(p_{ijt} - \frac{\Theta_t}{\tau_{ijt}}\right) y_{ijt} - c_f$$

and identify the firm with the largest negative value. We exclude this firm and recompute the aggregate equilibrium. We repeat this iterative procedure until all firms have non-negative profits (net of the fixed production cost).

For most parameter combinations, our model admits a unique equilibrium. However, when equilibrium multiplicity arises, this algorithm allows us to consistently select the equilibrium that features the largest number of firms.

For each pair $(\lambda, c)$, we then have the general equilibrium computed for all possible capital values. The steady-state(s) of our economy correspond to the value(s) of $K$ for with the rental rate $r_t$ is equal to $\frac{1}{\beta} - (1 - \delta)$.

We obtain the (sales-weighted) average level of markups and its standard-deviation for the largest steady-state, given our interpretation that the US economy was in the highest steady-state in both 1985 and 2007.

[33] Aggregate TFP $e^{z_t}$ is assumed to be constant and equal to one.

### D.1.2 Data Definitions

For the sales weighted-average markup, we use the series computed by De Loecker and Eeckhout (2017). The authors calculate price-cost markups for the universe of public firms, using data from COMPUSTAT. The markup of a firm $j$ in a 2-digit NAICS sector $s$ at time $t$ is calculated as

$$\mu_{sjt} = \xi_{st} \cdot \frac{\text{sale}_{sjt}}{\text{cogs}_{sjt}}$$

where $\xi_{st}$ is the elasticity of sales to the total variable input bundle, $\text{sale}_{sjt}$ is sales and $\text{cogs}_{sjt}$ is the cost of the goods sold, which measures total variable costs.

To measure markup dispersion, we compute the standard deviation of markups within 2-digit NAICS sectors. Treating $\xi_{st}$ as constant within a sector $s$ and time $t$, we can measure markup dispersion within this sector as

$$\text{sd}_s\left[\log\left(\mu_{sjt}\right)\right] = \text{sd}_s\left[\log\left(\frac{\text{sale}_{sjt}}{\text{cogs}_{sjt}}\right)\right]$$

We calculate this measure for all 23 sectors (2-digit NAICS). We then compute an average across all such sectors, weighted by the sector sales. Figure 27 shows the evolution of this measure.

In our model, we compute the standard deviation of (log) markups across all firms in the economy, i.e. we do not compute it industry by industry. We think of an industry in our model as a market at the possible level of disaggregation (e.g. 10-digit NAICS). We cannot however observe data at such a fine level of disaggregation – first because most data sets only provide industry information at the 6-digit, second because many large firms are multi-product an operate in different markets. We hence think of our final good $Y_t$ as one big-sector.

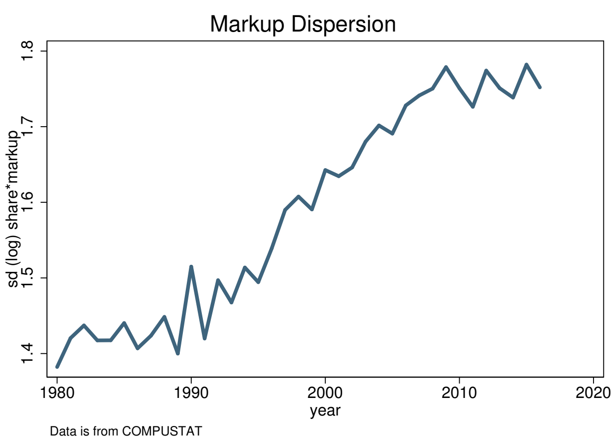



Figure 27: Markup Dispersion

### D.2 Solution Algorithm for the Dynamic Optimization Problem

We now explain the algorithm we use for the dynamic optimization problem of the representative household. We take the calibrated parameters $(\lambda, c)$ and form a grid for aggregate capital with $n_K = 30$ points. This grid is centered around the highest steady-state $K_H^{\mathrm{ss}}$, with a lower-bound $0.75 \times K_H^{\mathrm{ss}}$ and upper bound $1.25 \times K_H^{\mathrm{ss}}$.

We also form a grid for (log) aggregate TFP, $z$. We use Tauchen's algorithm with $n_z = 9$ points, autocorrelation parameter $\phi_Z$ and standard deviation for the innovations $\sigma_\varepsilon$ (the last two parameters are calibrated, as explained in the main text).We compute the aggregate equilibrium for each value of $K$ and $z$.

