Giving According To Agreement

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Abstract

We propose an axiom that we call Agreement to deal with changing preferences and derive its empirical implications. The resulting revealed preference condition generalises GARP when preferences are different but one preference relation is still informative about another one. We apply this idea to a social choice experiment, where a player can respond to another player being generous or relatively selfish. We find that people have consistent preferences for each case, but that preferences depend on the selfishness of the other player, and that subjects act in line with Agreement. We thus provide a microeconomic foundation for modelling and interpreting responses to the intentions of other players as a preference for reciprocity.

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1 Introduction

In many economic settings, economics actors can and do react to the decisions of others. Transactions often involve repeated interactions with owners, managers, employees, suppliers, service providers, or consumers. In such situations, reciprocity, a tendency to respond in kind to behaviour or the perceived intention behind that behaviour, can play an important role. Reciprocity can sustain cooperation or conflict where this would not occur otherwise. It introduces path dependency into choice settings: the evaluation of an alternative will depend on how pleasant or unpleasant previous interactions were.

Empirical findings of reciprocal behaviour (e.g. Falk et al., 2008, Falk et al., 2003, Charness, 2004, Cohn et al., 2015, Dohmen et al., 2009, Falk et al., 2018) are often interpreted as reflecting a preference, an interpretation Sobel (2005) calls *intrinsic reciprocity*. Many theories of reciprocity (e.g. Rabin, 1993; Dufwenberg and Kirchsteiger, 2004; Falk and Fischbacher, 2006; Segal and Sobel, 2007) model reciprocal tendencies as a preference, and economic theories about such diverse topics as efficiency wages (Akerlof and Yellen, 1990) and the capacity of states to tax citizens (Besley, 2020) are based on an assumption of reciprocal preferences. Whether reciprocity is a preference therefore has an immediate bearing on the validity of many important models, as well as the interpretation of empirical findings.

Whether or not reciprocity is a preference also has other important implications, for example how what sort of reciprocal behaviour may be observed in the economy and how we can study its welfare implications. If reciprocity is a preference, cooperation can be sustained even if it no longer has a immediate or strategic benefit, or where enforcement of a law or norm is not possible. It can exacerbate wage stickiness: not only might managers be hesitant to lower wages for fear of retaliation, they may not even *want* to do so if previous interactions with employees were pleasant. And if reciprocity is a preference, a method such as the one proposed by Bernheim and Rangel (2007) must be used for welfare analysis as there are different preference relations according to which welfare can be judged and these will lead to different answers.

In this paper, we test whether choices in an experiment are consistent with a preference for reciprocity using a revealed preference approach. The revealed preference approach allows us to test the hypothesis of reciprocal preferences with minimal assumptions. In particular, the approach does not require assuming a functional form for utility functions that represent reciprocal preferences. This is an important advantage as there are many utility functions that might represent social preferences and there is no consensus which should be used. By making minimal assumptions about social preferences, we avoid making incorrect inferences due to incorrectly specified functional forms.

The base hypothesis of reciprocal preferences that we test is that there should exist a preference relation according to which people make social choices, and this preference relation should be different depending on how generous or selfish another person's behaviour is. We do so in an experiment which is similar in spirit to Andreoni and Miller (2002), with subjects allocating money between themselves and another player where the price of giving varies between tasks (a modified dictator game) in response to a more or less generous choice by the other player. This allows us to test for the existence of preference relations that fit with observed choices and whether the preference relation depends on the generosity of the other.

Furthermore, we introduce an axiom to relate different preference relations. Reciprocity suggests behaviour changes in a particular direction depending on how generous or selfish another person has been. Our axiom gives empirical meaning to the idea that one is more or less generous depending on the behaviour of another person. We show the existence of utility functions, one of which is more selfish than the other in line with the axiom we propose, if and only if the revealed preference condition is satisfied. We use this to test whether people indeed respond with generosity to generosity and with selfishness to selfishness.

The concept underlying our axiom is that two preference relations, one of which is less selfish, *agree* on particular allocations. Specifically, if some allocation x is preferred to some allocation y according to the more selfish preference relation, even though allocation y gives the decision maker more than allocation x, then x should also be preferred to y according to the more generous preference relation. As the purely selfish motive would suggest picking y over x, choosing x means allocation y is too selfish even according to the more selfish preference relation, in which case x should certainly be preferred to y according to the more generous preference relation. This *Agreement axiom* captures that while different perceptions of others' intentions may make people more or less selfish, it should not affect how they trade off fairness and efficiency.

Many behavioural findings besides reciprocity show behaviour is sensitive to changes in the choice environment that do not directly affect final outcomes, such as framing or the salience of alternatives. The Agreement axiom and its revealed preference condition can be used to model such behaviour in a very general way. Although the explanation above interprets preferences as more or less selfish in response to more or less selfish behaviour, it can be applied generally whenever one preference relation likes one good or attribute more or less depending on the choice situation.

