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Resale and bundling in auctions☆

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ABSTRACT

The possibility of resale increases bidders' incentives to jointly reduce demand in multi-object auctions, because resale increases low-value bidders' willingness to pay and reduces high-value bidders' willingness to pay. Therefore, resale may reduce the seller's revenue. In a simple model with complete information, however, allowing resale and bundling the objects on sale are "complement strategies" for the seller (under reasonable conditions)—by bundling *and* allowing resale the seller earns a higher revenue than by selling the objects separately and/or forbidding resale. We also show why allowing resale may reduce efficiency.

1. Introduction

The 2002 "Cave Report," which was commissioned by the UK Government to review its spectrum policies, recommended to allow trading of spectrum licenses "as soon as possible."[1] Since 2003, the US Federal Communications Commission allows leasing and trading of the spectrum licenses it awards. The main rationale for this policy is that trading favors a more efficient allocation of the spectrum among its users. In this paper we analyze how bidders' strategies in multi-object auctions are affected by the possibility of trading in the aftermarket the objects acquired in the auction, and the effect of this possibility on the seller's revenue and on his incentive to bundle the units on sale.

When an auction is followed by a resale market, a losing bidder can still obtain the auction prize by purchasing it from a winning bidder. In single-object auctions, if bidders' relative valuations are known, the possibility of resale increases the seller's revenue because it gives a weak (i.e., low-value) bidder a chance to win the auction against a strong (i.e., high-value) bidder, and so it induces him to participate in the auction and bid more aggressively (Pagnozzi, 2007).[2]

In multi-object auctions, however, bidders also have an incentive to "reduce demand"—i.e., to bid for fewer objects than they actually want, in order to pay a lower price for the objects they do win. Demand reduction typically reduces the seller's revenue and results in an inefficient allocation of the objects on sale (Wilson, 1979).[3] But while demand reduction is generally profitable for a weak bidder–because he cannot win the auction if a higher-value competitor bids aggressively for all the objects on sale–a strong bidder may instead prefer to win more objects rather than reduce demand, even at the cost of paying a higher price for them. Therefore, when an auction is not followed by a resale market, demand reduction does not necessarily take place in equilibrium.

However, if the objects on sale are inefficiently allocated as a consequence of demand reduction, it is natural to expect bidders to trade among themselves, if they are allowed to do so.[4] Specifically, if trading in the aftermarket is allowed and a low-value bidder wins an object, he will resell it to a high-value bidder who reduced demand during the auction, with both bidders making a profit. So resale allows

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[1] The "Review of Radio Spectrum Management" was commissioned to Professor Martin Cave and also suggests that auctions should become the "default means" of assigning spectrum licenses.

[2] Garratt and Troger (2006), Gupta and Lebrun (1999), Hafalir and Krishna (2008), Haile (2000, 2003), Milgrom (1987), and Zheng (2002) analyze resale in single-object auctions with incomplete information.

[3] The literature on demand reduction also includes Ausubel and Cramton (1998), Back and Zender (1993), Engelbrecht-Wiggans and Kahn (1998), Kremer and Nyborg (2004), and Noussair (1995). Kagel and Levin (2001) and List and Lucking-Reiley (2000) provide experimental evidence of demand reduction; Weber (1997), Wolfram (1998) and Wolak (2003) show that demand reduction affected several FCC spectrum auctions, as well as the UK and the California electricity markets; Klemperer (2004) and Grimm et al. (2003) describe demand reduction in the 1999 GSM spectrum auction in Germany. Milgrom (2004) provides an excellent exposition of the literature on multi-unit auctions.

[4] Bikhchandani and Huang (1989) and Bose and Deltas (1999) show that bidders may also trade when some bidders cannot participate in the auction and can only acquire the objects on sale in the aftermarket.

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to correct an inefficient allocation due to demand reduction, thus affecting a strong bidder's incentive to reduce demand.

More precisely, when resale is allowed, a weak bidder is willing to pay a higher price in the auction (than when resale in not allowed) because he anticipates a positive surplus in the resale market if he *wins* an object in the auction; while a strong bidder is willing to pay a lower price in the auction (than when resale in not allowed) because she anticipates a positive surplus in the resale market if she *loses* an object in the auction (Haile, 2003). It follows that, when resale is allowed, for a strong bidder it is both more costly to outbid a weaker competitor, because the latter is willing to bid more aggressively, and less costly to lose an object in the auction, because the strong bidder can still acquire the object in the resale market. So the possibility of resale makes joint demand reduction–i.e., all bidders simultaneously reducing demand–more attractive for bidders.

Our first result is that, in a simple model of a uniform-price auction with complete information, while demand reduction is not always an equilibrium when resale is not allowed, when resale is allowed demand reduction is always an equilibrium, and it is the unique Pareto dominant equilibrium in undominated strategies (Proposition 1). So allowing resale may induce bidders to reduce demand, thus reducing the seller's revenue.[5]

Uniform-price auctions are often used to allocate multiple identical objects—for example, for on-line IPOs (including the one of Google in August 2004), electricity markets, markets for emission permits, and by the US Treasury Department to issue new securities. We analyze a uniform-price auction because this is the auction mechanisms in which the incentive to reduce demand arises more clearly (Ausubel and Cramton, 1998). But our qualitative result that resale may reduce the seller's revenue by making demand reduction more attractive for bidders also holds for any mechanism to allocate multiple objects in which bidders face a trade-off between winning more objects and paying lower prices. As explained above, the reason is that, by nearing bidders' actual valuations, resale makes it relatively more costly for a bidder to outbid his competitors and win more of the objects on sale.

How can the seller react to the risk of bidders reducing demand in an auction? Bundling the objects on sale appears a natural strategy for the seller, because bundling forces a bidder to win all objects, or none at all. So bundling makes it impossible for bidders to profitably reduce demand (Anton and Yao, 1992). Unfortunately, bundling actually reduces the seller's revenue whenever bidders do not reduce demand if the objects are sold separately, because it makes the auction price also depend on the lowest valuation of the losing bidder, rather than only on his highest one; and bundling may result in an inefficient allocation. Therefore, bundling has an ambiguous effect on the seller's revenue when resale is forbidden.[6] By contrast, we show that bundling always increases the seller's revenue when resale is allowed in our simple model, because bidders always reduce demand if resale is allowed and the objects are sold separately.

Moreover, we also show that bundling the objects on sale and allowing resale are "complement strategies" for the seller, provided bidders are not too asymmetric. Specifically, our second result is that bundling the objects and allowing resale always yields a higher seller's revenue than (*i*) bundling the objects and forbidding resale and (*ii*) selling the objects separately and allowing resale; while bundling the objects and allowing resale also yields a higher seller's revenue than selling them separately and forbidding resale if (*a*) the weak bidder has a sufficiently high valuation for at least one of the objects and/or (*b*) the strong bidder does not obtain too large a share of the gains from trade in the resale market (Proposition 2). The reason is that, when either condition (*a*) or condition (*b*) holds, allowing resale induces a weak bidder to bid much more aggressively (as in a single-object auction) and, at the same time, bundling prevents a strong bidder from reacting to this by reducing demand.

So our analysis suggests that a seller may prefer to bundle the objects on sale in order to increase his revenue, even if bundling may generate an inefficient initial allocation. And this is especially true when the seller cannot prevent resale. Moreover, if resale is allowed and there are no frictions to trading in the aftermarket, resale eventually allows bidders to correct an inefficient allocation achieved by the auction (e.g., because of bundling), and so ensures that the final allocation of the objects on sale is efficient.

However, if the resale market is not necessarily efficient–because, for example, bidders may be unable to trade after the auction even if they would like to–allowing resale may actually reduce efficiency, because it may result in an inefficient final allocation of the objects when an auction without resale would instead be efficient (Proposition 3). The reason is that resale may still induce a strong bidder to reduce demand, only then to find herself unable to acquire the object(s) owned by a weaker bidder in the resale market.[7] This casts some doubts on the argument that allowing resale always increases efficiency: even if the possibility of resale does increase efficiency *ex post*, it also affects bidders' strategies during the auction, and may thus reduce the efficiency of the auction's allocation compared to an auction without resale.

The analysis is focused on resale and bundling because these are simple and detail-free instruments that are often advocated in actual practice, and that the seller can implement without specific knowledge of bidders' valuations, or of the distribution of these valuations. Of course, the seller can also use other instruments to react to the risk of demand reduction. For example, a seller who is better informed about bidders' valuations may impose a reserve price. But sellers often lack the information and the commitment power to set optimal reserve prices.[8] Moreover, the seller may strategically reduce the quantity supplied to discourage demand reduction (see, e.g., McAdam, 2007). In many real auctions, however, sellers are committed for efficiency reasons to always selling all the objects available.[9]

Finally, we also analyze *unilateral* demand reduction by a strong bidder—i.e., the possibility that a bidder reduces demand alone, even though her opponent does not reduce demand. We show that resale may eliminate the incentive for a strong bidder to unilaterally reduce demand, because resale induces a weak bidder to bid relatively more aggressively on a marginal object, thus increasing the auction price when only the strong bidder reduces demand (Proposition 4).

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 introduces demand reduction without resale and Section 4 analyzes how demand reduction is affected by the possibility of resale. The effects of bundling on the seller's revenue are discussed in Section 5. Section 6 considers the possibility of an inefficient resale

[5] Pagnozzi (2008) analyzes how resale affects the seller's revenue when it also attracts speculators—i.e., bidders who have no use value for the objects on sale.

