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Sorry Winners

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Abstract Bidders who receive both “common-value” and “private-value” signals about the value of an auction prize cannot fully infer their opponents’ information from the bidding. So bidders may overestimate the value of the prize and, subsequently, regret winning. When multiple objects are on sale, bidding in an auction provides information relevant to the other auctions, and sequential auctions are more vulnerable to overpayment and winners’ regret than are simultaneous auctions. With information inequality among bidders, the seller’s revenue is influenced by two contrasting effects. On the one hand, simultaneous auctions reduce the winner’s curse of less informed bidders and allow them to bid more aggressively. On the other hand, sequential auctions induce less informed bidders to bid more aggressively in early auctions to acquire information.

Keywords asymmetric bidders · auctions · overpayment

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1 Introduction

After an auction terminates, winning bidders sometimes complain that they overbid—i.e., they bid a price higher than the actual value of the object on sale—and are reluctant

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to pay the auction price.[1] But when bidders have either common or private valuations, it is not clear how *rational* players can overbid in an auction.

However, in actual practice, bidders generally receive both *common-value* and *private-value* signals about the value of the object on sale (Milgrom and Weber 1982).[2] A common-value signal affects the valuation of all bidders alike (e.g., the demand estimate for the mobile-phone services that can be provided by the winner of a license sold in an auction) while a private-value signal only affects the valuation of the bidder who receives it (e.g., a firm's production efficiency and cost). When these different types of information influence the value of the auction prize, bidding behavior depends on all of them, but it does not fully reveal any of them. Then even a perfectly rational bidder cannot fully infer his opponents' information from the bidding and may overestimate the prize value, if he faces a rival with a large private signal. Therefore, a rational bidder may overbid and regret winning the auction after discovering that he overpaid.

Non-rational bidders may overbid because of the "winner's curse"—i.e., if they fail to take into account the information conveyed by winning an auction. However, at least in high-stake auctions, there is evidence that bidders do take into account the winner's curse when choosing their bids (see, e.g., Hendricks et al. 2003). But even rational bidders, who take into account all the information conveyed by winning, may overbid in auctions with both common-value and private-value signals.[3]

Of course, the prize value may be subject to pure ex-post risk, and so auction winners may obtain additional information from exogenous sources after the auction that make them reduce their initial estimate of the prize value (e.g., Eso and White 2004). For instance, a new technical analysis with pessimistic views about the possible use of a mobile-phone license sold in an auction may only become available after the end of the auction. By contrast, our point is that an auction winner also obtains information from his opponents' bidding in other auctions, and this can reveal that the auction winner overpaid, without any exogenous information becoming available.

When multiple objects with non-independent values are auctioned, the order of sale is crucial for information revelation, because prices in an auction are a source of information about a bidder's valuation in other auctions. In sequential auctions, winning a second auction at a low price after also winning a first auction conveys bad news, since it reveals that your opponents' estimate of the prize value is lower than you expected when winning the first auction. So an auction winner may discover that he

[1] For example, soon after winning the 1996 C-Block spectrum auction in the US, NextWave Telecom and General Wireless filed for bankruptcy to avoid paying their bids (Board 2007). And many winners of the European 3G licenses auctions in 2000 complained that the auction prices were higher than the value of the licenses ("The Telecoms Begging Bowl," *The Economist*, 3 May 2001).

[2] The literature on auctions with private and common signals also includes Compte and Jehiel (2002), De Frutos and Pechlivanos (2006), Goeree and Offerman (2003), Jehiel and Moldovanu (2000), Maskin (1992), Pagnozzi (2007), and Pesendorfer and Swinkels (2000).

[3] Rational bidders may also consciously overbid because of the presence of externalities (Jehiel and Moldovanu 1996), and in simultaneous ascending auctions because of the "exposure problem"—which arises when bidders consider two or more objects as complements—if there are budget constraints (Brusco and Lopomo 2007), or if there are bidders interested in winning only few objects (Zheng 2006). By contrast, we show that overbidding may arise even without externalities or complementarities.

overpaid, given the result of a later auction. And the evidence that an auction winner overpaid has political costs for the seller, induces litigation after the auction, and may create embarrassment to managers of bidding firms. Moreover, if managers suffer an explicit "embarrassment cost" when they win an auction but pay more than the prize is worth, they may bid less aggressively in early auctions, or even fail to participate.

In contrast to sequential auctions, in simultaneous auctions bidders obtain more information while they can still modify their bidding strategies. So simultaneous auctions greatly reduce the risk of winners' regret, and sellers may prefer them to sequential auctions.

But even if with sequential sales winners learn that they are losing money in one auction, aggregating across all auctions they may still earn positive profits ex-post. A lower price in a later auction conveys bad news about the profitability of an earlier auction but, at the same time, allows winners to obtain the prize relatively cheaply. So sellers should be cautious when evaluating bidders' complaints of overpayment, if these are based on lower prices than expected in some auctions.

Bidding strategies in multi-object auctions with non-independent values display some interesting characteristics. In simultaneous auctions, bidders bid more aggressively (than in independent single-object auctions), because more information is revealed during the bidding and this reduces their potential winner's curse. But, after winning one auction, a bidder may drop out of the other auction immediately, because observing an opponent dropping out conveys bad news about the prize value.

In sequential auctions, bidding strategies in later auctions depend on the result of earlier ones. And in early auctions, bidders have an incentive to bid more than their expected valuation conditional on winning, in order to obtain information that could be valuable in later auctions. (This is consistent with the "declining price anomaly" in the sequential sale of identical goods—see, e.g., Ashenfelter 1989.) Milgrom and Weber (2000) argue that, in sequential auction, "[t]o the extent that having 'better' information has value in later rounds, a bidder may choose to bid a bit higher in the first round in order to have a better estimate of the winning bid, should he lose," but the authors do not provide a proof. By analyzing a simpler model than the one that they consider, we show that this claim is indeed correct.

Therefore, with multiple objects the order of sale affects the seller's revenue.[4] We analyze a model in which information is unequally distributed among bidders and show that the timing of the auctions has two contrasting effects on the seller's revenue: (i) simultaneous auctions reduce information asymmetries more than do sequential auctions, and this allows less informed bidders to bid more aggressively; and (ii) sequential auctions induce less informed bidders to bid aggressively in early auctions to discover their opponents' information. When the second effect is absent, simultaneous auctions yield a higher expected revenue for the seller than do

[4] When bidders receive both private and common signals, the revenue equivalence theorem does not hold since the allocation of prizes among bidders depends on the auction format.

sequential auctions.[5] We also show that reducing information inequality among bidders in simultaneous auctions increases the seller's revenue.

When European governments independently auctioned "third-generation" (3G) mobile-phone licenses in 2000 and 2001, prices were generally higher in auctions run earlier, and much lower in later auctions.[6] Based in part on the evidence provided by the lower prices in later auctions, winners of earlier ones complained that they overpaid and successfully lobbied governments for improved license conditions.[7] Our analysis suggests that governments may have done better by organizing a simultaneous sale of all 3G licenses: It would then have been more difficult for winning firms to litigate and lobby after the auctions. Total revenue and efficiency may also have been higher.

The rest of the paper is organized as follows: Section 2 starts by considering a single-object auction with both private- and common-value signals and shows how a rational bidder may overpay for an auction prize. We then introduce a simple two-object auction model to investigate the effect of the order of sale on bidders' profits. Section 4 analyzes sequential auctions and shows that bidders may be "sorry winners." Section 5 analyzes simultaneous auctions and compares them to sequential ones in terms of possible winners' regret. The seller's revenue is discussed in Section 6. The last section concludes. All proofs are in the Appendix.

