Great Economists of the Past

Honors

- Nobel prize (1991)
- Died at 102

What he is (most) famous for

- **Coase theorem**
  - *The problem of Social Cost*, Journal of Law and Economics (1960)
  - As long as property rights are clearly allocated, bargaining yields efficient outcome
- **Coase conjecture**
  - *Durability and Monopoly*, Journal of Law and Economics (1972)
  - Durable good monopolist sells at competitive price “in the twinkling of an eye”

THE COASE CONJECTURE

Assume that a supplier owns the total stock of a completely durable good. At what price will he sell it? To take a concrete example, assume that one person owns all the land in the United States and, to simplify the analysis, that all land is of uniform quality. Assume also that the landowner is not able to work the land himself, that ownership of land yields no utility and that there are no costs involved in disposing of the land. If there were a large number of landowners and the price were competitively determined, the price would be that at which the amount demanded was equal to the amount of land in the United States. If we imagine this fixed supply of land to be various amounts either greater or smaller, and then discover what the competitively determined price would be, we can trace out the demand schedule for American land. Assume that this demand schedule is DD and that from this a marginal revenue schedule, MR, has been derived. Both schedules are shown in Figure

We now have to determine the price which the monopolistic landowner would charge for a unit of land in the assumed conditions. The diagram would seem to suggest (and has, I believe, suggested to some) that such a monopolistic landowner would charge the price OA, would sell the quantity of land OM, thus maximising his receipts, and would hold off the market the quantity of land, MQ. But suppose that he did this. MQ land and money equal to OA $\times$ OM would be in the possession of the original landowner while OM land would be owned by others. In these circumstances, why should the original landowner continue to hold MQ off the market? The original landowner could obviously improve his position by selling more land since he could by this means acquire more money. It is true that this would reduce the value of the land OM owned by those who had previously bought land from him—but the loss would fall on them, not on him. If the same assumption about his behaviour was made as before, he would then sell part of MQ. But this is not the end of the story, since some of MQ would still remain unsold. The process would continue as long as the original landowner retained any land, that is, until OQ had been sold. And if there were no costs of disposing of the land, the whole process would take place in the twinkling of an eye.

A loose formalization

- Seller owns as many goods as there are consumers. Goods are worthless for him, who instead wants the money
- Consumers have unit demand (consume at most one good), but heterogeneous willingness to pay for the good $\sim U[0,1]$
- One-shot monopolist. Quote a price p, those with valuation above p buy, those below don't. Profits

$$\Pi_1 = \max_p p(1-p) \Rightarrow p^\star = \frac{1}{2}$$

- Now add durability. There are still $\frac{1}{2}$ goods to be sold and agents $\left[0, \frac{1}{2}\right]$ that have positive valuation for it; solve

$$\Pi_2 = \max_p p\left(\frac{1}{2} - p\right) \Rightarrow p^\star = \frac{1}{4}$$

- ... but there are still $\frac{1}{4}$ goods to be sold and agents $\left[0, \frac{1}{4}\right]$ have positive valuation for it ...

"Behavioral" Buyers

- Keep going: At period t, quote

$$p_t = \left(\frac{1}{2}\right)^t$$

and sell to $[p_t, p_{t-1}] = \left[\left(\frac{1}{2}\right)^t, \left(\frac{1}{2}\right)^{t-1}\right]$ making profits

$$\Pi_t = \left(\frac{1}{2}\right)^t \left[\left(\frac{1}{2}\right)^{t-1} - \left(\frac{1}{2}\right)^t\right] = \left(\frac{1}{2}\right)^{2t-1} \left[1 - \left(\frac{1}{2}\right)\right] = \left(\frac{1}{2}\right)^{2t}$$

$$\Pi = \sum \Pi_t = \sum_{t=1}^{\infty} \left(\frac{1}{2}\right)^{2t} = \frac{1}{3}$$

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$$\text{Surplus (Price Discrim.)} = \frac{1}{2} > \frac{1}{3} > \frac{1}{4} = \text{Static monopolist}$$

Buyer Sophistication

- ... something is suspicious in the construction above
- Think about the guy $\theta = \frac{1}{2} + \epsilon$.
- On path he buys in the first period at price $\frac{1}{2}$, getting surplus ϵ
  - If he waited one period, could have purchased the good at $\frac{1}{4}$, surplus $\left(\frac{1}{4} + \epsilon\right) > \epsilon$
    - Adding discounting changes nothing, surplus deviation is $\beta\left(\frac{1}{4} + \epsilon\right)$ and for all $\beta > 0$ can find ϵ small such that...
- Buyers anticipate the seller will face a residual demand they dominate, so must expect a price drop in the future period
  - Never purchase today: wait for tomorrow when the price drops