[6] Various papers analyze the effects of bundling the objects on sale in an auction without resale. Palfrey (1983) shows that, when bidders are privately informed about their valuations, the seller's bundling decision should depend on the number of bidders: bundling increases (reduces) the seller's revenue when the number of bidders is small (large). See also Chakraborty (1999). Anton and Yao (1992) show that auctioning the objects on sale separately may reduce the seller's revenue, because it may allow bidders to coordinate their bids and accommodate each other. Armstrong (2000) and Avery and Hendershott (2000) show that optimal auctions for multiple objects display some form of bundling. Jehiel et al. (2007) show that "mixed bundling" increases the seller's revenue in a class of dominant-strategy mechanisms.

[7] See also Hafalir and Krishna (forthcoming) who show that resale may reduce efficiency in a single-object first-price auction with asymmetric bidders.

[8] Before the UK 3G spectrum auction in 2000, for example, the government and industry analysts estimated the licenses to be worth £2–5 billion. The licenses sold for £22,5 billion instead (Klemperer, 2004). With such a poor estimate of bidders' valuations, setting a meaningful reserve price is extremely difficult. See also the discussion in Bulow and Klemperer (forthcoming).

[9] Hazlett and Munoz (2008) estimate the welfare cost of withholding spectrum in auctions for wireless licenses and conclude that "restricting the use of spectrum inputs is a relatively expensive way to raise public funds."

market, and Section 7 analyzes unilateral demand reduction. The last section concludes. All proofs are in Appendix A.

2. The model

Consider a sealed-bid uniform-price auction for two units of the same good with two bidders. This is the simplest model that allows us to analyze the effects of interest. Each bidder simultaneously submits two non-negative bids, one for each unit. In a uniform-price auction, the two highest bids are awarded the units, and the winner(s) pay for each unit won a price equal to the third-highest bid. The reserve price is normalized to zero.

We let v_i^k be bidder i's valuation for the k^{th} unit he acquires. Bidder S is a strong (i.e., high-value) female bidder who has the highest valuation for one of the units on sale, while bidder W is a weak (i.e., low-value) male bidder. Bidders have decreasing marginal valuations for the units on sale–i.e., $v_i^1 \geq v_i^2$–and, without loss of generality, v_S^1 is the highest valuation. So bidders valuations are:

	1st unit	2nd unit
Bidder S	v_S^1	v_S^2
Bidder W	v_W^1	v_W^2

Notice that bidder S does not necessarily have the highest valuation for both units (i.e., each bidder may have one of the two highest valuations).

We make the following assumption on valuations, which is often used in the literature on demand reduction (e.g., Wilson, 1979).

Assumption 1. *Valuations are common knowledge among bidders, but the seller does not know bidders' valuations.*

This assumption implies that the identity of the strong bidder and the ex-post efficient allocation of the units on sale is common knowledge among bidders. Therefore, in our model resale is not caused by uncertainty in valuations, or by a change in the order of bidders' valuations after the auction (as in Haile, 2000, 2003). Moreover, Assumption 1 allows us to abstract from issues of information transmission between the auction and the resale market, that are not the focus of this paper.

We assume that the seller can allow or forbid resale and bundle the units on sale or sell them separately. These simple strategies do not require information about bidders' valuations and may be adopted by the seller to increase his revenue.

If resale is allowed, bidders always trade in the aftermarket when there are gains from trade.

Assumption 2. *When bidders trade in the resale market, bidder W obtains a share α of the gains from trade and bidder S obtains a share $(1-\alpha)$ of the gains from trade, where $0<\alpha\leq 1$.*

Therefore, the outcome of bargaining between the two bidders in the resale market is given by the Nash bargaining solution with weights α and $(1-\alpha)$, where the disagreement point is represented by bidders' outside options.[10] The parameter α is a measure of bidders' bargaining power; $\alpha=1$ represents the case in which bidder W can make a take-it-or-leave-it offer to bidder S in the resale market. In order to make the resale market relevant, we assume that $\alpha>0$ (i.e., that the weak bidder always obtains at least some of the gains from trade).

When bidders trade a unit in the resale market, the outside option of the bidder who is trying to acquire the unit is normalized to zero, while the outside option of the bidder who won the unit in the auction is equal to his valuation. So a bidder's valuation is relevant in the resale market and affects his bargaining surplus. This implies that the gains from trading a unit in the resale market are equal to the difference between the two bidders' valuations, and that the resale price is located somewhere between the two bidders' valuations, with the exact position determined by the parameter α.

We define a bidder's "willingness to pay" for a unit in the auction as the highest auction price the bidder is happy to pay for the unit. When resale is not allowed, a bidder's willingness to pay is equal to his valuation. When resale is allowed, a bidder's willingness to pay for a unit is represented by the price at which he can buy or sell the unit in the resale market (e.g., Milgrom, 1987).

In the auction, a strategy for bidder i is a vector $\mathbf{b}_i=(b_i^1; b_i^2)$, where b_i^1 is bidder i's bid for the first unit and b_i^2 is his bid for the second unit, $i=S, W$. We assume that bidding is costless and we only consider weakly undominated strategies.[11] All the results are robust to the introduction of a small fixed cost that bidders have to pay to learn their valuations and enter the auction, or of a small cost that they have to pay to trade in the resale market. There is *demand reduction* if a bidder's bid for a unit is lower than his willingness to pay for the unit. The following assumption requires that the quantity demanded by a bidder is not increasing in the auction price:

Assumption 3. *The bids of bidder i for the two units must be such that $b_i^1 \geq b_i^2$.*

This is a standard and natural requirement in auctions of identical units (see, e.g., Krishna, 2002), since typically the value of an additional unit is decreasing with the number of units already obtained. Because the possibility of resale may induce a bidder to have a higher willingness to pay for the second unit than for the first unit (see Section 4), this requirement may limit a bidder's bid for the second unit below his willingness to pay when resale is allowed. However, the equilibria and the results of Sections 4 and 5 do not hinge on Assumption 3.

Bidders jointly reduce demand if, for the second unit on sale, they both bid a price which is lower than their willingness to pay (for the second unit) and lower than their opponent's willingness to pay for the first unit. As we are going to show, when bidders jointly reduce demand each bidder wins one of the units and the auction price is equal to the highest between the two bidders' bids for the second unit.[12]

To simplify the analysis, we assume that, if in equilibrium bidders jointly reduce demand, they both bid zero for the second unit—i.e., they coordinate on the equilibrium with joint demand reduction that gives them the highest profit, which is the equilibrium with an auction price equal to zero. This is a natural assumption because such an equilibrium Pareto dominates, from the bidders' point of view, any other equilibrium with joint demand reduction but a positive auction price.[13] The results of the analysis do not hinge on this assumption.

In Section 6, in order to analyze the effects of an inefficient resale market, we assume that with positive probability bidders are unable to trade after the auction. In the analysis of unilateral demand reduction in Section 7, we further assume that there is an arbitrarily small cost that bidders have to pay to trade in the resale market.

[10] This can be interpreted as the limit, as the length of the bargaining periods goes to zero, of a strategic model of alternating offers where players face a small exogenous risk of breakdown of negotiations, that induces them to take their outside options (Binmore et al., 1986; Sutton, 1986). A bidder's "outside option" is the bidders' utility in the event that the bargaining process does break down.

[11] When there is no cost of bidding, bidders may make positive bids even if they know they will eventually lose the auction. Hence, for example, without resale each bidder follows his weakly dominant strategy of bidding his valuation for the first unit on sale, even when he knows that he will not win any unit (see Section 3). Moreover, when the units are bundled, each bidder always follows his weakly dominant strategy of bidding the sum of his valuations for the two units, as in a single-object second-price auction (see Section 5).

[12] Bidders coordinating their behavior to reduce demand in concert is often described as tacit collusion. *Unilateral* demand reduction, in which a bidder reduces demand to one unit even if the other bidder does not, is discussed in Section 7.

[13] An equilibrium Pareto dominates another equilibrium from the bidders' point of view if in the first equilibrium at least one bidder is strictly better off and no bidder is worse off than in the second equilibrium.

3. Equilibria without resale

Assume that the seller does not allow resale after the auction. In this case, a bidder's willingness to pay is equal to his valuation for a unit. It is well known that, in a uniform-price auction, it is a weakly dominant strategy for each bidder to bid his valuation for the first unit (see, e.g., Milgrom, 2004); hence bidders never reduce demand for the first unit.[14] Moreover, bidding more than one's willingness to pay for any unit is a weakly dominated strategy; hence $b_i^1 = v_i^1$ and $b_i^2 \le v_i^2$, $i = S,W$.[15]

But bidders may find it profitable to reduce demand and bid less than their willingness to pay for the second unit, in order to pay a lower price for the first unit and so obtain a higher profit (Wilson, 1979; Ausubel and Cramton, 1998). The logic is the same as the standard textbook logic for a monopsonist withholding demand: buying an additional unit increases the price paid for the first, inframarginal, units, and may thus reduce profits. We analyze the conditions required for a Nash equilibrium with joint demand reduction in undominated strategies, in which each bidder wins one of the unit on sale in the auction and bids zero for the second unit.

When $v_W^1 > v_S^2$, each bidder has one of the two highest valuations. In this case, it is a weakly dominant strategy for each bidder to reduce demand and bid zero for the second unit, given that her opponent bids his valuation for the first unit. The reason is that each bidder always wins a single unit (because her valuation for the second unit is lower than her opponent's valuation, and hence than her opponent's bid, for the first unit); hence the second-unit bid only affects the auction price and each bidder is better off making the lowest possible bid for the second unit.