2 Single-Object Auction

We start by analyzing a single-object ascending auction in order to illustrate that a winner may overestimate the value of the object on sale when observing his competitors bidding aggressively, and end up paying more than the object is actually worth.[8] Bidders' strategies in a single-object auction also provide a benchmark for the analysis of multi-object auctions in Sections 4 and 5.

Our model is a special case of the "general auction model" of Milgrom and Weber (1982). We assume that bidders' valuations have an additive form as in the "wallet game" of Klemperer (1998) and Bulow and Klemperer (2002), and as in Compte and Jehiel (2002).

[5] There are also reasons, which are not considered here, why a simultaneous auction may raise less revenue. For example, signalling and retaliation is easier during a simultaneous auction, and firms can use them to sustain collusion (e.g., Brusco and Lopomo 2002).

[6] For example, in euros per capita, prices were 650 and 615 in the UK and German auctions run in March and July 2000 respectively, but were less than 100 in all of the auctions in 2001 (Klemperer 2002).

[7] In Germany and the UK, the winning firms were allowed to share infrastructure building costs (although it is commonly argued that this makes collusion easier). In Germany, the winners also lobbied to be allowed to merge keeping all the licenses won (which was explicitly forbidden by the auction rules for competition reasons). Licences were also lengthened in other countries.

[8] In an ascending auction the price is raised continuously by the auctioneer, and bidders who wish to be active at the current price depress a button. When a bidder releases the button, he is withdrawn from the auction. The number of active bidders is continuously displayed, and the auction ends when only one active bidder is left.

An object—for example a mobile-phone license—is sold by an ascending auction with two risk-neutral bidders, called E and I_1, whose valuations are:

$$\begin{cases} V_E = \theta + t_E, \\ V_1 = \theta + t_1. \end{cases}$$

We call θ the common-value signal (or simply common signal) and t_i the private-value signal of bidder i (or simply private signal). Bidders' valuations are partly private and partly common: the signal θ affects all valuations alike while the signal t_i only affects the valuation of bidder i. We assume that each bidder is privately informed about his private signal and bidder I_1 (but not bidder E) also knows the common signal θ.[9] Signals are independently and identically distributed.

We can interpret t_i as expressing bidder i's production cost (or efficiency level) or the financial cost that bidder i has to pay in order to raise money to bid, which depends on the particular credit condition he can obtain from financial institution. On the other hand, θ represents an intrinsic characteristic of the object—for example, the future level of demand for mobile-phone services—which affects the profitability of the license for all bidders.

A bidder obtains a profit equal to the difference between his valuation and the auction price if he wins the auction, and zero otherwise. The next lemma describes equilibrium bidding strategies.

Lemma 1. *In the unique equilibrium in undominated strategies of the single-object auction, bidder I_1 bids up to $\theta + t_1$ and bidder E bids up to $2t_E$.*

We then have the following result:

Proposition 1. *Bidder E wins the auction but obtains negative profit if and only if:*

$$t_1 > t_E > \frac{1}{2}(\theta + t_1).$$

Knowing his opponent's bid does not tell bidder E all relevant information since bidder I_1's bid is not a sufficient statistic for his private information (as it is the case when private information has only one dimension). So bidder E's profit can be negative if bidder I_1's private signal is sufficiently higher than the common signal, because then I_1 bids aggressively and E rationally expects the common signal to be high.[10]

Definition 1. *A bidder is a "sorry winner" if he realizes after winning an auction that the value of the prize is lower than the price he paid.*

[9] Results analogous to those presented in this section would be obtained by assuming that both bidders receive a signal about θ, but no bidder knows θ. However, it is crucial for our results that bidder E does not know θ.

[10] Whenever bidder E overpays, the outcome of the auction is also inefficient because bidder E wins even if he has a lower valuation than bidder I_1. Maskin (1992), Jehiel and Moldovanu (2000), and Goeree and Offerman (2003) prove in much more general settings that, with multidimensional signals, auctions can be ex-post inefficient.

So a sorry winner regrets winning at a price he was willing to bid during the auction. However, in a single-object auction, bidder E never becomes aware that he overpaid (until he actually obtains his profit). But after an auction ends, the winner may learn additional negative information about the common signal or about his rival's private signal (even before he actually obtains his profit). Our general point is that, when multiple objects are on sale, other auctions become a source of information, and the sequence of sale is crucial for the revelation of information. Moreover, a bidder may find it convenient to bid more than the expected value of the prize in some auctions, and so willingly overpay, even before any additional information is revealed.

3 The Model

Consider two ascending auctions for two objects (A and B)—e.g., two mobile-phone licenses—with three risk-neutral bidders (E, I_1, and I_2). Let V_E be bidder E's valuation for each of the objects on sale, and let V_i^j be bidder I_i's valuation for object j. Bidders' valuations in the two auctions are:

		Bidder		
		E	I_1	I_2
Auction	A	$V_E = \theta + t_E$	$V_1^A = \theta + t_1$	$V_2^A = 0$
	B	$V_E = \theta + t_E$	$V_1^B = 0$	$V_2^B = \theta + t_2$

All bidders' signals (θ, t_E, t_1 and t_2) are independently drawn from a uniform distribution on [0, 1]. We assume that each bidder i is privately informed about his private signal t_i. Bidders I_1 and I_2 also know the common signal θ, while bidder E does not know it. So bidders are asymmetrically informed. We will think of bidders I_1 and I_2 as incumbent firms in market A and B respectively, who are better informed about their own market's characteristics (e.g., they are able to better estimate future demand) and are only interested in bidding in their home market. Therefore, bidder I_1 only participates in auction A, and bidder I_2 only participate in auction B. Bidder E is a potential new entrant who is less informed about the markets' characteristics and competes in both auctions. For simplicity, bidder E has the same valuation in both auctions.

Our results would not be affected by the presence of more entrants, because the strategy of an entrant does not convey information relevant to other bidders. By contrast, if there are more incumbent firms that are informed about θ, bidder E can obtain valuable information from their bidding strategies, and this reduces, but does not eliminate, the risk of overpayment.

We assume that bidder E faces two different incumbents in the two auctions, so that we do not have to take into account a possible signalling strategy by a single informed incumbent. For example, a single incumbent may want to manipulate his bid strategically in one auction in order to signal misleading information about θ to the entrant and affect his bidding strategy in the other auction (Ortega-Reichert 2000). We deliberately neglect these issues in order to focus on the effects of interest.

In each auction, a bidder's profit is equal to the difference between his valuation and the auction price if he wins the auction, and zero otherwise. Bidders I_1 and I_2 know their valuations for the objects on sale in the auction in which they participate. Therefore, in an ascending auction, it is a dominant strategy for them to bid up to their valuation.

Lemma 2. *It is a weakly dominant strategy for bidder I_1 to bid up to $\theta + t_1$ in auction A, and for bidder I_2 to bid up to $\theta + t_2$ in auction B.*

So the bidding strategies of bidders I_1 and I_2 do not depend on the order of sale. By contrast, the strategy of bidder E is affected by the order of sale. In each auction, bidder E's bidding behavior depends on two factors: (i) his expected valuation for the auction's prize, conditional on winning and on all the information available about the common signal; (ii) the information that the bidding process reveals, which could be valuable in the other auction. Both of these factors may be affected by the order of sale. In the following sections, we describe the different bidding strategies of bidder E, depending on the order of sale.

4 Sequential Auctions

Assume that the two objects are sold by sequential ascending auctions. Without loss of generality, the auction for object A is run first.

