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Information Design with Frame Choice

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Abstract
We study how framing interplays with information design. Whereas Sender conceives all contingencies separately, Receiver cannot initially distinguish among some of them, i.e., has a coarse frame. To influence Receiver's behavior, Sender first decides whether to refine Receiver's frame and then designs an information structure for the chosen frame. Sender faces a trade-off between keeping Receiver under the coarse frame — thus concealing part of the information structure — and reframing — hence inducing Receiver to revise preferences and prior beliefs after telling apart initially indistinguishable contingencies. Sender benefits from re-framing if this enhances persuasion possibilities or makes persuasion unnecessary. Compared to classical information design, Receiver's frame becomes more critical than preferences and prior beliefs in shaping the optimal information structure. Although a coarse worldview may open the doors to Receiver's exploitation, re-framing can harm Receiver in practice, thus questioning the scope of disclosure policies.

JEL Classification: D1; D8; D9; G2; G4; M3.

Keywords: Framing; Information Design; Disclosure Policies.

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1 Introduction

In communication, the meaning of words and facts depends on their interpretation. If agents disagree on what they are talking about because they frame the world differently, they misinterpret the information provided to them. The framing of payoff-relevant contingencies and information is widespread and has been recognized to have critical implications for decision-making since the seminal work of Thaler (1980); Tversky and Kahneman (1981); Hershey, Kunreuther, and Schoemaker (1982); Kahneman and Tversky (1984); Schkade and Kahneman (1998). Motivated by this evidence, we study the interplay of framing with information design to understand when and how information providers can take advantage of their finer worldview, which contingencies remain hidden in communication, and whether preventing the exploitation of coarse worldviews calls for policy interventions. Exploring these questions has implications for financial advice, product labeling, rating and test design, and consumer protection.

Formally, we allow for the asymmetric framing of payoff-relevant states in the classical information design model (Kamenica and Gentzkow, 2011; Bergemann and Morris, 2019). Whereas Sender conceives each element of the state space separately, Receiver has a coarse frame, conceiving two states as a single state that is a combination of the two actual states. Before designing information to manipulate Receiver's beliefs and behavior, Sender decides whether to refine Receiver's coarse frame so that the latter conceives all states (i.e., adopts a fine frame). If so, Receiver reacts to re-framing by forming new preferences and prior beliefs. To discipline the relation between Receiver's beliefs and preferences before and after re-framing, we assume that these satisfy sub-additivity (Tversky and Koehler, 1994) and the partition-dependent expected payoff model (Ahn and Ergin, 2010). Akin to canonical models of information design, Sender commits to a conditional probability distribution over signals for each state. Under the coarse frame, however, Receiver conceives the conditional probability distributions under indistinguishable states as a single distribution (in the spirit of Mullainathan, Schwartzstein, and Schleifer, 2008). We explore several new questions that arise in such a setting. How does Receiver's coarser worldview influence information design? Does Sender have an incentive to refine Receiver's frame? How do Sender's choices about Receiver's frame and information design affect Receiver's welfare? Does mandatory disclosure (i.e., re-framing) benefit Receiver?

By answering these questions, we uncover three main insights. First, we show that how Receiver frames the state space becomes crucial in shaping the optimal information structure. As in standard information design problems, under the fine frame, the Sender-optimal information structure depends on Receiver's preferences and prior beliefs. In contrast, under the coarse frame, Sender exploits Receiver's coarse conception of the state space to conceal part of the information structure and increase the likelihood of persuasion, i.e., of Receiver taking Sender's preferred action. Second, we characterize how

Sender solves the trade-off between keeping Receiver in the dark, thus concealing part of the information structure, and re-framing, hence inducing Receiver to revise preferences and beliefs after telling apart initially indistinguishable states. Sender finds re-framing optimal if this makes persuasion unnecessary or more likely. In turn, re-framing can affect persuasion possibilities through (the combination of) three channels: directly, through a change in preferences and prior beliefs, and indirectly, by shaping the optimal information structure. Third, we show that re-framing can harm Receiver in practice, thus questioning the scope of disclosure policies. In particular, upon re-framing, Receiver can adopt preferences and prior beliefs that increase the likelihood of persuasion more than what the concealment of the information structure does under the coarse frame. Moreover, Receiver may be better off under the coarse frame from a subjective viewpoint if, under such a frame, Sender finds it optimal to confirm Receiver's default action choice.

Road Map. The following two subsections provide an overview of our model and results with an example and discuss the related literature. Section 2 introduces the general model. Section 3 characterizes the optimal information structures and compares them across the coarse and fine frames. Section 4 characterizes Sender's optimal frame choice. Section 5 discusses welfare and the scope of disclosure policies. Section 6 discusses some natural extensions of the main model. Section 7 concludes.

1.1 Example: Financial Advice

Consider financial advice on bank loans for house purchases. The persuasion of households with a coarse understanding of loans' functioning may help to explain seemingly irrational behavior, thus connecting our work to other behavioral explanations of this phenomenon.[1] Suppose a household (Receiver) must choose between a fixed- or a floating-rate loan; a bank advisor (Sender) provides the household with recommendations. The advisor has a fine frame: he understands that interest rates can stay low, mildly increase because of muted economic conditions (e.g., faster growth), or sharply increase because of exogenous shocks (e.g., an inflation crisis). Should both agents understand states according to the fine frame, their common prior belief about future interest rates, denoted by μ, is

$$\mu(\text{Low Rate}) = \frac{3}{10}, \qquad \mu(\text{Mild Increase}) = \frac{13}{20}, \qquad \mu(\text{Sharp Increase}) = \frac{1}{20}.$$

The household initially has a coarse frame: in her mind, rates can only stay low or be-

[1] Starting with Campbell and Cocco (2003), the financial literature has explored demand and supply explanations behind loan choices (see Albertazzi, Fringuellotti, and Ongena, 2024, for a review). Among behavioral explanations, coarse framing may relate to the effect of financial literacy (Agarwal, Amromin, Ben-David, Chomsisengphet, and Evanoff, 2010; Fornero, Monticone, and Trucchi, 2011; Gathergood and Weber, 2017). Moreover, households are myopic and consider future interest rates only up to a finite horizon (Koijen, Van Hemert, and Van Nieuwerburgh, 2009; Badarinza, Campbell, and Ramadorai, 2018; Foà, Gambacorta, Guiso, and Mistrulli, 2019); during a period of stability, this is equivalent to assuming that households neglect or underweight the possibility of a sharp increase in interest rates, fitting our setting.

come high. The advisor is aware of his superior understanding (e.g., because of previous interactions with other households). To discipline the relation between the household's beliefs across frames, we assume that such beliefs satisfy sub-additivity: if the advisor refines the household's frame—by disclosing contextual information, i.e., explaining the difference between a mild and a sharp increase in interest rates—the sum of the probabilities the household assigns to these states is larger than the probability she assigned to the composite state. A parameter $\lambda \in [0,1]$ captures how the household subconsciously performs this operation, with λ and $1-\lambda$ capturing the prominence of each of the two states in the composite state. Formally, the household's prior beliefs under the coarse frame (denoted by μ_R) and the fine frame are related as follows: $\mu_R(\text{High Rate}) = \lambda\mu(\text{Mild Increase}) + (1-\lambda)\mu(\text{Sharp Increase})$. Suppose $\lambda = 1$, meaning that the household identifies a high rate with a mild increase; according to the numerical specification above, we have

$$\mu_R(\text{Low Rate}) = \frac{7}{20}, \qquad \mu_R(\text{High Rate}) = \frac{13}{20}.$$

We report preferences under the fine and coarse frames in Tables 1 and 2. Each column corresponds to a state; each row corresponds to a household's action. In each cell, the first entry is the household's payoff, and the second is the advisor's payoff. The advisor always prefers the floating-rate loan. Instead, the household prefers the floating-rate loan if the interest rates stay low, whereas she prefers the fixed-rate loan if the interest rates increase. Building on the partition-dependent expected payoff model, we assume that the household's payoff from each action depends on her frame. Consistently with her prior beliefs, the household's payoffs under the coarse frame (denoted by u_R) and the fine frame (denoted by u) are related as follows: $u_R(a, \text{High Rate}) = \lambda u(a, \text{Mild Increase}) + (1-\lambda)u(a, \text{Sharp Increase})$ for all actions a, where $\lambda = 1$ in this example.

Table 1: Preferences under the Fine Frame.

	Low Rate	Mild Increase	Sharp Increase
Floating Rate	1,1	-1,1	-10,1
Fixed Rate	0,0	0,0	0,0

Table 2: Preferences under the Coarse Frame with $\lambda = 1$.

	Low Rate	High Rate
Floating Rate	1,1	-1,1
Fixed Rate	0,0	0,0

Given her preferences and prior beliefs, in the absence of the advisor's recommendation, the household takes the fixed-rate loan under both frames. Thus, persuasion is necessary under both frames: the advisor must design information to manipulate the household's posterior beliefs and induce the choice of a floating-rate loan with positive probability. The optimal information structure π—a family of conditional probability distributions, one for each state, over loan recommendations—depends on the frame.

When the household has a fine frame, the optimal information structure directly follows from the standard arguments in Kamenica and Gentzkow (2011):

$$\pi(z_1 \,|\, \text{Low Rate}) = 1, \qquad \pi(z_1 \,|\, \text{Mild Increase}) = \frac{6}{13}, \qquad \pi(z_1 \,|\, \text{Sharp Increase}) = 0,$$

where signal z_1 corresponds to a recommendation to take the floating-rate loan that the household finds optimal to obey. Under this information structure, the household takes a floating-rate loan with probability $\frac{3}{5}$.

When the household has a coarse frame, the advisor can exploit his finer understanding to increase the probability of signal z_1 (i.e., of inducing the household to take the floating-rate loan). In this case, the optimal information structure is

$$\pi(z_1 \,|\, \text{Low Rate}) = 1, \qquad \pi(z_1 \,|\, \text{Mild Increase}) = \frac{7}{13}, \qquad \pi(z_1 \,|\, \text{Sharp Increase}) = 1,$$

inducing the household to take a floating-rate loan with probability $\frac{7}{10} > \frac{3}{5}$. Crucially, the household's interpretation of information depends on her inability to distinguish a mild from a sharp increase in interest rates. Consistently with her prior beliefs and payoffs, the household conception of the information structure, denoted by π_R, satisfies $\pi_R(z_1 \,|\, \text{High Rate}) = \lambda\pi(z_1 \,|\, \text{Mild Increase}) + (1-\lambda)\pi(z_1 \,|\, \text{Sharp Increase})$. Thus, the advisor can conceal part of the mapping from states to recommendations. For $\lambda = 1$, the household conceives the information structure as

$$\pi_R(z_1 \,|\, \text{Low Rate}) = 1, \qquad \pi_R(z_1 \,|\, \text{High Rate}) = \frac{7}{13}.$$

Next, we investigate the advisors' incentives to refine the household's frame and the implications of the advisor's choices on the household's welfare. The analysis sheds light on whether disclosure of contextual information, i.e., re-framing, prevents the household's exploitation. We consider two welfare perspectives. Under the objective criterion, we evaluate the household's welfare under both frames using her preferences and beliefs under the fine frame. Under the subjective criterion, we evaluate the household's welfare using her preferences and beliefs under the frame she has.

In the example above, the advisor is better off when the household has a coarse frame; hence, the advisor decides to keep the household under the coarse frame. The household, however, would be better off under the fine frame from the viewpoint of objective welfare. The reason is that, under the coarse frame, the household takes a suboptimal loan with a higher probability. In this case, mandatory re-framing would improve the household's welfare. We next show that this seemingly obvious conclusion is not generally true. Although a coarse worldview may open the doors to the household's exploitation, re-framing can harm her in practice, thus questioning the scope of disclosure policies. This insight is valid regardless of the adopted welfare perspective.

The advisor's choices—hence, the household's welfare—depend on how the latter conceives states and their likelihood under the coarse frame, as captured by the parameter λ. Consider the same setup as before but now with $\lambda = \frac{3}{4}$. This change affects only the analysis under the coarse frame, where the household's prior belief becomes

$$\mu_R(\text{Low Rate}) = \frac{1}{2} = \mu_R(\text{High Rate}).$$

We report preferences under the coarse frame in Table 3.

Table 3: Preferences under the Coarse Frame with $\lambda = \frac{3}{4}$.

	Low Rate	High Rate
Floating Rate	1,1	$-\frac{13}{4}$,1
Fixed Rate	0,0	0,0

When $\lambda = \frac{3}{4}$, the household (subconsciously) assigns positive weight to the possibility of a sharp increase in interest rates in her coarse conception. Compared to the case in which $\lambda = 1$, Receiver's prior belief of high future interest rates decreases from $\frac{13}{20}$ to $\frac{1}{2}$ by subadditivity. This change better aligns Receiver's prior beliefs with the advisor's goal. At the same time, Receiver's conceived payoff from taking a floating-rate loan in case of high interest rates decreases from -1 to $-\frac{13}{4}$. Hence, since a floating-rate loan now looks less appealing to the households, this change makes Receiver's preferences less aligned with the advisor's goal. Under the corresponding optimal information structure, which now is

$$\pi(z_1 \,|\, \text{Low Rate}) = 1, \qquad \pi(z_1 \,|\, \text{Mild Increase}) = \frac{16}{39}, \qquad \pi(z_1 \,|\, \text{Sharp Increase}) = 0,$$

the household takes a floating-rate loan with probability $\frac{17}{30}$, which is lower than $\frac{3}{5}$, the probability with which she does so under the fine frame. Hence, since the preference effect dominates that of prior beliefs, the advisor finds re-framing optimal. The household, however, is now better off under the coarse frame from the viewpoint of objective welfare. The optimal information structure under the fine frame induces the household to mistakenly take the floating-rate loan in the event of a mild increase in the interest rates with a higher probability: $\frac{6}{13} > \frac{16}{39}$. Hence, in this case, re-framing hurts the household.

Consider next the viewpoint of subjective welfare. Under this criterion, the household is indifferent between frames whenever persuasion is necessary under both frames. Thus, disclosure policies play no role in this case. Instead, if persuasion is unnecessary under one frame but necessary under the other, the advisor and the household are better off under the former frame. To see this, suppose $\lambda = 0$, so that the household's prior belief under the coarse frame becomes

$$\mu_R(\text{Low Rate}) = \frac{19}{20}, \qquad \mu_R(\text{High Rate}) = \frac{1}{20}.$$

We report preferences under the coarse frame in Table 4.

Table 4: Preferences under the Coarse Frame with $\lambda = 0$.

	Low Rate	High Rate
Floating Rate	1,1	-10,1
Fixed Rate	0,0	0,0

In this case, persuasion is necessary only under the fine frame. The advisor is better off and keeps the household under the coarse frame, under which she takes the floating-rate loan by default (and so with probability 1). From a subjective viewpoint, the household's expected payoff equals $\frac{9}{20}$ under the coarse frame. In contrast, under the fine frame, persuasion is necessary, and the household's expected payoff at the optimal information structure is 0. Thus, from a subjective viewpoint, ignorance is bliss: the household prefers a coarse worldview in which the advisor confirms her default action to a finer worldview in which the advisor provides information to steer her behavior. Hence, mandatory re-framing would hurt the household.

1.2 Related Literature

We contribute to the literature on Bayesian persuasion and information design (see, e.g., Kamenica, 2019; Bergemann and Morris, 2019, for recent surveys). In particular, our work complements recent research on information design with behavioral agents, which explores complementary channels that can affect persuasion, such as psychological concerns (Lipnowski and Mathevet, 2018) present bias (Mariotti, Schweizer, Szech, and von Wangenheim, 2023), non-Bayesian updating (de Clippel and Zhang, 2022) correlation neglect (Levy, Barreda, and Razin, 2022), and ambiguity aversion (e.g., Beauchêne, Li, and Li, 2019; Hedlund, Kauffeldt, and Lammert, 2021; Liu and Yannelis, 2021; Cheng, Klibanoff, Mukerji, and Renou, 2024). To our knowledge, we are the first to consider the interplay between framing and information design.

Closest to our paper is Galperti (2019), where agents' prior beliefs have different supports: whereas Sender has interior prior beliefs, Receiver deems some states impossible. Galperti (2019) characterizes when Sender finds it optimal to change Receiver's worldview by providing evidence in the form of a signal. Our study differs in three aspects. First, Receiver cannot initially distinguish between some states in our model, instead of assigning zero probability to them as in Galperti (2019). This feature allows Sender to conceal part of the information structure from Receiver. Second, in contrast to Galperti (2019), Receiver has frame-dependent preferences in our setting, as in Ahn and Ergin (2010). This feature creates an additional channel for Sender to influence Receiver's behavior. Third, Receiver reacts differently to unexpected information. In Galperti (2019), Receiver adopts an arbitrary interior prior after changing worldview; instead, we follow Tversky and Koehler (1994) and assume that Receiver is subject to sub-additivity in forming new beliefs upon re-framing. These differences make our findings not directly comparable.

Our paper also relates to the literature on strategic interactions under framing effects (see, e.g., Piccione and Spiegler, 2012; Spiegler, 2014; Salant and Siegel, 2018; Ostrizek and Shishkin, 2023; Burkovskaya and Li, 2024). In this context, the closest paper to ours is Mullainathan et al. (2008), which studies the effect of coarse thinking on communication. In their framework, Receiver simplifies complex information into broad mental categories, distorting beliefs and decision-making. Their approach to information processing resembles Receiver's coarse understanding of the information structure in our model. However, Mullainathan et al. (2008) differs significantly from our setting. First, Receiver fully conceives the state space but coarsely understands observable situations associated with different information-generating processes. Second, Sender neither commits to an information structure nor fully controls it. Third, Receiver actively chooses the coarse categorization to fit the information she receives. In Mullainathan et al. (2008), Sender anticipates and exploits Receiver's response in terms of coarse categorization. In contrast, in our model, Sender actively chooses Receiver's fame and then designs an information structure for the selected frame. Because of these differences, our results do not directly compare.

The existing literature shows that mandatory disclosure policies can improve welfare in some markets—e.g., housing (Myers, Puller, and West, 2022) and environmental regulation (Cohen and Santhakumar, 2007)—by reducing information asymmetry and incentivizing desirable behaviors. However, the effectiveness of these policies varies with market context, cognitive barriers, and unintended consequences. For instance, mandatory financial disclosure improves the efficiency of financial markets but may crowd out private information production, reducing firm learning and investment incentives (Goldstein and Yang, 2017; Jayaraman and Wu, 2019). In the context of financial advice, Inderst and Ottaviani (2012) shows that mandatory disclosure is likely beneficial when consumers of the advice are naive about the conflict of interest between them and the financial advisor. In our model, an opposite insight emerges: mandatory disclosure can actually be harmful even if consumers misperceive the conflict of interest. Our analysis provides insight into disclosure policies when the less informed agent is boundedly rational (see Loewenstein, Sunstein, and Golman, 2014, for an overview). Recent financial studies show that disclosure might have a limited effect due to inattention, which can arise from pessimistic beliefs (Adams, Hunt, Palmer, and Zaliauskas, 2021) or limited attention (Hillenbrand and Schmelzer, 2017). We show that forced disclosure can harm both agents, even if the less informed one becomes fully rational after disclosure.

2 Model

Primitives. There are two agents: Sender (he) and Receiver (she). Their payoffs depend on the state $\omega \in \Omega := \{\omega_1, \omega_2, \omega_3\}$ and Receiver's action $a \in A := \{a_1, a_2\}$. Agents are initially uncertain about the state and share a common prior belief $\mu \in \Delta_{++}(\Omega)$.

Sender has state-independent preferences and prefers action a_1 to action a_2. His preferences are represented by the payoff function $v\colon A \times \Omega \to \mathbb{R}$, where

$$v(a,\omega) := \begin{cases} 1 & \text{if } a = a_1 \\ 0 & \text{if } a = a_2 \end{cases}.$$

Receiver has state-dependent preferences, represented by the payoff function $u\colon A \times \Omega \to \mathbb{R}$, where

$$u(a,\omega) := \begin{cases} 1 & \text{if } a = a_1 \text{ and } \omega = \omega_1 \\ -1 & \text{if } a = a_1 \text{ and } \omega = \omega_2 \\ \gamma & \text{if } a = a_1 \text{ and } \omega = \omega_3 \\ 0 & \text{if } a = a_2 \end{cases},$$

and $\gamma \neq 0$. We normalize Receiver's payoff from action a_2 to 0. Receiver prefers action a_1 to action a_2 in state ω_1, and action a_2 to action a_1 in state ω_2. If $\gamma > 0$ (resp., $\gamma < 0$), Receiver prefers action a_1 (resp., a_2) to action a_2 (resp., a_1) in state ω_3.