Now assume that $v_W^1 < v_S^2$. First notice that it is still a weakly dominant strategy for bidder W to reduce demand and bid zero for the second unit, when bidder S bids his valuation for the first unit. This is because bidder W can never win more than one unit (because his valuation for the second unit is lower than bidder S's bid for the first unit). Therefore, even in this case bidder W's bid for the second unit can only affect the price he pays, and not whether he wins the second unit or not. So whether joint demand reduction is an equilibrium in this case crucially depends on bidder S. If she does not reduce demand, bidder S wins both units at price v_W^1 each (which is bidder W's bid for the first unit), and obtains a profit of $v_S^1 + v_S^2 - 2v_W^1$. While if bidder S reduces demand too, she wins only one unit at price zero and obtains a profit of v_S^1. Therefore, bidder S prefers to reduce demand together with bidder W if and only if $2v_W^1 > v_S^2$.

Lemma 1. *Consider a uniform-price auction in which resale is not allowed.*

(i) If $2v_W^1 > v_S^2$, the unique equilibrium that survives iterated deletion of weakly dominated strategies is for bidder S to bid $\mathbf{b}_S = (v_S^1;\ 0)$ and for bidder W to bid $\mathbf{b}_W = (v_W^1;\ 0)$—i.e., joint demand reduction. Moreover, this is also the Pareto dominant equilibrium in undominated strategies for bidders.

(ii) If $v_S^2 > 2v_W^1$, all equilibria in weakly undominated strategies are characterized by bidder S bidding $\mathbf{b}_S = (v_S^1;\ b_S^2)$ and bidder W bidding $\mathbf{b}_W = (v_W^1;\ b_W^2)$, where $v_W^1 < b_S^2 \le v_S^2$ and $b_W^2 \le v_W^2$. All these equilibria result in bidder S winning both units at per-unit price v_W^1.

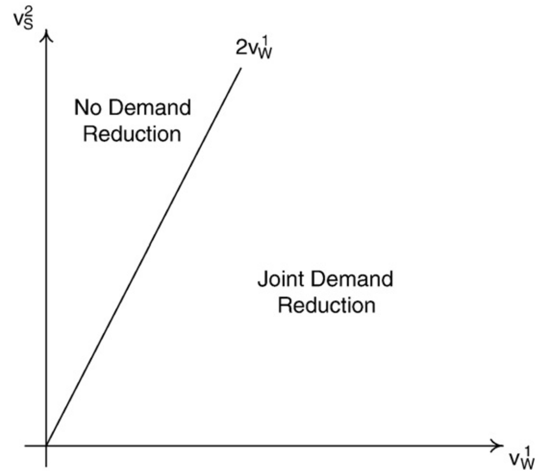

Fig. 1. Bidders' valuations for which joint demand reduction takes place in equilibrium when resale is not allowed.

Fig. 1 shows for which values of v_S^1 and v_W^2 the auction has the "joint demand reduction" equilibrium described in Lemma 1.[16] Bidder S wins both units on sale in equilibrium if and only if she has a much higher valuation than bidder W for both units, so that she prefers to win two units at a higher price, rather than reduce demand to one unit in order to keep the auction price low. Otherwise, bidders jointly reduce demand and each of them wins one of the units at price zero.

In other words, joint demand reduction takes place in equilibrium if bidders are relatively symmetric because their valuations are not too far from each other—i.e., demand reduction requires bidder W to have a relatively high valuation for the first unit and bidder S to have a relatively low valuation for the second unit. In this case, even if bidder S has the highest valuation for both units, it is not profitable for bidder S to win the second unit because she has to pay a high price to outbid bidder W in the auction, and the second unit is not particularly valuable for her anyway. So bidder S prefers to keep the auction price low by allowing bidder W to win one of the units on sale.[17]

Corollary 1. *When resale is not allowed, joint demand reduction takes place in equilibrium if and only if $2v_W^1 > v_S^2$.*

Demand reduction harms the seller. If $v_S^2 > 2v_W^1$, bidders do not reduce demand and the auction's price and the seller's revenue are equal to $2v_W^1$—i.e., twice the third-highest valuation for a unit. By contrast, if $2v_W^1 > v_S^2$, both bidders reduce demand to one unit, and the seller's revenue is equal to zero. However, with demand reduction, the auction ends with an inefficient allocation of the units, and bidders are willing to trade among themselves after the auction. As we are going to show in the next section, a bidder's incentive to reduce demand depends on whether he can acquire in the aftermarket a unit that he loses in the auction to a bidder with a lower valuation.

14 A bidder's first-unit bid affects the auction price only when it is the third-highest bid, in which case the bidder wins no unit and the price is irrelevant to her. Therefore, a bidder's first-unit bid only determines whether she wins the unit, and not the price she pays for it. And exactly as in a single-unit second-price auction, it is a weakly dominant strategy for a bidder to bid her valuation for the first unit, so that she wins the unit if and only if it is profitable for her to do so—i.e., if and only if her valuation is not lower than the auction price.

15 Because we exclude dominated strategies, we do not consider equilibria in which one bidder reduces demand because her opponent bids a very high price, higher than his own willingness to pay for a unit, expecting not to pay for it.

16 When $2v_W^1 > v_S^2$, there are multiple equilibria in undominated strategies that result in different outcomes. For example, depending on bidders' valuations, the auction may also have an equilibrium in which both bidders bid their valuations for both units on sale and bidder S wins two units. However, this equilibrium does not survives iterated deletion of weakly dominated strategies and is also Pareto dominated, from bidders' point of view, by the equilibrium with joint demand reduction, because both bidders obtain a strictly higher profit by bidding zero for the second unit. More generally, as shown in the Proof of Lemma 1, when it exists, the "joint demand reduction" equilibrium Pareto dominates, from bidders' point of view, all other possible equilibria in undominated strategies.

17 Notice that demand reduction is always an equilibrium if the units are perfectly divisible and bidders submit a continuous demand function (see Wilson, 1979, who considers the case of common values). By contrast, in our model demand reduction is not always an equilibrium because quantities and bids are discrete (Kremer and Nyborg, 2004).

4. Resale and demand reduction

After the auction, there are gains from trade whenever a bidder wins a unit even if his opponent has a higher valuation for that unit. In this case, if resale is allowed, the auction winner resells the unit to the loser, and bidders share the gains from trade, which are equal to the difference between their valuations. Therefore, bidders' willingness to pay in the auction depends on the price at which they can acquire or resell a unit in the aftermarket.

If bidder W wins both units, he always resells one of them to bidder S at price $v_W^2+\alpha(v_S^1-v_W^2)=\alpha v_S^1+(1-\alpha)v_W^2$ (since bidder W obtains a share α of the gains from trade). So this is the price at which bidder S can acquire the first unit in the aftermarket.

What about the other unit? First assume that $v_S^2>v_W^1$. If bidder W wins one unit, he resells it to bidder S at price $\alpha v_S^2+(1-\alpha)v_W^1$. So this is the price at which bidder S can acquire the second unit in the aftermarket. While if bidder S wins both units, there is no trade in the aftermarket. Hence, when resale is allowed and $v_S^2>v_W^1$, bidders' *total* surplus as a function of the number of units they win in the auction (including the surplus they anticipate from the resale market and excluding the auction price) is equal to:[18]

	No unit	One unit	Two unit
S	$(1-\alpha)(v_S^1+v_S^2-v_W^1-v_W^2)$	$v_S^1+(1-\alpha)(v_S^2-v_W^1)$	$v_S^1+v_S^2$
W	0	$\alpha v_S^2+(1-\alpha)v_W^1$	$\alpha(v_S^1+v_S^2)+(1-\alpha)(v_W^1+v_W^2)$

If, on the other hand, $v_S^2<v_W^1$ and bidder S wins both units, he resells one of them to bidder W at price $v_S^2+(1-\alpha)(v_W^1-v_S^2)=\alpha v_S^2+(1-\alpha)v_W^1$ (since bidder S obtains a share $(1-\alpha)$ of the gains from trade). So this is the price at which bidder W can acquire the second unit in the aftermarket. While there is no trade in the aftermarket if each bidder wins one unit. Hence, when resale is allowed and $v_S^2<v_W^1$, bidders' *total* surplus as a function of the number of units they win in the auction (including the surplus they anticipate from the resale market and excluding the auction price) is equal to:[19]

	No unit	One unit	Two unit
S	$(1-\alpha)(v_S^1-v_W^2)$	v_S^1	$v_S^1+\alpha v_S^2+(1-\alpha)v_W^1$
W	$\alpha(v_W^1-v_S^2)$	v_W^1	$v_W^1+\alpha v_S^1+(1-\alpha)v_W^2$

To determine a bidder's bid for a unit in the auction, we have to consider his *marginal* willingness to pay for the unit, which is equal to the *incremental* surplus obtained by winning the unit in the auction. So a bidder's willingness to pay in the auction for the k^{th} unit is the difference between his total surplus if he wins k units and his total surplus if he wins $k-1$ units—that is (both when $v_S^2>v_W^1$ and when $v_S^2<v_W^1$):[20]

	1st unit	2nd unit
S	$\alpha v_S^1+(1-\alpha)v_W^2$	$\alpha v_S^2+(1-\alpha)v_W^1$
W	$\alpha v_S^2+(1-\alpha)v_W^1$	$\alpha v_S^1+(1-\alpha)v_W^2$

Notice that the possibility of resale alters the structure of bidders' valuations. Indeed, due to resale: (i) one of the two bidders has a higher willingness to pay for the second unit than for the first unit (i.e., there are increasing marginal values), and (ii) regardless of bidders' actual valuations, no bidder has the highest willingness to pay for both units.

Recall from Section 3 that, when resale is not allowed, the auction does not have an equilibrium with joint demand reduction if bidders are relatively asymmetric, because in this case bidder S strictly prefers to outbid bidder W. But the possibility of resale reduces the asymmetry between bidders by nearing their willingness to pay. This makes joint demand reduction more attractive for bidders.

