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Franz Ostrizek and Elia Sartori

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University of Naples Federico II **University of Salerno** **Bocconi University, Milan**

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS AND STATISTICS – UNIVERSITY OF NAPLES FEDERICO II
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unina.it
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Franz Ostrizek[*] and Elia Sartori[†]

Abstract
We investigate the behavioral foundations of informed trade. We extend the canonical (Kyle, 1989) model to allow for wide range of misperception about the information environment (e.g. overconfidence and correlation delusion) as well as the market clearing condition (e.g. understatement of individual impact) and ask when a trading equilibrium can exist. We show that existence requires either i) the market clearing rule being perceived with (cognitive) noise of arbitrary size, or ii) sufficiently strong misperceptions that lead traders to overestimate the precision of their private information (relative to that of others) or underestimate their market impact. Following i) provides a cognitive foundation for the noise trader approach, while ii) yields a highly tractable linear model of (sufficiently) biased traders. Fixing the bias, a higher number of traders is beneficial for existence, though the economy is typically discontinuous in the countable-trader limit. In the latter case, equilibrium is characterized by limit uncertainty, a property which is satisfied if and only if traders perceive some correlation in their competitors' information.

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[*] Sciences Po. Email: franz.ostrizek@gmail.com

[†] CSEF and University of Naples Federico II. Email: elia.sartori@unina.it

1 Introduction

Models of informed trading lie at the heart of the literature on market microstructure in economics and finance. Across a wide range of such models, there is a well understood problem of equilibrium existence that dates back to the classic price paradox: If all dispersed information is aggregated through the market price then individual private information is superfluous and traders should not use it; but if traders don't use private information, the market price contains no fundamental information and traders should use their own. Aggregation through prices means that private information is superfluous if (and only if) it is used, making a trading (as well as a no-trade) equilibrium impossible.

A common way to resolve this vicious cycle – and to obtain equilibrium existence – is to assume that markets are also populated by a fringe of noise traders (Kyle, 1985; Black, 1986) that inject aggregate noise to the market price. They do so by trading based on non-fundamental information, usually justified as an exogenous liquidity or hedging demand or as the result of unsophisticated behavioral traders. In the words of Black (1986) "Noise makes financial markets possible, but also makes them imperfect".

In stark cognitive opposition to the random actions of noise traders, informed traders are assumed perfect Bayesians who correctly interpret their own information as well as the information contained in prices. This assumption has been challenged in the vast literature on behavioral finance, which documents and models the range of market imperfections caused by the biases and misperceptions of market participants, such as overconfidence (Odean, 1998), cursedness (Eyster et al., 2019), or misperceptions of correlation (Banerjee, 2011).

In this paper, we ask whether biases and misperceptions make financial markets not only imperfect, but also possible. We consider a workhorse model of informed trade (Kyle, 1989) in a version without noise traders, but a wide range of behavioral parameters. Our objective environment is completely symmetric and prices are determined in fully revealing, frictionless manner. Our agents, however, may misperceive the information environment, thinking themselves to be more informed than others. Similarly, they may misperceive the market clearing rule, believing themselves to be more (or less) important, the price to be set or communicated with noise, or the market to be deeper than it truly is.

As is well understood, there is no equilibrium in the rational benchmark. How does the strength and interaction of these biases affect equilibrium existence? First, we show that an equilibrium exists as long as traders perceive the price to be noisy. Indeed, we can interpret the noise trader in the classic model as cognitive noise: What matters (for existence and the equilibrium) is not the real market clearing condition, but market clearing condition perceived by the traders.[1] Second, in an economy without noise, we show that equilibrium existence requires sufficiently strong misperceptions. Existence is ensured when traders overestimate the precision of their private information (relative to that of others reflected in the market price) and underestimate their market impact (relative to the depth of the market) to a sufficient degree.

The fact that substantial biases are required in the absent of noise traders though an infinitesimal

[1] Of course, when analyzing the market, e.g. computing price volatility, it will make a difference whether the noise traders are present in the market or merely in the mind of the traders.

noise trader is sufficient reflects a structural difference between the model with and without noise. In the former, the signal-to-noise ratio of the price is endogenous, and trading activity scales to ensure an equilibrium, no matter how small variance of the noise. In the latter, the price is fully informative in every informative equilibrium, and a sufficiently strong bias is required to ensure that individual traders use their private information. Another consequence of this distinction is that the model without noise traders is highly tractable, as it preserves the linearity inherent in the Gaussian-CARA setting.

We also study the role of the number of traders. As long as the number is finite, adding more traders never threatens equilibrium existence. However, this monotonicity result is not preserved in the limit. In an economy with countably many traders, existence requires a condition we call limit uncertainty, namely that traders consider their private signal informative about the fundamental even conditioning on the infinite collection of competitors' signals. This condition is independent of misperceptions of market clearing, and only satisfied by misperceptions of correlation. Indeed, also the model with (cognitive) noise of fixed variance fails to satisfy it.

2 Model and Equilibrium Definition

We aim to model traders who may suffer from a range of biases in perceiving the stochastic environment and of market clearing. Before introducing these biases, let us describe the objective environment.

2.1 Objective Environment

We conduct our analysis in the workhorse model of Kyle (1989). There are N ex-ante identical traders with CARA utility and risk aversion parameter ρ. They trade a single risky zero net supply asset with liquidation value $\nu \sim \mathcal{N}(0, \tau_o^{-1})$ by submitting demand schedules $x_i(p)$. Before submitting their demand, every trader receives a private signal $s_i \sim \mathcal{N}(\nu, \tau_s^{-1})$ independently across traders. The price is set by a market maker to clear the market

$$\sum_i x_i(p) = 0$$

and payoffs, $x_i(\nu - p)$, realize.

We focus on linear equilibria in which traders use a linear demand schedule, $x_i = \beta s_i - \gamma p$. Given this demand schedule, the market clearing price can be written as

$$p = p_i + M_i x_i$$

where p_i is the intercept of the residual supply curve for trader i and M_i denotes her market impact. Conditioning on the price, then, allows traders to infer about the average signal of others, and we write $\mathbb{E}[\nu|s_i, p_i]$ and $\mathbb{V}[\nu|s_i, p_i]$ for her posterior expectation (resp. variance) of the fundamental value. The optimal demand is given by

$$x_i = \frac{\mathbb{E}\left[\nu|s_i, p\right] - p_0}{2M_i + \rho\mathbb{V}\left[\nu|s_i, p\right]}. \tag{2.1}$$

2.2 Misperceptions

We assume throughout that our traders have the above common prior about the fundamental value of the asset, the utility specification, and we only consider biases that preserve the jointly Gaussian structure of the environment. The above best response rule, therefore, also holds in our setting with misperceptions. The structure of the rule remains, though clearly the specific biases will affect the specification of the expectation, variance and perceived market power operators.

2.2.1 Perceived Information Environment

According to trader i, the signals of herself and the other traders are distributed according to

$$\begin{pmatrix} s_i \\ \boldsymbol{s_{-i}} \end{pmatrix} \sim \mathcal{N}\left(\boldsymbol{\nu}, \tau_s^{-1}\begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{pmatrix}\right) \tag{2.2}$$

where $\Sigma_{11} \in \mathbb{R}_+$ captures misperception of the trader's own signal such as overconfidence, $\Sigma_{22} \in \mathbb{R}^{N-1\times N-1}$ captures misperceptions about the signals of others, such as dismissiveness and correlation illusion and $\Sigma_{12} \in \mathbb{R}^{N-1}$ captures misperceptions about the link between the signal of the trader herself and that of others, e.g. information projection. The only restriction we impose on the perceptions of the agent is that $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{pmatrix}$ is a valid variance-covariance matrix, i.e. that it is positive semidefinite.

The perceived information environment is symmetric in the sense that all traders perception of their own information, as well as their perception of others is identical.

2.2.2 Perceived Market Clearing

To understand how the price is determined, the trader uses her perceived market clearing rule

$$\begin{pmatrix} \Lambda_1, & \Lambda_2, & \Lambda_z\tau_s^{-1/2}, & \Lambda_p \end{pmatrix} \cdot \begin{pmatrix} x_i \\ \boldsymbol{x}_{-i} \\ z \\ -\gamma p \end{pmatrix} = 0 \tag{2.3}$$

where the parameters $\Lambda_1 \in \mathbb{R}_+, \Lambda_2 \in \mathbb{R}_+^{N-1}$ measure the relative misperception of the impact of the demand of the traders in determining the market price; $z \sim \mathcal{N}(0, 1)$ is perceived noise in market clearing, e.g. because traders believe that there is noise trader demand, with $\Lambda_z \geq 0$ denoting the perceived standard deviation of this noise relative to private information and $\Lambda_p \geq 0$ denotes additional market depth perceived to be provided by uninformed traders.

