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Pandora's Box Problem with Correlations: Some Results for the Case of Stochastic Dominance

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Abstract
We consider a version of Pandora's box problem in which the distributions of the various alternatives' utilities are ranked by first-order stochastic dominance and possibly correlated. Under independence, Weitzman's optimal search rule prescribes inspecting the dominant alternative first. We show that, with correlation, this sampling order remains optimal if there are two alternatives, each with only two possible utility levels. Next, we show that, with three possible utility levels for each alternative, inspecting the dominated alternative first can be optimal: we provide sufficient conditions for this to happen.

JEL Classification: C6; D8.

Keywords: Sequential Search; Pandora's Box Problem; Correlation; Stochastic Dominance.

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1 Introduction

In Pandora's box problem (Weitzman, 1979), an agent must decide in which order to inspect a certain number of alternatives (or boxes) with unknown utilities and when to stop the search. Inspecting an alternative reveals its utility to the agent but is costly. This framework has many applications, such as searching for a product, a job, a school, or an employee (see, e.g., Armstrong, 2017; Chade, Eeckhout, and Smith, 2017; Beyhaghi and Cai, 2024). When the alternatives' utilities are stochastically independent, a simple reservation utility strategy, known as "Weitzman's rule", characterizes the optimal sampling sequence and stopping rule. Without independence, however, a general closed-form solution for the optimal search strategy is unknown. In response, recent work characterizes approximate solutions to the problem (see, e.g., Chawla, Gergatsouli, Teng, Tzamos, and Zhang, 2020).[1]

In this paper, we study a version of Pandora's box problem in which the utilities of two alternatives are correlated, and their marginal distributions are ranked by first-order stochastic dominance. Stochastic dominance is a strong ranking assumption that suggests a natural guess: inspecting the dominant alternative first may be optimal. Indeed, this is what Weitzman's rule would prescribe with independent utilities across alternatives. We show that this guess generally fails under correlation. The central insight is that correlation introduces a trade-off between starting the search from the alternative that is more likely to have a high utility and doing so from the one providing more information on the value of continuing the search.

Specifically, we make two contributions. First, we show that if each alternative takes only two possible utility levels, inspecting the dominant alternative first remains optimal (Proposition 1). Since all binary utility distributions can be ranked by first-order stochastic dominance, this result implies that the sampling sequence prescribed by Weitzman's rule remains optimal with two alternatives and binary utilities, independently of their correlation. Second, we show that, despite the strong assumption of the dominance ranking, Weitzman's rule fails if each alternative has three possible utility levels (Proposition 2), and we provide sufficient conditions on utility distributions under which inspecting the dominated alternative first is optimal.

In addition to its theoretical interest, the first-order stochastic dominance ordering is of independent interest because it naturally arises even from ex-ante identical alternatives under social learning (see, e.g., Mueller-Frank and Pai, 2016; Lomys, 2024). Given the theoretical difficulty in characterizing optimal sequential search under correlated utilities, existing work focuses on specific distributional assumptions. For instance, Ke and Lin (2020) and Bao, Li, and Yu (2023) characterize the optimal policy when utilities have a

[1]Chawla et al. (2020) and follow-up work focus on the loss minimization version of the problem, in which alternatives have a non-negative price, and the agent wants to minimize the selected price plus total cost. For exact optimization, which is our focus, utility maximization and loss minimization are equivalent problems (Beyhaghi and Cai, 2024).

common and an idiosyncratic component. Neither paper focuses on stochastic dominance.

2 Model

An agent must select an alternative in $X \coloneqq \{1,2\}$. Let u_x denote the utility of alternative $x \in X$ to the agent. For some $U \subseteq \mathbb{R}_+$, the utility vector (u_1, u_2) is drawn from a joint distribution F with support U^2. For our results, assuming $|U| \in \{2,3\}$ suffices. The agent wants to take the alternative with the highest realized utility; she knows F and can learn the realized value u_x via costly sequential search with recall, as follows:

- The agent decides which alternative x to inspect first.
- After learning u_x, the agent decides whether to inspect the remaining alternative, denoted $\neg x$, or to discontinue the search.[2]
- Each inspection costs $c \in \mathbb{R}_+$. The agent's utility is $u_x - c$ if she searches once, and $\max\{u_x, u_{\neg x}\} - 2c$ if she searches twice.