Lemma 2. *When resale is allowed, all Pareto dominant equilibria for bidders in weakly undominated strategies are characterized by bidder S bidding $\mathbf{b}_S=(b_S^1;\,0)$ and bidder W bidding $\mathbf{b}_W=(b_W^1;\,0)$, where $b_S^1\in(\frac{1}{2}[\alpha v_S^1+(1-\alpha)v_W^2],\alpha v_S^1+(1-\alpha)v_W^2]$ and $b_W^1\in(\frac{1}{2}\alpha v_S^2+(1-\alpha)v_W^1,\alpha v_S^2+(1-\alpha)v_W^1]$ – i.e., joint demand reduction.*

In all equilibria with joint demand reduction each bidder wins one of the units on sale and the auction price is 0. Each of these equilibria Pareto dominates, from bidders' point of view, any other possible equilibrium in undominated strategies.[21] The reason is that, when joint demand reduction is an equilibrium, each bidder obtains a strictly higher profit by winning one unit at price zero, rather than by paying a positive auction price, even if this allows her to win both units on sale.

The intuition for the result in Lemma 2 is straightforward. After reducing demand, when $v_W^1>v_S^2$ bidders do not trade after the auction. In this case, as when resale is not allowed, no bidder can increase his profit by outbidding her competitor; hence bidders strictly prefer to keep the auction price as low as possible. When $v_S^2>v_W^1$, bidder S buys the second unit from bidder W in the resale market. Recall from Lemma 1 that, when resale is not allowed, demand reduction takes place in equilibrium if and only if bidder W is willing to pay a high price for the first unit, and bidder S is willing to pay a low price for the second unit—i.e., if and only if $2v_W^1>v_S^2$. But the possibility of resale increases bidder W's willingness to pay for the first unit up to the price at which he can resell it in the aftermarket and, at the same time, reduces bidder S's willingness to pay for the second unit, because she has the option of buying the second unit in the aftermarket after losing it in the auction.[22] For these reasons, demand reduction is more profitable for bidder S. So bidder S always prefers to win one unit in the auction at price zero and purchase the second unit in the resale market if $v_S^2>v_W^1$, rather than raise the auction price to win both units in the auction. And clearly bidder W also prefers to win one unit in the auction at price zero and resell it in the aftermarket if $v_S^2>v_W^1$, rather than outbid bidder S and win two units (in which case he obtains a profit of zero on the second unit and reduces his profit on the first unit).

Notice that our result does not depend on bidders' bargaining power in the resale market, α. In fact, bidder S strictly prefers to reduce demand in the auction even if $\alpha=1$, in which case she obtains

[18] Let p be the auction price. If bidder W wins both units, bidder S acquires them in the aftermarket and obtains a total profit of $v_S^2+v_S^1-\alpha(v_S^1+v_S^2)-(1-\alpha)(v_W^1+v_W^2)$; while bidder W obtains a total profit of $\alpha(v_S^1+v_S^2)+(1-\alpha)(v_W^1+v_W^2)-2p$ from reselling the units. If each bidder wins one unit, bidder S obtains a profit of v_S^1-p from the unit she wins in the auction and a profit of $v_S^2-\alpha v_S^2-(1-\alpha)v_W^1$ from the unit she acquires in the aftermarket; while bidder W obtains a total profit of $\alpha v_S^2+(1-\alpha)v_W^1-p$ from reselling one unit. If bidder S wins both units, he does not trade in the aftermarket and obtains a total profit of $v_s^1+v_s^2-2p$; while bidder W obtains no profit.

[19] If any bidder i wins two units, her total profit is equal to her valuation for the first unit (i.e., v_i^1) minus the auction price, plus the resale price for the second unit (i.e., $\alpha v_S^1+(1-\alpha)v_W^2$ for bidder W and $\alpha v_S^2+(1-\alpha)v_W^1$ for bidder S). If any bidder i wins no unit, she then buys one unit in the aftermarket and her total profit is equal to her valuation for the first unit (i.e., v_i^1) minus the resale price at which she buys it (i.e., $\alpha v_S^2+(1-\alpha)v_W^1$ for bidder W and $\alpha v_S^1+(1-\alpha)v_W^2$ for bidder S).

[20] Equivalently, bidders' willingness to pay can be obtained by noticing that a bidder who has a lower valuation than his opponent for a unit is willing to pay for that unit an auction price equal to the price at which he can resell the unit in the aftermarket; while a bidder who has a higher valuation than his opponent for a unit is willing to pay for that unit an auction price equal to the price at which she can buy the unit in the aftermarket.

[21] The auction has other possible equilibria in undominated strategies. For example, it may be an equilibrium for both bidders to bid a strictly positive price for the second unit. And there are also equilibria in weakly dominated strategies—for example, bidder W bidding a price higher than bidder S's willingness to pay for both units and bidder S bidding zero and then buying in the resale market.

[22] In the terminology of Haile (2003), bidder W bids more aggressively because of the "resale seller effect" and bidder S bids less aggressively because of the "resale buyer effect."

no surplus in the resale market. The reason is that reducing demand still allows bidder S to win one unit at price zero in the auction, and this is always better than outbidding bidder W when resale is allowed. Therefore, bidder S is not induced to reduce demand by the positive surplus that she can obtain in the resale market, but rather by the effect of the possibility of resale on bidder W's willingness to pay in the auction. Moreover, our results are robust to the introduction of a strictly positive, but not too large, resale cost that bidders have to pay to trade in the resale market.

Lemma 2 shows that resale induces bidders to reduce demand even if they have no incentive to reduce demand when resale is not allowed. Therefore, if $v_S^2 > 2v_W^1$ the possibility of resale reduces the auction price and the seller's revenue from $2v_W^1$ to 0. By contrast, when $2v_W^1 > v_S^2$ bidders jointly reduce demand regardless of the presence of resale, yielding no revenue for the seller. So we have the following result.

Proposition 1. *In a multi-unit uniform-price auction, allowing resale (weakly) reduces the seller's revenue.*

As in a single-unit auction, resale induces a weak bidder to bid more aggressively (Pagnozzi, 2007). But with multiple units on sale, this increases bidder S's incentive to reduce demand jointly with bidder W, because outbidding bidder W becomes more costly. And demand reduction reduces the seller's revenue. Moreover, resale makes an inefficient allocation in the auction (i.e., bidder W winning a unit even if he has a lower valuation than bidder S) more attractive for bidders, because the inefficient allocation can be rectified in the aftermarket.

Example 1. *Assume $v_S^1 = v_S^2 = 10$, $v_W^1 = 2$, $v_W^2 = 0$, and $\alpha = \frac{1}{2}$. Without resale, bidder S prefers to outbid bidder W and win two units at price 2 each, rather than reduce demand and win one unit at price 0. So the seller's revenue is 4. With resale, bidder W is willing to pay up to 6 for the first unit and bidder S is willing to pay up to 5 for the first unit. Hence, it is an equilibrium for S to bid (5; 0) and for W to bid (6; 0), in which case each bidder wins one unit, the seller's revenue is 0 and bidder W resells to bidder S in the aftermarket at price 6. (If bidder S deviates and outbids bidder W to win 2 units in the auction, she raises the auction price to 6 and obtains a profit of 8 rather than 14.) Clearly, there is no other equilibrium in undominated strategies in which a bidder obtains a strictly higher total profit.*

5. Bundling

Bundling the units on sale appears a natural reaction for the seller to the risk of demand reduction, because bundling makes it impossible for bidders to profitably reduce demand in the auction (see, e.g., Anton and Yao, 1992). However, as we are going to show, without resale bundling has an ambiguous effect on the seller's revenue. In fact, when resale is not allowed, although bundling increases the seller's revenue if it prevents bidders from reducing demand, it reduces the seller's revenue if bidders do not reduce demand when the units are sold separately.

So should the seller bundle the units on sale when resale is allowed? And if the seller can credibly forbid resale, should he do so in order to prevent demand reduction by bidders? We address these questions in the following sections.[23]

[23] As discussed in the introduction, we assume the seller's only available strategies are to bundle the units on sale and to forbid resale, because the seller does not have enough information to use other instruments to increase his revenue. This is an extreme assumption. If the seller knows the exact bidders' valuations and has the commitment power to set a reserve price, his optimal strategy is to set a reserve price equal to the highest bidders' valuation for each of the two units, thus obtaining the whole bidders' surplus. And even if the seller does not know the exact bidders' valuations, there are perhaps more complex mechanisms that would allow him to extract more of the bidders' surplus. But, in actual practice, setting a credible reserve price is often extremely difficult, and more complex mechanisms are even harder to implement.

5.1. Bundling without resale

First assume that resale is not allowed. If the two units are sold separately (as assumed in Section 3), the seller's revenue depends on whether bidders reduce demand or not and is equal to:

$$\Pi_{NR}^{NB} = \begin{cases} 2v_W^1 & \text{if bidders do not reduce demand}\left(\text{i.e., if } v_S^2 > 2v_W^1\right); \\ 0 & \text{if bidders reduce demand}\left(\text{i.e., if } 2\,v_W^1 > v_S^2\right). \end{cases}$$

Suppose instead that the seller auctions the two units bundled together, awarding them to the bidder who submits the highest bid for the bundle at a price equal to the second-highest bid. (Basically, in this case the seller runs a second-price auction for a single object.) Bundling affects the seller's revenue because it makes the auction price also depend on bidder W's valuation for the second unit, rather than only on his highest valuation. However, bundling also eliminates bidders' incentives to reduce demand. Specifically, when the units are bundled and resale is not allowed, it is a weakly dominant strategy for each bidder to bid the sum of his valuations for the two units, and the seller's revenue is equal to the lowest bid:

$$\Pi_{NR}^{B} = min\left\{v_S^1 + v_S^2; v_W^1 + v_W^2\right\}.$$

So without resale, bundling does not necessarily increase the seller's revenue: the effect of bundling on the auction price depends on whether or not bidders jointly reduce demand when the units are sold separately (since $2v_W^1 > \min\{v_S^1 + v_S^2;\ v_W^1 + v_W^2\} > 0$).