Bidder E's strategy in auction B depends on the outcome of auction A. If bidder E wins auction A at price p_A, he learns that $\theta + t_1 = p_A$. Therefore, conditional on also winning auction B at price p_B, he expects the common signal to be equal to:[11,12]

$$\mathbb{E}\left[\theta \,|\theta + t_1 = p_A,\ \theta + t_2 = p_B\right] = \begin{cases} \frac{1}{2}\min\{p_A; p_B\} & \text{if } \max\{p_A; p_B\} \leq 1, \\ \frac{1}{2}(p_A + p_B - 1) & \text{if } \min\{p_A; p_B\} < 1 < \max\{p_A; p_B\}, \\ \frac{1}{2}\max\{p_A; p_B\} & \text{if } 1 \leq \min\{p_A; p_B\}. \end{cases}$$

And in auction B, as in a single-object auction, bidder E bids up to his expected valuation of the prize, conditional on all the information acquired.

Lemma 3. *In sequential auctions: (i) if bidder E wins auction A at price $p_A < 2t_E$, then in auction B he bids up to:*

$$\beta_1(t_E, p_A) = \begin{cases} t_E + \frac{1}{2}p_A & \textit{if} \;\; p_A \leq 2(1 - t_E), \\ 2t_E + p_A - 1 & \textit{if} \;\; 2(1 - t_E) < p_A \leq 1, \\ 2t_E & \textit{if} \;\; 1 < p_A; \end{cases}$$

[11] Throughout the paper, to save notation we denote by $\mathbb{E}[\theta|\mathcal{I}]$ the expectation computed by bidder E of the random variable whose realization is θ, given the information $\mathcal{I}$.

[12] If p_A and p_B are both lower than 1, the highest price is uninformative about θ; hence, $\mathbb{E}[\theta\,|\theta + t_1 = p_A,\ \theta + t_2 = p_B] = \mathbb{E}\left[\theta\,|\theta + t_i = \min\{p_A, p_B\}\right]$. Similarly, if p_A and p_B are both higher than 1, the lowest price is uninformative about θ; hence, $\mathbb{E}[\theta\,|\theta + t_1 = p_A,\ \theta + t_2 = p_B] = \mathbb{E}\left[\theta\,|\theta + t_i = \max\{p_A, p_B\}\right]$.

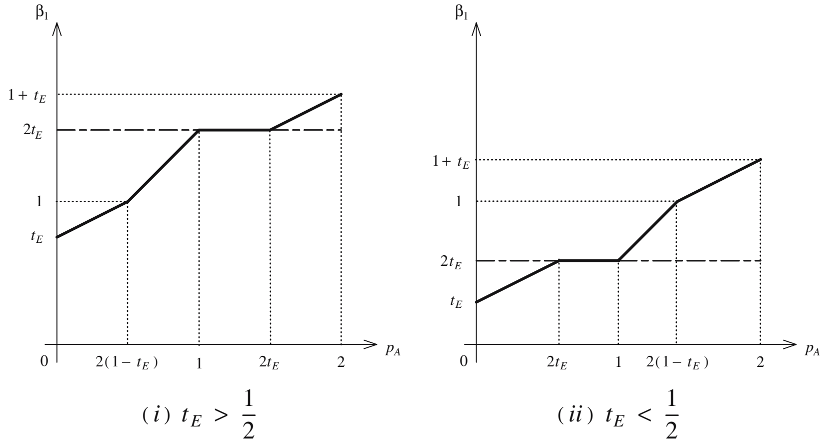

Fig. 1 Bidder E's bid in auction B after winning auction A

(ii) *if bidder E wins auction A at price* $p_A \geq 2t_E$*, then in auction B he bids up to:*

$$\beta_1(t_E, p_A) = \begin{cases} 2t_E & \textit{if} \;\; p_A \leq 1, \\ 2t_E + p_A - 1 & \textit{if} \;\; 1 < p_A \leq 2(1 - t_E), \\ t_E + \frac{1}{2}p_A & \textit{if} \;\; 2(1 - t_E) < p_A. \end{cases}$$

Figure 1 represents E's bidding strategy in auction B, as a function of the price p_A at which he wins auction A (in the two cases: $t_E < \frac{1}{2}$ and $t_E > \frac{1}{2}$). Notice that: (i) for $p_A < 2t_E$, $\beta_1(t_E, p_A) \leq 2t_E$; and (ii) for $p_A > 2t_E$, $\beta_1(t_E, p_A) \geq 2t_E$—winning auction A at a low (high) price is bad (good) news about θ and leads bidder E to bid less (more) aggressively in auction B (compared to a single-object auction).

Consider now the first auction. Raising the price in auction A provides bidder E with valuable information about the common signal. Moreover, after winning at a price lower than $2t_E$ (the price at which the expected value of the object conditional on winning is equal to the auction price), bidder E earns positive expected profits in auction A. Hence, bidder E never drops out at a price lower than $2t_E$. However, we cannot conclude that bidder E drops out at price $2t_E$ (as in a single-object auction), since he may have an incentive to bid higher in order to obtain more information about θ and, hence, have a better bid in auction B.

Lemma 4. *In sequential auctions, bidder E bids up to a price strictly higher than* $2t_E$ *in the first auction.*

The proof of Lemma 4 shows that bidder E is always better off bidding up to $2t_E + \varepsilon$, for ε small enough, rather than dropping out at $2t_E$, regardless of whether, as a consequence, he wins auction A or not. Firstly, if bidder E loses auction A at price $2t_E + \varepsilon$, he only learns valuable additional information about the common signal, and can bid more accurately in auction B. Secondly, if bidder E wins auction A at price $2t_E + \varepsilon$, then he loses, in expectation, an amount of order ε in auction A. But, given the additional information about the common signal, bidder E also reduces his bid in auction B by an amount of order higher than ε. And this prevents him from winning auction B at a price higher than his valuation, and losing an amount of order higher than ε (which happens with a probability of order higher than ε).

So bidder E does not drop out of auction A when the price reaches his expected valuation conditional on winning because, even when this leads him to win and overpay in auction A, he avoids the risk of overpaying a much larger amount in auction B.[13] Unfortunately, bidder E's equilibrium bidding function in auction A is intractable.

Summing up, compared to a single-object auction, in sequential auctions: (i) bidder E bids more aggressively in the first auction in order to learn information about the common signal which he can use in the second auction; (ii) if he wins the first auction at a price lower than $2t_E$, bidder E bids less aggressively in the second auction; (iii) if he wins the first auction at a price higher than $2t_E$ or if he loses the first auction, bidder E bids more aggressively in the second auction.

4.1 Sorry Winners

As in a single-object auction, in sequential auctions bidder E wins the first auction but pays more than the object is worth when $t_1 > t_E > \frac{1}{2}(\theta + t_1)$.[14] In addition, bidder E may also overpay in the first of two sequential auctions because of the incentive to bid more than his expected valuation conditional on winning.

There are two different reasons why bidder E may become a sorry winner in sequential auctions. Firstly, when bidder E deliberately bids more than $2t_E$ in auction A to obtain information about the common signal and then wins the auction, he is aware that he overpaid as soon as auction A terminates. In this case, bidder E does not even need the additional information provided by auction B to regret winning. When bidder E overpays out of a deliberate choice—in essence an investment—to trade off the potential loss in the first auction with the possibility of improving his bid in the second auction, we refer to him as a "strategic" sorry winner.

Secondly, bidder E also learns additional information about the common signal from bidder I_2's bidding in auction B. Hence, he may become a sorry winner only after auction B terminates, and even if he does not deliberately overbid in auction A. In general, winning auction B at a price lower than the price he paid in auction A is bad news for bidder E about the common signal and, hence, about his valuation for object A.

Proposition 2 (Sorry Winner in Expectation). *In sequential auctions, bidder E regrets winning the first auction at price p_A after winning the second auction at a price $p_B < p_A$ if and only if: (i) $t_E + \frac{1}{2}p_B < p_A < 1$; or (ii) $p_B \leq 1 \leq p_A$ and $p_A + 1 > 2t_E + p_B$; or (iii) $p_B > 1$ and $p_A > 2t_E$.*

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[13] When the same bidders compete in two sequential auctions, bidding strategies are also affected by consideration of information revelation. Specifically, a bidder has an incentive to shade his bid in the first auction in order not to reveal information that rival bidders will use to their advantage in the second auction (Ortega-Reichert 2000; De Frutos and Rosenthal 1998).

