CENTRO STUDI IN ECONOMIA E FINANZA

CENTRE FOR STUDIES IN ECONOMICS AND FINANCE

WORKING PAPER NO. 46

Money and Credit in a Production Economy

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July 2000

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Abstract

In this paper we combine liquidity constrained lenders and borrowers in a market for investment projects that is characterized by incomplete information. The assumption of different probability distributions of the investment projects creates an adverse selection problem which gives rise to credit rationing in the loan market. Monetary policy has real effects, interacts with both the degree of liquidity and the degree of credit rationing, and alters the aggregate level of capital stock and its marginal productivity.

Keywords: credit rationing, cash-in-advance constraints, investment.

JEL Classification: D45, D92, E44.

Acknowledgements: We ackowledge financial support from the Training and Mobility of Researchers Network Program (TMR) on "Savings, Pensions and Portfolio Choice".

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Table of contents

1 Introduction

Recent microeconomic literature on credit and financial instruments builds on the theory of incomplete information. The basic argument is that capital markets not only intermediate in a mechanical way between savers and investors, but in addition deal with a variety of problems that arise from asymmetric information about investment projects between borrowers and lenders. These informational problems both shape capital markets institutions and debt instruments and affect the way in which policy actions are transmitted to the goods markets.

It is often argued that the quantity of money is not the key variable in the determination of the price level and output, either because it is endogenous or because the financial system is sufficiently flexible to generate as much inside money as might be needed to finance any level of activity. For instance, if credit is rationed the interest rate is not anymore a sufficient indicator in appraising monetary and financial policy: other quantity variables, such as the amount of credit, should be checked.

In this paper we aim at developing in a general equilibrium framework the idea that monetary policy may affect in a non trivial way the performance of a financial market characterized by asymmetric information, with ambiguous effects on the economic performance as measured by output or the steady-state capital stock. In doing that, we explore the possibility that the interaction between borrowing constraints (as expressed by the occurrence of credit rationing) and liquidity constraints (cash-in-advance) is crucial in determining the level of the interest rate. We show that the mere presence of asymmetric information in credit markets may have major implications for the effects of monetary policy.

To the best of our knowledge, Azariadis and Smith [**?**], are the first to address the role of asymmetric information and adverse selection in credit markets within a general equilibrium economy where credit rationing arises endogenously. They achieve this by embedding the Rothschild and Stiglitz [4] insurance market model in a pure exchange OLG economy. They show that static incentive problems apart from causing dynamic inefficiencies, they create a role for outside money. Bencivenga and Smith [2], depart from Azariadis and Smith by assuming that the economy is productive and by setting up an endogenous growth model by which they study the effects of credit rationing on the real growth rate. In both papers the classic result of Rothschild and Stiglitz is preserved: all Nash equilibria of a two stage game between borrowers and lenders display self selection of borrowers according to the contracts offered by the lenders, in particular, the equilibrium contracts are always separating and they imply credit rationing for the borrowers with the safer projects.

We depart from Azariadis and Smith [**?**] by assuming a monetary economy. We introduce money demand via a cash in advance constraint in future consumption purchases, so that we can study the effect of monetary policy on the credit market. We embed the production framework of Bencivenga and Smith [2], in our monetary economy, without the growth features they employ, so that we can study the interaction of monetary variables, such as the money growth rate, and credit market variables, such as the credit rationing, in the determination of the level of capital stock and the real interest rate. Money growth is guaranteed by a government that uses seignorage revenue in order to finance the endogenous sequence of its expenditure. There are two types of agents (entrepreneurs and financiers) that interact in the credit market; entrepreneurs are privately informed about the probability distributions of their uncertain investment realization, while financiers no. This asymmetry in information creates an adverse selection problem in the credit market that results in credit rationing which identifies with unfulfilled demand for credit.

We study the effects on credit rationing, capital and interest rate arising from changes in the money growth rate and from the tightness of the liquidity constraints, performing comparative statics analysis. We show that a higher inflation rate has negative effects on the amount of per-firm available credit, increases the interest rate and leaves unchanged the degree of credit rationing. This is due to the easily interpretable fact that higher inflation reduces the amount of credit that liquidity constrained lender may offer. Conversely, increasing the amplitude of the liquidity constraint has ambiguous effects on the level of economic activity, on the real interest rate and on the degree of credit rationing. The intuition is as follows. On the one hand, a higher liquidity constraint reduces the amount of credit available and this entails a lower level of per firm capital and decreases borrowers' expected utility. On the other one, this allows a relaxation of the incentive compatibility constraint which becomes compatible with a lower degree of credit rationing. Competition among lenders will make actually effective this increase. The latter effect, in turn, will lower the per-firm labor force which will then contrast the initial fall in the capital intensity. It follows that the marginal productivity of capital, and thus the capital rental price, can increase as well as decrease.

This finding encourages us to think that we can use our model in order to explain real time phenomena, where ups and downs of the interest rates are not fully explained by existing theories. We want to stress at this point that this paper is a first step towards understanding the interaction of cash-in-advance monetary constraints with credit markets characterized by asymmetric information .

The remaining of the paper is structured as follows. In section two, we provide a full description of our economy, which is the monetary version of the model of Bencivenga and Smith [2]. In the two next sections we analyses the equilibrium issues of our economy (section three) and characterize the equilibrium (section four). Section five is the core of this paper, here we describe the comparative statics of our economy. We conclude by summarizing our results and stressing the fact that our task with this paper is simply a first step towards dealing questions of imperfect information in monetary economies with production.