Buyer Sophistication

- The "cost" is foregoing consumption today: will need to start eating from tomorrow onward
  - With discounting, indifference condition of the form: marginal buyer at t such that

$$\theta_t - p_t = \beta\left(\theta_t - p_{t+1}\right)$$

  - $\beta = 1 \Rightarrow$ all prices are equal, cannot credibly equal a positive number [loose!] so must be zero
- Horrible for the seller, has to quote $p = 0$, the competitive (Bertand) equilibrium outcome
  - Durable good monopolist is competing against his future self
  - If he could, in the first period he would sell at $p^\star$ and burn all the $F\left(p^\star\right)$ residual units of the good, so for his promise "I won't lower the price" to be credible

Two Periods Example

- Good lasts two periods, utility from consuming t periods is $t\theta$
  - "Overall valuation" $\sim U[0,2]$, monopolist sets price $p = 1$, profit $\frac{1}{2}$
    - Or, sell the stuff for two periods at price $\frac{1}{2}$ without observing/adjusting pricing after first period
  - Solve model for each possible marginal buyer in the first period, then optimize on this variable
- Assume in period 1 you sold to $[\theta_1, 1]$
- In period 2 left with a $[0, \theta_1]$ audience, so set $p_2 = \frac{\theta_1}{2}$ and make profit $\left(\frac{\theta_1}{2}\right)^2$
- Price in the first period must make θ_1 indifferent between buying today and tomorrow

$$2\theta_1 - p_1 = \theta_1 - \frac{\theta_1}{2} \Rightarrow p_1 = \frac{3}{2}\theta_1$$

Find θ_1

- Solving

$$\max_{\theta_1} \overbrace{(1-\theta_1)\frac{3\theta_1}{2}}^{\text{Sell today}} + \underbrace{\left(\frac{\theta_1}{2}\right)^2}_{\text{Sell tomorrow}}$$

$$\theta_1 = \frac{3}{5}, p_1 = \frac{9}{10}, p_2 = \frac{3}{10}$$

- Types > 0.6 buy today at 0.9. Types below wait for the price to drop to 0.3. Type 0.5 would have positive surplus $2 * 0.5 - 0.9$ from buying today, but waits and gets $0.5 - 0.3$.
  - **Ratchet effect**: In Period 1 we have (i) Less trade $(\frac{1}{2} < \frac{3}{5})$ and (ii) lower price $(\frac{9}{10} < 1)$ compared to one-shot monopoly.
- Value is $\frac{9}{20} < \frac{1}{2}$. Better-off in the "Sell today at 1 and that's it" strategy. Possibility to resell/observe whether guy purchased hurts.

Multiple Periods

- Three periods, one-shot monopolist sets price $p = \frac{3}{2}$, profit $\frac{3}{4}$

$$p_1 \approx 1.24$$

$$p_2 \approx 0.58$$

$$\theta_1 \approx 0.65$$

$$\theta_2 \approx 0.393$$

$$\theta_3 = p_3 \approx 0.196$$

$$\text{Rev} \approx 0.622 \ll \frac{3}{4} \approx 82\% \text{ of Potential}$$

- The price goes to 0.196 in 3 periods (twinkling of an eye...)
- Revenue: the longer t, the further we get from $\frac{1}{4}t$.
- Try to do it, i.e. find the path of prices/thresholds/seller's revenue as a function of t (numerically?)

A Real Formalization

- Stokey (1982), Bulow (1982), Kahn (1986), *Gul Sonnenschein and Wilson (1986)*

Let $\Sigma(f, \delta)$ denote the set of equilibria for the market (f, δ) and let $\Sigma^s(f, \delta)$ denote the subset of equilibria which satisfy the condition that the state of the market, after any price that is lower than all preceding prices, is independent of the earlier price history in the market. Equilibria in $\Sigma^s(f, \delta)$ are said to be stationary for the consumers, since the sets of those accepting and those rejecting depend only on the current price. The following is an immediate consequence of Theorem 1.

COROLLARY. *Generic markets satisfying* (B) *and* (L) *have a unique equilibrium path and this path leads to a determinate sequence of price offers and acceptances. Furthermore, the path is associated with an equilibrium that is stationary for the consumers, prices are decreasing along the equilibrium path, and all consumers are served after a finite number of offers.*[12]

A Real Formalization

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THEOREM 3 (Coase conjecture). *For each $\varepsilon > 0$ there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$ and for all equilibria $\sigma \in \Sigma^s(f, \delta)$, the first price prescribed by σ is less than ε.*

- *Gul (1987)*: Competing firms in durable goods market extract (static) monopolist surplus

Theorem 2. For all f, $N \geq 2$ and $\epsilon > 0$, there exists $\bar{\delta} \in (0, 1)$ such that $\delta \in (\bar{\delta}, 1)$ implies that there exists $\sigma \in E(f, \delta, N)$ such that $\sum_{j=1}^{N} \pi^j(\sigma) > \pi_f - \epsilon$.

Theorem 2 and the Coase conjecture show that while a monopolist who can make offers arbitrarily frequently is forced to behave competitively, two or more firms can extract the one-shot monopoly profit.

