

CSEF

Centre for Studies in Economics and Finance

WORKING PAPER NO. 725

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June 2024



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Abstract

Part of a ship's cargo is jettisoned in order to save the vessel and the remaining cargo from imminent peril. How should the loss be shared among the cargo owners? The law of general average, an ancient principle of maritime law, prescribes that the owners share the loss proportionally according to the respective values of their cargo. We analyze whether the law of general average is a truthful and efficient mechanism. That is, we investigate whether it induces truthful reporting of cargo values and yields a Pareto efficient allocation in equilibrium. We show that the law of general average is neither truthful nor efficient if owners have expected utility preferences, but is both truthful and efficient if owners have maxmin utility preferences. We discuss why maxmin behavior may be reasonable in the general average context.

JEL Classification: C72, D82, G22, K39.

Keywords: general average, loss sharing, maritime law, maxmin, mutual insurance, truthful equilibrium, Pareto efficiency.

Acknowledgments: The authors are grateful to Paolo Siconolfi, Danielle D'Onfro, Eric Talley, Holger Spamann, and Francesco Parisi for their extremely valuable advice and comments and to Hojoon Kim for his excellent research assistance. We also thank audiences at LET XII, SAET 2024, ALEA 2024, Georgetown, Columbia, and Cornell for their helpful feedback.

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1 Introduction

Part of a ship's cargo is voluntarily jettisoned in order to save the vessel and the remaining cargo from imminent peril. How should the loss be shared among the cargo owners?

The law of general average, an ancient principle of the general maritime law of nations, prescribes that the owners share the loss in proportion to the respective values of their cargo. Its roots can be traced back to a provision of Roman law, Digest XIV.2.1, which cites the Rhodian law of jettison: "The Rhodian law provides that if cargo has been jettisoned in order to lighten a ship, the sacrifice for the common good must be made good by common contribution" (Watson 1998, p. 419). Modern courts have interpreted this maxim to require pro rata contribution.¹ A contemporary statement of the law of general average, which is also known simply as general average, is set forth in *Zim Israel Navigation Co., Ltd. v. 3-D Imports, Inc.*, 29 F. Supp. 2d 186, 190 (S.D.N.Y. 1998) (citations omitted):

"General Average is an ancient doctrine, referring to rules apportioning loss suffered by cargo owners whose goods are sacrificed in a maritime adventure. . . . [W]hen one partner in the adventure sacrifices its cargo or incurs expenses for the general safety of the ship and other cargo, the loss is assessed against all participants in proportion to their respective share in the adventure. Today, contribution in General Average is recognized by all major maritime nations."

Prior work by Landes and Posner (1978, pp. 106-108) showed that the law of general average has important efficiency properties. Their model, however, assumed cargo values are public information, whereas they often are private information. Consequently, Landes and Posner analyzed only the incentives provided by the law of general average for the ship master's decisions regarding which and how much cargo to jettison. They showed that the general average principle gives the master the incentive to minimize the collective loss "by selecting the lowest-valued (per lb.) goods to jettison." But they did not consider the incentives it provides owners for truthful reporting of cargo values.

Epstein (1993, pp. 582-584) recognized that the "secret" to making the general average mechanism "work," in the sense of enabling the master to minimize the collective loss, is to get truthful reporting of cargo values.² (In fact, truthful reporting is sufficient, but not necessary, to enable the master to minimize the collective loss. What's necessary is *truthful ordering*, i.e., the owners' declared values must be in the same rank order as their true values.)

¹See, e.g., *Cia. Atlantica Pacifica, S.A. v. Humble Oil & Refining Co.*, 274 F. Supp. 884, 891 (D. Md. 1967) ("The principle embodied in this maxim [is] that loss for the common benefit which is incurred by one who partakes in a maritime venture should be shared ratably by all who participate in the venture. . . ."). See also *Empire Stevedoring Co. v. Oceanic Adjusters, Ltd.*, 315 F. Supp. 921, 927 (S.D.N.Y. 1970).

²See also Gregory, Kalven, and Epstein (1977, pp. 35-36).

Epstein offered intuition for why the law of general average gives owners the right incentives for truthful reporting. However, he did not provide a formal game-theoretic treatment or welfare analysis of the general average mechanism.

We model the general average game and analyze whether the law of general average is a truthful and efficient mechanism. That is, we investigate whether the law of general average induces truthful reporting of cargo values and yields a Pareto efficient allocation in equilibrium. We show that truthful reporting is not a Bayesian Nash equilibrium of the general average game if owners have heterogeneous expected utility preferences, but that truthful reporting is the unique Nash equilibrium if owners have maxmin utility preferences. We further show that in the expected utility case, (i) the law of general average does not yield a Pareto efficient allocation in equilibrium because it does not induce truthful ordering (let alone truthful reporting) in equilibrium,³ and (ii) even with truthful reporting, an allocation prescribed by the law of general average, which ipso facto entails proportional loss sharing, is Pareto efficient if and only if owners have identical (up to a positive scalar) CRRA utility functions.⁴ In the maxmin utility case, by contrast, the law of general average not only induces truthful reporting, it also (mechanically) yields a Pareto efficient allocation.⁵

In addition to contributing to the niche literature on the economics of general average, our paper makes contributions to two broader literatures in economics and law.

The first is the economics of mutual insurance. The paper most closely related to ours is Cabrales, Calvó-Armengol, and Jackson (2003) which analyzes a mutual fire insurance mechanism in Andorra called *La Crema*. In the *La Crema* game, each homeowner reports the value of his house. In case of a fire, one or more houses burn down, where nature determines which and how many houses burn. Each owner of a burned house receives his reported value, which is paid by all homeowners (including himself) in proportion to their reported values. The general average game is similar. Each cargo owner declares the value of her cargo. In an emergency, cargo is jettisoned in ascending order of declared value, where nature determines how much cargo must be jettisoned. Each owner of jettisoned cargo receives her declared value, which is paid by all cargo owners (including herself) in proportion to their declared values. The key difference between the two games is that in the *La Crema* game whether an owner's house burns down is independent of the reported values, whereas in the general average game whether an owner's cargo is jettisoned depends on all the declared values including her own.

³Without truthful ordering, the master cannot minimize the collective loss.

⁴The acronym CRRA stands for constant relative risk aversion.

⁵We say “mechanically” because there is no risk sharing in the maxmin case; there is only one utility-relevant state (the worst-case state) and the law of general average exhausts all resources in every state.

The second broader literature to which we contribute is the economics of ancient law. A prime example is Aumann and Maschler (1985) which presents a game-theoretic analysis of a bankruptcy problem in the Babylonian Talmud and shows that the Talmudic solution coincides with the nucleolus of the corresponding coalitional game. The principle underlying the Talmudic solution is not proportional division. Still, the Talmudic bankruptcy problem and the general average problem are similar in that the core question is how to divide a residual among claimants whose claims sum to more than the total value of the residual. Miller (2010) collects two dozen contributions to this literature which cover a wide range of topics including ancient liability systems, family law, land law, and criminal law.

The remainder of the paper proceeds as follows. Section 2 is a brief primer on the law of general average. Section 3 describes the general average game. Sections 4 and 5 present our equilibrium results for the cases where cargo owners have expected utility preferences (the Bayesian game) and where owners have maxmin utility preferences (the maxmin game), respectively. Section 6 presents our efficiency results for both cases. Section 7 concludes the paper with a discussion in which we compare and contrast our results with those of Cabrales, Calvó-Armengol, and Jackson (2003), suggest why maxmin behavior may be reasonable in the general average context, and point to directions for future research. The appendix collects selected proofs.

2 The Law of General Average

In maritime law, the term “average” means damage or loss (Shoenbaum 2011, p. 253). It is an anglicization of the French nautical term *avarie*, which in turn derives from the Arabic word *‘awār* through the Latin (and later Italian) *avaria* (Healy and Sharpe 1999, p. 760; Khalilieh 2006, p. 150). The term “general average” refers to a collective loss. It is the loss incurred when, for the benefit of all parties with a financial interest in the voyage, part of a ship or its cargo is voluntarily sacrificed to avoid a common imminent peril (Chamberlain 1940, p. 89). Under the law of general average, the parties share the collective loss in proportion to the values of their respective interests (Healy and Sharpe 1999, p. 761; Khalilieh 2006, p. 151).⁶

The principle embodied in the law of general average dates back to antiquity.⁷ The Babylonian Talmud, Bava Kamma 116b, articulates the principle in the context of land caravans: “If a caravan was traveling through the wilderness and a band of robbers threatened to plunder it, the apportionment [for buying them off] will have to be made according with

⁶See also, e.g., Rose (1997, p. 2); Robertson, Friedell, and Stuckey (2001, p. 426).