We next iterate on the policy function of the representative household. Recall that the representative household must solve (1), taking all aggregate variables (rental rate, wage rate and profits) as given. Specifically, he does not internalize the impact that his choice of $K$ can have on aggregate variables. We then start with a guess for the policy function $C_t = f_C(K_t, z_Z)$. We also start with a guess for the law of motion $K_{t+1} = f_K(K_t, z_t)$. The representative household takes this law of motion as given (so that he forms expectations about the evolution of aggregate variables that are independent of his choices of capital). We iterate simultaneously on the policy function and on the law of motion.

## D.3 Steady-State Multiplicity

Figure 28 shows the long-run demand and supply of capital under the 1985 and the 2007 parameters. It illustrates how multiple steady-state can arise. In our model the steady-state interest rate is pinned down by the discount factor, hence the flat supply. The demand for capital depends on the competition of the economy. In the 1985 calibration the demand for capital slopes down for low levels of capital, when the economy is large enough, entry occurs. As the degree of competition increases so do factor shares and output. This tilts the demand for capital, making it upward sloped. When a large enough number of new firms have entered the market, demand turns downward sloping again. These dynamics generate the existence of two steady states: one featuring few firms with high market power, low demand for capital and low output and a second one with more firms, more competition and higher output.

Similar dynamics occur in the 2007 calibration. However, due to the higher heterogeneity in firm level productivity, the demand slopes up to a lesser degree. It is clear from the graph that for even higher level of productivity heterogeneity, the high output steady state may disappear.

