Impossibility Theorems in Economics and Social Choice

(Some) Impossibility Theorems in Economics

- [Matching] *There is no stable matching that is strategy proof*
  - Roth 1982
- [Finance] *The impossibility of informationally efficient markets*
  - Diamond and Verrecchia 1981, Grossman and Stiglitz 1980
- [Trade] *There cannot be common knowledge that everyone expects to gain from trade*
  - Aumann 1976, Milgrom Stokey 1986, Morris 1991
- [Contract] *No efficient trading institution with two-sided asymmetric information*
  - Myerson and Satterthwaite 1981
- [Social Choice] *Only dictatorships make sense, only dictatorships can happen*

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  - Arrow 1951, Gibbard (1973) and Satterthwaite (1975)

No Price, No Trade

- *The impossibility of informationally efficient markets*

Price paradox: *If prices perfectly reflect the information contained in individual actions, then individuals will ignore their private information (and cease to acquire any), thereby eliminating the very source of information contained in prices.*

- Fundamental uncertainty θ, continuum of agents ...

  i) see $s_i = \theta + \epsilon_i$ *ii*) choose $a_i(s_i, \cdot)$ *iii*) induce $\bar{a} = \mathbb{E}[a_i]$ *iv*) see $\bar{a}$

$$\text{Use } s_i \quad \Rightarrow \quad \bar{a} = \theta \quad \Rightarrow \quad \text{Do not use } s_i \quad \Rightarrow \bar{a} \perp \theta \quad \Rightarrow \quad \text{Use } s_i$$

- *No trade theorems*
  - Aumann 1976: You cannot agree to disagree: If there is common knowledge of agentsí posteriors, then those posteriors must be equal
  - Milgrom Stokey 1986: There cannot be common knowledge that everyone expects to gain from trade $\Rightarrow$ cannot trade based on private information
  - Morris 1991/94: No trade if and only if common prior

Impossibility Theorems: What do we learn

1. Some properties cannot coexists $\Rightarrow$ formalize tension between conflicting objectives
   - Matching: Tension between efficiency and stability ($\approx$ trilemma in monetary economics)
   - Gale and Shapley mechanism is stable but not strategy proof (for receiver), there are (Pareto) efficient matching mechanisms that are strategy proof but not stable
2. Implausibility of the conclusion $\Rightarrow$ restrictive implicit assumptions in our model
   - We might not know whether there is efficient trade, but for sure we observe a lot of trade!
   - What is "wrong" with the model? What features must we add/change to get reasonable predictions?
     - No common prior (trade on "fundamental worldview differences")
     - Liquidity constraints / noise traders
     - Biases in processing information and/or making inference from others' actions

Depressing Results?

- Is "nothing works" really bad or do we learn something?
  - Nothing works in this environment, so the "right" environment should have those features

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- [General Equilibrium] Sonnenschein–Mantel–Debreu: Microeconomic rationality has no implications for aggregate outcomes
- [Implementation] Abreu Matsushima: Give us $\epsilon-$ freedom, every function can be implemented in iterative deletion of dominated strategies

Impossibility Theorem in Social Choice

from Stephen Morris' slides

Honors

Doctoral students	David F. Bradford
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	Nancy Gordon
	Gillian Hadfield
	John Harsanyi
	Jan Kmenta
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	Michael Spence
	Nancy Stokey[2]
	Menahem Yaari

Setting

- Agents $1, 2, ..., I$
- Outcomes X; $\#X \geq 3$.
- All agent preference profiles possible (identify types with preferences)

Constitutions

- A *constitution* specifies a societal preference relation (complete and transitive) for every profile of agent preferences
- A constitution satisfies...
  - *unanimity* if society ranks α strictly above β whenever every agent ranks α strictly above β
  - *independence of irrelevant alternatives* (IIA) if society's ranking of any pair of alternatives α and β depends only on the agents' rankings of α and β
  - *dictatorship by agent n* if society strictly prefers α to β whenever agent *n* strictly prefers α to β.

Arrow's Theorem

Any constitution satisfying unanimity and independence of irrelevant alternatives is a dictatorship.