This perceived market clearing condition is used by the traders both to extract information from the price as well as to evaluate their market impact. Rearranging (2.3) we obtain the following linear subjective pricing equation

$$p = \lambda x_i + p_i \tag{2.4}$$

where

$$\lambda = \frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} \tag{2.5}$$

represent how much agents' think their demand will impact the price, while

$$p_i = \frac{\beta\Lambda_2 \boldsymbol{s}_{-i} + \Lambda_z \tau_s^{-1/2} z}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} \tag{2.6}$$

denotes the subjective intercept (the price that would realize if she chooses to refrain from trading).

The economy is characterized by the set of behavioral parameters $\vartheta = (\Sigma, \Lambda)$, and an objective environment $\tau = [\tau_0, \tau_s, \rho, N]$. We refer to $\vartheta^R = \left(I_{N\times N}, \left(\begin{array}{cccc} 1 & ,\mathbf{1}, & 0, & 0 \end{array}\right)\right)$ as the rational economy and to $\vartheta^K_{\Lambda_z} = \left(I_{N\times N}, \left(\begin{array}{cccc} 1 & ,\mathbf{1}, & \Lambda_z, & 0 \end{array}\right)\right)$ as the Λ_z-Kyle economy for some $\Lambda_z \in (0, \infty)$.

We introduce a useful taxonomy of the class of economies we will consider.

Definition 1. An economy is **regular** if $\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T \neq 0$. An economy is **linear** if it is regular and $\Lambda_z = 0$.

Regularity is a mild assumption that ensures that the price (subjectively) contains some information about the fundamental even after conditioning on the private signal. As their analysis is qualitatively different, we study non-regular economies separately in Appendix B.1. Linearity further requires that traders do not perceive an exogenous shifter in the market clearing equation. The reason for the moniker will be apparent in the next section. For the moment, we just notice that the rational economy is linear while the Kyle economy is not linear.

2.2.3 Discussion: our class of misperceptions

The misspecified economies (2.2)-(2.3), summarized by the parameters $\vartheta = (\Sigma, \Lambda)$ allows to consider a broad class of misperceptions about the information as well as institutional environment. As the examples in Section clarify, many commonly studied biases fall within our setup. However, our construction is not without loss of generality: in particular, we consider economies with a representative trader that is a misspecified Bayesian agent. Misspecified Bayesian traders have equilibrium awareness, i.e. implicit in the fixed-point formulation is that the traders know the equilibrium loadings (β, γ) when extracting information from prices to compute their best-response. In other words, our traders know that competitors put weight β on their private signal, though they might misunderstand (in an essentially arbitrary way) the joint distribution of the competitors' signals as well as the way their action determine the market price. What is more, even if our construction does not impose any symmetry assumption on traders' information and market impact, we are still studying linear and symmetric equilibria of the misspecified economy, thus assuming that traders make inference *as if* every other trader played the same strategy (which needs to be confirmed in the best response correspondence).

Understanding the types of biases that conduce to existence even outside the misspecified Bayesian paradigm is a natural extension of our analysis. We make a first step towards this generalization in Appendix B.2 where we study existence when traders form beliefs according to the mixture model (not representable with (2.2)-(2.3)) of cursed equilibrium — i.e. a frictionless version of the model considered by (Eyster et al., 2019).

2.3 Inference and Equilibrium

To obtain $\mathbb{E}(\nu|\{s_i, p\}), \mathbb{V}(\nu|\{s_i, p\})$ it is convenient to transform the price into to an independent signal $\hat{p}_i$ that is (subjectively) mean ν and conditionally independent of s_i, that is

$$\begin{pmatrix} s_i \\ \hat{p}_i \end{pmatrix} \sim \mathcal{N}\left(\boldsymbol{\nu}, \begin{pmatrix} \hat{\tau}_s & 0 \\ 0 & \hat{\tau}_p \end{pmatrix}\right) \tag{2.7}$$

so that

$$\mathbb{E}(\nu|\{s_i, p\}) = \frac{s_i\hat{\tau}_s + \hat{p}_i\hat{\tau}_p}{\hat{\tau}_p + \hat{\tau}_s + \tau_0}, \qquad \mathbb{V}(\nu|\{s_i, p\}) = \frac{1}{\hat{\tau}_p + \hat{\tau}_s + \tau_0}.$$

Lemma 1. *The conditionally independent representation (2.7) results from letting*

$$\hat{p}_i = \frac{\frac{(\mathbf{1}\Lambda_2^T + \Lambda_p)\gamma}{\beta} p_i - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T s_i}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} \tag{2.8}$$

and

$$\hat{\tau}_p^{-1} = \tau_s^{-1}\frac{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T + \frac{\Lambda_z^2}{\beta^2}}{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)^2} \tag{2.9}$$

$$\hat{\tau}_s^{-1} = \tau_s^{-1}\Sigma_{11} \tag{2.10}$$

We therefore arrive at the consistency condition

$$\beta s_i - \gamma p = \frac{\mathbb{E}(\nu|\{s_i, p_i\}) - p_i}{2\lambda + \rho\mathbb{V}(\nu|\{s_i, p_i\})} = \frac{\frac{s_i\hat{\tau}_s + \hat{p}_i\hat{\tau}_p}{\hat{\tau}_p + \hat{\tau}_s + \tau_0} - p_i}{2\lambda + \frac{\rho}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}} \tag{2.11}$$

where the variables on the right hand side are given as a functions of (β, γ) above.

We are now ready to define a trading equilibrium.

Definition 2. Given an economy ϑ, τ an equilibrium is a pair (β, γ) such that

i) (β, γ) satisfy the matching coefficients condition (2.11)

ii) the second order condition $2\lambda(\tau_0 + \hat{\tau}_s + \hat{\tau}_p) + \rho > 0$ is satisfied.

Essentially, an equilibrium exists if the matching coefficient operator has a fixed point that satisfies the (subjective) second-order condition. In the rational economy no equilibrium exists while the Kyle economy always has an equilibrium.

Lemma 2. *If $\vartheta = \vartheta^R$, then there is no equilibrium. For any $\Lambda_z \in (0, \infty)$, $\vartheta^K_{\Lambda_z}$ admits a unique equilibrium.*

3 Finite N

The mechanics of equilibrium existence differ between the linear and the non-linear case. In the nonlinear case, traders perceive the presence of noise traders. By Lemma (1), the precision of the residual information contained in the price is given by

$$\hat{\tau}_p(\beta) = \tau_s \left[\frac{\Lambda_2 \left(\Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12} \right) \Lambda_2^T}{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T \right)^2} + \frac{\Lambda_z^2}{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T \right)^2} \frac{1}{\beta^2} \right]^{-1} \tag{3.1}$$

The extent the private signal is used by traders (β) determines the signal-to-noise ratio of the price. Notice that, whenever $\Lambda_z > 0$ then as $\beta \to 0$ the second addendum explodes and $\hat{\tau}_p(\beta)$ vanishes: no matter its size, if traders perceive an exogenous shifter in the market clearing equation then the precision of the price goes to zero if the private signal is just barely used. This endogenous precision adjustment is key to ensure equilibrium existence: Intuitively, if the price were too informative and crowded out the use of private, its precision would be endogenously depressed, causing traders to rely more on private information and thereby supporting equilibrium existence. This will be possible as long as the perceived residual supply curve for the asset is sufficiently flat.

In the linear case ($\Lambda_z = 0$), the perceived precision of the price is instead independent of β,

$$\hat{\tau}_p = \tau_s \frac{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T \right)^2}{\Lambda_2 \left(\Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12} \right) \Lambda_2^T}. \tag{3.2}$$

Since there is no (perceived) noise, the price fully reveals the weighted sum of signal realizations $\Lambda_2 \boldsymbol{s}_{-i}$, independently of β, as long as it is nonzero. The perceived precision of prices is the same in all trading equilibria and depends solely on the (primitive) biases. Existence relies on a sufficient magnitude of those biases.