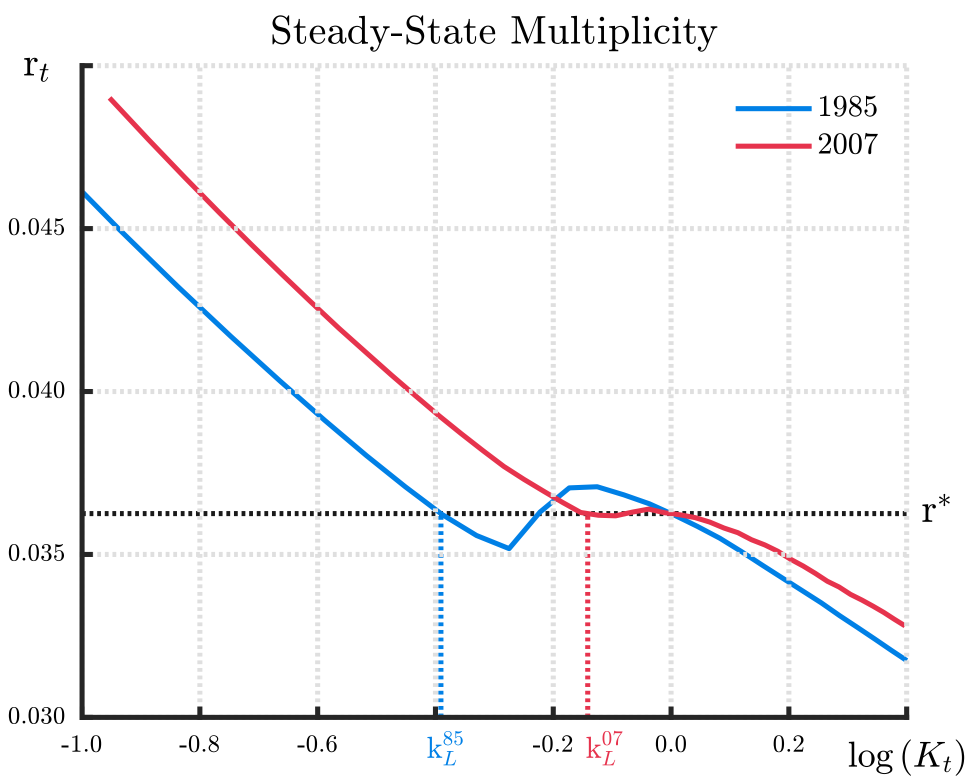



Figure 28: Steady-State Multiplicity

### D.4 Alternative Measures of Aggregate Markups

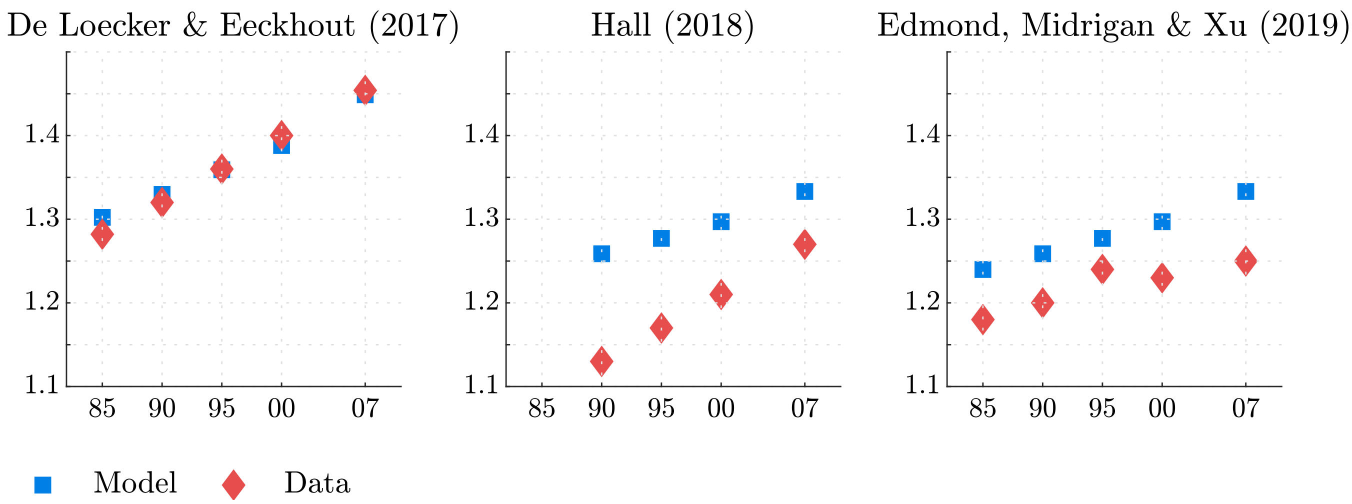



Figure 29: Aggregate markup: alternative measures

## D.5 Model with Fixed Market Structure: IRF and Business Cycle Moments

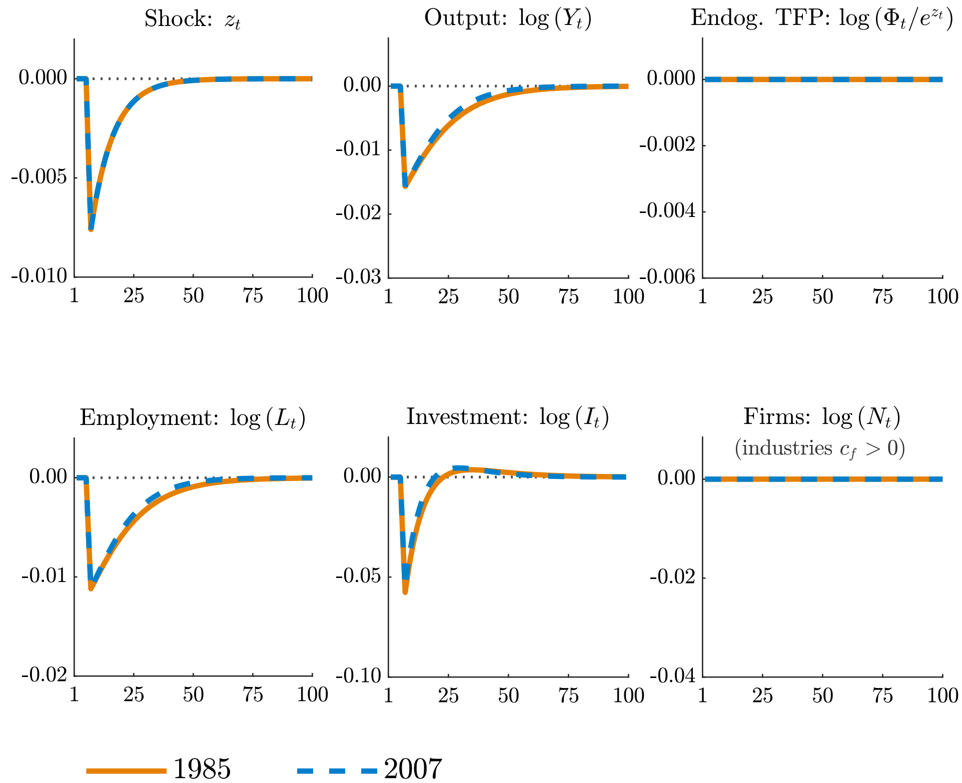



Figure 30: Impulse Responses: Small Shock

% Deviation of $\log(Y_t)$ from High Steady-State

| Calibration | $T = 5$ | $T = 10$ | $T = 20$ | $T = 100$ |
|---|---|---|---|---|
| 1985 | -0.014 | -0.011 | -0.006 | 0.000 |
| 2007 | -0.014 | -0.010 | -0.005 | 0.000 |