Sender conceives each state in Ω separately. At the beginning of the game, Receiver cannot distinguish state ω_2 from state ω_3: she conceives the state space Ω as $\Omega_R := \{\omega_1, \omega_{23}\}$, where we write $\omega = \omega_{23}$ if and only if $\omega \in \{\omega_2, \omega_3\}$. We refer to Ω_R and Ω as Receiver's *coarse* and *fine frames*, respectively, and to ω_{23} as the *coarse state*.

Under the coarse frame, Receiver's preferences are represented by the payoff function $u_R\colon A \times \Omega_R \to \mathbb{R}$, where

$$u_R(a,\omega) := \begin{cases} 1 & \text{if } a = a_1 \wedge \omega = \omega_1 \\ \alpha_{23} & \text{if } a = a_1 \wedge \omega = \omega_{23} \\ 0 & \text{if } a = a_2 \end{cases},$$

and, for some $\lambda \in [0,1]$,

$$\alpha_{23} := -\lambda + (1-\lambda)\gamma. \tag{1}$$

Receiver's conceived payoff from action a_1 in the coarse state ω_{23}, denoted by α_{23}, is a convex combination of her payoffs in states ω_2 and ω_3. The parameter λ disciplines how prominent, albeit subconsciously, state ω_2 is relative to state ω_3 in Receiver's coarse conception. Receiver prefers action a_1 to action a_2 in state ω_{23} if and only if $(1-\lambda)\gamma \geq \lambda$. Hence, Receiver's favorite action in state ω_{23} depends on her preferences under the fine frame—i.e., the parameter γ—and her coarse conception—i.e., the parameter λ.

Receiver's prior beliefs under the coarse frame, $\mu_R \in \Delta_{++}(\Omega_R)$, are sub-additive, and

$$\mu_R(\omega_{23}) := \lambda\mu(\omega_2) + (1-\lambda)\mu(\omega_3). \tag{2}$$

The parameter λ in condition (2) tailors Receiver's conception of prior beliefs under the coarse frame to that of the payoff function. Condition (2) implies $\mu_R(\omega_1) > \mu(\omega_1)$: Receiver conceives state ω_1 as more likely under the coarse frame than under the fine frame.

Sender knows everything about the game, including Receiver's initial coarse frame Ω_R, and her preferences and prior beliefs under both frames. Receiver only knows the part of the game within her frame and is initially unaware of Sender's fine frame. Otherwise, the game is common knowledge within Receiver's frame.

Frame Choice and Information Design. Sender can refine Receiver's frame and design the information structure to steer her behavior. The interaction proceeds as follows:

1. Sender decides whether to refine Receiver's frame by explicitly describing states ω_2 and ω_3 to her. Let $\Omega' \in \{\Omega_R, \Omega\}$, $u' \in \{u_R, u\}$, and $\mu' \in \{\mu_R, \mu\}$ be Receiver's frame, payoff function, and prior belief after Sender's decision, respectively.

2. Sender provides evidence about state ω by committing to an *information structure* $\pi := \big(Z, \{\pi(\cdot \,|\, \omega)\}_{\omega \in \Omega}\big)$, where $\{\pi(\cdot \,|\, \omega)\}_{\omega \in \Omega}$ is a family of probability distributions on the finite set of signals Z. Receiver's conception of π, denoted by $\pi' \in \{\pi_R, \pi\}$, depends on her frame Ω'. If $\Omega' = \Omega$, then $\pi' = \pi$. If $\Omega' = \Omega_R$, then Receiver conceives the signal distribution under the coarse state ω_{23} as a convex combination of the signal distributions under ω_2 and ω_3. In particular, we have $\pi' = \pi_R$, where

$$\pi_R(z \,|\, \omega_1) := \pi(z \,|\, \omega_1) \quad \text{and} \quad \pi_R(z \,|\, \omega_{23}) := \lambda \pi(z \,|\, \omega_2) + (1-\lambda)\pi(z \,|\, \omega_3). \tag{3}$$

   Again, the parameter λ in condition (3) tailors Receiver's conception of the signal distribution under the coarse frame to that of the payoff function and prior beliefs.

3. A signal z from the information structure π is publicly realized.

4. Receiver takes an action $a \in A$, and payoffs are realized.

Within their frames, the agents are Bayesian and maximize their (subjective) expected payoff.

Let $p'(\cdot \,|\, z) \in \Delta(\Omega')$ denote Receiver's posterior belief induced by signal z under frame Ω'. We write $p'(\cdot \,|\, z) = p(\cdot \,|\, z)$ if $\Omega = \Omega$ and $p'(\cdot \,|\, z) = p_R(\cdot \,|\, z)$ if $\Omega' = \Omega_R$. By Bayes rule,

$$p'(\omega \,|\, z) = \frac{\pi'(z \,|\, \omega)\mu'(\omega)}{\sum_{\tilde{\omega} \in \Omega'} \pi'(z \,|\, \tilde{\omega})\mu'(\tilde{\omega})} \quad \text{for all } z \in Z \text{ and } \omega \in \Omega'. \tag{4}$$

We denote by $p' \in \Delta(\Omega')$ a typical (prior or posterior) Receiver's belief under frame Ω'. We write $p' = p$ if $\Omega' = \Omega$ and $p' = p_R$ if $\Omega' = \Omega_R$.

Sender's trade-off when deciding whether to refine Receiver's frame relates to how the latter conceives the state space and, hence, preferences, beliefs, and information structure. On the one hand, re-framing induces Receiver to revise preferences and beliefs after telling apart states she could not distinguish. Depending on primitives, this may make Receiver more inclined to take Sender's preferred action a_1. On the other hand, under the coarse frame, Sender can conceal part of the information structure from Receiver and focus on the provision of evidence (i.e., signals) in favor of action a_1 in one of the two states Receiver cannot distinguish, ω_2 and ω_3. This freedom is lost under the fine frame because Receiver can tell the evidence produced in state ω_2 apart from that produced in state ω_3.

Equilibrium. We focus on the unique perfect Bayesian equilibrium in which: (i) Receiver takes action a_1 if indifferent; (ii) Sender does not refine Receiver's frame if indifferent; (iii) Receiver's action depends only on her posterior belief induced by the signal realization.

Preferences and Beliefs upon Re-Framing. We discipline the relation between Receiver's coarse and fine worldviews by making two related assumptions. First, we assume that Receiver's beliefs satisfy sub-additivity: when Sender refines Receiver's frame by explaining that a state consists of two states, the sum of the probabilities Receiver assigns to these states is larger than the probability she assigns to the original composite state. Tversky and Koehler (1994) propose a theory of subjective probability—called support theory—according to which different descriptions of the same event can give rise to different judgments. This theory accounts for sub-additive beliefs, where individuals conceive the joint probability of two disjoint events to be lower when they are presented together—as under Receiver's coarse frame—as opposed to when they are presented separately—as under Receiver's fine frame. Several studies provide evidence in favor of this theory (see, e.g., Fischhoff, Slovic, and Lichtenstein, 1978; Fox and Clemen, 2005; Sonnemann, Camerer, Fox, and Langer, 2013). We impose a special form of sub-additivity tailored to our model's specifics to guarantee the tractability of its analysis. Our specification maintains all qualitative features of more general formulations.

Second, we relate Receiver's preferences and beliefs before and after re-framing by assuming that they satisfy the partition-dependent expected payoff (PDEP) model by Ahn and Ergin (2010). Drawing upon Tversky and Koehler (1994), Ahn and Ergin (2010) propose an axiomatic approach to introduce the PDEP model in which agents' preferences and beliefs depend on their frame as summarized by a partition of the state space. Their work provides a decision-theoretic foundation for support theory. We rely on Ahn and Ergin (2010)'s framework as a foundation for Receiver's preferences and beliefs in our model. We extensively discuss such a foundation in the context of our model in Appendix A.

Discussion of Other Modeling Choices. We discuss in Section 6 why we focus on three states and alternative frames of the state space. The interested reader can read the discussion in Section 6 before our main results, which we present in Sections 3–5.

3 Information Design

In this section, we characterize Sender's optimal information structure under the fine and coarse frames, providing insights into how Sender exploits Receiver's coarse conception when designing information.

3.1 Preliminary Observations

Receiver's Optimal Action. Receiver's optimal action $a\colon \Delta(\Omega') \to A$ satisfies $a(p') \in \arg\max_{a\in A} \sum_{\omega\in\Omega'} u'(a,\omega)p'(\omega)$, and is equal to a_1 if the maximizer is not unique. The following lemma, whose proof is in Appendix B.1, characterizes how Receiver's optimal action depends on her frame, preferences, and beliefs.

Lemma 1. *Receiver's optimal action $a\colon \Delta(\Omega') \to A$ satisfies:*

1. *If $\Omega' = \Omega$, then $a(p) = a_1$ if and only if $p(\omega_1) + \gamma p(\omega_3) \geq p(\omega_2)$.*
2. *If $\Omega' = \Omega_R$, then $a(p_R) = a_1$ if and only if $p_R(\omega_1) \geq -\alpha_{23} p_R(\omega_{23})$.*

Lemma 1 is intuitive: Receiver takes Sender's preferred action a_1 if and only if her belief that the state is one in which she also prefers action a_1 is sufficiently large.

Next, we introduce the notion of persuasion necessity under a given frame.

Definition 1. Persuasion is necessary *under frame Ω' if Receiver's optimal action at prior belief μ' differs from Sender's preferred action, i.e., $a(\mu') \neq a_1$, and* unnecessary *otherwise.*

By Lemma 1, persuasion is necessary under frame Ω if and only if

$$\mu(\omega_1) + \gamma\mu(\omega_3) < \mu(\omega_2), \tag{5}$$

and under frame Ω_R if and only if

$$\mu_R(\omega_1) < -\alpha_{23}\mu_R(\omega_{23}). \tag{6}$$

Condition (6) requires that $\alpha_{23} < 0$, i.e., Receiver prefers action a_2 in the coarse state ω_{23}.

Basic Properties of Optimal Information Structures. Three standard properties routinely hold in this class of models (see Kamenica and Gentzkow, 2011, for the details).

1. Bayes Plausibility. By designing an information structure π, Sender can induce any posterior beliefs such that Receiver's expected posterior belief equals her prior belief:

$$\sum_{z\in Z} p'(\cdot \mid z)\left(\sum_{\omega\in\Omega'} \pi(z \mid \omega)\mu'(\omega)\right) = \mu'(\cdot). \tag{7}$$

2. Straightforward Information Structures. There is a straightforward information structure, i.e., with only two signals, $Z = \{z_1, z_2\}$, which is outcome-equivalent to any optimal information structure. Let signal z_1 (resp., z_2) correspond to Sender's recommendation to take action a_1 (resp., a_2). Hence, z_1 corresponds to evidence in favor of Sender's goal.

We focus on straightforward information structures under which Receiver finds it optimal to obey Sender's recommendation. Sender's expected payoff under any such information structure π, denoted by $\mathbb{E}_\pi[v(a,\omega)]$, equals the probability of signal z_1 under π:

$$\mathbb{E}_\pi[v(a,\omega)] = \sum_{\omega\in\Omega} \pi(z_1 \,|\, \omega)\mu(\omega). \tag{8}$$

3. Posterior Beliefs. If persuasion is unnecessary under frame Ω', without loss, we assume that Sender leaves Receiver's prior beliefs unchanged. If persuasion is necessary, the posterior beliefs induced by the optimal information structure satisfy the following properties:

(a) Signal z_1 makes Receiver indifferent between actions a_1 and a_2. If $\Omega' = \Omega$, this property implies that

$$2p(\omega_1 \,|\, z_1) + (1+\gamma)p(\omega_3 \,|\, z_1) = 1. \tag{9}$$

If $\Omega' = \Omega_R$, this property implies that

$$p_R(\omega_1 \,|\, z_1) = -\frac{\alpha_{23}}{1-\alpha_{23}}. \tag{10}$$

(b) Signal z_2 makes Receiver certain that the state is one in which action a_2 is uniquely optimal to her. If $\Omega' = \Omega$, this property implies that

$$p(\omega_1 \,|\, z_2) = 0, \qquad \text{and, if } \gamma > 0, \qquad p(\omega_3 \,|\, z_2) = 0. \tag{11}$$

If $\Omega' = \Omega_R$, this property implies that

$$p_R(\omega_1 \,|\, z_2) = 0. \tag{12}$$

3.2 Optimal Information Structure

Assuming persuasion is necessary, we characterize the optimal information structure in each frame. We first introduce the notions of favorable and unfavorable states to Sender.

Definition 2. State ω is favorable to Sender *if Receiver prefers action a_1 in state ω, and* unfavorable *otherwise.*

State ω_1 is favorable to Sender under both frames. By equation (4) and the optimality conditions (11) and (12), the optimal information structure satisfies $\pi(z_1 \,|\, \omega_1) = 1$: Sender recommends Receiver to take Sender's preferred action a_1 with probability 1 in state ω_1.

Characterizing the optimal signal distributions conditional on states ω_2 and ω_3 is more interesting. Since Receiver cannot distinguish ω_2 from ω_3 under the coarse frame, comparing the properties of $\pi(\cdot \,|\, \omega_2)$ and $\pi(\cdot \,|\, \omega_3)$ across frames gives insights into how Sender exploits Receiver's coarse conception when designing information. To help the analysis, we introduce the notions of persuasion focus and possible persuasion.

Definition 3. *Under the optimal information structure, for $k, \ell \in \{2,3\}$ with $k \neq \ell$:*

- Sender focuses persuasion on state ω_k *if he maximizes $\pi(z_1 \,|\, \omega_k)$ subject to Bayes plausibility and the feasibility constraints $\pi(z_1 \,|\, \omega_2) \in [0,1]$ and $\pi(z_1 \,|\, \omega_3) \in [0,1]$.*

- *If Sender focuses persuasion on state ω_k,* persuasion is possible in state ω_ℓ *if* $\pi(z_1 \mid \omega_\ell) > 0$*, and* impossible *otherwise.*

Sender focuses persuasion on state ω_k if he maximizes the likelihood of signal z_1 (hence, action a_1) in state ω_k and leaves producing evidence in favor of his goal (i.e., signal z_1) in state ω_ℓ as a residual task. This task is viable if persuasion is possible in state ω_ℓ, which requires $\pi(z_1 \mid \omega_k) = 1$.

We summarize the optimal information structure under each frame with the following propositions. Their formal statements and proofs are in Appendices B.2 and B.3.

Proposition 1. *Suppose $\Omega' = \Omega$ and persuasion is necessary. Under the optimal information structure, $\pi(z_1 \mid \omega_1) = 1$, and:*

1. *Sender focuses persuasion on state ω_3 if*
$$\gamma \geq -1, \tag{13}$$
and on state ω_2 otherwise.
2. *Given persuasion focus on state ω_3, persuasion is possible in state ω_2 if and only if*
$$\frac{\mu(\omega_1)}{\mu(\omega_3)} \geq -\gamma. \tag{14}$$
3. *Given persuasion focus on state ω_2, persuasion is possible in state ω_3 if and only if*
$$\frac{\mu(\omega_1)}{\mu(\omega_2)} \geq 1. \tag{15}$$

The optimal information structure in Proposition 1 corresponds to the solution of a standard Bayesian persuasion problem. Its interpretation is as follows. First, suppose $\gamma > 0$. If so, only state ω_2 is unfavorable to Sender, and conditions (13) and (14) are satisfied. Hence, Sender focuses persuasion on state ω_3, and persuasion in state ω_2 is possible.

Next, suppose $\gamma < 0$. If so, states ω_2 and ω_3 are unfavorable to Sender. By Bayes plausibility (condition (7)) and the optimality condition (9), there is a persuasion trade-off: increasing the likelihood of recommending action a_1 in state ω_2 (i.e., $\pi(z_1 \mid \omega_2)$) requires decreasing that in state ω_3 (i.e., $\pi(z_1 \mid \omega_3)$), and vice-versa. By condition (13), how Sender resolves this trade-off depends on how Receiver's payoff from action a_1 in state ω_2 compares to her payoff from action a_1 in state ω_3. If the former is larger than the latter (i.e., $-1 > \gamma$), inducing Sender to take action a_1 in state ω_2 is "easier" than doing so in state ω_3; hence, Sender focuses persuasion on state ω_2. Otherwise, Sender focuses persuasion on state ω_3.

For $k, \ell \in \{2, 3\}$ with $k \neq \ell$, by conditions (14) and (15), when Sender focuses persuasion on state ω_k, whether persuasion is possible in state ω_ℓ depends on Receiver's prior beliefs and preferences. Persuasion is possible in state ω_2 if and only if, after being told that the state is not ω_2, Receiver's expected benefit from action a_1, equal to $\mu(\omega_1)/[\mu(\omega_1)+$

$\mu(\omega_3)]$, is larger than Receiver's expected cost from action a_1, equal to $-\gamma\mu(\omega_3)/[\mu(\omega_1)+\mu(\omega_3)]$. Similarly, persuasion is possible in state ω_3 if and only if, after being told that the state is not ω_3, Receiver's expected benefit from action a_1, equal to $\mu(\omega_1)/[\mu(\omega_1)+\mu(\omega_2)]$, is larger than Receiver's expected cost from action a_1, equal to $\mu(\omega_2)/[\mu(\omega_1)+\mu(\omega_2)]$.

Proposition 2. *Suppose $\Omega' = \Omega_R$ and persuasion is necessary. Under the optimal information structure, $\pi(z_1 \,|\, \omega_1) = 1$, and:*

1. *Sender focuses persuasion on state ω_3 if*
$$\lambda\mu(\omega_3) \geq (1-\lambda)\mu(\omega_2), \tag{16}$$
and on state ω_2 otherwise.

2. *Given persuasion focus on state ω_3, persuasion is possible in state ω_2 if and only if*
$$-\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \geq 1-\lambda. \tag{17}$$

3. *Given persuasion focus on state ω_2, persuasion is possible in state ω_3 if and only if*
$$-\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \geq \lambda. \tag{18}$$

Proposition 2 shows how Sender exploits Receiver's inability to distinguish state ω_2 from state ω_3 when designing the optimal information structure under the coarse frame.

By Bayes plausibility (condition (7)) and the optimality condition (10), increasing $\pi(z_1 \,|\, \omega_2)$ requires decreasing $\pi(z_1 \,|\, \omega_3)$, and vice-versa. By condition (16), how Sender resolves this persuasion trade-off depends on his prior beliefs about states ω_2 and ω_3, i.e., $\mu(\omega_2)$ and $\mu(\omega_3)$, and the prominence of state ω_2 in Receiver's conception of state ω_{23}, i.e., the parameter λ. Sender focuses persuasion on state ω_3 (resp., ω_2) if $\mu(\omega_3)$ (resp., $\mu(\omega_2)$) is sufficiently large—since this makes Receiver more likely to take action a_1 from Sender's viewpoint—and λ is sufficiently large (resp., small)—since evidence in favor of action a_1 produced in state ω_3 (resp., ω_2) is "discounted" less than that in state ω_2 (resp., ω_3) in Receiver's conception of the information structure (see condition (3)).

For $k, \ell \in \{2,3\}$ with $k \neq \ell$, by conditions (17) and (18), when Sender focuses persuasion on state ω_k, whether persuasion is possible in state ω_ℓ depends on Receiver's prior beliefs, preferences, and conception of state ω_{23} under the coarse frame. Persuasion is possible in state ω_ℓ if and only if Receiver's conceived expected benefit from action a_1, equal to $\mu_R(\omega_1)$, is sufficiently large relative to Receiver's conceived expected cost from action a_1, equal to $-\alpha_{23}\mu_R(\omega_{23})$. How large such a payoff difference must be depends on how prominent state ω_k is relative to state ω_ℓ in Receiver's conception of state ω_{23}.

Comparing Information Structures across Frames. By conditions (13) and (16), the first difference between the optimal information structures across frames is what drives Sender's persuasion focus. Under the fine frame, persuasion focus depends solely on Receiver's preferences. In contrast, under the coarse frame, persuasion focus depends on Sender's prior beliefs and Receiver's conception of the coarse state ω_{23}. That is, Sender exploits the coarse conception of Receiver, ignoring her preferences and beliefs, which are instead crucial in most standard information design problems.

By conditions (14)-(15) and (17)-(18), given Sender's persuasion focus on state ω_k, whether persuasion is possible in state ω_ℓ (where $k, \ell \in \{2, 3\}$ with $k \neq \ell$) depends on Receiver's preferences and beliefs under both frames. Under the coarse frame, however, Receiver's conception of the coarse state ω_{23} also plays a decisive role.

