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Anna Maria C. Menichini and Peter J. Simmons

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University of Naples Federico II | University of Salerno | Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS AND STATISTICS – UNIVERSITY OF NAPLES FEDERICO II
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unina.it
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Optimal Multiple Loan Contracting under Sequential Audits and Contagion Losses

Anna Maria C. Menichini[*] and Peter J. Simmons[†]

Abstract
We propose a rationale for the joint financing of two independent projects based on the reduction in audit costs resulting from endogenous sequential veri.cation. This cost reduction occurs not only when joint financing offers coinsurance bene.ts, but, remarkably, also in the presence of contagion losses -where the failure of one project negatively impacts the other. This is because the benefits from endogenous verification - namely, the cost saving from audits optimally decreasing in the reported outcome - may offset the additional cost arising from contagion, specifically, the potential need to audit a successful project due to the failure of the other. We provide a detailed characterisation of the optimal contract, showing that under certain conditions it may take the form of standard debt. Furthermore, we conduct a comparative static analysis relating the optimality of joint financing to the quality of accounting information. Importantly, we find that with fully transparent accounting information, joint financing always dominates single financing even under contagion. The results remain robust across scenarios involving simultaneous audits and multiple projects.

JEL Classification: D82, D86, G32, G34.

Keywords: financial contracts, auditing, joint financing, project finance, conglomerates.

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[*] University of Salerno and CSEF. Email: amenichini@unisa.it. *(Corresponding author)*

[†] University of York. Email: peter.simmons@york.ac.uk

1 Introduction

Economic theory has extensively explored how costly audits deter ex-post moral hazard in principal-agent relationships. In financial contracting, lenders may audit a firm's output to ensure repayment (Townsend, 1979; Gale and Hellwig, 1985), while in tax compliance, tax authorities may audit income reports to discourage evasion (Border and Sobel, 1985; Mookherjee and Png, 1989). These classic models typically focus on single reporting dimensions, such as total income, and the design of audit mechanisms to prevent misstatements. However, many real-world scenarios involve more complex relationships requiring audits across multiple dimensions. For instance, a bank extending loans to subsidiaries of a conglomerate may need to audit each subsidiary to minimize the overall default risk. Similarly, tax authorities may need to audit individual components of a multidimensional income report to reduce tax evasion. This complexity raises a critical question: how does the need to audit multiple, rather than single items, shape optimal audit policies and the scope of the principal-agent relationship?

To address these questions we focus on a credit relationship in which a borrower/firm needs to finance two independent projects in a competitive credit market. The projects may either be financed as stand-alones or jointly, with their returns - high or low - ex-post privately observed by the borrower. Upon the returns' realisation, the borrower sends a report to the lender who can verify its truthfulness with a costly audit. Audits of jointly financed projects are assumed to be sequential, that is, the lender selects one project to audit first - possibly at random - and then, based on the results, decides whether to audit the other.[1] We explore how far the possibility of carrying out audits on multidimensional rather than on single reports of income generates endogenous positive or negative synergies that shape the optimal scope of the firm, namely its organisational structure as stand-alone or conglomerate.[2]

In a setting with deterministic audits, prior literature has shown that joint finance brings about positive synergies only in the presence of coinsurance, i.e., when losses in one project are offset by gains in another and the conglomerate fails only when all projects fail (Diamond, 1984). In the presence of contagion, i.e., when losses in one project exceed gains

[1] Audits sequentiality is introduceds as a working hypothesis. It will be relaxed in Section 7.1, where we show that all results are robust to audits being simultaneous.

[2] The model could be adapted to analyse the tax authority optimal audit strategy when taxpayers report several components of taxable income.

in another and the conglomerate fails when at least one project does, risk-contamination losses arise from joining projects together, making separate financing more attractive than joint financing (Banal-Estañol, Ottaviani and Winton, 2013).

In line with the literature, we find that when the audit strategy is optimally chosen, the firm always prefers joint over separate financing under coinsurance. Surprisingly, and contrary to previous studies, we also find that joint financing may be preferred over separate financing even under contagion. The reason is that, by optimising over the audit strategy, joint financing allows a saving in audit cost due to joint audit frequency optimally decreasing in the reported outcome. This saving may offset the extra deadweight loss of joint financing, namely, the potential need to audit a successful project due to the failure of the other.

To understand the drivers of our results, consider that both in individual and joint finance some audit is necessary to stop the borrower from always reporting the low revenue outcome. With stand-alone finance there are only two possible reports for each project, fail or success. To maximise the borrower's incentives to report truthfully (and minimise the frequency of audit), it is optimal to audit a fail report stochastically, pledging to the lender the entire returns from failure, plus as much as necessary of the revenues of a successful project to meet its expected costs, leaving the residual revenues to the borrower. Under joint finance, considering the combined outcomes of the two projects -two fails, one success and one fail, zero fails- there are three possible joint reports: a bottom report of two fails, an intermediate report of one fail (and one success) and a top report of zero fails. To maximise the reporting incentives, the returns from two fails are entirely pledged to the lender. As this amount is, by assumption, insufficient to let the lender break even, it is necessary to pledge also (part or all of) the returns from one success and one fail. By cross-pledging the returns from one success, the borrower gives up (part of) the rent she could have obtained if each project was financed as a stand-alone, thereby slackening the reporting constraints. If the amount pledged covers the investment and the expected audit cost, audits can be concentrated on bottom reports of two fails. Because these occur with lower joint probability than reports of individual fails in single finance contracts, there is an audit cost saving. In addition, because the failing project of an intermediate outcome is not audited, a further saving in audit cost arises relative to single finance where a failing project is always audited with positive probability. These two cost savings constitute the coinsurance benefit of joint finance. As a result, joint finance always dominates. We refer to this subsequently as a *joint finance contract with coinsurance*.[3]

If the combined returns from two fails and one fail (and one success) do not cover the investment and the expected audit cost, the borrower must additionally pledge part of the returns from two successes. This implies that an audit has to occur not only upon a joint report of two fails, but also upon an intermediate report of one fail (and one success). In particular, to ensure truthful reporting at lower cost, a joint report of two fails - less likely to occur - is audited deterministically, while an intermediate one is audited randomly, with an overall audit cost saving relative to single finance. However, differently from the coinsurance case, following an intermediate report, not only the failing project will be audited, but possibly, depending on the quality of the accounting information, also the successful one. Compared with single finance, where a successful project is never audited, joint finance may therefore bring about an extra audit cost, the contagion loss of joint finance. We refer to this as a *joint finance contract with contagion*, where contagion in our setting refers to the possibility that also the successful project of an intermediate outcome is audited.

The main contribution of our paper is to show that the extra audit cost that joint financing may bring about may be offset by the saving arising from the joint audit frequency optimally decreasing in the reported state. Joint financing may therefore dominate separate financing even under contagion, a result that is novel in the literature and remains robust to the sequencing of verification, in particular to the possibility that audits are carried out simultaneously rather than sequentially.

The extent of the trade-off depends on the quality of the firm's accounting information, which determines the informativeness of the borrower's intermediate report. Under poor/opaque accounting information, intermediate reports give only a coarse indication of which of the two projects failed and which one succeeded. This makes the audit of the successful project more likely, with a subsequent extra audit cost. Conversely, under transparent accounting information, intermediate reports are fully informative, revealing with certainty which of the two projects succeeded. This allows audit only of the failing project, with no extra cost of audit. In this case joint financing comes only with benefits, those arising from endogenous audit, and always dominates separate financing.[4]

[3] In the special case in which the returns from two fails and one fail (and one success) exactly cover investment and expected audit cost, bottom reports of two fails are audited deterministically, while intermediate reports are still never audited. Thus, a standard debt contract is the optimal joint financing arrangement in this case.

[4] The extent of the trade-off also depends on market conditions and the size of audit cost. In particular, coeteris paribus, we show that joint financing is more likely to arise the higher the probability of success

The discussion so far has concerned the relative profitability of separate and joint financing when both financing regimes are feasible. However, there are also situations in which projects with positive net present value (NPV) can be financed either only separately or only jointly. We characterise such situations showing that, when coinsurance prevails, joint financing mitigates credit rationing, widening the area of feasible financial contracts. Conversely, when contagion prevails, depending on the quality of the accounting information, the area of feasible joint financial contracts may shrink, resulting in projects that can be financed separately but not jointly.

Related literature. The idea that misreporting incentives can be controlled by costly audits started in the costly state verification (CSV, henceforth) literature (Townsend, 1979; Gale and Hellwig, 1985) in a world with deterministic audits and a single project with continuous revenue outcomes. Here the solution is a standard debt contract. The range of possible audit strategies was extended in Border and Sobel (1985) and Mookherjee and Png (1989), who allow stochastic audit and show that generally the socially optimal audit probabilities are interior and fall with the profitability of the state, with the highest revenue state not audited.

The idea that joint financing may reduce monitoring cost has been highlighted by Diamond (1984) who, with multiple lenders financing several independent projects, shows the optimality of delegating monitoring to an intermediary. The incentive of the intermediary to misreport to lenders is controlled by a debt contract and diversification minimises the risk of the intermediary failing. Thus, the pooling of risks across projects drives the intermediary's default risk to zero as the number of projects rises.[5] As highlighted by Banal-Estañol, Ottaviani and Winton (2013) however, this reduced risk of bankruptcy works only if the pooled returns from one success and one failure cover the total loan cost (Diamond, 1996), i.e., under coinsurance. If the losses from the failing project exceed the profits from the successful project, contagion losses can occur, leading to the firm's overall bankruptcy despite having one successful venture. If default costs are proportional to total projects returns, joint financing involves extra bankruptcy costs that would not be incurred under stand-alone finance, thus overturning the benefits arising under coinsurance.[6]

and the lower the audit cost.

[5] The incentive effects of multiple projects have been explored in Laux (2001) who, in a setting with moral hazard, shows that joining projects together, by relaxing limited liability constraints, allows to elicit high effort at lower cost.

[6] The coinsurance benefits of joint financing may also be overturned because of the lower market discipline

Despite providing a more thorough understanding of the role of multiple projects in financial contracting, both these papers assume audits to be deterministic and do not consider the possibility of optimising over the audit strategy. Our main contribution is to show that a novel trade-off emerges when the audit strategy is endogenously chosen. In particular, optimising the audit policy across multiple projects may bring about a cost saving that offsets the extra cost from contagion risk, thus overturning the conventional wisdom that coinsurance is a prerequisite for joint financing.

In proposing a cost-efficient audit structure that enhances coinsurance benefits and mitigates contagion losses, the paper complements the findings of Stein (1997, 2003), offering a mechanism similar to the "smarter-money" effect of internal capital markets. It also aligns with empirical evidence showing that diversification improves financial outcomes by reducing risk through imperfectly correlated returns across divisions (Duchin, 2010; Boutin et al., 2013; Hann, Ogneva and Ozbas, 2013; Matvos and Seru, 2014; Kuppuswamy and Villalonga, 2016; Benz and Hoang, 2021).[7]

A further contribution of our paper is that the audit cost saving is higher, the better the quality of the accounting information, thus highlighting a role for information in supporting joint financing. This role, while recognised in several empirical works (Rossi and Volpin, 2004; Bris and Cabolis, 2008; Zhang, 2008; among others), has not been fully incorporated into theoretical models of financial contracting. These models have traditionally focused on how better information helps monitoring individual projects, rather than explicitly addressing how it facilitates the joint financing of multiple projects. Our work makes a step forward in this respect by showing that higher quality accounting information, by reducing audit costs, makes it more likely that projects are financed jointly rather than separately.