For a given action by the other player, we find that choices satisfy (or come close to satisfying) standard revealed preference conditions, meaning choices are consistent with maximising a well-behaved preference relation. Yet we find that revealed preferences are different depending on the action of the other player, which means choices can only be explained by people maximising different preference relations depending on the action of the other player, in line with a preference for reciprocity. We furthermore find that choices largely satisfy the Agreement axiom, meaning that preferences are more selfish when the other player's action is more selfish, as expected from reciprocal preferences.

2 Agreement

2.1 Preferences

Suppose a decision maker has two preference relations for allocations of money for herself and someone else. For interpretation, we assume the other to always be the same person. Let $\mathbf{X} = \mathbb{R}^2_+$ be the preference domain and for all $z = (z_1, z_2) \in \mathbf{X}$, let z_1 be the money amount she gives to herself and z_2 be the money amount for the other person. Let $\mathbf{P} = \mathbb{R}^2_{++}$[1] be the set of all prices. The decision maker is characterised by a pair of preference relations $(\succsim_G, \succsim_S)$ on $\mathbf{X}$. The preference relation $\succsim_G$ reflects a more generous self and $\succsim_S$ a more selfish self. Both preference relations are complete and transitive.

[1] We take the convention that $\mathbb{R}^2_+ = \{x \in \mathbb{R}^2 : x_1 \geq 0 \text{ and } x_2 \geq 0\}$ and $\mathbb{R}^2_{++} = \{x \in \mathbb{R}^2 : x_1 > 0 \text{ and } x_2 > 0\}$.

'Generous' and 'selfish' are taken as relative terms here; the selfish set of preferences need not be perfectly selfish.[2]

The decision maker faces two contexts to make choices, which we call *context* G and *context* S, where the decision maker is expected to be more generous in context G than in context S, for example for reasons of reciprocity. The following axiom gives the relation between these two preference relations.

Axiom 1 (Agreement)**.** *For all* $x, y \in \mathbf{X}$ *with* $x = (x_1, x_2)$ *and* $y = (y_1, y_2)$,

$$[x \succsim_S (\succ_S)\ y \quad \text{and} \quad x_1 \leq y_1] \quad \text{implies} \quad x \succsim_G (\succ_G)\ y.$$

Intuitively, Agreement states that if the decision maker (strictly) prefers x to y when choosing according to $\succsim_S$, and she keeps less for herself in choice x than y, then she also (strictly) prefers x to y according to the more generous preferences ($\succsim_G$).[3] An equivalent formulation is presented in Proposition 1 (all proofs are in the Appendix).

Proposition 1. *The Agreement axiom is equivalent to the condition that for all* $x, y \in \mathbf{X}$ *with* $x = (x_1, x_2)$ *and* $y = (y_1, y_2)$, $[x \succsim_G (\succ_G)\ y \quad \text{and} \quad x_1 \geq y_1]$ *implies* $x \succsim_S (\succ_S)\ y$.

2.2 Empirical Implications of Agreement for Revealed Preferences

We start this section with a review of basic concepts of revealed preferences.

Definition 1. *A* set of observations Ω *is a finite collection of pairs* $\{(z^i, r^i)\}_{i=1}^k \subset \mathbf{X} \times \mathbf{P}$.

An observation $(z^i, r^i) = ((z_1^i, z_2^i), (r_1^i, r_2^i))$ denotes how much the decision maker chooses to keep, z_1^i, and to give to the other, z_2^i, given the choice set $\{z^i \in \mathbb{R}_+^2 : r_1^i z_1^i + r_2^i z_2^i \leq 1\}$. We use superscripts to indicate observations and subscripts to indicate coordinates.

[2] Of course, a decision maker may have more than two preference relations. As the interest of this paper is in comparing preferences, we limit the analysis to two preference relations, but multiple such comparisons can be made between any number of preference relations.

[3] Agreement is in the same spirit as the MAT relation in Cox et al. (2008). If one preference relation is *MAT (more altruistic than)* the other preference relation then they satisfy Agreement, but the reverse does not hold. For a proof, see Proposition 3 in the Appendix.

We now define the revealed preference relations on Ω. We say that z^i is *directly revealed preferred* to z, written as $z^i \boldsymbol{R}^0 z$, if $p^i z^i \geq p^i z$; it is *indirectly revealed preferred* to z, written as $z^i \boldsymbol{R} z$, if there exist $z^{i_1}, z^{i_2}, \cdots, z^{i_m} \in \Omega$ such that $z^i \boldsymbol{R}^0 z^{i_1} \boldsymbol{R}^0 z^{i_2} \boldsymbol{R}^0 \cdots \boldsymbol{R}^0 z^{i_m} \boldsymbol{R}^0 z$. We use $\boldsymbol{P}^0$ ($\boldsymbol{P}$) to denote the strict preference relation: z^i is *strictly directly revealed preferred* to z, written $z^i \boldsymbol{P}^0 z$, if $p^i z^i > p^i z$. We say z^i is *strictly revealed preferred* to z, written $z^i \boldsymbol{P} z$, if there exist $z^{i_1}, z^{i_2}, \cdots, z^{i_m} \in \Omega$ such that $z^i \boldsymbol{R}^0 z^{i_1} \boldsymbol{R}^0 z^{i_2} \boldsymbol{R}^0 \cdots \boldsymbol{R}^0 z^{i_m} \boldsymbol{R}^0 z$ and at least one of these revealed preference relations is strict.