2 The Economy

We consider a discrete time economy consisting of an infinite sequence of two-period lived overlapping generations, with time indexed by $t = 1, 2, ...$It is assumed the presence of an initial old generation at $t = 1$ and zero population growth. Each generation is represented by a continuum of agents whose size is normalized to one. The economic structure of each generation is exogenously given and its members are distinguished in two groups of equal size, the entrepreneurs and the financiers. All young agents are born endowed with one unit of labor that they may supply to the production of the final consumption good and earn a labor income. What distinguishes agents is that entrepreneurs are endowed with the ability to become providers of capital needed to produce the final consumption good. They are the owners of technologies that transform income into productive capital which, in turn is used as an input to produce the final consumption good while, financiers are the owners of income. This distinction between young agents generates a credit market where *financiers* supply their funds (they become *lenders*) to the *entrepreneurs (borrowers)* who constitute the demand size of that market.

Both entrepreneurs and financiers optimize utilities from consumption of the single good. It is assumed that financiers are constraint to express demand for money as a medium of exchange. This assumption is introduced through a proportional cash in advance constraint on the entrepreneurs' second period consumption. This assumption places as in an economy with *"liquidity constraints",* the latter defined as conditions, represented by cash in advance constraints, responsible for the transactions demand for money. The money supply side role is played by a government that mechanically issues money and finances its expenditure with seignorage revenues. On the other hand, production of the single consumption good needs two production factors: labor and capital. Entrepreneurs are the economic agents that can produce capital because they own the capital technologies, they also own labor and they look for the other input, income that accrues in units of the consumption good, in the credit markets, they express a demand for loans. The assumed informational structure is responsible for a second type of constraints faced by our agents, the *"borrowing constraints",* expressed as excess demand for credit, not all entrepreneurs are granted a loan. Agents are asymmetrically informed, there are two different types of entrepreneurs, each entrepreneur knows his type ex-ante, but this information is private. This informational structure is responsible for an adverse selection problem in the credit market, in particular in generating credit rationing.

Production of capital takes time and it is subject to uncertainty. One unit of income put in type i entrepreneur's technology at time t gives, with probability p_i, Q units of productive capital at $t + 1$, and with probability $(1 - p_i)$ fails to give any productive capital. Entrepreneurs are supposed to have different abilities and this difference is measured by the probability of success of their technology. It is assumed that there are two different types of entrepreneurs indexed by $i \in \{H, L\}$ with $1 \geq p_L > p_H \geq 0$, H stands for "high risk", the probability of failure of the entrepreneur H is higher than for the entrepreneur L,"low risk". These probabilities measure also the probability of *solvency* of each of the entrepreneur-borrower type. The distribution of the abilities in the borrowers' population is exogenously given, a fraction $\lambda \in (0,1)$ of the borrowers are assumed to be of the type H and the rest $1-\lambda$ of the type L. The successful entrepreneurs (those that produce positive quantities of capital) at each date become 'firm owners'. Firms are behaving competitively, they rent capital at the competitively determined rental rate ρ_t and they hire labor at the competitively determined real wage rate w_t.

2.1 Financiers and Government

We assume that young financiers are risk neutral and evaluate consumption in both periods of their life, according to the linear intertemporal utility function $U^l\left(c_{1t}^l, c_{2t+1}^l\right) = c_{1t}^l + c_{2t+1}^l$, with c_{1t}^l, c_{2t+1}^l denoting young and old age consumption respectively (the index l stands for lenders). They have no endowment of the single producible consumption good at either date. Young lenders at time t sell their labor to firms and earn a real wage w_t. The proceeds can either be consumed during young age or loaned to other agents. Lenders are "liquidity constrained" with respect to their old age consumption, in the sense that they must finance at least a share $\mu^l \in [0,1)$ of their consumption during old age out of money holdings. Given that only financiers express a demand for money we drop, from now on, the index l from the monetary variables and we refer to $(1-\mu)$ as the economy's *"degree of liquidity"*.

A young financier born at time t faces therefore, the following sequence of constraints

$$c_{1t}^l + q_{it} + m_t = w_t, \tag{1}$$

$$c_{2t+1}^l = R_{it} q_{it} p_i + m_t \gamma_t \tag{2}$$

and

$$\mu c_{2t+1}^l \leq m_t \gamma_t, \tag{3}$$

where $m_t^l \equiv M_t^l / P_t$ are real balances held by each financier, q_{it}, $i \in \{H, L\}$, is the amount of loan granted to type i entrepreneur, R_{it} is the interest rate on granted loans, p_i is the probability that type i entrepreneur is solvent, $\gamma_t = P_t / P_{t+1}$ is the deflation factor between period t and period $t+1$.

As soon as the price level P_t grows fast enough so that the effective real interest rate $R_{it} p_i$ is always greater than the deflation factor γ_t, money holdings are always dominated by an investment in the credit market. Actually, we restrict ourselves in solutions that satisfy (4) so that constraint (3) is binding:

$$\gamma_t < R_{it} p_i \qquad i \in \{H, L\}, \quad all\ t \tag{4}$$

As we will show in the sequel, free entry in the supply of contracts ensures that the relative price between c_{1t} and c_{2t+1} is equal to 1 and the financier's intertemporal budget constraint has the form

$$c^l_{1t} + [(1-\mu)/R_{it}p_{it} + \mu/\gamma_t]\, c^l_{2t+1} = w_t. \tag{5}$$

The binding liquidity constraint implies an upper bound $\widehat{q}_{it}$ on the amount q_{it} of loan availability. We denote $\widehat{q}_{it} \equiv a_t w_t$, where

$$a_t \equiv \left[1 + (\mu/(1-\mu))\,(R_{it}p_i/\gamma_t)\right]^{-1} \in (0,1) \tag{6}$$

denotes *the share of the wage available for credit.* The value of a_t depends on the amplitude μ of the liquidity constraint. When $\mu = 0$ the whole real wage can be rented since $a_t = 1$ (financier's avail all their income to credit and our economy collapses in its *non-monetary version*), when $\mu = 1$ the credit market collapses, $a_t = 0$ and we obtain the *purely monetary version* of our economy. We admit values for a_t in the open interval $(0, 1)$ in order to avoid the two extreme cases of our economy because we want to study the interaction of the money and the credit markets. Moreover, a_t is decreasing with $R_{it}p_{it}/\gamma_t$: *a higher relative return on loans decreases the availability of loans, the faster the price level grows with respect to the effective interest rate the more we approach to the purely monetary economy and the credit market collapses endogenously.*