It is clearly possible to imagine other scenarios in which economies of scale in auditing arise, for example due to the possibility of using internal control processes on several projects, with a joint total audit cost that is smaller than twice the audit cost on the single project. However, rather than pointing to a pure efficiency gain in auditing from joint financing, our paper highlights a novel and unexpected reason why economies of scale in auditing arise: endogenising the audit policy in a joint financing setting results in an inten-

of the conglomerate (Boot and Schmeits, 2000; Inderst and Müller, 2003), its reduced probability of refinancing (Faure-Grimaud and Inderst, 2005), or its lower tax benefits (Leland, 2007). Luciano and Nicodano (2014) instead focus on the possibility of mitigating the potential for risk contamination by introducing conditional guarantees which, preserving the guarantor's limited liability, do not trigger its default.

[7] The importance of this argument is also supported by recent survey evidence (Hoang, Gatzer and Ruckes, 2024).

sive audit of the collective worst outcomes, less likely to occur, and a minimal or no audit of the intermediate outcomes, with an audit cost saving relative to separate financing, where each fail report is always audited with positive probability.

The remainder of the article is organised as follows. Section 2 lays out the model assumption. Section 3 develops a standard CSV model in which two individual projects are financed as stand-alones in the competitive banking sector. Section 4 considers the case of two independent projects to illustrate the basic role of joint financing in reducing the deadweight loss of audits both in the case in which coinsurance gains and contagion losses between projects arise. Section 5 compares individual and joint financing in these two settings. Section 6 presents a comparative static analysis on the effects of changes in market conditions, audit costs, and accounting information quality. Section 7 addresses robustness issues, in particular the role of the sequencing of audits and the number of projects. Section 8 concludes. All the proofs, unless otherwise specified, are in the Appendix.

2 The Model Assumptions

An entrepreneur/borrower has two investment projects with uncorrelated returns, each costing I, which can be funded from a risk neutral investor. Each project gives a random return, H or L, with $H > I > L > 0$. Outcome H (L) occurs with probability p $(1-p)$. Each project is socially profitable, i.e., the expected return covers the investment cost: $pH + (1-p)L > I$. The return of each project is freely observable only to the entrepreneur, who, once it is realised, reports the outcome to the investor. Because of output unobservability, the borrower has an incentive to report the low outcome L on each. But because $I > L$, the only way for the investor to recoup the investment cost on a single project is to carry out an audit. This has a cost $c > 0$ per project and its result is observable and verifiable.

The possible ex-post outcomes ς vary with how projects are grouped in their financing, as stand-alones or joint. With stand-alone projects there are only two outcomes to the contract on each project, $\varsigma^S = \{L, H\}$. With the two projects jointly financed in a single contract four outcomes are possible, $\varsigma^J \in \{LL, HL, LH, HH\}$. Two successes occur with probability p^2, two failures, with probability $(1-p)^2$, one success and one failure, with probability $2p(1-p)$.

Reports. Upon the outcome, depending on the financing regime, separate (S) or joint (J), the borrower sends an observable report σ^i, $i = \{S, J\}$, indicating the number of successful projects. Any report must be feasible, in that the borrower has to have funds to make the appropriate repayment. With single finance, the borrower sends two different reports, one for each project and each one independent of the other, $\sigma^S \in \{0,1\}$, where $\sigma^S = 0$ denotes a report of zero successes and $\sigma^S = 1$ a report of one success. With joint finance, the borrower sends a joint report $\sigma^J \in \{0,1,2\}$, with $\sigma^J = 0$ denoting zero successes (and thus an aggregate return $2L$), $\sigma^J = 1$ one success (and one failure) (and an aggregate return $H + L$), and $\sigma^J = 2$ two successes (and an aggregate return $2H$). Besides revealing the aggregate return, a report $\sigma^J = 1$ may also provide indications regarding the identity of the failing project. The accuracy of these indications - the transparency/quality of the firm's accounting information - depends on the regulatory context in which the firm operates. The transparency/quality of the firm's accounting information is captured by the exogenous parameter $s \in [1/2, 1]$. When the accounting information is fully transparent ($s = 1$), a report of one success also indicates the project that has succeeded. When the accounting information is opaque, a report of one success provides no indication, or only a coarse indication of which project has succeeded, which implies that both projects may in principle be audited ($s \geq 1/2$).

Audits. For each financing regime, following a report σ^i, $i = \{S, J\}$, a costly audit may occur to verify its truthfulness. We assume there is commitment in the contract, so the lender has to carry through the audit policy even knowing that this will never catch a cheat.

Under single finance, an audit of each project may occur, following a report of no success ($\sigma^S = 0$), with probability m_0^S.

Under joint finance, audits may occur following either a joint report of zero successes ($\sigma^J = 0$) or of one success ($\sigma^J = 1$). Audits of jointly financed projects are sequential, that is, the lender selects one project to audit first and then, based on the results, decides whether to audit the other. Upon a joint report of zero successes ($\sigma^J = 0$), m_0^J denotes the first stage audit probability and $m_{0,i}$ the second stage audit probability conditional on the outcome $i = \{L, H\}$ of a first stage audit.

Upon a joint report of one success ($\sigma^J = 1$), whether one or both projects are audited depends on the quality of the firm's accounting information, captured by the exogenous parameter $s \in [1/2, 1]$. Denoting with m_1 the first stage audit probability of a report $\sigma^J = 1$, if $s = 1/2$, the accounting information is opaque and it is equally likely that a first stage audit will pick a success or a fail. If it picks a success, a second stage audit will occur with probability $m_{1,H}$, while if it picks a fail, the second stage audit probability $m_{1,L}$ is zero. Indeed, having received a report of one success and having discovered a fail upon first stage audit, the remaining project can only be a success and so it is not audited. If $s > 1/2$ it is more likely that a first stage audit will pick a fail, thus making a second stage audit less likely. The closer is s to one, the more transparent the accounting information and the more likely that a first stage audit will pick a fail. If $s = 1$, a first stage audit will certainly pick a fail, thus making unnecessary a second stage audit. Thus, s can be interpreted as the probability with which the lender detects a fail upon first stage audit of a report of one success.

3 Single finance

When each project is funded as a stand-alone, a contract specifies repayments and the probability with which an audit will occur, if any. Because reports must be feasible, reports of one success, $\sigma^S = 1$, are never audited. Let R_1 be the corresponding repayment. Let m_0^S be the probability of auditing a report $\sigma = 0$. Let $R_{0|\varsigma}$ be the repayment due following a report $\sigma = 0$, and an audit which reveals that the state is $\varsigma \in \{L, H\}$, and $R_{0|\cdot}$ be the repayment with report $\sigma^S = 0$, but no audit. All repayments are non-negative and the borrower has limited liability.

The sequence of events, depicted in the game tree in Fig. 1, is as follows.

1. A financing contract is offered and, if accepted, the borrower is committed to the investment.
2. Nature (N) chooses the project outcome, $\varsigma^S = \{L, H\}$. This is only observed by the borrower (A), who makes a report $\sigma^S \in \{0, 1\}$ to the investor (P).
3. Conditional on the report, the audit decision is taken.
4. Conditional on the report and audit decisions, repayments are made.

The contract $\mathcal{P}^S$ sets repayments $R_{0|H}$, R_1, $R_{0|\cdot}$, $R_{0|L}$ and audit probability m_0^S to

$$\max EP^S = p(H - R_1) + (1-p)\left[m_0^S\left(L - R_{0|L}\right) + \left(1 - m_0^S\right)\left(L - R_{0|\cdot}\right)\right] \tag{1}$$

$$\text{st } pR_1 + (1-p)\left[\left(1 - m_0^S\right)R_{0|\cdot} + m_0^S\left(R_{0|L} - c\right)\right] \geq I \tag{2}$$

$$R_1 \leq m_0^S R_{0|H} + \left(1 - m_0^S\right)R_{0|\cdot} \tag{3}$$

$$0 \leq R_1, R_{0|H} \leq H \text{ and } 0 \leq R_{0|\cdot}, R_{0|L} \leq L \tag{4}$$

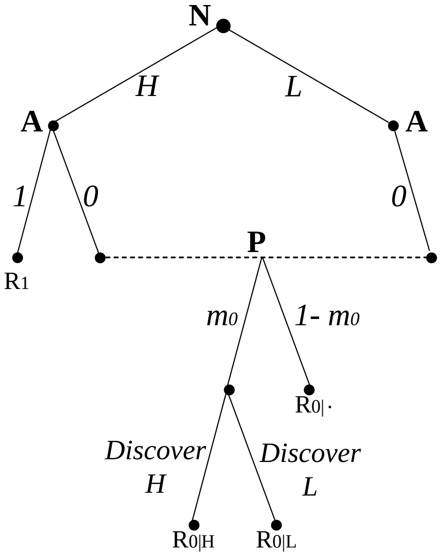

Figure 1: Game tree with project finance

where (1) is the borrower's expected profit per-project, (2) is the participation constraint, ensuring that the lender breaks even in expected terms on each project, (3) is the truth-telling constraint, ensuring that upon a high state the borrower prefers to report truthfully rather than cheating and be audited with probability m_0^S, and (4) the limited liability conditions.

The solution to programme $\mathcal{P}^S$ is described in Proposition 1:

Proposition 1 *Suppose two identical and independent projects are financed separately. The second-best contract has:*

(i) *maximum punishment for detected false low state report:* $R_{0|H} = H$;

(ii) *zero low state return for the borrower:* $R_{0|L} = R_{0|\cdot} = L$;

(iii) *random audit of low state reports,* m_0^S :

$$m_0^S = \frac{I - L}{p\,(H - L) - (1 - p)\,c} < 1; \tag{5}$$

(iv) *lender repayment following a high state report equal to* $R_1 = \frac{(H-L)I-(1-p)L(H-L+c)}{p(H-L)-(1-p)c} < H$, *and expected return to the borrower equal to*

$$EP^S = pH + (1 - p)\,L - I - \frac{(1 - p)\,(I - L)\,c}{p\,(H - L) - (1 - p)\,c} > 0. \tag{6}$$

From (5), the single finance contract is feasible when

$$pH + (1-p)\,L - I \geq (1-p)\,c. \qquad \text{(Condition 1)}$$

This condition can be represented in the space of $p\,(H-L)$ and $I-L$ in Fig. 2 by a linear function with intercept $(1-p)\,c$ and slope 1.

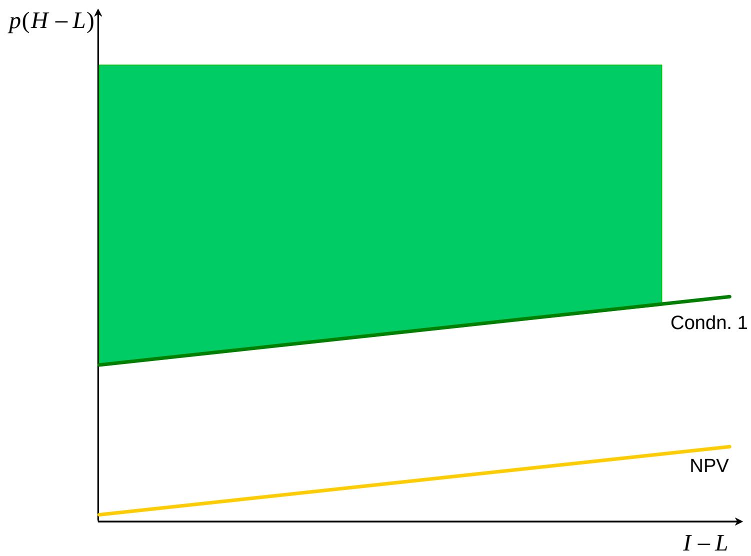

Figure 2: Area of single finance contracts

Thus, a single finance contract is feasible in the green area above the Condition 1 line. The line from the origin in Fig. 2 instead represents the locus of exogenous parameters where $NPV = pH + (1-p)\,L - I = 0$. Thus, although all projects with positive NPV (those above the orange line) are socially profitable, only those generating enough returns to cover also the expected audit cost are financed (those above the green line), and there is credit rationing.