Let F_x denote the marginal distribution of alternative x. Let f (resp., f_x) denote the probability mass function of F (resp., F_x). Hence, $f(i,j)$ is the probability that alternative 1 has utility i and alternative 2 has utility j, and $f_x(i)$ is the probability that alternative x has utility i. We write $F_1 \succsim F_2$ to indicate that F_1 first-order stochastically dominates F_2: $F_1(u) \leq F_2(u)$ for all $u \in U$, with strict inequality for some u.

Hereafter, following Armstrong (2017), we assume $\mathbb{E}[u_x] > c$ for all $x \in X$, ensuring that the agent searches at least once.

Weitzman's Rule. The *reservation value* of alternative x is the unique $r_x \in \mathbb{R}_+$ satisfying $c = \mathbb{E}[\max\{0, u_x - r_x\}]$. If u_1 and u_2 are independent, Weitzman (1979)'s optimal search strategy is: "*Inspect alternatives in descending order of reservation value; discontinue the search when finding an alternative whose utility exceeds the reservation value of any uninspected alternative.*" Weitzman's rule implies the following result, whose proof is in Appendix A.1.

Lemma 1. *Suppose u_1 and u_2 are independent and $F_1 \succsim F_2$. Then, either the agent is indifferent between alternatives or inspecting alternative 1 first is optimal.*

3 Two Utility Levels

Suppose $|U| = 2$. We show that inspecting the stochastically dominant alternative first is optimal, independently of the correlation between u_1 and u_2. When $|U| = 2$, either $F_1 \succsim F_2$ or $F_2 \succsim F_1$. Hence, our result implies that the sampling sequence prescribed by

[2] The behavior when the agent is indifferent on searching again is irrelevant to our results.

Weitzman's rule remains optimal with two alternatives and binary utilities, independently of their correlation.

Proposition 1. *Suppose* $U \coloneqq \{\ell, h\}$*, with* $0 \leq \ell < h$*, and* $F_1 \succsim F_2$*. Then, inspecting alternative* 1 *first is optimal.*

Proof. We proceed in steps.

Step 1. Let $\Delta \coloneqq h - \ell$. With two utilities,

$$F_1 \succsim F_2 \iff \mathbb{P}(u_1 = h) > \mathbb{P}(u_2 = h). \tag{1}$$

Step 2. Suppose the agent inspects alternative x first. If $u_x = h$, the expected gain from the second search is 0, and the agent discontinues the search. If $u_x = \ell$, the expected gain from the second search is

$$G_x \coloneqq \mathbb{P}(u_{\neg x} = h \mid u_x = \ell)\Delta, \tag{2}$$

and the agent inspects alternative $\neg x$ if and only if $c \leq G_x$.

Step 3. By step 2, the value of the search problem of an agent with search cost c who inspects alternative x first is

$$V(x, c) \coloneqq \begin{cases} \mathbb{E}[u_x] - c & \text{if } c > G_x \\ \mathbb{E}[u_x] - c + [\mathbb{P}(u_{\neg x} = h \mid u_x = \ell)\Delta - c]\mathbb{P}(u_x = \ell) & \text{if } c \leq G_x \end{cases}. \tag{3}$$

Step 4. When $c = 0$, the agent is indifferent between which alternative to inspect first: $V(x, 0) = V(\neg x, 0)$ or, equivalently,

$$\begin{aligned} &\mathbb{E}[u_x] + \mathbb{P}(u_{\neg x} = h \mid u_x = \ell)\mathbb{P}(u_x = \ell)\Delta \\ &\quad = \mathbb{E}[u_{\neg x}] + \mathbb{P}(u_x = h \mid u_{\neg x} = \ell)\mathbb{P}(u_{\neg x} = \ell)\Delta. \end{aligned} \tag{4}$$

Step 5. Assume $c > 0$. We distinguish between four exclusive and exhaustive cases. For each case, we show that $V(1, c) - V(2, c) > 0$, from which the desired result follows.

- If $c > \max\{G_1, G_2\}$,

  $$V(1, c) - V(2, c) = \mathbb{E}[u_1] - \mathbb{E}[u_2] > 0,$$

  where: the equality holds by definition (3) and the assumption $c > \max\{G_1, G_2\}$; the inequality holds by assumption (1).