Definition 2. *A utility function* $u : \mathbf{X} \to \mathbb{R}$ rationalises *a set of observations* Ω *if* $u(z^i) \geq u(z^j)$ *whenever* $z^i \boldsymbol{R} z^j$.

Afriat (1967) and Varian (1982) provide an easily testable condition which is necessary and sufficient for the existence of a utility function that rationalises a set of observations.

Axiom 2 (GARP)**.** *A set of observations* Ω *satisfies the* Generalised Axiom of Revealed Preference (GARP) *if for all* $z^i \boldsymbol{R} z^j$ *not* $z^j \boldsymbol{P}^0 z^i$.

Varian (1982) provides a construction of a utility function that rationalises Ω when Ω satisfies GARP. We use the following representation that can be derived directly from Varian (1982).

Proposition 2. *A set of observations* Ω *satisfies GARP if and only if there exists a continuous, strictly increasing and quasiconcave utility function* u *that rationalises* Ω *where* $u((a,a)) = a$ *for all* $a \in \mathbb{R}$.

In our model, we have two sets of observations, one from context G and the other from context S, denoted by $\Omega_G = \{x^i, p^i\}_{i=1}^n$ and $\Omega_S = \{y^j, q^j\}_{j=1}^m$. Let $\boldsymbol{R}_G$ and $\boldsymbol{R}_S$ be the revealed preference relation on Ω_G and Ω_S respectively. If Agreement holds then the observations Ω_G (Ω_S) reveal information about $\succsim_S$ ($\succsim_G$). We explain this point with Figure 1.

In the left image, given the budget line in context G, the decision maker chooses x^i, so x^i is revealed preferred to all allocations on and below the budget line in context G. Every allocation from the area indicated by the red vertical lines gives the decision maker less than x^i does. Thus, by Agreement, x^i is also revealed preferred to the allocations in the area indicated by the red vertical lines in context S. In the right graph, the decision

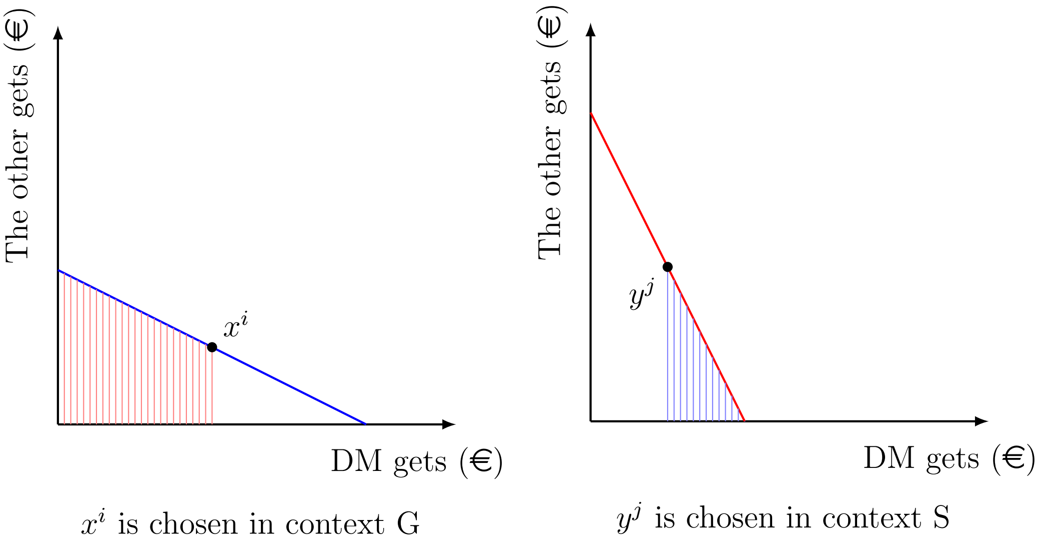


Figure 1: Revealed preference implications of the Agreement axiom. The observations are revealed preferred to the area below the budget line in the same context and to the area indicated with the vertical lines in the other context.

maker chooses y^j in context S, and any allocation in the area indicated with the blue vertical lines gives the decision maker more than y^j does. Thus, by Agreement, she should also prefer y^j over any allocation in the blue area in context G.