Besides guaranteeing that the audit probability (5) is in the unit interval, Condition 1 also has an economic interpretation, namely, that to be feasible, each project must be sufficiently profitable to cover the investment and certain audit cost of a low report. Indeed, Condition 1 can be obtained by assuming that in the participation constraint (2) the revenue from zero and one success outcomes (L and H) is sufficient to meet the investment cost plus certain audit of a low report.

The intuition behind these results is the following. When Condition 1 does not hold, the expected revenue from the project cannot cover the investment and expected audit cost. Thus, no contract is signed, despite the project having positive NPV. If Condition 1 does hold, the frequency of audit is positive.[8] The deadweight loss of audit is minimised by raising $R_{0|H}$ to H and reducing the audit probability until the incentive constraint (3) holds with equality. In addition, low state repayments, whether audited or not, are set to give zero surplus to the borrower: $R_{0|\cdot} = R_{0|L} = L$. However, because $R_1 < H$, the borrower gets a rent in the high state.

4 Joint finance

When the two projects are jointly financed, a contract specifies the probability with which an audit will occur and repayments conditional on reports and audit, if any.

Under a joint report of zero successes ($\sigma^J = 0$), both projects may be audited and the lender can randomly choose which one, if any, to audit first with probability 1/2 on each (by the principle of insufficient reason). Denote with m_0^J the probability to audit one of the two projects, and with $1 - m_0^J$ the probability of auditing neither. In cases in which the lender does not audit, he demands a repayment $R_{0|\cdot}$ and the game ends. When the lender does audit and discovers the outcome i for the selected project, he can decide whether to go further and audit the remaining project with probability $m_{0,i}$, $i \in \{L, H\}$, where the second subscript denotes the outcome of the first audit, or to stop with probability $1 - m_{0,i}$. Denote with $R_{0|ij}$, $i, j = \{L, H\}$, the repayment the lender gets upon receiving a report of zero successes when he audits both projects and discovers the true state to be i on the first and j on the second, and with $R_{0|i\cdot}$ the repayments in case he audits just one project and discovers the true state to be i, but does not audit the other.

Upon a joint report of one success ($\sigma^J = 1$), depending on the quality of the firm's accounting information, both projects may in principle be audited sequentially. In particular, the lender selects the first project to audit randomly at the endogenously chosen rate m_1, or does not audit at all. In the case in which he does not audit, with probability $1 - m_1$, he demands a repayment $R_{1|\cdot}$ in total on the two projects and the game ends. If he does audit, $s \in [1/2, 1]$ is the chance that the first stage audit reveals a fail, and $1 - s$ that it reveals a success. If it reveals a fail, then the lender stops auditing as he knows the other project must be a success ($m_{1,L} = 0$), and gets a repayment $R_{1|L}$. If the first audit reveals a success,

[8] If not, from (3), $R_1 = R_{0|\cdot} \leq L$ and there is insufficient revenue to meet the investment cost.

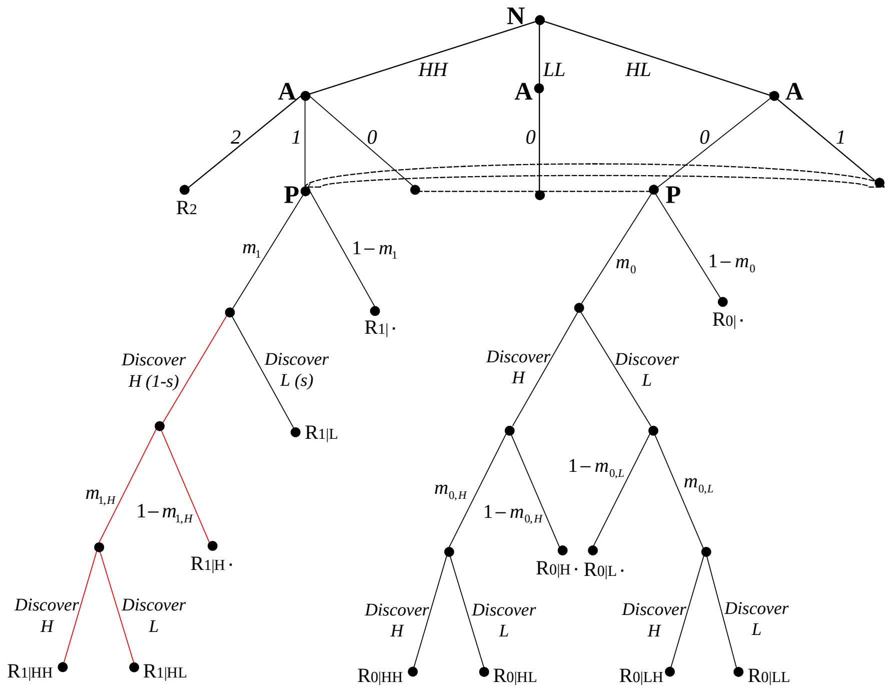

Figure 3: Game tree with joint finance and sequential audit

the lender either goes on to audit the second project at the endogenously chosen rate $m_{1,H}$, or does not audit. If he does audit, he demands a repayment $R_{1|HH}$ if he discovers a success and $R_{1|HL}$ if he discovers a fail. If he does not audit, with probability $1 - m_{1,H}$, he gets $R_{1|H\cdot}$.

Last, because reports must be feasible, a joint report of two successes ($\sigma^J = 2$) must be truthful. So, neither project is audited. Let R_2 be the corresponding repayment.

The sequence of events, depicted in the game tree in Fig. 3, is as follows.

1. A financing contract is offered and, if accepted, the borrower is committed to the investment.

2. Nature (N) chooses the projects' joint outcome, $\varsigma^J = \{LL, HL, LH, HH\}$. This is observed by the borrower (A), who makes a report $\sigma^J = \{0, 1, 2\}$ to the lender (P).

3. Conditional on the report, the audit decision is taken.

4. Conditional on the report and audit decisions, repayments are made.

Notice that if, upon a report of one success ($\sigma^J = 1$), the transparency of the accounting information is maximum ($s = 1$), then the first stage audit certainly reveals a fail and a successful project will never be audited. The game ends following the first stage audit (no red branches in the game tree in Fig. 3).

4.1 The contract problem

In this section we set up the contract problem under joint financing to maximise the borrower's expected profits, subject to the lender getting a non-negative return, to the incentive constraints guaranteeing that the borrower does not cheat on the reports and to the limited liability conditions.

The borrower's joint payoff function with truthtelling is

$$EP^J(s) = p^2(2H - R_2) + 2p(1-p)\{H + L - (1-m_1)R_{1|\cdot} - m_1[sR_{1|L} + (1-s)(m_{1,H}R_{1|HL} + (1-m_{1,H})R_{1|H\cdot})]\} + (1-p)^2\{2L - (1-m_0^J)R_{0|\cdot} - m_0^J[m_{0,L}R_{0|LL} + (1-m_{0,L})R_{0|L\cdot}]\}. \tag{7}$$

The participation constraint requires the expected return to the lender from financing both projects to cover the joint loan costs and the expected audit costs:

$$EP^L(s) = p^2R_2 + 2p(1-p)\{(1-m_1)R_{1|\cdot} + m_1[(sR_{1|L} + (1-s)(m_{1,H}(R_{1|HL} - c) + (1-m_{1,H})R_{1|H\cdot})) - c]\} + (1-p)^2\{(1-m_0^J)R_{0|\cdot} + m_0^J[m_{0,L}(R_{0|LL} - c) + (1-m_{0,L})R_{0|L\cdot} - c]\} \geq 2I. \tag{8}$$

As regards the incentive constraints, cheating may occur when one or two successes realise. In particular, with two true successes ($\varsigma = HH$), there are two ways of cheating. To report zero successes, or to report one. The incentive constraint that ensures that a borrower with two successes prefers to make a truthful report $\sigma^J = 2$ rather than a false one $\sigma^J = 0$ requires that the repayment due by reporting truthfully, R_2, is no higher than what is due by cheating and reporting two fails:

$$R_2 \leq (1-m_0^J)R_{0|\cdot} + m_0^J[m_{0,H}R_{0|HH} + (1-m_{0,H})R_{0|H\cdot}]. \tag{9}$$

Upon receiving a false report of two fails, the lender has to audit both projects sequentially. Because the borrower has cheated, a first stage audit by the lender, which occurs with probability m_0^J, always reveals a success. Any second stage audit, which occurs with probability $m_{0,H}$, also reveals a success, and has associated repayment for the lender $R_{0|HH}$. If no second stage audit occurs, with probability $1 - m_{0,H}$, the associated repayment for the lender is $R_{0|H\cdot}$.

The incentive constraint that ensures that a borrower with two successes prefers to make a truthful report $\sigma^J = 2$ rather than a false report $\sigma^J = 1$ is:

$$R_2 \leq (1 - m_1) R_{1|\cdot} + m_1 \left[m_{1,H} R_{1|HH} + (1 - m_{1,H}) R_{1|H\cdot} \right] \tag{10}$$

By constraint (10), the repayment due after reporting truthfully two successes, R_2, is no higher than what is due by cheating and reporting one success. To get this latter amount, consider that, when reporting one success, the borrower reports just the aggregate return $H + L$, and the truthfulness of the report can only be ascertained by auditing both projects sequentially. In particular, because the borrower has cheated, a first stage audit by the lender, which occurs with probability m_1, always reveals a success. Any second stage audit, which occurs with probability $m_{1,H}$, given that a first-stage audit has certainly revealed a success, also reveals a success, and an associated repayment for the lender $R_{1|HH}$. If no second stage audit occurs, with probability $1 - m_{1,H}$, the associated repayment for the lender is $R_{1|H\cdot}$.

Cheating may also occur when one success realises ($\varsigma = HL, LH$). A borrower with one success prefers to report truthfully $\sigma^J = 1$ rather than falsely $\sigma^J = 0$ if:

$$\begin{aligned} (1 - m_1) R_{1|\cdot} + m_1 \left[s R_{1|L} + (1 - s) \left(m_{1,H} R_{1|HL} + (1 - m_{1,H}) R_{1|H\cdot} \right) \right] \leq \left(1 - m_0^J \right) R_{0|\cdot} \\ + m_0^J \left[\frac{1}{2} \left(m_{0,H} R_{0|HL} + (1 - m_{0,H}) R_{0|H\cdot} \right) + \frac{1}{2} \left(m_{0,L} R_{0|LH} + (1 - m_{0,L}) R_{0|L\cdot} \right) \right]. \end{aligned} \tag{11}$$

The expected compensation associated with a truthful report $\sigma^J = 1$ (left hand side of constraint (11)) takes into account that a first stage audit of one of the projects can occur with probability m_1 and discover either a success or a fail, depending on the transparency of the accounting information. In particular, with probability s a first stage audit will reveal a fail, thus calling for no further audit, while with probability $1 - s$ it will reveal a success, thus calling for a second stage audit with probability $m_{1,H}$. With a false report of zero successes a first stage audit may occur with prob. m_0^J and the lender can randomly choose which project to audit, if any, with probability 1/2 on each (by the principle of insufficient reason). Since an audit may discover either a success or a fail, the lender may go on to audit at the second stage with probability $m_{0,H}$ or $m_{0,L}$, getting $R_{0|HL}$ or $R_{0|LH}$, respectively.

Last, the limited liability conditions are:

$$\begin{aligned} R_2, R_{1|HH}, R_{0|HH} &\leq 2H, \\ R_{1|\cdot}, R_{1|L}, R_{1|H\cdot}, R_{1|HL}, R_{0|H\cdot}, R_{0|HL}, R_{0|LH} &\leq H + L, \\ R_{0|\cdot}, R_{0|L\cdot}, R_{0|LL} &\leq 2L. \end{aligned} \tag{12}$$

Summing up, the contract $\mathcal{P}^{\mathcal{J}}$ sets repayments R_2, $R_{1|\cdot}$, $R_{1|H\cdot}$, $R_{1|HH}$, $R_{1|HL}$, $R_{1|L}$, $R_{0|H\cdot}$, $R_{0|HL}$, $R_{0|LH}$, $R_{0|\cdot}$, $R_{0|L\cdot}$, $R_{0|LL}$, $R_{0|HH}$, and audit probabilities m_0^J, $m_{0,H}$, $m_{0,L}$, m_1, $m_{1,H} \in [0,1]$ to maximise the objective function (7), subject to the participation constraint (8), the incentive constraints (9), (10), and (11), and the limited liability conditions (12).