Failed Example 1: Majority Rule

- Society strictly prefers α to β if the number of people strictly preferring α to β is greater than the number of people strictly preferring β to α
- Consider following strict preferences:

1	2	3
α	γ	β
β	α	γ
γ	β	α

- Now $\alpha \succ \beta \succ \gamma \succ \alpha$, contradicting transitivity
- "Condorcet cycle" [Marquis de Condorcet, 18th century French mathematicion]
- Widely used: importance of agenda setting

Failed Example 2: Borda Count

- Rank outcomes
- Add up ranks across players for each outcome
- Order outcomes from lowest sum of ranks to highest
- Transitive
- Jean-Charles de Borda (another 18th century mathematician)

Failed Example 2: Borda Example

1	2	3
α	α	β
β	β	α

- Borda says society prefers α (total rank 4) to β (total rank 5)

Adding Alternatives

1	2	3
α	α	β
β	β	γ
γ	γ	δ
δ	δ	α

- Borda says society prefers β (total rank 5) to α (total rank 6)
- Thus IIA violated

Failed Example 2: Borda Count

Used by....

- Roman senate
- French academy of sciences (Borda)
- (wikipedia) Parliament of Nauru, Icelandic parliamentary elections, selecting presidential election candidates in Kiribati
- ...and the Econometric Society

Proof

- "Three Brief Proofs of Arrow's Theorem" by John Geanakoplos
- Lucky for you, we will do only one

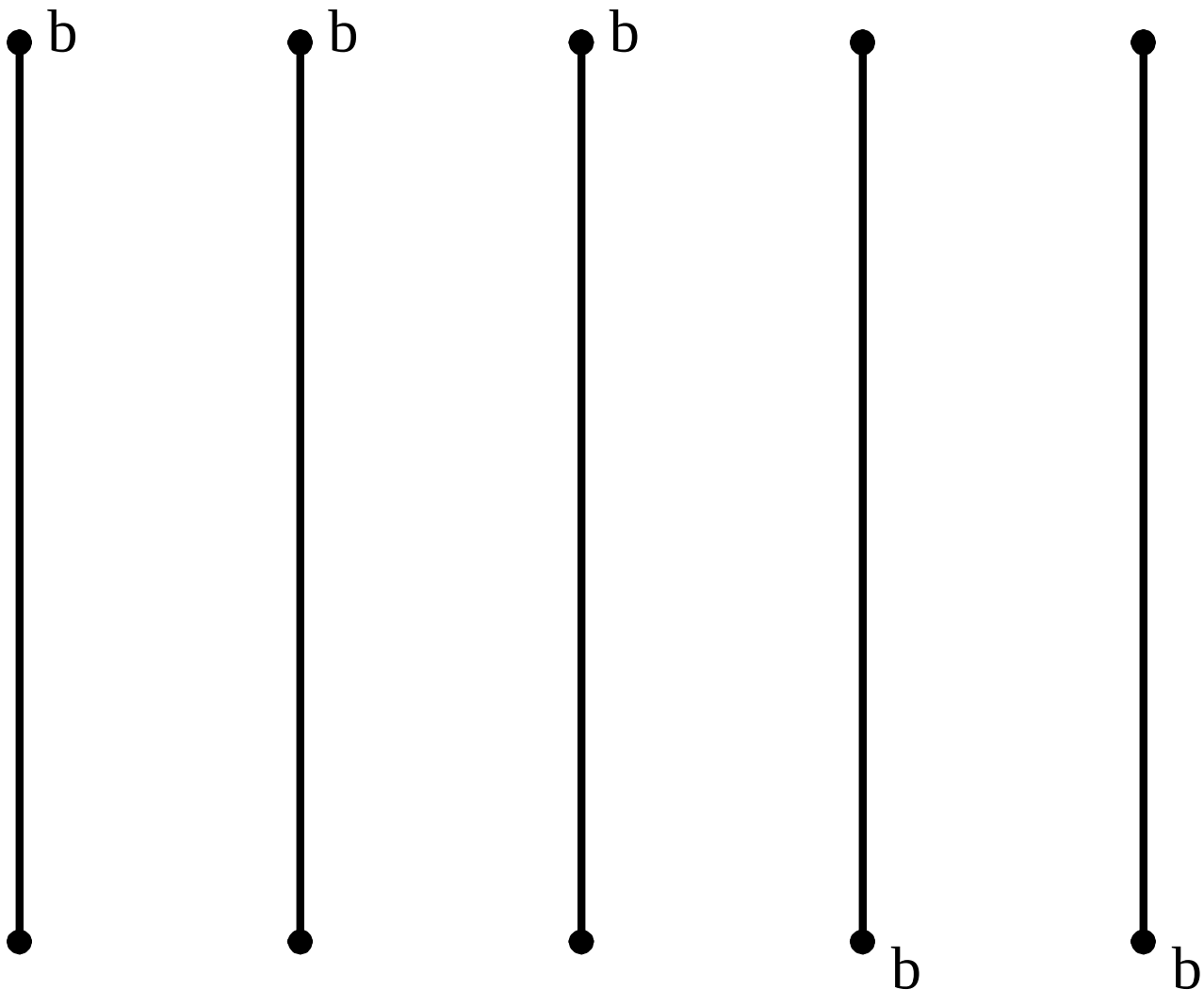
Claim 1: Extreme Outcomes

Fix any alternative b.