⁷Some commentators suggest that general average may go back to the Phoenicians (circa 1200-800 BCE), the Babylonians (circa 2000 BCE), or even earlier. See, e.g., *Orient Mid-East Lines, Inc. v. Shipment of Rice on Board S.S. Orient Transporter*, 496 F.2d 1032, 1034 (5th Cir. 1974); Kruit (2017, pp. 23-24).

the [value] of possessions [in the caravan]” (Friedell 1996, p. 656). A snippet of Roman law, Digest XIV.2.1, which references the Rhodian law of jettison, is the earliest statement of the principle in the maritime context: “The Rhodian law provides that if cargo has been jettisoned in order to lighten a ship, the sacrifice for the common good must be made good by common contribution” (Watson 1998, p. 419). The principle was incorporated into Islamic legal codes from the eighth century (Khalilieh 2006, p. 160). Later statements appear in medieval European maritime codes such as the Rolls of Oleron and the Laws of Wisby, and also in early modern European maritime codes such as the Guidon de la Mer and the Ordonnance de la Marine (Barclay 1891; Lowndes, de Hart, and Rudolf 1912, pp. 4-16).

By the turn of the nineteenth century, the law of general average had been incorporated into the English common law (Lowndes, de Hart, and Rudolf 1912, pp. 18 & 21). Justice Lawrence of the King’s Bench gave the following definition in *Birkley v. Presgrave*, [1801] 1 East 220, 228: “All loss which arises in consequence of extraordinary sacrifices made or expenses incurred for the preservation of the ship and cargo comes within general average, and must be borne proportionably by all who are interested” (Lowndes, de Hart, and Rudolf 1912, p. 21).⁸

In *McAndrews v. Thatcher*, 70 U.S. 347, 366 (1865), the United States Supreme Court defined the law of general average as follows:

“[W]here two or more parties are in a common sea risk, and one of them makes a sacrifice or incurs extraordinary expenses for the general safety, the loss or expenses so incurred shall be assessed upon all in proportion to the share of each in the adventure; or, in other words, the owners of the other shares are bound to make contribution in the proportion of the value of their several interests.”⁹

Three requirements must be met for a loss to qualify for general average contribution: “First, that the ship and cargo should be placed in a common imminent peril; secondly, that there

⁸See also *The Copenhagen*, [1799] 1 Chr. Rob. 289, in which Lord Stowell of the Admiralty Court wrote: “General average is for a loss incurred, towards which the whole concern is bound to contribute pro rata, because it was undergone for the general benefit and preservation of the whole” (Lowndes, de Hart, and Rudolf 1912, p. 18).

⁹See also *Star of Hope*, 76 U.S. 203, 228 (1869); *Fowler v. Rathbones*, 79 U.S. 102, 114 (1870); *Hobson v. Lord*, 92 U.S. 397, 404 (1875); *Ralli v. Troop*, 157 U.S. 386, 395 (1895). The earliest general average cases in the U.S. Supreme Court were *Columbian Ins. Co. v. Ashby*, 38 U.S. 331, 338 (1839), and *Barnard v. Adams*, 51 U.S. 270 (1850). The first reported general average case in an American court was *Brown v. Cornwell*, 1 Root (Conn.) 60 (1773). The vessel in that case sprung a leak during storm on Christmas Eve while sailing to St. Croix. Five horses stowed on deck were thrown overboard in order to save the ship and the cargo stowed below. The court held that the owner of the jettisoned horses was entitled to general average contribution from the owners of the saved cargo.

should be a voluntary sacrifice of property to avert that peril; and, thirdly, that by that sacrifice the safety of the other property should be presently and successfully attained.”¹⁰

The archetypal general average case involves the jettison of cargo.¹¹ The following are a handful of examples from reported cases in American courts:¹²

- The steamship *Allianca* declared general average after jettisoning 100 cases of turpentine during her voyage from New York to Rio de Janeiro in June 1889.¹³
- The schooner *Ernestina*, which sailed from San Juan, Puerto Rico in November 1917 laden with general cargo of merchandise, declared general average after jettisoning a portion of the cargo to refloat the ship after it sprung a leak during a violent storm.¹⁴
- The vessel *Odysseus III*, while on a voyage from Cuba to Tampa, Florida in 1946, jettisoned part of its cargo of pineapples in order to save the ship during a storm. The owner of the jettisoned cargo sued the vessel for general average contribution.¹⁵
- The vessel *William G. Osment*, while transporting lumber from Nicaragua to Curaçao in June 1951, jettisoned its deck cargo to save the ship and other cargo from total loss. The owner of the deck cargo sued the vessel for general average contribution.¹⁶
- The steamship *Columbia Brewer* declared general average on June 24, 1970 after jettisoning part of its cargo of bulk sugar in order to lighten the vessel after running aground on a shoal off Old Providence Island while en route from Hawaii to New Orleans.¹⁷

Although the jettison of cargo is the classic example, the law of general average applies to other losses as well,¹⁸ including sacrifices of part of the vessel such as the cutting away of the mast,¹⁹ and extraordinary expenses incurred for the joint benefit of the vessel and cargo

¹⁰ *Columbian Ins. Co. v. Ashby*, 38 U.S. 331, 338 (1839). See also, e.g., *Barnard v. Adams*, 51 U.S. 270 (1850); *Ralli v. Troop*, 157 U.S. 386 (1895).

¹¹ See *Ralli v. Troop*, 157 U.S. 386, 393 (1895); Rose (1997, p. 5); Kruit (2017, p. 23).

¹² Another example is *Brown v. Cornwell*, 1 Root (Conn.) 60 (1773), discussed supra note 9.

¹³ See *Commercial Union Ins. Co. v. Proceeds of the Allianca*, 64 F. 871 (S.D.N.Y. 1894).

¹⁴ See *Bravo v. St. Paul Fire & Marine Ins. Co.*, 259 F. 772 (1st Cir. 1919).

¹⁵ See *The Odysseus III*, 77 F. Supp. 297 (S.D. Fla. 1948).

¹⁶ See *Nicaraguan Long Leaf Pine Lumber Co. v. Moody*, 211 F.2d 715 (5th Cir. 1954). A similar case is *The Hettie Ellis*, 20 F. 393 (E.D. La. 1884), aff'd, 20 F. 507 (C.C.E.D. La. 1884).

¹⁷ See *California & Hawaiian Sugar Co. v. Columbia Steamship Co., Inc.*, 391 F. Supp. 894 (E.D. La. 1972), aff'd, 510 F.2d 542 (5th Cir. 1975).

¹⁸ For a non-exhaustive list, see Rose (1997, p. 5).

¹⁹ See *Ralli v. Troop*, 157 U.S. 386, 393 (1895).

such as those sustained in freeing the ship from the strand of a river,²⁰ capture by pirates,²¹ or the wreckage of a collapsed bridge.²²

While the law of general average is part of the general maritime law of nations, international maritime interests have created a set of rules to harmonize general average practice around the world (Robertson, Friedell, and Stuckey 2001, p. 426). The first version of these rules was known as the Glasgow Resolutions 1860 (Lowndes, de Hart, and Rudolf 1912, pp. 41-44). The current version is known as the York-Antwerp Rules 2016. These rules have never been adopted by treaty and do not have the force of law; however, they are widely incorporated in bills of lading and courts generally enforce them as binding terms of contract between the parties (Gilmore and Black 1975, pp. 252-253; Robertson, Friedell, and Stuckey 2001, p. 426; Shoenbaum 2011, pp. 256-257).²³

General average is recognized by marine commentators as a “risk spreading or burden sharing mechanism. One could even say that it is a kind of mutual insurance for the parties interested in the maritime adventure, which already existed before the insurance concept as we know it today was introduced” (Kruit 2017, p. 22).²⁴ Marine insurance proper was first developed in the thirteenth and fourteenth centuries (Wilson and Cooke 1997, p. 683). Notwithstanding the development of modern marine insurance, the law of general average continues to operate today. Moreover, general average rights and obligations exist and are determined independently of any marine insurance held by the parties. As Justice Gorell Barnes stated in *The Brigella*, [1893] P. 189, 195-196: “[T]he obligation to contribute in general average exists between the parties to the adventure, whether they are insured or not.

²⁰See *Navigazione Generale Italiana v. Spencer Kellogg & Sons*, 92 F.2d 41 (2d Cir. 1937).

²¹See *Mitsui & Co. Ltd. v. Beteiligungsgesellschaft LPG Tankerflotte MBH & Co. KG*, [2017] UKSC 68.

²²The cargo ship *Dali* declared general average on April 17, 2024 after knocking down the Francis Scott Key Bridge in Baltimore three weeks prior (Gardner 2024).

²³The York-Antwerp Rules 2016 are available at comitemaritime.org. They have three parts. Two prefatory rules form the first part. The second part comprises seven rules lettered A through G that specify basic principles. The third part contains 23 rules numbered I through XXIII that cover specific types of losses. Because our general average game features the jettison of cargo, we highlight two rules pertaining to cargo lost by sacrifice. First, Rules XVI(a)(i) & XVII(a)(i) together provide that the amount to be allowed as general average, and the contribution to a general average, shall be based on the value at the time of shipment, unless there is a commercial invoice rendered to the receiver, in which case it shall be based on the value at the time of discharge. Second, Rule XIX(b) provides that “[w]here goods have been wrongfully declared at the time of shipment at a value which is lower than their real value, any general average loss or damage shall be allowed on the basis of their declared value, but such goods shall contribute on the basis of their actual value.” In our general average game, we consider the situation where the true values are the cargo owners’ private information, and hence we posit that recovery amounts for jettisoned cargo, and contribution amounts for all cargo, are based on the declared values at the time of shipment. This is consistent with Rules XVI(a)(i) & XVII(a)(i), but seemingly inconsistent with Rule XIX(b); however, when cargo values are private information, Rule XIX(b) is facially inoperable.