By solving Programme $P^{\mathcal{J}}$ two possible cases may arise, depending on whether a common repayment for the lender after one or two successes covers the investment and expected audit cost. If it does, only bottom reports of two fails are audited, while intermediate reports of one success and one fail are not. The subsequent audit cost saving that joint finance brings about relative to single finance is the coinsurance benefit of joint finance. However, if pooling the top two repayments does not cover the investment and expected audit cost, then an audit must involve also intermediate reports. This may then lead to the audit of the truly failing project, but possibly, depending on the quality of the accounting information, also of the successful one. The subsequent extra audit cost from the audit of the successful project is the contagion loss of joint finance.

The properties of the joint second-best contract are described in Proposition 2.

Proposition 2 *Suppose two identical and independent projects are jointly financed. The second-best contract has:*

(i) maximum punishment for detected false reporting: $R_{0|HH} = R_{1|HH} = 2H$; $R_{0|H\cdot} = R_{0|HL} = R_{0|LH} = H + L$;

(ii) zero rent to the borrower in the lowest true state: $R_{0|L\cdot} = R_{0|\cdot} = R_{0|LL} = 2L$.

Moreover, the second-best contract has:

1. *deterministic audit of reports of zero successes at first stage or at second stage having discovered a cheat by first stage audit,* $m_0^J = m_{0,H} = 1$; *random audit of reports of*

zero successes at second stage having discovered a truthful report at first stage:

$$m_{0,L} = \frac{4(I-L) + 2(1-p)^2 c - p(2-p)(H-L)}{p(2-p)(H-L) - 2(1-p)^2 c} \leq 1; \tag{13}$$

2. *repayments pooled in the top two states,* $R_{1|\cdot} = R_2 = 2L + \frac{2(I-L)(H-L)}{p(2-p)(H-L)-2(1-p)^2 c} \leq H + L$, *so that the borrower with at least one success is indifferent between truthfully reporting one or two successes;*

3. *no audit following an intermediate report of one fail, i.e.,* $m_1 = m_{1,H} = 0$;

4. *borrower's expected returns:*

$$EP^J = 2[pH + (1-p)L - I] - \frac{4(1-p)^2 (I-L)c}{p(2-p)(H-L) - 2(1-p)^2 c}. \tag{14}$$

when $m_{0,L}$ *(13) is in the unit interval.*
When $m_{0,L} = \frac{4(I-L)+2(1-p)^2 c - p(2-p)(H-L)}{p(2-p)(H-L)-2(1-p)^2 c} > 1$, *the second-best contract has:*

5. *deterministic audit for reports of two fails:* $m_0^J = m_{0,L} = m_{0,H} = 1$;

6. *zero rent to the borrower reporting one success, whether audited or not:* $R_{1|\cdot} = R_{1|L} = R_{1|HL} = R_{1|H\cdot} = H + L$;

7. *random first stage auditing for single fail reports*

$$m_1(s) = \frac{2(I-L) - p(2-p)(H-L) + 2(1-p)^2 c}{p[p(H-L) - 2(1-p)(2-s)c]} \leq 1 \tag{15}$$

and deterministic second stage auditing when first stage auditing has revealed a success, $m_{1,H} = 1$;

8. *repayment after a report of two successes higher than* $H + L$:

$$R_2(s) = 2H - 2(H-L)\frac{pH + (1-p)L - I - \{1 - p[s + p(1-s)]\}c}{p^2(H-L) - 2p(1-p)(2-s)c}; \tag{16}$$

9. *borrower's expected returns*

$$EP^J(s) = 2[pH + (1-p)L - I] - 2(1-p)c[1 - p + p(2-s)m_1(s)], \tag{17}$$

lower than the expected profits in (14), and increasing in s, *with* $m_1(s)$ *as defined in 15.*

The intuition behind the results in Proposition 2 is the following. Maximum punishment and zero rent to the borrower in the lowest truthfully reported states (results (i) and (ii)) maximise the incentive for truth-telling whilst also keeping the audit cost as small as possible.

Moreover, when (13) is in the unit interval (results 1 to 4) it is possible to pool repayments in the top two states and concentrate audits on report of two fails. A strictly positive probability of auditing a report of two fails (result 1) is required to prevent the borrower from always reporting zero successes and getting away with cheating, leading to repayments which do not cover the investment cost. Moreover, because first stage audit m_0^J has two incentive effects, one working directly at the first stage and the other combining with $m_{0,L}$ at the second stage, first stage audit is a more powerful control on potential cheating than second stage audit. Thus $m_0^J = 1$ and $m_{0,L} \leq 1$. The incentive to cheat between a report of one or two successes is controlled by pooling the repayments, $R_2 = R_{1|\cdot}$ (result 2). These must be above $2L$, as otherwise there would be insufficient revenue to the lender to recoup the loan cost, and no higher than $H + L$, the highest revenue available if only one project succeeds. With flat repayments for one or two successes, audit of projects following one fail report is unnecessary as the borrower has no incentive to cheat, $m_1 = m_{1,H} = 0$ (result 3). Because the failing project of an intermediate outcome is never audited, joint finance allows a saving in audit cost relative to single finance. When this occurs, we say that the second-best contract displays coinsurance.

When (13) is not in the unit interval (results 5 to 9 of Proposition 2), additional revenues in excess of $H+L$ must be raised from the report of two successes to cover the investment plus audit cost of the two projects. But to ensure truthful reports of two successes, intermediate reports must sometimes be audited ($m_1, m_{1,H} \geq 0$), which implies that, besides the failing project, also the successful project may end up being audited. It follows that joint finance brings about an extra audit cost relative to single finance. When this occurs, we say that the second-best contract displays contagion.

The optimal audit probabilities are nevertheless decreasing in the profitability of the state. In particular, whereas a report of zero successes is audited deterministically ($m_0^J = m_{0,L} = m_{0,H} = 1$), a report of one success is audited with a lower intensity. This improves the incentive to truthfully declare one success instead of no successes and it is efficient as it minimises the audit cost. Indeed, because all intermediate repayments are equal to $H + L$, the borrower with only one successful project might have an incentive to report zero successes rather than one, so as to bet on the possibility of not being audited. To make sure that this does not happen, the lender always audits reports of zero successes ($m_0^J = m_{0,L} = m_{0,H} = 1$). Since the borrower gets zero anyway by reporting 0 or 1 successes, she might then be indifferent between cheating and telling the truth. However, she is still better off by telling the truth because the audit costs are lower upon a one success report and so the ex-ante profits are higher. Thus, audits are concentrated on the worst state report which is more likely to reflect cheating and on which strong audit will have more power in ensuring truthtelling, whereas intermediate state reports are audited residually.

As regards the intensity of audits following an intermediate report of one success, this varies depending on whether the first stage audit reveals a success or a fail, which in turn depends on the transparency of the accounting information. If it reveals a fail (with probability s), the second project must be a success and there is no further audit ($m_{1,L} = 0$). But if the first audit reveals a success (with probability $1 - s$), because the second project may also be a success, there may still be an audit. If so, using a low probability of auditing the first project ($m_1(s) > 0$) but maximum probability of auditing the second ($m_{1,H} = 1$) gives the most powerful truthtelling incentive and economises on wasteful audit cost.

As regards the social efficiency of the second-best contract it varies with coinsurance or contagion, by comparing the borrower's expected returns, those arising under contagion (17) fall short of those under coinsurance (14). The difference is driven by the lower frequency with which audits occur when coinsurance rather than contagion prevails, with a subsequent lower audit cost.

Last, we have seen that the borrower's expected return (17) is increasing in the transparency of the accounting information s. This is because a better quality of the accounting information reduces the contagion loss of joint finance, increasing the firm's profits.

Proposition 2 has described the properties of the second-best contract when the coinsurance or contagion arise. Proposition 3 states the conditions under which each of these scenarios are feasible.

Proposition 3 *From (13), the joint finance contract with coinsurance is feasible when*

$$p(2-p)(H-L) - 2(I-L) \geq 2(1-p)^2 c. \qquad \text{(Condition 2)}$$

From (15), the joint finance contract with contagion is feasible when

$$2\left[pH+(1-p)L-I\right]\geq 2(1-p)c\left[1+p(1-s)\right]. \qquad \text{(Condition 3)}$$

Besides guaranteeing that the audit probabilities (13) and (15) are in the unit interval, Conditions 2 and 3 also have an economic interpretation, namely, that to be feasible, the projects taken together must be sufficiently profitable to cover the investment and certain audit cost of the bottom and the intermediate report, respectively. Indeed, Condition 2 can be obtained by assuming that in the participation constraint (8) the revenue from zero or one success outcomes ($2L$ and $H+L$) is sufficient to meet the investment cost plus certain audit of the lowest report. This allows there to be no audit of the intermediate report as there is a common repayment after one and two successes. If Condition 2 is violated and the revenue from zero or one success outcomes is insufficient to meet the investment plus certain audit cost of the lowest report, then extra-resources must be raised from a two success outcome ($2H$), which implies that also the intermediate report must be audited to stop a borrower with two successes reporting one. Condition 3 is obtained by assuming that collecting the revenue from zero, one and two successes outcomes ($2L$, $H+L$ and $2H$) yields enough expected revenue to cover the investment cost and the expected cost incurred by auditing deterministically all reports involving two fails or, with a reported one fail, at least one of the projects, depending on the quality of the accounting information (s).

Condition 2 can be represented in the space of $p(H-L)$ and $I-L$ by a linear function with intercept $2(1-p)^2 c/(2-p)$ and slope $2/(2-p)$ (Fig. 4). Thus, a coinsurance contract is feasible in the red area to the left of the Condition 2 line, where the high state return is sufficiently high relative to the investment cost.