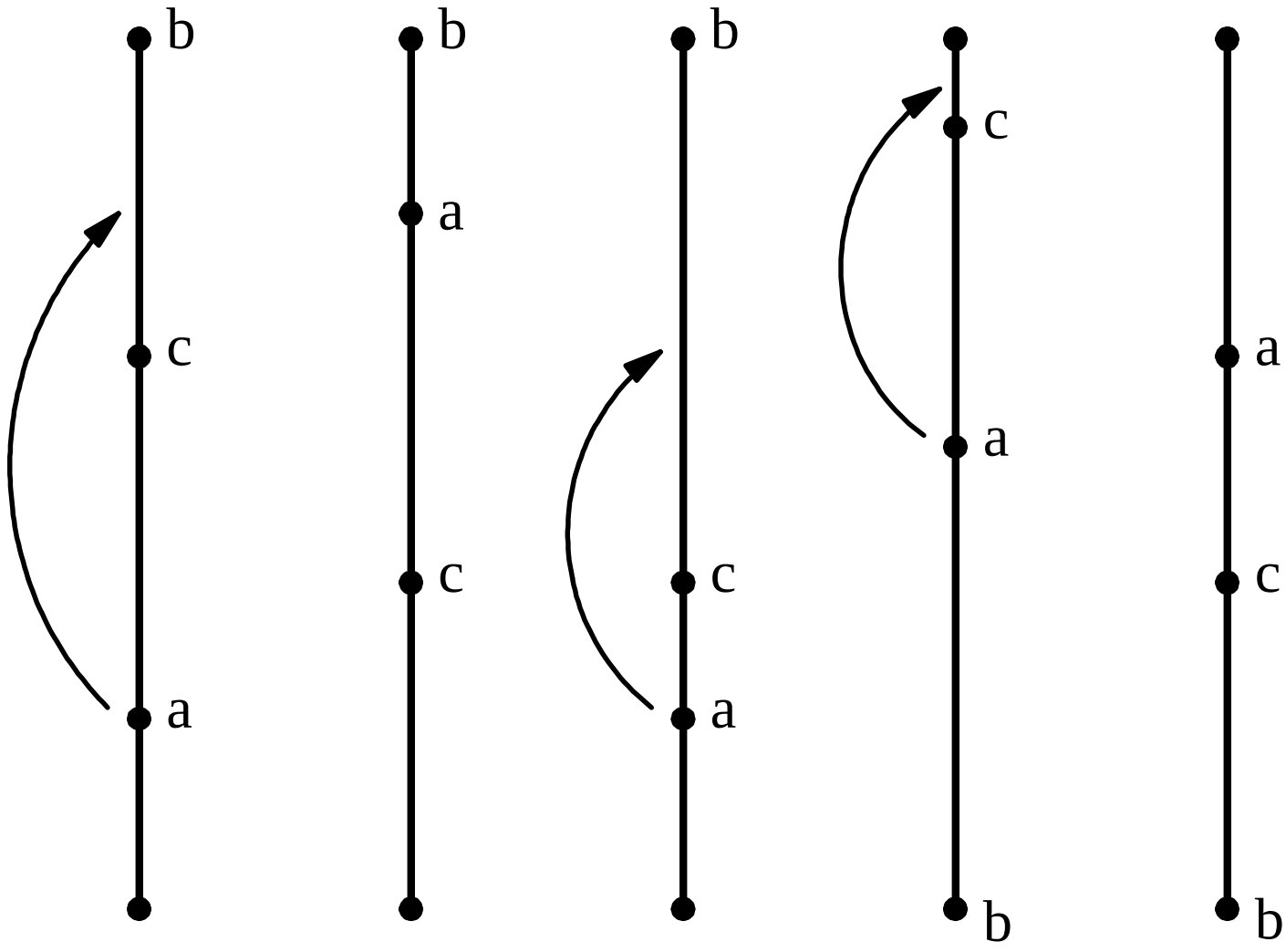
Claim 1: If every agent ranks b to be the "strictly best" or "strictly worst" outcome (e.g., profile 1), then constitution must also.