The lack of equilibrium feedback in the (perceived) precision of the price greatly simplifies the specification of the equilibrium system. Using this linear structure we arrive at a characterization of the candidate equilibrium, in closed form for the linear case, implicitly for all regular economies.

Proposition 1. *If an equilibrium exists, then it satisfies*

$$\beta = \frac{1}{\rho} \left[\hat{\tau}_s \left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p} \right) - \hat{\tau}_p(\beta) \left(\frac{2\Lambda_1 + \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T \left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p} \right)}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T} \right) \right] \tag{3.3}$$

$$\gamma = \frac{\tau_0 + \hat{\tau}_s + \hat{\tau}_p(\beta)}{\hat{\tau}_s + \hat{\tau}_p(\beta) \left(1 + \frac{\Lambda_p}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T} \right)} \beta \tag{3.4}$$

Our main result in this section characterizes the condition for equilibrium existence in terms of the behavior of the precision of the price in an setting where traders only use an infinitesimal amount of their private information. For linear economies, we can plug (3.2) into (3.3) and obtain a (unique) candidate equilibrium

$$b(\vartheta)=\frac{\tau_s}{\rho}\left[\Sigma_{11}^{-1}\left(1-\frac{\Lambda_1}{\mathbf{1}\Lambda_2^T+\Lambda_p}\right)-\frac{\left(\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)}{\Lambda_2\left(\Sigma_{22}-\Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T}\left(2\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1-\frac{\Lambda_1}{\mathbf{1}\Lambda_2^T+\Lambda_p}\right)\right)\right] \tag{3.5}$$

though the expression seems analytically cumbersome it much simplifies when considering specific biases (see Section 3.1 for examples). Crucially, (3.5) maps each linear economy ϑ into a single number, representing the only possible equilibrium loading on private information.

Theorem 1. *An equilibrium exists in the economy $\vartheta=(\Sigma,\Lambda)$ if and only if*

1. *the market clearing rule is perceived with (cognitive) noise, $\Lambda_z>0$, or*
2. *in the linear economy, $\Lambda_z=0$ the bias pushes forward into an equilibrium loading that either*
   (a) *is positive*
   $$b(\vartheta)>0,$$
   or
   (b) *is negative*
   $$b(\vartheta)<0$$
   and
   $$\Lambda_2\left(\Sigma_{22}-\mathbf{1}^T\Sigma_{12}\right)\Lambda_2^T<0. \tag{3.6}$$

Following Definition 2, an equilibrium is a solution to the matching coefficients equations that satisfies the traders' second-order condition. The matching coefficients equations always have a solution, so the question of existence reduces to the second-order condition which, using (3.3)-(3.4) can be rearranged to

$$\frac{\kappa(\vartheta)}{\beta}+\rho\mathbb{V}\left(\nu|\{s_i,p\}\right)>0$$

where $\kappa(\vartheta)$ is a positive constant. Since $\mathbb{V}(\nu|\{s_i,p\})$ is always positive, any positive solution to the matching coefficient equation is a valid equilibrium. Whenever $\Lambda_z>0$ the matching coefficient equation has a positive solution, which is therefore an equilibrium. Intuitively this is because $\Lambda_z>0\Rightarrow\lim_{\beta\to0}\hat{\tau}_p(\beta)=0$: whenever the market clearing equation is perceived with noise, the volume of informed trading can shrink enough that prices contain almost no information and it is therefore sufficient that the constant term in (3.3) is positive, which in turns requires that $1-\frac{\Lambda_1}{\mathbf{1}\Lambda_2^T+\Lambda_p}>0$. Notice that in the Kyle economy (rational but with $\Lambda_z>0$) the condition reduces to the trivial inequality $1>\frac{1}{N-1}$.

If $\Lambda_z=0$, instead $\hat{\tau}_p$ is no more endogenous and there is a unique solution (3.5) to the matching coefficient equation: if the solution is negative it has to be large in absolute value so that the positive variance term can dominate, and (3.6) ensures that this is the case.

Notice that, if $\Sigma_{12}=0$ then the condition (3.6) for a negative candidate b to be an equilibrium reads

$$\Lambda_2\Sigma_{22}\Lambda_2^T<0, \tag{3.7}$$

which can never be satisfied as Λ_2 is a positive vector and Σ_{22} is a positive semidefinite matrix. Therefore, an immediate corollary of Theorem (1) is that

Corollary 1. *In linear economies where traders have no misperception about the correlation of their individual signal with the other traders' signals, there is an equilibrium if and only if the candidate loading on private information is positive.*

In other words, because linear economies lack the adjustment margin as $\hat{\tau}_p(\beta)$ is constant, existence requires that traders are sufficiently biased in their perception of the information environment or of the market clearing, as we will explore shortly in a series of examples. Notice that the rational economy induces $b\left(\vartheta^R\right) = \frac{N-2}{2} - N - 1 < 0$ and (3.7) further simplifies to $\mathbf{1}I\mathbf{1} < 0$, which does not hold reaffirming that it cannot sustain existence in the linear (no-noise-trader) case.

3.1 Examples

All derivations from the general case to the specific examples are straightforward but rather algebraically tedious. We provide detailed calculations only for the example of information projection, in Appendix A.1.

3.1.1 Overconfidence

The model with overconfidence deviates from the rational model only in setting $\Sigma_{11} = \xi > 1$. A version of this model constituted the static benchmark in Kyle et al. (2018). Notice that this restriction satisfies the hypothesis of Corollary 1, hence an equilibrium exists if and only if the candidate loading on private information is positive. By direct substitution in (3.5), we obtain

$$b_\xi = \frac{\tau_s}{\rho}\left[\frac{N-2}{N-1}\xi - 2\right]$$

and therefore we have existence if and only if $\xi > 2\frac{N-1}{N-2}$. In particular, with bilateral trade, overconfidence alone is not capable of restoring existence; for large N, traders need to perceive their signal to be at least doubly as informative as it truly is. Compared to the rational candidate, the overconfident agent puts excessive weight on his private signal.

To see why a modicum of overconfidence is not enough, note that a rational agent wants load on all signals equally, which can be achieved through the price p. With slight overconfidence, the agents wants to put a small additional weight on his private signal. In equilibrium, this unravels as all traders want to put a greater weight on the price than on their signal. Intuitively, up to an adjustment for market power, at least a factor of two is needed to make sure the direct weight on the private signal exceeds its indirect weight through conditioning on the price.

Remark. Note that the above reasoning does not support equilibrium existence without noise traders in a rational model with heterogeneous traders. While market participants whose information is at least twice as precise as that of the average trader would be willing to use it sufficiently, in a rational model there has to be at least one trader with below average precision, from where any candidate equilibrium unravels. It is only in a behavioral model (as in the real world) that every trader can believe that they are at least twice as precise as their average competitor.

3.1.2 Under-Appreciation of the Information of Others

Consider the setting with $\Lambda = \Lambda^r$, $\Sigma_{11} = 1, \Sigma_{12} = 0$ and $\Sigma_{22} = I\delta(1-\phi) + \mathbf{1}^T\delta\phi\mathbf{1}$, where the parameter δ captures dismissal of other's precision and ϕ is correlation delusion; $\delta = 1$, $\phi = 0$ represent the rational benchmark. Since still $\Sigma_{12} = 0$, Corollary 1 applies and existence is characterized by a positive loading on private information.[2] In this setting, equation (3.5) simplifies to

$$b_{\delta,\phi} = \frac{\tau_s}{\rho}\left[\frac{N-2}{N-1} - 2\frac{(N-1)}{\delta(1+(N-2)\phi)}\right]$$

which is positive if and only if

$$\delta(1+(N-2)\phi) > \frac{2(N-1)^2}{N-2}$$

that is, if δ and ϕ are sufficiently large. If traders dismiss the information possessed by their competitor, either because it is heavily correlated (large ϕ) or because it is poor to begin with (large δ), then they are willing to load on their private signal and an equilibrium exists. More generally, we can consider the subjective precision of the price, $\hat{\tau}_p$, in effective units of other informed traders. That is, we parametrize $\hat{\tau}_p = (n-1)\tau_s$ where n is the equivalent number of informed traders. Then the candidate equilibrium is

$$b_n = \frac{\tau_s}{\rho}\left[\frac{N-2n}{N-1}\right]$$

which is positive if and only if $n < \frac{N}{2}$. Hence, to restore existence, we require that traders think that the price contains independent signals of half other traders.