Through Agreement we can thus extend the revealed relation $\boldsymbol{R}_G$ by incorporating information from Ω_S and extend $\boldsymbol{R}_S$ by incorporating information from Ω_G. Let $\Omega = \Omega_G \cup \Omega_S = \{z^i, r^i\}_{i=1}^k = \{x_i, p^i\}_{i=1}^n \cup \{y_j, q^j\}_{i=1}^m$, where (z^i, r^i) is an observation from either Ω_G or Ω_S. Writing the extensions as $\tilde{\boldsymbol{R}}_G$ and $\tilde{\boldsymbol{R}}_S$, it follows that $z^i \tilde{\boldsymbol{R}}_G^0 z^j$, if $z^i \boldsymbol{R}_G^0 z^j$ or if $z^i \boldsymbol{R}_S^0 z^j$ and $z_1^i \leq z_1^j$; and that $z^i \tilde{\boldsymbol{R}}_S^0 z^j$, if $z^i \boldsymbol{R}_S^0 z^j$ or if $z^i \boldsymbol{R}_G^0 z^j$ and $z_1^i \geq z_1^j$. $\tilde{\boldsymbol{P}}_G$ and $\tilde{\boldsymbol{P}}_S$ are defined analogously. We next define rationalisation in our model.

Definition 3. *An altruistic utility function $u : \mathbf{X} \to \mathbb{R}$ and a selfish utility function $v : \mathbf{X} \to \mathbb{R}$* Agreement-rationalise (AG-rationalise) *Ω, if $u(z^i) \geq u(z^j)$ whenever $z^i \tilde{\boldsymbol{R}}_G z^j$ and if $v(z^i) \geq v(z^j)$ whenever $z^i \tilde{\boldsymbol{R}}_S z^j$.*

The altruistic and selfish utility functions represent the extended revealed preference relations inferred from the observations Ω. AG-rationalisation captures what it means to choose according to one's preferences if these preferences satisfy Agreement.

Axiom 3 (AG-GARP)**.** *A set of observations Ω satisfies* Agreement-GARP (AG-GARP), *if* $z^i \tilde{\boldsymbol{R}}_G z^j$ *implies not* $z^j \tilde{\boldsymbol{P}}^0_G z^i$ *and* $z^i \tilde{\boldsymbol{R}}_S z^j$ *implies not* $z^j \tilde{\boldsymbol{P}}^0_S z^i$.

Theorem 1. *Given a set of observations* $\Omega = \Omega_G \cup \Omega_S$ *with* $\Omega_G = \{x^i, p^i\}_{i=1}^n$ *and* $\Omega_S = \{y^j, q^j\}_{j=1}^m$, *the following conditions are equivalent:*

(a) The set of observations Ω *satisfies AG-GARP.*

(b) There exist a generous utility function u *and selfish utility function* v *that are continuous, strictly increasing and quasiconcave and AG-rationalise* Ω. *Moreover, for all* $x, y \in \mathbf{X}$, $u(x) \geq u(y)$ *with* $x_1 \geq y_1$ *implies that* $v(x) \geq v(y)$ *and for all* $x, y \in \mathbf{X}$, $v(x) \geq v(y)$ *with* $x_1 \leq y_1$ *implies that* $u(x) \geq u(y)$.

Theorem 1 describes that if the set of observations satisfies AG-GARP, the decision maker's choices can be represented by two utility functions, one of which is more generous than the other. These two utility functions represent the extended revealed preference relations. Theorem 1 thus captures the empirical implications of Agreement and rationalisation.

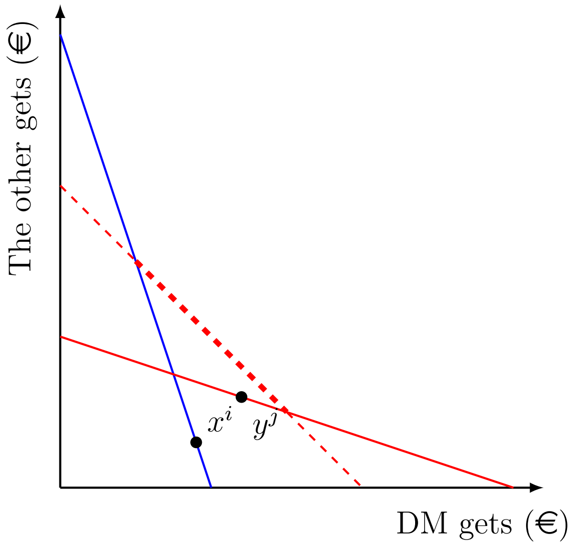
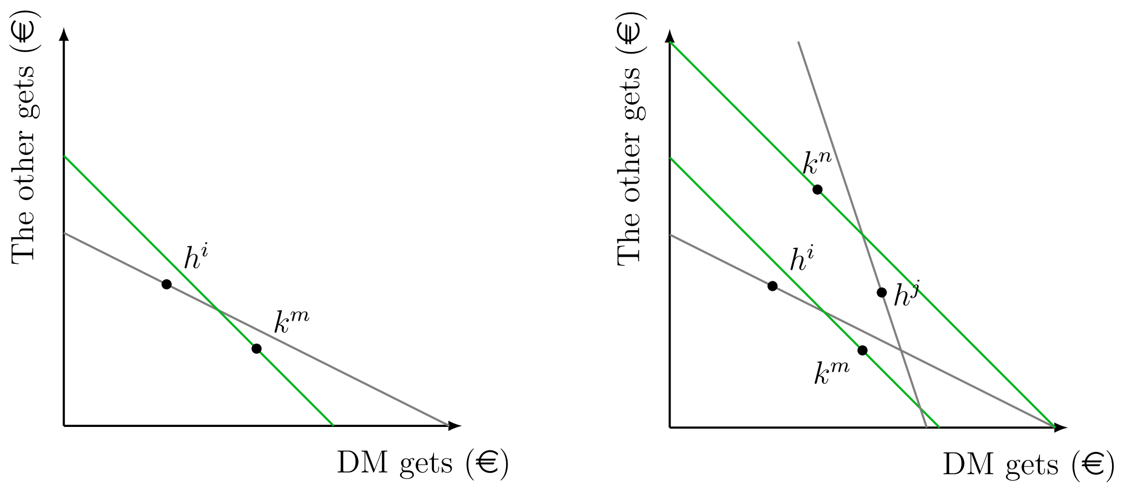