Summing up, Propositions 1 and 2 highlight that Receiver's framing of the state space can become more critical than her preferences and beliefs in shaping the optimal information structure. First, by affecting persuasion focus, Receiver's frame determines what states Sender pools together (ω_1 and ω_2 as opposed to ω_1 and ω_3) when providing evidence in favor of his goal. As a result, Sender's persuasion focus can differ across frames. Second, for a given persuasion focus, persuasion in the other state may be possible under one frame but impossible under the other. We show in the following sections that these features have rich implications for Sender's choice of Receiver's frame and equilibrium welfare.

4 Frame Choice

In this section, we characterize Sender's optimal frame choice and how this relates to Receiver's conception and the optimal information structures under the two frames.

Value of Re-Framing. Let π_c (resp., π_f) be the optimal information structure when Receiver's frame is Ω_R (resp., Ω). The *value of re-framing* is

$$V := \mathbb{E}_{\pi_f}[v(a, \omega)] - \mathbb{E}_{\pi_c}[v(a, \omega)], \tag{19}$$

where $\mathbb{E}_\pi[v(a, \omega)]$ is Sender's expected payoff under information structure $\pi \in \{\pi_c, \pi_f\}$, as defined by equation (8). Sender refines Receiver's frame if and only if $V > 0$.

4.1 Optimal Frame Choice

The following proposition characterizes when Sender finds it optimal to refine Receiver's frame. We refer to Appendix B.4 for the formal statement and its proof.

Proposition 3. *Sender refines Receiver's frame if and only if:*

1. *Persuasion is unnecessary only under the fine frame;*
2. *Persuasion is necessary under both frames, but signal z_1 under the optimal information structure is more likely under the fine frame than under the coarse frame.*

By Proposition 3, Sender refines Receiver's frame in two cases. The first is when persuasion is necessary under the coarse frame and unnecessary under the fine frame (part

1). In this case, re-framing alone is enough to induce Receiver to take Sender's preferred action a_1 with probability 1. The second is when persuasion is necessary under both frames, but Receiver's preferences or prior beliefs after re-framing make it more likely for Sender to induce Receiver to take action a_1 (part 2).

Graphical Illustration. Given optimal information structure, Figure 1 shows how the optimal frame choice depends on the parameter λ. In blue, we represent the region of the parameter space where persuasion is unnecessary under the coarse frame. In green, we represent the region of the parameter space where persuasion is necessary under both frames and signal z_1 is more likely under the coarse frame than under the fine frame. The value of re-framing cannot be positive in these regions.

In yellow, we represent the region of the parameter space where persuasion is necessary under the coarse frame and unnecessary under the fine frame. This region corresponds to part 1 of Proposition 3. In gold, we represent the region of the parameter space where persuasion is necessary under both frames and signal z_1 is more likely under the fine frame than under the coarse frame. This region corresponds to part 2 of Proposition 3. The value of re-framing is positive in these regions.

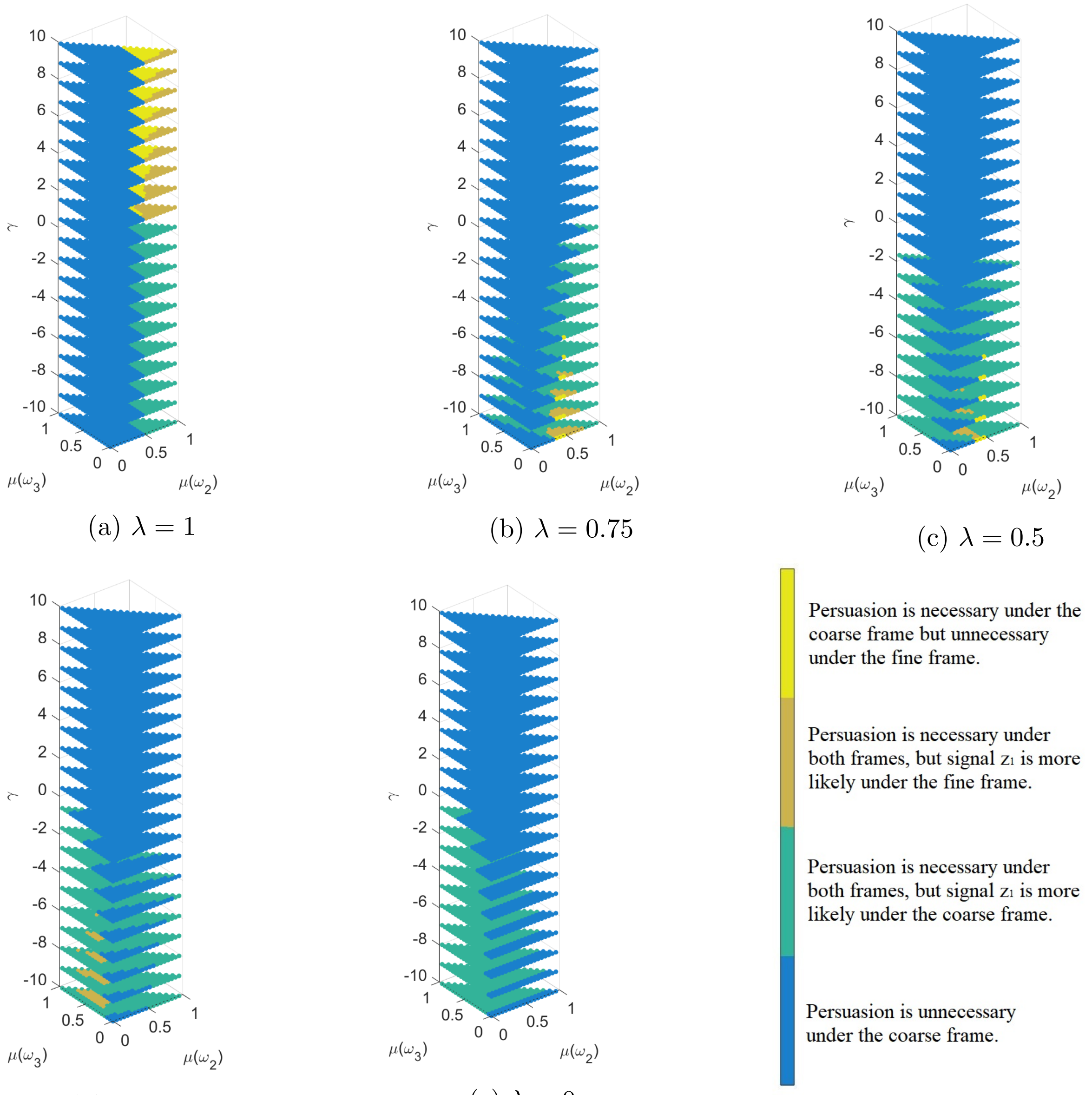
Figure 1 displays a complex relationship between the region of the parameter space where the value of re-framing is positive and the value of λ. Recall that, when $\lambda = 1$, Receiver conceives the coarse state ω_{23} as state ω_2—which is unfavorable to Sender—while ignoring state ω_3, which is favorable to Sender if $\gamma > 0$, and unfavorable otherwise. The opposite occurs when $\lambda = 0$. For any $\lambda \in (0, 1)$, Receiver's conception of the coarse state ω_{23} is a non-trivial combination of states ω_2 and ω_3.

First, suppose $\lambda = 1$. In this case, Sender can completely conceal the signal distribution in state ω_3 under the coarse frame, whereas no concealing is possible upon re-framing. Given this, the value of re-framing is positive if persuasion is necessary under the coarse frame (which happens if $\mu(\omega_2) > \frac{1}{2}$ by condition (6)) and $\gamma > 1$. To gain intuition, we distinguish two cases depending on the value of γ:

1. If $\gamma < 0$, state ω_3 is unfavorable to Sender. Under the coarse frame, Sender can recommend action a_1 with probability 1 in state ω_3 without jeopardizing persuasion in state ω_2. Under the fine frame, instead, the persuasion trade-off we describe after Proposition 1 forces Sender to focus persuasion on either state ω_2 or state ω_3, leaving persuasion in the other state as a residual task. Hence, re-framing is not optimal.

2. If $\gamma > 0$, state ω_3 is favorable to Sender. In this case, Sender can recommend action a_1 with probability 1 in state ω_3 under both frames. Hence, Sender's re-framing decision depends on the likelihood of producing evidence in favor of his goal (i.e., signal z_1) in state ω_2. If $\gamma \leq 1$, signal z_1 is more likely under the coarse frame because sub-additivity makes state ω_1 more likely and because of the properties of the optimal information structure. As a result, re-framing is not optimal. To coun-

Figure 1: Optimal Frame for Different Values of λ.



(a) $\lambda = 1$ (b) $\lambda = 0.75$ (c) $\lambda = 0.5$ (d) $\lambda = 0.25$ (e) $\lambda = 0$ (f) Legend

terbalance sub-additivity and make the value of re-framing positive, it must be that Receiver's payoff from action a_1 is greater in state ω_3 than in state ω_1, i.e., $\gamma > 1$.

When λ decreases, the region of the parameter space where the value of re-framing is positive for $\gamma > 1$ shrinks and eventually disappears, as panels (b)–(e) show. The reason is that, for all $\lambda \in (0, 1)$, Receiver conceives state ω_{23} as a non-trivial combination of states ω_2 and ω_3. Hence, her conceived payoff from Sender's preferred action a_1 in state ω_{23}, $\alpha_{23} = -\lambda + (1-\lambda)\gamma$, increases as λ decreases. Due to this increase, the region of the parameter space where persuasion is unnecessary under the coarse frame for $\gamma > 1$ expands.

Lower values of λ, however, also correspond to new regions of the parameter space where the value of re-framing is positive. For example, when $\lambda = 0.75$, if $\gamma < 0$ and is large in absolute value, the region where persuasion is unnecessary under the coarse frame shrinks because α_{23} decreases. This fact creates a new region where, for small values of $\mu(\omega_3)$, the value of re-framing is positive, as panel (b) shows. The reason is that, as γ becomes negative and large in absolute value, signal z_1 becomes less likely under the coarse frame. At the same time, such a decrease in γ has little impact on the optimal information structure under the fine frame if $\mu(\omega_3)$ is sufficiently small. Similar insights apply to the regions of the parameter space where the value of re-framing is positive when $\gamma < 0$ and is large in absolute value in panels (c) and (d).

Summing up, Sender refines Receiver's frame in two circumstances. First, when state ω_3 is (sufficiently) favorable to Sender, and Receiver ignores it under the coarse frame. Second, when state ω_3 is unlikely and (sufficiently) unfavorable to Sender, and Receiver assigns an excessive weight to it under the coarse frame. Crucially, Receiver's coarse conception indirectly affects the frame choice via its impact on the optimal information structure under the coarse frame. When $\lambda = 1$, Sender focuses persuasion on state ω_3 (since he can conceal it from Receiver), and persuasion is possible in state ω_2. As λ decreases, Sender finds focusing persuasion on state ω_3 progressively less attractive up to a threshold (see condition (16)) where Sender begins focusing persuasion on state ω_2. Similarly, as λ moves away from 1, the possibility of persuading in state ω_2 is challenged (see condition (17)). At the other extreme, when $\lambda = 0$, Sender focuses persuasion on state ω_2 (since he can conceal it from Receiver), and persuasion is possible in state ω_3. Therefore, since the optimal information structure under the fine frame does not depend on λ, Sender often faces a choice between frames under which he finds it optimal to focus persuasion on different states. The direct effect of λ on preferences and beliefs and its indirect effect through the optimal information structure jointly determine Sender's choice of Receiver's frame.

5 Welfare

In this section, we study Receiver's equilibrium welfare to understand the scope of disclosure (i.e., re-framing) policies. Given the information structure designed by Sender, our analysis focuses on determining under which frame Receiver is better off. We motivate the focus on Receiver's frame by observing that policymakers are more likely to mandate disclosure of all relevant contextual information (that is, mandate re-framing) rather than regulating the communication content.

We evaluate Receiver's welfare by considering a subjective and an objective criterion. According to the subjective criterion, we evaluate Receiver's welfare using her preferences, beliefs, and conception of the information structure conditional on her frame. Hence, Receiver's subjective welfare is a function of u, μ, and π under the fine frame and u_R, μ_R, and π_R under the coarse frame. According to the objective criterion, we evaluate Receiver's welfare from the viewpoint of an outside observer with the fine frame (e.g., a policymaker). In particular, we assume that Receiver's correct preferences and beliefs are u and μ.

The following proposition, whose proof is in Appendix B.5, summarizes our findings about Receiver's welfare and shows that re-framing can harm Receiver.

Proposition 4. *According to the subjective criterion:*

(a) If persuasion is necessary under both frames, Receiver is indifferent between frames.

(b) If persuasion is necessary under one frame but unnecessary under the other, Receiver is better off under the latter.

(c) If persuasion is unnecessary under both frames, Receiver can be better off under either frame.

According to the objective criterion, Receiver is better off under the fine frame if and only if

$$\mu(\omega_2)[\pi_c(z_1 \,|\, \omega_2) - \pi_f(z_1 \,|\, \omega_2)] + \gamma\mu(\omega_3)[\pi_f(z_1 \,|\, \omega_3) - \pi_c(z_1 \,|\, \omega_3)] > 0. \tag{20}$$

First, consider the subjective criterion. If persuasion is unnecessary under frame $\Omega' \in \{\Omega_R, \Omega\}$, Sender leaves Receiver's beliefs unchanged, thus confirming her default action a_1. Given Receiver's payoffs, persuasion is unnecessary under frame Ω' if and only if action a_1 gives her an expected payoff larger than 0 (and equal to 0 in a non-generic region of the parameter space). In contrast, if persuasion is necessary under frame Ω', the optimal information structure makes Receiver indifferent between the two actions after signal z_1 and Receiver prefers action a_2 after signal z_2. Since Receiver's payoff from action a_2 is equal to 0, Receiver's expected payoff if persuasion is necessary under frame Ω' equals 0. Parts 1–(a) and 1–(b) of the proposition follow. If persuasion is unnecessary under both frames, Receiver prefers the frame under which her subjective expected payoff from the default action is the largest; depending on the primitives, this may occur under either frame. Part 1–(c) of the proposition follows.

Next, consider the objective criterion. Receiver is better off under the frame in which Sender's recommendations induce her to take her optimal action given the state with the highest objective probability. This probability depends on preferences and beliefs (i.e., u and μ) and the optimal information structures (i.e., π_f or π_c). Condition (20) in part 2 of the proposition summarizes this relationship.

To fix ideas, suppose first $\gamma < 0$. If so, Receiver prefers action a_2 in states ω_2 and ω_3. In this case, for Receiver to be better off under the fine frame, by condition (20), it suffices that $\pi_c(z_1 \,|\, \omega_k) > \pi_f(z_1 \,|\, \omega_k)$ for all $k \in \{2, 3\}$. In other words, Receiver is better off under the fine frame if Sender exploits the coarse frame to send signal z_1 more often, inducing Receiver to take action a_1 even when sub-optimal for her. Next, suppose $\gamma > 0$. If so, Receiver prefers action a_1 in state ω_3. In this case, $\pi_f(z_1 \,|\, \omega_3) = 1$ by Proposition 1. Hence, by condition (20), Receiver can be better off under the fine frame even if $\pi_c(z_1 \,|\, \omega_2) < \pi_f(z_1 \,|\, \omega_2)$. In other words, Receiver can be better off under the fine frame even if Sender finds it optimal to send signal z_1 in state ω_2—thus inducing Receiver to take action a_1 even when sub-optimal for her—more often than under the coarse frame.

Also according to the objective criterion, however, there are situations in which Receiver is better off under the coarse frame, as the following example shows. Suppose $\lambda = 1$ and $\mu(\omega_1) + \gamma\mu(\omega_3) < \mu(\omega_2)$, so that persuasion is necessary under both frames (by conditions (5) and (6)). In this scenario, whenever $\gamma > 1$, Receiver is better off under the coarse frame. The reason is that, under the coarse frame, Sender cannot leverage state ω_3, favorable to him, to send signal z_1 in state ω_2 more often (by condition (3) for $\lambda = 1$). At the same time, he can do so under the fine frame. Formally:

- Sender recommends action a_1 with probability 1 in state ω_3 under both frames, i.e., $\pi_f(z_1 \,|\, \omega_3) = 1 = \pi_c(z_1 \,|\, \omega_3)$;
- Sender recommends action a_1 in state ω_2 under the fine frame with higher probability than under the coarse frame: $\pi_f(z_1 \,|\, \omega_2) = \frac{\mu(\omega_1)+\gamma\mu(\omega_3)}{\mu(\omega_2)} > \frac{\mu(\omega_1)+\mu(\omega_3)}{\mu(\omega_2)} = \pi_c(z_1 \,|\, \omega_2)$.

Thus, Sender finds it optimal to refine Receiver's frame. At the same time, by equation (20), the difference in Receiver's objective welfare under the fine and the coarse frames is proportional to $\pi_c(z_1 \,|\, \omega_2) - \pi_f(z_1 \,|\, \omega_2)$, the difference in the probability of signal z_1 in state ω_2. Since such a difference is negative, Receiver is better off under the coarse frame.

Implications for Disclosure Policies. The previous analysis has implications for the scope of policy interventions. Call a conflict of interest a situation in which Sender and Receiver are better off under different frames. According to the subjective criterion:

- If persuasion is necessary under both frames, Receiver is indifferent between frames, whereas Sender can be better off under either frame (Proposition 3).
- If persuasion is necessary under one frame but unnecessary under the other, Receiver and Sender are better off under the frame where persuasion is unnecessary.
- If persuasion is unnecessary under both frames, Sender chooses the coarse frame, whereas Receiver can be better off under either frame.

Hence, a conflict of interest can arise only in the latter case. This case, however, is knife-edge, as we select the equilibrium in which Sender does not refine Receiver's frame if indifferent. If we impose that Sender chooses Receiver's preferred frame whenever indifferent as an alternative selection criterion, any conflict of interest would disappear.

From the viewpoint of the objective criterion, the previous discussion shows that Sender is better off under the frame where he can send signal z_1 more often under unfavorable states, whereas the opposite holds for Receiver. Hence, a conflict of interest often arises. In particular, re-framing is not always in Receiver's best interest.

Summing up, under both welfare criteria, there are situations under which Receiver is better off under the coarse frame. Hence, an effective disclosure policy would require a careful understanding of the interplay between framing and information design.

Graphical Illustration. Figures 2 and 3 illustrate for which values of parameters there exists a conflict of interest between Sender and Receiver and whether mandating re-framing is a solution to this problem. In red, we represent the region of the parameter space where there is a conflict of interest. In green, we represent the region of the parameter space where Sender and Receiver are better off under the same frame. We use a darker (resp., lighter) color tone to represent the region of the parameter space where Receiver is better off under the fine (resp., coarse) frame.

Figure 2 considers the subjective criterion. A conflict of interest can arise only when persuasion is unnecessary under both frames. In this case, there is a conflict of interest if Receiver is better off under the fine frame, as we select the equilibrium under which Sender chooses not to re-frame whenever indifferent. This case corresponds to the dark red region. In all other scenarios, mandating re-framing is either unnecessary—which is the dark green region—or harms Receiver (and Sender)—which is the light green region.

Figure 3 considers the objective criterion. Here, a conflict of interest is more likely. Whereas Sender is better off under the frame where he can recommend action a_1 with the highest probability, this harms Receiver because of the higher likelihood that the recommendation is misleading. There is no conflict of interest when persuasion is unnecessary under both frames—which is the light green region. Mandating re-framing can either help—which is the dark red region—or harm—which is the light red region—Receiver.