Table 13: Impulse Response Functions of $\log(Y_t)$. This table shows the values $\log(Y_t)$ in deviation from its steady-state, after a negative shock to $\varepsilon_t$. This shock is equal to $\varepsilon_t = -\sigma_\varepsilon$ and lasts for two quarters.

| | Output | Consumption | Investment | Hours |
|---|---|---|---|---|
| | Correlation with Output | | | |
| Data: 1985-2018 | 1.00 | 0.99 | 0.87 | 0.89 |
| Model: 1985 calibration | 1.00 | 0.95 | 0.74 | 1.00 |
| Model: 2007 calibration | 1.00 | 0.94 | 0.70 | 1.00 |
| | Standard Deviation Relative Output | | | |
| Data: 1985-2018 | 1.00 | 1.00 | 2.53 | 1.07 |
| Model: 1985 calibration | 1.00 | 0.93 | 2.18 | 0.71 |
| Model: 2007 calibration | 1.00 | 0.94 | 2.39 | 0.71 |

Table 14: Business Cycle Moments: Fixed Market Structure

## D.6 The Great Recession

### Welfare

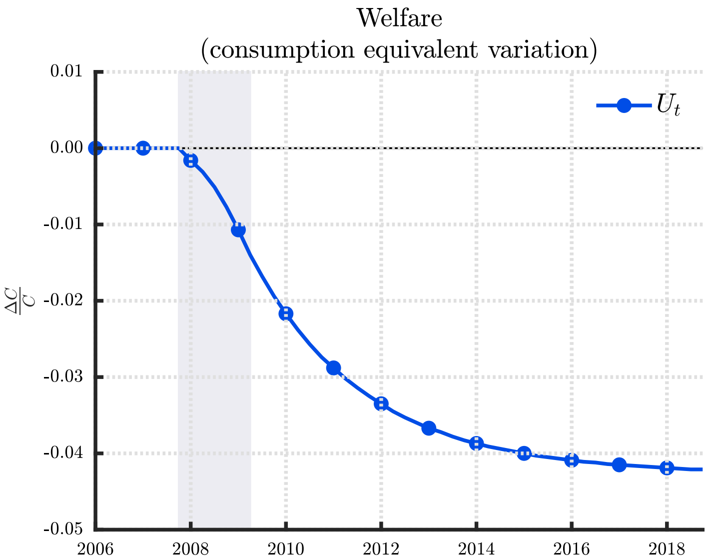
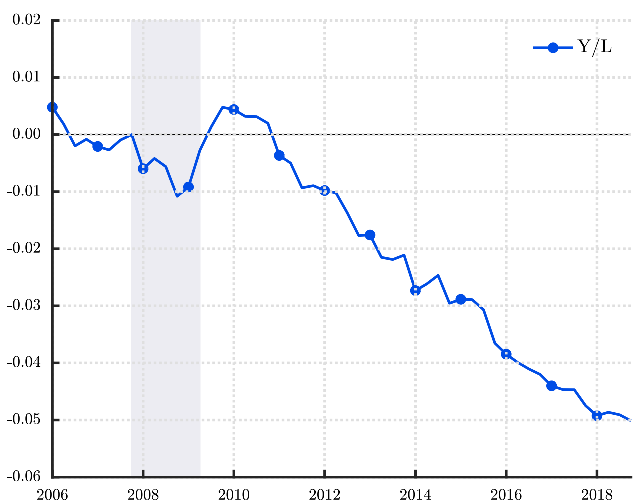
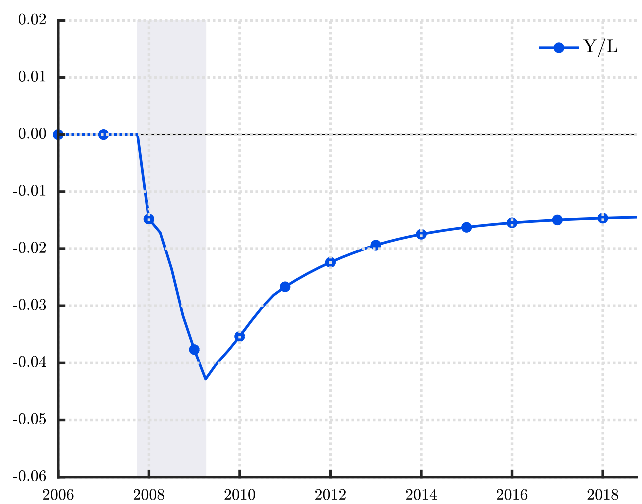