1. Suppose to the contrary that, for some such preference profile, constitution ranks $c \geq b$ and $b \geq a$.
2. Suppose that we move a strictly above c (profile 2)
   - 2.1 By IIA, we continue to have $c \geq b$ and $b \geq a$
   - 2.2 By transitivity, we have $c \geq a$
   - 2.3 By unanimity $a > c$

PROFILE 1



PROFILE 2



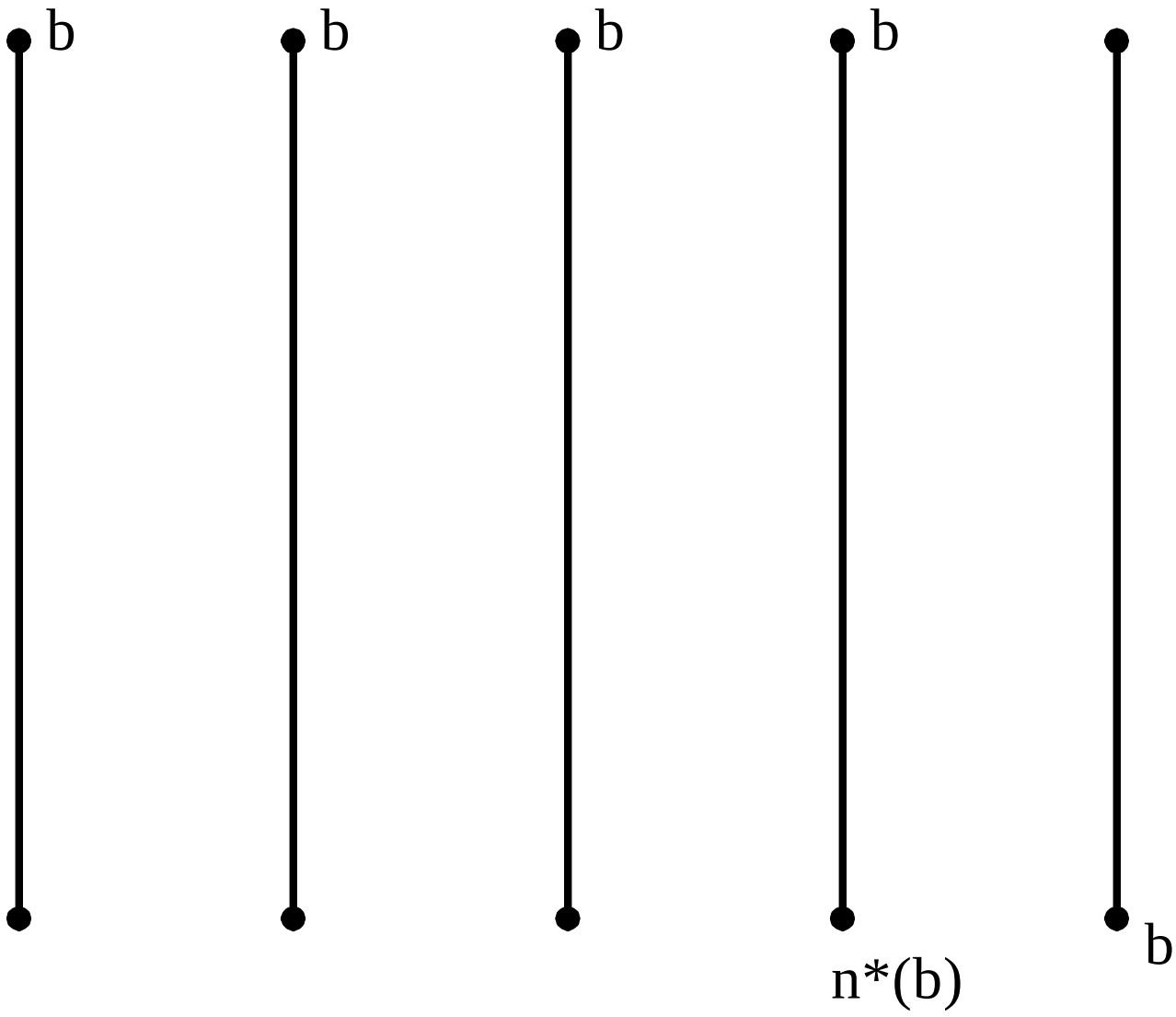
Claim 2: Extreme Pivotality

Fix any alternative b. An agent is *extremely pivotal* (for b) if changing his preference profile at some profile can move b from strictly worst to strictly best under the constitution.