The only government activity consists in financing the endogenous public spending g_t thorough the emission of new money at the constant rate σ: therefore nominal money supply evolves according to $M_{t+1} = (1+\sigma)M_t$ and one has $g_t = \sigma m_t$, where σm_t is seignorage and $m \equiv M/p$ is total current real balances.

2.2 Entrepreneurs and firms

It is assumed that the agents endowed (apart from their labor endowment) with productive capital technologies, namely the entrepreneurs, care to maximizing old age consumption c^b_{2t+1} (the index b stands for borrowers since they will constitute the demand side of the credit market). Consistently their utility function is represented by $U^b\left(c^b_{1t}, c^b_{2t+1}\right) = c^b_{2t+1}$. Young entrepreneurs in period t can either sell their labor to firms, in which case they also earn the real wage w_t, or they can apply their labor to the operation of an "investment project". As already mentioned, investment projects concern the production of capital used further by firms producing the single consumption good. Real labor income (here in terms of the single consumption good) together with units of labor are the productive factors for capital which takes one period to obtain. Entrepreneurs of both types H, and L express the demand for income and they become potential borrowers in the credit market. An entrepreneur of type i,

$i \in \{H, L\}$, which receives a loan of q_{it} units of the consumption good, with probability p_i his project is successful and he obtains Qq_{it} units of capital at $t+1$. He has to pay back the gross interest rate R_{it} per unit of the loan received and he becomes at the same time a supplier of capital to the firms, earning a rental price ρ_{t+1} per unit of capital sold, and a firm owner. Firm owners of time t can rent capital at the competitively determined rental rate ρ_t, and can hire labor at the competitively determined real wage w_t. A firm employing k_t units of capital and l_t units of labor at t can produce the consumption good in amount y_t, with y_t given by a constant returns to scale Cobb-Douglas technology:

$$y_t = k_t^{\theta} l_t^{1-\theta}, \tag{7}$$

where $\theta \in (0, 1)$ is the share of capital in total income.

With probability $(1 - p_{it})$ his project fails, and nothing is paid back to the lender. If type i borrower is denied credit, he supplies his labor to the firms and earns the real wage w_t. This income, which accrues in the form of the consumption good, can be stored for future consumption. We assume that each type i agent has indeed access to a storage technology which yields $1 \geq \beta_i$ units of consumption at time $t+1$ for each unit stored at t.

The values β_i, $i \in \{H, L\}$, satisfy[1]

$$\beta_L / p_L > \beta_H / p_H. \tag{8}$$

2.3 The credit market

Although labor and capital markets are perfectly competitive, rental and wage rates are taken as given by all agents, the behavior in the credit market is marked by the presence of the informational asymmetries discussed above. This point deserves detailed description: at each time t financiers announce loan contracts. The terms of the contract specify the loan quantity offered, q_t, the gross real interest rate, R_t, and some quality prerequisites that have to be met by the borrower in order the loan to be granted, represented by the probability π_t that an entrepreneur applying for a loan is also receiving it. Loan contracts are thus represented by a triple (R_t, q_t, π_t)

We will assume that each borrower can contact only a single lender (and each lender can be contacted only by one borrower). This assumption is necessary because it introduces an upper bound on loan size. A more general assumption being that each lender is supposed to match with at most a finite number of lenders, we keep the *one-to-one* assumption throughout since it is without loss of generality and simplifies the analysis. Summarizing, financiers are viewed as announcing loan contracts $(R_{it}, q_{it}, \pi_{it})$ to be offered to type i entrepreneurs at

[1]The values β_H and β_L must satisfy assumption (8) in order for different types of entrepreneurs to perceive appropriately different opportunity costs of being denied credit. It is used to guarantee that credit rationing induces a separating self selection equlibrium in the credit market.

time t. At each time t, each lender announces a loan contract taking as given the announcements of other lenders, this strategic behavior of the financiers qualifies the credit market as a game between Nash competitors (the lenders) and its outcome as a Nash Equilibrium for the time period t. If these announced contract terms are not dominated by those of another lender, he is approached by a potential borrower. With probability π_{it} a type i borrower satisfies the quality prerequisites and he is granted the loan and operates his capital productive technology. With probability $1-\pi_{it}$ the type i borrower is denied credit, he supplies his labor to the firms, he stores his wage earnings w_t and consumes at time $t+1$ $\beta_i w_t$ units of the consumption good. Hence, the expected utility of a type i borrower is:

$$p_i \pi_{it}(Q\rho_{t+1} - R_{it})q_{it} + (1-\pi_{it})\beta_i w_t. \tag{9}$$

We will assume that at each date entrepreneurs prefer operating their investment project to selling their labor, i.e. that expected utility (9) is an increasing function of π_{it}, condition which can stated as