For a given value of $s\in[1/2,1)$, Condition 3 can be represented in the space of $p(H-L)$ and $I-L$ by a linear function $p(H-L)=(1-p)c\left[1+p(1-s)\right]+(I-L)$ with intercept $(1-p)c\left[1+p(1-s)\right]$ and slope 1 (Fig. 5). As s increases the Condition 3 line shifts down, coinciding with the Condition 1 line when $s=1$. Thus, a contract with contagion is feasible in the purple area above the Condition 3 line. Whether it arises depends on whether in the same space of parameters a contract with coinsurance is also feasible, i.e., on whether Condition 2 is satisfied or violated. When both Conditions 2 and 3 are satisfied, a coinsurance contract arises due to its lower audit cost relative to a contagion contract. When Condition 2 is violated, a coinsurance contract is not feasible and the only feasible joint

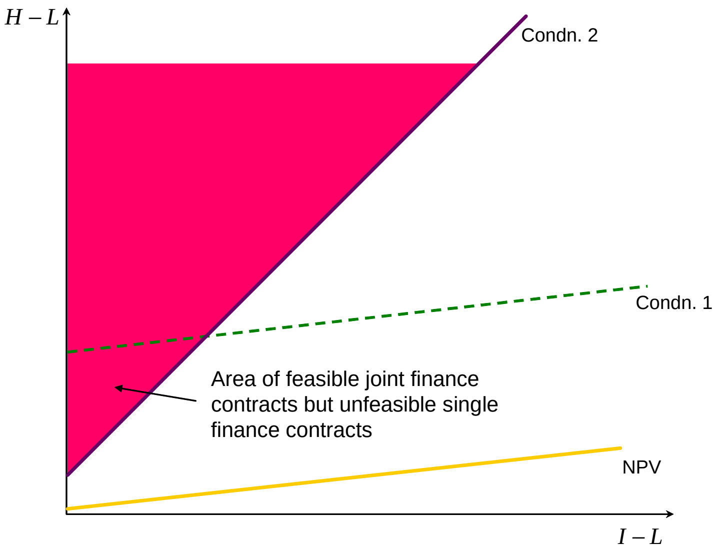

Figure 4: Area of joint finance contract with coinsurance

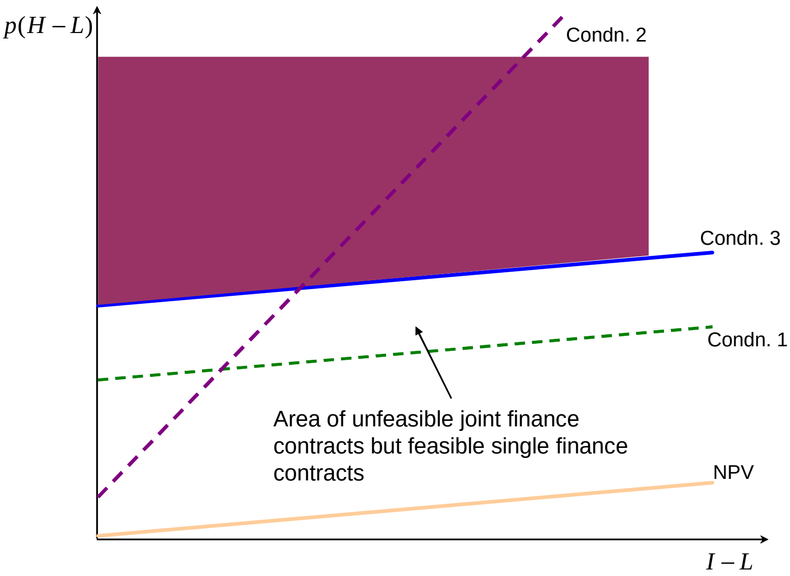

Figure 5: Area of feasible joint finance contract with contagion

finance contract involves contagion. Graphically, this occurs to the right of the intersection between Condition 2 and 3 lines in Fig. 5, where $p(H-L) = 2(1-p)(2-s)c$ and $I-L = (1-p)(1+(2-p)(1-s))c$.

From the above, we can state the following proposition:

Proposition 4 *The joint second-best contract displays coinsurance when Condition 2 holds, while it displays contagion when Condition 3 holds, but Condition 2 does not.*

Last, notice that in the joint second-best contract reported cash flows are verified only if they fall short of a given threshold, $R_{1|\cdot} = R_2 \leq H+L$ if there is coinsurance, and $R_2 \leq 2H$ under contagion as in a debt contract. In the special case in which either Condition 2 or Condition 3 hold with equality, the optimal contract is a standard debt contract as verification becomes deterministic. In particular, when Condition 2 holds with equality (coinsurance), if reported cash flows fall short of $H+L$, verification always occurs for bottom reports and never for intermediate and top ones, i.e., $m_0^J = m_{0,L} = m_{0,H} = 1$, and $m_1 = m_{1,H} = 0$. When Condition 3 holds with equality (contagion), if reported cash flows fall short of $2H$, verification always occurs both for bottom and intermediate reports, i.e., $m_0^J = m_{0,L} = m_{0,H} = m_{1,H} = m_1(s) = 1$, and never for top ones.

5 Efficiency

In the following we contrast the efficiency properties of single and joint finance contracts. To this aim, consider that when two projects are financed as stand-alone, the expected profits are twice the profits obtainable from each project as defined in Eq. (6). In the space $p(H-L), I-L$, the feasible single finance contracts are those above the Condition 1 line in Fig. 2. Under joint financing, the expected profits are defined in Eq. (17) and the feasible second-best contracts are those above Condition 2 and Condition 3 lines in Figs. 4 and 5.

5.1 Single vs. joint finance contract under coinsurance

We first consider the case in which the joint second-best contract displays coinsurance (Condition 2 is satisfied).

The first thing to notice by comparing Conditions 1 and 2 in Fig. 4 is that, relative to single finance, joint finance widens the area of the feasible contracts and mitigates credit rationing. In particular, there are projects that are feasible when funded jointly, in the sense that their expected returns cover the investment and expected audit cost, but not separately, namely those in the area to the left of the intersection between Condition 1 and 2 lines, where $p(H-L) = 2(1-p)c$ and $I - L = (1-p)c$, as shown in Fig. 4. Thus, projects that are not viable if financed as stand-alones become viable when financed jointly.

We next consider the case in which both single and joint financing regimes are feasible and compare their profitability. From Proposition 2, we know that when coinsurance effects prevail reports of one success are never audited, $m_1 = m_{1,H} = 0$, and only reports of zero successes are audited with probability $m_0^J = 1$ and $m_{0,L} > 0$ as defined in the proposition. By comparing Eqs. (14) and (6) we get:

$$\underbrace{(1-p)\,c\frac{2\,(I-L)}{p\,(H-L)-(1-p)\,c}}_{\text{exp. audit cost under single finance}} - \underbrace{(1-p)^2\,c\frac{4\,(I-L)}{p\,(2-p)\,(H-L)-2\,(1-p)^2\,c}}_{\text{exp. audit cost under joint finance and coinsurance}}.$$

The difference reduces to $\frac{2p^2(1-p)(H-L)(I-L)c}{[p(H-L)-(1-p)c]\left[p(2-p)(H-L)-2(1-p)^2c\right]}$, which is always positive. Thus, when coinsurance effects prevail joint financing has higher profits (or lower expected audit cost) than single finance**.**

To determine the driver of this result, we compare the audit probabilities in the two scenarios, $m_0^J\,(1+m_{0,L}) - 2m_0^S = \frac{2p(I-L)[p(H-L)-2(1-p)c]}{[p(H-L)-(1-p)c][p(2-p)(H-L)-2(1-p)^2c]}$. This difference is positive, thus indicating that there is more intensive audit under joint financing. It follows that the dominance of joint finance with coinsurance over project finance can be ascribed to the lower probability with which default occurs ($(1-p)^2$ under joint finance vs. $1-p$ under single finance) - and thus the lower frequency with which an audit occurs - along with the pooling of returns implied by Condition 2 that allows to target audits only on reports of two fails (coinsurance benefit of joint finance). This result in which an intensive audit is applied with a low frequency is reminiscent of Becker (1968) in which maximum deterrence is obtained at minimal cost by inflicting a high punishment with a sufficiently low probability. We can thus state the following proposition:

Proposition 5 *When the joint second-best contract displays coinsurance, joint finance mitigates credit rationing and always dominates single finance.*

These results are in line with those obtained by the early literature highlighting the diversification benefits of joint financing (Lewellen, 1971; Diamond, 1984, among others), and also with Banal-Estañol, Ottaviani and Winton (2013) for the case in which coinsurance gains arise from joint financing, except that we allow for optimal random audits.

5.2 Single vs. joint finance contract under contagion

We next consider the case in which the joint second-best contract displays contagion (Condition 3 is satisfied, while Condition 2 is violated). This means that it is not possible to meet the lender's participation constraint by pooling the top two returns and auditing only reports of zero successes, even deterministically ($m_0^J = m_{0,L} = 1$). Extra-resources must then be raised from the two successes outcome, which implies that also reports of one success must be audited: $m_1, m_{1,H} > 0$.

By comparing Conditions 1 and 3 in Fig. 5, we may notice that for any given $s < 1$, the area of feasible single finance contracts is larger than that of joint finance contracts. This means that, unlike the case in which the contract displays coinsurance, under contagion joint financing makes the credit rationing problem more severe, i.e., there are projects that can be financed separately but not jointly. However, as the quality of the accounting information improves (s increases), due to the lower deadweight loss of audit, the area of feasible joint finance contract widens, coinciding with the area of feasible single finance contract when $s = 1$.[9]

We next focus on the case in which both financing regimes are feasible, comparing profits under joint and single finance contract , i.e., Eqs. (17) and (6), respectively:

$$\underbrace{2(1-p)c\frac{(I-L)}{p(H-L)-(1-p)c}}_{\text{exp. audit cost under single finance}} - \underbrace{2(1-p)c[1-p+p(2-s)m_1(s)]}_{\text{exp. audit cost under joint finance and contagion}} \tag{18}$$

with $m_1(s)$ as defined in 15. The sign of (18) depends on the quality of the accounting information, that impacts on the probability that a first stage audit following a report of one success detects a fail, s.[10] Since from Proposition 2, the borrower's expected profits under contagion (17) are increasing in s, joint finance has the least advantage when the quality of the accounting information is the poorest, i.e., when $s = 1/2$.[11] In this case the difference in profits (18) reduces to $\frac{\{(2-p/2)[p(H-L)-(1-p)c]-2(I-L)\}2p(H-L)(1-p)c}{[p(H-L)-3(1-p)c][p(H-L)-(1-p)c]}$, which is positive, given

[9] The efficiency analysis for this case is postponed to Section 7.1.

[10] Using m_0^S as defined in (5), $m_0^J = m_{0,L} = m_{1,H} = 1$ and $m_1(s)$ as defined in (15), the difference in profits (18) can be written as $\frac{2p(H-L)\{(1+(2-p)(1-s))[p(H-L)-(1-p)c]-(1+2(1-s))(I-L)\}(1-p)c}{[p(H-L)-(1-p)c][p(H-L)-2(1-p)(2-s)c]}$.

[11] This can also be proven by working out the derivative of (18) with respect to s: $\frac{[p(H-L)-(1-p)c]\left\{2(I-L)-p(2-p)(H-L)+2(1-p)^2c\right\}}{[p(H-L)-2(1-p)(2-s)c]^2}$, whose sign depends on the sign of the term in curly brackets in the numerator. Since this coincides with the numerator of $m_1(s)$ (15), which is positive when Condition 3 holds, it follows that the difference in profits increases as the quality of accounting information improves.

that the denominator is positive, if:

$$\left(2-\frac{p}{2}\right)\left[p\left(H-L\right)-\left(1-p\right)c\right]-2\left(I-L\right)>0. \tag{19}$$

From the above, we can thus state Proposition 6.

Proposition 6 *When the contract displays contagion and the quality of the accounting information is poor ($s = 1/2$), joint financing makes credit rationing more severe. However, when both financing regimes are feasible, joint finance dominates single finance if condition (19) holds.*

The result that joint financing may dominate single even when contagion effects prevail is novel in the literature. To disentangle its determinants, notice that a novel trade-off emerges under contagion. On one side there is a higher cost due to the audit of successful projects (the contagion loss of joint finance). On the other side, there is a saving in audit costs due to the optimally chosen random audit. But rather than being driven by the lower probability with which default occurs, as in the coinsurance case, this audit cost saving is driven by the minimal audit of reports of one success. Indeed, the probability with which an audit occurs when contagion prevails is actually higher under joint than single financing (from (18), $(1-p)^2 + 2p(1-p) > 1-p$). Given that a report of zero successes is audited deterministically ($m_0^J = m_{0,L} = 1$), it turns out that the saving in expected audit costs relative to single financing may be ascribed to the random (and minimal) audit of reports of one success and one fail.

These results contrast with Banal-Estañol, Ottaviani and Winton (2013) who show that when the contract displays contagion single always dominates joint financing. This is to be ascribed to the different assumptions regarding the audit strategy. In Banal-Estañol, Ottaviani and Winton (2013), audits are deterministic. In particular, any time the borrower cannot repay the loan in full, the corporation defaults and the entire projects' realised returns are transferred to the creditor who is only able to recover a fraction of them. The default costs are then given by the fraction of returns that cannot be recovered and includes a fraction of the high state returns, a loss that would never occur if each project were financed separately.[12] In our setting, audits are chosen optimally, they are maximal, i.e., deterministic, in the bottom state, less likely to occur, but minimal in the intermediate

[12] However, their results also hold with a more general structure of default costs, provided there are not too extreme diseconomies of scale in default (Banal-Estañol and Ottaviani, 2013).

state. This lower frequency of audits in intermediate states allows a saving in audit cost relative to separate financing that might offset the extra cost of auditing successful projects.