Lemma 3. *When resale is not allowed, bundling reduces the seller's revenue if $v_S^2 > 2v_W^1$ (i.e., if bidders do not reduce demand without bundling); bundling increases the seller's revenue if $2v_W^1 > v_S^2$ (i.e., if bidders reduce demand without bundling).*

In addition to its effect on the auction price, another potential drawback of bundling is that it can reduce efficiency. Indeed, bundling generates an inefficient allocation of the units on sale if a bidder has a higher valuation than his opponent for one of the units, but a lower valuation for the bundle. In this case, when the units are bundled this bidder wins no unit, while it would be efficient to award one unit to each bidder.[24]

5.2. Bundling with resale

Now consider the seller's revenue when resale is allowed. To make the analysis interesting, we assume that bidders can trade the two units separately in the resale market, even if the units are bundled in the auction.[25] Hence, if the seller bundles the two units and a bidder with a lower valuation than his opponent for any of the units wins the auction, the two bidders trade in the resale market.

When bidder S has the highest valuations for both units on sale, she can buy them in the resale market at prices $\alpha v_S^1 + (1-\alpha)v_W^2$ and $\alpha v_S^2 + (1-\alpha)v_W^1$ respectively. And bidder W can resell the two units at these same prices. On the other hand, when bidder W has a higher valuation than bidder S for one of the units on sale, bidder W can buy the first unit in the resale market at price $\alpha v_S^2 + (1-\alpha)v_W^1$ and sell the second unit in the resale market at price $\alpha v_S^1 + (1-\alpha)v_W^2$; while bidder S can buy the first unit in the resale market at price $\alpha v_S^1 + (1-\alpha)v_W^2$ and sell

[24] For example, bundling generates an inefficient allocation if $v_S^1 + v_S^2 > v_W^1 + v_W^2$ but $v_W^1 > v_S^2$, because bidder W wins no unit with bundling, while it would be efficient for him to win one.

[25] If the units cannot be sold separately in the resale market, our model is analogous to a single object auction with resale (see, e.g., Pagnozzi, 2007).

the second unit in the resale market at price $\alpha v_S^2 + (1-\alpha)v_W^1$. Therefore, both when $v_S^2 > v_W^1$ and when $v_S^2 < v_W^1$, the two bidders are both willing to pay $\alpha(v_S^1 + v_S^2) + (1-\alpha)(v_W^1 + v_W^2)$ for the two units in the auction. And because it is a weakly dominant strategy for each bidder to bid her willingness to pay in the auction, the seller's revenue is also equal to:

$$\Pi_R^B = \alpha\left(v_S^1 + v_S^2\right) + (1-\alpha)\left(v_W^1 + v_W^2\right).$$

By contrast, if the seller auctions the units separately and resale is allowed, by Lemma 2 both bidders reduce demand and the seller's revenue is equal to 0. Hence, we have the following result.

Lemma 4. *When resale is allowed, bundling strictly increases the seller's revenue.*

So, when resale is allowed, the seller always obtains a higher revenue by bundling the two units, because bundling eliminates bidders' incentives to reduce demand, while this incentive is always present if the units are sold separately. In other words, in contrast to a situation in which resale is not allowed, when resale is allowed bundling always prevents bidders from jointly reducing demand, thus raising the auction price.

5.3. Bundling and allowing resale

Assume now that the seller can prevent bidders from reselling after the auction. Should the seller do so to discourage demand reduction or should he instead bundle the units on sale?

The answer is that, typically, the seller should not prevent resale and should bundle the units, because bundling and allowing resale are *complement strategies* for the seller. First, as shown by Lemma 4, when resale is allowed bundling increases the seller's revenue. Second, exactly as in a single-object auction, when the units are bundled allowing resale increases the seller's revenue because it induces the bidder with the lowest total valuation to bid more aggressively (Pagnozzi, 2007). Third, as proven in the next proposition, the seller's revenue is also higher in an auction with resale and bundling than in an auction without resale in which the units are sold separately if: (1) bidder W has a sufficiently high valuation for at least one of the units or (2) bidder W can obtain a sufficiently large share of the gains from trade in the resale market.

Proposition 2. *Bundling the units on sale and allowing resale yields a higher seller's revenue than: (i) selling the units separately and allowing resale, and (ii) bundling and forbidding resale. Bundling the units on sale and allowing resale also yields a higher seller's revenue than selling the units separately and forbidding resale if: (1) $2v_W^1 > v_S^2$ or (2) $\alpha > \frac{v_W^1 - v_W^2}{v_S^1 + v_S^2 - v_W^1 - v_W^2}$.*

The intuition is that, by simultaneously bundling and allowing resale, the seller induces the weak bidder to bid more aggressively because of the option to resell in the aftermarket and, at the same time, he prevents the strong bidder from reacting to this strategy by reducing demand. This always increases the seller's revenue when bidders reduce demand without resale and without bundling—i.e., when condition (1) is satisfied. Moreover, even if bidders do not reduce demand without resale and without bundling, and although bundling makes the auction price also depend on the weak bidder's lowest willingness to pay for a single unit, allowing resale compensates this effect by sufficiently increasing the willingness to pay of the weak bidder if he has enough bargaining power in the resale market—i.e., if condition (2) is satisfied. Therefore, if bidders are not too asymmetric, the seller can obtain the advantages of both resale and bundling, without suffering from the drawbacks that these strategies may create.

Notice that condition (2) is always satisfied if, for example, bidders equally share the gains from trade in the resale market (i.e., if $\alpha = \frac{1}{2}$) or if bidder W has the same valuation for both units on sale (i.e., if $v_W^1 = v_W^2$).[26]

Of course, if both bidder W's valuations *and* his bargaining power in the resale market are much lower than bidder S's, bidder W is unable to obtain a large surplus by reselling to bidder S; hence allowing resale does not induce him to bid much more aggressively than without resale. In this case, bundling and allowing resale may reduce the seller's revenue. The reason is that bidder S does not reduce demand when the units are sold separately and resale is not allowed, and bidder W's marginal losing bid (per unit) is higher when the units are sold separately and resale is not allowed than when the units are bundled and resale is allowed, because his marginal losing bid in the former case only depends on his highest valuation (rather than on both his valuations) and the option to resell after the auction is not particularly valuable.

As regards the additional potential drawback of bundling, even if bundling results in an inefficient allocation in the auction, resale allows bidders to correct the allocation in the aftermarket and eventually achieve efficiency. So resale also eliminates the risk of inefficiency due to bundling.

6. Inefficient resale market

In Section 4 we have shown that resale may reduce the seller's revenue. It is usually claimed, however, that the possibility of resale increases efficiency, because it allows bidders to exploit further gains from trade after the auction, thus ensuring that the units on sale are efficiently allocated eventually.

In the previous sections, we have assumed that the resale market is always efficient, because bidders are capable of exploiting all profitable trade opportunities after the auction. In this section we make the more realistic assumption that the resale market is not necessarily efficient. We introduce the possibility of inefficiency in the simplest possible way, by assuming that with a strictly positive probability $(1-p)$ bidders are unable to trade after the auction—i.e., that bidders can only trade in the resale market with probability $p<1$ if they are willing to do so. To simplify the analysis, we also assume that $v_S^2 > v_W^1$, so that it is efficient to allocate both units to bidder S. All other assumptions are as in our main model.

If bidder W wins one of the units on sale in the auction, with probability p he resells it to bidder S at price $\alpha v_S^2 + (1-\alpha)v_W^1$. And if bidder W also wins a second unit in the auction, with probability p he resells it to bidder S at price $\alpha v_S^1 + (1-\alpha)v_W^2$. Therefore, bidder W's willingness to pay for the first unit in the auction is increased by an amount equal to his expected surplus in the resale market if he wins one unit—i.e., by the resale price minus his valuation for the first unit, $\alpha v_S^2 + (1-\alpha)v_W^1 - v_W^1$, times the probability that resale takes place, p. And bidder W's willingness to pay for the second unit is increased by an amount equal to the surplus he expects to obtain from the second unit in the resale market—i.e., by the resale price minus his valuation for the second unit, $\alpha v_S^1 + (1-\alpha)v_W^2 - v_W^2$, times the probability that resale takes place, p.

By contrast, bidder S's willingness to pay for the second unit in the auction is reduced by an amount equal to her expected surplus in the resale market if bidder W wins one unit—i.e., by her valuation for the second unit minus the resale price, $v_S^2 - \alpha v_S^2 - (1-\alpha)v_W^1$, times the probability that resale takes place, p. And bidder S's willingness to pay for the first unit is reduced by an amount equal to her additional expected surplus in the resale market if she does not win the first

[26] And the closer are bidder W's valuations for the two units, the lower is the value of α needed to satisfy condition (2).

unit—i.e., by her valuation for the first unit minus the resale price, $v_S^1 - \alpha v_S^1 - (1-\alpha)v_W^2$, times the probability that resale takes place, p.

Summing up, bidders' willingness to pay for each unit in the auction is equal to:

	1st unit	2nd unit
S	$v_S^1 - p \cdot (1-\alpha)(v_S^1 - v_W^2)$	$v_S^2 - p \cdot (1-\alpha)(v_S^2 - v_W^1)$
W	$v_W^1 + p \cdot \alpha(v_S^2 - v_W^1)$	$v_W^2 + p \cdot \alpha(v_S^1 - v_W^2)$

Compared to a situation in which bidders are always able to trade after the auction (i.e., $p=1$), if the resale market is not necessarily efficient bidder W is willing to pay a lower price in the auction while bidder S is willing to pay a higher price in the auction, because both bidders expect to obtain a lower surplus in the resale market. This reduces, but does not eliminate, bidder S's incentive to reduce demand to one unit and then try to acquire the other unit in the resale market. Specifically, bidder S still prefers to reduce demand and win only one unit in the auction at price zero rather than outbid bidder W to win two units, if the sum of her valuation for the first unit and her expected surplus in the resale market is higher than her profit from winning two units in the auction at the cost of raising the auction price up to bidder W's willingness to pay for the first unit.