6 Discussion and Extensions

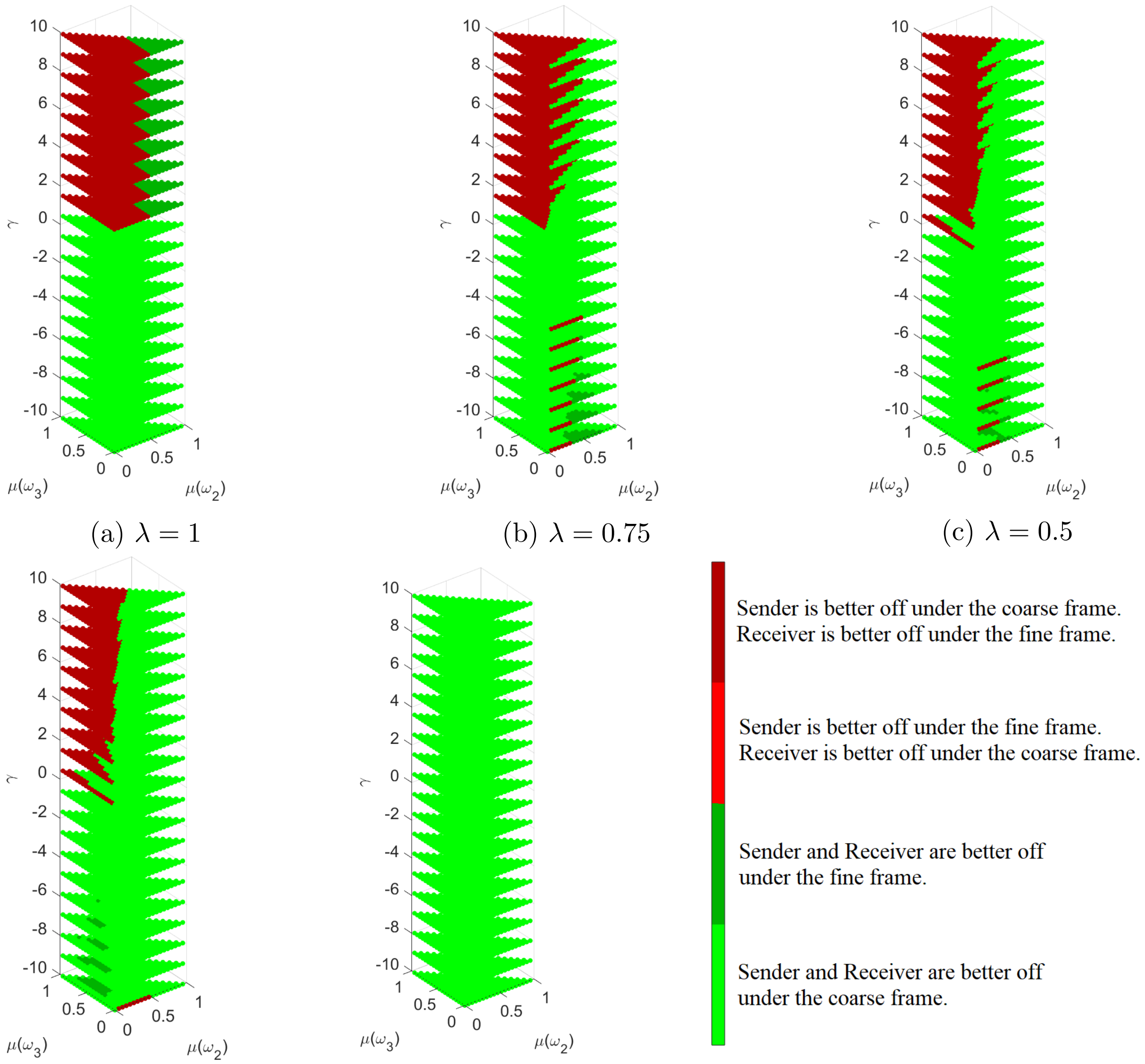
In this section, we discuss our modeling choices and a few natural extensions.

Three States. We assume that Receiver cannot distinguish between two out of three states and that Sender can "certifiably" prove there are no other states beyond ω_1, ω_2, and ω_3 upon re-framing. In other words, Receiver is initially unaware of her coarse conception. However, upon re-framing, although Receiver becomes aware of her initial coarse conception, she becomes sure that no other state beyond ω_1, ω_2, and ω_3 exists. These assumptions allow us to focus on how framing interplays with information design in the simplest possible non-trivial setting. Therefore, we can transparently analyze Sender's incentives and trade-off between keeping Receiver in the dark and re-framing and characterize the welfare implications of Sender's choices.

With more than three states, if Sender chooses a frame that contains more states than the original Sender frame but is not the same as Sender's frame, Receiver may begin reasoning about the possibility that Sender's frame is finer than hers. These considerations would naturally lead to a (non-standard) informed principal problem (Myerson, 1983). Although certainly of interest, these considerations are beyond the scope of this paper.

Alternative Frames. In Appendix D, we consider alternative coarse frames and show that qualitatively analogous insights emerge. The case in which Receiver cannot distinguish between states ω_1 and ω_3, i.e., $\Omega_R = \{\omega_{13}, \omega_2\}$, is specular to our main specification. Under the coarse frame, Receiver conceives ω_2, the unfavorable state to Sender. At the

Figure 2: Conflict of Interest under the Subjective Criterion for Different Values of λ.



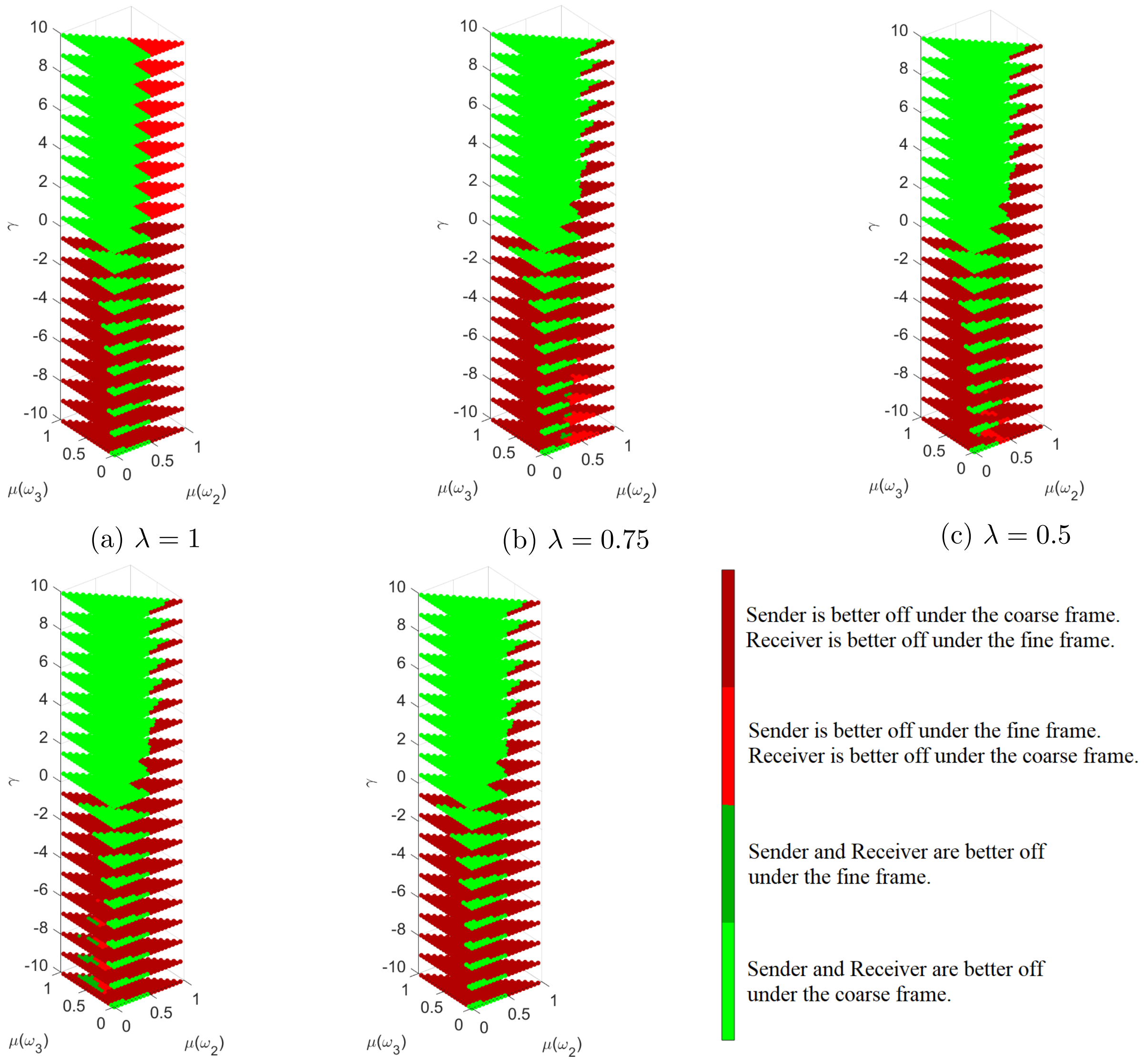
(a) $\lambda = 1$ (b) $\lambda = 0.75$ (c) $\lambda = 0.5$ (d) $\lambda = 0.25$ (e) $\lambda = 0$ (f) Legend

same time, she cannot distinguish ω_1, the favorable state to Sender, from ω_3, which is a favorable or unfavorable state to Sender depending on the sign of γ. Hence, in contrast to the main specification, under the coarse frame, Sender cannot conceal evidence in favor of his goal produced in the unfavorable state ω_2. In addition, under sub-additivity, Receiver conceives state ω_2 as more likely under the coarse frame than under the fine frame. These forces strengthen Sender's incentives to re-frame. Hence, re-framing becomes optimal in a larger region of the parameter space. However, the welfare implications remain the same: Receiver may be better off under the coarse frame.

If Receiver cannot distinguish between ω_1 and ω_2, i.e., $\Omega_R = \{\omega_{12}, \omega_3\}$, the insights resemble those when $\Omega_R = \{\omega_1, \omega_{23}\}$ if $\gamma > 0$, and those when $\Omega_R = \{\omega_2, \omega_{13}\}$ if $\gamma < 0$.

Unawareness Interpretation. Our model also accommodates asymmetric awareness and the expansion of Receiver's state space. In particular, when $\lambda \in \{0, 1\}$, Receiver is

Figure 3: Conflict of Interest under the Objective Criterion for Different Values of λ.



(a) $\lambda = 1$ (b) $\lambda = 0.75$ (c) $\lambda = 0.5$ (d) $\lambda = 0.25$ (e) $\lambda = 0$ (f) Legend

unaware of one of the states in the state space. Hence, when Sender refines her conception, Receiver becomes aware of the state she was initially unaware of. In this sense, our paper is close to recent work exploring belief formation under growing awareness (see, e.g., Karni and Vierø, 2013; Piermont, 2021). We stick to the approach in Ahn and Ergin (2010), which accommodates growing awareness in a setting where agents' preferences and beliefs may depend on their level of awareness summarized by how they partition the state space.

7 Conclusion

We allow for asymmetric framing of the state space in the classical information design model. Whereas Sender conceives all states separately, Receiver cannot initially distinguish among some of them. Our analysis provides three main insights. First, we show that Receiver's framing of the state space becomes more critical than preferences and beliefs in shaping the optimal information structure. Second, we characterize how Sender resolves the trade-off between keeping Receiver in the dark, thus concealing part of the information structure, and re-framing, hence inducing Receiver to revise preferences and prior beliefs after telling apart initially indistinguishable states. Third, we show that, although a coarse worldview may open the doors to Receiver's exploitation, re-framing can harm Receiver in practice, thus questioning the scope of disclosure policies.

Several questions may be worth future research. First, one could consider the case in which Sender is uncertain about Receiver's reaction to re-framing, i.e., Receiver's beliefs and preferences upon re-framing. Studying such a model would require adopting a robust approach to information design (e.g., in the spirit of Dworczak and Pavan, 2022; Kosterina, 2022). Second, one could consider competing Senders. Whereas competition between Senders who provide information about the same state would lead to complete revelation, non-trivial insights may emerge if the information provision is about distinct states. Third, one could consider multiple Receivers, some of which have a coarse frame while others do not. In this case, under public information design, Sender must persuade Receivers who are heterogeneous in their worldviews.

A Foundations for Preferences and Beliefs

In this section, we introduce Ahn and Ergin (2010)'s partition-dependent expected payoff (PDEP) model. For each element of the PDEP model, we present the corresponding element in our setting.

- There is a decision maker.
  - In our setting, Receiver is the decision maker.
- There is a state space S and a set Φ of finite partitions of S. A partition $\phi \in \Phi$ corresponds to a description of the state space S. We assume that the trivial partition $\{S\}$ is an element of Φ, i.e., $\{S\} \in \Phi$. For all $\phi \in \Phi$, let $\sigma(\phi)$ be the algebra on S generated by ϕ. We say that partition ϕ is finer than partition ϕ', denoted by $\phi \geq \phi'$, if $\sigma(\phi') \subseteq \sigma(\phi)$. Finally, let $\mathcal{C} := \bigcup_{\phi\in\Phi} \phi$.
  - In our setting, we have $S = \Omega$. Moreover, let $\phi_{\mathrm{f}} := \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}\}$, $\phi_{\mathrm{c}} := \{\{\omega_1\}, \{\omega_2, \omega_3\}\}$, and $\phi_{\mathrm{t}} := \{\Omega\}$; then, we have $\Phi = \{\phi_{\mathrm{t}}, \phi_{\mathrm{c}}, \phi_{\mathrm{f}}\}$. A partition of Ω corresponds to a frame; in particular, ϕ_{c} corresponds to the coarse frame Ω_R, and ϕ_{f} corresponds to the fine frame Ω. We include the trivial partition $\phi_{\mathrm{t}} \in \Phi$ for consistency with the PDEP model, but ϕ_{t} will not play a role in the following analysis. Here, we have $\sigma(\phi_{\mathrm{t}}) = \{\emptyset, \Omega\}$, $\sigma(\phi_{\mathrm{c}}) = \{\emptyset, \{\omega_1\}, \{\omega_2, \omega_3\}, \Omega\}$, and $\phi_{\mathrm{f}} = 2^\Omega$, where 2^Ω denotes the power set of Ω; therefore, $\phi_{\mathrm{f}} \geq \phi_{\mathrm{c}} \geq \phi_{\mathrm{t}}$. Finally, we have $\mathcal{C} = \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_2, \omega_3\}, \Omega\}$.
- There is a finite set X of consequences or prizes. An act is a function $f : S \to \Delta(X)$ that maps states to lotteries (i.e., probability distributions) on X. The act f is $\sigma(\phi)$-measurable if $f^{-1}(p) \in \sigma(\phi)$ for all $p \in \Delta(X)$; equivalently, the act f is $\sigma(\phi)$-measurable if, for all $E \in \phi$ and $s, s' \in E$, we have $f(s) = f(s')$. We denote by $\mathcal{F}_\phi$ the set of $\sigma(\phi)$-measurable acts. Informally, $\mathcal{F}_\phi$ is the set of acts that can be described using the descriptive power of ϕ; an act $f \notin \mathcal{F}_\phi$ requires a finer categorization than is available in ϕ.
  - In our setting, consider the set of prizes $X = \{x_0, x_1, x_2, x_3\}$. Moreover, consider the following acts:

f_1	ω_1	ω_2	ω_3
x_0	0	0	0
x_1	1	0	0
x_2	0	λ	λ
x_3	0	$1-\lambda$	$1-\lambda$

f_2	ω_1	ω_2	ω_3
x_0	1	1	1
x_1	0	0	0
x_2	0	0	0
x_3	0	0	0

f_1'	ω_1	ω_2	ω_3
x_0	0	0	0
x_1	1	0	0
x_2	0	1	0
x_3	0	0	1

The previous tables read as follows. Act f_1 gives prize x_1 in state ω_1 with probability 1; moreover, act f_1 gives prize x_2 with probability λ and prize x_3 with probability $1-\lambda$ in states ω_2 and ω_3. Act f_2 gives prize x_0 in all states with probability 1. For all

$i \in \{1, 2, 3\}$, act f_1' gives prize x_i with probability 1 in state ω_i. Note that $f_1, f_2 \in \mathcal{F}_{\phi_c}$ and $f_1', f_2 \in \mathcal{F}_{\phi_f}$. Given partition ϕ_c (or, equivalently, frame Ω_R), act f_1 corresponds to action a_1 and act f_2 corresponds to action a_2. Given partition ϕ_f (or, equivalently, frame Ω), act f_1' corresponds to action a_1 and act f_2 corresponds to action a_2.

- The primitive is a family of preferences $\{\succsim_\phi\}_{\phi \in \Phi}$, where each $\succsim_\phi$ is defined over the set of $\sigma(\phi)$-measurable acts $\mathcal{F}_\phi$. The interpretation of $f \succsim_\phi g$ is that f is weakly preferred to g when the state space is described as the partition ϕ.
  - In our setting, the family of preferences is $\{\succsim_{\phi_t}, \succsim_{\phi_c}, \succsim_{\phi_f}\}$.
- The PDEP studies the following utility representation. The decision maker has a non-additive set function $\nu\colon \mathcal{C} \to \mathbb{R}_+$. Presented with description ϕ of the state space, the decision maker places a weight $\nu(E)$ on each $E \in \phi$. We refer to ν as a support function. Normalizing by the sum,

$$\mu_\phi(E) = \frac{\nu(E)}{\sum_{F \in \phi} \nu(F)} \quad \text{for all } E \in \phi,$$

indices a probability measure over $\sigma(\phi)$. The decision's maker utility for the $\sigma(\phi)$-measurable act f is expressed as

$$\sum_{E \in \phi} u(f(E))\mu_\phi(E),$$

where $u\colon \Delta(X) \to \mathbb{R}$ is an affine utility function over lotteries. Here, with some abuse of notation, we denote by $f(E)$ the lottery $p \in \Delta(X)$ such that $p = f(s)$ for all $s \in E$ (since f is $\sigma(\phi)$-measurable, for all $E \in \phi$ and $s, s' \in E$, we have $f(s) = f(s')$).

Definition 4 (Ahn and Ergin (2010)). *The family of preferences $\{\succsim_\phi\}_{\phi \in \Phi}$ admits a partition-dependent expected payoff (PDEP) representation if there exists a non-constant affine von Neumann–Morgenstern payoff function $u\colon \Delta(X) \to \mathbb{R}$ and a support function $\nu\colon \mathcal{C} \to \mathbb{R}_+$ such that, for all $\phi \in \Phi$ and $f, g \in \mathcal{F}_\phi$,*

$$f \succsim_\phi g \iff \int_S u \circ f \mathrm{d}\mu_\phi \geq \int_S u \circ g \mathrm{d}\mu_\phi,$$

where μ_ϕ is the unique probability measure on $(S, \sigma(\phi))$ such that, for all $E \in \phi$,

$$\mu_\phi(E) = \frac{\nu(E)}{\sum_{F \in \phi} \nu(F)}.$$

- In our setting, consider the Bernoulli utility function $\overline{u}\colon X \to \mathbb{R}$ defined pointwise as

$$\overline{u}(x_0) \coloneqq 0, \qquad \overline{u}(x_1) \coloneqq 1, \qquad \overline{u}(x_2) \coloneqq -1, \qquad \overline{u}(x_3) \coloneqq \gamma.$$

For any lottery $\alpha \in \Delta(X)$, let α_i denote the probability of prize x_i under lottery α. Hence, the von Neumann–Morgenstern payoff function $u\colon \Delta(X) \to \mathbb{R}$ is defined pointwise as

$$u(\alpha) \coloneqq \sum_{i=0}^{3} \overline{u}(x_i)\alpha_i.$$

Moreover, consider the support function $\nu\colon \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_2, \omega_3\}, \Omega\} \to \mathbb{R}_+$ defined pointwise as

$$\nu(\{\omega_1\}) \coloneqq \mu(\omega_1), \qquad \nu(\{\omega_2\}) \coloneqq \mu(\omega_2), \qquad \nu(\{\omega_3\}) \coloneqq \mu(\omega_3), \qquad \nu(\Omega) \coloneqq 1,$$

and

$$\nu(\{\omega_2, \omega_3\}) \coloneqq \mu(\omega_1)\frac{\lambda\mu(\omega_2) + (1-\lambda)\mu(\omega_3)}{1 - [\lambda\mu(\omega_2) + (1-\lambda)\mu(\omega_3)]}.$$

Using this support function, we obtain a probability distribution (corresponding to Receiver's prior belief under the fine frame) over $\sigma(\phi_{\mathrm{f}})$ that satisfies

$$\mu_{\phi_{\mathrm{f}}}(\{\omega_1\}) = \mu(\omega_1), \qquad \mu_{\phi_{\mathrm{f}}}(\{\omega_2\}) = \mu(\omega_2), \qquad \mu_{\phi_{\mathrm{f}}}(\{\omega_3\}) = \mu(\omega_3). \tag{A.1}$$

Moreover, we obtain a probability distribution (corresponding to Receiver's prior belief under the coarse frame) over $\sigma(\phi_{\mathrm{c}})$ that satisfies

$$\mu_{\phi_{\mathrm{c}}}(\{\omega_1\}) = 1 - [\lambda\mu(\omega_2) + (1-\lambda)\mu(\omega_3)], \tag{A.2}$$

$$\mu_{\phi_{\mathrm{c}}}(\{\omega_2, \omega_3\}) = \lambda\mu(\omega_2) + (1-\lambda)\mu(\omega_3), \tag{A.3}$$

By definition (2) and observing that $\mu_R(\omega_1) = 1 - \mu_R(\omega_{23})$, equalities (A.2) and (A.3) read as

$$\mu_{\phi_{\mathrm{c}}}(\{\omega_1\}) = \mu_R(\omega_1) \qquad \text{and} \qquad \mu_{\phi_{\mathrm{c}}}(\{\omega_2, \omega_3\}) = \mu_R(\omega_{23}). \tag{A.4}$$

Using the above specifications of the von Neumann–Morgenstern payoff function and of the support function (and the induced probability measure), we have

$$\begin{aligned} f_1 \succsim_{\phi_{\mathrm{c}}} f_2 &\iff \int_\Omega u \circ f_1 \mathrm{d}\mu_{\phi_{\mathrm{c}}} \geq \int_\Omega u \circ f_2 \mathrm{d}\mu_{\phi_{\mathrm{c}}} \\ &\iff \overline{u}(x_1)\mu_{\phi_{\mathrm{c}}}(\{\omega_1\}) + [\lambda\overline{u}(x_2) + (1-\lambda)\overline{u}(x_3)]\mu_{\phi_{\mathrm{c}}}(\{\omega_2, \omega_3\}) \\ &\qquad \geq \overline{u}(x_0)[\mu_{\phi_{\mathrm{c}}}(\{\omega_1\}) + \mu_{\phi_{\mathrm{c}}}(\{\omega_2, \omega_3\})] \\ &\iff \mu_R(\omega_1) + \alpha_{23}\mu_R(\omega_3) \geq 0, \end{aligned}$$

where: the second equivalence holds by definition of f_1, f_2 and u; the third equivalence holds by the definition of $\overline{u}$, equalities (A.4), and definition (1). Moreover, we have

$$\begin{aligned} f_1' \succsim_{\phi_{\mathrm{f}}} f_2 &\iff \int_\Omega u \circ f_1' \mathrm{d}\mu_{\phi_{\mathrm{f}}} \geq \int_\Omega u \circ f_2 \mathrm{d}\mu_{\phi_{\mathrm{f}}} \\ &\iff \overline{u}(x_1)\mu_{\phi_{\mathrm{f}}}(\{\omega_1\}) + \overline{u}(x_2)\mu_{\phi_{\mathrm{f}}}(\{\omega_2\}) + \overline{u}(x_3)\mu_{\phi_{\mathrm{f}}}(\{\omega_3\}) \\ &\qquad \geq \overline{u}(x_0)[\mu_{\phi_{\mathrm{f}}}(\{\omega_1\}) + \mu_{\phi_{\mathrm{f}}}(\{\omega_2\}) + \mu_{\phi_{\mathrm{f}}}(\{\omega_3\})] \end{aligned}$$

$$\iff \mu(\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3) \geq 0,$$

where: the second equivalence holds by definition of f_1', f_2 and u; the third equivalence holds by the definition of $\overline{u}$ and equalities (A.1).