3.1.3 Information Projection and Inverted Equilibria

Suppose the only bias in information processing is information projection, that is, given s_i, trader i thinks that trader j observes signal

$$s_j = \alpha s_i + (1-\alpha)(\nu + \epsilon_j)$$

which fits into our general framework once we let $\Sigma_{11} = 1$, $\Sigma_{22} = I\left(1-\alpha^2\right) + \mathbf{1}^T\alpha^2\mathbf{1}$ and $\Sigma_{12} = \alpha\mathbf{1}$ for $\alpha \in (0,1)$. As for the market clearing rule, we maintain $\Lambda_1 = \Lambda_2 = \mathbf{1}$, $\Lambda_p = 0$ and consider both the case with and without perceived liquidity traders. Henceforth, we consider projective economies $\vartheta_{\alpha,\Lambda_z}$ described by the projection parameter α and the perceived noise Λ_z. Detailed derivations are relegated to Appendix A.1.

In the linear model, where Corollary 1 does not apply and we have to check for potential inverted equilibria, the residual perceived precision contained in the price is

$$\hat{\tau}_p = \frac{(1-\alpha)(N-1)}{1+\alpha}\tau_s.$$

[2] The model without correlation delusion ($\phi = 0$) is essentially equivalent to the model with overconfidence analyzed in the previous section since, by suitably adjusting τ_0, the model can be reparametrized in terms of *relative* overconfidence $\xi \cdot \delta$ alone.

If $\alpha = 0$ we are back to the linear rational benchmark $\hat{\tau}_p = (N-1)\,\tau_s$, while if $\alpha = 1$ prices contain no residual information since the signal of others simply replicate the private signal. Using (3.3), we get the candidate equilibrium

$$b_\alpha = -\frac{\tau_s}{\rho}\left[\frac{N + (N-2)^2\,\alpha}{(N-1)\,(1+\alpha)}\right] \tag{3.8}$$

which is always negative. Using the sufficient condition (3.6), we get that (3.8) constitutes an equilibrium iff $\alpha > \frac{1}{N-2}$. There is an equilibrium in a market with (sufficiently) projective traders even in absence of noise traders, but this equilibrium is inverted: traders load negatively on their private information (and positively on the price).

Let us now consider the model where there is exogenous $\Lambda_z > 0$. Because τ_p is endogenous, (3.3) now defines a candidate equilibrium only implicitly as a solution to the cubic equation

$$\beta = \frac{\tau_s}{\rho}\left[\left(1 - \frac{1}{N-1}\right) - \frac{2 + (N-1)\,\alpha\left(1 - \frac{1}{N-1}\right)}{(1+\alpha) + \frac{\Lambda_z}{\beta^2}}\right]. \tag{3.9}$$

We show that (3.9) always admits a real positive solution, which per Theorem 1 then constitutes an equilibrium; as for its negative root(s), the SOC is satisfied at those candidate equilibria iff

$$|\beta| > \sqrt{\frac{\Lambda_z}{(N-2)\,\alpha - 1}}. \tag{3.10}$$

Combining (3.9) and (3.10) defines a region in the (α, Λ_z) space where a projective (negative β) equilibrium exists. Studying this region, and comparing the properties of (eventual) projective equilibria with the normal (positive roots of (3.9)) gives important insights.

Remark 1. [Existence of projective equilibria] Recall for $\Lambda_z = 0$ a projective equilibrium exists as long as $\alpha > \frac{1}{N-2}$. On the contrary, there is a threshold $\bar{\Lambda}$ such that if $\Lambda_z > \bar{\Lambda}$ then for no $\alpha \in (0,1)$ the economy $\vartheta_{\alpha,\Lambda_z}$ admits a projective equilibrium. In general, there exists a $\bar{\alpha}\,(\Lambda_z)$ such that the economy $\vartheta_{\alpha,\Lambda_z}$ admits a projective equilibrium iff $\alpha > \bar{\alpha}\,(\Lambda_z)$; $\bar{\alpha}$ is increasing (from $\frac{1}{N-2}$) in Λ_z and reaches 1 for finite Λ_z.

Remark 2. [Multiplicity and selection local to no-noise]. Fix some $\alpha^\star > \frac{1}{N-2}$ and a small $\Lambda_z^\star$. In an open ball around $(\alpha^\star, \Lambda_z^\star)$ there are have two equilibria: the noise trader (small positive β^N) and the projective one (large negative β^P). Both equilibria are continuous in the ball, because so is the cubic equation that defines them. However, on the boundary where $\Lambda_z = 0$, only the projective equilibrium exists. Moreover, for each α in the ball,

$$b_\alpha = \lim_{\Lambda_z \to 0} \beta^P\left(\vartheta_{\alpha,\Lambda_z}\right) < 0 = \lim_{\Lambda_z \to 0} \beta^N\left(\vartheta_{\alpha,\Lambda_z}\right),$$

which suggests that it is more natural to select the projective equilibria around $\Lambda_z = 0$, and that the approach of adding projection in a (noise trader) setting where an equilibrium already exists might lead to a significant misunderstanding and wrong predictions in model of trade with projective investors. This is because the instrument (noise trading) employed to ensure existence also renders the bias irrelevant at the candidate equilibrium, while the bias herself can lead to existence.

Remark 3. [Equivalence of conditionally-uninformative-price equilibria] In the limit $\alpha = 1$, $\Lambda_z \to \infty$, we have the same, irregular equilibrium: the price is perceived to be uninformative conditional on s_i, so p_i is not invertible and so $\beta = \frac{\tau_s}{\rho}\frac{N-2}{N-1}$. The traders perceive the price to be completely determined by noise, either because the signal is overturned by the infinite noise trader variance or because there is no signal (conditional on s_i) to begin with.

3.1.4 Misperceptions of Market Impact

Now consider traders who correctly perceive the information environment (Σ is the identity matrix), but misperceive their position within the market. Instead of the correct market clearing rule $\sum_{j\in[N]} x_j = 0$, each trader i believes that the price is determined according to $m_1 x_i + m_2 \sum_{j\neq i} x_j = 0$, where $m = \frac{m_1}{m_2}$ represent individual's perception of the relative importance of own's demand (relative to that of other) in determining the market price.[3]

$$\frac{\beta}{\gamma} = \frac{N\tau_s}{\tau_0 + N\tau_s}.$$

The loading on private information is

$$\beta = \frac{\tau_s}{\rho}\left(1 - m\frac{2N-1}{N-1}\right)$$

If the agent ignores his own market impact, i.e. if $m = 0$, we have $\beta = \frac{\tau_s}{\rho}, \gamma = \frac{\tau_s}{\rho\left(\frac{\tau_0}{N}+\tau_s\right)}$ and an equilibrium exists in which each trader correctly extracts the information available but disregards the fact that his actions move the market against him. More generally, equilibrium existence requires

$$m < \frac{N-1}{2N-1} < \frac{1}{2}.$$

In particular, underestimating the impact of other traders, m_2, is not helpful for existence. In the linear model, the information content of the price is independent of market clearing parameters and m_2 affects the equilibrium solely by amplifying the perceived market power.

The case of $m_2 = 0$, by contrast, is fundamentally different as it makes the economy nonregular. In this case, the price has no subjective information content and the analysis is equivalent to the case $\hat{\tau}_p = 0$. The candidate solution is

$$\beta = \left(1 - \frac{\Lambda_1}{\Lambda_p}\right)\frac{\tau_s}{\rho}$$
$$\gamma = \left(1 - \frac{\Lambda_1}{\Lambda_p}\right)\frac{\tau_0 + \tau_s}{\rho}$$

and to ensure existence, we require that the residual market demand is not too step relative to individual trader's impact, i.e. $\Lambda_p > \Lambda_1$.[4]

[3] We consider a restriction of (2.3), letting $\Lambda_1 = m_1$ and $\Lambda_2 = m_2\mathbf{1}^T$ to capture with a single parameter individuals' relative misperception of market power.

[4] In Appendix B.1, we show that this intuition generalizes to all non-regular economies.