$$p_i \left(Q\rho_{t+1} - R_{it}\right) q_{it} > \beta_i w_t. \tag{10}$$

Assumption (8) guarantees that type H and type L borrowers face different opportunity costs from being denied credit and, therefore, at equilibrium self selection occurs. Equilibrium contracts $(R_{Ht}, q_{Ht}, \pi_{Ht}) \neq (R_{Lt}, q_{Lt}, \pi_{Lt})$ satisfy:

$$\begin{aligned} & p_H \pi_{Ht}(Q\rho_{t+1} - R_{Ht})q_{Ht} + (1-\pi_{Ht})\beta_H w_t \\ \geq \; & p_H \pi_{Lt}(Q\rho_{t+1} - R_{Lt})q_{Lt} + (1-\pi_{Lt})\beta_H w_t \end{aligned} \tag{11}$$

$$\begin{aligned} & p_L \pi_{Lt}(Q\rho_{t+1} - R_{Lt})q_{Lt} + (1-\pi_{Lt})\beta_L w_t \\ \geq \; & p_L \pi_{Ht}(Q\rho_{t+1} - R_{Ht})q_{Lt} + (1-\pi_{Ht})\beta_L w_t \end{aligned} \tag{12}$$

We can now define an equilibrium in the credit market.

Definition 1 *A Nash equilibrium in credit markets at time t is a pair of contracts* $\{(R_{it}, q_{it}, \pi_{it})\}$, $i = H, L$, *satisfying (11) and (12), and such that no lender has an incentive to offer an alternative contract, taking the offers of other lenders and the values* w_t *and* ρ_{t+1}*as given.*

2.4 Self Selection Equilibrium Contracts

By now it should have become clear that: a) any equilibrium displays self selection, b) in equilibrium contracts earn zero expected profits and, c), the contracts received by type i borrowers at t are maximal for them among the set of all contracts satisfying the self-selection constraints (11) and (12), given the contracts received by the borrowers of the other type. This is the same reasoning as in

Rothschild and Stiglitz (1976). If we look at (5), we immediately see that zero profits condition requires R_{Ht} and R_{Lt} to satisfy

$$p_i R_{it} = (1-\mu) b_t, \quad i \in \{H, L\}, \; t \geq 1 \tag{13}$$

where

$$b_t \equiv \gamma_t / (\gamma_t - \mu). \tag{14}$$

Moreover, condition (13) insures that lenders are indifferent between consuming when young or when old and between granting loans to type H agents or to type L agents, since the relative price of their young age consumption with respect to that of their old age consumption is equal to one.

A mere inspection of (13) delivers an important observation: the expected interest rate $p_i R_{it}$, $i \in \{H, L\}$, is increasing in the amplitude μ of the liquidity constraint. In particular, when $\mu = 0$, $p_i R_{it} = 1$ whereas, when μ converges to γ_t, R_{it} diverges to $+\infty$. *This feature of the expected interest rate is rather intuitive: the stronger the liquidity constraint (the higher the degree of liquidity μ), the smaller the quantity of loanable funds, the higher the interest rate charged on them, given the zero profit condition of lenders.*

We observe, in addition, that in the light of condition (13), the parameter a_t in (6) can be rewritten

$$a_t = (\gamma_t - \mu)/\gamma_t = b_t^{-1}. \tag{15}$$

Hence, *the share of the wage available for credit a_t is a function of the deflation factor and the degree of liquidity of the economy.*

The zero profit condition (13), together with the assumption (10) that borrowers prefer to operate their projects than to supply their labor, and together condition (15), delivers the following condition that we will assume throughout to hold:

$$p_i Q \rho_{t+1} a_t > (1-\mu) + \beta_i, \quad i \in \{H, L\}. \tag{16}$$

Now, we can characterize the candidate equilibrium loan contracts. In accordance with Rothschild and Stiglitz (1976), the contract $(R_{Ht}, q_{Ht}, \pi_{Ht})$ is not affected by considerations of self selection. Competition among lenders for borrowers implies that high risk entrepreneurs receive their most preferred contract. Given condition (16) the terms of the high risk equilibrium contract $(R_{Ht}, q_{Ht}, \pi_{Ht})$ should be given by:

$$R_{Ht} = (1-\mu) b_t / p_H, \; q_{Ht} = a_t w_t, \; \pi_{Ht} = 1 \tag{17}$$

For the low risk entrepreneurs the equilibrium contract should entail a gross interest rate of $R_{Lt} = (1-\mu) b_t / p_L$, and has to be maximal among all contracts that satisfy the self selection condition (11) holding with equality at equilibrium. Substituting R_{Ht}, q_{Ht} and π_{Ht} from (17) the self selection condition we get

$$q_{Lt} = \frac{\left[p_H Q \rho_{t+1} - (1-\mu) b_t\right] - (1 - \pi_{Lt}) \beta_H}{p_H \pi_{Lt} \left(Q \rho_{t+1} - (1-\mu) b_t / p_L\right)} w_t. \tag{18}$$

Obviously, it must hold also $q_{Lt} \leq a_t w_t$. Combining this with (18) we get a necessary and sufficient condition for π_{Lt}

$$\pi_{Lt} \geq \frac{p_H Q\rho_{t+1} - (1-\mu)b_t - \beta_H}{p_H Q\rho_{t+1} - \frac{p_H}{p_L}(1-\mu) - \beta_H}. \tag{19}$$

Then π_{Lt} and q_{Lt} must maximize the expected utility (9) of type L borrower which can be rewritten as

$$\pi_{Lt}\left[p_L Q\rho_{t+1} - (1-\mu)b_t\right] q_{Lt} + (1-\pi_{Lt})\beta_L w_t \tag{20}$$

subject to (18) and (19). Assumptions 1 and 2 imply that the solution to this problem has (19) holding with equality,[2] and so, taking into account (15),

$$\pi_{Lt} = \frac{p_H Q\rho_{t+1} - (1-\mu)b_t - \beta_H}{p_H Q\rho_{t+1} - p_H(1-\mu)/p_L - \beta_H} < 1 \tag{21}$$

We call π_{Lt} *the credit rationing factor* since $1 - \pi_{Lt}$ represents the measure, itself, of credit rationing in a continuum population. Substituting (21) into (18) we get at equilibrium $q_{Lt} = a_t w_t$.