[14] In this case, as well as when bidder I_1 wins but has a lower valuation than bidder E, the auction is also inefficient ex-post.

Example 1. *Assume* $t_1 = \frac{3}{4}$, $\theta = t_2 = \frac{1}{4}$ *and* $t_E = \frac{1}{2} + \varepsilon$, *where* ε *is small. Then bidder E wins auction A at price* $p_A = 1$ *and auction B at price* $p_B = \frac{1}{2}$. *However, after auction B, bidder E expects his valuation for object A to be equal to* $t_E + \mathbb{E}\left[\theta \,\middle|\, p_A = 1,\ p_B = \frac{1}{2}\right] = \frac{1}{2} + \varepsilon + \frac{1}{4} < p_A$.

Hence, bidder E may expect to have paid more than the prize value in the first auction if, conditional on the information obtained in both auctions, his expected valuation for object A is less than the price he paid. In this case, bidder E's expected profit in auction A is negative. (But notice that bidder E may be a sorry winner in expectation even if he does not actually overpay for object A.) Moreover, after winning auction B, bidder E also learns that θ is at most equal to p_B (since he knows that $\theta + t_2 = p_B$), and hence that his valuation is at most equal to $t_E + p_B$. So the price in auction B may be so low that bidder E learns that he overpaid with certainty in auction A.[15]

Proposition 3 (Sorry Winner with Certainty). *In sequential auctions, bidder E wins both auctions and, after the second auction terminates, he is certain that he paid more than his valuation in the first auction if:*

$$t_E - \theta > t_1 - t_E > t_2.$$

Example 2. *Assume* $t_1 = 1$, $\theta = t_2 = 0$ *and* $t_E = \frac{1}{2} + \varepsilon$, *where* ε *is small. Then bidder E wins auction A at price* $p_A = 1$ *but, after also winning auction B, he learns that* $\theta = 0$, *and hence that he lost* $\frac{1}{2} - \varepsilon$ *in auction A.*

So the auction sequence provides additional information about the common signal and, therefore, about bidder E's value for object A. Given the result of a later auction, bidder E may discover that his valuation is lower than he expected and become a sorry winner. Of course, bidder E may also discover that he lost money in auction A if some information about θ is exogenously revealed after the auction. But our model suggests that, with sequential auctions, the order of sale endogenously reveals relevant information and can induce bidders' regret, even if no exogenous information is revealed.

The evidence that an auction winner overpaid may have serious political consequences. For instance, overpayment may create embarrassment to a government selling a public asset or to managers of bidding firms who have to justify their behavior to shareholders. Moreover, if managers suffer an explicit "embarrassment cost" when it is revealed that they overpaid, they may bid less aggressively or even refuse to participate in the auction.

But although in sequential auctions a bidder may discover he overpaid in the first auction after winning the second one at a low price, his total expected profit (aggregating across both auctions) may still be positive. Consider the following example.

Example 3. *Assume bidder E wins auction A at price* $p_A < 2t_E$ *and then wins auction B at price* $p_B \leq p_A$, *so that he may be a sorry winner in auction A. Bidder E's*

[15] This result is independent of the distribution of signals.

total expected profit in the two auctions is:[16]

$$\mathbb{E}\left[\pi_E \mid p_A, p_B\right] = 2\left(\mathbb{E}\left[\theta \mid \theta + t_1 = p_A,\ \theta + t_2 = p_B\right] + t_E\right) - p_A - p_B$$
$$\geq 2\left(\tfrac{1}{2} p_B + t_E\right) - p_A - p_B > 0.$$

Our point is that the bad news about the common signal conveyed by a low price in the second auction is also itself good news about profits in the second auction.[17] For example, in the European 3G mobile-phone licenses auctions, telecom firms may have regretted winning in early auctions at relatively high prices given the outcome of later auctions; but the prices that these firms paid in later auctions were so low that, on aggregate, they may arguably still have earned positive profit. So governments should be suspicious of winning firms complaining that they overpaid, especially when the evidence is based on the lower prices paid by the same winners in a later auction.

Example 3 also shows that bidder E may end up earning negative total expected profits only if he willingly pays more than his expected valuation in the first auction and becomes a "strategic" sorry winner. This happens, for instance, if he willingly overpays in auction A and then just breaks even in auction B (i.e., if he loses auction B or wins at a price equal to his expected valuation). However, this outcome requires that the price in auction B is not much lower than the price paid by bidder E in auction A.[18]

5 Simultaneous Auction

Consider a simultaneous ascending auction for the two objects. For simplicity, we assume that the prices in the two auctions rise simultaneously and continuously. Each auction terminates (independently of the other) when one of the bidders drops out.

Bidder E's bidding in each auction depends on whether the other auction is still running or not. If bidder E wins one auction, say auction A, at price p while the other auction is still running, he knows that $\theta + t_2$ is higher than p, and hence he expects the common signal to be equal to:[19]

$$\mathbb{E}\left[\theta \mid \theta + t_1 = p,\ \theta + t_2 > p\right] = \begin{cases} \frac{3p - p^2}{6 - 3p} & \text{if} \quad p < 1, \\ \frac{4 - 3p^2 + p^3}{3(2-p)^2} & \text{if} \quad p \geq 1. \end{cases} \tag{1}$$

---

[16] The first inequality follows because, when p_B is lower than p_A, $\mathbb{E}\left[\theta \mid \theta + t_1 = p_A,\ \theta + t_2 = p_B\right] \geq \mathbb{E}\left[\theta \mid \theta + t_1 = p_B,\ \theta + t_2 = p_B\right] = \frac{1}{2} p_B$.

[17] In the working paper of this article (Pagnozzi 2005), we analyze various simplified versions of our model that allow us to compute closed-form equilibrium bidding strategies. Specifically, we assume bidder E's valuation in auction B does not depend on θ. Then in auction A bidder E has no incentive to bid more than his expected valuation conditional on winning (although he still obtains valuable information about θ from bidder I_2's behavior in auction B). We show that, in these models, although bidder E may be a sorry winner in auction A, if he wins both auctions he always expects to earn positive total profit ex-post.

[18] If bidder E wins auction A at a price slightly higher than $2t_E$, in auction B he bids up to a price approximately equal to $2t_E$. See the proof of Lemma 4 in the appendix.

[19] See the appendix for details about the derivation of equation (1).

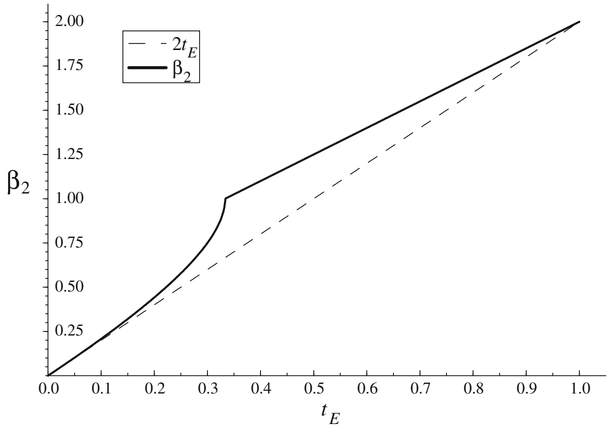

Fig. 2 Bidder E's bid in simultaneous auctions if he wins no object

In each auction, bidder E bids up to his expected valuation, conditional on the information conveyed by winning.[20]

Lemma 5. *In simultaneous auctions, if bidders I_1 and I_2 are still active, bidder E drops out of both auctions simultaneously at price:*

$$\beta_2(t_E) = \begin{cases} \frac{3}{4}\left(1 + t_E - \sqrt{1 - \frac{10}{3}t_E + t_E^2}\right) & \textit{if} \quad t_E < \frac{1}{3}, \\ \frac{1}{2} + \frac{3}{2}t_E & \textit{if} \quad t_E \geq \frac{1}{3}. \end{cases}$$

Figure 2 represents bidder E's bidding strategy in simultaneous auctions, as a function of his signal, conditional on both his opponents being still active. Notice that $\beta_2(t_E) > 2t_E$: Compared to a single-object auction, simultaneous auctions allow bidder E to bid more aggressively because, by observing both his opponents' bidding at the same time, bidder E learns additional valuable information about θ, and this reduces his potential winner's curse. The reason is as follows: Even if bidder E wins one auction, he knows that his opponent's signals in the other auction are higher than the current price, and this is good news about his valuation in the first auction.