²⁴See also Wilson and Cooke (1997, p. 683).

The circumstance of a party being insured can have no influence upon the adjustment of general average, the rules of which . . . are entirely independent of insurance.”²⁵

3 The General Average Game

There is a set $\mathcal{N} = \{1, \dots, N\}$ of $N > 2$ cargo owners. Each owner $i \in \mathcal{N}$ ships one cargo box with unit weight on the same vessel. Without loss of generality, we normalize each owner’s initial wealth to zero, apart from her cargo box. Each owner i has a utility function U_i that is strictly increasing, strictly concave, and twice differentiable. To capture heterogeneity in risk preferences, we assume that none of the owners’ utility functions are cardinally equivalent. That is, we assume U_i is not a positive affine transformation of U_j for all $i, j \in \mathcal{N}$, $i \neq j$.²⁶

The true value of owner i ’s cargo box is $t_i \in [0, \bar{t}]$ where $\bar{t} > 0$. The true value t_i is owner i ’s private information. The true values are independent and identically distributed according to a positive probability density function f_t with support $[0, \bar{t}]$. Consequently, with probability one, there are no ties among the true values. Let $t = (t_1, \dots, t_N)$ denote the vector of true values and $T = \sum_{i=1}^N t_i$ denote the total true value of the cargo boxes. For the vector of true values, we sometimes use the notation $t = (t_i, t_{-i})$ where $t_{-i} \in [0, \bar{t}]^{N-1}$ is the subvector of true values other than t_i .

At the outset of the venture, the owners privately declare the values of their cargo boxes to the master of the vessel. We assume that the master subsequently publishes the declared values. This assumption is without loss of generality, however, because no further strategic decisions are made in the game. Other than the true values, which are the owners’ private information, we assume that all other aspects of the game are common knowledge.

Let $v_i \in [0, \bar{t}]$ denote the declared value of owner i ’s cargo box, $v = (v_1, \dots, v_N)$ denote the vector of declared values, and $V = \sum_{i=1}^N v_i$ denote the total declared value of the cargo

²⁵See also *Shaver Transp. Co. v. Travelers Indem.*, 481 F. Supp. 892, 897 (D. Or. 1979) (“General average contribution exists independently of marine insurance and is owed even in the absence of cargo insurance.”); Parks (1987, p. 620) (“[G]eneral average is not part of the law of marine insurance; . . . rights to contribution are not affected by the presence or absence of marine insurance coverage.”). Nowadays, cargo owners typically insure themselves against liability for general average contribution and loss from general average sacrifice of their cargo (in which case the insurer is subrogated to the owner’s right to general average contribution). See generally Parks (1987, pp. 620-628); Wilson and Cooke (1997, sec. 7); Rose (1997, ch. 7). See also Young et al. (1995, sec. 66). We abstract from cargo insurance in the model; however, we do not believe that adding it would change the basic insights provided that insurers have the same information as owners.

²⁶This assumption is actually stronger than we need. For purposes of our results, it would suffice to assume that at least one owner’s utility function is cardinally unique from the rest. The case of heterogeneity is arguably the interesting case, as there is ample evidence of heterogeneity in risk preferences in insurance settings (see, e.g., Cohen and Einav 2007; Einav et al. 2012; Barseghyan, Prince, and Teitelbaum 2011; Barseghyan et al. 2013; Barseghyan, Molinari, and Teitelbaum 2016; Barseghyan et al. 2021), including in mutual insurance settings (e.g., Mazzocco and Saini 2012; Chiappori et al. 2014).

boxes. For the vector of declared values, we sometimes use the notation $v = (v_i, v_{-i})$ where $v_{-i} \in [0, \bar{t}]^{N-1}$ is the subvector of declared values other than v_i .

The owners' declarations—and the realization of a random tie-breaking rule r applied to break any ties among them—induce a strict ordering of the cargo boxes. Assume r is distributed according to a probability mass function f_r with support $\Psi(\mathcal{N})$, where $\Psi(\mathcal{N})$ denotes the set of all permutations of \mathcal{N} . A realization of r is a randomly selected ordering of the cargo owners.²⁷ Thus, for any and all sets of tied declarations, a realization of r can be applied to strictly order such declarations. Let $n_i(v, r)$ denote the place of owner i 's cargo box in the ascending order of declared values. (For the avoidance of doubt, all references to “the ascending order of declared values” or “in ascending order of declared value” refer to such order with any ties broken.) We assume that the master labels each cargo box with its owner's identity, declared value, and place in the ascending order of declared values.

In an emergency, the master jettisons cargo boxes in ascending order of declared value. Let $\Theta = \{0, \dots, N\}$ and $\theta \in \Theta$ denote the number of cargo boxes that must be jettisoned in order to save the vessel and the remaining cargo. Assume θ is distributed according to a probability mass function f_θ with support Θ . Owner i 's cargo box is jettisoned if and only if $n_i(v, r) \leq \theta$. Let $J_v(v, \theta)$ denote the total declared value of the cargo boxes that are jettisoned (i.e., the sum of their declared values) and $P_v(v, \theta) = J_v(v, \theta)/V$ denote the proportion of the total declared value that is jettisoned. Similarly, let $J_t(t, v, r, \theta)$ denote the total true value of the cargo boxes that are jettisoned (i.e., the sum of their true values) and $P_t(t, v, r, \theta) = J_t(t, v, r, \theta)/T$ denote the proportion of the total true value that is jettisoned.²⁸ Note that while $P_v(v, \theta)$ is common knowledge, $P_t(t, v, r, \theta)$ is unknown to all.

Under the law of general average, which prescribes proportional sharing of the collective loss, owner i 's final wealth equals $v_i - v_i P_v(v, \theta)$ if her cargo box is jettisoned and $t_i - v_i P_v(v, \theta)$ if her cargo box is not jettisoned. That is, owner i 's payoff is

$$c_i = \begin{cases} v_i - v_i P_v(v, \theta) & \text{if } n_i(v, r) \leq \theta \\ t_i - v_i P_v(v, \theta) & \text{if } n_i(v, r) > \theta \end{cases}. \quad (1)$$

In what follows, we sometimes refer to the first component of owner i 's payoff, v_i or t_i (as the case may be), as her *recovery*, and to the second component, $v_i P_v(v, \theta)$, as her *contribution*. Observe that the law of general average is a strongly budget-balanced mechanism. Accord-

²⁷The only requirement that f_r must satisfy is that all permutations must have positive probability.

²⁸Note that the total declared value of the cargo boxes that are jettisoned does not depend on r because it only affects the ordering of cargo boxes with equal declared values. By contrast, the total true value of the cargo boxes that are jettisoned does depend on r because, with probability one, cargo boxes with equal declared values have unequal true values.

ingly, summing the payoffs across all owners for any given θ , we obtain $(1 - P_t(t, v, r, \theta))T$.²⁹ Thus, the outcome of the general average game is an allocation among the owners of the total true value of the cargo boxes that are not jettisoned.

4 Equilibrium of the Bayesian Game

Assume cargo owners have expected utility preferences. In this case the general average game is a standard Bayesian game. Each owner $i \in \mathcal{N}$ knows the true value of her cargo box $t_i \in [0, \bar{t}]$ (and all other aspects of the game other than t_{-i}). Her declaration, therefore, is a function $b_i : [0, \bar{t}] \rightarrow [0, \bar{t}]$. The declaration function b_i is effectively owner i 's strategy. Let $b = (b_1, \dots, b_N)$ denote the vector of declaration functions. We sometimes use the notation $b = (b_i, b_{-i})$ where b_{-i} refers to the subvector of all declaration functions other than b_i , and we sometimes write b_i as $b_i(\cdot)$ to emphasize that it is a function.³⁰ In equilibrium, for every owner $i \in \mathcal{N}$, (I) it is *as if* she knows the declaration functions $b_{-i}(\cdot)$ of the other owners, and hence knows their declarations $v_{-i} = b_{-i}(t_{-i})$ conditional on any $t_{-i} \in [0, \bar{t}]^{N-1}$, and (II) her declaration $v_i = b_i(t_i)$ must maximize her expected payoff given $b_{-i}(\cdot)$.

As we note in section 1, *truthful reporting*, i.e., $b_i(t_i) = t_i$ for all $i \in \mathcal{N}$ and $t_i \in [0, \bar{t}]$, is sufficient, but not necessary, to enable the master to minimize the owners' collective loss. What's necessary is *truthful ordering*, i.e., the owners' declared values must be in the same rank order as their true values.