Summarizing, for type L agents equilibrium contracts are given by the triple:

$$R_{Lt} = (1-\mu)b_t/p_L,\ q_{Lt=}a_t w_t,\ \pi_{Lt} = \frac{p_H Q\rho_{t+1} - (1-\mu)b_t - \beta_H}{p_H Q\rho_{t+1} - p_H(1-\mu)/p_L - \beta_H}. \tag{22}$$

Finally it is easy to verify that these contracts satisfy the self selection condition (12) as well. As in Rothchild and Stiglitz [4] no equilibrium in pure strategies need exist. Existence issues, which are similar to those in Bencivenga and Smith [2], are analyzed in the sequel.

2.5 Capital, Labor and, Money Markets

We proceed by determining the equilibria in the capital and labour market. As mentioned above, all agents behave competitively in both labor and capital markets. The number of firms is equal to the number of borrowers with positive quantities of capital, there are $0.5\left[\lambda p_H + (1-\lambda)p_L\pi_{Lt-1}\right]$ firms per capita at t, $\forall t \geq 2$. This is because half of all agents are borrowers. Among them, the fraction λ who are type H all receive credit, resulting in λp_H successful investment projects. Similarly, a fraction π_{Lt-1} of the $1-\lambda$ type L borrowers receive loans at $t-1$, resulting in $(1-\lambda)p_L\pi_{Lt-1}$ successful projects. The per capita supply of labor at t is $0.5\left[1 + (1-\lambda)(1-\pi_{Lt})\right]$; this is because half of the agents are lenders, all of whom supply labor, and in addition, the $0.5(1-\lambda)(1-\pi_{Lt})$ type L borrowers who are denied credit are also in the labor force. Since in equilibrium

[2] See Appendix 1 for a full demonstration of the proof.

all firms must employ equal amounts of labour (and capital), l_t (the quantity of labor employed by each firm) is given by

$$l_t(\pi_{Lt}) = \frac{1 + (1-\lambda)(1-\pi_{Lt})}{\lambda p_H + (1-\lambda)p_L\pi_{Lt-1}}, \quad t \geq 2 \tag{23}$$

where the numerator of (23) is the total supply of labor, whereas the denominator represents the total number of firms.

The equilibrium values of ρ_t and w_t are given by the following marginal productivity relations

$$w_t = (1-\theta)k_t^{\theta}l_t^{-\theta} \tag{24}$$

$$\rho_t = \theta k_t^{\theta-1}l_t^{1-\theta}. \tag{25}$$

The rental rate ρ_t and the wage rate w_t are functions of the capital stock employed at time t and of the credit rationing factor π_{Lt}. Substituting (23) into (25) yields

$$\rho_t = \theta k_t^{\theta-1}l_t(\pi_{Lt})^{1-\theta} \quad t \geq 2. \tag{26}$$

Equation (26), together with equation (21), determine the sequence $\left\{\rho_{t+1}, \pi_{Lt}\right\}_{t=2}^{\infty}$ given k_t.

The capital stock of each firm at $t+1$ is given by $k_{t+1} = Qa_tw_t$, since $q_{it} = a_tw_t$, $i = H, L$. By (24) we get

$$k_{t+1} = Q(1-\theta)a_tk_t^{\theta}l_t^{-\theta} \tag{27}$$

Equilibrium in money market requires all money to be held by lenders. Therefore, the deflation factor $\gamma_t \equiv P_t/P_{t+1}$ at equilibrium satisfies

$$\gamma_t = m_{t+1}/m_t(1+\sigma) \tag{28}$$

where $m_t = m_t^l$ for all $t \geq 1$ denotes the total amount of real balances. Let us observe that the value of the credit rationing factor π_{Lt} at time t is determined simultaneously with the value of the capital rental rate ρ_{t+1} of the next period and both depend on the liquidity factor μ and the money growth rate σ, the latter influencing the amplitude of a.

3 The Steady state

A steady state Nash Equilibrium (SSNE) for our economy is defined as follows:

Definition 2 *A Steady State Nash equilibrium in credit markets is a sequence of contracts $\{(R_{it} = R_i, q_{it} = q_i, \pi_{it} = \pi_i)\}_{t=1}^{\infty}$, $i = H, L$, satisfying the steady state self selection conditions, (11) and (12) (with the time index suppressed), and such that no lender has an incentive to offer an alternative contract, taking the offers of other lenders and the steady state sequence of capital rental rate $\{\rho_t = \rho\}_{t=2}^{\infty}$ as given.*

We are talking basically about an infinite replica of the two periods Nash equilibrium defined above. Henceforth, we consider the economy at the steady state, i.e. with $\rho_t = \rho$ and $\pi_{Lt} = \pi_L$ for $t \geq 2$. From equation (27) we get the steady state capital stock,

$$k = (Q(1-\theta)a)^{1/(1-\theta)}\, l\,(\pi_L)^{-\theta/(1-\theta)} \tag{29}$$

and from (27) the stationary deflation factor $\gamma = 1/\left(1+\sigma\right)$. It is immediate that constraint (3) binds if and only if $\sigma > 0$ (i.e. $\gamma < 1$).