Figure 2: AG-GARP predicts that a third choice on the dashed red budget line must be on the thick red part.

We can use AG-GARP to predict choices. This is illustrated in Figure 2. x^i is chosen in context G and y^j is chosen in context S. Suppose a person is next presented with the dashed red budget in context S. A person who chooses according to preferences which



h^i must be context G, k^m must be context S   Cannot be rationalised with Agreement

Figure 3: Identifying contexts (preference relations) and testing AG-GARP without assuming which context is G or S.

satisfy AG-GARP must then choose on the thick red part of the budget line. Choosing to keep more violates GARP within context S (and therefore violates AG-GARP); choosing to keep less violates Agreement.[4] Thus, we get predictions beyond that choices must be different or that the decision maker must keep more in the selfish context on the same budget.

We get clear welfare implications from AG-GARP using the behavioural welfare definitions of Bernheim and Rangel (2007). They define an alternative x to be a *strict individual welfare improvement* over an alternative y if a decision maker sometimes chooses x but never y, or never chooses either, when x and y are both available. When $z^i \boldsymbol{P}_S z$ and $z_1^i \leq z_1$, we have $z^i \tilde{\boldsymbol{P}}_G z$ and $z^i \tilde{\boldsymbol{P}}_S z$. A decision maker who satisfies AG-GARP will then never choose z over z^i, hence z^i is a strict individual welfare improvement over z. Similarly, if $z^i \boldsymbol{P}_G z$ and $z_1^i \geq z_1$, we have $z^i \tilde{\boldsymbol{P}}_G z$ and $z^i \tilde{\boldsymbol{P}}_S z$ and z^i is a strict individual welfare improvement over z for a decision maker who satisfies AG-GARP.

Although we have assumed that we have a dataset from context G where the decision maker is more generous and a dataset from context S where the decision maker is more

[4]Such predictions can easily be extended to situations where a person does not choose in perfect accordance with AG-GARP by correcting for efficiency. Efficiency is discussed in Section 4.2.

selfish, it is not essential that we know for sure in which context the decision maker is more generous. If we want to model changing preferences without specifying in which context the preferences should be more or less generous we can let the data speak for itself (as long as we have two distinct sets of data). Satisfying Agreement means violations of GARP can only go in one direction (see Figure 10 in the Appendix). We can use this to infer from the data which is context G and which is context S. If two observations from two different contexts violate GARP, then the observation where the decision maker keeps less must come from context G, and hence all observations from that set must belong to Ω_G, as illustrated by the left image of Figure 3. If we do not wish to assume which set of observations is more generous, we can also still test for AG-GARP: AG-GARP is then violated if we find violations of GARP that go in different directions. This is illustrated by the right image of Figure 3 for two sets of observations, $\{h^i, h^j\}$ and $\{k^n, k^m\}$. The violation of GARP by h^i and k^m suggests the set $\{k^n, k^m\}$ must come from the more selfish context, but the violation of GARP by h^j and k^n suggests $\{h^n, h^m\}$ must be the observations from the more selfish context. This violates AG-GARP no matter which set of observations is taken as the more selfish one.

In our setting, a decision maker has preferences over $X = \mathbb{R}^2_+$ and every point in X is taken as an allocation of money. The preference domain X can have other interpretations. For example, $x = (x_1, x_2) \in X$ can be consumption and leisure, and a person may prefer leisure (good 1) more when they are older than younger, or more in the summer than in the winter. Our method can be applied very generally.