Definition 1 *The cargo owners' declared values have a truthful order if and only if*

$$t_i < t_j \iff b_i(t_i) < b_j(t_j) \tag{2}$$

for all $i, j \in \mathcal{N}$ and $t_i, t_j \in [0, \bar{t}]$.

The following lemma, the formal proof of which is set forth in section A.2 of the appendix, establishes that if the declared values have a truthful order, then every owner must have the same declaration function.

Lemma 1 *If the declared values have a truthful order, then $b_i(\cdot) = b_j(\cdot)$ for all $i, j \in \mathcal{N}$. Moreover, $b_i(\cdot)$ is a strictly increasing function for all $i \in \mathcal{N}$.*

We now can state the following result.

²⁹See section A.1 in the appendix.

³⁰For the same reason, we sometimes write other functions in this way as well—e.g., $U_i(\cdot)$.

Proposition 1 *If cargo owners have expected utility preferences, then there does not exist a Bayesian Nash equilibrium of the general average game in which the owners' declared values have a truthful order.*

Observe that truthful reporting implies truthful ordering (but not vice versa). Hence, if we do not have truthful ordering in equilibrium, then we do not have truthful reporting either.

Corollary 1 *If cargo owners have expected utility preferences, then there does not exist a Bayesian Nash equilibrium of the general average game in which all owners truthfully declare the values of their cargo boxes.*

The proof of proposition 1 is set forth in section A.3 of the appendix. The following is a sketch of the argument. Assume the owners' declared values have a truthful order. With probability one, therefore, there are no ties among the declared values (which renders moot the tie-breaking rule r). Take any owner $i \in \mathcal{N}$ and any declaration functions $b_{-i}(\cdot)$ for the other owners. Owner i 's declaration problem is

$$\max_{v_i \in [0, \bar{v}]} \Pi_i(v_i, t_i) = E_{t_{-i}, \theta} [U_i(c_i(v_i, t_i, b_{-i}(t_{-i}), \theta))] \quad (3)$$

where we now write owner i 's payoff c_i as $c_i(v_i, t_i, b_{-i}(t_{-i}), \theta)$ to make explicit the variables on which it depends and to reflect that the other owners' declared values v_{-i} depend on their true values t_{-i} via their declaration functions $b_{-i}(\cdot)$:

$$c_i(v_i, t_i, b_{-i}(t_{-i}), \theta) = \begin{cases} v_i - v_i P_v(v_i, b_{-i}(t_{-i}), \theta) & \text{if } n_i(v_i, v_{-i}) \leq \theta \\ t_i - v_i P_v(v_i, b_{-i}(t_{-i}), \theta) & \text{if } n_i(v_i, v_{-i}) > \theta \end{cases}. \quad (4)$$

Observe that owner i can compute her expected payoff $\Pi_i(v_i, t_i)$ for any declaration v_i as she knows the distributions of t_{-i} and θ . The solution to problem (3) is $v_i^* = b_i^*(t_i)$ and must satisfy a set of first-order conditions.³¹ However, because U_i is not cardinally equivalent to U_j for any other owner $j \in \mathcal{N}$, the solution for another owner $j \in \mathcal{N}$ is $v_j^* = b_j^*(t_j)$ where $b_j^*(\cdot) \neq b_i^*(\cdot)$, which contradicts lemma 1. Hence, the owners' declared values cannot have a truthful order in equilibrium, and so we do not have truthful reporting in equilibrium either.

Remark Proposition 1 continues to hold when the number of cargo owners is “large,” provided there is still heterogeneity in risk preferences. (This is clear from the proof as well as in the intuition above.) In a large society, all that is needed is a positive measure of owners with utility functions that are cardinally unique from the rest.

³¹See equation (A.2) in the appendix.

5 Equilibrium of the Maxmin Game

In the previous section, we showed that there does not exist a truthful equilibrium of the general average game if cargo owners have expected utility preferences. In this section, by contrast, we establish the following result.

Proposition 2 *If cargo owners have maxmin utility preferences, then truthful declarations by all owners is the unique Nash equilibrium of the general average game.*

Assume owners have maxmin utility preferences. Given any declarations v_{-i} by the other owners, owner i 's declaration problem is

$$\max_{v_i \in [0, \bar{t}]} \min_{\substack{v_{-i} \in [0, \bar{t}]^{N-1} \\ r \in \Psi(\mathcal{N}) \\ \theta \in \Theta}} \begin{cases} U_i(v_i - v_i P_v(v_i, v_{-i}, \theta)) & \text{if } n_i(v_i, v_{-i}, r) \leq \theta \\ U_i(t_i - v_i P_v(v_i, v_{-i}, \theta)) & \text{if } n_i(v_i, v_{-i}, r) > \theta \end{cases}.$$

In other words, the owner chooses her declaration v_i to maximize her payoff assuming the worst-case combination of the declarations by the other owners v_{-i} , the realization of the tie-breaking rule r , and the number of cargo boxes that are jettisoned θ .

Observe that if $\theta \in \{0, N\}$, the owner's payoff does not depend on her declaration v_i . After all, if no cargo boxes are jettisoned then her payoff is t_i , and if all cargo boxes are jettisoned then her payoff is zero.³² Thus, defining $\bar{\Theta} = \{1, \dots, N-1\}$, the owner's non-degenerate problem is

$$\max_{v_i \in [0, \bar{t}]} \min_{\substack{v_{-i} \in [0, \bar{t}]^{N-1} \\ r \in \Psi(\mathcal{N}) \\ \theta \in \bar{\Theta}}} \begin{cases} U_i(v_i - v_i P_v(v_i, v_{-i}, \theta)) & \text{if } n_i(v_i, v_{-i}, r) \leq \theta \\ U_i(t_i - v_i P_v(v_i, v_{-i}, \theta)) & \text{if } n_i(v_i, v_{-i}, r) > \theta \end{cases}. \quad (5)$$

Looking at problem (5), we can see that, given v_i , whether or not owner i 's cargo box is jettisoned, the worst-case combination of v_{-i} and θ is the combination that yields the maximum value of $P(v_i, v_{-i}, \theta)$. The following lemma establishes that this value is $(N-1)/N$.

Lemma 2 *Take any $v_i \in [0, \bar{t}]$. Suppose $v_{-i} \in [0, \bar{t}]^{N-1}$, $r \in \Psi(\mathcal{N})$, and $\theta \in \bar{\Theta}$. Then the maximum value that $P_v(v_i, v_{-i}, \theta)$ can achieve is $(N-1)/N$. This value is achieved by setting $v_j = v_i$ for all $j \in \mathcal{N}$, $j \neq i$, and $\theta = N-1$.*

The proof of lemma 2 is set forth in section A.4 in the appendix.

³²Formally, if $\theta = 0$ then $P_v(v_i, v_{-i}, \theta) = 0$ and hence $t_i - v_i P_v(v_i, v_{-i}, \theta) = t_i$, and if $\theta = N$ then $P_v(v_i, v_{-i}, \theta) = 1$ and hence $v_i - v_i P_v(v_i, v_{-i}, \theta) = 0$.

In what follows, we consider two collectively exhaustive cases, $v_i \geq t_i$ and $v_i \leq t_i$, and apply lemma 2 to show that in each case the solution to problem (5) is $v_i^* = t_i$.

First, suppose the owner declares $v_i \geq t_i$. For any given combination of v_i and θ , this implies $v_i - v_i P_v(v_i, v_{-i}, \theta) \geq t_i - v_i P_v(v_i, v_{-i}, \theta)$. It follows that the worst-case combination of v_i , r , and θ is the combination that yields $P_v(v_i, v_{-i}, \theta) = (N-1)/N$ and $n_i(v_i, v_{-i}, r) > \theta$. In this case, problem (5) amounts to

$$\max_{v_i \in [t_i, \bar{v}]} U_i \left(t_i - v_i \frac{N-1}{N} \right).$$

Observe that $t_i - v_i[(N-1)/N]$ is strictly decreasing in v_i . Because $U_i(\cdot)$ is strictly increasing, this implies that the solution to the owner's problem in this case is $v_i^* = t_i$.

Next, suppose the owner declares $v_i \leq t_i$. For any given combination of v_i and θ , this implies $v_i - v_i P_v(v_i, v_{-i}, \theta) \leq t_i - v_i P_v(v_i, v_{-i}, \theta)$. It follows that the worst-case combination of v_i , r , and θ is the combination that yields $P_v(v_i, v_{-i}, \theta) = (N-1)/N$ and $n_i(v_i, v_{-i}, r) \leq \theta$. In this case, problem (5) amounts to

$$\max_{v_i \in [0, t_i]} U_i \left(v_i - v_i \frac{N-1}{N} \right).$$

Observe that $v_i - v_i[(N-1)/N]$ is strictly increasing in v_i . Because $U_i(\cdot)$ is strictly increasing, this implies that the solution to the owner's problem in this case is also $v_i^* = t_i$.

The foregoing establishes that making a truthful declaration is the unique solution to problem (5), assuming owner i has maxmin utility preferences. Because owner i is arbitrary, this implies that if cargo owners have maxmin utility preferences, then truthful declarations by all owners is the unique Nash equilibrium of the general average game.