3.2 Comparative Statics of Existence

From now on, we will work with the model restrictions introduced in the examples and adopt the parametrization $\Sigma_{11} = \xi$, $\Sigma_{22} = I\delta(1-\phi) + \mathbf{1}^T\delta\phi\mathbf{1}$, $\Sigma_{12} = 0$, and $\Lambda_1 = m_1$, $\Lambda_2 = m_2\mathbf{1}$. Results extend to the general setting considered in the previous sections though $\Sigma_{12} \neq 0$, i.e. perceived correlation between individual and others' signals, induces interdependence of information and market clearing misperceptions and thus make it impossible to summarize the existence comparative statics in two functions H, E, one pertaining misperceptions of the information environment, one pertaining misperceptions of the market clearing equation.

Recall that existence is guaranteed in non-linear economies (Theorem 1- point 1.). In this section we turn to the question of what biases relax or make more stringent the existence condition for linear economies (Theorem 1- point 2.).

Define the Hybris and Egocentrism functions $E, H : \Theta \to \mathbb{R}$ as follows[5]

$$H(\vartheta) = \frac{\xi\delta(1+(N-2)\phi)}{(N-1)}, \qquad E(\vartheta) = \frac{1 - \frac{m_1}{\Lambda_p + m_2(N-1)}}{2\frac{m_1}{m_2(N-1)}} \tag{3.11}$$

Proposition 2. *Consider two linear economies ϑ, ϑ' and suppose an equilibrium exists in ϑ. Then, if $H(\vartheta') \geq H(\vartheta)$ and $E(\vartheta') \geq E(\vartheta)$, an equilibrium exists in ϑ'.*

Existence, therefore, is determined by two dimensions of the traders' misperception. First, how much a trader overestimates the precision of her private signal relative to the information of others contained in the market price (hybris, H). In particular, economies with more overconfident, dismissive and correlation-delusional traders are conducive for equilibrium existence: An equilibrium exists only if each individual trader thinks he is sufficiently more informed that the rest. Second, how much a trader underestimates his impact on the price relative to market depth (egocentrism E). In particular, it is helpful for existence if traders believe that their impact on price formation is small, that the information of others is heavily priced and that the market is deep, either because others strongly react to the price or because there is additional uninformed capacity. Consider indeed the further restriction where $\Lambda_p = 0$, i.e. there is no uninformed capacity. Then, $E(\vartheta)$ is proportional to $\frac{m_2}{m_1}$, the relative misperception of others' to own market impact. In short, an equilibrium exists in an economy of traders who think they are much smarter than anybody else but do not move the price, while existence fails in an economy of traders who want to listen to the price and think the price listens to them.

Another important question is whether a larger number of traders is beneficial for equilibrium existence. On the one hand, more traders increase the information content of the price. On the other hand, it dilutes market power. The second effect dominates.

Proposition 3. *If ϑ_N admits an equilibrium, then $\vartheta_{N'}$ admits an equilibrium for all $N' > N$.*

[5] Proposition 2 extends to general economies under the appropriate redefinition of the hybris and egocentrism functions. In particular, $H(\vartheta) = \frac{\hat{\tau}_s(\vartheta)}{\hat{\tau}_p(\vartheta)}$, $E(\vartheta) = \frac{b_1(\vartheta)}{b_2(\vartheta)}$, substituting (3.2) for the price precision and $b_1 = \frac{2\Lambda_1 + \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}\right)}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}$, $b_2 = 1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}$ give the Proposition. Such general formulation, however, confounds the intuition of what misperceptions are conducive to existence due to the interaction of information and market impact of the different traders.

The proof of this result is a straightforward algebraic argument that a larger number of traders relaxes the condition $b(\vartheta) > 0$ which, in light of Corollary 1, is the only route to existence. The perceived information content of the price grows as more traders add their private information, but this effect is overpowered by the reduction in market power. The following example illustrates that this result could be overpowered if a bias leads the agent to perceive the precision of the price in a way that is steeply increasing in N.

Example 1 (Perceived Negative Correlation)**.** Suppose that the agent believes that the signals of the other traders are negatively correlated. Note that there is a bound on this perceived correlation, $\phi > -\frac{1}{N-2}$, as there cannot be too many signals of any given pairwise negative correlation. This means that we cannot even perform the analysis for an arbitrary number of traders. Even when correlation is physically possible, it implies that the subjective precision of the price grows quickly in the number of traders, and hence Lemma (3) does not extend to this case. To see why, consider a three trader model with perceived correlation $\phi = -1$ (coinciding with the lower bound at $N = 3$) so that each agent i thinks that $\epsilon_j = -\epsilon_k$ and therefore $s_j + s_k = \nu$. Because of the perfect negative correlation, the price subjectively reveals the fundamental and an equilibrium cannot exist in a regular economy. For $N = 2$, instead, the subjective correlation has no impact, and we can have an equilibrium with a sufficient underestimation of market power. This failure of monotonicity in existence is general in the case of negative ϕ, up to integer constraints.

Definition 3. An economy ϑ has a **limit equilibrium** if there exists an N such that ϑ_N has an equilibrium.

By Proposition 3, the definition implies that ϑ has an equilibrium for all $N' > N$. Furthermore, the sequence of equilibria converges. A natural question is now whether the limit equilibrium corresponds to an equilibrium of an economy with countably many traders.

4 Limit Economy

Formally, the limit economy, denoted by $\vartheta_{\mathbb{N}}$, corresponds to our model with a set of traders $\mathbb{N}$, with all remaining properties as defined in the finite case.

Definition 4. An economy ϑ has an **equilibrium in the limit** if $\vartheta_{\mathbb{N}}$ admits an equilibrium.

There are two crucial differences compared to the finite N setting. First, objectively, the collection of private signals of all traders now fully reveals the state. Second, traders become atomistic and market power is no more a concern. Including behavioral biases and assuming an equilibrium exists, the subjective joint distribution of (p_i, s_i, ν) in the regular case is given by

$$\begin{pmatrix} p_i \\ s_i \\ \nu \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\beta}{\gamma}\tau_0^{-1} + \phi\delta & \frac{\beta}{\gamma}\tau_0^{-1} & \frac{\beta}{\gamma}\tau_0^{-1} \\ \frac{\beta}{\gamma}\tau_0^{-1} & \tau_0^{-1} + \tau_s^{-1}\xi^{-1} & \tau_0^{-1} \\ \frac{\beta}{\gamma}\tau_0^{-1} & \tau_0^{-1} & \tau_0^{-1} \end{pmatrix}\right). \tag{4.1}$$

Absent a perceived correlation in the others' information, infinitely many signals of arbitrarily small perceived precision $(\frac{\tau_s}{\delta})$ reveal the state and render a single signal of arbitrarily high perceived precision $(\xi\tau_s)$ superfluous.

Definition 5. An economy ϑ has **limit uncertainty** if $\mathbb{E}\left[\nu | \{s_j\}_{j\in\mathbb{N}\setminus\{i\}}\right] \neq \mathbb{E}\left[\nu | \{s_j\}_{j\in\mathbb{N}}\right]$ and $\mathbb{V}\left[\nu | \{s_j\}_{j\in\mathbb{N}}\right] > 0$.

From the subjective variance-covariance matrix (4.1), it follows that we have limit uncertainty if and only if $\phi > 0$.[6] In that case, the agents believe that the errors of others have a common component, which does not wash out in the aggregate and is not present in their signal. Therefore, s_i is informative about the fundamental even conditional on infinitely many signals of others.

Limit uncertainty is a crucial property because it characterizes equilibrium existence.

Theorem 2. *A regular economy ϑ admits an equilibrium in the limit if and only if it has limit uncertainty and a limit equilibrium.*

Notice that this result holds in all regular economies. In particular, this proves a second discontinuity of the Kyle model: Fix σ and let $\overline{\vartheta^K_\sigma} \coloneqq \lim_{N\to\infty} \vartheta^K_\sigma$. Then for no positive Λ_z, $\overline{\vartheta^K_{\Lambda_z}}$ admits an equilibrium (though all elements along the sequence do). This mirrors the observation that for any fixed N, $\vartheta^K_{\Lambda_z}$ has an equilibrium, but its limit as $\Lambda_z \to 0$, i.e. the rational economy, does not. This sheds light on the structure of the noise trader equilibrium, which hinges on informed trading volume adjusting to the match the relative magnitude of noise trader demand, $\frac{\Lambda_z}{N}$. Whenever this quantity is zero, instead, the adjustment is ineffective and an equilibrium cannot exist.

In the non-regular case, the agent perceives the price to be uninformative independently of the number of traders. Existence, therefore, only depends on his perceived market power and the economy is continuous in the limit.