We use a graphical analysis to show the parameter space in which, when the joint finance contract displays contagion, joint dominates single finance (Fig. 6). To do this, notice that, for the comparisons to be meaningful, both standalone finance and joint finance with contagion must be feasible, whereas joint finance with coinsurance must be infeasible. Thus, in the space of $p\,(H-L)$ and $I-L$, because expression (19) must satisfy both Conditions 1 and 3, whilst Condition 2 must be violated, we are focusing on the area to the right of Condition 2 and to the left of Condition 3. The locus of exogenous parameters where single and joint finance are indifferent (the pink line labelled indifference line in Fig. 6) has intercept $(1-p)\,c/p$ and slope $4/p\,(4-p)$. Thus, for $p\,(H-L)$ high relative to $I-L$, i.e., above the indifference line in the red and purple areas, for given p, c, joint finance is superior to single. For $I-L$ high relative to $p\,(H-L)$, i.e., in the green areas, single finance is instead superior.

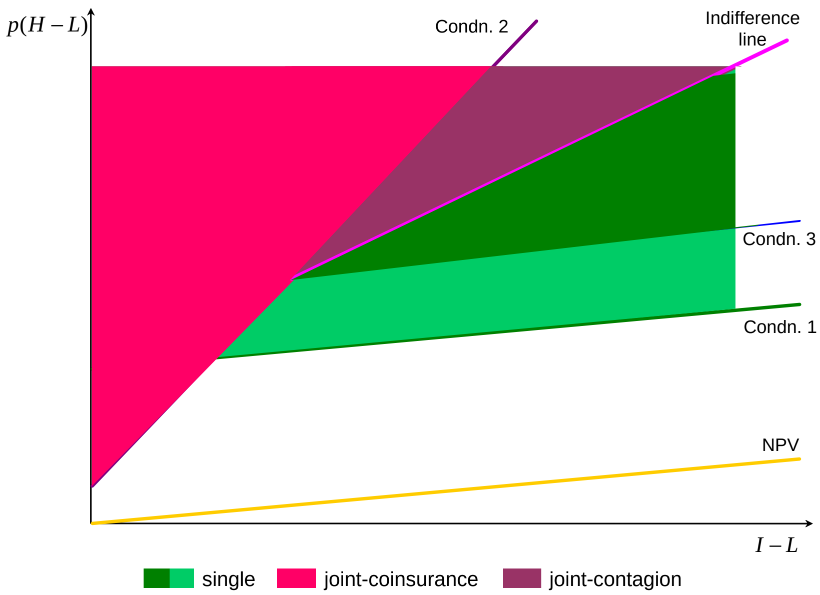

Figure 6: Optimal financial contracts under sequential audit

6 Comparative statics

Proposition 6 shows that the choice between single or joint financing is affected by four key parameters: the size of the high state return H relative to the investment cost I, the probability of success p, the audit cost c, and the quality of the accounting information, s. To assess the impact on the equilibrium outcome of a change in these parameters, in the following we carry out a comparative static analysis.

6.1 The role of market conditions

We start by looking at the effects of a change in the probability of success, p. As p increases, this affects both the intercept and the slope of Conditions 1, 2 and 3, widening the area where both joint and separate financing arise. Intuitively, the expected return pledgeable to creditors also increases and it becomes easier to finance projects, even when joint finance brings about higher risk through contagion. What is most relevant for our analysis is that also the intercept and the slope of the indifference condition (19) decrease, which implies that joint finance is optimal for a larger region of parameters. This is consistent with a vast academic literature showing that merger waves occur in periods of economic recovery (see Martynova and Renneboog (2008b), for a survey).

6.2 The role of audit costs

As regards audit costs c, their increase determines a shift upwards of the intercepts of Conditions 1, 2, 3 and 19, and a reduction of the area where both joint and separate financing arise. If we interpret audit costs as capturing investor vulnerability, this finding is consistent with evidence that merger activity is more likely in countries with stronger investor protection (Rossi and Volpin, 2004; Martynova and Renneboog, 2008a; Bris and Cabolis, 2008; Bris, Brisley and Cabolis; 2008). It is also consistent with the finding of Subramanian and Tung (2016) showing that project financing is more frequent in countries with weak investor protection and weaker creditor rights in bankruptcy.

6.3 The role of accounting information

We have seen so far that the optimal financing regime depends on a trade off between the benefit and cost of joint financing. The cost of joint financing, in particular, depends on the quality of the accounting information, that determines the probability s that, following an intermediate report, a first stage audit picks a fail. When the accounting information is poor ($s = 1/2$), there are projects that can be financed separately but not jointly, namely those in the light green area in Fig. 6. In addition, when both financing regimes are feasible, there is a region of parameters where joint dominates single finance, the purple area in Fig. 6, and a region where single finance dominates, the dark green area in Fig. 6.

In this section we want to explore the impact that a change in the quality of the accounting information (s) has both on the area of feasible joint finance contracts and the optimal financing regime. To this aim, consider that, as s increases, the Condition 3 line in Figures 5 and 6 shifts downwards and joint finance is feasible for a larger region of parameters, until it coincides with Condition 1 line for $s = 1$. Thus, any pair of projects that can be financed separately can be financed also jointly. Moreover, as s increases, also the area of dominance of joint over single finance delimited by the indifference line in Fig. 6 widens. This can be seen by considering that the difference in expected profits (18) is increasing in s. Indeed, differentiating (18) with respect to s we get $\frac{[p(H-L)-(1-p)c]\left\{2(I-L)-p(2-p)(H-L)+2(1-p)^2c\right\}}{[p(H-L)-2(1-p)(2-s)c]^2}$, whose sign depends on the sign of the term in curly brackets. Since this coincides with the numerator of $m_1(s)$ (15), which is positive, it follows that the difference in profits between joint and single finance contract increases as the quality of accounting information improves, widening the area of dominance of joint finance contracts. In the extreme case in which $s = 1$, the difference in profits between the two regimes (18) reduces to $\frac{p(H-L)[pH+(1-p)L-I-(1-p)c]2(1-p)c}{[p(H-L)-2(1-p)c][p(H-L)-(1-p)c]}$, which is strictly positive under Condition 1. Thus, when $s = 1$, joint financing always dominates single finance.

We can thus state the following proposition:

Proposition 7 *With fully transparent accounting information ($s = 1$), joint financing always dominates single financing.*

The above results show that an improvement in the quality of the accounting information mitigates credit rationing and widens the region of dominance of joint over single finance. Intuitively, disentangling the successful project from the failing one allows target of audits only on fail reports, saving the monetary loss associated with contagion. Thus, so long as audits are chosen optimally and the accounting information is accurate, joint financing always dominates separate financing.

We can thus conclude that the optimal financing regime may be affected by the quality of the accounting information. This is consistent with a large body of empirical evidence showing that better accounting information increases M&A activity and outcomes (Rossi and Volpin, 2004; Zhang, 2008; Bris and Cabolis, 2008; Erel, Liao and Weisbach, 2012; Skaife and Wangerin, 2013; Amel-Zadeh and Zhang, 2015: McNichols and Stubben, 2015;

Francis, Huang and Khurana, 2016; among others).

7 Robustness and extensions

In this section we consider the relevance of the arguments to more general settings.

7.1 Simultaneous audit

In Section 5.2, we have shown that the superiority of joint over single finance depends on a trade-off between the saving in audit cost due to random audit within an enlarged state space and the possible extra audit cost arising, under a not fully transparent accounting information ($s < 1$), from the first stage audit of a successful project following an intermediate report. This extra-cost does not arise when the first stage audit picks a fail, as, by feasibility of reports, the other project must necessarily be a success. The leakage of information from sequential audits determines therefore an information gain, with a subsequent saving in audit cost. It might then appear that the benefits of joint financing are primarily driven by the sequential nature of the audits. In this section we show that this is not the case and that the main result remains valid even under simultaneous audits.

When audits are simultaneous, there is no leakage of information and, if the quality of the accounting information is poor, the extra audit cost following an intermediate report is incurred with certainty. To see whether the benefit of joint financing is offset by this extra-cost, we compare the gains from joint finance with a required simultaneous audit of the projects and those from single finance. We find that, although the advantage of joint finance is reduced due to the impossibility of using the first audit to inform the second, joint finance may still dominate single. In particular, for $s = 1/2$, the difference in profits under joint finance with contagion and simultaneous audit and single finance is equal to $\frac{2p(H-L)\{(3-p)[p(H-L)-(1-p)c]-3(I-L)\}(1-p)c}{[p(H-L)-4(1-p)c)][p(H-L)-(1-p)c)]}$, whose sign, given that the denominator is positive, depends on the sign of the numerator.[13]

We can portray this geometrically in Fig. 7, which extends Fig. 6 by showing that the indifference line captured by condition (19), at which single finance is as costly as joint finance (pink dashed line), shifts to the left (light blue line). Thus, under simultaneous audits and low quality of the accounting information, joint finance may still dominate,

[13] The full proof of the analysis is available upon request.

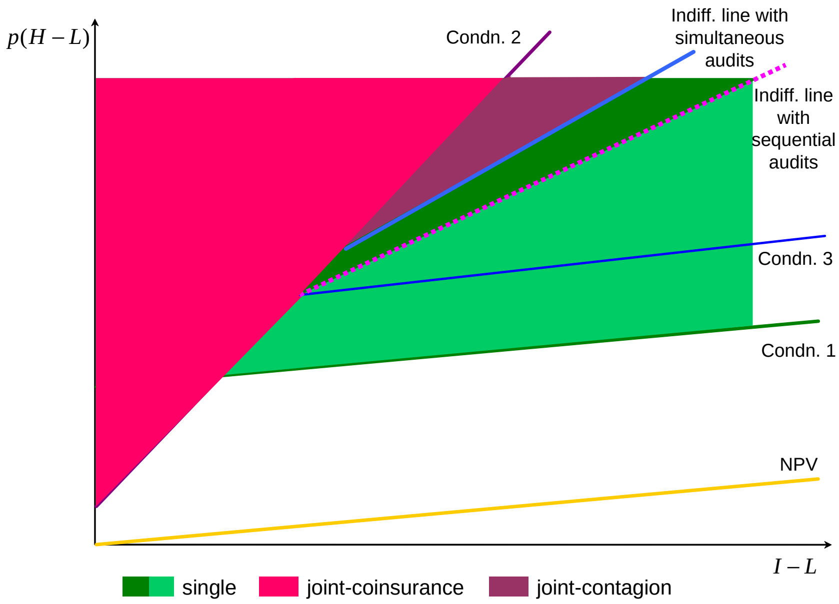

Figure 7: Optimal financial contracts under simultaneous audit

although for a smaller region of parameters.[14] We can thus conclude that joint financing allows a saving in audit cost, no matter whether audits are simultaneous or sequential.

In real world, the relevance of sequential versus simultaneous audits is an empirical matter that depends on the specific situation being audited. Simultaneous audits are often necessary for conglomerates with several subsidiaries during the preparation of consolidated financial statements to obtain an overview of the company's financial health. Sequential audits, instead, may be required when considerations based on the risk profile of individual subsidiaries prevail. Thus, while our model shows joint financing to be robust to simultaneous audits, ultimately, the actual timing of audits aligns with the particular needs and characteristics of the context under scrutiny.