Remark The key to understanding owner i 's maxmin behavior lies in her "conjecture" that whatever declaration v_i she chooses, the other owners and nature will "conspire" to minimize her payoff. This entails them choosing v_{-i} and θ to maximize $P_v(v_i, v_{-i}, \theta)$, which increases her contribution, $v_i P_v(v_i, v_{-i}, \theta)$, all else equal. By lemma 2, these values are $v_{-i} = v_i$ and $\theta = (N-1)/N$. Moreover, owner i conjectures that (i) if she overdeclares then the tie-breaking rule will result in her cargo box not being jettisoned, in which case she receives her true value minus her contribution (i.e., she is denied the benefit of her overdeclaration (overrecovery) but suffers the cost (higher contribution)), and (ii) if she underdeclares then the tie-breaking rule will result in her cargo box being jettisoned, in which case she receives her declared value minus her contribution (i.e., she suffers the cost of her underdeclaration (underrecovery) which exceeds the benefit (lower contribution)). In the former case she

gains by decreasing her declaration to her true value (which does not affect her recovery but decreases her contribution), and in the latter case she gains by increasing her declaration to her true value (which increases her recovery by more than it increases her contribution).

6 Pareto Efficiency

Recall that the outcome of the general average game is an allocation among the cargo owners of the total true value of the cargo boxes that are not jettisoned, $(1 - P_t(t, v, r, \theta))T$. In this section, we investigate the conditions under which an allocation prescribed by the law of general average, to which we refer as a general average allocation, is Pareto efficient.

As a threshold matter, Pareto efficiency requires that the proportion of the total true value that is jettisoned, $P_t(t, v, r, \theta)$, is the minimum necessary to save the vessel and the remaining cargo. We refer to this requirement as *resource efficiency*. Given that cargo boxes are jettisoned in ascending order of declared value, resource efficiency is attained if and only if the declared values have a truthful order.

6.1 Expected Utility Preferences

Suppose cargo owners have expected utility preferences. Recall that in this case there does not exist a Bayesian Nash equilibrium of the general average game in which the declared values have a truthful order. An immediate implication is that the law of general average does not yield a Pareto efficient allocation in equilibrium, because without truthful ordering (let alone truthful reporting) resource efficiency cannot be attained. Even with truthful reporting, however, a general average allocation, which ipso facto entails proportional loss sharing, is Pareto efficient if and only if owners have identical (up to a positive scalar) CRRA utility functions. We prove this claim in section A.5 of the appendix.

The following proposition recaps the foregoing.

Proposition 3 *If cargo owners have expected utility preferences, then the law of general average does not yield a Pareto efficient allocation in equilibrium. This is because there is no Bayesian Nash equilibrium of the general average game in which there is a truthful ordering of declared values (let alone truthful reporting), and hence resource efficiency is not attained in equilibrium. Even assuming truthful reporting, a general average allocation is Pareto efficient if and only if owners have identical (up to a positive scalar) CRRA utility functions.*

6.2 Maxmin Utility Preferences

Now suppose cargo owners have maxmin utility preferences. Recall that in this case truthful reporting is the unique Nash equilibrium of the general average game. In equilibrium, therefore, the law of general average yields the following allocation as a function of θ :

$$c_i(\theta) = (1 - P_t(t, v, r, \theta))t_i \quad \forall i \in \mathcal{N}, \forall \theta \in \Theta.$$

Call this allocation c^* and denote its components by $c_i^*(\theta)$.

With maxmin utility preferences, the utility that owner i derives from any allocation c is the utility assuming the worse-case state, $\min_{\theta \in \Theta} U_i(c_i(\theta))$. Let $\bar{\theta}$ denote the worse-case state in the non-degenerate problem.³³ Then owner i 's utility from allocation c is $U_i(c_i(\bar{\theta}))$.

Given resource efficiency, which is implied by truthful reporting, an allocation is Pareto efficient if and only if there does not exist a reallocation of resources that makes at least one owner better off without making at least one other owner worse off. Take allocation c^* . The only utility-relevant components of c^* are the payoffs in state $\bar{\theta}$,

$$c_i^*(\bar{\theta}) = (1 - P_t(t, v, r, \bar{\theta}))t_i \quad \forall i \in \mathcal{N},$$

and the only relevant resource constraint is the one pertaining to state $\bar{\theta}$,

$$\sum_{i=1}^N c_i^*(\bar{\theta}) = (1 - P_t(t, v, r, \bar{\theta}))T.$$

Because c^* exhausts all available resources in each state,³⁴ it follows that any reallocation c' which increases the payoff in state $\bar{\theta}$ to owner i , $c'_i(\bar{\theta}) > c_i^*(\bar{\theta})$, necessarily decreases the payoff in state $\bar{\theta}$ to some other owner j , $c'_j(\bar{\theta}) < c_j^*(\bar{\theta})$, in order to obey the resource constraint for state $\bar{\theta}$. Because $U_i(\cdot)$ is strictly increasing, this implies that there does not exist a reallocation of resources that makes at least one owner better off without making at least one other owner worse off. Hence, allocation c^* is Pareto efficient. To recap:

Proposition 4 *If cargo owners have maxmin utility preferences, then the law of general average yields a Pareto efficient allocation in equilibrium.*

³³Lemma 2 establishes that $\bar{\theta} = N - 1$. The argument that follows, however, applies given any value of $\bar{\theta}$.

³⁴For all $\theta \in \Theta$, $\sum_{i=1}^N c_i^*(\theta) = \sum_{i=1}^N (1 - P_t(t, v, r, \theta))t_i = (1 - P_t(t, v, r, \theta))T$.

7 Discussion

We have shown that the law of general average does not induce truthful ordering (let alone truthful reporting) in equilibrium, and consequently does not yield a Pareto efficient allocation in equilibrium, if cargo owners are expected utility maximizers and there is heterogeneity in owners' risk preferences. However, we have also shown that if cargo owners choose their declarations according to the maxmin criterion, then the law of general average both induces truthful reporting in equilibrium and yields a Pareto efficient allocation in equilibrium.

To help situate our results and contributions in the literature, it is useful to compare and contrast the properties of the general average game with those of the *La Crema* game in Cabrales, Calvó-Armengol, and Jackson (2003). In both the *La Crema* game and the general average game, nature determines how much property is destroyed—i.e., the number of houses burned (k) or cargo boxes jettisoned (θ), as the case may be—and the owners share the collective loss in proportion to the declared values of their property. In each game, therefore, the owners' conditional payoffs (i.e., their payoffs conditional on their property being destroyed or not destroyed) are affected by their declarations (and in precisely the same way).³⁵ The key difference is that in the *La Crema* game nature randomly determines which houses are burned (given k , nature uniformly randomly selects k houses), whereas in the general average game the master determines which cargo boxes are jettisoned based on the declared values (given θ , the master selects θ boxes in ascending order of declared value). Thus, in the general average game, but not in *La Crema* game, the owners' declarations affect not only their conditional payoffs but also the probability that their property is destroyed. The general average game would reduce to the *La Crema* game if the true values were common knowledge, the tie-breaking rule were uniformly distributed, and the master were to disregard the declared values and solely apply the tie-breaking rule to select which cargo boxes to jettison.

The similarities between the general average game and the *La Crema* game lead to some convergent results. Most notably, neither game has an equilibrium in which the owners report the true values of their property, assuming that owners have expected utility preferences and there is relevant heterogeneity among them,³⁶ and consequently neither game yields a Pareto efficient allocation in equilibrium. At the same time, the key difference between the two games leads to an important divergence when the number of owners is “large.”

³⁵Generally speaking, if they overdeclare they will overrecover if their property is destroyed but overcontribute if their property is not destroyed, and if they underdeclare they will underrecover if their property is destroyed but undercontribute if their property is not destroyed.

³⁶The general average game relies on heterogeneity in risk preferences while the *La Crema* game relies on heterogeneity in property values.

In sufficiently large societies, the (nondegenerate) Nash equilibria of the *La Crema* game are nearly truthful and approximately Pareto efficient. By contrast, the truth-telling and efficiency properties of the general average game continue to hold in large societies.

What do we take away from our analysis of the law of general average? On the one hand, we might conclude, based on our expected utility results, that the law of general average is inefficient and should be abandoned. This is the usual move in law and economics—if a law is found to be inefficient under standard assumptions (such as expected utility maximization), the analyst calls for the law to be reformed.