The Economics of Commitment

- Every good is durable "in the twinkling of an eye" $\Rightarrow$ Timing of production and sale
- Seller wants commitment, credibly promising he will not exploit the buyers he excludes "in the first round"
- "Coasian dynamics" of unraveling because cannot promise you won't revise your action
  - Arbitrary punishment ($\Rightarrow$ efficient outcome in moral hazard problems, Mirrlees 1974) are not ex-post credible
    - Banking regulation "I would like to let you default until I have to let you default"
  - Rules rather than discretion (Kydland & Prescott) and time inconsistency in monetary policy / bank regulation
  - Optimal audit rule (Sanchez & Sobel) audits only honest firms

THE COASE THEOREM

What is the Coase Theorem?

- In Hurwicz (1995, *What is the Coase Theorem?*) words ...
  - **The Coase Theorem is interpreted as asserting that the equilibrium level of an externally (e.g., pollution) is independent of institutional factors (in particular, assignment of liability for damage), except in the presence of transaction costs.**
  - It is shown here [i.e. in the Hurwicz paper] that absence of income effects (due to parallel preferences or quasi-linear utility functions) is not only sufficient (which is well known) but also necessary for this to be true.

What is the Coase Theorem?

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  - **The Coase Theorem is interpreted as asserting that the equilibrium level of an externally (e.g., pollution) is independent of institutional factors (in particular, assignment of liability for damage), except in the presence of transaction costs.**

- We read [extracts] from the original paper, then give a formalization in a slightly different setting (bilateral trading & property rights vs. social cost & liability for damage).
  - Make sure you are able to make the connection; formalize the setting that is "discussed" and discuss the setting is formalized
  - Ability of abstracting from the specifics of the setting and isolating the "economic forces" at play is fundamental for a researcher

I. THE PROBLEM TO BE EXAMINED[1]

THIS paper is concerned with those actions of business firms which have harmful effects on others. The standard example is that of a factory the smoke from which has harmful effects on those occupying neighbouring properties. The economic analysis of such a situation has usually proceeded in terms of a divergence between the private and social product of the factory, in which economists have largely followed the treatment of Pigou in *The Economics of Welfare*. The conclusions to which this kind of analysis seems to have led most economists is that it would be desirable to make the owner of the factory liable for the damage caused to those injured by the smoke, or alternatively, to place a tax on the factory owner varying with the amount of smoke produced and equivalent in money terms to the damage it would cause, or finally, to exclude the factory from residential districts (and presumably from other

II. The Reciprocal Nature of the Problem

The traditional approach has tended to obscure the nature of the choice that has to be made. The question is commonly thought of as one in which A inflicts harm on B and what has to be decided is: how should we restrain A? But this is wrong. We are dealing with a problem of a reciprocal nature. To avoid the harm to B would inflict harm on A. The real question that has to be decided is: should A be allowed to harm B or should B be allowed to harm A? The problem is to avoid the more serious harm. I instanced in my previous article[2] the case of a confectioner the noise and vibrations from whose machinery disturbed a doctor in his work. To avoid harming the doctor would inflict harm on the confectioner. The problem posed by this case was essentially whether it was worth while, as a result of restricting the methods of production which could be used by the confectioner, to secure more doctoring at the cost of a reduced supply of confectionery products. Another example is

Cows paying beacuse they ate crops

I propose to start my analysis by examining a case in which most economists would presumably agree that the problem would be solved in a completely satisfactory manner: when the damaging business has to pay for all damage caused *and* the pricing system works smoothly (strictly this means that the operation of a pricing system is without cost).

A good example of the problem under discussion is afforded by the case of straying cattle which destroy crops growing on neighbouring land. Let us suppose that a farmer and a cattle-raiser are operating on neighbouring proper-

Crops selling themselves to be eaten by cows

IV. THE PRICING SYSTEM WITH NO LIABILITY FOR DAMAGE

I now turn to the case in which, although the pricing system is assumed to work smoothly (that is, costlessly), the damaging business is not liable for any of the damage which it causes. This business does not have to make a payment to those damaged by its actions. I propose to show that the allocation of resources will be the same in this case as it was when the damaging business was liable for damage caused. As I showed in the previous case that the allocation of resources was optimal, it will not be necessary to repeat this part of the argument.

I return to the case of the farmer and the cattle-raiser. The farmer would suffer increased damage to his crop as the size of the herd increased. Suppose

The same crops will be eaten by the same cows

It is necessary to know whether the damaging business is liable or not for damage caused since without the establishment of this initial delimitation of rights there can be no market transactions to transfer and recombine them. But the ultimate result (which maximises the value of production) is independent of the legal position if the pricing system is assumed to work without cost.

... and that's the Coase Theorem...