Some algebra delivers the equation

$$\rho = \theta\left[Q(1-\theta)a\right]^{-1} l(\pi_L) \tag{30}$$

which together with

$$\pi_L = \frac{p_H Q\rho a - (1-\mu) - \beta_H}{p_H Q\rho a - \frac{p_H}{p_L}(1-\mu) - \beta_H}. \tag{31}$$

determines the steady state values $\{\rho, \pi_L\}$ of the real rental rate and the credit rationing factor.

3.0.1 Determination of $\{\rho, \pi_L\}$.

For given values of μ and σ (and thus γ) as well as of all the other structural parameters Q, θ, p_H, p_L, λ and β_H, equations (30) and (31) define two continuous functions $\rho_1(\pi_L)$ and $\rho_2(\pi_L)$, respectively. Let us plot them in the (π_L, ρ) plane, as shown in Figure 1. The locus defined by $\rho_1(\pi_L)$ intersects the vertical axis at $\rho = A\left(2-\lambda\right)/\lambda p_H$, where $A \equiv \theta\left[Q(1-\theta)a\right]^{-1}$, is monotonically decreasing in the (π_L, ρ) plane[3] and attaint $\rho = A\left[\lambda p_H + (1-\lambda)p_L\right]$ when $\pi_L = 1$. The locus $\rho_2(\pi_L)$ defined by (31) intersects the vertical axis $(\pi_L = 0)$ at $\rho = \left[1-\mu+\beta_H\right]/\left[p_H Q a\right]$. In addition π_L is monotonically increasing in ρ and converges to one when $\rho \rightarrow +\infty$.

[3] Its slope is $\frac{\partial\rho}{\partial\pi_L} = A\frac{\partial l(\pi_L)}{\partial\pi_L} < \mathbf{0}$.

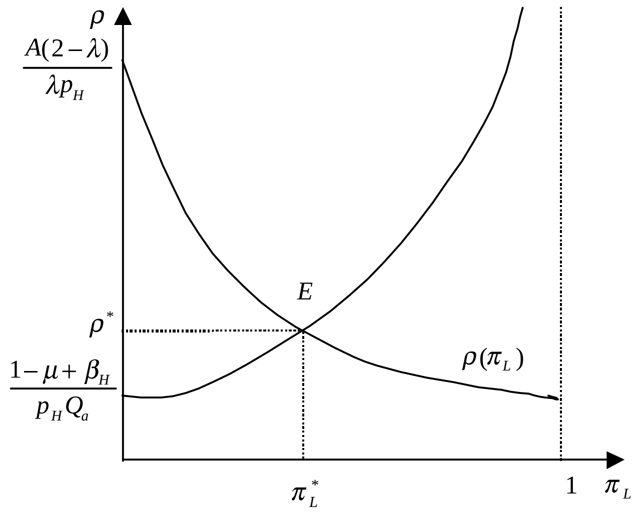

Figure 1

Comparing the two loci $\rho_1(\pi_L)$ and $\rho_2(\pi_L)$ (see Figure 1), it follows that there exists exactly one non trivial stationary equilibrium (π_L^*, ρ^*) satisfying $0 < \pi_L^* < 1$ and $\rho^* > 0$ if and only if

$$(1 - \mu + \beta_H) < \theta(1 - \theta)^{-1} (2 - \lambda) / \lambda. \tag{32}$$

We will assume throughout condition (32) to hold.

Let us now observe that, evaluated at the steady state, inequality (16) becomes

$$p_i Q \rho a > 1 - \mu + \beta_i \quad i \in \{H, L\}. \tag{33}$$

As it is apparent, ρ^* satisfies inequality (33) for[4] $i = H$. Conversely, satisfaction of the other half of (33) is not guaranteed, but $\rho^* > [1 - \mu + \beta_L] / [p_L Q a]$ will clearly hold if β_L / p_L is sufficiently closed to β_H / p_H.

Assuming (33) is satisfied by ρ^*, a candidate equilibrium has been derived in which type L borrowers face credit rationing. The existence of this rationing affects the level of output for this economy. In particular, if we take in (29) the derivative of capital with respect to π_L we obtain

$$\partial k / \pi_L = \frac{\theta}{1 - \theta} A^{-\frac{1}{1-\theta}} \left(L\left(\pi_L\right)\right)^{-\frac{1}{1-\theta}} (1 - \lambda) p_L \left[\lambda \left(p_H / p_L\right) + 2 - \lambda\right] > 0.$$

Therefore policy that can *reduce credit rationing (increase* π_L*)* will have the effect of increasing the per firm capital level and therefore per capita capital.

[4]Indeed, (33) holds with equality for $\rho = [1 - \mu + \beta_H] / [p_H Q a] < \rho^*$.

3.0.2 Ruling out pooling contracts

Even, assuming $\rho^* > [1 - \mu_l + \beta_L] / [p_L Qa]$, it remains to investigate whether lender have an incentive to offer an alternative pooling contract in the presence of contracts described previously. Let (R, q_L, π) be such a pooling contract which is announced taking ρ^* as given. The most preferred pooling contract, from the point of view of a type L borrower that is consistent with lender earning non negative expected profit has $R = [\lambda p_H + (1 - \lambda) p_L]^{-1}$ and selects q_L and π in order to maximize

$$\pi p_L (Q\rho^* - R) q_{Lt} + (1 - \pi) d_L w_t \tag{34}$$

subject to $q_L \leq aw_t$. Since $Q\rho^* > R$, $q_L = aw_t$ must hold. Then there are two cases to consider.