On the other hand, we might conclude, given the survival of the law of general average across time and cultures, that perhaps our maxmin utility results suggest that maxmin behavior may be reasonable in the general average context.³⁷ Sea voyages are subject to numerous risks, including but not limited to those related to weather, carrier error (e.g., overloading, structural failure, navigation error), international politics, and violence (e.g., piracy, terrorism, war). Maximization of expected utility may be reasonable when a decision maker has a credible basis for placing a prior probability distribution on the unknown features of her decision problem. However, when the decision maker faces Knightian uncertainty (ambiguity)—which very well may be the case in the general average context, where each voyage and its risks may be idiosyncratic, singular, or otherwise hard to know or quantify—she may feel that she lacks a credible basis for forming a prior. In these circumstances, the decision maker may reasonably evaluate her alternatives by the worst utility that they may yield and choose an alternative that yields the best worst utility.³⁸

The maxmin criterion is a deeply rooted idea in social science. Wald (1950) developed it as a solution of a statistical decision problem when a prior probability distribution is unknown. Rawls (1971) famously invoked it as part of a normative theory of justice. More to the point, Gilboa and Schmeidler (1989) proposed it as a model of choice under uncertainty when the decision maker has too little information to form a single prior and is uncertainty averse. In their model, known as maxmin expected utility (MMEU), uncertainty is captured by a set of priors, and the decision maker evaluates alternatives according to the minimal expected utility over the priors in the set. At one extreme, when the set contains a single prior, the MMEU model corresponds to the expected utility model. At the other extreme, when the set contains all possible priors, the MMEU model corresponds to the maxmin utility model.³⁹

³⁷That is, instead of economics informing the law, perhaps the law can inform economics—in this case, about the correct model of decision making under uncertainty.

³⁸Our argument here echoes that of Manski (2013, p. 122-123).

³⁹Because it selects alternatives that perform best in worst-case scenarios, the maxmin criterion is also central to theories of robust control (Hansen and Sargent 2008), robust optimization (Ben-Tal, El Ghaoui, and Nemirovski 2009), and robust contracting and mechanism design (Carroll 2019).

In the end, maxmin behavior may make quite a bit of sense for a cargo owner who must decide what to declare under a veil of ignorance about nature’s true distribution.

As a final observation, we reiterate that one can view the general average problem as a bankruptcy problem—i.e., the problem of how to divide a residual among claimants whose total claims exceed the value of the residual. Numerous rules other than proportional division have been proposed and studied as solutions to the bankruptcy problem (see generally Thompson 2003, 2015), including the Talmudic rule studied in Aumann and Maschler (1985). In future work it would be interesting to explore the truth-telling and efficiency properties of rules other than proportional division for sharing a general average loss.

Appendix

A.1 Summation of Payoffs

Take any $\theta \in \Theta$. Let $\mathcal{J}(\theta) \subseteq \mathcal{N}$ denote the set of owners whose cargo boxes are jettisoned. The law of general average is a strongly budget-balanced mechanism: the total transfers paid to owners whose cargo boxes are jettisoned,

$$\sum_{i \in \mathcal{J}(\theta)} v_i - v_i P_v(v, \theta) = \sum_{i \in \mathcal{J}(\theta)} v_i - \sum_{i \in \mathcal{J}(\theta)} v_i P_v(v, \theta) = (1 - P_v(v, \theta)) J_v(v, \theta),$$

are equal to the total transfers paid by owners whose cargo boxes are not jettisoned,

$$\sum_{i \in \mathcal{N} \setminus \mathcal{J}(\theta)} v_i P_v(v, \theta) = (V - J_v(v, \theta)) P_v(v, \theta) = (1 - P_v(v, \theta)) J_v(v, \theta).$$

Accordingly, summing the payoffs in equation (1) across all owners, we have

$$\sum_{i \in \mathcal{J}(\theta)} v_i - v_i P_v(v, \theta) + \sum_{i \in \mathcal{N} \setminus \mathcal{J}(\theta)} t_i - v_i P_v(v, \theta) = \sum_{i \in \mathcal{N} \setminus \mathcal{J}(\theta)} t_i = (1 - P_t(t, v, r, \theta)) T.$$

Hence, the outcome of the general average game is an allocation among the owners of the total true value of the cargo boxes that are not jettisoned.

A.2 Proof of Lemma 1

We proceed by contradiction for both claims. Assume that the declared values have a truthful order. That is, assume condition (2) holds.

First, suppose that for some $i, j \in \mathcal{N}$ there exists some $t_0 \in [0, \bar{t}]$ such that $b_i(t_0) \neq b_j(t_0)$. Without loss of generality, suppose $b_i(t_0) < b_j(t_0)$. By condition (2), this implies $t_0 < t_0$, which is impossible. Hence, there cannot exist any $t_0 \in [0, \bar{t}]$ such that $b_i(t_0) \neq b_j(t_0)$. It follows that $b_i(\cdot) = b_j(\cdot)$ for all $i, j \in \mathcal{N}$.

Now let $b_0(\cdot) = b_i(\cdot) = b_j(\cdot)$ for all $i, j \in \mathcal{N}$ but suppose that $b_0(\cdot)$ is not strictly increasing. Then there must exist some $t_i, t_j \in [0, \bar{t}]$ such that $t_i > t_j$ and $b_0(t_i) \leq b_0(t_j)$. This, however, contracts condition (2), under which $t_i > t_j$ implies $b_0(t_i) > b_0(t_j)$. Hence, $b_0(\cdot)$ must be strictly increasing. It follows that $b_i(\cdot)$ is a strictly increasing function for all $i \in \mathcal{N}$.

A.3 Proof of Proposition 1

By way of contradiction, suppose that the declaration functions $b_1^*(\cdot), \dots, b_N^*(\cdot)$ constitute a truthful order equilibrium of the Bayesian game.

By lemma 1, we know that for some $b_0^*(\cdot)$ we have that $b_0^*(\cdot) = b_i^*(\cdot) = b_j^*(\cdot)$ for all $i, j \in \mathcal{N}$ and that $b_0^*(\cdot)$ is strictly increasing. By standard results, because it is monotone over an interval, $b_0^*(\cdot)$ is differentiable almost everywhere on its domain.

Because ties among the true values have probability zero, lemma 1 implies that ties among the declared values have probability zero in a truthful order equilibrium. Hence, we can safely ignore ties in what follows and drop the tie-breaking rule r from the notation.

Now consider a cargo owner $i \in \mathcal{N}$ whose cargo box has true value $t_i \in [0, \bar{t}]$ and who takes as given that all other owners make declarations according to $b_0^*(\cdot)$. Given t_i , owner i chooses her declaration $v_i \in [0, \bar{t}]$ to maximize her expected payoff as in problem (3), where $b_{-i}(t_{-i})$ is now given by $b_{-i}^*(t_{-i}) = (b_0^*(t_1), \dots, b_0^*(t_{i-1}), b_0^*(t_{i+1}), \dots, b_0^*(t_N))$.

Take any $v_i \in [0, \bar{t}]$. Let $\mathcal{B}(v_i) = \{j \in \mathcal{N}, j \neq i : b_0^*(t_j) < v_i\}$ denote the set of other cargo owners whose declarations are below v_i , and let $|\mathcal{B}(v_i)|$ denote the cardinality of the set $\mathcal{B}(v_i)$. Given any $\theta \in \Theta$, let

$$\mathcal{T}(v_i, \theta) = \{t_{-i} \in [0, \bar{t}]^{N-1} : |\mathcal{B}(v_i)| \geq \theta\}$$

denote the set of other true value vectors t_{-i} such that at least θ other owners declare values below v_i . Hence, given v_i and θ , the probability that owner i 's cargo box remains on board (i.e., is not jettisoned) is given by

$$R(v_i, \theta) = \int_{t_{-i} \in \mathcal{T}(v_i, \theta)} f_{t_{-i}}(t_{-i}) dt_{-i}$$

where, with a slight abuse of notation, $f_{t_{-i}}(\cdot)$ denotes the joint density of t_{-i} .⁴⁰

We can now write the expected payoff to cargo owner i of declaring value v_i given her true value t_i and the hypothetical truthful order equilibrium characterized by $b_0^*(\cdot)$:

$$\begin{aligned} \Pi_i(v_i, t_i) = & \sum_{\theta=0}^N \{ (1 - R(v_i, \theta)) E_{t_{-i}} [U_i(v_i - v_i P_v(v_i, b_{-i}^*(t_{-i}), \theta)) \mid t_{-i} \notin \mathcal{T}(v_i, \theta)] \\ & + R(v_i, \theta) E_{t_{-i}} [U_i(t_i - v_i P_v(v_i, b_{-i}^*(t_{-i}), \theta)) \mid t_{-i} \in \mathcal{T}(v_i, \theta)] \} f_\theta(\theta). \end{aligned} \quad (\text{A.1})$$

Our contradiction hypothesis that $b_0^*(\cdot)$ constitutes a truthful order equilibrium now says that for every given t_i it must be that $b_0^*(t_i)$ maximizes owner i 's expected payoff $\Pi_i(v_i, t_i)$.

Because $b_0^*(\cdot)$ is strictly increasing, if t_i is interior then so is $b_0^*(t_i)$. Observe that t_i is interior almost surely and that $b_0^*(\cdot)$ is differentiable almost everywhere. Hence we can limit our attention to examining the first-order condition necessary for a maximum of $\Pi_i(v_i, t_i)$ at any interior t_i where $b_0^*(\cdot)$ is differentiable.