The Coase Theorem in a Bilateral Trading Setting

Gains from Bilateral Trade

- Two agents, one object, and money
- Agents have heterogeneous valuation θ_1, θ_2 for the object and quasilinear (transferable) utility
- Efficiency benchmark: who has the highest valuation consumes the good

$$\max_{\{x_1+x_2=1, m_1+m_2=m\}} \sum_{i\in\{1,2\}} x_i\theta_i - t_i \Rightarrow x_i = \mathbb{I}\left[\theta_i > \theta_{-i}\right]$$

- Compensate the other with money $\approx$ allocate resources to the guy that puts them to the most efficient use
- Property rights: someone owns the object (seller)
  - How does the (ex-ante) allocation of property rights affect the efficiency of the (final) allocation?
  - If trade is prohibited (ex-ante=final), efficiency loss of $(\theta_B - \theta_S)^+$

Gains from Bilateral Trade

- Guy with valuation $\frac{1}{2}$ owns (S) an object that the other (B) guy values 1
- No trade $\Rightarrow$ $\frac{1}{2}$ efficiency loss = gains from trade
- Say the two can meet in a venue (market) and bargain
- TIOLI offer (who is the crop, who is the cow?)
  - S says: give me 1\$, I give you the object. $\Rightarrow$ B consumes, S extracts surplus
  - B says: give me the object, I give you $\frac{1}{2}$\$. $\Rightarrow$ B consumes, B extracts surplus
- Other bargaining protocols
  - Rubinstein alternating offer $\Rightarrow$ B consumes, allocation of surplus depends on agent's impatience
  - Nash bargaining $\Rightarrow$ abstract from details of the bargaining protocol, represent the set of efficient outcomes

Nash Bargaining [Detour]

- Bargaining problem: allocation of utilities $x \in \mathbb{R}^2$ among a feasible set $F \subset \mathbb{R}^2$, disagreement utilities $v \in \mathbb{R}^2$
- $\phi(F, v)$ gives bargaining outcome as a function of possibility F and disagreement point v
- Loosely,

$$\text{Efficiency} \Leftrightarrow \phi(F, v) = \arg \max_{x \in F, x \geq v} (x_1 - v_1)(x_2 - v_2)$$

  In the example (check) $x_1 = \frac{3}{4}$ equal surplus splitting.
- More formally $\phi = \arg\max \ldots$ if and only if
  1. Rationality $\phi(F, v) \geq v$
  2. Efficiency $x \geq \phi(F, v) \Rightarrow x \notin F$
  3. Symmetry $[v_1 = v_2 \text{ and } (x_1, x_2) \in F \Rightarrow (x_2, x_1) \in F] \Rightarrow \phi_1(F, v) = \phi_2(F, v)$
  4. IIA $\{v, \phi(F, v)\} \subseteq G \Rightarrow \phi(F, v) = \phi(G, v)$ for all G closed convex subsets of F
  5. Scale invariance...

Outside Seller

- Back to the $\frac{1}{2} - 1$ guy economy. The object is now owned by an outside seller.
- He goes to the 1 guy and says
  - Give me 1\$, I give you the object. $\Rightarrow$ B consumes, O extracts surplus
- **Coase Theorem** *"If trade can occur, then bargaining will lead to an efficient outcome no matter how property rights are allocated (as long as they are clearly allocated)."*
- In the end efficiency is all about enforcing property rights and opening up frictionless trading venues.
  - Still, surplus allocation depends on who has the object to start with, endowment reallocation $\approx$ redistribution without efficiency losses

- Fundamental functioning of efficient bilateral trade (what about the problem of social cost?)

... the guy with the highest valuation gives some money (how much depends on who has bargaining power) to get the good.

BUT...

… Thou Shall Know Your Opponent's Value

 

Efficient mechanism under asymmetric information

- Two agents one object, agent i has valuation $\theta_i \sim U[0,1]$ independent
- Quasilinear preferences:

$$u_i = x_i\theta_i - t_i$$

- Valuation is private information: the owner cannot target his TIOLI offer to opponent's valuation
- A mechanism in this framework consists of a message space M_i, outcomes are allocation and transfer pairs

$$(x,t) : M_1 \times M_2 \to \Delta(2) \times \mathbb{R}_+^2$$

- Efficient allocation $x \in \Delta(2)$ (probability distributions over binary states) that solves

$$f(\theta) = \arg\max_{x\in\Delta(2)} \sum_{i=1,2} x_i\theta_i$$

$\Rightarrow x_i = 1 \iff \theta_i > \theta_j$

Outside Seller

- Suppose object is owned by an outside seller (zero reservation utility)
- Auctions partially implement the efficient allocation under asymmetric information
- Partially implement = there exists an equilibrium under which the allocation is efficient
  - Monotone equilibrium in (any) auction $\Rightarrow$ highest bidder gets the obejct, payments depend on the specifics of the mechanism
  - SPA: Partial implementation in (weakly) dominant strategies. Belongs to family of VCG mechanisms
  - Recall SPA also admits inefficient equilibria in weakly dominated actions. (1 always bids 0, 2 always bids 1)

One Agent owns the good

- Property rights + asymmetric information: 1 is the owner of the good. An incentive compatible mechanism satisfies

$$\mathbb{E}\left(u_1(x,t)\,|\theta_1\right) \geq \theta_1, \qquad \forall \theta_1 \in [0,1]$$

$$\mathbb{E}\left(u_2(x,t)\,|\theta_2\right) \geq 0, \qquad \forall \theta_2 \in [0,1]$$

- Is there an efficient allocation rule (trade iff $\theta_1 < \theta_2$) satisfying those constraints?