Lemma 5. *When resale is allowed but bidders are only able to trade after the auction with probability $p<1$, it is an equilibrium for bidder S to bid $\mathbf{b}_S = (v_S^1 - p(1-\alpha)(v_S^1 - v_W^1);\ 0)$ and for bidder W to bid $\mathbf{b}_W = (v_W^1 + p\alpha(v_S^2 - v_W^1);\ 0)$–i.e., joint demand reduction–if and only if $p > \frac{v_S^2 - 2v_W^1}{(1+\alpha)(v_S^2 + v_W^1)}$.*

Not surprisingly, demand reduction requires that the probability of inefficiency in the resale market is not too large. Otherwise bidder S strictly prefers to outbid bidder W in the auction, rather than allow him to win one unit to keep the auction price low, because the risk of being unable to acquire that unit in the resale market is too high. Therefore, if the resale market is not necessarily efficient, it is less likely that bidders reduce demand in equilibrium and that the seller's revenue is reduced to zero.

However, if the resale market is not necessarily efficient, allowing resale may actually reduce efficiency. To see this, recall from Section 3 that, when resale is not allowed, bidders do not reduce demand and the auction is efficient if $v_S^2 > 2v_W^1$. But in this case allowing resale may still induce bidders to reduce demand during the auction, even if they may then be unable to trade in the resale market and so the final allocation of the units may be inefficient. So an auction without resale may yield a more efficient final allocation than an auction with resale.

Proposition 3. *If $v_S^2 > 2v_W^1$ and $p > \frac{v_S^2 - 2v_W^1}{(1+\alpha)(v_S^2 + v_W^1)}$, allowing resale reduces efficiency with probability $(1-p)$ (compared to an auction without resale).*

This result suggests that it is not necessarily true that allowing resale increases efficiency. Although resale may increase efficiency after the auction, it also affects bidders' strategies during the auction. And allowing resale may result in an inefficient allocation at the end of the auction, even when bidders may be unable to trade and achieve an efficient allocation in the aftermarket.

7. Unilateral demand reduction

In this section, we analyze how resale affects a strong bidder's incentive to *unilaterally* reduce demand–i.e., to bid zero for the second unit–when her opponent does not reduce demand and bids his willingness to pay for both units. Clearly, bidding his willingness to pay for both units is not an equilibrium strategy for the weak bidder, because when a strong bidder reduces demand it is a best reply for a weak bidder to reduce demand too. However, in the real world bidders are often unable or unwilling to coordinate their strategy and simultaneously reduce demand, and cannot always act on the expectation that their opponents will reduce demand.[27] And there may also be exogenous reasons that induce a weak bidder not to reduce demand.

In order to explore this issue, we assume that $v_S^2 > v_W^1$ and that bidder W never reduces demand–i.e., that he follows a strategy of always bidding his willingness to pay for both units, although this is not a profit-maximizing strategy–and we analyze whether bidder S has an incentive to reduce demand unilaterally anyway.

We also assume that there is an arbitrarily small resale cost c that bidder S pays for each unit traded in the resale market. This can be interpreted as either a transaction cost or a waiting cost (due to discounting of future surplus) that a bidder pays if she buys a unit later in the resale market, rather than earlier in the auction. This assumption allows us to simplify the analysis because it implies that, for a given resale price, bidder S has a higher willingness to pay in the auction than bidder W. We assume that $c \approx 0$, so that trading in the resale market is always profitable after bidder W wins a unit in the auction. All other assumptions are as in our main model (and, in particular, bidders can always trade in the resale market if they are willing to do so).

With a resale cost, bidders' willingness to pay in the auction is:[28]

	1st unit	2nd unit
S	$\alpha v_S^1 + (1-\alpha)v_W^2 + c$	$\alpha v_S^2 + (1-\alpha)v_W^1 + c$
W	$\alpha v_S^2 + (1-\alpha)v_W^1 - c$	$\alpha v_S^1 + (1-\alpha)v_W^2 - c$

It may be expected that, when resale is allowed, unilateral demand reduction is more profitable for bidder S, because resale allows bidder S to purchase the second unit in the aftermarket if she does not win it in the auction. Therefore, it may be expected that allowing resale always increases bidder S's incentive to unilaterally reduce demand. But this is not the case.

The reason is that resale increases bidder W's willingness to pay for the second unit, and it increases it relatively more than his willingness to pay for the first unit, because the price at which bidder W can resell a second unit to bidder S depends on v_S^1. It follows that, when resale is allowed, it is less profitable for bidder S to unilaterally reduce demand, because when she does so she can only reduce the auction price down to bidder W's bid for the second unit, which is relatively high due to his high willingness to pay.

Lemma 6. *Assume bidder W does not reduce demand. When resale is allowed, bidder S has no incentive to reduce demand unilaterally.*

By contrast, when resale is not allowed, bidder S may strictly prefer to unilaterally reduce demand. To see this, assume that bidder W bids his valuation for both units—i.e., v_W^1 for the first unit and v_W^2 for the

[27] For example, in the German 3G spectrum auction in 2000 bidders seem to have been unable to coordinate their strategies on a mutually profitable demand reduction (Klemperer, 2004). Cramton (2002) writes that, in the US Nationwide Narrowband spectrum auction in 1994, "[t]he largest bidder, PageNet reduced its demand from three of the large licences to two, at a point when prices were still well below its marginal valuation for the third unit. [It] felt that if it continued to demand a third license, it would drive up the prices on all the others to disadvantageously high levels." This appears to have been unilateral behavior, since there is no suggestion that PageNet expected any other bidder to respond by reducing demand, nor that any other bidder did so. Cramton (2002) also provides evidence of unilateral demand reduction in the US C-Block spectrum auction in 1995. Wolak (2003) analyzes the California Electricity Crisis in January 2001 and shows that suppliers had an incentive to unilaterally raise prices, although there is no evidence that they coordinated their actions. This suggests that the crisis may have been generated by a unilateral exercise of market power.

[28] The analysis of bidders' willingness to pay is analogous to the one in Section 4.

second unit. If bidder S does not reduce demand unilaterally, she wins both units at price v_W^1 each, and obtains a profit of $v_S^1+v_S^2-2v_W^1$. While if bidder S unilaterally reduces demand, she wins one unit only at price v_W^2 and obtains a profit of $v_S^1-v_W^2$. So bidder S prefers to unilaterally reduce demand when resale is not allowed if and only if $2v_W^1>v_S^2+v_W^2$. Hence, we have the following result.

Proposition 4. *When $2v_W^1>v_S^2+v_W^2$, allowing resale eliminates bidder S's incentive to unilaterally reduce demand in the auction.*

The intuition for this result is that, when resale is not allowed, unilateral demand reduction by bidder S requires bidder W to have a relatively low willingness to pay for the second unit, because in this case bidder S can reduce the auction price by a large amount if she reduces demand, even if bidder W bids his valuation for both units. But introducing the possibility of resale increases bidder W's willingness to pay for the second unit; hence it may induce bidder S to increase her demand (when bidder W does not reduce demand).[29]

Example 2. *Assume $v_S^1=10$, $v_S^2=6$, $v_W^1=4$, $v_W^2=0$, and $\alpha=\frac{1}{2}$, and assume that bidder W bids his willingness to pay for both units. Without resale, bidder S prefers to unilaterally reduce demand (in order to obtain a profit of* 10 *rather than* $16-2\times4=8$*). With resale, bidder W is willing to pay up to* 5–c *for each unit. Therefore, bidder S can win two units in the auction and obtain profit* $6+2c$*. If bidder S unilaterally reduces demand instead, she wins one unit at price* 5–c *in the auction and buys the second unit in the resale market at price* 5*, paying the cost c. Hence, she obtains a total profit of* 6*. So bidder S strictly prefers not to reduce demand unilaterally when resale is allowed.*

8. Conclusions

It is sometimes argued that resale should always be permitted because, by allowing bidders to exploit gains from trade after the auction, it favors an efficient allocation of the objects on sale in the auction.

But resale also affects bidders' strategies during an auction. Resale increases the willingness to pay of a low-value bidder, because it gives him an option to resell in the aftermarket to a high-value bidder and, at the same time, resale reduces the willingness to pay of a high-value bidder, because it gives her an option to buy in the aftermarket a unit she loses in the auction. When multiple units are on sale, this favors demand reduction by a high-value bidder. Therefore, unlike in single-unit auctions, resale may reduce the seller's revenue in multi-unit auctions.

Moreover, our analysis also suggests that, if the resale market is not necessarily efficient, allowing resale may even reduce efficiency (compared to an auction without resale), because the possibility of resale may induce bidders to reduce demand during the auction, only then to find themselves unable to trade in the resale market.

But when resale is allowed, the seller can always increase his revenue by bundling the units on sale (rather than selling them separately). Moreover, bundling the units on sale at the same time as allowing resale also yields a higher seller's revenue than bundling the units on sale and forbidding resale, or selling the units separately and forbidding resale, provided bidders are not too asymmetric.

Appendix A

Proof of Lemma 1. It is a weakly dominant strategy for each bidder to bid his valuation for the first unit (e.g., Milgrom, 2004), and to bid at most his valuation for the second unit.