7.2 More than two projects

We have so far considered the case of two identical projects with independent returns, success or fail, where, depending on whether pooling returns from one success and one failure cover the total loan cost and expected audit cost, coinsurance or contagion arises. One obvious extension considers more than two projects. For n iid projects that can each

[14]Notice, however, that under high quality of the accounting information ($s = 1$), Proposition 7 holds: joint financing always dominates single financing even under simultaneous audits. In particular, the difference in profits when $s = 1$ reduces to $\frac{p(H-L)[pH+(1-p)L-I-(1-p)c]}{[p(H-L)-2(1-p)c][p(H-L)-(1-p)c]}$, strictly positive under Condition 1.

either succeed or fail, we can define the minimum number of successes $k^* = \{0, 1, 2, 3, ..., n\}$ such that the revenues $k^*H + (n - k^*)L$ can cover the cost of the investment, nI, i.e., $\min\{k|kH + (n-k)L \geq nI\} = k^*$.

In this case, the forces of the two-projects case are still at work. In particular, only reports of $\sigma \leq k^*$ successes will be audited, with the highest probability for those lower than k^* ($m_\sigma = 1$ for $\sigma < k^*$), and with probability strictly less than one for report equal to k^* ($m_\sigma < 1$ for $\sigma = k^*$). Reports of $\sigma > k^*$ successes will never be audited, with a debt contract for the conglomerate emerging endogenously.

As with two projects, conglomeration yields a cost saving due to endogenous audit and the concentration of audit on states which are less likely to occur.[15] The dominance of joint over single finance turns out to depend on the quality of the accounting information, i.e., on whether reports of $n - k$ fails are sufficiently detailed to identify the failing projects, and thus target audit just on those. If they are fully informative, then joint audits comes only with benefits and no cost, and joint finance always dominates single finance. If they are (partially) uninformative, then, a report $0 < \sigma \leq k^*$, also comes with the costs of the unnecessary audit of succeeding projects. As in the case with two-projects, the extent of the trade-off, and thus the optimal financing regime, depends on the parameters of the model.

8 Conclusion

The article shows that joint financing multiple projects with uncorrelated returns reduces audit costs. This happens not only when joint financing generates coinsurance benefits, but may also hold when it brings about contagion costs. This depends on a trade-off between the cost saving from endogenous audits - with an intensive audit of the collective worst outcomes, less likely to occur, and a lower or no audit of the intermediate outcomes - and the extra-cost coming from the audit that a successful project may undergo when jointly financed. As a result, debt may be the optimal joint contractual arrangement. The results are robust to the sequencing of audit and, consistent with the empirical evidence, stronger the better the quality of the accounting information.

With several independent projects, we have seen that the forces identified should remain and the type of mechanism in which the reduced probability of a joint failure goes together with the highest audit frequency should lead to deterministic audit of the worst outcomes

[15] A report of zero successes, for example, has a probability of occurring $(1-p)^n$ which is lower than the probability of auditing n individually financed projects $n(1-p)$.

and no audit of the remaining ones, with a standard debt contract emerging endogenously. Another possible extension could consider correlated returns. This should even favour joint financing through the audit cost saving due to sequential audit, as knowing the outcome on one project is informative about the outcome of the other and saves on audit cost. We leave the development of these extensions for future research.

A Appendix

Proof of Proposition 1 Using maximum punishment ($R_{0|H} = H$) in the optimisation problem $\mathcal{P}^{\mathcal{S}}$ and forming a Lagrangian with multiplier λ and μ, the FOC's wrt R_1, $R_{0|\cdot}$, $R_{0|L}$ and m_0^S are

$$\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial R_1} &: (\lambda - 1)\, p - \mu \geq 0, R_1 \leq H \\
\tfrac{\partial \mathcal{L}}{\partial m_0^S} &: (1-p)\left(R_{0|\cdot} - R_{0|L}\right)(1-\lambda) - \lambda (1-p)\, c + \mu \left(H - R_{0|\cdot}\right) \geq 0, m_0^S \leq 1 \\
\tfrac{\partial \mathcal{L}}{\partial R_{0|L}} &: (\lambda - 1)\, m_0^S (1-p) \geq 0, R_{0|L} \leq L \\
\tfrac{\partial \mathcal{L}}{\partial R_{0|\cdot}} &: \left(1 - m_0^S\right)\left[(\lambda - 1)(1-p) + \mu\right] \geq 0, R_{0|\cdot} \leq L
\end{aligned}$$

1. $\lambda > 1$.
   Suppose $\lambda = 1$. Then by $\frac{\partial \mathcal{L}}{\partial R_1}$, $\mu = 0$. By $\frac{\partial \mathcal{L}}{\partial m_0^S}$, this implies $-\lambda (1-p)\, c \leq 0$, a contradiction, as $\frac{\partial \mathcal{L}}{\partial m_0^S} \geq 0$.

2. $R_{0|L} = R_{0|\cdot} = f_L$.
   By $\lambda > 1$, $\frac{\partial \mathcal{L}}{\partial R_{0|L}}, \frac{\partial \mathcal{L}}{\partial R_{0|\cdot}} > 0$ and $R_{0|L} = R_{0|\cdot} = f_L$.

3. $R_1 < H$
   Using $R_{0|L} = R_{0|\cdot} = L$, $R_{0|H} = H$ and $m_0^S = \frac{R_1 - L}{H - L}$ from the incentive constraint, the contract problem becomes to choose R_1 to $\max p\,(H - R_1) \mid \text{st } pR_1 + (1-p)\left(L - \frac{R_1 - L}{H - L} c\right) = I$. The objective function is decreasing in R_1, whereas the participation constraint is increasing in it, provided Condition 1 holds ($\frac{\partial PC}{\partial R_1} = \frac{1}{H-L}\left[p\,(H-L) - (1-p)\, c\right]$). R_1 is then obtained by solving the participation constraint, giving $R_1 = \frac{(H-L)I - (1-p)L(H-L+c)}{p(H-L) - (1-p)c}$. Substituting out in m_0^S, gives m_0^S (5). For $m_0^S < 1$, $pH + (1-p)\,L - I - (1-p)\,c > 0$, which certainly holds under Condition 1. This in turn implies from (3) that $R_1 < H$. The expected return to the borrower (6) is obtained using the solutions to the programme set out above in the objective function.

■

Proof of Proposition 2

1. Maximum punishment for false reports
   From programme $\mathcal{P}^{\mathcal{J}}$ we see that the punishment repayments $R_{1|HH}$, $R_{0|HH}$, $R_{0|H\cdot}$, $R_{0|HL}$, $R_{0|LH}$ only enter the incentive constraints. So, by setting maximum punishment, the right hand side of these increases and either m_0^J or m_1, $m_{1,H}$ or both

can be reduced. For example if $R_{0|HH} < 2H$, then we can increase $R_{0|HH}$ and reduce m_0^J keeping $m_0^J m_{0,H} R_{0|HH}$ constant. This raises the right hand side of (9) because it raises $\left(1 - m_0^J\right) R_{0|\cdot}$ and slackens (8) due to the decreased frequency of the audit cost $m_0 c$. In turn this allows a reduction in R_2. Similar arguments apply to increases in $R_{0|H\cdot}$ and $m_{0,H}$ keeping $(1 - m_{0,H})R_{0|H\cdot}$ constant and variations in $R_{1|HH}$ (increase) and m_1 (decrease) keeping $m_1 m_{1,H} R_{1|HH}$ constant, in $R_{0|HL}$ (increase) and m_0^J (decrease) keeping $m_0^J m_{0,H} R_{0|HL}$ constant, and $R_{0|LH}$ (increases) and $m_{0,L}$ (decreases) keeping $m_0^J m_{0,L} R_{0|LH}$ constant. Thus, $R_{1|HH} = R_{0|HH} = 2H$, $R_{0|H\cdot} = R_{0|HL} = R_{0|LH} = H + L$. Given these, $m_{0,H}$ only enters the right hand side of (9) and is increasing in it. So we can set $m_{0,H} = 1$.

2. $R_{0|L\cdot} = R_{0|\cdot} = R_{0|LL} = 2L$
If $R_{0|L\cdot} < 2L$ and $R_2 > 0$ we can reduce R_2 and raise $R_{0|L\cdot}$ so as to kee constant $p^2 R_2 + (1-p)^2 m_0^J (1 - m_{0,L}) R_{0|L\cdot}$, leaving both the objective function and the participation constraint unchanged. This slackens the incentive constraints, allowing a reduction in m_0. Similarly, we can reduce R_2 and raise $R_{0|\cdot}$ so that $p^2 R_2 + (1-p)^2 \left(1 - m_0^J\right) R_{0|\cdot}$ stays constant, leaving both the objective function and the participation constraint unchanged, whilst slackening the incentive constraints. We know $R_2 > 2L > 0$ because if $R_2 \leq 2L$ there is insufficient revenue to recoup the investment cost. Hence, such reductions in R_2 are always possible. The result is $R_{0|L\cdot} = R_{0|\cdot} = 2L$. $R_{0|LL}$ only appears in the objective function and the participation constraint. Using $R_{0|\cdot} = R_{0|L\cdot} = 2L$, we have that lowering R_2 and raising $R_{0|LL}$ so as to keep $p^2 R_2 + (1-p)^2 m_0 m_{0,L} R_{0|LL}$ constant leaves both the objective and the participation constraint unchanged, whilst slackening the first and second incentive constraint. So, also $R_{0|LL} = 2L$.

3. $m_0 > 0$
Using the results of points 1 and 2, the contract problem becomes $(\mathcal{P}^{\mathcal{J}'})$:

$$\max EP^J(s) = p^2(2H - R_2) + 2p(1-p)\left\{H + L - (1 - m_1) R_{1|\cdot} - m_1\left[sR_{1|L} + (1-s)\left(m_{1,H} R_{1|HL} + (1 - m_{1,H}) R_{1|H\cdot}\right)\right]\right\}, \tag{20}$$

$$EP^L(s) = p^2 R_2 + 2p(1-p)\left\{(1 - m_1) R_{1|\cdot} + m_1\left[\left(sR_{1|L} + (1-s)\left(m_{1,H}\left(R_{1|HL} - c\right) + (1 - m_{1,H}) R_{1|H\cdot}\right)\right) - c\right]\right\} + (1-p)^2\left[2L - m_0^J(1 + m_{0,L})c\right] \geq 2I. \tag{21}$$

$$R_2 \leq 2\left(1 - m_0^J\right)L + 2m_0^J H \tag{22}$$

$$R_2 \leq (1 - m_1) R_{1|\cdot} + m_1\left[2m_{1,H} H + (1 - m_{1,H}) R_{1|H\cdot}\right] \tag{23}$$

$$(1 - m_1) R_{1|\cdot} + m_1\left[sR_{1|L} + (1-s)\left(m_{1,H} R_{1|HL} + (1 - m_{1,H}) R_{1|H\cdot}\right)\right] \leq \left(1 - m_0^J\right) R_{0|\cdot} + \frac{m_0^J}{2}\left[H + L + m_{0,L}(H + L) + (1 - m_{0,L}) R_{0|L\cdot}\right]. \tag{24}$$

If $m_0 = 0$, the first incentive constraint would give $R_2 \leq 2L$, which is less than

$2I$. So we must have $m_0 > 0$. Moreover, constraint (24) must be binding. If not, it would be possible to lower $m_{0,L}$ slackening the participation constraint, thus allowing a reduction in R_2.