Theorem

(Myerson-Satterthwaite 1983) *No.*

Implications of the MS Theorem

- There exists no efficient bilateral trading mechanism
- Fundamental result: when there is two-sided asymmetric information, no institution can achieve economic efficiency if one part owns the good
- Together with SPA being an efficient mechanism under two-sided asymmetric information, this gives the insight that the way goods are initially allocated is essential for ultimate efficiency
  - Contrary to the prediction of the Coase Theorem, randomly allocating the good and letting players retrade does not ensure efficiency
  - Key argument to use auctions to allocate radio spectrum

Plan of the Proof

- **Step 1**: State and prove the Revelation Principle in this context.
  - There exists no mechanism ... If there were such a mechanism, by the Revelation Principle the direct mechanism would do the job.
- **Step 2**: Play with Incentive Constraints to prove necessary conditions
  1. Monotonicity of the allocation
  2. Payoff Formula (Transfers are pinned down by allocations)
- **Step 3**: Show monotonicity and payoff formula are sufficient for truthtelling.
- **Step 4**: Use the simplification to derive a contradiction
  - An efficient individually rational trading mechanism satisfying monotonicty and payoff formula would give both agents a surplus weakly larger than total surplus, contradiction.

The Funny Part Ends Here

Revelation Principle

Theorem

(Revelation Principle) *If there exists a mechanism* $\left(\mathcal{M}, \widehat{x}, \widehat{t}\right)$ *that implements the efficient allocation (i.e. admitting a Nash equilibrium* $\sigma : [0,1]^2 \to \mathcal{M}$ *under which* $\widehat{x}\left(\sigma\left(\theta\right)\right) = \mathbb{I}_{\theta_2 \geq \theta_1}$*), then there exists a direct mechanism* $\left([0,1]^2, x, t\right)$ *that implements the efficient allocation.*

- The proof works through a "replication argument": direct mechanisms asks the type and plays the non-direct mechanism as if it was that type. (Same idea as canonical correlated equilibrium).

Revelation Principle (Proof)

- Consider any mechanism $\left(\mathcal{M}, \widehat{x}, \widehat{t}\right)$ and let $(\sigma_i)_{i\in\{1,2\}}$ denote the NE of interest, so that

$$\mathbb{E}\left[u_i\left(\sigma_i\left(\theta_i\right), \sigma_j\left(\theta_j\right)\right)|\theta_i\right] \geq \mathbb{E}\left[u_i\left(m_i, \sigma_j\left(\theta_j\right)\right)|\theta_i\right] \qquad (1)$$

- Now define the direct mechanism $\left([0,1]^2, x, t\right)$ with

$$\forall\theta_i, \theta_j \in [0,1]^2,\ x_i\left(\theta_i, \theta_j\right) = \widehat{x}_i\left(\sigma_i\left(\theta_i\right), \sigma_j\left(\theta_j\right)\right)$$

$$t_i\left(\theta_i, \theta_j\right) = \widehat{t}_i\left(\sigma_i\left(\theta_i\right), \sigma_j\left(\theta_j\right)\right)$$

- Equation (1) implies that truthtelling is an equilibrium of this new mechanism. It yields the sam distribution of outcomes.

Revelation principle (Detour from MS)

- Key result in **partial** implementation
- Contrapositive statement: If we cannot find any *direct* mechanism implementing a SCF, then that SCF cannot be partially implemented even if we consider more complex mechanisms
- So, **study properties of the direct mechanism, of the SCF!!!**
- For dominant strategies implementation (not this case, here Nash), DSIC is necessary to have some mechanism implementing f in dominant strategies. If it works, go with the direct mechanism, if it fails don't bother looking for something better
- It does **not** say that every mechanism implementing a SCF must be direct

Monotonicity

- Back to MS. Let

$$\overline{x}_i(\theta_i) = \mathbb{E}_{[0,1]}\left[x_i(\theta_i, \theta_j)\right]$$

$$\overline{t}_i(\theta_i) = \mathbb{E}_{[0,1]}\left[t_i(\theta_i, \theta_j)\right]$$

$$\overline{u}_i(\theta_i) = \overline{x}_i(\theta_i)\,\theta_i - \overline{t}_i(\theta_i)$$

Expected probability of winning, transfer and utility induced by the allocation and transfer rules when you are of type θ_i