5 Conclusion

We investigate the behavioral foundations of informed trade. We augment the classic Kyle model to encompass a variety of biases in how traders perceive the information environment, as well as the market clearing condition which determines both the way individual think their actions affect the price and what information the price conveys. The classic noise trader model generates an adjustment margin of the endogenous precision of prices which ensures equilibrium existence making it possible for the private signal to be precise enough relative to the information contained in prices. We develop within our general framework a reinterpretation of the noise traders' shock as a cognitive error in conceptualizing the pricing functional; this model results in the same equilibrium behavior, but features different comparative statics of, say, price efficiency.

In the lens of the more general framework, this cognitive noise model represents only one very specific parametric restriction that guarantees existence. Moreover, existence comes at the cost of breaking the linearity that otherwise characterizes the model and much simplifies its analysis. The cost of conducting the analysis in the linear restriction of the general framework is, instead, that substantial biases are needed for existence: no equilibria exist in a neighborhood of the rational model. We do not see such requirement as an inherent problem of the linear restriction, but rather

[6] An extension of cursed equilibrium to our setting (Eyster et al., 2019; Ostrizek & Sartori, 2021) exhibits limit uncertainty for every positive level of partial cursedness. This is a direct consequence of the specification of cursedness as a mixture model, which also puts it outside of our misspecified jointly-Gaussian framework (2.2).

as informative of the type of biases that are prevalent in the market. Indeed, although equilibria exists for arbitrarily small noise trader shock, those close to rationality ($\Lambda_z \approx 0$) entail essentially no-trade ($\beta \approx 0$) as the informed trading volume needs to shrink to the match the relative magnitude of noise trader demand. Hence, local to rationality we are essentially comparing linear models that yield no equilibrium with non-linear models where an equilibrium exists but features (essentially) no trade. Indeed to get plausible levels of informed trade and price volatility, the size of noise traders is usually calibrated to match the (substantial) share of retail investors. At this point the "literal" noise trader story falls back into the conceptual pitfall highlighted in the introduction: to have the erratic behavior have impact in determining the price we either need to assume that the non-fundamental demand results from a "big" noise trader, or that there is a measure of atomistic traders who act based on a coordination device which the informed side of the market has no way to observe. Either rationalization is, in our view, problematic. The interpretation where the noise is in the mind of informed traders is instead immune to large-or-correlated critique and indeed the correlation structure of the cognitive noise shock is completely immaterial for the equilibrium characterization.

We show that existence in the linear model requires instead that traders are strongly hybristic, i.e. they perceive the precision of their signal (relative to the price) to be large, and/or little egocentric, i.e. think that their actions (relative to the actions of others) does not have a significant impact on the price. Biases that make traders overestimate the precision of their information conduce to existence, as they they make them more willing to bet. By contrast, biases that make traders overestimate their impact on the market are detrimental for existence, since thinking that the price will move strongly against them makes them more reluctant to act. In finite economies, more traders dilute market power and therefore relax the equilibrium existence condition: adding traders never breaks existence. We show that this result does not extend in a limit economy with countably many traders. Instead, existence relies on a property, limit uncertainty, which requires that traders think their private signal is informative about the fundamental even conditioning on the infinite collection of competitors' signals. In particular, misperceptions of market power are powerless in the limit economy which instead requires that traders perceive some positive correlation in their competitors' information.

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A Appendix: Proofs

Proof of Lemma 1. Recall that, subjectively, $\begin{pmatrix} s_i \\ \boldsymbol{s_{-i}} \end{pmatrix} \sim \mathcal{N}\left(\boldsymbol{\nu}, \tau_s^{-1}\begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{pmatrix}\right)$. Then,

$$\begin{pmatrix} s_i \\ \Lambda_2 \cdot \boldsymbol{s_{-i}} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \nu \\ \Lambda_2 \cdot \boldsymbol{\nu} \end{pmatrix}, \tau_s^{-1}\begin{pmatrix} \Sigma_{11} & \Lambda_2\Sigma_{12}^T \\ \Lambda_2\Sigma_{12}^T & \Lambda_2\Sigma_{22}\Lambda_2^T \end{pmatrix}\right)$$

using the conditional distribution of jointly normal random variables we obtain

$$\begin{aligned} \Lambda_2 \cdot \boldsymbol{s_{-i}}|s_i &\sim \mathcal{N}\left(\Lambda_2\boldsymbol{\nu} + \Lambda_2\Sigma_{12}\Sigma_{11}^{-1}(s_i - \nu), \tau_s^{-1}\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Lambda_2\right)\Lambda_2^T\right) \\ &= \mathcal{N}\left(\left(\mathbf{1}\Lambda_2^T - \Lambda_2\Sigma_{12}\Sigma_{11}^{-1}\right)\nu + \Lambda_2\Sigma_{12}\Sigma_{11}^{-1}s_i, \tau_s^{-1}\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Lambda_2\right)\Lambda_2^T\right) \end{aligned}$$

and

$$\frac{\Lambda_2 \cdot \boldsymbol{s_{-i}} - \Lambda_2\Sigma_{12}\Sigma_{11}^{-1}s_i}{\left(\mathbf{1}\Lambda_2^T - \Lambda_2\Sigma_{12}\Sigma_{11}^{-1}\right)} \sim \mathcal{N}\left(\nu, \tau_s^{-1}\frac{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Lambda_2\right)\Lambda_2^T}{\left(\mathbf{1}\Lambda_2^T - \Lambda_2\Sigma_{12}\Sigma_{11}^{-1}\right)^2}\right)$$

and finally,

$$\begin{aligned} \frac{\frac{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)\gamma}{\beta}p_i - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T s_i}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} &= \frac{\Lambda_2\boldsymbol{s}_{-i} - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T s_i + \frac{\Lambda_z}{\beta}\tau_s^{-1/2}z}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} \\ &\sim \mathcal{N}\left(\nu, \tau_s^{-1}\frac{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Lambda_2\right)\Lambda_2^T + \frac{\Lambda_z^2}{\beta^2}}{\left(\mathbf{1}\Lambda_2^T - \Lambda_2\Sigma_{12}\Sigma_{11}^{-1}\right)^2}\right) \end{aligned}$$

and conditionally independent of s_i, proving expressions (2.8)-(2.9)-(2.10). $\square$

Proof of Proposition 1. The candidate loadings in the linear equilibrium $x_i = \beta s_i - \gamma p$ are obtained matching coefficient of the best response function $x_i = \frac{\mathbb{E}(\nu|\{s_i,p_i\}) - p_i}{2\lambda_i + \rho\mathbb{V}(\nu|\{s_i,p_i\})}$, substituting for p_i, λ_i their expressions (2.6) and (2.5), and the misspecified expectations and variances derived using equations (2.8)-(2.9)-(2.10) in Lemma 1.

$$\frac{\mathbb{E}\left(\nu|\{s_i,p_i\}\right) - p_i}{2\lambda_i + \rho\mathbb{V}\left(\nu|\{s_i,p_i\}\right)} = \frac{\frac{s_i\hat{\tau}_s + \hat{p}_i\hat{\tau}_p}{\hat{\tau}_p + \hat{\tau}_s + \tau_0} - p_i}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}}$$

$$= \frac{\frac{\hat{\tau}_s}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}s_i + \frac{\hat{\tau}_p}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}\frac{\frac{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)\gamma}{\beta}p_i - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T s_i}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} - p_i}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}}$$

$$= \frac{\frac{\hat{\tau}_s}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}s_i + \frac{\hat{\tau}_p}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}\frac{\frac{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)\gamma}{\beta}\left(p - \frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)}(\beta s_i - \gamma p)\right) - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T s_i}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} - \left(p - \frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)}(\beta s_i - \gamma p)\right)}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}}$$

collecting the loadings on s_i, p we obtain

$$\beta = \frac{\frac{\hat{\tau}_s}{\hat{\tau}_s+\hat{\tau}_p+\tau_0} - \frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} + \frac{\beta\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}} \tag{A.1}$$

and

$$-\gamma = \frac{\frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\frac{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)\gamma}{\beta}\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} - \left(1+\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}}.$$

dividing the two, we obtain

$$\frac{\beta}{\gamma} = -\frac{\frac{\hat{\tau}_s}{\hat{\tau}_s+\hat{\tau}_p+\tau_0} - \frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} + \frac{\beta}{\gamma}\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}}{\frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}{\left(\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)\frac{\beta}{\gamma}} - \left(1+\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}$$

$$\frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\mathbf{1}\Lambda_2^T+\Lambda_p+\Lambda_1}{\left(\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)} - \frac{\beta}{\gamma} = \frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} - \frac{\hat{\tau}_s}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}$$

which can be rearranged to yield (3.4). Substituting this in (A.1) gives also (3.3).