Summing up, from Definition 4 and the previous analysis, we conclude that Receiver's preferences and beliefs follow a PDEP representation in our model. Similar arguments apply to the alternative coarse frames we discuss in Appendix D.

B Formal Statements and Proofs

B.1 Proof of Lemma 1

Part 1. Let $\Omega' = \Omega$. Receiver's expected payoff given belief $p \in \Delta(\Omega)$ is equal to $p(\omega_1) - p(\omega_2) + \gamma p(\omega_3)$ if $a = a_1$, and equal to 0 if $a = a_2$. Hence, the desired result follows by observing that $a(p) = a_1 \iff p(\omega_1) - p(\omega_2) + \gamma p(\omega_3) \geq 0 \iff p(\omega_1) + \gamma p(\omega_3) \geq p(\omega_2)$. ■

Part 2. Let $\Omega' = \Omega_R$. Receiver's expected payoff given belief $p_R \in \Delta(\Omega_R)$ is equal to $p_R(\omega_1) + p_R(\omega_{23})\alpha_{23}$ if $a = a_1$, and equal to 0 if $a = a_2$. Hence, the desired result follows by observing that $a(p_R) = a_1 \iff p_R(\omega_1) + p_R(\omega_{23})\alpha_{23} \geq 0 \iff p_R(\omega_1) \geq -\alpha_{23} p_R(\omega_{23})$. ■

B.2 Proposition 1

Formal Statement. *If $\Omega' = \Omega$ and persuasion is necessary, we distinguish four cases depending on whether the following conditions hold:*

$$\gamma \geq -1, \tag{B.1}$$

$$\frac{\mu(\omega_1)}{\mu(\omega_3)} \geq -\gamma, \tag{B.2}$$

$$\frac{\mu(\omega_1)}{\mu(\omega_2)} \geq 1. \tag{B.3}$$

1. *If conditions (B.1) and (B.2) hold, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}, \qquad \pi(z_1 \,|\, \omega_3) = 1,$$

   and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = 1 + \mu(\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3). \tag{B.4}$$

2. *If condition (B.1) holds and condition (B.2) does not hold, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = 0, \qquad \pi(z_1 \,|\, \omega_3) = -\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)},$$

and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = \frac{\gamma - 1}{\gamma}\mu(\omega_1). \tag{B.5}$$

3. *If condition (B.1) does not hold and condition (B.3) holds, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = 1, \qquad \pi(z_1 \,|\, \omega_3) = \frac{\mu(\omega_2) - \mu(\omega_1)}{\gamma\mu(\omega_3)},$$

and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = 1 - \frac{\mu(\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3)}{\gamma}. \tag{B.6}$$

4. *If neither condition (B.1) nor condition (B.3) holds, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = \frac{\mu(\omega_1)}{\mu(\omega_2)}, \qquad \pi(z_1 \,|\, \omega_3) = 0,$$

and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = 2\mu(\omega_1). \tag{B.7}$$

Proof. By equation (4) and since $p(\omega_1 \,|\, z_2) = 0$ (by condition (11)), we have $\pi(z_2 \,|\, \omega_1) = 0$, from which it follows that the optimal information structure must satisfy

$$\pi(z_1 \,|\, \omega_1) = 1. \tag{B.8}$$

By Bayes plausibility (condition (7)) and since $p(\omega_1 \,|\, z_2) = 0$ (by condition (11)), we have

$$p(\omega_1 \,|\, z_1)\pi(z_1) = \mu(\omega_1), \tag{B.9}$$

where $\pi(z_1) := \sum_{\omega\in\Omega} \pi(z_1 \,|\, \omega)\mu(\omega)$. Since $2p(\omega_1 \,|\, z_1) + (1+\gamma)p(\omega_3 \,|\, z_1) = 1$ by condition (9), and $\sum_{\omega\in\Omega} p(\omega \,|\, z_1) = 1$, we have

$$p(\omega_1 \,|\, z_1) = p(\omega_2 \,|\, z_1) - \gamma p(\omega_3 \,|\, z_1). \tag{B.10}$$

By conditions (B.9) and (B.10), we have

$$[p(\omega_2 \,|\, z_1) - \gamma p(\omega_3 \,|\, z_1)]\pi(z_1) = \mu(\omega_1). \tag{B.11}$$

By Bayes rule, for all $\omega \in \Omega$, we have

$$p(\omega \,|\, z_1) = \frac{\pi(z_1 \,|\, \omega)\mu(\omega)}{\pi(z_1)}. \tag{B.12}$$

Therefore, by conditions (B.11) and (B.12), we have

$$\pi(z_1 \,|\, \omega_2) = \frac{\mu(\omega_1)}{\mu(\omega_2)} + \frac{\gamma\pi(z_1 \,|\, \omega_3)\mu(\omega_3)}{\mu(\omega_2)}, \tag{B.13}$$

where for now we ignore the feasibility constraints $\pi(z_1 \,|\, \omega_2) \in [0,1]$ and $\pi(z_1 \,|\, \omega_3) \in [0,1]$.

By conditions (8), (B.8), and (B.13), Sender's expected payoff is $\mathbb{E}_\pi[v(a,\omega)] = 2\mu(\omega_1) + \mu(\omega_3)(1+\gamma)\pi(z_1 \,|\, \omega_3)$. As $\mathbb{E}_\pi[v(a,\omega)]$ is linear in $\pi(z_1 \,|\, \omega_3)$, the optimal information structure depends on the sign of the coefficient on $\pi(z_1 \,|\, \omega_3)$. In particular, we have two cases.

- If the coefficient on $\pi(z_1 \,|\, \omega_3)$ is non-negative, i.e., $\gamma \geq -1$, Sender maximizes $\pi(z_1 \,|\, \omega_3)$ s.t. the feasibility constraints $\pi(z_1 \,|\, \omega_2) \in [0,1]$ and $\pi(z_1 \,|\, \omega_3) \in [0,1]$. We further distinguish two cases.

  First, suppose $\gamma \in [-1, 0)$. By condition (B.13), we have

$$\pi(z_1 \,|\, \omega_2) \geq 0 \iff \pi(z_1 \,|\, \omega_3) \leq -\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)},$$
$$\pi(z_1 \,|\, \omega_3) \leq 1 \iff \pi(z_1 \,|\, \omega_2) \geq \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}.$$

  Therefore, we have

$$\pi(z_1 \,|\, \omega_2) = \min\left\{1, \max\left\{0, \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}\right\}\right\}, \tag{B.14}$$
$$\pi(z_1 \,|\, \omega_3) = \max\left\{0, \min\left\{1, -\frac{\mu_R(\omega_1)}{\gamma\mu_R(\omega_3)}\right\}\right\}. \tag{B.15}$$

  Note that

$$\frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)} < 1 \iff \mu(\omega_1) + \gamma\mu(\omega_3) < \mu(\omega_2). \tag{B.16}$$

  Since persuasion is necessary (condition (5)), the inequality on the right-hand side of equivalence (B.16) is satisfied, and so, by condition (B.14), we have

$$\pi(z_1 \,|\, \omega_2) = \max\left\{0, \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}\right\}. \tag{B.17}$$

  Since $\gamma < 0$, we have $-\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)} > 0$, and so, by condition (B.15), we have

$$\pi(z_1 \,|\, \omega_3) = \min\left\{1, -\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)}\right\}. \tag{B.18}$$

  Next, note that

$$\frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)} \geq 0 \iff \frac{\mu(\omega_1)}{\mu(\omega_3)} \geq -\gamma, \tag{B.19}$$
$$-\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)} \leq 1 \iff \frac{\mu(\omega_1)}{\mu(\omega_3)} \leq -\gamma. \tag{B.20}$$

  Therefore, by conditions (B.17)–(B.20), we have the following. If $\frac{\mu(\omega_1)}{\mu(\omega_3)} \geq -\gamma$, we have

$$\pi(z_1 \,|\, \omega_2) = \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)} \qquad \text{and} \qquad \pi(z_1 \,|\, \omega_3) = 1, \tag{B.21}$$

  and the optimality of the information structure in part 3 of the proposition for $\gamma \in$

$[-1, 0)$ follows from conditions (B.8) and (B.21). If, instead, $\frac{\mu(\omega_1)}{\mu(\omega_3)} < -\gamma$, we have

$$\pi(z_1 \,|\, \omega_2) = 0 \qquad \text{and} \qquad \pi(z_1 \,|\, \omega_3) = -\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)}, \tag{B.22}$$

and the optimality of the information structure in part 3 of the proposition for $\gamma \in [-1, 0)$ follows from conditions (B.8) and (B.22).

Second, suppose $\gamma > 0$. If so, condition (B.2) holds. Hence, we need to show that the optimal information structure is that in part 1 of the proposition. By condition (B.13), we have

$$\pi(z_1 \,|\, \omega_2) \geq 0 \iff \pi(z_1 \,|\, \omega_3) \geq -\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)}.$$

Therefore, we have

$$\pi(z_1 \,|\, \omega_2) = \max\left\{0, \min\left\{1, \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}\right\}\right\}. \tag{B.23}$$

Next, note that

$$\frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)} < 1 \iff \mu(\omega_1) + \gamma\mu(\omega_3) < \mu(\omega_2). \tag{B.24}$$

Since persuasion is necessary (condition (5)), the inequality on the right-hand side of equivalence (B.24) is satisfied, and so, by condition (B.23), we have

$$\pi(z_1 \,|\, \omega_2) = \max\left\{0, \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}\right\}. \tag{B.25}$$

Moreover, note that

$$\frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)} \geq 0 \iff \frac{\mu(\omega_1)}{\mu(\omega_3)} \geq -\gamma. \tag{B.26}$$

Since $\gamma > 0$, the inequality on the right-hand side of equivalence (B.26) is satisfied, and so, by condition (B.25), we have

$$\pi(z_1 \,|\, \omega_2) = \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}. \tag{B.27}$$

By equation (4) and since $p(\omega_3 \,|\, z_2) = 0$ (by condition (11)), we have $\pi(z_2 \,|\, \omega_3) = 0$, from which it follows that the optimal information structure must satisfy

$$\pi(z_1 \,|\, \omega_3) = 1. \tag{B.28}$$

The optimality of the information structure in part 1 of the proposition for $\gamma > 0$ follows from conditions (B.8), (B.27), and (B.28).

- If the coefficient on $\pi(z_1 \,|\, \omega_3)$ is negative, i.e., $\gamma < -1$, Sender minimizes $\pi(z_1 \,|\, \omega_3)$ s.t. the feasibility constraints $\pi(z_1 \,|\, \omega_2) \in [0, 1]$ and $\pi(z_1 \,|\, \omega_3) \in [0, 1]$.

By condition (B.13), we have

$$\pi(z_1 \,|\, \omega_2) \leq 1 \iff \pi(z_1 \,|\, \omega_3) \geq \frac{\mu(\omega_2) - \mu(\omega_1)}{\gamma\mu(\omega_3)},$$
$$\pi(z_1 \,|\, \omega_3) \geq 0 \iff \pi(z_1 \,|\, \omega_2) \leq \frac{\mu(\omega_1)}{\mu(\omega_2)}.$$

Therefore, we have

$$\pi(z_1 \,|\, \omega_2) = \max\left\{0, \min\left\{1, \frac{\mu(\omega_1)}{\mu(\omega_2)}\right\}\right\}, \tag{B.29}$$
$$\pi(z_1 \,|\, \omega_3) = \min\left\{1, \max\left\{0, \frac{\mu(\omega_2) - \mu(\omega_1)}{\gamma\mu(\omega_3)}\right\}\right\}. \tag{B.30}$$

Since $\frac{\mu(\omega_1)}{\mu(\omega_2)} > 0$, by condition (B.29), we have

$$\pi(z_1 \,|\, \omega_2) = \min\left\{1, \frac{\mu(\omega_1)}{\mu(\omega_2)}\right\}. \tag{B.31}$$

Note that

$$\frac{\mu(\omega_2) - \mu(\omega_1)}{\gamma\mu(\omega_3)} < 1 \iff \mu(\omega_1) + \gamma\mu(\omega_3) < \mu(\omega_2). \tag{B.32}$$

Since persuasion is necessary (condition (5)), the inequality on the right-hand side of equivalence (B.32) is satisfied, and so, by condition (B.30), we have

$$\pi(z_1 \,|\, \omega_3) = \max\left\{0, \frac{\mu(\omega_2) - \mu(\omega_1)}{\gamma\mu(\omega_3)}\right\}. \tag{B.33}$$

Next, note that

$$\frac{\mu(\omega_2) - \mu(\omega_1)}{\gamma\mu(\omega_3)} \geq 0 \iff \frac{\mu(\omega_1)}{\mu(\omega_2)} \geq 1. \tag{B.34}$$

Hence, by conditions (B.31), (B.33), and (B.34), we have the following. If $\frac{\mu(\omega_1)}{\mu(\omega_2)} \geq 1$, we have

$$\pi(z_1 \,|\, \omega_2) = 1 \qquad \text{and} \qquad \pi(z_1 \,|\, \omega_3) = \frac{\mu(\omega_2) - \mu(\omega_1)}{\gamma\mu(\omega_3)}, \tag{B.35}$$

and the optimality of the information structure in part 3 of the proposition for $\gamma < -1$ follows from conditions (B.8) and (B.35). If, instead, $\frac{\mu(\omega_1)}{\mu(\omega_2)} < 1$, we have

$$\pi(z_1 \,|\, \omega_2) = \frac{\mu(\omega_1)}{\mu(\omega_2)} \qquad \text{and} \qquad \pi(z_1 \,|\, \omega_3) = 0, \tag{B.36}$$

and the optimality of the information structure in part 4 of the proposition for $\gamma < -1$ follows from conditions (B.8) and (B.36).

For all four parts of the proposition, Sender's expected payoff follows from simple algebra by evaluating equation (8) at the corresponding optimal information structure. ■

B.3 Proof of Proposition 2

Formal Statement. *If $\Omega' = \Omega_R$ and persuasion is necessary, we distinguish four cases depending on whether the following conditions hold:*

$$\lambda\mu(\omega_3) \geq (1-\lambda)\mu(\omega_2), \tag{B.37}$$

$$-\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \geq 1-\lambda, \tag{B.38}$$

$$-\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \geq \lambda. \tag{B.39}$$

1. *If conditions (B.37) and (B.38) hold, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} - \frac{1-\lambda}{\lambda}, \qquad \pi(z_1 \,|\, \omega_3) = 1,$$

and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = 1 - \mu(\omega_2)\left[\frac{1}{\lambda} + \frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}\right]. \tag{B.40}$$

2. *If condition (B.37) holds and condition (B.38) does not hold, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = 0, \qquad \pi(z_1 \,|\, \omega_3) = -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})},$$

and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = \mu(\omega_1) - \frac{\mu_R(\omega_1)\mu(\omega_3)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}. \tag{B.41}$$

3. *If condition (B.37) does not hold and condition (B.39) holds, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = 1, \qquad \pi(z_1 \,|\, \omega_3) = -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\lambda}{1-\lambda},$$

and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = 1 - \mu(\omega_3)\left[\frac{1}{1-\lambda} + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}\right]. \tag{B.42}$$

4. *If neither condition (B.37) nor condition (B.39) holds, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}, \qquad \pi(z_1 \,|\, \omega_3) = 0,$$

and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = \mu(\omega_1) - \frac{\mu_R(\omega_1)\mu(\omega_2)}{\lambda\alpha_{23}\mu_R(\omega_{23})}. \tag{B.43}$$

Proof. By equation (4) and since $p_R(\omega_1 \,|\, z_2) = 0$ (by condition (12)), we have $\pi_R(z_2 \,|\, \omega_1) = 0$, from which it follows that the optimal information structure must satisfy

$$\pi_R(z_1 \,|\, \omega_1) = 1 = \pi(z_1 \,|\, \omega_1), \tag{B.44}$$

where the second equality holds by condition (3).

By Bayes plausibility (condition (7)) and since $p_R(\omega_1 \,|\, z_2) = 0$ (by condition (12)), we have

$$p_R(\omega_1 \,|\, z_1)[\pi_R(z_1 \,|\, \omega_1)\mu_R(\omega_1) + \pi_R(z_1 \,|\, \omega_{23})\mu_R(\omega_{23})] = \mu_R(\omega_1). \tag{B.45}$$

Since $p_R(\omega_1 \,|\, z_1) = -\frac{\alpha_{23}}{1-\alpha_{23}}$ by condition (10) and $\pi_R(z_1 \,|\, \omega_1) = 1$ by condition (B.44), simplifying and rearranging, condition (B.45) becomes

$$\frac{\pi_R(z_1 \,|\, \omega_{23})\mu_R(\omega_{23})}{\mu_R(\omega_1)} = -\frac{1}{\alpha_{23}}. \tag{B.46}$$

Moreover, by condition (3), we have $\pi_R(z_1 \,|\, \omega_{23}) := \lambda\pi(z_1 \,|\, \omega_2) + (1-\lambda)\pi(z_1 \,|\, \omega_3)$. Therefore, condition (B.46) becomes

$$\lambda\pi(z_1 \,|\, \omega_2) = -\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} - (1-\lambda)\pi(z_1 \,|\, \omega_3), \tag{B.47}$$

where for now we ignore the feasibility constraints $\pi(z_1 \,|\, \omega_2) \in [0,1]$ and $\pi(z_1 \,|\, \omega_3) \in [0,1]$.