To keep notation the notation simple, let $\Pi'_i(v_i, t_i)$ denote the partial derivative of $\Pi_i(v_i, t_i)$ with respect to v_i . At any interior t_i where $b_0^*(\cdot)$ is differentiable, our contradiction hypothesis implies that for owner i and every other owner $j \in \mathcal{N}$,

$$\Pi'_i(b_0^*(t_i), t_i) = 0 \quad \text{and} \quad \Pi'_j(b_0^*(t_i), t_i) = 0. \quad (\text{A.2})$$

However, because $U_i(\cdot)$ is not cardinally equivalent to $U_j(\cdot)$ for any $j \in \mathcal{N}$, $j \neq i$, condition (A.2) does not hold in general. Substantiating this claim in its most comprehensive meaning would involve abstract arguments from differential topology.⁴¹ Instead, we present a simple (but generalizable) argument that relies on linear perturbations of utility that form a field of cardinally unique utility functions.⁴² Using the notation we establish in equation (4), pick a small $\alpha > 0$ and define

$$\tilde{U}_i(c_i(v_i, t_i, b_{-i}(t_{-i}), \theta)) = U_i(c_i(v_i, t_i, b_{-i}(t_{-i}), \theta)) + \alpha v_i.$$

Note that $U_i(\cdot)$ and $\tilde{U}_i(\cdot)$ are not cardinally equivalent but that as α becomes arbitrarily small they become arbitrarily close (in the sup norm). Using equation (A.1), define the

⁴⁰Because the true values are independent and identically distributed according to $f_i(\cdot)$, we have that $f_{t_{-i}}(\cdot) = \underbrace{f_t(\cdot) \times \cdots \times f_t(\cdot)}_{N-1}$.

⁴¹Two key references are Guillemin and Pollack (1974) and Mas-Colell (1985).

⁴²It is standard practice to use linear perturbations of utility to prove (or disprove) genericity and stability results with respect to optima. See, e.g., Mas-Colell (1985, ch. 8); Mas-Colell, Whinston, and Green (1995, ch. 17.D).

expected payoffs given $U_i(\cdot)$ and $\tilde{U}_i(\cdot)$ by $\Pi_i(v_i, t_i)$ and $\tilde{\Pi}_i(v_i, t_i)$, respectively. We then have

$$\tilde{\Pi}'_i(b_0^*(t_i), t_i) = \Pi'_i(b_0^*(t_i), t_i) + \alpha. \quad (\text{A.3})$$

From equation (A.3) it is immediate that if $\Pi'_i(b_0^*(t_i), t_i) = 0$ then $\tilde{\Pi}'_i(b_0^*(t_i), t_i) \neq 0$.

A.4 Proof of Lemma 2

Take any $v_i \in [0, \bar{t}]$. Suppose $v_{-i} \in [0, \bar{t}]^{N-1}$, $r \in \Psi(\mathcal{N})$, and $\theta \in \bar{\Theta}$ are such that owner i 's cargo box is jettisoned. Let $\mathcal{J}(\theta) \subseteq \mathcal{N}$ denote the set of owners whose cargo boxes are jettisoned. Note that $i \in \mathcal{J}(\theta)$. Then

$$P_v(v_i, v_{-i}, \theta) = \frac{v_i + \sum_{j \in \mathcal{J}(\theta), j \neq i} v_j}{v_i + \sum_{j \in \mathcal{N}, j \neq i} v_j}. \quad (\text{A.4})$$

Observe that the numerator of equation (A.4) is strictly increasing in θ while the denominator is independent of θ . Hence, to maximize equation (A.4), we must set $\theta = N - 1$. Given this, equation (A.4) becomes

$$P_v(v_i, v_{-i}, \theta) = \frac{v_i + \sum_{j \in \mathcal{J}(\theta), j \neq i, j \neq N} v_j}{v_i + v_N + \sum_{j \in \mathcal{N}, j \neq i, j \neq N} v_j}. \quad (\text{A.5})$$

Note that equation (A.5) is strictly increasing in the summation term in the numerator. Thus, to maximize (A.5), we must set each declared value in the summation equal to \bar{t} , which implies that we also must set v_N equal to \bar{t} (because v_N is the highest declared value), and we conclude that the maximum value of $P_v(v_i, v_{-i}, \theta)$ in this case is

$$P_v(v_i, v_{-i}, \theta) = \frac{v_i + (N - 2)\bar{t}}{v_i + (N - 1)\bar{t}}. \quad (\text{A.6})$$

Take the same $v_i \in [0, \bar{t}]$. But now suppose $v_{-i} \in [0, \bar{t}]^{N-1}$, $r \in \Psi(\mathcal{N})$, and $\theta \in \bar{\Theta}$ are such that owner i 's cargo box is not jettisoned. Let $\mathcal{J}(\theta) \subseteq \mathcal{N}$ denote the set of owners whose cargo boxes are jettisoned. Note that now $i \notin \mathcal{J}(\theta)$. Then

$$P_v(v_i, v_{-i}, \theta) = \frac{v_i + \sum_{j \in \mathcal{J}(\theta)} v_j}{v_i + \sum_{j \in \mathcal{N}, j \neq i} v_j}. \quad (\text{A.7})$$

Observe that the numerator of equation (A.7) is strictly increasing in θ while the denominator is independent of θ . Hence, to maximize equation (A.7), we must set $\theta = N - 1$. Given this, the summations in the numerator and denominator are both the summation of all declared values other than v_i . Thus, to maximize equation (A.7), we must set these declared values as

high as possible without violating the condition $n_i(v_i, v_{-i}, r) > \theta$. We can do this by setting them all equal to v_i . (With all declared values equal, the tie-breaking rule can be set such that owner i 's cargo box is the only one not jettisoned.) We therefore conclude that the maximum value of $P_v(v_i, v_{-i}, \theta)$ in this case is

$$P_v(v_i, v_{-i}, \theta) = \frac{\sum_{j \in \mathcal{N}, j \neq i} v_j}{v_i + \sum_{j \in \mathcal{N}, j \neq i} v_j} = \frac{(N-1)v_i}{v_i + (N-1)v_i} = \frac{(N-1)}{N}. \quad (\text{A.8})$$

Note that because the maximum value of $P_v(v_i, v_{-i}, \theta)$ set forth in equation (A.8), namely $(N-1)/N$, is achieved with all declared values being equal, it follows that it can be achieved with the tie-breaking rule yielding either that owner i 's cargo box is not the one jettisoned or that owner i 's cargo box is the one jettisoned. Therefore, to conclude the proof, it is sufficient to show that $(N-1)/N$ is weakly greater than the maximum value of $P_v(v_i, v_{-i}, \theta)$ set forth in equation (A.6). Indeed, this is immediate by noting that equation (A.6) is strictly increasing in v_i and, in fact, is equal to $(N-1)/N$ when $v_i = \bar{t}$.

A.5 Proof of Claim in Section 6.1

In this section we prove the claim, made in section 6.1, that if cargo owners have expected utility preferences, then even with truthful reporting, a general average allocation is Pareto efficient if and only if owners have identical (up to a positive scalar) CRRA utility functions.

Assume cargo owners have expected utility preferences. Let $c_i(\theta)$ denote the payoff to owner i in state θ . An allocation is an array $c = [c_i(\theta)]_{i \in \mathcal{N}, \theta \in \Theta}$ of payoffs to all owners $i \in \mathcal{N}$ in all states $\theta \in \Theta$. With truthful reporting, the law of general average prescribes the following allocation:

$$c_i(\theta) = (1 - P_t(t, v, r, \theta))t_i \quad \forall i \in \mathcal{N}, \forall \theta \in \Theta.$$

Thus, a truthful general average allocation is characterized by

$$\frac{c_j(\theta)}{c_i(\theta)} = \frac{t_j}{t_i} \quad \forall i, j \in \mathcal{N}, \forall \theta \in \Theta, \quad (\text{A.9})$$

$$\frac{c_i(\theta'')}{c_i(\theta')} = \frac{1 - P_t(t, v, r, \theta'')}{1 - P_t(t, v, r, \theta')} \quad \forall i \in \mathcal{N}, \forall \theta', \theta'' \in \Theta. \quad (\text{A.10})$$

Give the assumptions on U_i , the interior of the set of Pareto efficient allocations coincides with the solutions to the planner's problem with positive Pareto weights:

$$\max_c \sum_{i=1}^N \sum_{\theta=0}^N \alpha_i [U_i(c_i(\theta))\mu(\theta)], \quad \alpha_1, \dots, \alpha_N > 0,$$

subject to the resource constraints

$$\sum_{i=1}^N c_i(\theta) = (1 - P_t(t, v, r, \theta))T \quad \forall \theta \in \Theta,$$

which are satisfied here. The necessary and sufficient first-order conditions are

$$U'_i(c_i(\theta)) = \frac{\lambda_\theta}{\mu(\theta)\alpha_i} \quad \forall i \in \mathcal{N}, \forall \theta \in \Theta,$$

where λ_θ denotes the Lagrange multiplier pertaining to the θ -constraint. It follows that the set of Pareto efficient allocations is characterized by

$$\frac{U'_i(c_i(\theta))}{U'_j(c_j(\theta))} = \frac{\alpha_j}{\alpha_i} \quad \forall i, j \in \mathcal{N}, \forall \theta \in \Theta, \quad (\text{A.11})$$

$$\frac{U'_i(c_i(\theta'))}{U'_i(c_i(\theta''))} = \frac{\lambda_{\theta'}\mu(\theta'')}{\lambda_{\theta''}\mu(\theta')} \quad \forall i \in \mathcal{N}, \forall \theta', \theta'' \in \Theta. \quad (\text{A.12})$$