Assume that bidder W makes her weakly dominant bid for the first unit, and bids $b_W^2\le v_W^2$ for the second unit. If bidder S bids more than v_W^1 for the second unit, she wins two units at price v_W^1 and her profit is $v_S^1+v_S^2-2v_W^1$. If instead bidder S bids less than v_W^1 for the second unit (and bids her valuation for the first unit), she wins one unit only at price max $\{b_S^2;\ b_W^2\}$. In this case, her profit is $v_S^1-\max\{b_S^2;\ b_W^2\}$. Therefore, if $v_S^2>2v_W^1$, bidder S strictly prefers not to reduce demand and win both units on sale by bidding strictly more than v_W^1 for the second unit, regardless of bidder W's bid for the second unit. While bidder W is indifferent among all bids $b_W^i<v_S^2$, $i=1, 2$, because it is never profitable for him to win a unit in the auction. Hence, if $v_S^2>2v_W^1$, all equilibria in weakly undominated strategies are characterized by bidder S bidding $\mathbf{b}_S=(v_S^1;\ b_S^2)$ where $v_W^1<b_S^2\le v_S^2$, and bidder W bidding $\mathbf{b}_W=(v_W^1;\ b_W^2)$ where $b_W^2\le v_W^2$.

To analyze equilibrium bidding strategies when $2v_W^1>v_S^2$, we proceed by iterated deletion of weakly dominated strategies. Given that bidder S makes her weakly dominant bid for the first unit, it is a weakly dominant strategy for bidder W to reduce demand and bid 0 for the second unit, because it is never profitable for him to win two units (since bidder S's bid for the first unit is higher than bidder W's valuation for the second unit) and, therefore, his second-unit's bid can only affect the auction price.

When $v_W^1>v_S^2$, given that bidder W makes his weakly dominant bid for the first unit, it is also a weakly dominant strategy for bidder S to reduce demand and bid 0 for the second unit, because her second-unit's bid can only affect the auction price. Hence, if $v_W^1>v_S^2$, the unique equilibrium that survives iterated deletion of weakly dominated strategies is for bidder S to bid $\mathbf{b}_S=(v_S^1;\ 0)$ and for bidder W to bid $\mathbf{b}_W=(v_W^1;\ 0)$—i.e., joint demand reduction.

Assume now that $v_S^2>v_W^1$ and that bidder W bids $\mathbf{b}_W=(v_W^1;\ 0)$, which is the unique strategy that survives iterated deletion of weakly dominated strategies. If bidder S reduces demand and bid 0 for the second unit, she wins one unit only at price 0 and her profit is v_S^1. If instead bidder S does not reduce demand and bids more than v_W^1 for the second unit, she wins two units at price v_W^1 and her profit is $v_S^1+v_S^2-2v_W^1$. Therefore, if and only if $2v_W^1>v_S^2$, bidder S strictly prefers to reduce demand. Hence, if $2v_W^1>v_S^2>v_W^1$, the unique equilibrium that survives iterated deletion of weakly dominated strategies is for bidder S to bid $\mathbf{b}_S=(v_S^1;\ 0)$ and for bidder W to bid $\mathbf{b}_W=(v_W^1;\ 0)$—i.e., joint demand reduction.

When $2v_W^1>v_S^2$ there are also many other equilibria in undominated strategies (but that do not survive iterated deletion of weakly dominated strategies) in which either each bidder wins one unit and the auction price is positive, or bidder S wins both units by outbidding bidder W. Specifically, it is an equilibrium for bidder S to bid $\mathbf{b}_S=(v_S^1;\ x)$ and for bidder W to bid $\mathbf{b}_W=(v_W^1;\ x)$, $\forall x\in[0,\ \min\{2v_W^1-v_S^2;\ v_W^2\}]$. (This is because bidder S prefers to win one unit at price x rather than outbid bidder W to win two units at price v_W^1 if and only if $v_S^1-x>v_S^1+v_S^2-2v_W^1$.) However, all these equilibria are Pareto dominated, from bidders' point of view, by the equilibrium with $x=0$–i.e., with joint demand reduction and an auction price equal to 0–because both bidders win the same number of units in all these equilibria and only the auction price differs. There are also equilibria in undominated strategies (but that do not survive iterated deletion of weakly dominated strategies) in which bidders S wins both units on sale. For example, it is an equilibrium for bidder S to bid $\mathbf{b}_S=(v_S^1;\ v_S^2)$ and for bidder W to bid $\mathbf{b}_W=(v_W^1;\ x)$, $\forall x\in[2v_W^1-v_S^2;\ v_W^2]$ (because bidder S prefers to outbid bidder W and win two units at price v_W^1, rather than win one unit only at price x if and only if $v_S^1+v_S^2-2v_W^1>v_S^1-x$). But also these equilibria are Pareto dominated by the equilibrium with joint demand reduction and an auction price equal to 0, because (as we have previously shown) bidder S obtains a higher profit by winning one unit at price 0 rather than outbidding bidder W when $2v_W^1>v_S^2$ (and, clearly, also bidder W obtains a higher profit with joint demand reduction). □

Proof of Lemma 2. First, we are going to show that no bidder has a profitable deviation from the equilibria described in the statement

[29] This suggests that, if resale is allowed, even though joint demand reduction is an equilibrium, it may be more difficult to achieve it when a bidder has to adopt a unilateral behavior because she does not expect her opponent to reduce demand and/or bidders are unable to coordinate their strategies.

both when $v_S^2<v_W^1$ and when $v_S^2>v_W^1$. In the candidate equilibria, the auction price is equal to zero and each bidder wins one unit. Notice that the only bids for the first unit that are not dominated are such that $b_W^1\leq\alpha v_S^2+(1-\alpha)v_W^1$ and $b_S^1\leq\alpha v_S^1+(1-\alpha)v_W^2$—i.e., lower than bidders' willingness to pay.[30]

Case (i): $v_W^1>v_S^2$. In this case no bidder resells the unit won in the auction and each bidder i obtains a profit equal to his valuation for the first unit, v_i^1. In order to win a second unit, a bidder has to raise the auction price for both units up to the price at which he will resell the second unit in the aftermarket. This clearly reduces his profit. (And a bidder also earns a lower profit by winning no unit.) Hence, no bidder has an incentive to deviate from the candidate equilibria.

Case (ii): $v_S^2>v_W^1$. In this case bidder W resells the unit won in the auction and bidder S obtains a total profit of:

$$\pi_S^* = \underbrace{v_S^1}_{\text{auction profit}} + \underbrace{v_S^2-\alpha v_S^2-(1-\alpha)v_W^1}_{\text{resale surplus}} = v_S^1 + (1-\alpha)\left(v_S^2 - v_W^1\right).$$

In order to win two units, bidder S has to pay an auction price of $2\cdot b_W^1$ to outbid bidder W. In this case, she obtains a total profit of:

$$\pi'_S = v_S^1 + v_S^2 - 2b_W^1.$$

Clearly:

$$\pi_S^* > \pi'_S \Leftrightarrow b_W^1 > \frac{1}{2}\left[\alpha v_S^2 + (1-\alpha)v_W^1\right]. \tag{A.1}$$

Therefore, when condition A.1 is satisfied, bidder S prefers not deviate from the equilibria described by winning two units. Moreover, if bidder S wins no units, she earns a profit of $(1-\alpha)[(v_S^1+v_S^2)-(v_W^1+v_W^2)]$, which is also lower than π_S^*. So bidder S has no incentive to deviate when condition A.1 is satisfied.

Now consider bidder W. In the candidate equilibria, he obtains a profit equal to the resale price at which he resells one unit to bidder S in the aftermarket, that is:

$$\pi_W^* = \alpha v_S^2 + (1-\alpha)v_W^1.$$

In order to win two units (that he resells to bidder S in the aftermarket), bidder W has to pay an auction price of $2\cdot b^1{}_S$ to outbid bidder S in the auction. In this case, he obtains a profit of:

$$\pi'_W = \alpha\left(v_S^1 + v_S^2\right) + (1-\alpha)\left(v_W^1 + v_W^2\right) - 2b_S^1.$$

Clearly:

$$\pi_W^* > \pi'_W \Leftrightarrow b_S^1 > \frac{1}{2}\left[\alpha v_S^1 + (1-\alpha)v_W^2\right]. \tag{A.2}$$

Therefore, when condition A.2 is satisfied, bidder W prefers not deviate from the equilibrium described by winning two units. (Clearly, winning no units also yields a lower profit for bidder W.) This proves that the strategies described in the statement constitute an equilibrium in undominated strategies.

Even when resale is allowed, the auction has other possible equilibria in weakly undominated strategies, in which either each bidder wins one unit and the auction price is positive, or one bidder wins both units by outbidding his competitor. However, by the same arguments of Lemma 1, all other equilibria are Pareto dominated, from bidders' point of view, by each of the equilibria described, in which bidders jointly reduce demand and the auction price is equal to 0. □

[30] Because with resale one bidder has a higher willingness to pay for the second unit than for the first unit, bidding his willingness to pay for the first unit is not necessarily a dominant strategy anymore.

Proof of Proposition 1. By Lemma 2, when resale is allowed bidders always jointly reduce demand and the seller's revenue is equal to zero in equilibrium. By contrast, by Lemma 1, when resale is not allowed the seller's revenue is strictly positive when bidders do not reduce demand—i.e., when $v_S^2>2v_W^1$. □

Proof of Lemma 3. Follows from the discussion preceding the statement. □

Proof of Lemma 4. If bidder W wins the auction, he always resells the second unit to bidder S at price $\alpha v_S^1+(1-\alpha)v_W^2$. Assume first that $v_S^2>v_W^1$. Then bidder W also resells the first unit to bidder S at price $\alpha v_S^2+(1-\alpha)v_W^1$; hence he is willing to pay $\alpha(v_S^1+v_S^2)+(1-\alpha)(v_W^1+v_W^2)$ for the two units in the auction. And bidder S is also willing to pay $\alpha(v_S^1+v_S^2)+(1-\alpha)(v_W^1+v_W^2)$ for the two units in the auction, since this is the price at which she can buy them in the resale market.