□

Proof of Theorem 1: After substituting for candidate equilibrium β, γ obtained in Proposition 1, the SOC of the trader's problem reads

$$2\frac{\Lambda_1}{\beta\frac{\hat{\tau}_s+\hat{\tau}_p+\tau_0}{\hat{\tau}_s+\hat{\tau}_p\frac{\mathbf{1}\Lambda_2^T+\Lambda_p-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}}\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)} + \frac{\rho}{\hat{\tau}_p+\hat{\tau}_s+\tau_0} > 0$$

$$2\frac{\Lambda_1\left(\hat{\tau}_s+\hat{\tau}_p\frac{\mathbf{1}\Lambda_2^T+\Lambda_p-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)}{\left[\hat{\tau}_s\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right) - \hat{\tau}_p\left(\frac{2\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)\right]\beta\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)} + 1 > 0 \tag{A.2}$$

Note that if $\beta > 0$, the SOC is satisfied. Otherwise, the denominator above is negative, and we need to flip the inequality. Lengthy but straightforward manipulations show that (A.2) is equivalent to

$$\Lambda_1\hat{\tau}_s - \Lambda_1\hat{\tau}_p\frac{\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} < -\left[\hat{\tau}_s - \hat{\tau}_p\left(\frac{\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)\right]\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)$$

$$\Leftrightarrow \left(\mathbf{1}\Lambda_2^T+\Lambda_p+\Lambda_1\right)\left[\hat{\tau}_s - \hat{\tau}_p\left(\frac{\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)\right] < 0$$

Using $\left(\mathbf{1}\Lambda_2^T + \Lambda_p + \Lambda_1\right) > 0$ and substituting (3.2) for $\hat{\tau}_p$, the inequality simplifies to

$$\left(1 - \frac{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)\Sigma_{12}\Lambda_2^T}{\Lambda_2^T\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2}\right)\Sigma_{11}^{-1} < 0$$

and in turns to $\Lambda_2\left(\Sigma_{22} - \mathbf{1}^T\Sigma_{12}\right)\Lambda_2^T < 0$, which is equation (3.6) in the text.

□

Proof of Proposition 2. Recall from now we are working in the restricted economies $\Sigma_{11} = \xi$, $\Sigma_{22} = I\delta\left(1-\phi\right) + \mathbf{1}^T\delta\phi\mathbf{1}$, $\Sigma_{12} = 0$, and $\Lambda_1 = m_1$, $\Lambda_2 = m_2\mathbf{1}$. Since Corollary 1 applies to such restriction, an equilibrium exists if and only if (3.5) is positive. Notice that (3.5) is equivalent to

$$\left(1 - \frac{m_1}{\Lambda_p + m_2\left(N-1\right)}\right)\xi\tau_s - 2\frac{m_1}{m_2}\frac{\tau_s}{\delta + \left(N-2\right)\phi\delta} > 0$$

$$\iff \frac{\left(1 - \frac{m_1}{\Lambda_p + m_2\left(N-1\right)}\right)}{2\frac{m_1}{m_2}} > \frac{1}{\xi\delta\left[1 + \left(N-2\right)\phi\delta\right]}$$

using definitions (3.11), the latter condition is equivalent to

$$H\left(\vartheta\right)E\left(\vartheta\right) > 1,$$

from which the statement follows. □

Proof of Proposition 3. In the non-linear case there is always an equilibrium so the statement is vacuous. In the linear case, fix a misperceived economy ϑ and suppose an equilibrium exists with N traders. By Theorem 1, this implies that $b_N\left(\vartheta\right) > 0$. Whenever $N' > N$ then

$$\begin{aligned} b_{N'}\left(\vartheta\right) &= \left(1 - \frac{m_1}{\Lambda_p + m_2\left(N'-1\right)}\right)\xi\tau_s - 2\frac{m_1}{m_2}\frac{\tau_s}{\delta + \left(N'-2\right)\phi\delta} \\ &> \left(1 - \frac{m_1}{\Lambda_p + m_2\left(N-1\right)}\right)\xi\tau_s - 2\frac{m_1}{m_2}\frac{\tau_s}{\delta + \left(N-2\right)\phi\delta} = b_N\left(\vartheta\right) > 0 \end{aligned}$$

and, by Theorem 1, an equilibrium exists with N' traders. □

Proof of Theorem 2: Note that in a regular economy with $\beta, \gamma \neq 0$, the price is a sufficient statistic for ν with respect to $\left\{s_j\right\}_{j\in\mathbb{N}\setminus\{i\}}$, including subjectively.

Suppose there is no limit uncertainty and $\beta, \gamma \neq 0$. Then, the best-response of trader i, (2.1), is independent of s_i. Therefore, $\beta = 0$ and we cannot have an informative equilibrium. Conversely, if $\beta = 0$, the price is uninformative and the agent's best response loads on s_i (unless we are in the no-trade case, which we ruled out). Therefore, this also not an equilibrium.

Conversely, suppose that there is limit uncertainty. Then, β is characterized by (3.3) and this equation is continuous in N. Therefore, if a limit equilibrium exists, it constitutes an equilibrium of the limit economy. □

A.1 Derivations for Economies with Information Projection

Suppose the only bias in information processing is information projection, that is $\Sigma_{11} = 1$, $\Sigma_{22} = I\left(1-\alpha^2\right) + \mathbf{1}^T\alpha^2\mathbf{1}$ and $\Sigma_{12} = \alpha\mathbf{1}$ for $\alpha \in (0,1)$. As for the market clearing rule, we maintain $\Lambda_1 = 1, \Lambda_2 = \mathbf{1}$, $\Lambda_p = 0$ and consider both the case with and without perceived liquidity traders (Λ_z equal or larger than zero). Under those restrictions, the price precision (3.1) reads

$$\begin{aligned}\hat{\tau}_p^{-1} &= \tau_s^{-1}\frac{\Lambda_2\left(\Sigma_{22}-\Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T + \frac{\Lambda_z}{\beta^2}}{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)} = \tau_s^{-1}\frac{\mathbf{1}\left(I\left(1-\alpha^2\right)+\mathbf{1}^T\alpha^2\mathbf{1}-\mathbf{1}^T\alpha^2\mathbf{1}\right)\mathbf{1}^T + \frac{\Lambda_z}{\beta^2}}{\left(\left(N-1\right)\left(1-\alpha\right)\right)^2} \\ &= \tau_s^{-1}\frac{\left(1+\alpha\right)+\frac{\Lambda_z}{\beta^2}}{\left(N-1\right)\left(1-\alpha\right)}\end{aligned}$$

In the linear economy $\Lambda_z = 0$ the (exogenous) price precision (3.2) becomes

$$\hat{\tau}_p = \frac{1-\alpha}{1+\alpha}\left(N-1\right)\tau_s$$

and the candidate linear equilibrium (3.5) reads

$$\begin{aligned}b_\alpha &= \frac{\tau_s}{\rho}\left[\left(1-\frac{1}{N-1}\right) - \frac{\left(N-1\right)\left(1-\alpha\right)}{1+\alpha}\left(\frac{2+\left(N-1\right)\alpha\left(1-\frac{1}{N-1}\right)}{\left(N-1\right)\left(1-\alpha\right)}\right)\right] \\ &= -\frac{\tau_s}{\rho}\left[\frac{N+\left(N-2\right)^2\alpha}{\left(N-1\right)\left(1+\alpha\right)}\right]\end{aligned}$$

which is always negative, so per Point 2.b of Theorem 1 there is an equilibrium if and only if (3.6) is satisfied. Under projection (3.6) reads

$$\begin{aligned}1 - \frac{\left(N-1\right)\left(1-\alpha\right)}{1+\alpha}\left(\frac{\left(N-1\right)\alpha}{\left(N-1\right)\left(1-\alpha\right)}\right) < 0 \\ \Leftrightarrow \frac{\alpha}{1+\alpha} > \frac{1}{N-1} \Leftrightarrow \alpha > \frac{1}{N-2}\end{aligned}$$

hence a fully projective inverted equilibrium exists in the absence of noise trader under substantial ($\alpha > \frac{1}{N-2}$) projection.