- If $c \leq \min\{G_1, G_2\}$,

  $$\begin{aligned} V(1, c) - V(2, c) &= \mathbb{E}[u_1] + [\mathbb{P}(u_2 = h \mid u_1 = \ell)\Delta - c]\mathbb{P}(u_1 = \ell) \\ &\quad - \mathbb{E}[u_2] + [\mathbb{P}(u_1 = h \mid u_2 = \ell)\Delta - c]\mathbb{P}(u_2 = \ell) \\ &= [\mathbb{P}(u_2 = \ell) - \mathbb{P}(u_1 = \ell)]c \\ &> 0, \end{aligned}$$

where: the first equality holds by definition (3) and the assumption $c \le \min\{G_1, G_2\}$; the second equality holds by condition (4); the inequality holds by assumption (1), which implies $\mathbb{P}(u_2 = \ell) < \mathbb{P}(u_1 = \ell)$, and the assumption $c > 0$.

- If $G_1 < c \le G_2$,

$$\begin{aligned}
V(1,c) - V(2,c) &= \mathbb{E}[u_1] - \mathbb{E}[u_2] - [\mathbb{P}(u_1 = h \mid u_2 = \ell)\Delta - c]\mathbb{P}(u_2 = \ell) \\
&> \mathbb{E}[u_1] - \mathbb{E}[u_2] - \mathbb{P}(u_1 = h \mid u_2 = \ell)\mathbb{P}(u_2 = \ell)\Delta \\
&+ \mathbb{P}(u_2 = h \mid u_1 = \ell)\mathbb{P}(u_2 = \ell)\Delta \\
&= [\mathbb{P}(u_2 = \ell) - \mathbb{P}(u_1 = \ell)]\mathbb{P}(u_2 = h \mid u_1 = \ell)\Delta \\
&> 0,
\end{aligned}$$

where: the first equality holds by definition (3) and the assumption $G_1 < c \le G_2$; the first inequality holds by the assumption $c > G_1$ and the definition of G_1 in (2); the second equality holds by condition (4); the second inequality holds by assumption (1), which implies $\mathbb{P}(u_2 = \ell) > \mathbb{P}(u_1 = \ell)$, and the assumption $\Delta > 0$.

- If $G_2 < c \le G_1$,

$$\begin{aligned}
V(1,c) - V(2,c) &= \mathbb{E}[u_1] + [\mathbb{P}(u_2 = h \mid u_1 = \ell)\Delta - c]\mathbb{P}(u_1 = \ell) - \mathbb{E}[u_2] \\
&\ge \mathbb{E}[u_1] + \mathbb{P}(u_2 = h \mid u_1 = \ell)\mathbb{P}(u_1 = \ell)\Delta \\
&- \mathbb{E}[u_2] - \mathbb{P}(u_2 = h \mid u_1 = \ell)\mathbb{P}(u_1 = \ell)\Delta \\
&= \mathbb{E}[u_1] - \mathbb{E}[u_2] \\
&> 0,
\end{aligned}$$

where: the first equality holds by definition (3) and the assumption $G_2 < c \le G_1$; the first inequality holds by the assumption $c \le G_1$ and the definition of G_1 in (2); the last inequality holds by assumption (1). ■

4 Three Utility Levels

For $|U| = 3$, we provide sufficient conditions under which $F_1 \succsim F_2$, but inspecting alternative 2 first is optimal; hence, Weitzman's rule fails.

Proposition 2. *Suppose $U \coloneqq \{\ell, m, h\}$, where $0 \le \ell < m < h$, and:*

(a) $f(m,h) - f(h,m) = f(h,\ell) - f(\ell,h) > 0$;

(b) $f(m,\ell) = f(\ell,m) > 0$.

Then, $F_1 \succsim F_2$. Moreover, for all $c > 0$, one can choose ℓ, m, h so that inspecting alternative 2 first is optimal.

With correlation, inspecting an alternative allows the agent to learn its utility *and* the conditional distribution of the remaining alternative. Correlation introduces a trade-off between starting the search from the alternative that is more likely to have a high utility and from the one providing more information about the value of continuing the search. Conditions (a) and (b) capture this informational effect while keeping stochastic dominance in favor of action 1 and ensuring that the agent solves the trade-off by inspecting alternative 2 first.