Assume now that $v_S^2<v_W^1$. In this case, any bidder who wins the auction resells one unit in the aftermarket. If bidder W wins the auction at price p, he resells one unit at price $\alpha v_S^1+(1-\alpha)v_W^2$ and makes total profit $v_W^1+\alpha v_S^1+(1-\alpha)v_W^2-p$; while if bidder W loses the auction, he buys one unit in the aftermarket at price $\alpha v_S^2+(1-\alpha)v_W^1$ and makes total profit $v_W^1+\alpha v_S^2-(1-\alpha)v_W^1$. Therefore, bidder W is willing to pay $\alpha(v_S^1+v_S^2)+(1-\alpha)(v_W^1+v_W^2)$ for the two units in the auction, which is the price at which he is indifferent between winning the auction and losing it. Similarly, if bidder S wins the auction, she resells one unit at price $\alpha v_S^2+(1-\alpha)v_W^1$ and makes total profit $v_S^1+\alpha v_S^2+(1-\alpha)v_W^1-p$; while if bidder S loses the auction, she buys one unit in the aftermarket at price $\alpha v_S^1+(1-\alpha)v_W^2$ and makes total profit $v_S^1-\alpha v_S^1-(1-\alpha)v_W^2$. Therefore, bidder S is also willing to pay $\alpha(v_S^1+v_S^2)+(1-\alpha)(v_W^1+v_W^2)$ for the two units in the auction, which is the price at which she is indifferent between winning the auction and losing it. Hence, both bidders have exactly the same willingness to pay, both when $v_S^2>v_W^1$ and when $v_S^2<v_W^1$.

Since it is a weakly dominant strategy in a second-price auction to bid one's willingness to pay, it follows that the seller's revenue when resale is allowed and the units are bundled is equal to:

$$\Pi_R^B = \alpha\left(v_S^1 + v_S^2\right) + (1-\alpha)\left(v_W^1 + v_W^2\right).$$

by contrast, if resale is allowed but the units are sold separately, by Lemma 2 bidders jointly reduce demand and the seller's revenue is always equal to 0. This is lower than Π_R^B. □

Proof of Proposition 2. To prove the statement, we compare the seller's revenue with bundling and resale, $\Pi_R^B=\alpha(v_S^1+v_S^2)+(1-\alpha)(v_W^1+v_W^2)$, with: (1) the seller's revenue without bundling and with resale, Π_R^{NB}, (2) the seller's revenue with bundling and without resale, Π_{NR}^B, and (3) the seller's revenue without bundling and without resale, Π_{NR}^{NB}.

From Lemma 4 it follows that $\Pi_R^B>\Pi_R^{NB}$. From the discussion in Section 5.1, the seller's revenue with bundling and without resale is equal to:

$$\Pi_{NR}^B = min\left\{v_S^1 + v_S^2; v_W^1 + v_W^2\right\}.$$

This is clearly (weakly) lower than Π_R^B. (Notice that $\Pi_R^B=\Pi_{NR}^B$ if and only if $\alpha=1$ and $v_S^1+v_S^2<v_W^1+v_W^2$) Finally, from Lemma 1, the seller's revenue without bundling and without resale is equal to:

$$\Pi_{NR}^{NB} = \begin{cases} 2v_W^1 & \text{if bidders do not reduce demand } \left(\text{i.e., if } v_S^2 > 2v_W^1\right); \\ 0 & \text{if bidders reduce demand } \left(\text{i.e., if } 2v_W^1 > v_S^2\right). \end{cases}$$

When $2v_W^1 > v_S^2$ this is clearly lower than Π_R^B. When $v_S^2 > 2v_W^1$,

$$\Pi_R^B > \Pi_{NR}^{NB} \Leftrightarrow \alpha\left(v_S^1 + v_S^2\right) + (1-\alpha)\left(v_W^1 + v_W^2\right) > 2v_W^1$$

$$\Leftrightarrow \alpha > \frac{v_W^1 - v_W^2}{v_S^1 + v_S^2 - v_W^1 - v_W^2}. \qquad \square$$

Proof of Lemma 5. We are going to show that, if and only if $p > \frac{v_S^2 - 2v_W^1}{(1+\alpha)(v_S^2 + v_W^1)}$, no bidder has a profitable deviation from the bidding strategies $\mathbf{b}_S = (v_S^1 - p(1-\alpha)(v_S^1 - v_W^2);\ 0)$ and $\mathbf{b}_W = (v_W^1 + p\alpha(v_S^2 - v_W^1);\ 0)$. First notice that bidders' bids for the first unit are not dominated, because they are not higher than bidders' willingness to pay.

In the candidate equilibrium, the auction price is equal to zero and each bidder wins one of the units on sale. Then, with probability p, bidder W resells his unit in the resale market at price $\alpha v_S^2 + (1-\alpha)v_W^1$. Hence, bidder S obtains a total expected profit of:

$$\pi_S^* = v_S^1 + p(1-\alpha)\left(v_S^2 - v_W^1\right).$$

By contrast, if bidder S outbids bidder W, she wins two units but raises the auction price for both units up to bidder W's bid for the first unit—i.e., $v_W^1 + p\alpha(v_S^2 - v_W^1)$. Hence, her total profit is:

$$\pi'_S = v_S^1 + v_S^2 - 2\left[v_W^1 + p\alpha\left(v_S^2 - v_W^1\right)\right].$$

It follows that bidder S does not deviate from the strategies described if and only if:

$$\pi_S^* > \pi'_S \Leftrightarrow p(1-\alpha)\left(v_S^2 - v_W^1\right) > v_S^2 - 2v_W^1 - p2\alpha\left(v_S^2 - v_W^1\right)$$

$$\Leftrightarrow p > \frac{v_S^2 - 2v_W^1}{(1+\alpha)(v_S^2 + v_W^1)}.$$

Now consider bidder W. In the candidate equilibrium, with probability p bidder W obtains a profit equal to the resale price at which he resells one unit to bidder S; while with probability $(1-p)$ he does not resell the unit and obtains a profit equal to his valuation. Hence, his total expected profit is:

$$\pi_W^* = (1-p)v_W^1 + p\left[\alpha v_S^2 + (1-\alpha)v_W^1\right].$$

In order to outbid bidder S and win two units to resell in the aftermarket, bidder W has to raise the auction price up to $v_S^1 - p(1-\alpha)(v_S^1 - v_W^2)$, which is bidder S's bid for the first unit. In this case, his total expected profit is:

$$\pi'_W = (1-p)\left(v_W^1 + v_W^2\right) + p\left[\alpha\left(v_S^1 + v_S^2\right) + (1-\alpha)\left(v_W^1 + v_W^2\right)\right]$$
$$- 2\left[v_S^1 - p(1-\alpha)\left(v_S^1 - v_W^2\right)\right].$$

It follows that bidder W does not deviate from the equilibrium described if and only if:

$$\pi_W^* > \pi'_W \Leftrightarrow (1-p)v_W^2 + p\alpha v_S^1 + p(1-\alpha)v_W^2 - 2\left[v_S^1 - p(1-\alpha)\left(v_S^1 - v_W^2\right)\right] < 0$$

$$\Leftrightarrow p < \frac{2v_S^1 - v_W^2}{(2-\alpha)v_S^1 - (2-\alpha)v_W^2}.$$

But because $\frac{2v_S^1 - v_W^2}{(2-\alpha)v_S^1 - (2-\alpha)v_W^2} > 1$, it is never profitable for bidder W to deviate from the strategies described. $\square$

Proof of Proposition 3. By Lemma 1, if $v_S^2 > 2v_W^1$ bidders do not reduce demand when resale is not allowed. By Lemma 5, if $p > \frac{2(v_S^2 - 2v_W^1)}{3(v_S^2 + v_W^1)}$ bidders reduce demand when resale is allowed. In this case, each bidder wins one unit but, with probability $(1-p)$, bidders are unable to trade in the resale market and the allocation is inefficient. $\square$

Proof of Lemma 6. Let $\alpha v_S^1 + (1-\alpha)v_W^2 = x$ and $\alpha v_S^2 + (1-\alpha)v_W^1 = y$, so that bidders' willingness to pay is:

	1st unit	2nd unit
S	$x+c$	$y+c$
W	$y-c$	$x-c$

We are going to prove that, if bidder W does not reduce demand, then bidder S has no incentive to reduce demand either.

Firstly assume that $x > y$. Let $b_W^1 \in [x+c;\ y+c]$ be bidder W's bid for the first unit. Since bidder W has a higher willingness to pay for the second unit and she does not reduce demand, her bid for the second unit, b_W^2, is never lower than b_W^1. Hence, bidder S can outbid bidder W and win two units in the auction at price b_W^1 each. If instead bidder S unilaterally reduces demand, she wins one unit in the auction at price b_W^2 and she purchases the second unit from bidder W in the resale market at price y, paying also the resale cost c. Therefore, bidder S strictly prefers not to reduce demand.

Secondly assume that $x < y$. Since bidder W does not reduce demand, he bids $\mathbf{b}_W = (y-c;\ x-c)$. In this case, regardless of whether she reduces demand or not, bidder S always wins one unit in the auction (because her bid for the first unit is higher than bidder W's bid for the second unit). If bidder S reduces demand, she buys the second unit in the resale market at price y, and also pays the resale cost c. Therefore, bidder S has no incentive to unilaterally reduce demand. $\square$

Proof of Proposition 4. Follows from Lemma 6 and the discussion preceding the statement. $\square$

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