- Domain of x, t is inprinciple M but we already used the revelation principle
- In the mechanism, $\overline{x}_i$ is the probability of winning when you are θ_i, assuming everyone (yourself included) report truthfully.
- Want to show

$$\theta_i \geq \theta_i' \implies \overline{x}_i(\theta_i) \geq \overline{x}_i(\theta_i')$$

Monotonicity

- Take any $\theta_i \geq \theta_i'$.
- $IC : \theta_i \rightarrow \theta_i'$

$$\overline{x}_i(\theta_i)\,\theta_i - \overline{t}_i(\theta_i) \geq \overline{x}_i(\theta_i')\,\theta_i - \overline{t}_i(\theta_i')$$

- $IC : \theta_i' \rightarrow \theta_i$

$$\overline{x}_i(\theta_i')\,\theta_i' - \overline{t}_i(\theta_i') \geq \overline{x}_i(\theta_i)\,\theta_i' - \overline{t}_i(\theta_i)$$

- Summing them the transfers drop and

$$\left[\overline{x}_i(\theta_i) - \overline{x}_i(\theta_i')\right]\theta_i \geq \left[\overline{x}_i(\theta_i) - \overline{x}_i(\theta_i')\right]\theta_i'$$

$$\left[\overline{x}_i(\theta_i) - \overline{x}_i(\theta_i')\right](\theta_i - \theta_i') \geq 0 \overset{\theta_i \geq \theta_i'}{\Longrightarrow} \overline{x}_i(\theta_i) - \overline{x}_i(\theta_i') \geq 0$$

Payoff Formula

- When choosing his message $\theta_i' \in [0,1]$, type θ_i solves

$$\overline{u}_i(\theta_i) = \max_{\theta_i'} \overline{x}_i(\theta_i')\,\theta_i - \overline{t}_i(\theta_i')$$

- Truthtelling requires θ_i is in the arg max, therefore by the Envelope Theorem

$$\frac{\mathrm{d}}{\mathrm{d}\theta_i}\overline{u}_i(\theta_i) = \overline{x}_i(\theta_i)$$

- So

$$\overline{u}_i(\theta_i) = \overline{u}_i(0) + \int_0^{\theta_i} \frac{\mathrm{d}}{\mathrm{d}\theta_i}\overline{u}_i(\theta_i')\,\mathrm{d}\theta_i' =$$

$$= \overline{u}_i(0) + \int_0^{\theta_i} \overline{x}_i(\theta_i')\,\mathrm{d}\theta_i'$$

proving the payoff formula.

Sufficiency

- So far, we proved monotonicity and the payoff formula are *necessary* conditions for (x, t) to have truthtelling as incentive compatible

Lemma

(Sufficiency) *If the direct mechanism* (x, t) *satisfies monotonicity and the payoff formula, then truthtelling is incentive compatible*

- We must show that for each type θ_i, her expected payoff from sending message θ_i is (weakly) greater than the expected payoff from sending any alternative message θ_i'.
- Let
$$\overline{u}_i\left(\theta_i, \theta_i'\right) = \overline{x}_i\left(\theta_i'\right)\theta_i - \overline{t}_i\left(\theta_i'\right)$$
So that
$$\overline{u}_i\left(\theta_i, \theta_i\right) - \overline{u}_i\left(\theta_i, \theta_i'\right) = \left[\overline{x}_i\left(\theta_i\right) - \overline{x}_i\left(\theta_i'\right)\right]\theta_i - \left[\overline{t}_i\left(\theta_i\right) - \overline{t}_i\left(\theta_i'\right)\right]$$

Sufficiency

- Want to show that expression is (weakly) positive
- Suppose $\theta_i > \theta_i'$. Then

$$\bar{t}_i(\theta_i) - \bar{t}_i(\theta_i') = \bar{x}_i(\theta_i)\theta_i - \bar{u}_i(\theta_i) - [\bar{x}_i(\theta_i')\theta_i' - \bar{u}_i(\theta_i')]$$

$$= \bar{x}_i(\theta_i)\theta_i - \bar{x}_i(\theta_i')\theta_i' - [\bar{u}_i(\theta_i) - \bar{u}_i(\theta_i')]$$

$$= \bar{x}_i(\theta_i)\theta_i - \bar{x}_i(\theta_i')\theta_i' - \left[\int_0^{\theta_i} \bar{x}_i(\theta_i'')\,\mathrm{d}\theta_i'' - \int_0^{\theta_i'} \bar{x}_i(\theta_i'')\,\mathrm{d}\theta_i''\right]$$

$$= \bar{x}_i(\theta_i)\theta_i - \bar{x}_i(\theta_i')\theta_i' - \int_{\theta_i'}^{\theta_i} \bar{x}_i(\theta_i'')\,\mathrm{d}\theta_i''$$