Now suppose that bidder E wins one auction—say auction A—at price p_A, and hence learns that $\theta + t_1 = p_A$. Conditional on also winning auction B at price $p_B \geq p_A$, he expects the common signal to be equal to $\mathbb{E}[\theta \,|\theta + t_1 = p_A, \ \theta + t_2 = p_B]$. In auction B, bidder E bids up to his expected valuation, conditional on all the information acquired. However, bidder E *immediately* drops out of auction B if his expected

[20] If prices do not rise simultaneously in all auctions and bidder E can control the pace of the auctions (for instance, by bidding only on one object), then bidder E may have an incentive first to raise the price in only one of the auctions (and possibly even bid more than his expected valuation conditional on winning), in order to obtain more precise information about θ. However, simultaneous ascending auctions used in the real world (e.g., to sell spectrum licenses) typically include an "activity rule," which is designed precisely to prevent bidders from strategic slowing the pace of some of the auctions (by initially not bidding in them).

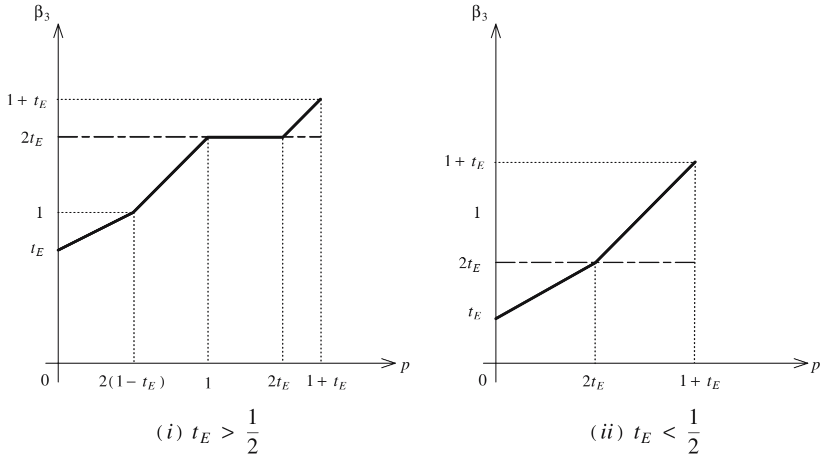

Fig. 3 Bidder E's bid in the remaining simultaneous auction, after winning one auction at price p

valuation is lower than the current price, because now the good news of the higher price in auction A is terminated.

Lemma 6. *In simultaneous auctions:* (i) *if bidder E wins one auction at price* $p < 2t_E$*, then in the other auction he bids up to:*

$$\beta_3(t_E, p) = \begin{cases} t_E + \frac{1}{2}p & \textit{if} \;\; p \leq 2(1-t_E), \\ 2t_E + p - 1 & \textit{if} \;\; 2(1-t_E) < p < 1, \\ 2t_E & \textit{if} \;\; 1 \leq p; \end{cases}$$

(ii) *if bidder E wins one auction at price* $p \geq 2t_E$*, then he immediately drops out of the other auction.*

Figure 3 represents bidder E's bidding strategy in one auction, conditional on his winning the other auction at price p. Notice that, for $p < 2t_E$, $\beta_3(t_E, p) \leq 2t_E$—as in sequential auctions, winning one auction at a price lower than $2t_E$ is bad news about θ and results in bidder E bidding less aggressively in the other auction (compared to a single-object auction).

Summing up, in a simultaneous auction: (i) bidder E bids more aggressively (compared to a single-object auction) in both auctions as long as both his competitors remain active; (ii) after winning one auction at a relatively low price, bidder E bids less aggressively in the remaining auction; (iii) after winning one auction at a relatively high price, bidder E *immediately* drops out of the remaining auction.

In contrast to sequential auctions, in simultaneous auctions bidder E's bidding strategy in each auction depends on all information revealed by the bidding of both of his competitors. Hence, bidder E learns whether any of them has low signals while the auctions are still running and he can still modify his bidding strategy. So the outcome of one of the auctions can never induce bidder E to regret winning the other auction, and bidder E always expects a positive profit in each of the auctions he wins, after all the auctions are over.

Proposition 4. *In simultaneous auctions, bidder E is never a sorry winner.*

Notice that bidder E may overpay even in a simultaneous auction, because he can still only estimate the value of the common signal. But a simultaneous auction does not endogenously provide evidence of overpayment and, hence, bidder E cannot realize he overpaid during the auction process. So the risk of embarrassment and political costs for sellers and managers of bidding firms is lower. Moreover, in the presence of an explicit "embarrassment cost," simultaneous auction may lead to more aggressive bidding and higher participation.

6 Seller's Revenue

In this section, we compare the two auction formats analyzed—simultaneous and sequential auctions—in terms of seller's revenue. We restrict attention to simultaneous and sequential auctions because these are commonly used auction formats, but neither format is an optimal selling mechanism in our model. In principle, there are more complex mechanisms through which the seller can exploit the interdependence of bidders' valuations to extract more of the bidders' surplus.[21]

In our model, bidders are asymmetrically informed about the common value of the objects on sale—bidder E does not know the value of the common signal θ, while his opponents do. Our analysis highlighted two contrasting effects of the order of sale on the seller's revenue. On the one hand, simultaneous auctions reveal more information than do sequential auctions during the bidding process and so allow a poorly informed bidder to bid more aggressively. On the other hand, in sequential auctions a poorly informed bidder has an incentive to bid more aggressively in the first auction, in order to obtain useful information for the second auction. (Notice that the second effect can also help explain the "declining price anomaly" in sequential auctions of identical goods—see, e.g., Ashenfeleter 1989.)

Our model does not allow us to evaluate the seller's expected revenue in sequential auctions, because bidder E's equilibrium strategy is intractable. However, when bidder E has no incentive to bid more than his expected valuation conditional on winning in the first of the two sequential auctions (e.g., because his valuation in auction B is independent of θ), simultaneous auctions yield a higher expected revenue than sequential auctions.[22] This is shown in the next result, which is based on numerical simulations.

Proposition 5. *Assume that bidder E's valuation in auction B is equal to* $1+t_E$ *(while all other valuations are as described in Section 3). Then the seller's total expected revenue is higher in simultaneous auctions than in sequential auctions. Moreover, expected efficiency is also higher in simultaneous auctions than in sequential auctions.*

[21] We also assume that there is no "embarrassment cost" for a bidder who overpays in an auction. With an explicit embarrassment cost (paid, for instance, by managers of bidding firms), bidders bid less aggressively if there is a risk of being sorry winners, and hence simultaneous auctions may yield an even higher seller's revenue than sequential auctions.

[22] By contrast, when bidders' signals are affiliated and bidders are symmetrically informed, Milgrom and Weber (2000) show that, because of the "linkage principle," sequential auctions yield a higher expected revenue for the seller than do simultaneous auctions.