By conditions (8), (B.44), and (B.47), Sender's expected payoff satisfies

$$\lambda\mathbb{E}_\pi[v(a,\omega)] = \lambda\mu(\omega_1) - \frac{\mu_R(\omega_1)\mu(\omega_2)}{\alpha_{23}\mu_R(\omega_{23})} + \Big[\lambda\mu(\omega_3) - (1-\lambda)\mu(\omega_2)\Big]\pi(z_1 \,|\, \omega_3).$$

Since $\mathbb{E}_\pi[v(a,\omega)]$ is linear in $\pi(z_1 \,|\, \omega_3)$, the optimal information structure depends on the sign of the coefficient on $\pi(z_1 \,|\, \omega_3)$. In particular, we have two cases.

- If the coefficient on $\pi(z_1 \,|\, \omega_3)$ is non-negative, i.e., $\lambda\mu(\omega_3) \geq (1-\lambda)\mu(\omega_2)$, Sender maximizes $\pi(z_1 \,|\, \omega_3)$ s.t. the feasibility constraints $\pi(z_1 \,|\, \omega_2) \in [0,1]$ and $\pi(z_1 \,|\, \omega_3) \in [0,1]$.

  By condition (B.47), we have

$$\pi(z_1 \,|\, \omega_2) \geq 0 \iff \pi(z_1 \,|\, \omega_3) \leq -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})},$$
$$\pi(z_1 \,|\, \omega_3) \leq 1 \iff \pi(z_1 \,|\, \omega_2) \geq -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} - \frac{1-\lambda}{\lambda}.$$

  Therefore, we have

$$\pi(z_1 \,|\, \omega_2) = \min\left\{1, \max\left\{0, -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} - \frac{1-\lambda}{\lambda}\right\}\right\}, \tag{B.48}$$
$$\pi(z_1 \,|\, \omega_3) = \max\left\{0, \min\left\{1, -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}\right\}\right\}. \tag{B.49}$$

  Note that

$$-\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} - \frac{1-\lambda}{\lambda} < 1 \iff \frac{\mu_R(\omega_1)}{\mu_R(\omega_{23})} < -\alpha_{23}. \tag{B.50}$$

Since persuasion is necessary (condition (6)), the inequality on the right-hand side of equivalence (B.50) is satisfied, and so, by condition (B.48), we have

$$\pi(z_1 \,|\, \omega_2) = \max\left\{0, -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} - \frac{1-\lambda}{\lambda}\right\}. \tag{B.51}$$

Since $\alpha_{23} < 0$, we have $-\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} > 0$, and so, by condition (B.49), we have

$$\pi(z_1 \,|\, \omega_3) = \min\left\{1, -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}\right\}. \tag{B.52}$$

Next, note that

$$-\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} - \frac{1-\lambda}{\lambda} \geq 0 \iff -\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \geq 1-\lambda, \tag{B.53}$$

$$-\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} \leq 1 \iff -\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \leq 1-\lambda. \tag{B.54}$$

Therefore, by conditions (B.51)–(B.54), we have the following. If $-\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \geq 1-\lambda$, we have

$$\pi(z_1 \,|\, \omega_2) = -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} - \frac{1-\lambda}{\lambda} \qquad \text{and} \qquad \pi(z_1 \,|\, \omega_3) = 1, \tag{B.55}$$

and the optimality of the information structure in part 1 of the proposition follows from conditions (B.44) and (B.55). If, instead, $-\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} < 1-\lambda$, we have

$$\pi(z_1 \,|\, \omega_2) = 0 \qquad \text{and} \qquad \pi(z_1 \,|\, \omega_3) = -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}, \tag{B.56}$$

and the optimality of the information structure in part 1 of the proposition follows from conditions (B.44) and (B.56).

- If the coefficient on $\pi(z_1 \,|\, \omega_3)$ is negative, that is, $\lambda\mu(\omega_3) < (1-\lambda)\mu(\omega_2)$ Sender minimizes $\pi(z_1 \,|\, \omega_3)$ s.t. the feasibility constraints $\pi(z_1 \,|\, \omega_2) \in [0,1]$ and $\pi(z_1 \,|\, \omega_3) \in [0,1]$.

  By condition (B.47), we have

$$\pi(z_1 \,|\, \omega_2) \leq 1 \iff \pi(z_1 \,|\, \omega_3) \geq -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\lambda}{1-\lambda},$$

$$\pi(z_1 \,|\, \omega_3) \geq 0 \iff \pi(z_1 \,|\, \omega_2) \leq -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}.$$

  Therefore, we have

$$\pi(z_1 \,|\, \omega_2) = \max\left\{0, \min\left\{1, -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}\right\}\right\}, \tag{B.57}$$

$$\pi(z_1 \,|\, \omega_3) = \min\left\{1, \max\left\{0, -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\lambda}{1-\lambda}\right\}\right\}. \tag{B.58}$$

Since $\alpha_{23} < 0$, we have $-\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} > 0$, and so, by condition (B.57), we have

$$\pi(z_1 \,|\, \omega_2) = \min\left\{1, -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}\right\}. \tag{B.59}$$

Note that

$$-\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\lambda}{1-\lambda} < 1 \iff \frac{\mu_R(\omega_1)}{\mu_R(\omega_{23})} < -\alpha_{23}. \tag{B.60}$$

Since persuasion is necessary (condition 6), the inequality on the right-hand side of equivalence (B.60) is satisfied, and so, by condition (B.58), we have

$$\pi(z_1 \,|\, \omega_3) = \max\left\{0, -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\lambda}{1-\lambda}\right\}. \tag{B.61}$$

Next, note that

$$-\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} \le 1 \iff -\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \le \lambda, \tag{B.62}$$

$$-\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\lambda}{1-\lambda} \ge 0 \iff -\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \ge \lambda. \tag{B.63}$$

Hence, by conditions (B.59) and (B.61)–(B.63), we have the following. If $-\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} \ge \lambda$, we have

$$\pi(z_1 \,|\, \omega_2) = 1 \qquad \text{and} \qquad \pi(z_1 \,|\, \omega_3) = -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\lambda}{1-\lambda}, \tag{B.64}$$

and the optimality of the information structure in part 3 of the proposition follows from conditions (B.44) and (B.64). If, instead, $-\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} < \lambda$, we have

$$\pi(z_1 \,|\, \omega_2) = -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} \qquad \text{and} \qquad \pi(z_1 \,|\, \omega_3) = 0, \tag{B.65}$$

and the optimality of the information structure in part 3 of the proposition follows from conditions (B.44) and (B.65).

For all four parts of the proposition, Sender's expected payoff follows from simple algebra by evaluating equation (8) at the corresponding optimal information structure. ■

B.4 Proposition 3

When persuasion is unnecessary under frame Ω_R, the value of re-framing V cannot be positive. The following statement characterizes when Sender finds it optimal to refine Receiver's frame when persuasion is necessary under frame Ω_R.

Formal Statement. *Suppose that persuasion is necessary under frame Ω_R, and let V be the value of re-framing in definition (19).*

1. *If persuasion is unnecessary under frame Ω, then $V > 0$.*
2. *Suppose that persuasion is necessary under frame Ω and conditions (B.1) and (B.2) hold.*

(a) *If conditions* (B.37) *and* (B.38) *hold, then*

$$V > 0 \iff \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)} > -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} - \frac{1-\lambda}{\lambda}.$$

(b) *If condition* (B.37) *holds and condition* (B.38) *does not hold, then* $V > 0$.

(c) *If condition* (B.37) *does not hold and condition* (B.39) *holds, then*

$$V > 0 \iff \frac{\mu(\omega_3)}{\mu(\omega_2)} > (1-\lambda)\left(\frac{1 - \frac{\mu(\omega_1)+\gamma\mu(\omega_3)}{\mu(\omega_2)}}{1 + \frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})}}\right)$$

(d) *If neither condition* (B.37) *nor condition* (B.39) *holds, then*

$$V > 0 \iff \frac{\mu(\omega_3)}{\mu(\omega_2)} > -\frac{\mu_R(\omega_1)}{\alpha_{23}\mu_R(\omega_{23})} - \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}.$$

3. *Suppose that persuasion is necessary under frame* Ω, *condition* (B.1) *holds, and condition* (B.2) *does not hold.*

(a) *If conditions* (B.37) *and* (B.38) *hold, then* $V \leq 0$.

(b) *If condition* (B.37) *holds and* (B.38) *does not hold, then*

$$V > 0 \iff \frac{\mu(\omega_1)}{\gamma\mu(\omega_3)} < \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}.$$

(c) *If condition* (B.37) *does not hold and condition* (B.39) *holds, then*

$$V > 0 \iff \mu(\omega_2) + \frac{1}{\gamma}\mu(\omega_1) - \frac{\mu(\omega_3)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}\mu_R(\omega_1) - \frac{\lambda}{1-\lambda} < 0.$$

(d) *If neither condition* (B.37) *nor condition* (B.39) *holds, then*

$$V > 0 \iff \frac{\mu(\omega_1)}{\mu(\omega_2)} > \frac{\gamma\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}.$$

4. *Suppose that persuasion is necessary under frame* Ω, *condition* (B.1) *does not hold, and condition* (B.3) *holds.*

(a) *If conditions* (B.37) *and* (B.38) *hold, then*

$$V > 0 \iff \frac{\gamma\mu(\omega_3) + \mu(\omega_1) - \mu(\omega_2)}{\alpha_{23}\mu_R(\omega_{23}) + \mu_R(\omega_1)} < \frac{\gamma\mu(\omega_2)}{\lambda\alpha_{23}\mu_R(\omega_{23})}$$

(b) *If condition* (B.37) *holds and condition* (B.38) *does not hold, then*

$$V > 0 \iff \mu(\omega_2) + \mu(\omega_3)\left[\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\mu(\omega_1) - \mu(\omega_2)}{\gamma\mu(\omega_3)}\right] > 0.$$

(c) *If condition* (B.37) *does not hold and condition* (B.39) *holds, then*

$$V > 0 \iff \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} - \frac{\mu(\omega_1) - \mu(\omega_2)}{\gamma\mu(\omega_3)} + \frac{\lambda}{1-\lambda} > 0.$$

(d) *If neither condition* (B.37) *nor condition* (B.39) *holds, then* $V > 0$.

5. *Suppose that persuasion is necessary under frame* Ω *and neither condition* (B.1) *nor condition* (B.3) *holds.*

   (a) *If conditions* (B.37) *and* (B.38) *hold, then*

$$V > 0 \iff \mu(\omega_3) - \mu(\omega_2)\left[\frac{\mu(\omega_1)}{\mu(\omega_2)} + \frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} + \frac{1-\lambda}{\lambda}\right] < 0.$$

   (b) *If condition* (B.37) *holds and condition* (B.38) *does not hold, then*

$$V > 0 \iff \frac{\mu(\omega_1)}{\mu(\omega_3)} > -\frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}.$$

   (c) *If condition* (B.37) *does not hold and condition* (B.39) *holds, then* $V > 0$.

   (d) *If neither condition* (B.37) *nor condition* (B.39) *hold, then*

$$V > 0 \iff \frac{\mu(\omega_1)}{\mu(\omega_2)} > -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}.$$

Proof. The desired result follows by evaluating the value of re-framing V in each case.

- Part 1 of the proposition is obvious.
- For part 2 of the proposition, we have the following. Part (a) follows from equations (B.4) and (B.40). Part (b) follows from equations (B.4) and (B.41). Part (c) follows from equations (B.4) and (B.42). Part (d) follows from equations (B.4) and (B.43).
- For part 3 of the proposition, we have the following. Part (a) follows from equations (B.5) and (B.40). Part (b) follows from equations (B.5) and (B.41). Part (c) follows from equations (B.5) and (B.42). Part (d) follows from equations (B.5) and (B.43).
- For part 4 of the proposition, we have the following. Part (a) follows from equations (B.6) and (B.40). Part (b) follows from equations (B.6) and (B.41). Part (c) follows from equations (B.6) and (B.42). Part (d) follows from equations (B.6) and (B.43).
- For part 5 of the proposition, we have the following. Part (a) follows from equations (B.7) and (B.40). Part (b) follows from equations (B.7) and (B.41). Part (c) follows from equations (B.7) and (B.42). Part (d) follows from equations (B.7) and (B.43). ■

B.5 Proof of Proposition 4

Receiver's welfare corresponds to her expected payoff. Recall that π_f (resp., π_c) denotes the optimal information structure when Receiver's frame is Ω (resp., Ω_R). See Appendix B.2 for the characterization of π_f and Appendix B.3 for the characterization of π_c.

Subjective Criterion. According to the subjective criterion, we evaluate Receiver's welfare using her preferences, beliefs, and conception of the information structure conditional on her frame $\Omega' \in \{\Omega_R, \Omega\}$. If $\Omega' = \Omega$, Receiver's welfare, denoted by U, is

$$U = \pi(z_1 \,|\, \omega_1)\mu(\omega_1) - \pi(z_1 \,|\, \omega_2)\mu(\omega_2) + \gamma\pi(z_1 \,|\, \omega_3)\mu(\omega_3), \tag{B.66}$$

where $\pi = \pi_f$. If $\Omega' = \Omega_R$, Receiver's welfare, denoted by U_R, is

$$U_R = \pi_R(z_1 \,|\, \omega_1)\mu_R(\omega_1) + \alpha_{23}\pi_R(z_1 \,|\, \omega_{23})\mu_R(\omega_{23}),$$

where π_R is Receiver's conception of π_c.

If persuasion is unnecessary under frame $\Omega' \in \{\Omega_R, \Omega\}$, Sender leaves Receiver's prior beliefs unchanged, and Receiver takes action a_1. If $\Omega' = \Omega$, this fact implies that

$$U = \mu(\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3) \geq 0,$$

where the inequality holds because, by condition (5), persuasion is unnecessary under frame Ω if and only if $\mu(\omega_1) + \gamma\mu(\omega_3) \geq \mu(\omega_2)$. If, instead, $\Omega' = \Omega_R$, this fact implies that

$$U_R = \mu_R(\omega_1) + \alpha_{23}\mu_R(\omega_{23}) \geq 0,$$

where the inequality holds because, by condition (6), persuasion is unnecessary under frame Ω_R if and only if $\mu_R(\omega_1) \geq -\alpha_{23}\mu_R(\omega_{23})$.

If persuasion is necessary under both frames, $U = U_R = 0$ follows immediately from two observations. First, the optimal information structure under any frame Ω' is such that: (i) Receiver is indifferent between actions after signal z_1; (ii) Receiver prefers action a_2 after signal z_2. Second, Receiver's payoff from action a_2 equals 0 under both frames.

From the previous observations, Part 1–(a) of the proposition immediately follows. Part 1–(b) of the proposition immediately follows, with Receiver having a payoff equal to zero under the frame where persuasion is necessary and a positive payoff under the frame where persuasion is unnecessary. Part 1–(c) of the proposition follows by noting that, depending on the parameters, we may have $\mu(\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3) \geq \mu_R(\omega_1) + \alpha_{23}\mu_R(\omega_{23})$, i.e., $U \geq U_R$, or $\mu_R(\omega_1) + \alpha_{23}\mu_R(\omega_{23}) \geq (\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3)$, i.e., $U_R \geq U$. ■

Objective Criterion. According to the objective criterion, we evaluate Receiver's welfare from the viewpoint of an outside observer with the fine frame. To do so, we use equation (B.66), where we replace π with either π_f or π_c depending Receiver's frame.

Denote by $U(\pi)$ equation (B.66) evaluated at information structure $\pi \in \{\pi_c, \pi_f\}$. Define

$$\begin{aligned} \Delta U &:= U(\pi_f) - U(\pi_c) \\ &= \mu(\omega_2)[\pi_c(z_1 \,|\, \omega_2) - \pi_f(z_1 \,|\, \omega_2)] + \gamma\mu(\omega_3)[\pi_f(z_1 \,|\, \omega_3) - \pi_c(z_1 \,|\, \omega_3)], \end{aligned} \tag{B.67}$$

where the equality holds because $\pi_f(z_1 \,|\, \omega_1) = \pi_c(z_1 \,|\, \omega_1) = 1$. Since Receiver is better off under the fine frame if and only if $\Delta U > 0$, part 2 of the proposition follows. ■

We refer to Appendix C for additional details on Receiver's welfare according to the objective criterion.

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Supplemental Appendix

C Objective Criterion: Further Details

The following proposition provides a characterization of the comparison between Receiver's objective welfare under the coarse and the fine frame that we use to obtain Figure 3.

Proposition 5. *Consider Receiver's welfare from the viewpoint of the objective criterion, and let ΔU be as in definition (B.67). We distinguish four cases:*

1. *If persuasion is unnecessary under both frames, then $\Delta U = 0$.*
2. *If persuasion is necessary under frame Ω and unnecessary under frame Ω_R, then $\Delta U > 0$.*
3. *Suppose that persuasion is necessary under frame Ω_R and unnecessary under frame Ω.*
   - *(a) If $\gamma < 0$, then $\Delta U < 0$.*
   - *(b) If $\gamma > 0$, we have the following:*
     - *(i) If conditions (B.37) and (B.38) hold, then $\Delta U < 0$.*
     - *(ii) If condition (B.37) holds and condition (B.38) does not hold, then*

$$\Delta U < 0 \iff \gamma \frac{\mu(\omega_3)}{\mu(\omega_2)} \left[1 + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}\right] < 1.$$

     - *(iii) If condition (B.37) does not hold and condition (B.39) holds, then $\Delta U > 0$.*
     - *(iv) If neither condition (B.37) nor condition (B.39) holds, then*

$$\Delta U < 0 \iff \gamma \frac{\mu(\omega_3)}{\mu(\omega_2)} < \left[1 + \frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}\right].$$

4. *Suppose that persuasion is necessary under both frames.*
   - *(a) If conditions (B.1) and (B.2) hold, we have the following:*
     - *(i) If conditions (B.37) and (B.38) hold, then*

$$\Delta U < 0 \iff \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)} + \frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} + \frac{1-\lambda}{\lambda} > 0.$$

     - *(ii) If condition (B.37) holds and condition (B.38) does not hold, then*

$$\Delta U < 0 \iff \mu(\omega_2)\left[-\frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}\right] + \gamma\mu(\omega_3)\left[1 + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}\right] < 0.$$

     - *(iii) If condition (B.37) does not hold and condition (B.39) holds, then*

$$\Delta U < 0 \iff \mu(\omega_2)\left[1 - \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}\right] + \gamma\mu(\omega_3)\left[1 + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} + \frac{\lambda}{1-\lambda}\right] < 0.$$

(iv) If neither condition (B.37) *nor condition* (B.39) *holds, then*

$$\Delta U < 0 \iff \gamma\mu(\omega_3) < \mu(\omega_2)\left[\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} + \frac{\mu(\omega_1)+\gamma\mu(\omega_3)}{\mu(\omega_2)}\right].$$

(b) If condition (B.1) *holds and condition* (B.2) *does not hold, we have the following:*

(i) If conditions (B.37) *and* (B.38) *hold, then* $\Delta U > 0$.

(ii) If condition (B.37) *holds and* (B.38) *does not hold, then*

$$\Delta U < 0 \iff -\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)} + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} > 0.$$

(iii) If condition (B.37) *does not hold and* (B.39) *holds, then*

$$\Delta U < 0 \iff \mu(\omega_2) + \gamma\mu(\omega_3)\left[-\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)} + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} + \frac{\lambda}{1-\lambda}\right] < 0.$$

(iv) If neither condition (B.37) *nor condition* (B.39) *holds, then*

$$\Delta U < 0 \iff -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} < \frac{\mu(\omega_1)}{\mu(\omega_2)}.$$

(c) If condition (B.1) *does not hold and condition* (B.3) *holds, we have the following:*

(i) If conditions (B.37) *and* (B.38) *hold, then*

$$\Delta U < 0 \iff -\mu(\omega_2)\left[1+\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})}+\frac{1-\lambda}{\lambda}\right]-\gamma\mu(\omega_3)\left[1-\frac{\mu(\omega_2)-\mu(\omega_1)}{\gamma\mu(\omega_3)}\right] < 0.$$

(ii) If condition (B.37) *holds and condition* (B.38) *does not hold, then*

$$\Delta U < 0 \iff -\mu(\omega_2) + \gamma\mu(\omega_3)\left[\frac{\mu(\omega_2)-\mu(\omega_1)}{\gamma\mu(\omega_3)} + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})}\right] < 0.$$

(iii) If condition (B.37) *does not hold and* (B.39) *holds, then*

$$\Delta U < 0 \iff \frac{\mu(\omega_2)-\mu(\omega_1)}{\gamma\mu(\omega_3)} + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} + \frac{\lambda}{1-\lambda} > 0.$$

(iv) If neither condition (B.37) *nor condition* (B.39) *holds, then* $\Delta U < 0$.