- Definition, rearrangement, payoff formula, rearrangement

Sufficiency

- Then,

$$
\begin{aligned}
\overline{u}_i(\theta_i, \theta_i) - \overline{u}_i(\theta_i, \theta_i') &= [\overline{x}_i(\theta_i) - \overline{x}_i(\theta_i')]\,\theta_i \\
&\quad - \left[\overline{x}_i(\theta_i)\,\theta_i - \overline{x}_i(\theta_i')\,\theta_i' - \int_{\theta_i'}^{\theta_i} \overline{x}_i(\theta_i'')\,\mathrm{d}\theta_i''\right] \\
&= -\overline{x}_i(\theta_i')(\theta_i - \theta_i') + \int_{\theta_i'}^{\theta_i} \overline{x}_i(\theta_i'')\,\mathrm{d}\theta_i'' \\
&= -\int_{\theta_i'}^{\theta_i} (\overline{x}_i(\theta_i') - \overline{x}_i(\theta_i''))\,\mathrm{d}\theta_i''
\end{aligned}
$$

- By monotonicity,

$$\overline{x}_i\left(\theta_i'\right) - \overline{x}_i\left(\theta_i''\right) \geq 0, \qquad \forall \theta_i'' \in \left[\theta_i', \theta_i\right]$$

- Which implies

$$\overline{u}_i(\theta_i, \theta_i) - \overline{u}_i\left(\theta_i, \theta_i'\right) \geq 0$$

Proving IC. The argument for $\theta_i < \theta_i'$ is specular and omitted.

Let's pause

- Monotonicity is a global IC, obtained from considering one types incentives to imitate the reports of any other type.
- The payoff formula is also derived from IC, but is local and takes the form of a first order condition (truthtelling is immune to local deviations).
- The efficient design problem thus reduces to

$$\max_{\overline{x},\overline{t}} \sum \overline{x}_i(\theta_i)\,\theta_i$$

$$\overline{x}_i \text{ feasible and satisfies M}$$

$$\overline{x}_i, \overline{t}_i \text{ satisfies PF}$$

The optimal mechanism

- Often we solve the relaxed problem without the monotonicity constraint and then check (hope) the obtained allocation is indeed monotonic (First Order Approach)
- Luckly, to prove that no efficient trading mechanism exists, we don't need to solve for the efficient mechanism.
- We just need to show that there cannot be a mechanism that satisfies monotonicity, the payoff formula, and the seller-dependent IR conditions.

Our Problem

- Efficiency requires

$$\forall\theta, \quad x_1(\theta) + x_2(\theta) = 1 \text{ (the good is allocated)}$$

$$x(\theta) = \mathbb{I}_{\theta_2 > \theta_1}$$

- The surplus S created by the mechanism is

$$S = \int_0^1 \int_0^1 (\theta_2 - \theta_1)\, \mathbb{I}_{\theta_2 > \theta_1} \mathrm{d}\theta_1 \mathrm{d}\theta_2$$

We complete the proof by showing that any efficient IC mechanism would give each player the entire surplus, which is of course absurd.

Seller's Surplus

- Consider first the surplus accruing to the seller.
- Using the payoff formula,

$$\overline{u}_1(\theta_1) = \overline{u}_1(1) - \int_{\theta_1}^{1} \overline{x}_1(\theta_1')\, d\theta_1'$$

- Using efficiency

$$\int_{\theta_1}^{1} \overline{x}_1(\theta_1')\, d\theta_1' = \int_{\theta_1}^{1} \underbrace{\int_0^1 \mathbb{I}_{\theta_2 \le \theta_1'} d\theta_2}_{=\overline{x}_1(\theta_1')} d\theta_1'$$

$$= \int_0^1 \int_{\theta_1}^1 \mathbb{I}_{\theta_2 \le \theta_1'} d\theta_1' d\theta_2 = \int_0^1 [1 - \max(\theta_1, \theta_2)]\, d\theta_2$$

Seller's Surplus

- Now, using $\overline{u}_1(1) \geq 1$ and the lines above we have

$$S_1 \geq \int_0^1 \left[1 - \int_0^1 \left[1 - \max(\theta_1, \theta_2)\right] \mathrm{d}\theta_2 - \theta_1\right] \mathrm{d}\theta_1$$

$$= \int_0^1 \int_0^1 \left[\max(\theta_1, \theta_2) - \theta_1\right] \mathrm{d}\theta_2 \mathrm{d}\theta_1$$

$$= \int_0^1 \int_0^1 (\theta_2 - \theta_1) \mathbb{I}_{\theta_2 > \theta_1} \mathrm{d}\theta_1 \mathrm{d}\theta_2 = S$$

Buyer's Surplus

- Consider now the surplus going to the buyer

$$S_2 = \int_0^1 \overline{u}_2(\theta_2)\,\mathrm{d}\theta_2$$

$$\overline{u}_2(\theta_2) = \overline{u}_2(0) + \int_0^{\theta_2} \overline{x}_2(\theta_2')\,\mathrm{d}\theta_2'$$