3 Experiment

To measure how social preferences depend on the generosity or selfishness of another person we ran an interactive experiment where pairs of subjects made choices. One player (FM) made a single choice between two allocations: either giving €13 to the other player and keeping €27 or giving €18 and keeping €18. In our experiment, these two possibilities represent the contexts G and S that the second player faced. Using the strategy method, the second player made 14 choices from 14 linear budgets for each of these two contexts (thus making a total of 28 choices). We call the second player SM, for

Second Mover (or after the fact their choices were elicited with the Strategy Method). The first player we name FM, for First Mover. For the purposes of this paper, we are only interested in the choices made by the second mover. The first mover was only part of the experiment to incentivise player SM.

Crucially, both players were informed that only one choice would be implemented for real: either the choice of the first mover (FM) would be implemented or one of the choices of the second mover (SM) would be implemented. In the latter case, one budget would be randomly selected from the set of budgets corresponding to the choice made by FM. Thus, player SM's choice was only practically relevant as a response to FM's *intention*, never to a practically implemented choice by FM.

Player FM was informed that player SM could divide money between them at different rates, that the minimum SM could give them was €0 and that the maximum differed per budget but was never more than €60. Player FM was informed that SM made 14 choices for both of FM's possible choices. Player SM was informed that player FM was presented with this information. Both players were informed how they would be matched to each other and were informed about the payment procedure, including that both players would be paid either according to FM's choice or to one of SM's choices and that this was determined randomly.

After the instructions, both player SM and player FM were asked to answer three multiple choice comprehension questions. If they answered a question incorrectly they were given immediate feedback as to why their answer was wrong. They could only continue once they had answered the question correctly. Player SM was also given some practice tasks to familiarise them with the interface.[5]

Player FM could indicate their choice simply by selecting the desired allocation and confirming their choice (they could revise their choice before confirming). The order of the alternatives was randomised and the same in the instructions and in the actual choice situation.

The 14 budgets player SM was faced with in each context (shown in Figure 4) were all linear budgets where SM could give some money to FM at different rates, from giving

[5]Player FM was not provided with practice tasks because their task was a very simple binary choice which required no more than clicking on the desired option.

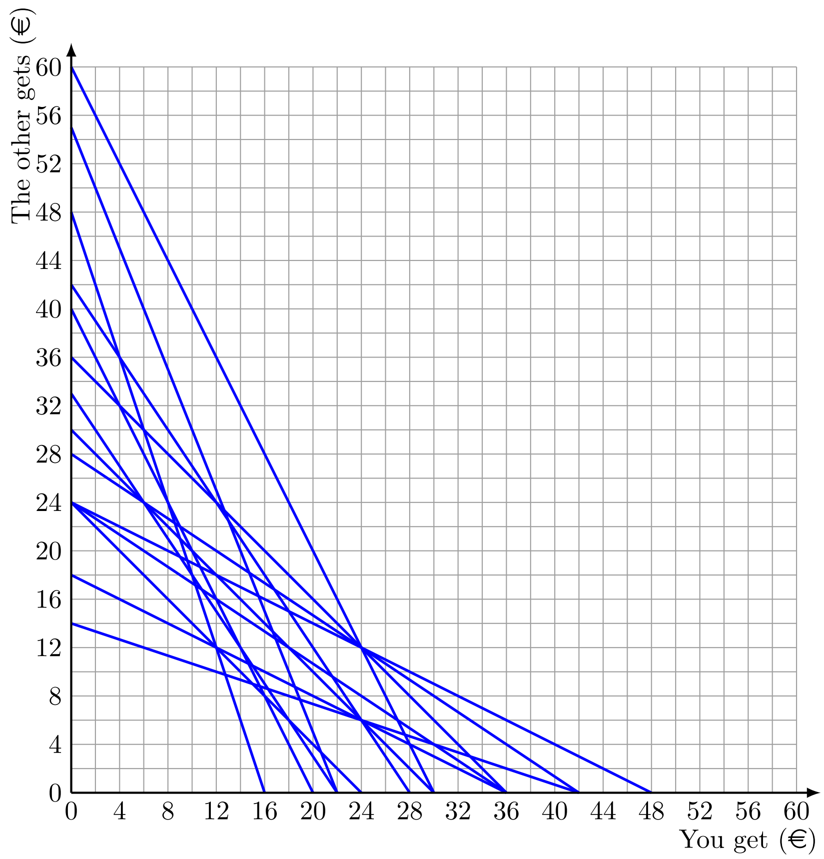


Figure 4: Budgets used in the experiment.

FM €0.33 to giving FM €3 for every euro SM gave up. The minimum SM could keep was always €0, as was the minimum SM could give away to FM. The maximum amount SM could keep or give to FM varied by budget and was never higher than €48, respectively €60. Budgets were chosen such that they intersected at many points to get good test power. The average slope was bigger than 1, meaning that on average giving up €1 increased FM's payoff by more than €1, to make it attractive for player SM to give at least some money to FM (if all choices are on the axis, power is zero).

Which of the contexts (player FM's two possible choices) for which player SM made choices was presented first was randomised at the start of the experiment and then kept the same in the instructions and in the choice tasks. The order of the 14 budgets was randomised for each of the contexts separately, so that the order of the budgets was different between the two contexts.