Case 1. $p_L(Q\rho^* - R) \leq \beta_L$. In this case the expression in (34) attains a maximum value of $\beta_L w_t$. There is then no pooling contract that attracts type L borrowers and earns a non negative expected profit.

Case 2. $p_L(Q\rho^* - R) > \beta_L$. In this case the most preferred pooling contract for type L borrowers has $\pi = 1$. The expected utility obtained by those borrowers under such a pooling contract is $p_L(Q\rho^* - R)w_t$. Then there is no pooling contract that attracts all borrowers and earns a non negative expected profit if

$$\pi_L (p_L Q\rho - 1) w_t + (1 - \pi_L) \beta_L w_t \geq p_L (Q\rho - R) w_t,$$

or equivalently if

$$\pi_L \geq (p_L Q\rho - p_L R - \beta_L) / (p_L Q\rho - 1 - \beta_L).$$

Noting that ρ^* is a continuous function of λ, it is readily verified that (34) is satisfied as a strict inequality for $\lambda = 1$, and hence for all values of λ sufficiently close to one. Thus, just as in Rothchild and Stiglitz [4], sufficiently large values of λ guarantee the existence of an equilibrium.

4 Comparative statistics

Equations (29), (30) and (31) are continuously differentiable functions of the parameters of the model Q, p_L, p_H, β_H, β_L, λ, θ, $\mu$and σ. We are interested in the consequences on the endogenous variables of changes in the money growth rate σ and in the amplitude μ of the liquidity constraint. This is our purpose in what follows[5]. We carry out our comparative statics analysis with the help of two pictures that follow, Figure 2 and Figure 3.

[5] Throughout it is assumed that an equilibrium exists both before and after the change in the relevant parameter.

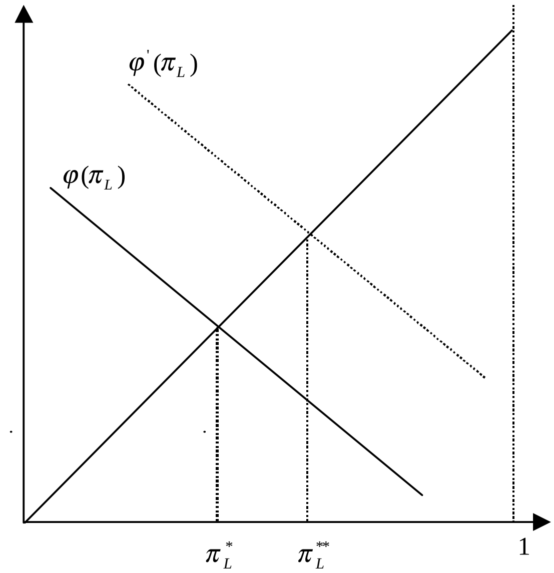

Figure 2

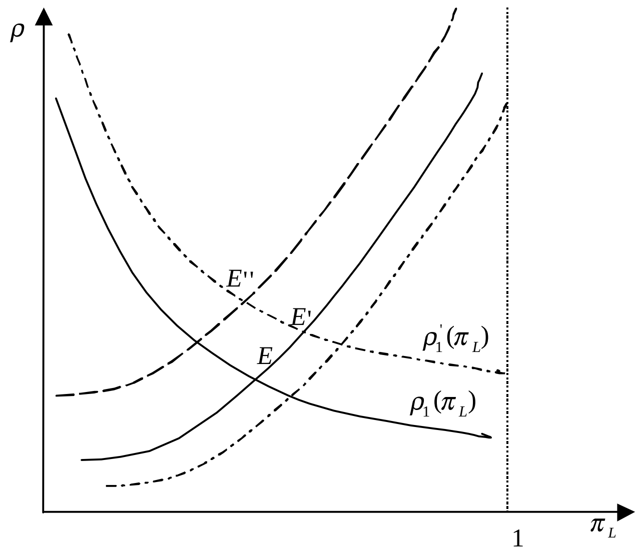

Figure 3

4.0.3 Changes in the money growth rate

In order to study how π_L varies when the rate σ of money growth supply increases (which implies a reduction of the stationary deflation factor γ), let substitute ρ from (30) into (31) in order to obtain

$$\pi_L = \varphi(\pi_L) \equiv \frac{p_H\theta(1-\theta)^{-1}l(\pi_L) - (1-\mu_l) - \beta_H}{p_H\theta(1-\theta)^{-1}l(\pi_L) - (p_H/p_L)(1-\mu_l) - \beta_H}. \tag{35}$$

Since σ does not appear in (35), the steady state value of π_L is unaffected by a change in the monetary rule σ. Conversely, by a direct inspection of (30), and taking into account that a is decreasing in σ, it is possible to verify that the stationary interest rate ρ increases. This in turn implies, in view of (29), a fall in the per capital steady state capital. This result should not be very surprising: an higher inflation rate entails indeed a lower amount of credit available for the borrowers, since lenders now need to invest in money an higher quantity of their wage income, and this implies a fall in the level of per-firm capital intensity and as a consequence, an increase in capital marginal productivity reflected in the capital rental price ρ. At the same time, the increase of ρ is exactly high enough to compensate the reduction in a, and therefore the credit rationing factor π_L does not undergoes any change. The effects on the degree of credit rationing, interest rate and capital of an increase in the monetary rule σ are summarized in the following proposition.