- As $\overline{u}_2(0) \geq 0$ from IR, it follows

$$\overline{u}_2(\theta_2) \geq \int_0^{\theta_2} \underbrace{\int_0^1 \mathbb{I}_{\theta_2' > \theta_1} \mathrm{d}\theta_1}_{=\overline{x}_2(\theta_2')} \mathrm{d}\theta_2'$$

$$= \int_0^1 \int_0^{\theta_2} \mathbb{I}_{\theta_2' > \theta_1} \mathrm{d}\theta_1 \mathrm{d}\theta_2' = \int_0^1 (\theta_2 - \theta_1)\,\mathbb{I}_{\theta_2 > \theta_1} \mathrm{d}\theta_1$$

- So

$$S_2 \geq \int_0^1 \int_0^1 (\theta_2 - \theta_1)\,\mathbb{I}_{\theta_2 > \theta_1} \mathrm{d}\theta_1 \mathrm{d}\theta_2 = S$$

Wrapping up

- Hence an efficienct mechanism satisfying the payoff formula and the interim IR would give each player the entire surplus. Contradiction.
- We used uniform distribution for convenience. Easy to extend to distributions with common support.

Theorem

(MS precise statement) Suppose buyers and sellers valuations, $b \in \left[\underline{b}, \overline{b}\right] \subset \mathbb{R}$ and $s \in [\underline{s}, \overline{s}] \subset \mathbb{R}$ are drawn independently according to smooth positive densities and $\left[\underline{b}, \overline{b}\right] \cap [\underline{s}, \overline{s}] \neq \emptyset$. Then there is no incentive compatible efficient individually rational mechanism.

A (quick) Many Agents Detour

Many Agents

- *"The Coase Theorem and the empty core"* V. Aivazian and J. Callen, Journal of Law and Economics 1981

THE Coase theorem, which states that in the absence of transaction costs resource allocation is neutral with respect to liability rules, is usually demonstrated in a two-participant scenario of either two firms or two consumers.[1] This is so in Coase's original verbal discussion, as well as in the more recent literature that treats the Coase argument within the confines of the neoclassical paradigm.[2] As we will show, this is an unfortunate state of affairs, because with more than two participants the Coase theorem cannot always be demonstrated.

Many Agents

We have provided an example in which liability rules have an asymmetrical impact on resource allocation. It was seen that when firms A and B are liable for damages they will stop producing and firm C earns \$40,000 per day. On the other hand, when firms A and B are not liable, negotiations cannot be consummated because the core is empty. In a world of zero transaction costs, negotiations could be endless. Even if some merger is negotiated, it will be the grand coalition only by happenstance.

It could be argued that, if the core is empty and no solution is forthcoming, the participants may agree to accept a set of allocation rules that will yield the desired outcome. The Shapley and Von Neumann-Morgenstern allocations are just two of many game-theory solutions which do not depend on the existence of the core.[11] Moreover, since these solutions are Pareto optimal (by assumption), the grand coalition obtains and the allocative outcome is independent of the liability rule. Thus, it may be argued the Coase theorem is valid once more.

Many Agents

The problem with this approach is that it makes a tautology out of the Coase theorem. If the core does not exist, the participants may accept an alternative solution concept; then again they may simply stop negotiating. It is an empirical question as to what happens when the core is empty. We do not know. Of course, one can impose a Pareto-optimal solution on the participants such as the Shapley allocation. To do so, however, would be tantamount to assuming what Coase was trying to prove, namely, that a Pareto-optimal solution will be forthcoming independent of the liability rule *and* independent of the bargaining structure. We have demonstrated that for one bargaining structure at least, the Coase theorem cannot be proved.

The Coase theorem fundamentally derives its importance from a world with positive transaction costs. It makes it clear that the existence and

THE COASE THEOREM AND THE EMPTY CORE: A COMMENT

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I would not presume to discuss the value of the theory of the core for economic analysis. What I will do is to explain why study of the example which Professors Aivazian and Callen use as the basis of their argument[1] has not led me to modify my views.

entered into than it would disappear. If my argument is to be taken seriously, it has to be shown that the "grand coalition" is stable. It might be argued that no firm would withdraw from this arrangement to enter into a two-party agreement since this would set in motion a process which would lead ultimately to lower profits. But I will not insist on this. I would, however, draw attention to the peculiar character of the contracts which are made in the recontracting process described by Aivazian and Callen, assuming that they are contracts and are not proposals to be superseded by other proposals, so that independent operation continues with the accompaniment of talk about ending it. If these agreements are contracts, they are peculiar in that their terms can be broken at will. The contracts last only so long as the parties are willing to be bound. Thus, if C

Many Agents / Coase Reply

I would not wish to conclude without observing that, while consideration of what would happen in a world of zero transaction costs can give us valuable insights, these insights are, in my view, without value except as steps on the way to the analysis of the real world of positive transaction costs. We do not do well to devote ourselves to a detailed study of the world of zero transaction costs, like augurs divining the future by the minute inspection of the entrails of a goose.