To make performing the tasks as easy as possible for player SM we developed an

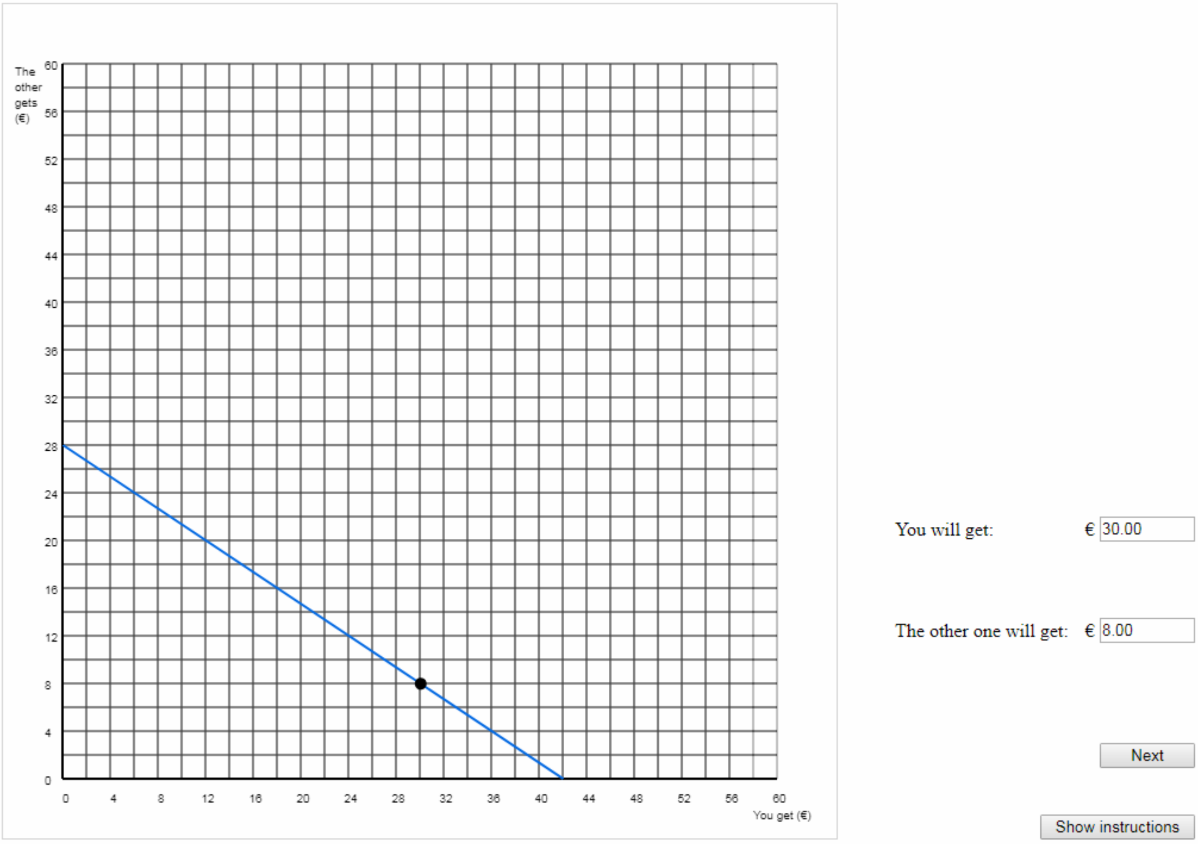


Figure 5: A screenshot of the interface for one of the player SM's tasks.

interface which graphically displayed the budget (shown in Figure 5). Player SM could make choices by clicking on any point on the budget, by typing the amount they wanted to keep, or by typing the amount they wanted to give to player FM. When either of these three methods had been used, a dot would appear on the budget line to indicate their choice and the amounts that player FM and SM would receive were displayed automatically in number fields. Player SM could then revise their choice if they were not happy with the resulting allocation by clicking somewhere else on the budget line, by dragging the dot around on the budget line, or by entering a different number in either of the fields displaying how much they would keep or give away.

The software automatically calculated a minimum step size in multiples of €0.05 and any choice was automatically converted to the closest step. This ensured that all choices were exactly on the budget line. For example, where player SM could give away €3 for every €1 they gave up, the minimum amount SM could give up other than 0 was €0.05 and the minimum amount they could give away other than 0 was €0.15. Giving up €0.03

or giving away €0.10 to FM was automatically rounded to giving up €0.05 and giving away €0.15. Entering an amount not on the budget line (negative amounts or amounts greater than the maximum amount that could be kept or given to FM) resulted in a message indicating that the amount entered was invalid. Player SM could only continue to the next task after choosing a valid allocation.