(d) If conditions (B.1) *and* (B.3) *do not hold, we have the following:*

(i) If conditions (B.37) *and* (B.38) *hold, then*

$$\Delta U < 0 \iff -\mu(\omega_2)\left[\frac{\mu(\omega_1)}{\mu(\omega_2)} + \frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} + \frac{1-\lambda}{\lambda}\right] - \gamma\mu(\omega_3) < 0.$$

(ii) If condition (B.37) *holds and condition* (B.38) *does not hold, then*

$$\Delta U < 0 \iff -\frac{\mu(\omega_1)}{\gamma\mu(\omega_3)} + \frac{\mu_R(\omega_1)}{(1-\lambda)\alpha_{23}\mu_R(\omega_{23})} > 0.$$

(iii) If condition (B.37) *does not hold and condition* (B.39) *holds, then* $\Delta U > 0$.

(iv) If neither condition (B.37) *nor condition* (B.39) *holds, then*

$$\Delta U < 0 \iff -\frac{\mu_R(\omega_1)}{\lambda\alpha_{23}\mu_R(\omega_{23})} < \frac{\mu(\omega_1)}{\mu(\omega_2)}.$$

Proof. The desired result follows by evaluating ΔU in each case.

- Part 1 of the proposition follows by observing that $\pi(z_1 \,|\, \omega) = 1$ for all $\pi \in \{\pi_c, \pi_f\}$ and $\omega \in \Omega$.

- For part 2 of the proposition, we have the following. Since persuasion is unnecessary under frame Ω_R, $\pi_c(z_1 \,|\, \omega) = 1$ for all $\omega \in \Omega$. First, suppose $\gamma > 0$. In this case, we have $\pi_f(z_1 \,|\, \omega_3) = 1$, and so $\Delta U = \mu(\omega_2)[1 - \pi_f(z_1 \,|\, \omega_2)] > 0$ because $\pi_f(z_1 \,|\, \omega_2) < 1$. Second, suppose $\gamma < 0$. In this case, we have $\Delta U = \mu(\omega_2)[1 - \pi_f(z_1 \,|\, \omega_2)] - \gamma\mu(\omega_3)[1 - \pi_f(z_1 \,|\, \omega_3)] > 0$ because $\pi_f(z_1 \,|\, \omega_2) < 1$ or $\pi_f(z_1 \,|\, \omega_3) < 1$ (or both).

- For part 3 of the proposition, we have the following. Since persuasion is unnecessary under frame Ω, $\pi_f(z_1 \,|\, \omega) = 1$ for all $\omega \in \Omega$. Therefore, we rewrite equation (B.67) as $\Delta U = \gamma\mu(\omega_3)[1 - \pi_c(z_1 \,|\, \omega_3)] - \mu(\omega_2)[1 - \pi_c(z_1 \,|\, \omega_2)]$. Part 3–(a) is trivial since $\gamma < 0$, and $\pi_c(z_1 \,|\, \omega_2) < 1$ or $\pi_c(z_1 \,|\, \omega_3) < 1$ (or both). Part 3–(b) follows from plugging in ΔU the expressions for $\pi_c(z_1 \,|\, \omega_2)$ and $\pi_c(z_1 \,|\, \omega_3)$ specified in parts 1–4 of the formal statement of Proposition 2 in Section B.3, respectively.

- For part 4 of the proposition, we have the following. In each part (a)–(d), we use the expressions for $\pi_f(z_1 \,|\, \omega_2)$ and $\pi_f(z_1 \,|\, \omega_3)$ specified in parts 1–4 of the formal statement of Proposition 1 in Section B.2, respectively. In each subpart (i)–(iv), we use the expressions for $\pi_c(z_1 \,|\, \omega_2)$ and $\pi_c(z_1 \,|\, \omega_3)$ specified in parts 1–4 of the formal statement of Proposition 2 in Section B.3, respectively. ■

D Alternative Coarse Frames

D.1 Coarse Understanding of States ω_1 and ω_3

D.1.1 Model

At the beginning of the game, Receiver cannot distinguish state ω_1 from state ω_3: her coarse frame is $\Omega_R := \{\omega_{13}, \omega_2\}$, where we write $\omega = \omega_{13}$ if and only if $\omega \in \{\omega_1, \omega_3\}$. We describe below the elements of the model that differ from our main specification in Section 2; otherwise, the model is as in Section 2.

Under the coarse frame, Receiver's preferences are represented by the payoff function $u_R \colon A \times \Omega_R \to \mathbb{R}$, where

$$u_R(a, \omega) := \begin{cases} \alpha_{13} & \text{if } a = a_1 \wedge \omega = \omega_{13} \\ -1 & \text{if } a = a_1 \wedge \omega = \omega_2 \\ 0 & \text{if } a = a_2 \end{cases},$$

and $\alpha_{13} := \lambda + (1-\lambda)\gamma$ for some $\lambda \in [0,1]$.

Receiver's prior beliefs under frame Ω_R are sub-additive, and $\mu_R(\omega_{13}) := \lambda\mu(\omega_1) + (1-\lambda)\mu(\omega_3)$. This condition implies $\mu_R(\omega_2) > \mu(\omega_2)$: Receiver conceives state ω_2 as more likely under the coarse frame than under the fine frame.

Under frame Ω_R, Receiver conceives the signal distribution in state ω_{13} as a convex combination of the signal distributions chosen by the Sender under ω_1 and ω_3. That is, $\pi_R(z \,|\, \omega_{13}) := \lambda\pi(z \,|\, \omega_1) + (1-\lambda)\pi(z \,|\, \omega_3)$, whereas $\pi_R(z \,|\, \omega_2) := \pi(z \,|\, \omega_2)$.

D.1.2 Information Design

The following lemma characterizes Receiver's optimal action under the coarse. Its proof mimics the proof of Lemma 1 in Appendix B.1.

Lemma 2. *If $\Omega' = \Omega_R$, Receiver's optimal action $a\colon \Delta(\Omega_R) \to A$ satisfies $a(p_R) = a_1$ if and only if $\alpha_{13}p_R(\omega_{13}) \geq p_R(\omega_2)$.*

Next, we define the notion of possible persuasion under the coarse frame.

Definition 5. Persuasion is impossible *under frame Ω_R if $a(p_R) = a_2$ for all $p_R \in \Delta(\Omega_R)$, and* possible *otherwise.*

By Lemma 2 and Definitions 1 and 5, we have the following:

- Persuasion is necessary under frame Ω_R if and only if

$$\alpha_{13}\mu_R(\omega_{13}) < \mu_R(\omega_2). \tag{D.1}$$

- Persuasion is possible under frame Ω_R if and only if $\alpha_{13} \geq 0$.

If persuasion is unnecessary or impossible, without loss, we assume that Sender leaves Receiver's prior beliefs unchanged. If persuasion is necessary and possible under frame Ω_R, the posterior beliefs induced by the optimal information structure satisfy the following properties:

(a) Signal z_1 makes Receiver indifferent between actions a_1 and a_2, implying that

$$p_R(\omega_{13} \,|\, z_1) = \frac{1}{1+\alpha_{13}}. \tag{D.2}$$

(b) Signal z_2 makes Receiver certain that the state is one in which action a_2 is uniquely optimal to her, implying that

$$p_R(\omega_{13} \,|\, z_2) = 0. \tag{D.3}$$

Proposition 6 characterizes the optimal information structure under the coarse frame.

Proposition 6. *Suppose $\Omega' = \Omega_R$ and persuasion is necessary and possible. Then, the optimal information structure is*

$$\pi(z_1 \,|\, \omega_1) = 1, \qquad \pi(z_1 \,|\, \omega_2) = \frac{\alpha_{13}\mu_R(\omega_{13})}{\mu_R(\omega_2)}, \qquad \pi(z_1 \,|\, \omega_3) = 1, \tag{D.4}$$

and Sender's expected payoff under the optimal information structure is

$$\mathbb{E}_\pi[v(a,\omega)] = 1 - \mu(\omega_2)\left[1 - \frac{\mu_R(\omega_{13})\alpha_{13}}{\mu_R(\omega_2)}\right]. \tag{D.5}$$

Proof. By equation (4) and since $p_R(\omega_{13} \,|\, z_2) = 0$ (by condition (D.3)), we have $\pi_R(z_2 \,|\, \omega_{13}) = 0$, from which it follows that

$$\pi_R(z_1 \,|\, \omega_{13}) = 1. \tag{D.6}$$

Since $\pi_R(z_1 \,|\, \omega_{13}) := \lambda\pi(z_1 \,|\, \omega_1) + (1-\lambda)\pi(z_1 \,|\, \omega_3) = 1$ and $\lambda \in [0,1]$, condition (D.6) implies that the optimal information structure must satisfy

$$\pi(z_1 \,|\, \omega_1) = \pi(z_1 \,|\, \omega_3) = 1. \tag{D.7}$$

By Bayes plausibility (equation (7)) and since $p_R(\omega_{13} \,|\, z_2) = 0$ (by condition (D.3)), we have

$$p_R(\omega_{13} \,|\, z_1)\big[\pi_R(z_1 \,|\, \omega_{13})\mu_R(\omega_{13}) + \pi_R(z_1 \,|\, \omega_2)\mu_R(\omega_2)\big] = \mu_R(\omega_{13}). \tag{D.8}$$

Since $p_R(\omega_{13} \,|\, z_1) = \frac{1}{1+\alpha_{13}}$ by condition (D.2) and $\pi_R(z_1 \,|\, \omega_{13}) = 1$ by condition (D.6), simplifying and rearranging, condition (D.8) becomes

$$\frac{\pi_R(z_1 \,|\, \omega_2)\mu_R(\omega_2)}{\mu_R(\omega_{13})} = \alpha_{13}. \tag{D.9}$$

In turn, condition (D.9) implies that

$$\pi_R(z_1 \,|\, \omega_2) = \pi(z_1 \,|\, \omega_2) = \frac{\alpha_{13}\mu_R(\omega_{13})}{\mu_R(\omega_2)}. \tag{D.10}$$

The optimality of the information structure described by the equalities in (D.4) follows from conditions (D.7) and (D.10). Equality (D.5) follows by evaluating equation (8) at the optimal information structure. ■

Proposition 6 shows how Sender exploits Receiver's inability to distinguish state ω_1 from state ω_3 under this alternative coarse frame. Unlike our main specification, there is no persuasion trade-off. If persuasion is possible, i.e., $\alpha_{13} \geq 0$, it is optimal for Sender to recommend action a_1 with probability 1 in state ω_{13}. In other words, Sender pools states ω_1 and ω_3: $\pi(z_1 \,|\, \omega_1) = \pi(z_1 \,|\, \omega_3) = 1$.

D.1.3 Frame Choice

When persuasion is unnecessary (resp., impossible) under frame Ω_R, the value of reframing V cannot be positive (resp., negative). Proposition 7 characterizes when Sender finds it optimal to refine Receiver's frame when persuasion is necessary and possible under frame Ω_R.

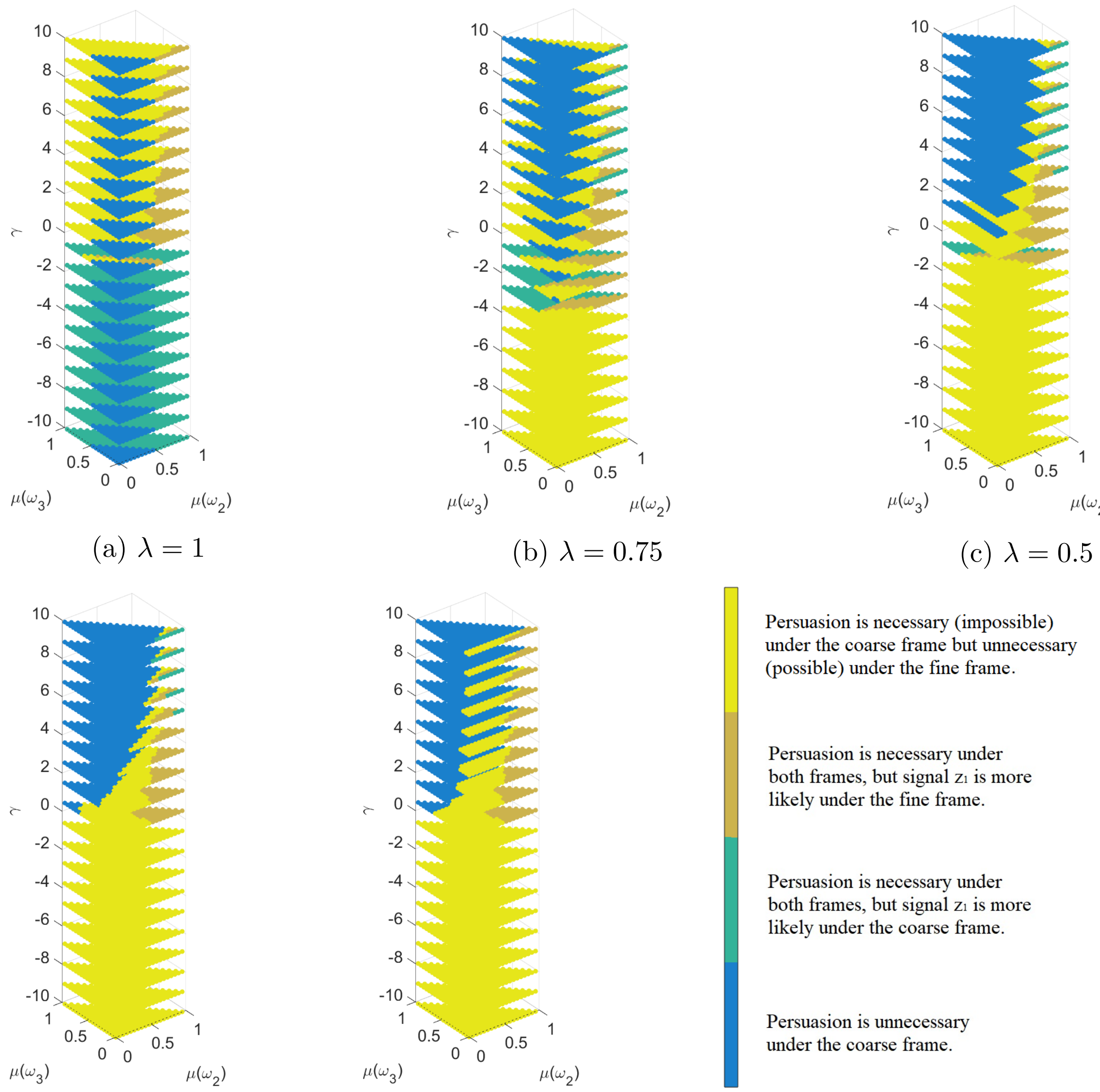
Proposition 7. *Suppose that persuasion is necessary and possible under frame Ω_R, and let V be the value of re-framing in definition (19).*

1. *If persuasion is unnecessary under frame Ω, then $V > 0$.*
2. *Suppose that persuasion is necessary under frame Ω and conditions (B.1) and (B.2) hold. Then,*
$$V > 0 \iff \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)} > \frac{\mu_R(\omega_{13})\alpha_{13}}{\mu_R(\omega_2)}.$$
3. *Suppose that persuasion is necessary under frame Ω, condition (B.1) holds, and condition (B.2) does not hold. Then, $V < 0$.*
4. *Suppose that persuasion is necessary under frame Ω, condition (B.1) does not hold, and condition (B.3) holds. Then,*
$$V > 0 \iff \frac{\mu(\omega_3)}{\mu(\omega_2)} < \frac{1 - \frac{\mu_R(\omega_{13})\alpha_{13}}{\mu_R(\omega_2)}}{1 - \frac{\mu(\omega_2)-\mu(\omega_1)}{\gamma\mu(\omega_3)}}.$$
5. *Suppose that persuasion is necessary under frame Ω and neither condition (B.1) nor condition (B.3) holds. Then,*
$$V > 0 \iff \frac{\mu(\omega_1)}{\mu(\omega_2)} - \frac{\mu_R(\omega_{13})\alpha_{13}}{\mu_R(\omega_2)} > \frac{\mu(\omega_3)}{\mu(\omega_2)}.$$

Proof. The desired result follows by evaluating the value of re-framing V in each case. Part 1 of the proposition is obvious. Part 2 follows from equations (B.4) and (D.5). Part 3 follows from equations (B.5) and (D.5). Part 4 follows from equations (B.6) and (D.5). Part 5 follows from equations (B.7) and (D.5). ■

Graphical Illustration. Figure D.1 shows how the optimal frame depends on the parameter λ. In blue, we represent the region of the parameter space where persuasion is unnecessary under the coarse frame. In green, we represent the region of the parameter space where persuasion is necessary under both frames but signal z_1 under the optimal information structure is more likely under the coarse frame than under the fine frame. The value of re-framing cannot be positive in these regions.

In yellow, we represent the region of the parameter space where persuasion is necessary (resp., impossible) under the coarse frame and unnecessary (resp., possible) under the fine frame. This region corresponds to part 1 of Proposition 7. In gold, we represent the region of the parameter space where persuasion is necessary under both frames but signal z_1 is more likely under the fine than under the coarse frame at the optimal information structure. This region corresponds to part 2 of Proposition 7. The value of re-framing is positive in these regions.



(a) $\lambda = 1$ (b) $\lambda = 0.75$ (c) $\lambda = 0.5$

(d) $\lambda = 0.25$ (e) $\lambda = 0$ (f) Legend

Figure D.1: The optimal frame when as a function of λ.

Figure D.1 displays a complex relationship between the value of λ and the region of the parameter space where the value of re-framing is positive. When $\lambda = 1$, Receiver conceives the coarse state ω_{13} as state ω_1—which is favorable to Sender—ignoring state ω_3, which is also favorable to Sender if $\gamma > 0$, and unfavorable otherwise. The opposite occurs when $\lambda = 0$. For any $\lambda \in (0, 1)$, Receiver's conception of the coarse state ω_{13} is a non-trivial combination of states ω_1 and ω_3.

First, suppose $\lambda = 1$. In this case, Sender can completely conceal state ω_3 under the coarse frame, whereas no concealing is possible upon re-framing. Given this, the value of re-framing is positive if persuasion is necessary under the coarse frame—which happens if $\mu(\omega_1) \leq \frac{1}{2}$ by condition (D.1)—and $\gamma \geq 0$. To gain intuition, we distinguish two cases depending on the value of γ:

- If $\gamma > 0$, state ω_3 is favorable to Sender. In this case, Sender can recommend action a_1 with probability 1 in state ω_3 under both frames. Hence, Sender's decision upon re-framing depends on the likelihood of producing evidence in favor of his goal (i.e., signal z_1) in state ω_2. Since sub-additivity makes state ω_2 more likely under the coarse frame, and $\gamma \geq 0$ implies that the expected payoff associated with action a_1 is increasing in the likelihood of state ω_3 under the fine frame, signal z_1 is more likely under the fine frame. As a result, the value of re-framing is positive.
- If $\gamma < 0$, state ω_3 is unfavorable to Sender. We distinguish two cases: $\gamma \in (-1, 0)$ and $\gamma \leq -1$. When $\gamma \in (-1, 0)$, Sender faces a trade-off: sub-additivity worsens Sender's ability to persuade under the coarse frame as before, but preferences make it harder to persuade under the fine frame because Sender cannot conceal ω_3, which is now unfavorable. Re-framing is usually sub-optimal, unless state ω_1 is sufficiently likely, particularly $\frac{1}{2} \geq \mu(\omega_1) \geq -\frac{\gamma}{1-\gamma}$. Instead, when $\gamma \leq -1$, the value of re-framing is negative for all $\mu(\omega_1) \leq \frac{1}{2}$.

When λ decreases, the region of the parameter space where the value of re-framing is positive expands for $\gamma < 0$, while it shrinks for $\gamma > 0$. For $\gamma < 0$, as λ decreases, the region where persuasion is unnecessary under the coarse frame shrinks because the lower λ, the lower is α_{13}, and thus the left-hand side of equation (D.1). If γ is sufficiently small, such that $\alpha_{13} < 0$, action a_2 is optimal for Receiver under the coarse frame, independently of the state—that is, persuasion is impossible—forcing Sender to re-frame. For instance, when $\lambda = 0.75$, persuasion becomes impossible under the coarse frame for $\gamma \leq -3$. When $\gamma \in (-3, 0)$, persuasion is possible under the coarse frame but is also necessary in a vaster region. Therefore, the region where persuasion is necessary under the coarse frame but unnecessary under the fine frame becomes larger. This region exists because of sub-additivity. Consider, for instance, $\gamma = -1$. Persuasion is unnecessary under the fine frame if and only if $\mu(\omega_1) > \frac{1}{2}$. At the same time, persuasion is necessary under the coarse frame if and only if $3\mu(\omega_1) + \mu(\omega_3) \leq \frac{8}{3}$. Instead, for $\gamma > 0$, the region where persuasion is unnecessary under coarse frame expands through an increase in α_{13}. For instance, consider $\lambda = 0.75$ and $\gamma = 2$. Persuasion is unnecessary under the coarse frame if $3\mu(\omega_1) + \mu(\omega_3) > \frac{16}{9}$. The region becomes larger as γ increases. Outside of this region, re-framing is optimal because of sub-additivity.