Figure 31: The great recession and its aftermath: welfare

### Output per Hour




(a) Data




(b) 2007 Model

Figure 32: The great recession and its aftermath: real output per hour

### D.7 Robustness: Calibration of the share of concentrated industries in 1985

Parameter Values

| Description | Parameter | Value | Source/Target |
|---|---|---|---|
| Between-Industry ES | $\sigma_I$ | 1.5 | Mongey (2019) |
| Within-Industry ES | $\sigma_G$ | 10 | Mongey (2019) |
| Elasticity of Labor Supply | $\nu$ | 0.4 | Jaimovich and Rebelo (2009) |
| Capital Elasticity | $\alpha$ | 1/3 | Standard value |
| Depreciation Rate | $\delta$ | $1 - 0.9^{1/4}$ | Standard value |
| Discount Factor | $\beta$ | $0.96^{1/4}$ | Standard value |
| Coefficient of Risk Aversion | $\gamma$ | 1 | log utility |
| Persistence of $z_t$ | $\rho_z$ | 0.90 | Autocorrelation of log output |
| Standard Deviation of $\varepsilon_t$ | $\sigma_\varepsilon$ | 0.004 | Standard deviation of log output |
| Number of Industries | $I$ | 5,000 | See text |
| Maximum Number of Firms per Industry | $N$ | 100 | See text |
| Fraction Competitive Industries 1985 | $f_{\text{comp},85}$ | 0.810 | Employment Share in Concentrated Industries |
| Pareto Tail 1985 | $\lambda_{85}$ | 7.35 | Markup Dispersion 1985 |
| Fixed Cost 1985 | $c_{85}$ | $4.73 \times 10^{-3}$ | Average Markup 1985 |
| Fraction Competitive Industries 2007 | $f_{\text{comp},07}$ | 0.785 | Employment Share in Concentrated Industries |
| Pareto Tail 2007 | $\lambda_{07}$ | 5.43 | Markup Dispersion 2007 |
| Fixed Cost 2007 | $c_{07}$ | $10.1 \times 10^{-3}$ | Average Markup 2007 |

Table 15: Targeted Moments and Model Counterparts

| | Markups: Average | | Markups: Std. Deviation | | Emp. Share Concent. Ind. | |
|---|---|---|---|---|---|---|
| | Data | Model | Data | Model | Data | Model |
| 1985 | 1.27 | 1.32 | 1.44 | 1.21 | - | 9.40% |
| 2007 | 1.46 | 1.45 | 1.74 | 1.69 | 7.62% | 9.48% |

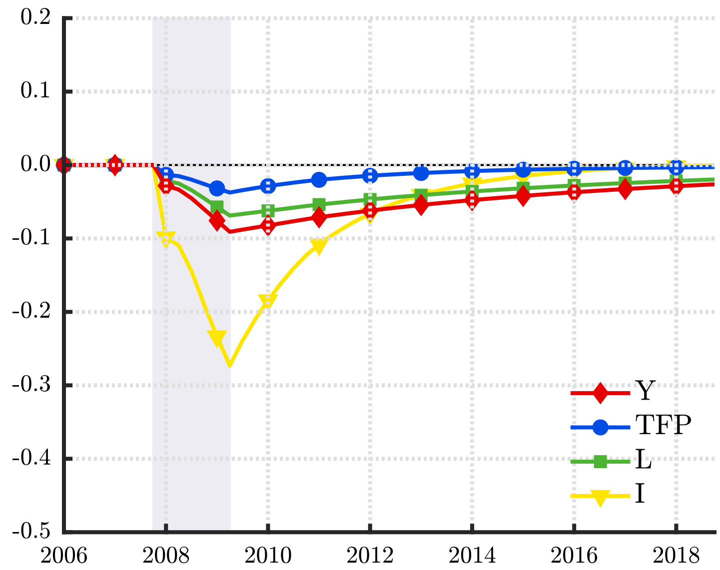



Figure 33: The *great recession* in the 1985 Model

This figures replicates Figure 16(a), under the new calibration strategy.