4. $m_0 = 1$, $m_1 = m_{1,H} = 0$
The variables are R_2, $R_{1|\cdot}$, $R_{1|H\cdot}$, $R_{1|HL}$, $R_{1|L}$, m_0, $m_{0,L}$, m_1, $m_{1,H}$. We know that $R_2 > 2L$ to provide sufficient expected revenue to repay the debt. Moreover, $m_{0,L} \geq 0$. So we can eliminate these two variables from the binding participation constraint and the binding incentive constraint (24), obtaining

$$m_{0,L} = 2\frac{m_1\left[sR_{1|L}+(1-s)\left(m_{1,H}R_{1|HL}+\left(1-m_{1,H}\right)R_{1|H\cdot}\right)\right]+(1-m_1)R_{1|\cdot}-2L}{m_0(H-L)} - 1$$

$$\begin{aligned} R_2 = &-\frac{2(1-p)[p(H-L)-(1-p)c]\left\{m_1\left[sR_{1|L}+(1-s)\left(m_{1,H}R_{1|HL}+\left(1-m_{1,H}\right)R_{1|H\cdot}\right)\right]+(1-m_1)R_{1|\cdot}\right\}}{p^2(H-L)} \\ &+\frac{2(1-p)\left[1+(1-s)m_{1,H}\right]m_1c}{p}+\frac{2\left(I-(1-p)^2L\right)}{p^2}-\frac{4(1-p)^2Lc}{p^2(H-L)} \end{aligned}$$

Substituting them out in the objective function (20) and in the incentive constraints (22) and (23) (IC_1, IC_2) leaves the variables $R_{1|\cdot}$, $R_{1|L}$, $R_{1|H\cdot}$, $R_{1|HL}$, m_0, m_1, $m_{1,H}$. Starting from any feasible position in the variables, we can locally vary all the variables in ways which keep each constraint unchanged (thus requiring $dIC_1 = dIC_2 = 0$) and see which directions of change will improve the objective function (dEP^J). This requires the variations to satisfy

$$\begin{aligned} dIC_i = &\frac{\partial IC_i}{\partial R_{1|\cdot}}dR_{1|\cdot} + \frac{\partial IC_i}{\partial R_{1|H\cdot}}dR_{1|H\cdot} + \frac{\partial IC_i}{\partial R_{1|HL}}dR_{1|HL} + \frac{\partial IC_i}{\partial R_{1|L}}dR_{1|L} \\ &+\frac{\partial IC_i}{\partial m_0}dm_0 + \frac{\partial IC_i}{\partial m_1}dm_1 + \frac{\partial IC_i}{\partial m_{1,H}}dm_{1,H} = 0, \; i = 1, 2 \end{aligned}$$

We use this to express local variations in $R_{1|\cdot}$, $R_{1|L}$ in terms of the variations in $R_{1|H\cdot}$, $R_{1|HL}$, m_0, m_1, $m_{1,H}$. Finally, we see the effect on the objective function:

$$\begin{aligned} dEP^J \;\; = \;\; &\frac{\partial EP^J}{\partial R_{1|L}}dR_{1|L} + \frac{\partial EP^J}{\partial R_{1|\cdot}}dR_{1|\cdot} + \frac{\partial EP^J}{\partial R_{1|HL}}dR_{1|HL} + \frac{\partial EP^J}{\partial R_{1|H\cdot}}dR_{1|H\cdot} + \\ &\frac{\partial EP^J}{\partial m_0}dm_0 + \frac{\partial EP^J}{\partial m_1}dm_1 + \frac{\partial EP^J}{\partial m_{1,H}}dm_{1,H} \end{aligned}$$

Substituting in the variations in $dR_{1|\cdot}$ and $dR_{1|L}$ which ensure that IC_1 (22) and IC_2 (23) hold, we get:

$$\frac{dEP^J}{dm_0} = \frac{2p^2\left(1-p\right)\left(H-L\right)c}{p\left(H-L\right)-\left(1-p\right)c} > 0$$

$$\frac{dEP^J}{dm_1} = -\frac{2p^2\left(1-p\right)\left[1+m_{1,H}\left(1-s\right)\right]c}{p\left(H-L\right)-\left(1-p\right)c} < 0$$

$$\frac{dEP^J}{dm_{1,H}} = -\frac{2p^2\left(1-p\right)m_1\left(1-s\right)\left(H-L\right)c}{p\left(H-L\right)-\left(1-p\right)c} < 0$$

Thus, the objective function can be increased by increasing m_0 and reducing m_1 and $m_{1,H}$.

The solution has $m_0 = 1$ and $m_1 = m_{1,H} = 0$, so long as the implied $R_{1|\cdot}$, $R_{1|L}$, $R_{1|H\cdot}$, $R_{1|HL}$, $R_2 \geq 0$, $R_2 < 2H$, $R_{1|L}$, $R_{1|\cdot}$, $R_{1|H\cdot}, R_{1|HL} \leq H + L$, $m_{0,L} \leq 1$, and there are sufficient revenues to repay the debt cost.

Using $m_0 = 1$ and $m_1 = m_{1,H} = 0$ in the incentive constraints (22) and (23), we get $R_{1|\cdot} \leq 2H$ and $R_2 \leq R_{1|\cdot}$.Because $R_{1|\cdot} \leq H + L < 2H$, constraint (22) is always slack and can be ignored. Moreover, because of monotonicity of repayments, from constraint (23) we deduce that $R_2 = R_{1|\cdot}$. Last, because $m_1 = 0$, $R_{1|L}, R_{1|HL}$ and $R_{1|H\cdot}$ are never paid and can be set to any value between 0 and $H + L$.

Using $m_0 = 1$ and $m_1 = m_{1,H} = 0$ in the solved out values of $m_{0,L}$ and R_2 and using $R_2 = R_{1|\cdot}$, we get $m_{0,L} = \frac{4(I-L)-p(2-p)(H-L)+2(1-p)^2 c}{p(2-p)(H-L)-2(1-p)^2 c}$, $R_{1|\cdot} = \frac{2(I-L)(H-L)}{p(2-p)(H-L)-2(1-p)^2 c} + 2L$, as reported in points 1 and 2 of the proposition. We next verify that $m_{0,L} \leq 1$ and $R_{1|\cdot} \leq H+L$. For these we need $p\,(2-p)\,(H-L) - 2\,(I-L) - 2\,(1-p)^2\, c \geq 0$, .which always holds under Condition 2.

Substituting out $R_2 = R_{1|\cdot}$ derived above in the objective function (7) we get the expected profits (14) as reported in point 4 of the proposition.

5. If equating R_2 and $R_{1|\cdot}$ (so allowing $m_1 = m_{1,H} = 0$) and setting $R_{1|\cdot} = H + L$ and $m_{0,L} = m_0 = 1$ fails to raise the revenue to meet the participation constraint (i.e., $p\,(2-p)\,(H-L) - 2\,(I-L) - 2\,(1-p)^2\, c < 0$), then extra revenue must be raised from a two successes outcome, which in turn requires $m_1, m_{1,H} > 0$ and $R_2 > R_{1|\cdot} = H+L$. The problem is to choose $R_2, R_{1|L}, R_{1|HL}, R_{1|H\cdot}$ and the minimal $m_1, m_{1,H}$ which allows the participation constraint to be satisfied. This will minimise the deadweight loss of audit whilst meeting the participation constraint. Setting $m_{0,L} = m_0 = 1$ and $R_{1|\cdot} = H + L$ and allowing for $R_2 > R_{1|\cdot}$ and $m_1, m_{1,H} > 0$ in problem $\mathcal{P}^{\mathcal{J}'}$, the contract problem becomes:

$$\max p^2\,(2H - R_2) + 2p\,(1-p)\, m_1 \left\{H + L - sR_{1|L} - \right.$$
$$\left. + \,(1-s)\left(m_{1,H} R_{1|HL} + (1 - m_{1,H})\, R_{1|H\cdot}\right)\right\}$$

$$\text{st } p^2 R_2 + 2\,(1-p)^2\,(L-c) + 2p\,(1-p)\,\{(1-m_1)\,(H+L) + \tag{25}$$
$$+ \, m_1\left[sR_{1|L} + (1-s)\left(m_{1,H}\left(R_{1|HL} - c\right) + (1-m_{1,H})\, R_{1|H\cdot}\right) - c\right]\} \geq 2I$$

$$R_2 \leq 2H \tag{26}$$

$$R_2 \leq (1-m_1)\,(H+L) + m_1\left[2m_{1,H} H + (1-m_{1,H})\, R_{1|H\cdot}\right] \tag{27}$$

$$m_1\left[sR_{1|L} + (1-s)\left(m_{1,H} R_{1|HL} + (1-m_{1,H})\, R_{1|H\cdot}\right)\right] +$$
$$+ \,(1-m_1)\,(H+L) \leq H + L. \tag{28}$$

6. $R_{1|H\cdot} = R_{1|HL} = R_{1|L} = H + L$.

If $R_{1|H\cdot} < H + L$ and $R_2 > H + L$ we can reduce R_2 and raise $R_{1|H\cdot}$ so that $p^2 R_2 +$

$2p(1-p)(1-m_{1,H})(1-s)R_{1|H\cdot}$ stays constant, i.e., both the objective function and the participation constraint are unchanged. This slackens (26), whilst not violating (28), which is satisfied when $R_{1|H\cdot}$ is evaluated at its highest value, $H+L$. Thus, $R_{1|H\cdot}=H+L$. For a similar argument, $R_{1|L}, R_{1|HL}$ can be increased up to $H+L$, whilst reducing R_2 in a way to keep both the objective function and the participation constraint (25) unchanged. This does not violate (28), which is still satisfied when $R_{1|L}, R_{1|HL}$ are evaluated at their highest value, $H+L$. In each case, the relaxation of the incentive constraints, especially (27), allows a reduction in m_1. Thus, $R_{1|H\cdot}=R_{1|HL}=R_{1|L}=H+L$.

Notice that if constraint (27) is satisfied, then certainly constraint (26) is. So we can ignore (26). Moreover, using, $R_{1|H\cdot}=R_{1|HL}=R_{1|L}=H+L$, constraint (28) is satisfied (it becomes: $H+L\leq H+L$) and can be dropped.

The contract problem can then be written as:

$$\max p^2(2H-R_2)$$

$$\text{st } p^2R_2+2p(1-p)\{(H+L)-m_1[1+(1-s)m_{1,H}]c\} \qquad (29)$$
$$+2(1-p)^2(L-c)=2I$$

$$R_2\leq(H+L)+m_1m_{1,H}(H-L) \qquad (30)$$

7. $m_{1,H}=1$

The monitoring probabilities $m_1, m_{1,H}$ do not enter the objective function, but only the participation and the incentive constraint (29 and 30). We know that both $m_1, m_{1,H}$ must be positive. An increase in either m_1 or $m_{1,H}$ slackens the incentive constraint, but increases the expected audit cost in the participation constraint. However, such increase is lower when $m_{1,H}$ is increased rather than m_1, as can be seen by differentating (29) wrt m_1 and $m_{1,H}$:

$$\frac{\partial PC}{\partial m_1}=-1-(1-s)m_{1,H},$$
$$\frac{\partial PC}{\partial m_{1,H}}=-(1-s)m_1.$$

Thus, it is optimal to increase $m_{1,H}$ to the maximum, $m_{1,H}=1$.

To determine the remaining variables notice that (30) must bind as otherwise m_1 could be reduced, allowing a reduction in R_2 without violating (29). Solving (30) for m_1 gives $m_1=\frac{R_2-H-L}{H-L}$, which, substituted out in the participation constraint (29), gives $R_2(s)$ and $m_1(s)$ as reported in points 7 and 8 of the proposition. We next verify that $m_1(s)\leq 1$ and $R_2(s)\leq 2H$. For these, we need $pH+(1-p)L\geq I+(1-p)c[1+p(1-s)]$, which always holds under Condition 3.

Last, substituting out R_2 in the objective function we get the expected profits (17) as reported in point 9 of the proposition. They are strictly lower than obtainable when coinsurance effects prevail (14), as can be seen by comparing (17) with (14). To show that they are increasing in s, we differentiate (17) with respect to s :

$$\frac{\partial EP(s)}{\partial s} = \frac{p[p(H-L) - 2(1-p)c] - 2[pH + (1-p)L - I - (1-p)c]}{[p(H-L) - (2-s)2(1-p)c]^2}.$$

The sign of the derivative depends on the sign of the term in the numerator, which coincides with the numerator of $m_1(s)$ (15). Since this is positive, the sign of the derivative is positive.

■

Proof of Proposition 3 This follows from using the results from Proposition 2 that $m_{0,L} \leq 1$ (13) and $m_1(s) \leq 1$ (15). ■

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