Proposition 3 *An increase in the money growth rate σ yields to a decrease in the per firm steady state capital and to an increase in the stationary capital rental price ρ. Conversely, the degree of credit rationing π_L does not undergo any modification.*

Proposition (3) claims that inflation has a negative impact on economic activity, measured in terms of per firm capital. This finding is in accord with a vast literature on the subject and seems to reinforce the idea that the well known Tobin's effect does not find widespread theoretical comfort. Indeed, the growth enhancing effects of higher nominal interest rates due to the way agents change the compositions of their portfolios by substituting more costly liquid asset with illiquid ones (namely productive capital), are not strong enough to offset the growth reducing effects induced by a general fall in present value lifetime wealth.

One should expect at this point that analogous growth reducing effects should derive from an increase in the amplitude of lenders' liquidity constraint: indeed, it could be reasonably argued that this would reduce the amount of credit available to the borrowers and thus the average rate of productive investment. Surprisingly, this is not always true since, as we are going to show, the one described is not the only mechanism at work induced by strengthening the liquidity constraint. Actually, one has to take also into account the relaxing effects of such an experiment on the incentive compatibility constraint, which in turn imply a lower degree of credit rationing, i.e. a higher amount of credit available. Which of these contrasting effects will prevail on the determination of economic activity depends on the relative sensitivities of agents' behavior and the incentive compatibility constraint with respect to the amplitude of the liquidity constraint.

4.0.4 Changes in the amplitude of the liquidity constraint

In order to study the change in π_L when lenders' liquidity degree $1-\mu$ decreases, let us inspect once more (35) and remember that its right hand side $\varphi(\pi_L)$ is decreasing in π_L. It is then immediate to verify that, as depicted in *Figure 2*, the new equilibrium value of π_L will be higher and the degree of credit rationing lower. In order to study the consequences of an increase in μ on the steady state levels of the interest rate and capital, we go back to *Figure 3*, in which the point E represents the initial equilibrium in the $\{\pi_L, \rho\}$ plane. As one can readily verify, when μ increases, the share a of lenders' wage income that is loaned decreases. From equation (30) it is easy to verify that $d\rho_1/d\mu > 0$ and that when μ increases the curve $\rho_1(\pi_L)$ is upward shifting in the (π_L, ρ) plane (see the new curve $\rho_1'(\pi_L)$ in figure 3). From equation (31), one also easily verifies that π_L is increasing in a. The sign of $d\pi_L/d\mu$ (and thus $d\rho_2/d\mu$) is therefore ambiguous and, a priori, the curve $\rho_2(\pi_L)$ can be upward as well as downward shifting. This implies that at the new equilibrium E' the new stationary interest rate ρ^* can be higher or lower. An analogous ambiguity, in the light of (29), concerns the change in the capital steady state level.

The basic intuition of the results described above is the following. If lenders are subject to an higher degree of liquidity, they need to invest in money an higher quantity of their wage income and therefore the amount of credit available to borrowers will be lower. This entails a lower level of per firm capital and decreases borrowers' expected utility. But this implies a relaxation of the incentive compatibility constraint (11) in the sense that the latter is now compatible with an higher π_L. Competition among lenders will make actually effective this increase. The latter effect, in turn, will lower the per-firm labor force $l(\pi_L)$ which will then contrast the initial fall in the capital intensity. For that reason the marginal productivity of capital, and thus the capital rental price, can increase as well as decrease. The effects on the degree of credit rationing, interest rate and capital of an increase in the liquidity degree μ of lenders are summarized in the following proposition.

Proposition 4 *An increase in the amplitude μ of the liquidity constraint yields an increase in π_L^*, whereas the effect on the per-firm capital steady state k^* and on the interest rate ρ^* are ambiguous.*

5 Concluding remarks

Economists largely agree on the fact that in order to deeply understand the role of credit rationing in determining the level of economic activity, one should be able to integrate monetary, financial and real aspects, possibly in a general equilibrium framework. In particular, it should be analyzed more in depth how borrowing constraints (as expressed by the occurrence of credit rationing) interact with liquidity (cash-in-advance) constraints in determining the amount of credit available.

In this paper, in the context of a monetary overlapping generations model (OLG) in which money demand is introduced via cash-in-advance constraint, we have studied the effects on credit rationing, capital and interest rate due to changes in the money growth rate or in the tightness of the liquidity constraints. We have shown, in particular, that an increase in the money growth rate as well as an increase in the liquidity degree of lenders, yields to a decrease in credit rationing, whereas the effects on the per-firm capital steady state and on the interest rate are ambiguous. In addition, we have seen that increasing the degree of liquidity of borrowers yields a reduction in the interest rate and in the degree of credit rationing and to an increase in capital.

There are at least three interesting extensions of the model. The first one concerns the study of government policies aiming at reducing the degree of credit rationing and thus to promote growth. The second one relies on the dynamical features of the model, in particular on the interactions between the presence of credit rationing and the possibility that the economy displays deterministic as well as expectations-driven fluctuations (see, e.g., Greenwald and Stiglitz [3]). Another extension would consist in the inclusion of financial intermediaries like banks. The credit market activity, is that case, would be better described by a game played between bank and firms.

References

[1] Azariadis, C. and B. Smith, (1993), Adverse Selection in the Overlapping Generations Model: The case of pure exchange, *Journal of Economic Theory* 60, 277-305.

[2] Bencivenga, V. and B. Smith, (1993), Some consequences of credit rationing in an endogenous growth model, *Journal of Economic Dynamics and Control* 17, 97-122.

[3] Greenwald B. and J. Stiglitz, 1993, Financial market imperfections and Business Cycles, *Quarterly Journal of Economics*, 108, 77-114.

[4] Rothschild, M. and J. Stiglitz, 1976, Equilibrium in competitive insurance markets: An essay on the economics of imperfect information, *Quarterly Journal of Economics*, 90, 629-650.