For illustrative purposes, we construct an example that satisfies conditions (a) and (b) with a small departure from independence. Suppose utilities are initially drawn independently from some $\overline{F}_1$ and $\overline{F}_2$ satisfying condition (a); further assume $\overline{f}_1(h) = \overline{f}_2(h)$. By stochastic dominance, this implies $\overline{f}_1(\ell) < \overline{f}_2(\ell)$ and $\overline{f}_1(m) > \overline{f}_2(m)$, and so $\overline{f}(m, \ell) > \overline{f}(\ell, m)$. Next, for $u_1 = \ell$ and $u_2 = m$, swap utilities across alternatives with some probability that make (m, ℓ) and (ℓ, m) ex-post equiprobable, thus satisfying condition (b). This change does not affect the joint probability of any other utility pair, so condition (a) still holds. The joint distribution F obtained in this way satisfies conditions (a) and (b).

Proof. We outline the main steps of the proof. The omitted details are in Appendix A.2.

Step 1. Assumptions (a) and (b) imply $f_1(\ell) < f_2(\ell)$ and $f_1(h) = f_2(h)$. In turn, these conditions imply $F_1 \succsim F_2$.

Step 2. Rescaling the utility levels, we can always ensure $\mathbb{E}[u_x] > c$. Assume the agent searches twice only when the first inspected alternative has utility ℓ; in Step 3, we derive conditions on $\{\ell, m, h\}$ for this to hold. Under this assumption, inspecting alternative 2 first is optimal if and only if

$$h - m > c. \tag{5}$$

Step 3. The agent searches twice only when the first inspected alternative has utility ℓ if:

$$\frac{f(\ell, h)(h-\ell) + f(\ell, m)(m-\ell)}{f(\ell, h) + f(\ell, m) + f(\ell, \ell)} > c, \tag{6}$$

$$\frac{f(h, \ell)(h-\ell) + f(m, \ell)(m-\ell)}{f(h, \ell) + f(m, \ell) + f(\ell, \ell)} > c, \tag{7}$$

$$\frac{f(m, h)}{f(m, h) + f(m, \ell) + f(m, m)}(h-m) < c, \tag{8}$$

$$\frac{f(h, m)}{f(h, m) + f(\ell, m) + f(m, m)}(h-m) < c. \tag{9}$$

Step 4. Since $f(m, \ell) = f(\ell, m) > 0$ by assumption (a), we have

$$\max\left\{\frac{f(m, h)}{f(m, h) + f(m, \ell) + f(m, m)}, \frac{f(h, m)}{f(h, m) + f(\ell, m) + f(m, m)}\right\} < 1.$$

Therefore, for all $c > 0$, there exists m and h such that

$$\max\left\{\frac{f(h, m)}{f(h, m) + f(\ell, m) + f(m, m)}, \frac{f(m, h)}{f(m, h) + f(m, \ell) + f(m, m)}\right\}(h-m) < c < h-m,$$

satisfying conditions (5), (8), and (9). Moreover, since $f(m,\ell) = f(\ell,m) > 0$ by assumption (b), conditions (6) and (7) are satisfied by choosing ℓ so that $m - \ell$ is large enough.

Step 5. Summing up, under assumptions (a) and (b), $F_1 \succsim F_2$ and, for all $c > 0$, there exists ℓ, m, h with $0 \leq \ell < m < h$ such that inspecting alternative 2 first is optimal, as desired.■

5 Conclusion

Weitzman's optimal sampling sequence generalizes beyond independence with two alternatives and two utility levels but fails with three utility levels, even under the strong assumption of first-order stochastic dominance. We leave characterizing the most general setting in which Weitzman's rule applies under correlation to future research.

References

Armstrong, M. (2017): "Ordered Consumer Search," *Journal of the European Economic Association*, 15, 989–1024.

Bao, R., S. Li, and J. Yu (2023): "Search with Interdependent Values," *Working Paper.*

Beyhaghi, H. and L. Cai (2024): "Recent Developments in Pandora's Box Problem: Variants and Applications," *ACM SIGecom Exchanges*, 21, 20–34.