Suppose owners have identical (up to a positive scalar) CRRA utility functions. That is,

$$U_i(x) = \begin{cases} \beta_i \frac{x^{1-\eta}}{1-\eta} & \text{if } \eta \neq 1 \\ \beta_i \ln(x) & \text{if } \eta = 1 \end{cases} \quad \forall i \in \mathcal{N},$$

where $\beta_i > 0$ and $\eta \geq 0$. Then conditions (A.11) and (A.12) become

$$\frac{c_j(\theta)}{c_i(\theta)} = \left[\frac{\alpha_j}{\alpha_i} \right]^{\frac{1}{\eta}} \quad \forall i, j \in \mathcal{N}, \forall \theta \in \Theta, \quad (\text{A.13})$$

$$\frac{c_i(\theta'')}{c_i(\theta')} = \left[\frac{\lambda_{\theta'}\mu(\theta'')}{\lambda_{\theta''}\mu(\theta')} \right]^{\frac{1}{\eta}} \quad \forall i \in \mathcal{N}, \forall \theta', \theta'' \in \Theta. \quad (\text{A.14})$$

Comparing conditions (A.9)-(A.10) and conditions (A.13)-(A.14), we can see that there exist positive Pareto weights and Lagrange multipliers such that the two pairs of conditions are equivalent. Moreover, this is not the case for utility functions outside the CRRA family, because only CRRA utility implies that payoff ratios across owners depend only on relative wealth levels and that payoff ratios across states depend only on relative shadow prices.

References

- Aumann, Robert J. and Michael Maschler. 1985. “Game Theoretic Analysis of a Bankruptcy Problem from the Talmud.” *Journal of Economic Theory* 36 (2):195–213.
- Barclay, Thomas. 1891. “The Definition of General Average.” *Law Quarterly Review* 7 (1):22–42.
- Barseghyan, Levon, Maura Coughlin, Francesca Molinari, and Joshua C. Teitelbaum. 2021. “Heterogeneous Choice Sets and Preferences.” *Econometrica* 89 (5):2015–2048.
- Barseghyan, Levon, Francesca Molinari, Ted O’Donoghue, and Joshua C. Teitelbaum. 2013. “The Nature of Risk Preferences: Evidence from Insurance Choices.” *American Economic Review* 103 (6):2499–2529.
- Barseghyan, Levon, Francesca Molinari, and Joshua C. Teitelbaum. 2016. “Inference under Stability of Risk Preferences.” *Quantitative Economics* 7 (2):367–409.
- Barseghyan, Levon, Jeffrey Prince, and Joshua C. Teitelbaum. 2011. “Are Risk Preferences Stable Across Contexts? Evidence from Insurance Data.” *American Economic Review* 101 (2):591–631.
- Ben-Tal, Aharon, Laurent El Ghaoui, and Arkadi Nemirovski. 2009. *Robust Optimization*. Princeton, NJ: Princeton University Press.
- Cabrales, Antonio, Antoni Calvó-Armengol, and Matthew O. Jackson. 2003. “La Crema: A Case Study of Mutual Fire Insurance.” *Journal of Political Economy* 111 (2):425–458.
- Carroll, Gabriel. 2019. “Robustness in Mechanism Design and Contracting.” *Annual Review of Economics* 11:139–166.
- Chamberlain, Roy W. 1940. *Maritime Law for Seamen*. New York: D. Van Nostrand.
- Chiappori, Pierre-André, Krislert Samphantharak, Sam Schulhofer-Wohl, and Robert M. Townsend. 2014. “Heterogeneity and Risk Sharing in Village Economies.” *Quantitative Economics* 5 (1):1–27.
- Cohen, Alma and Liran Einav. 2007. “Estimating Risk Preferences from Deductible Choice.” *American Economic Review* 97 (3):745–788.
- Einav, Liran, Amy Finkelstein, Iuliana Pascu, and Mark R. Cullen. 2012. “How General are Risk Preferences? Choice under Uncertainty in Different Domains.” *American Economic Review* 102 (6):2606–2638.

- Epstein, Richard A. 1993. "Holdouts, Externalities, and the Single Owner: One More Salute to Ronald Coase." *Journal of Law and Economics* 36 (1):553–586.
- Friedell, Steven F. 1996. "Admiralty and the Sea of Jewish Law." *Journal of Maritime Law and Commerce* 27 (4):647–660.
- Gardner, Hayes. 2024. "Dali's Owner Declares 'General Average' in Key Bridge Disaster." *Baltimore Sun*. <https://www.baltimoresun.com/2024/04/17/key-bridge-dali-general-average>.
- Gilboa, Itzhak and David Schmeidler. 1989. "Maxmin Expected Utility with Non-unique Prior." *Journal of Mathematical Economics* 18 (2):141–153.
- Gilmore, Grant and Charles L. Black. 1975. *The Law of Admiralty*. 2d ed. Mineola, NY: Foundation Press.
- Gregory, Charles O., Harry Kalven, and Richard A. Epstein. 1977. *Cases and Materials on Torts*. 3d ed. Boston, MA: Little Brown.
- Guillemin, Victor and Alan Pollack. 1974. *Differential Topology*. Englewood Cliffs, NJ: Prentice Hall.
- Hansen, Lars Peter and Thomas J. Sargent. 2008. *Robustness*. Princeton, NJ: Princeton University Press.
- Healy, Nicholas J. and David J. Sharpe. 1999. *Cases and Materials on Admiralty*. 3d ed. St. Paul, MN: West.
- Khalilieh, Hassan S. 2006. *Admiralty and Maritime Laws in the Mediterranean Sea (ca. 800-1050)*. Leiden: Brill.
- Kruit, Jolien. 2017. *General Average, Legal Basis and Applicable Law*. Zutphen: Paris Legal Publishers.
- Landes, William M. and Richard A. Posner. 1978. "Salvors, Finders, Good Samaritans, and Other Rescuers: An Economic Study of Law and Altruism." *Journal of Legal Studies* 7 (1):83–128.
- Lowndes, Richard, Edward L. de Hart, and George Rupert Rudolf. 1912. *The Law of General Average: English and Foreign*. 5th ed. London: Stevens & Sons.
- Manski, Charles F. 2013. *Public Policy in an Uncertain World: Analysis and Decisions*. Cambridge, MA: Harvard University Press.

- Mas-Colell, Andreu. 1985. *The Theory of General Economic Equilibrium: A Differentiable Approach*. Cambridge: Cambridge University Press.
- Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green. 1995. *Microeconomic Theory*. New York: Oxford University Press.
- Mazzocco, Maurizio and Shiv Saini. 2012. “Testing Efficient Risk Sharing with Heterogeneous Risk Preferences.” *American Economic Review* 102 (1):428–468.
- Miller, Geoffrey P. 2010. *Economics of Ancient Law*. Cheltenham: Edward Elgar.
- Parks, Alex L. 1987. *The Law and Practice of Marine Insurance and Average*, Vol. 1. Centreville, MD: Cornell Maritime Press.
- Rawls, John. 1971. *A Theory of Justice*. Cambridge, MA: Belknap Press of Harvard University Press.
- Robertson, David W., Steven F. Friedell, and Michael E. Stuckey. 2001. *Admiralty and Maritime Law in the United States*. Durham, NC: Carolina Academic Press.
- Rose, F. D. 1997. *General Average: Law and Practice*. London: LLP.
- Shoenbaum, Thomas J. 2011. *Admiralty and Maritime Law*, Vol. 2. 5th ed. St. Paul, MN: West.
- Thompson, William. 2003. “Axiomatic and Game-theoretic Analysis of Bankruptcy and Taxation Problems: A Survey.” *Mathematical Social Sciences* 45 (3):249–297.
- . 2015. “Axiomatic and Game-theoretic Analysis of Bankruptcy and Taxation Problems: An Update.” *Mathematical Social Sciences* 74:41–59.
- Wald, Abraham. 1950. *Statistical Decision Functions*. New York: John Wiley.
- Watson, Alan. 1998. *The Digest of Justinian*, Vol. 1. Philadelphia, PA: University of Pennsylvania Press.
- Wilson, D. J. and J. H. S. Cooke. 1997. *Lowndes and Rudlof: The Law of General Average and the York-Antwerp Rules*. 12th ed. London: Sweet & Maxwell.
- Young, Christopher, Shannon S. Sanfilippo, James F. Moseley, Jr., Douglas M. Muller, Susan R. Bogart, Stephen C. Smith, and Robert Bocko. 1995. “A Comparison of United States Law to the Marine Insurance Act of 1906.” *Tulane Maritime Law Journal* 20 (1):5–103.