The intuition for this result is that, when bidders are unequally informed about their valuations, the less informed bidder has to bid more cautiously to avoid the winner's curse. So information inequality among bidders reduces the seller's revenue. But then revealing information about θ reduces information inequality, allows the less informed bidder to bid more aggressively, and increases the seller's revenue. And simultaneous auctions reveal more information than do sequential ones.[23] Indeed, in simultaneous auctions, bidder E's strategy in each auction also depends on the information revealed in the other auction; while in sequential auctions, it is only bidder E's strategy in the second auction that depends on the information revealed in the first auction. And revealing more information about θ also reduces the risk of an inefficient allocation.

The conclusion that information inequality reduces the seller's revenue is confirmed by comparing simultaneous second-price sealed-bid auctions (i.e., a "static format") to simultaneous ascending auctions (i.e., a "dynamic format"), when bidder E's valuation in auction B is equal to $\theta + t_E$ (as in our main model). In each of two simultaneous second-price auctions, bidder E bids up to $2t_E$, his expected valuation conditional on winning, and the seller's total expected revenue in the two auctions is equal to 1.4.[24]

By contrast, in simultaneous ascending auctions, if the sum of the signals of bidder E's competitor in one auction is lower (higher) than $2t_E$, bidder E bids less (more) than $2t_E$ in the other auction. But numerical simulations show that the seller's total expected revenue in simultaneous ascending auctions is approximately equal to 1.46; hence, it is higher than in simultaneous second-price auctions. Again, the intuition is that simultaneous ascending auctions reduce information inequality among bidders, and this allows poorly informed bidders to bid more aggressively. Finally, with no information inequality (i.e., if all bidders know θ), the seller's revenue is even higher.[25]

Summing up, our results suggest that, with information inequality, an auction mechanism that reveals more information during the bidding process increases the seller's revenue. Notice that, because in our model bidders' signals are independent, this effect is different from the "linkage principle" of Milgrom and Weber (1982), which instead arises when bidders' signals are affiliated.

7 Conclusions

If players receive both private- and common-value signals about the value of the object on sale, rational bidders may regret winning an auction and become sorry winners. And sorry winners create political costs to sellers.

23 So better informed bidders should prefer sequential auctions, because simultaneous auctions reduce their information rent.

24 In simultaneous second-price auctions, no information about θ is revealed to bidder E before the auctions terminate and, conditional on winning one auction, bidder E only learns that the sum of his opponent's signals in that auction is equal to the price he pays, exactly as in a single-object auction. And in a single-object auction the seller's expected revenue is $\mathbb{E}\left[2t_E \,|\theta + t_i > 2t_E\right] \cdot \Pr\left(\theta + t_i > 2t_E\right) + \mathbb{E}\left[\theta + t_i \,|2t_E > \theta + t_i\right] \cdot \Pr\left(2t_E > \theta + t_i\right) = 0.7$.

25 When θ is known, it is a dominant strategy for each bidder to bid up to his known valuation in both second-price and ascending auctions. Hence, the seller's expected revenue is equal to $2\left(\mathbb{E}\left[\theta\right] + \mathbb{E}\left[t_i \,\middle|t_i < t_j\right]\right) = 1.\overline{6}$.

When multiple objects are on sale, in sequential auctions lower prices in later auctions can provide proof of overpayment in earlier auctions (as they supposedly did in the European 3G auctions), but winners may still expect positive profits on aggregate, because lower prices also imply higher profits in later auctions. So sellers should be cautious when evaluating winners' complaints of overpayment in sequential auctions. Nonetheless, sellers may prefer simultaneous auctions since they reduce the risk of sorry winners and lobbying. And simultaneous auctions, by reducing information inequality, may also increase the seller's revenue and efficiency.

However, given the possibility of independent auctions, each seller has an incentive to run its auction first to maximize revenue (since poorly informed bidders bid more aggressively in earlier auctions to acquire information), although this increases the risk of litigation (since overbidding is more likely in earlier auctions).

A. Appendix

Proof of Lemma 1. First notice that it is a weakly dominant strategy for bidder I_1 to bid up to his valuation. If bidder I_1 drops out of the auction at price p, then bidder E expects the common signal to be equal to:

$$\mathbb{E}\left[\theta | \theta + t_1 = p\right] = \tfrac{1}{2} p,$$

because signals are $i.i.d.$ Therefore, conditional on winning, bidder E expects his valuation to be equal to:

$$\mathbb{E}\left[V_E | E \text{ wins at price } p\right] = t_E + \tfrac{1}{2} p.$$

In an ascending auction, a player bids up to the expected value of the prize, conditional on winning. Hence, bidder E bids up to p^* such that:

$$p^* = \mathbb{E}\left[V_E | E \text{ wins at price } p^*\right] \quad \Rightarrow \quad p^* = 2t_E. \qquad \blacksquare$$

Proof of Proposition 1. Bidder E wins the auction if and only if $2t_E > \theta + t_1$. His profit is negative if and only if $t_1 > t_E$. Rearranging yields the statement. $\blacksquare$

Proof of Lemma 3. Assume that bidder E wins auction A at price p_A. Then in auction B he bids up to:

$$p^* = t_E + \mathbb{E}\left[\theta \,\middle|\, \theta + t_1 = p_A,\ \theta + t_2 = p^*\right].$$

For $p_A \leq 2t_E$,[26]

$$p^* = t_E + \begin{cases} \frac{1}{2} p_A & \text{if} \quad p_A < p^* \leq 1, \\ \frac{1}{2}(p^* + p_A - 1) & \text{if} \quad p_A < 1 < p^*, \\ \frac{1}{2} p^* & \text{if} \quad 1 \leq p_A < p^*. \end{cases} \tag{A.1}$$

If $2(1 - t_E) < 1 < 2t_E$—i.e., if $t_E > \frac{1}{2}$—(A.1) implies:

$$p^* = \begin{cases} t_E + \frac{1}{2} p_A & \text{if} \quad p_A \leq 2(1 - t_E), \\ 2t_E + p_A - 1 & \text{if} \quad 2(1 - t_E) < p_A < 1, \\ 2t_E & \text{if} \quad 1 \leq p_A \leq 2t_E. \end{cases}$$

If, on the other hand, $2t_E < 1 < 2(1 - t_E)$, (A.1) implies $p^* = t_E + \frac{1}{2} p_A$.

For $p_A > 2t_E$,

$$p^* = t_E + \begin{cases} \frac{1}{2} p^* & \text{if} \quad p^* < p_A \leq 1, \\ \frac{1}{2}(p^* + p_A - 1) & \text{if} \quad p^* < 1 < p_A, \\ \frac{1}{2} p_A & \text{if} \quad 1 \leq p^* < p_A. \end{cases} \tag{A.2}$$

If $2t_E < 1 < 2(1 - t_E)$, (A.2) implies:

$$p^* = \begin{cases} 2t_E & \text{if} \quad p_A \leq 1, \\ 2t_E + p_A - 1 & \text{if} \quad 1 < p_A < 2(1 - t_E), \\ t_E + \frac{1}{2} p_A & \text{if} \quad 2(1 - t_E) \leq p_A. \end{cases}$$

If, on the other hand, $2(1 - t_E) < 1 < 2t_E$, (A.2) implies $p^* = t_E + \frac{1}{2} p_A$. ■

Proof of Lemma 4. Notice that:

$$p \leq \mathbb{E}\left[V_E \middle| E \text{ wins } A \text{ at price } p \right] = t_E + \mathbb{E}[\theta | \theta + t_1 = p] \quad \Leftrightarrow \quad p \leq 2t_E.$$

Therefore, by winning at any price up to $2t_E$, bidder E expects to earn positive profits in auction A since he pays less than his expected valuation, conditional on winning. Moreover, dropping out earlier would only damage him in auction B, since it would prevent him from learning valuable information about θ. So bidder E has no incentive to drop out of auction A before the price reaches $2t_E$.