Chade, H., J. Eeckhout, and L. Smith (2017): "Sorting through Search and Matching Models in Economics," *Journal of Economic Literature*, 55, 493–544.

Chawla, S., E. Gergatsouli, Y. Teng, C. Tzamos, and R. Zhang (2020): "Pandora's Box with Correlations: Learning and Approximation," in *2020 IEEE 61st Annual Symposium on Foundations of Computer Science (FOCS)*, IEEE, 1214–1225.

Ke, T. T. and S. Lin (2020): "Informational Complementarity," *Management Science*, 66, 3699–3716.

Lomys, N. (2024): "Collective Search in Networks," *Working Paper.*

Mueller-Frank, M. and M. M. Pai (2016): "Social Learning with Costly Search," *American Economic Journal: Microeconomics*, 8, 83–109.

Weitzman, M. L. (1979): "Optimal Search for the Best Alternative," *Econometrica*, 43, 641–654.

A Supplementary Material

A.1 Proof of Lemma 1

The function $\phi_x\colon \mathbb{R} \to \mathbb{R}$, defined pointwise as

$$\phi_x(y) \coloneqq \mathbb{E}[\max\{0, u_x - y\}],$$

is strictly decreasing. Note that $\phi_x(y) \to 0$ as $y \to \infty$, and $\phi_x(0) > c$ by the assumption $\mathbb{E}[u_x] > c$. Hence, by the intermediate value theorem, there exists a unique $r_x \in \mathbb{R}_+$ such that

$$\phi_x(r_x) = c. \tag{10}$$

Moreover, under the assumption $F_1 \succsim F_2$, we have

$$\phi_1(y) \leq \phi_2(y) \quad \text{for all } y \in \mathbb{R}. \tag{11}$$

Conditions (10) and (11), together with the fact that ϕ_2 is decreasing, imply $r_1 \geq r_2$. By Weitzman's rule, we conclude that inspecting alternative 1 first is optimal. ■

A.2 Details for the Proof of Proposition 2

Step 1. $F_1 \succsim F_2$ is equivalent to $f_1(\ell) \leq f_2(\ell)$ and $f_1(h) \geq f_2(h)$, with at least one strict inequality. In turn,

$$f_1(\ell) \leq f_2(\ell) \iff \cancel{f(\ell,\ell)} + f(\ell,m) + f(\ell,h) \leq \cancel{f(\ell,\ell)} + f(m,\ell) + f(h,\ell), \tag{12}$$

and

$$f_1(h) \geq f_2(h) \iff \cancel{f(h,h)} + f(h,m) + f(h,\ell) \geq \cancel{f(h,h)} + f(m,h) + f(\ell,h). \tag{13}$$

The assumptions $f(h,\ell) - f(\ell,h) > 0$ and $f(m,\ell) = f(\ell,m)$ imply that conditions (12) hold with a strict inequality. The assumption $f(h,m) - f(m,h) = f(\ell,h) - f(h,\ell)$ implies that conditions (13) hold with equality. Hence, we conclude that $F_1 \succsim F_2$.

Step 2. Let $V(x,c)$ be the value of the search problem of an agent with search cost c who inspects alternative x first. Assuming that the agent inspects the remaining alternative only when the first inspected alternative has utility ℓ, the values of the search problems are

$$\begin{aligned}
V(1,c) =& f_1(h)h + f_1(m)m + f_1(\ell)[f_2(h \mid \ell)h + f_2(m \mid \ell)m + f_2(\ell \mid \ell)\ell - c] \\
=& f_1(h)h + f_1(m)m + f(\ell,h)h + f(\ell,m)m + f(\ell,\ell)\ell - f_1(\ell)c, \\
V(2,c) =& f_2(h)h + f_2(m)m + f_2(\ell)[f_1(h \mid \ell)h + f_1(m \mid \ell)m + f_1(\ell \mid \ell)\ell - c] \\
=& f_2(h)h + f_2(m)m + f(h,\ell)h + f(m,\ell)m + f(\ell,\ell)\ell - f_2(\ell)c,
\end{aligned}$$

where $f_x(i \mid j)$ denotes the conditional probability that alternative x has utility i given that alternative $\neg x$ has utility j.