D.1.4 Welfare

The following proposition summarizes our findings about Receiver's welfare. Again, re-framing can harm Receiver.

Proposition 8. *According to the subjective criterion:*

(a) If persuasion is necessary under both frames, Receiver is indifferent between frames.

(b) If persuasion is necessary or impossible under one frame but unnecessary under the other, Receiver is better off under the latter.

(c) If persuasion is unnecessary under both frames, Receiver can be better off under either frame.

According to the objective criterion, Receiver is better off under the fine frame if and only if

$$\mu(\omega_2)[\pi_c(z_1 \,|\, \omega_2) - \pi_f(z_1 \,|\, \omega_2)] - \gamma\mu(\omega_3)[1 - \pi_f(z_1 \,|\, \omega_3)] > 0. \tag{D.11}$$

Proof. Receiver's welfare corresponds to her expected payoff. Moreover, recall that π_f (resp., π_c) denotes the optimal information structure when Receiver's frame is Ω (resp., Ω_R). See Appendix B.2 for the characterization of π_f and Proposition 6 for the characterization of π_c. ■

Subjective Criterion. According to the subjective criterion, we evaluate Receiver's welfare using her preferences, beliefs, and conception of the information structure conditional on her frame $\Omega' \in \{\Omega_R, \Omega\}$. If $\Omega' = \Omega$, Receiver's welfare, denoted by U, is

$$U = \pi(z_1 \,|\, \omega_1)\mu(\omega_1) - \pi(z_1 \,|\, \omega_2)\mu(\omega_2) + \gamma\pi(z_1 \,|\, \omega_3)\mu(\omega_3),$$

where $\pi = \pi_f$. If $\Omega' = \Omega_R$, Receiver's welfare, denoted by U_R, is

$$U_R \coloneqq \alpha_{13}\pi_R(z_1 \,|\, \omega_{13})\mu_R(\omega_{13}) - \pi_R(z_1 \,|\, \omega_2)\mu_R(\omega_2),$$

where π_R is Receiver's conception of π_c.

If persuasion is unnecessary under frame $\Omega' \in \{\Omega_R, \Omega\}$, Sender leaves Receiver's prior beliefs unchanged, and Receiver takes action a_1. If $\Omega' = \Omega$, this fact implies that

$$U = \mu(\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3) \geq 0,$$

where the inequality holds because, by condition (5), persuasion is unnecessary under frame Ω if and only if $\mu(\omega_1) + \gamma\mu(\omega_3) \geq \mu(\omega_2)$. If, instead, $\Omega' = \Omega_R$, this fact implies that

$$U_R = \alpha_{13}\mu_R(\omega_{13}) - \mu_R(\omega_2) \geq 0,$$

where the inequality holds because, by condition (D.1), persuasion is unnecessary under frame Ω_R if and only if $\alpha_{13}\mu_R(\omega_{13}) \leq \mu_R(\omega_2)$.

If persuasion is necessary under both frames, $U = U_R = 0$ follows immediately from two observations. First, the optimal information structure under any frame Ω' is such that: (i) Receiver is indifferent between actions after signal z_1; (ii) Receiver prefers action a_2 after signal z_2. Second, Receiver's payoff from action a_2 equals 0 under both frames.

From the previous observations, Part 1–(a) of the proposition immediately follows. Part 1–(b) of the proposition immediately follows, with Receiver having a payoff equal to zero under the frame where persuasion is necessary or impossible and a positive payoff under the frame where persuasion is unnecessary. Part 1–(c) of the proposition follows by noting that, depending on the parameters, we may have $\mu(\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3) \geq \alpha_{13}\mu_R(\omega_{13}) - \mu_R(\omega_2)$, i.e., $U \geq U_R$, or $\alpha_{13}\mu_R(\omega_{13}) - \mu_R(\omega_2) \geq \mu(\omega_1) - \mu(\omega_2) + \gamma\mu(\omega_3)$, i.e., $U_R \geq U$. ■

Objective Criterion. According to the objective criterion, we evaluate Receiver's welfare from the viewpoint of an outside observer with the fine frame. To do so, we use (B.66), where we replace π with either π_f or π_c depending Receiver's frame.

Denote by $U(\pi)$ equation (B.66) evaluated at information structure $\pi \in \{\pi_c, \pi_f\}$. Define

$$\begin{aligned} \Delta U &:= U(\pi_f) - U(\pi_c) \\ &= \mu(\omega_2)[\pi_c(z_1 \,|\, \omega_2) - \pi_f(z_1 \,|\, \omega_2)] - \gamma\mu(\omega_3)[1 - \pi_f(z_1 \,|\, \omega_3)], \end{aligned} \tag{D.12}$$

where the equality holds because $\pi_f(z_1 \,|\, \omega_1) = \pi_c(z_1 \,|\, \omega_1) = \pi_c(z_1 \,|\, \omega_3) = 1$. Since Receiver is better off under the fine frame if and only if $\Delta U > 0$, part 2 of the proposition follows. ■

The following proposition provides a characterization of the comparison between Receiver's objective welfare under the coarse and the fine frame that we use to obtain Figure D.3.

Proposition 9. *Consider Receiver's welfare from the viewpoint of the objective criterion, and let ΔU be as in definition (B.67). We distinguish four cases:*

1. *If persuasion is unnecessary under both frames, then $\Delta U = 0$.*
2. *If persuasion is impossible under frame Ω_R, then $\Delta U \geq 0$.*
3. *If persuasion is necessary under frame Ω_R and unnecessary under frame Ω, then $\Delta U < 0$.*
4. *If persuasion is necessary under frame Ω and unnecessary under frame Ω_R, then $\Delta U > 0$.*
5. *Suppose that persuasion is necessary under both frames.*
   (a) *If conditions (B.1) and (B.2) hold, then*
   $$\Delta U < 0 \iff \frac{\alpha_{13}\mu_R(\omega_{13})}{\mu_R(\omega_2)} < \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{\mu(\omega_2)}.$$
   (b) *If condition (B.1) holds and condition (B.2) does not hold, then $\Delta U > 0$.*
   (c) *If condition (B.1) does not hold and condition (B.3) holds, then*
   $$\Delta U < 0 \iff -\gamma\mu(\omega_3)\left[1 - \frac{\mu(\omega_2) - \mu(\omega_1)}{\gamma\mu(\omega_3)}\right] < \mu(\omega_2)\left[1 - \frac{\alpha_{13}\mu_R(\omega_{13})}{\mu_R(\omega_2)}\right].$$
   (d) *If conditions (B.1) and (B.3) do not hold, then*
   $$\Delta U < 0 \iff \frac{\alpha_{13}\mu_R(\omega_{13})}{\mu_R(\omega_2)} < \frac{\mu(\omega_1) + \gamma\mu(\omega_3)}{.}\mu(\omega_2)$$

Proof. The desired result follows by evaluating ΔU in each case.

- Part 1 of the proposition follows by observing that $\pi(z_1 \,|\, \omega) = 1$ for all $\pi \in \{\pi_c, \pi_f\}$ and $\omega \in \Omega$.

- Part 2 of the proposition follows trivially from the observation that when persuasion is impossible—Receiver chooses action a_2 independently of beliefs—then $U(\pi_c) = 0$ for all π_c and $\Delta U = U(\pi_f) \geq 0$.

- For part 3 of the proposition, we have the following. Since persuasion is unnecessary under Ω, $\pi_f(z_1 \,|\, \omega) = 1$ for all $\omega \in \Omega$. Therefore, we rewrite equation (D.12) as $\Delta U = -\mu(\omega_2)[1 - \pi_c(z_1 \,|\, \omega_2)] < 0$ since $\pi_c(z_1 \,|\, \omega_2) < 1$.

- For part 4 of the proposition, we have the following. Since persuasion is unnecessary under Ω_R, $\pi_c(z_1 \,|\, \omega) = 1$ for all $\omega \in \Omega$. First, suppose $\gamma > 0$. In this case, we have $\pi_f(z_1 \,|\, \omega_3) = 1$, and so $\Delta U = \mu(\omega_2)[1 - \pi_f(z_1 \,|\, \omega_2)] > 0$ because $\pi_f(z_1 \,|\, \omega_2) < 1$. Second, suppose $\gamma < 0$. In this case, we have $\Delta U = \mu(\omega_2)[1 - \pi_f(z_1 \,|\, \omega_2)] - \gamma\mu(\omega_3)[1 - \pi_f(z_1 \,|\, \omega_3)] > 0$ because $\pi_f(z_1 \,|\, \omega_2) < 1$ or $\pi_f(z_1 \,|\, \omega_3) < 1$ (or both).

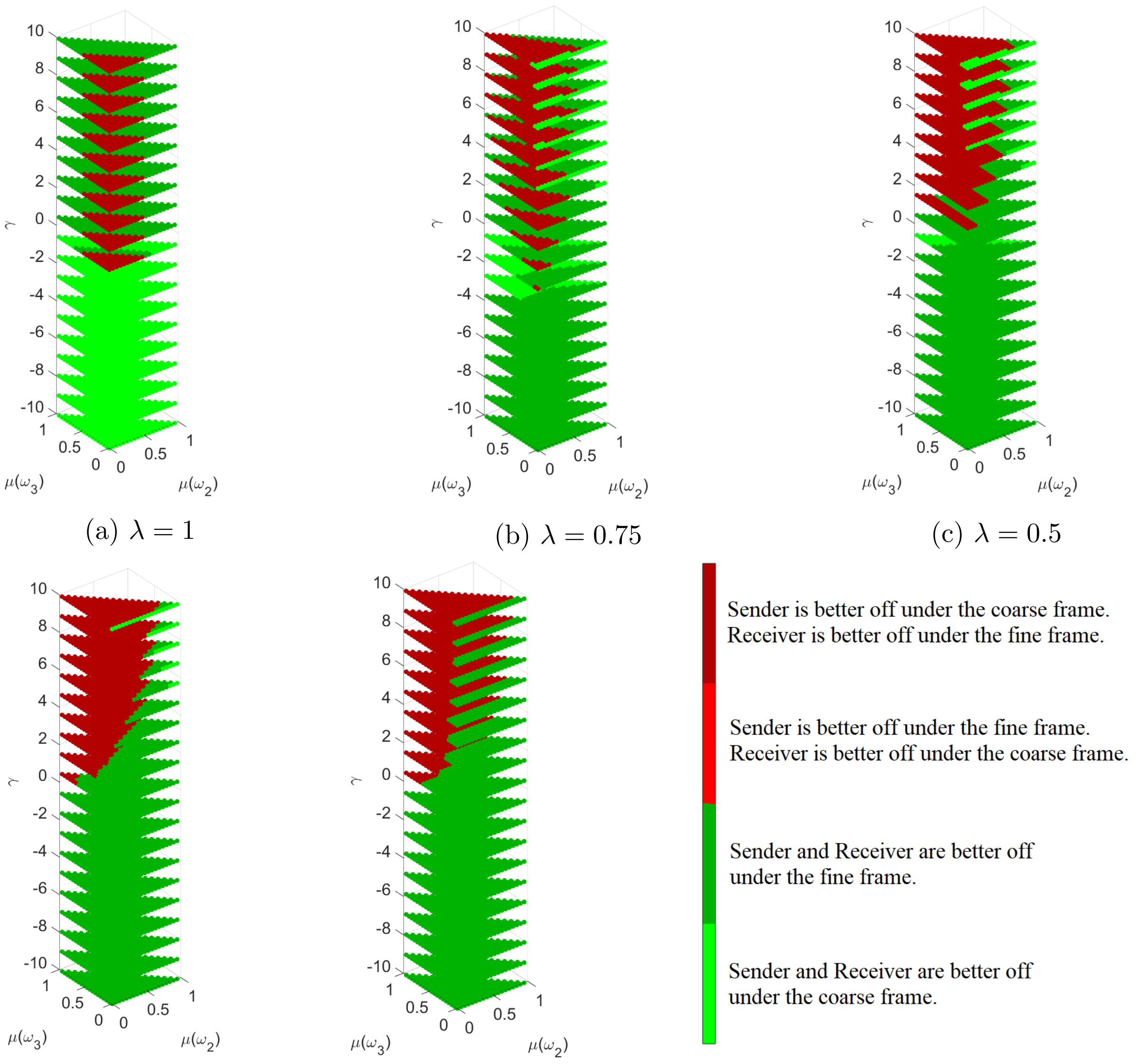
- For part 5 of the proposition, we have the following. In each part (a)–(d), we use: the expressions for $\pi_f(z_1 \,|\, \omega_2)$ and $\pi_f(z_1 \,|\, \omega_3)$ specified in parts 1–4 of the formal statement of Proposition 1 in Section B.2, respectively; the expressions for $\pi_c(z_1 \,|\, \omega_2)$ and $\pi_c(z_1 \,|\, \omega_3)$ specified in Proposition 6. ■

Our considerations following Proposition 4 extend to this alternative coarse frame. Remarkably, there are situations where persuasion is impossible under the coarse frame. In these cases, the expected payoff of Receiver is zero by definition. Therefore, Receiver is better off (resp., indifferent) if persuasion is unnecessary (resp., necessary) under the fine frame. Sender always refines Receiver's frame if persuasion is impossible under the coarse frame. Thus, there is no further conflict of interest.

Graphical Illustration. Figures D.2 and D.3 illustrate for which values of parameters there exists a conflict of interest between Sender and Receiver and whether mandating re-framing is a solution to this problem. In red, we represent the region of the parameter space where there is a conflict of interest. In green, we represent the region of the parameter space where Sender and Receiver are better off under the same frame. We use a darker (resp., lighter) color tone to represent the region of the parameter space where Receiver is better off under the fine (resp., coarse) frame.

Figure D.2 considers the subjective criterion. A conflict of interest can arise only when persuasion is unnecessary under both frames. In this case, there is a conflict of interest if Receiver is better off under the fine frame, as we select the equilibrium under which Sender chooses not to re-frame whenever indifferent. This case corresponds to the dark red region. In all other scenarios, mandating re-framing is either unnecessary—which is the dark green region—or harms Receiver (and Sender)—which is the light green region.

Figure D.2: Conflict of Interest under the Subjective Criterion for Different Values of λ.



(a) $\lambda = 1$ (b) $\lambda = 0.75$ (c) $\lambda = 0.5$

(d) $\lambda = 0.25$ (e) $\lambda = 0$ (f) Legend

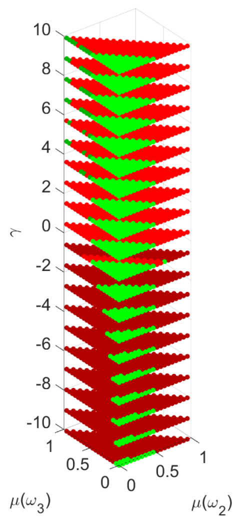
Figure D.3 considers the objective criterion. In this case, a conflict of interest is more likely. Whereas Sender is better off under the frame where he can recommend action a_1 with a higher probability, this harms Receiver because of the higher likelihood that the recommendation is misleading. There is no conflict of interest when persuasion is unnecessary under both frames—which is the light green region. Mandating re-framing can either help—which is the dark red region—or harm—which is the light red region—Receiver.

D.2 Coarse Understanding of States ω_1 and ω_2

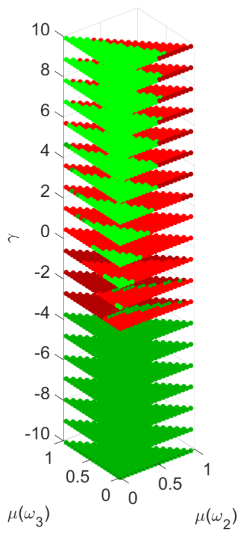
Model. At the beginning of the game, Receiver cannot distinguish state ω_1 from state ω_2: her coarse frame is $\Omega_R \coloneqq \{\omega_{12}, \omega_3\}$, where we write $\omega = \omega_{12}$ if and only if $\omega \in \{\omega_1, \omega_2\}$.

Under the coarse frame, Receiver's preferences are represented by the payoff function

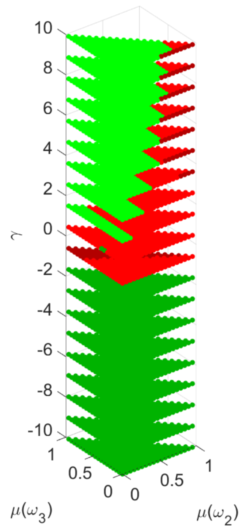
Figure D.3: Conflict of Interest under the Objective Criterion for Different Values of λ.



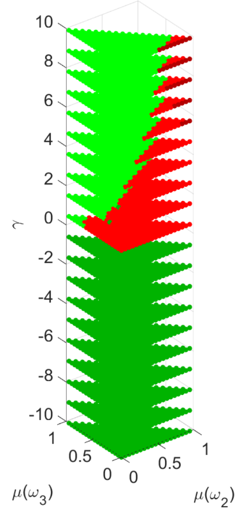
(a) $\lambda = 1$



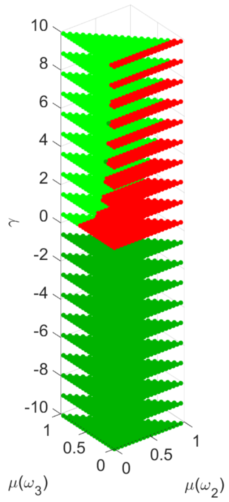
(b) $\lambda = 0.75$



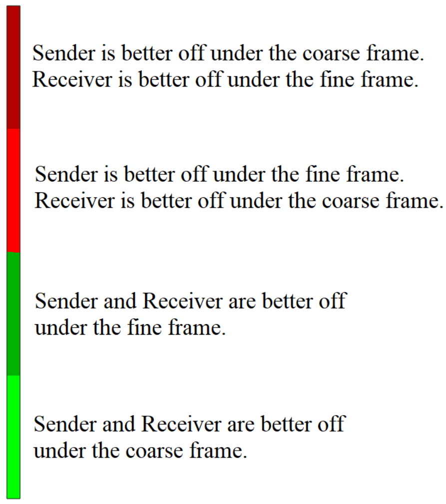
(c) $\lambda = 0.5$



(d) $\lambda = 0.25$



(e) $\lambda = 0$



(f) Legend

$u_R \colon A \times \Omega_R \to \mathbb{R}$, where

$$u_R(a, \omega) := \begin{cases} \alpha_{12} & \text{if } a = a_1 \wedge \omega = \omega_{12} \\ \gamma & \text{if } a = a_1 \wedge \omega = \omega_3 \\ 0 & \text{if } a = a_2 \end{cases},$$

and $\alpha_{12} := 2\lambda - 1$ for some $\lambda \in [0, 1]$.

Receiver's prior beliefs under frame Ω_R are sub-additive, and $\mu_R(\omega_{12}) := \lambda\mu(\omega_1) + (1-\lambda)\mu(\omega_2)$. This condition implies $\mu_R(\omega_3) > \mu(\omega_3)$: Receiver conceives state ω_3 as more likely under the coarse frame than under the fine frame.

Under frame Ω_R, Receiver conceives the signal distribution in state ω_{12} as a convex combination of the signal distributions chosen by the Sender under ω_1 and ω_2. That is, $\pi_R(z \,|\, \omega_{12}) := \lambda\pi(z \,|\, \omega_1) + (1-\lambda)\pi(z \,|\, \omega_2)$, whereas $\pi_R(z \,|\, \omega_3) := \pi(z \,|\, \omega_3)$.

Analysis and Results. When Receiver's coarse frame is $\Omega_R = \{\omega_{12}, \omega_3\}$, we distinguish two cases. If $\gamma > 0$, the analysis and results are qualitatively analogous to those when Receiver's coarse frame is $\Omega_R = \{\omega_1, \omega_{23}\}$. If, instead, $\gamma < 0$, the analysis and the results are qualitatively analogous to those when Receiver's coarse frame is $\Omega_R = \{\omega_{13}, \omega_2\}$. For this reason, we omit the formal analysis of this model and refer to the main text and Appendix D.1 for the main insights.