[26] For $X, Y, W \backsim U[0, 1]$ and $0 \leq a < b \leq 2$,

$$\mathbb{E}[X | X + Y = a, \; X + W = b] = \begin{cases} \frac{1}{2} a & \text{if} \quad a < b \leq 1, \\ \frac{1}{2}(a + b - 1) & \text{if} \quad a < 1 < b, \\ \frac{1}{2} b & \text{if} \quad 1 \leq a < b. \end{cases}$$

Assume that the price has reached $2t_E$ and E bids up to $p = 2t_E + \varepsilon$, for ε small. If E loses the auction anyway, then he learns valuable additional information about θ (i.e., that $\theta + t_1 > 2t_E + \varepsilon$) and he can bid more accurately in auction B. Therefore, given that he loses auction A anyway, bidder E prefers not to drop out at price $2t_E$.

However, by bidding up to $p = 2t_E + \varepsilon$, bidder E runs the risk of winning auction A and losing, in expectation,

$$p - \mathbb{E}\left[V_E | \theta + t_1 = 2t_E + \varepsilon\right] = 2t_E + \varepsilon - \left(t_E + \frac{2t_E+\varepsilon}{2}\right) = \frac{\varepsilon}{2}.$$

But, also in this case, given the additional information about θ, bidder E is able to bid more accurately in auction B.

If bidder E drops out at price $2t_E$, then in auction B he bids up to p' such that:[27]

$$\begin{aligned} p' &= \mathbb{E}\left[V_E \,\middle|\theta + t_1 > 2t_E,\ \theta + t_2 = p'\right] \\ &= t_E + \mathbb{E}\left[\theta \,\middle|\theta + t_1 > 2t_E,\ \theta + t_2 = p'\right] \end{aligned}$$

$$\Rightarrow \quad \begin{cases} 3p'^2 - \left(12t_E^2 + 6t_E\right)p' + 20t_E^3 = 0 & \text{if } 2t_E < p' \le 1, \\ p'^3 + (3 - 9t_E)\,p'^2 + \left(24t_E^2 - 9\right)p' - 20t_E^3 - 12t_E^2 + 15t_E + 2 = 0 & \text{if } 2t_E < 1 < p', \\ p'^3 - 3\,(3t_E - 1)\,p'^2 - 12\left(1 - t_E - t_E^2\right)p' - 24t_E^2 + 12t_E + 4 = 0 & \text{if } 1 \le 2t_E < p'. \end{cases}$$

Figure 4 represents bidder E's bid in auction B, conditional on his losing auction A at price $2t_E$. The figure shows that (apart from $t_E = 0$ and $t_E = 1$) $p'(t_E) > 2t_E$.

On the other hand, after winning auction A at price $2t_E + \varepsilon$, in auction B bidder E only bids up to:

$$\begin{aligned} p'' &= \mathbb{E}\left[V_E \,\middle|\theta + t_1 = 2t_E + \varepsilon,\ \theta + t_2 = p''\right] \\ &= t_E + \mathbb{E}\left[\theta \,\middle|\theta + t_1 = 2t_E + \varepsilon,\ \theta + t_2 = p''\right] \end{aligned}$$

$$\Rightarrow \quad p''(t_E) \cong 2t_E.$$

So given the additional information about θ, bidder E is able to reduce his bid in auction B by $\left(p' - p''\right)$. This reduction is almost always (i.e., apart from $t_E = 0$ and $t_E = 1$) of order higher than ε.

[27] It can be proven that, for $X, Y, W \backsim U[0,1]$ and $0 \le b < a \le 2$,

$$\begin{aligned} \mathbb{E}\left[X\,|X + Y = a,\ X + W > b\right] &= \int x \frac{\Pr\left(X + Y = a,\ X + W > b\,|X \in [x, x + dx]\right)\Pr\left(X \in [x, x + dx]\right)}{\Pr\left(X + Y = a,\ X + W > b\right)} \\ &= \begin{cases} \frac{3a^2 - b^3}{6a - 3b^2} & \text{if } b < a \le 1 \\ \frac{3 - b^3 - 3(1-b)(a-1)^2 - 2(a-1)^3}{9 - 3(a-b)^2 - 6b} & \text{if } b < 1 < a \\ \frac{4 + 3a^2 + 3a^2 b - 6ab - 2a^3}{3\left[(2-b)^2 - (a-b)^2\right]} & \text{if } 1 \le b < a. \end{cases} \end{aligned}$$

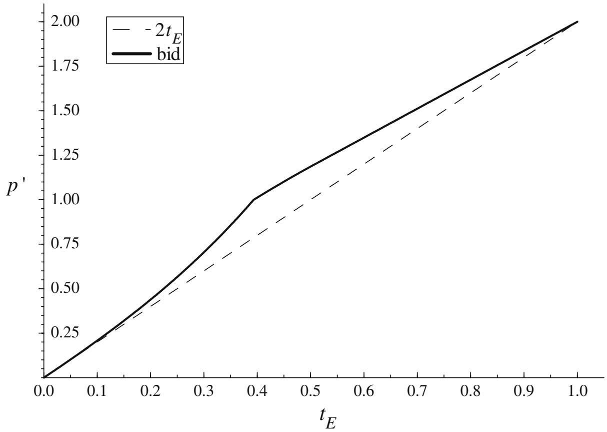

Fig. 4 Bidder E's bid in auction B after losing A at price $2t_E$

Therefore, whenever bidder I_2's bid in auction B—i.e., $\theta + t_2$—lies in the interval $\left[p'', p'\right]$ (which happens with a probability of order higher than ε), bidder E avoids overpaying by the difference between I_2's bid and his expected valuation, conditional on bidder I_1's signals being equal to $2t_E + \varepsilon$ and on I_2's bid, that is by:

$$
\begin{aligned}
\mathbb{E}\left[\text{Overpayment}\right] &= (\theta + t_2) - \mathbb{E}\left[V_E \,|\theta + t_1 = 2t_E + \varepsilon,\ \ (\theta + t_2)\right] \\
&\cong (\theta + t_2) - t_E - \mathbb{E}\left[\theta \,|\theta + t_1 = 2t_E,\ \ (\theta + t_2)\right] \\
&= \begin{cases} (\theta + t_2) - 2t_E & \text{if } 2t_E < (\theta + t_2) \le 1, \\ 1 + \frac{1}{2}(\theta + t_2) - 2t_E & \text{if } 2t_E < 1 < (\theta + t_2)\,, \\ \frac{1}{2}(\theta + t_2) - t_E & \text{if } 1 \le 2t_E < (\theta + t_2)\,, \end{cases}
\end{aligned}
$$

and this is also of order higher than ε.

Summing up, bidder E also prefers to win auction A at price $2t_E + \varepsilon$ instead of dropping out at price $2t_E$ since he only suffers a loss of order ε in auction A, but his expected gain in auction B, in terms of expected reduction in overpayment, is of order higher than ε.