With cognitive noise $\Lambda_z > 0$ then equilibrium is characterized by the cubic (3.9) which always admits a positive solution (Theorem 1 point 1.). We ask whether there are negative solutions that satisfy (3.6) which now simplifies to

$$\frac{\Lambda_z}{\beta^2} < \left(N-2\right)\alpha - 1$$

delivering, for $\Lambda_z = 0$ the condition above and for $\Lambda_z > 0$ condition (3.10) in the text. Combining (3.10) and (3.9) defines a region in the (α, Λ_z) space where an inverted trade equilibrium exists.

To see why there is a threshold Λ_z above which no negative equilibria can exist, notice that that the cubic (3.9) always has negative intercept, it has an inflection point at $\tilde{b} = -\frac{\alpha(N-2)^2+N}{3(1+\alpha)(N-1)}\frac{\tau_s}{\rho}$ and the first derivative at this inflection point is $\left(N-1\right)\rho - \frac{\left(\alpha(N-2)^2+N\right)^2\tau_s^2}{3(1+\alpha)(N-1)\rho}\tau_z$. If $\Lambda_z > \bar{\Lambda} :=$

$\left[\frac{\left(\alpha(N-2)^2+N\right)^2\tau_s^2}{3(1+\alpha)(N-1)^2\rho^2}\right]^{-1}$ then this derivative is positive, so there cannot be negative solutions to the cubic hence no candidate projective equilibria.

B Extensions

B.1 Irregular Economies

In regular economies (see Definition 1) traders perceive that the market price conveys some information about the state through the signal of others. If this is not the case then the normalization argument to extract information from p_i is invalid as we are dividing by zero. This happens either because other traders do not possess any information to begin with ($\delta = \infty$) or because their action does not have aggregate impact. In this non-regular case, the agent perceives $\hat{\tau}_p = 0$ and the rest of the analysis goes through, yielding

$$\beta = \left(1 - \frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)}\right)\frac{\hat{\tau}_s}{\rho}$$
$$\gamma = \left(1 - \frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)}\right)\frac{\tau_0 + \hat{\tau}_s}{\rho}$$

irrespective of the source of non-regularity. If $\Lambda_2 = \mathbf{0}$ then it is needed $\Lambda_p \neq 0$ so it the traders' misperception does not rule out market clearing. Essentially, traders think they are bidding against a fixed (non-stochastic) market clearing condition. Therefore, the only threat to existence is the traders market power. An equilibrium exists if perceived market power is sufficiently small or, conversely, the residual supply sufficiently flat

$$\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} < 1.$$

B.2 Cursed Economies

Following the equilibrium notion introduced in Eyster and Rabin '05, cursed agents form expectations taking a convex combination of the rational (weight $1 - \chi$) expectation and the one formed disregarding the information content of prices.

$$\mathbb{E}_\chi\Big[\nu \mid \{p_i, s_i\}\Big] = \chi\mathbb{E}^C\Big[\nu \mid \{p_i, s_i\}\Big] + (1-\chi)\,\mathbb{E}^R\Big[\nu \mid \{p_i, s_i\}\Big]$$

where

$$\mathbb{E}^R\Big[\nu \mid \{p_i, s_i\}\Big] = \frac{\left[\beta s_i + \gamma\,(N-1)\,p_i\right]\tau_s}{\beta\,(\tau_0 + N\tau_s)}$$

is the correct conditional expectation and

$$\mathbb{E}^C\Big[\nu \mid \{p_i, s_i\}\Big] = \frac{\mu_0\tau_0 + s_i\tau_s}{\tau_0 + \tau_s}$$

is obtained disregarding the information content in prices. Similarly,

$$\mathbb{V}_\chi\Big[\nu \mid \{p_i, s_i\}\Big] = \chi\mathbb{V}^C\Big[\nu \mid \{p_i, s_i\}\Big] + (1-\chi)\,\mathbb{V}^R\Big[\nu \mid \{p_i, s_i\}\Big]$$

where $\mathbb{V}^C\Big[\nu \mid \{p_i, s_i\}\Big] = (\tau_0+\tau_s)^{-1}$ and $\mathbb{V}^R\Big[\nu \mid \{p_i, s_i\}\Big] = (\tau_0+N\tau_s)^{-1}$. Plugging expectation and variance in the best-response function and matching coefficients, we obtain the following system of equations

$$\beta = \frac{\beta\left(\beta\rho\left(\tau_0+\tau_s+\chi\left(N-1\right)\tau_s\right)\right)+\tau_s\left(N\left(\tau_s+\tau_0\right)+\tau_s\left(N-3\right)\tau_s\chi-2\chi\tau_0\right)}{N\left[\left(\beta\rho\left(\tau_0+\tau_s+\chi\left(N-1\right)\tau_s\right)\right)+2\left(1-\chi\right)\tau_s\left(\tau_0+\tau_s\right)\right]}$$

$$\gamma = \frac{\beta(N-2)(\tau_0+\tau_s)(\tau_0+N\tau_s)}{(N-1)\left((1-\chi)N\tau_s(\tau_0+\tau_s)+\beta\rho(\tau_0+\tau_s+\chi(N-1)\tau_s)\right)}$$

One solution is $\beta=\gamma=0$ which clearly violates the SOC. The other solution is

$$\beta = \frac{\tau_s\left(\chi(N-1)(2\tau_0+N\tau_s)-N(\tau_0+\tau_s)\right)}{(N-1)\rho(\tau_0+\tau_s+\chi(N-1)\tau_s)} \tag{B.1}$$

Note that

$$\beta|_{\chi=0} = -\frac{N}{N-1}\frac{\tau_s}{\rho} < 0, \beta|_{\chi=1} = \frac{N-2}{N-1}\frac{\tau_s}{\rho} > 0$$

and that

$$\frac{d}{d\chi}\beta = \frac{2\tau_s(\tau_0+\tau_s)(\tau_0+N\tau_s)}{\rho(\tau_0+\tau_s+\chi(N-1)\tau_s)^2} > 0.$$

with rational traders ($\chi=0$, where we know no equilibrium exists) the candidate loading is negative, while the fully cursed economy always gives a positive candidate. Since the candidate equilibrium is increasing, there is a threshold above which it is positive. To check whether the candidate β actually constitute an equilibrium we need to check the SOC, which reads

$$\frac{2}{(N-1)\gamma} + \rho\left[\chi\left(\tau_0+N\tau_s\right)^{-1} + \left(1-\chi\right)\left(\tau_0+\tau_s\right)^{-1}\right] > 0.$$

Substituting the expressions for the equilibrium γ, we obtain[7]

$$\chi(N-1)(2\tau_0+N\tau_s) - N(\tau_0+\tau_s) > 0$$

$$\Leftrightarrow \quad \chi > \bar{\chi} \coloneqq \frac{N(\tau_0+\tau_s)}{(N-1)(2\tau_0+N\tau_s)}.$$

hence, cursedness restores equilibrium existence even without noise, provided that it is large enough. Notice that

$$\frac{\mathrm{d}}{\mathrm{d}N}\bar{\chi} = -\frac{(\tau_0+\tau_s)(2\tau_0+N^2\tau_s)}{(N-1)^2(2\tau_0+N\tau_s)^2} < 0$$

and $\lim_{N\to\infty}\bar{\chi} = 0$; the individual amount of cursedness required to sustain an equilibrium is decreasing in the number of traders, and any amount of cursedness suffices if we consider arbitrary number of traders: a limit equilibrium (Definition 3) exists for every $\chi > 0$, and so according to Theorem 1, we have an equilibrium in the limit (Definition 3) if and only if cursedness displays limit uncertainty. This is indeed the case as whenever $\chi > 0$: because a χ fraction of a (partially) cursed trader completely disregards the information content of prices altogether, even if the price

[7] Imposing that the candidate β (B.1) is positive delivers the same condition $\chi > \bar{\chi}$; a positive candidate β is necessary and sufficient for existence.

fully reveals the fundamental (as is the case when $\Lambda_z = 0$), the private signal is still conditionally informative about the fundamental and, absent market power issues (because of the countable trader limit) used in equilibrium.