## D.8 Robustness: Different Elasticities of Substitution

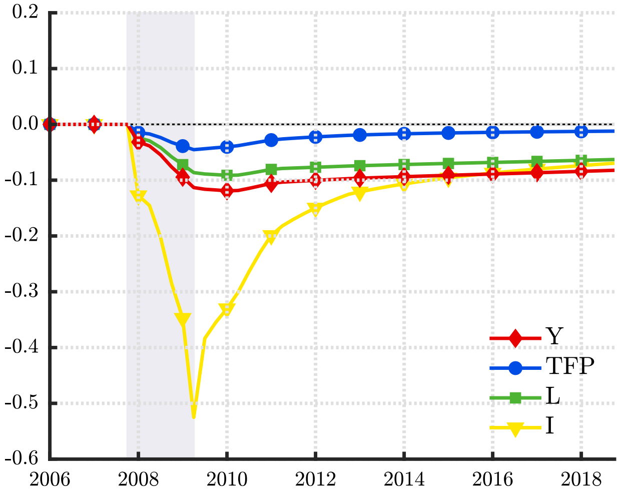
### D.8.1 $\sigma_I = 1.25$

Parameter Values

| Description | Parameter | Value | Source/Target |
|---|---|---|---|
| Between-Industry ES | $\sigma_I$ | 1.25 | Mongey (2019) |
| Within-Industry ES | $\sigma_G$ | 10 | Mongey (2019) |
| Elasticity of Labor Supply | $\nu$ | 0.4 | Jaimovich and Rebelo (2009) |
| Capital Elasticity | $\alpha$ | 1/3 | Standard value |
| Depreciation Rate | $\delta$ | $1 - 0.9^{1/4}$ | Standard value |
| Discount Factor | $\beta$ | $0.96^{1/4}$ | Standard value |
| Coefficient of Risk Aversion | $\gamma$ | 1 | log utility |
| Persistence of $z_t$ | $\rho_z$ | 0.90 | Autocorrelation of log output |
| Standard Deviation of $\varepsilon_t$ | $\sigma_\varepsilon$ | 0.004 | Standard deviation of log output |
| Number of Industries | $I$ | 5,000 | See text |
| Maximum Number of Firms per Industry | $N$ | 100 | See text |
| Fraction of Competitive Industries | $f_{\text{comp}}$ | 0.870 | Employment Share in Concentrated Industries |
| Pareto Tail 1985 | $\lambda_{85}$ | 7.40 | Markup Dispersion 1985 |
| Fixed Cost 1985 | $c_{85}$ | $5.25 \times 10^{-3}$ | Average Markup 1985 |
| Pareto Tail 2007 | $\lambda_{07}$ | 4.76 | Markup Dispersion 2007 |
| Fixed Cost 2007 | $c_{07}$ | $17.5 \times 10^{-3}$ | Average Markup 2007 |

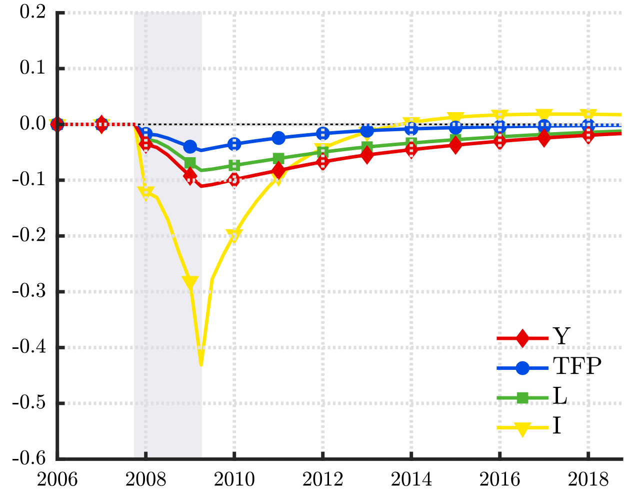
Table 16: Targeted Moments and Model Counterparts

| | Markups: Average | | Markups: Std. Deviation | | Emp. Share Concent. Ind. | |
|---|---|---|---|---|---|---|
| | Data | Model | Data | Model | Data | Model |
| 1985 | 1.27 | 1.33 | 1.44 | 1.54 | - | 6.23% |
| 2007 | 1.46 | 1.55 | 1.74 | 1.88 | 7.62% | 4.88% |




(a) 2007 Model

This figures replicates Figure 15(b), under the new calibration strategy




(b) 1985 Model

This figures replicates Figure 16(a), under the new calibration strategy

Figure 34: The *great recession* and its aftermath