Concluding, in auction A bidder E prefers *both* to win *and* to lose at price $2t_E + \varepsilon$, for ε small enough, instead of dropping out at price $2t_E$; hence, he bids strictly more than $2t_E$. ■

Proof of Proposition 2. After winning auction B at a price lower than the price at which he won auction A, bidder E regrets winning auction A if and only if:

$$
p_A > \mathbb{E}\left[V_E \,|p_A,\ p_B\right] = t_E + \begin{cases} \frac{1}{2}p_B & \text{if } \ p_A \le 1, \\ \frac{1}{2}(p_A + p_B - 1) & \text{if } \ p_B < 1 < p_A, \\ \frac{1}{2}p_A & \text{if } \ 1 \le p_B. \end{cases}
$$

Rearranging yields the statement. ■

Proof of Proposition 3. If $2t_E > \theta + t_1$, bidder E wins auction A at price $p_A = \theta + t_1$. In auction B, he bids up to $\beta_1 (t_E, \theta + t_1) > \theta + t_1$. If $\beta_1 (t_E, \theta + t_1) > \theta + t_2$, bidder E also wins auction B at price $p_B = \theta + t_2$. And if $p_A > t_E + p_B$, bidder E is certain that he overpaid in auction A. Rearranging these inequalities yields the statement. ■

Derivation of Equation (5.1). Let $X, Y, W \backsim U\,[0, 1]$. Then:

$$\begin{aligned}
&\mathbb{E}\,[X\,|X + Y = p, X + W > p] \\
&\quad = \int x \Pr\left(X \in [x, x + dx]\,|X + Y = p,\ X + W > p\right) \\
&\quad = \int x \frac{\Pr\left(X + Y = p,\ X + W > p\,|X \in [x, x + dx]\right) \Pr\left(X \in [x, x + dx]\right)}{\Pr\left(X + Y = p,\ X + W > p\right)} \\
&\quad = \frac{\int x f_Y\,(p - x)\,[1 - F_W\,(p - x)]\, f_X\,(x)\,dx}{\int f_Y\,(p - x)\,[1 - F_W\,(p - x)]\, f_X\,(x)\,dx}.
\end{aligned}$$

The above expression is equal to $\frac{\int_0^p x(1-p+x)dx}{\int_0^p (1-p+x)dx} = \frac{3p-p^2}{6-3p}$ for $p < 1$, and to $\frac{\int_{p-1}^1 x(1-p+x)dx}{\int_{p-1}^1 (1-p+x)dx} = \frac{4-3p^2+p^3}{3(2-p)^2}$ for $p \geq 1$. ■

Proof of Lemma 5. Given that one auction is still running, in the other auction bidder E bids up to p^* such that:

$$\begin{aligned}
p^* &= \mathbb{E}\left[V_E\,\middle|\theta + t_i = p^*,\ \theta + t_j > p^*\right] \\
&= t_E + \mathbb{E}\left[\theta\,\middle|\theta + t_i = p^*,\ \theta + t_j > p^*\right], \quad i, j = 1, 2, \quad i \neq j; \\
&\Rightarrow p^* = t_E + \begin{cases} \frac{3p^*-(p^*)^2}{6-3p^*} & \text{if} \quad p^* < 1, \\ \frac{4-3(p^*)^2+(p^*)^3}{3(2-p^*)^2} & \text{if} \quad p^* \geq 1. \end{cases}
\end{aligned} \tag{A.3}$$

For $p^* < 1$, (A.3) yields $p^* = \frac{3}{4}\left(1 + t_E - \sqrt{1 - \frac{10}{3}t_E + t_E^2}\right)$, which is the only root lower than 1, for $t_E < \frac{1}{3}$. For $p^* \geq 1$, (A.3) yields $p^* = \frac{1}{2} + \frac{3}{2}t_E$, which is the only root such that $1 \leq p^* \leq 1 + t_E$,[28] for $t_E \geq \frac{1}{3}$.

Moreover, once bidder E quits one auction at price p^*, he prefers to quit the other auction too. To see this, assume by contradiction that he does not. Then it must be that, for some $p' > p^*$,

$$\mathbb{E}\left[V_E\,\middle|\theta + t_i = p',\ \theta + t_j > p^*\right] > p', \quad i, j = 1, 2, \quad i \neq j.$$

[28] Bidder E never wants to bid more than the highest possible value of the prize.

But since $\mathbb{E}\left[V_E \left|\theta + t_i = p', \theta + t_j > p\right.\right]$ is weakly increasing in p, this implies that:

$$\mathbb{E}\left[V_E \left|\theta + t_i = p', \theta + t_j > p'\right.\right] > p',$$

which is inconsistent with the fact that bidder E chooses to quit one auction at a price lower than p'. ■

Proof of Lemma 6. The proof is analogous to the proof of Lemma 3 for sequential auctions. Notice only that if bidder E wins one auction at a price $\overline{p} \geq 2t_E$, then his expected valuation conditional on also winning the remaining auction at any price $p > \overline{p}$, is lower than p. Hence, bidder E prefers to lose the remaining auction, and so he drops out immediately.[29] ■

Proof of Proposition 4. In a simultaneous auction, bidder E keeps bidding in each auction if and only if his expected valuation, conditional on all the information revealed in both auctions, is higher than the current price. Hence, bidder E is never a sorry winner at the moment at which he wins one of the auctions. However, additional information may be revealed in the other auction, which makes bidder E revise his expected valuation.

But if bidder E wins one auction at a price lower than $2t_E$, then any outcome of the other auction leads him to expect a valuation higher than the price of the first auction. If, on the other hand, he wins one auction at a price higher than $2t_E$, then he drops out of the other auction immediately, so he has no chance to acquire additional information. Therefore, with simultaneous auctions the outcome of a second auction can never reveal bidder E's valuation to be lower than the price that he paid in a first auction. ■

Proof of Proposition 5. First notice that it is a weakly dominant strategy for bidder E to bid up to $1 + t_E$ in auction B. It follows that, with sequential auctions, in auction A bidder E bids up to $2t_E$ (as in a single-object auction), because he can obtain no useful information for auction B.

Consider now simultaneous auctions. (In this case, auction A is analogous to a single-object auction in which bidder E learns when the current price is equal to $\theta + t_2$.) If bidder E wins auction A at price p_A while auction B is still running, he knows that $\theta + t_2$ is higher than p_A and, hence, he expects the common signal to be $\mathbb{E}\left[\theta \left|\theta + t_1 = p_A, \theta + t_2 > p_A\right.\right]$. Therefore, if bidder I_2 is still active in auction B, bidder E drops out of auction A at price $\beta_2(t_E)$, which is defined in Lemma 5.

[29] This also follows from Lemma 3, which shows that, in sequential auctions, bidder E's bid in the second auction, after winning the first one at price $\overline{p} \geq 2t_E$, is strictly lower than $\overline{p}$. But since prices rise simultaneously in simultaneous auctions, when bidder E wins one auction at price $\overline{p} \geq 2t_E$, the price in the remaining auction is also equal to $\overline{p}$. Hence, bidder E would prefer to drop out of the remaining auction at a price strictly lower than the current one, and so he drops out immediately.

If, on the other hand, bidder E wins auction A at price p_A after winning auction B at price p_B, he expects the common signal to be $\mathbb{E}\left[\theta \,|\theta + t_1 = p_A,\ \theta + t_2 = p_B\right]$. Therefore, exactly as in Lemma 6: (i) if bidder E wins auction B at price $p_B < 2t_E$, in auction A he bids up to $\beta_3\,(t_E,\, p_B)$; (ii) if bidder E wins object B at price $p_B \geq 2t_E$, he drops out of auction A immediately. (More details about this model are in Pagnozzi, 2005—the working paper version of this article).

Since we can compute bidder E's equilibrium bidding strategies both in simultaneous and in sequential auctions, we can also compare the seller's revenue in the two auction formats. With sequential auctions, the seller's expected revenue in auction A is 0.7. Numerical simulation show that the seller's expected revenue in auction B is approximately 0.958, and the seller's expected revenue in auction A with simultaneous auctions is approximately 0.73. Therefore, the seller's total expected revenue is approximately 1.688 in simultaneous auctions and 1.658 in sequential auctions.

Finally, the outcome of auction A is inefficient whenever bidder E wins but his private signal is lower than his opponent's private signal. Numerical simulations show that the probability of an inefficient allocation is 0.167 in sequential auctions and 0.135 in simultaneous auctions; while the expected difference between the loser's and the winner's valuation in an inefficient allocation (which represents the magnitude of the inefficiency) is 0.125 in sequential auctions and 0.106 in simultaneous auctions.■

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