Claim 2: There is an extremely pivotal agent (call him $n^*(b)$)

1. take any preference profile where b is strictly worst for all agents
2. by unanimity, b must be strictly worst for society as well
3. consider the sequence of preference profiles where at each step we switch b from strictly worst to strictly best for some agent
4. by unanimity, b will eventually stop being strictly worst
5. let $n^*(b)$ be the agent whose switch shifts society's preference
6. write 1 for the preference profile before switch and 3 for the preference profile after switch
7. by claim 1, going from profile 1 to profile 3 must move b from strictly worst to strictly best under constitution

PROFILE 3



Claim 3: Extremely Pivotal Guy is Dictator part 1

Claim 3: The extremely pivotal agent $n^*(b)$ is a dictator over every pair of outcomes ac not including b

1. start from profile 3 and move a above b for $n^*(b)$, so

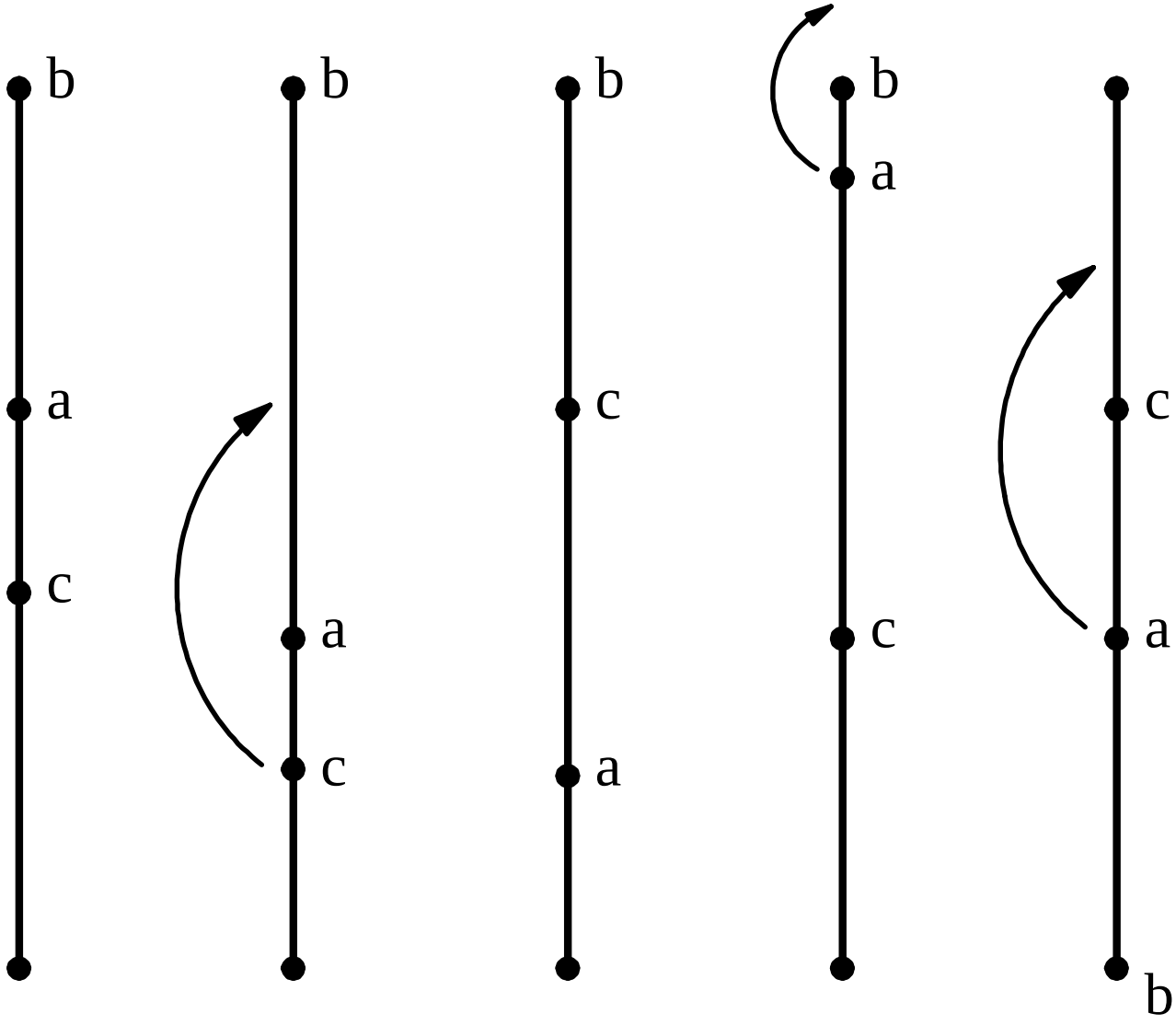
$$a >_{n^*(b)} b >_{n^*(b)} c$$

   and have $n \neq n^*(b)$ rearrange rankings of a and c leaving b in its extreme position; call this profile 4.

2. By IIA....
   - 2.1 constitution at profile 4 has $a > b$ (as a/b rankings are as in profile 1)
   - 2.2 constitution at profile 4 had $b > c$ (as b/c ranking are as in profile 3)

3. By transitivity, constitution at 4 has $a > c$
4. By IIA, constitution's ranking of a and c must depend only $n^*(b)$'s ranking (because we set everyone else's ranking of a and c arbitrarily in constructing profile 4)

PROFILE 4



Claim 4: Extremely Pivotal Guy is Dictator part 2

Claim 4: The extremely pivotal agent $n^*(b)$ is a dictator over every pair ab

1. For some c other than a and$b$, define $n^*(c)$ as in claim 2
2. By claim 3, $n^*(c)$ is a dictator over every pair of outcomes not involving c
3. In particular, $n^*(c)$ is a dictator over ab
4. Since $n^*(b)$ can influence ab ranking, $n^*(c)$ must equal $n^*(b)$

Intuition? Leveraging IIA

We iteratively get a lot of action out of independence of irrelevant alternatives, showing that it implies....

1. ....always agent extremal preferences over one outcome imply constitution has extremal outcome over that outcome
2. ....there exists extremely pivotal agent (for each outcome)
3. ....extremely pivotal agent must be dictatorial

Gibbard-Sattherwaite

- Relation to Gibbard-Sattherwaite Theorem:
  - Arrow's Theorem: no incentives, constitution (mapping from preference profiles to preference profiles), normative/welfare
  - G-S: incentives, social choice function, positive
- Muller-Sattherwaite: if a social choice function (mapping from preference profiles to outcomes) is efficient and monotonic, then dictatorship
  - monotonicity $\approx$ independent of irrelevant alternatives
  - M - S $\Rightarrow$ G-S

Positive Results?

Must drop an assumption....we'll do two examples...

1. $\#X \geq 3$
2. full domain of preferences

Number of Outcomes Matters

1. Let $\#X = 2$.
2. Majority Rule is a constitution that satisfies independence of irrelevant alternatives, unanimity and is not a dictatorship
3. Majority rule has other nice properties....

Positive Result 1: May's Theorem (1952)

Let $\#X = 2$

- A constitution is...
  - *anonymous* if agents' names do not matter
  - *neutral* if alternatives' names do not matter
  - *responsive* if constitutional weak preference is converted to constitutional strict preference if at least one agent switches preference in the right direction

May's theorem: Majority rule is the unique constitution satisfying unanimity, anonymity, neutrality and responsiveness

Positive Result 2: Median Voter Theorem

- A relation $\geq$ is a *linear order* if it is reflexive, transitive and total (i.e., no indifference, for distinct x, y, $x \geq y$ or $y \geq x$ but not both).
- A preference is *single peaked* with respect to linear order $\geq$ if there is an alternative $x \in X$ such that the preference is increasing on $\{y : x \geq y\}$ and decreasing on $\{y : y \geq x\}$, so that

$$\begin{aligned} x &\geq z > y \Rightarrow z \succ y \\ y &> z \geq x \Rightarrow z \succ y \end{aligned}$$

- Example: $X = [\underline{x}, \overline{x}]$ and preferences represented by a strictly concave utility function $u : X \to \mathbb{R}$
- Condorcet cycle preferences are not single peaked with respect to any linear order

Positive Result 2: Median Voter Theorem

- write x_i for agent i's most preferred outcome ("peak")
- agent h is the *median agent* if

$$\#\{i|x_i \geq x_h\} \geq \frac{I}{2} \text{ and } \#\{i|x_h \geq x_i\} \geq \frac{I}{2}$$

Median Voter Theorem: If preferences are single peaked (with respect to some common linear order), then the median agent's peak is a Condorcet winner and thus selected by majority rule.