The agent inspects alternative 2 first if $V(2,c) > V(1,c)$. Using that $f_1(h) = f_2(h)$ by Step 1, we have

$$
\begin{aligned}
&V(2,c) > V(1,c) \\
&\qquad \iff f_2(m)m + f(h,\ell)h + \cancel{f(m,\ell)m + f(\ell,\ell)\ell} - f_2(\ell)c \\
&\qquad\qquad\qquad - \Big[f_1(m)m + f(\ell,h)h + \cancel{f(\ell,m)m + f(\ell,\ell)\ell} - f_1(\ell)c\Big] > 0 \\
&\qquad \iff [f(h,\ell) - f(\ell,h)]h + [f_2(m) - f_1(m)]m - [f_2(\ell) - f_1(\ell)]c > 0,
\end{aligned}
\tag{14}
$$

where we use that $f(m,\ell) = f(\ell,m)$ by assumption (b). To further simplify the expression, note that assumption (b) also implies

$$
\begin{aligned}
f_2(\ell) - f_1(\ell) &= \cancel{f(\ell,\ell)} + \cancel{f(m,\ell)} + f(h,\ell) - [\cancel{f(\ell,\ell)} + \cancel{f(\ell,m)} + f(\ell,h)] \\
&= f(h,\ell) - f(\ell,h).
\end{aligned}
\tag{15}
$$

Moreover, we know that $f_1(h) = f_2(h)$ from Step 1, which implies

$$
\begin{aligned}
f_2(\ell) - f_1(\ell) &= 1 - f_2(m) - \cancel{f_2(h)} - [1 - f_1(m) - \cancel{f_1(h)}] \\
&= -[f_2(m) - f_1(m)].
\end{aligned}
\tag{16}
$$

In turn, equations (15) and (16) imply

$$
f_2(m) - f_1(m) = -[f(h,\ell) - f(\ell,h)]. \tag{17}
$$

Using equivalence (14) and equations (15) and (17), we obtain

$$
V(2,c) > V(1,c) \iff [f(h,\ell) - f(\ell,h)](h - m - c) > 0 \iff h - m > c,
$$

where the last equivalence holds because $f(h,\ell) - f(\ell,h) > 0$ by assumption (a). Thus, condition (5) in the main text follows.

Step 3. Search behavior.

- After inspecting an alternative with utility h first, the expected gain from the second search is 0 and the agent discontinues the search for all $c > 0$, independently of the identity of the alternative and the values of the other parameters.

- Suppose the agent inspects alternative 1 first. When $u_1 = \ell$, the agent inspects the remaining alternative if $\mathbb{E}[\max\{u_2 - \ell, 0\} \mid u_1 = \ell] > c$ or, equivalently,

  $$
  \frac{f(\ell,h)(h-\ell) + f(\ell,m)(m-\ell)}{f(\ell,h) + f(\ell,m) + f(\ell,\ell)} > c.
  $$

  The previous inequality corresponds to condition (6) in the main text.

  Suppose the agent inspects alternative 2 first. When $u_2 = \ell$, the agent inspects the remaining alternative if $\mathbb{E}[\max\{u_1 - \ell, 0\} \mid u_2 = \ell] > c$ or, equivalently,

$$\frac{f(h,\ell)(h-\ell) + f(m,\ell)(m-\ell)}{f(h,\ell) + f(m,\ell) + f(\ell,\ell)} > c.$$

The previous inequality corresponds to condition (7) in the main text.

- Suppose the agent inspects alternative 1 first. When $u_1 = m$, the agent discontinues the search if $\mathbb{E}[\max\{u_2 - m, 0\} \mid u_1 = m] < c$ or, equivalently,

  $$\frac{f(m,h)}{f(m,h) + f(m,\ell) + f(m,m)}(h-m) < c.$$

  The previous inequality corresponds to condition (8) in the main text.

  Suppose the agent inspects alternative 2 first. When $u_2 = m$, the agent discontinues the search if $\mathbb{E}[\max\{u_1 - m, 0\} \mid u_2 = m] < c$ or, equivalently,

  $$\frac{f(h,m)}{f(h,m) + f(\ell,m) + f(m,m)}(h-m) < c.$$

  The previous inequality corresponds to condition (9) in the main text.

Steps 4 and 5. These steps are in the main text. ■