On entering the lab, subjects were assigned a cubicle and asked to wait for the start of the experiment (if there was an odd number of subjects the last subject to arrive was given a show-up fee and did not participate). At the start of the experiment, subjects were handed an envelope containing two slips of paper containing their subject ID. After using their subject ID to log on, we collected their envelopes, leaving one of their IDs at their cubicle, and sorted them into one pile containing player FM IDs and one containing player SM IDs. Subjects who finished early were then asked to randomly match players by choosing an envelope from each pile (without seeing the ID codes within) and to put the ID code of player SM into the envelope containing the ID of player FM. Next, subjects were asked to roll a die to determine whether each pair would be paid according to player FM or player SM's choice (where the probability of FM or SM being selected was equal). This was marked on the envelope. If the pair was paid according to SM's choice, a subject was asked to draw a ball from a bag with 14 balls numbered 1 to 14 inclusive to select according to which budget the pair would be paid out. This too was marked on the envelope. When all subjects had finished subjects were asked one by one to come to the front desk to be paid.

The experiment was run in the ESE-econlab of Erasmus University Rotterdam. There were 9 sessions of roughly 20 subjects each, with a total of 170 subjects participating. The experiment lasted about an hour and the average payment was €16.47.

4 Analysis

In this section, we first show a few descriptive statistics for the different treatments. We then present our analysis of the data based on the revealed preference approach outlined in Section 2, and finally we present the results of more conventional parametric analysis assuming a CES utility function.

4.1 Descriptives

Player FM chose the more selfish allocation (€27 for themselves, €13 for player SM) roughly as often as the more equal allocation (€18, €18), with frequencies of 55% and 45%, respectively. This shows both options were attractive to player FM, which is important because if one option was very unattractive and therefore unlikely to be chosen, this case would not be well-incentivised for player SM.

Table 1: Choices by players SM conditional on the choice by FM.

	FM generous			FM selfish		
	Self (€)	Other (€)	Share self (%)	Self (€)	Other (€)	Share self (%)
Min	5	0	18.9	6	0	25.0
Median	19	10	70.4	22	6.90	78.3
Max	48	40	100	48	32.25	100

Throughout the analysis, we will treat player FM choosing €27 for themselves and €13 for player SM as player FM being (relatively) selfish, and player FM choosing €18 for themselves and player SM as FM being (relatively) generous. Table 1 shows various statistics on the choices of players SM conditional on the choice by player FM. The median choice gave away about 30% of their endowment to player FM if FM is generous, and about 22% if FM is selfish. The median amount kept by player SM was about €3 higher and SM gave FM about €3 less if FM is selfish. These differences are all significant with p-values < 0.001 according to Wilcoxon signed-rank tests.[6] Player SM is thus less generous when FM chooses allocation (€27, €13) than when FM chooses (€18, €18), consistent with reciprocity if we take the former as being a more selfish action. For the budgets with a price of 1, corresponding to the classic dictator game, the share of what was kept is 67.8% when FM's choice is generous and 77.3% when FM's choice is relatively selfish.

[6]This is true both when we take each individual choice as an observation, and when we take the average per subject as an observation.

4.2 Revealed preference analysis

In our analysis, we only use the data of player SM's choices. We first present the revealed preference analysis. We take the set of player SM's choices responding to player FM's possible selfish choice of keeping €27 and giving €13 as context S and the set of player SM's choices responding to player FM's possible more generous choice of keeping €18 and giving €18 as context G. Based on Proposition 2 and Theorem 1, we have the following hypotheses.

Hypothesis 1. *Player SM has different preferences in both contexts:* $\succsim_G \neq \succsim_S$. *This implies the following testable conditions: GARP holds on* Ω_G *and on* Ω_S*, but not on* Ω*.*

Hypothesis 1 is the simplest hypothesis generated by reciprocal preferences: people choose according to some (social) preference relation, but this preference relation is different depending on how generous another player has been. This requires GARP to be satisfied on the data from each context separately, as otherwise people are not maximising any utility function, but not on the data together, because people do not maximise the same utility function in both contexts.

Hypothesis 2. *Player SM has different preferences in each context, which are connected by Agreement. Player SM uses preferences* $\succsim_G$ *in context G and* $\succsim_S$ *in context S. Testable condition: AG-GARP holds on* Ω*, with the choices from* Ω_S *being more selfish.*

This second hypothesis captures that reciprocal preferences have a direction: we expect a person to be more generous when the recipient was relatively generous. The Agreement axiom as captured by AG-GARP gives empirical meaning to being more generous (or more selfish) for budgets with different prices and endowments (see Section 2).

As satisfying GARP is a binary variable, where one either satisfies it or does not and minor and economically unimportant violations of GARP are treated the same as very significant violations, we use Afriat's *Critical Cost Efficiency Index* (CCEI) (Afriat, 1972) instead. This captures how far away someone's choices are from utility maximisation. The index is bounded between 0 and 1. When choices satisfy GARP, the index is 1. When GARP is violated, the index is smaller than 1, with bigger violations (relative to the budget) resulting in lower indices. We construct a similar index for AG-GARP, which we call the *Agreement Efficiency Index (AGEI)* or simply Agreement efficiency.

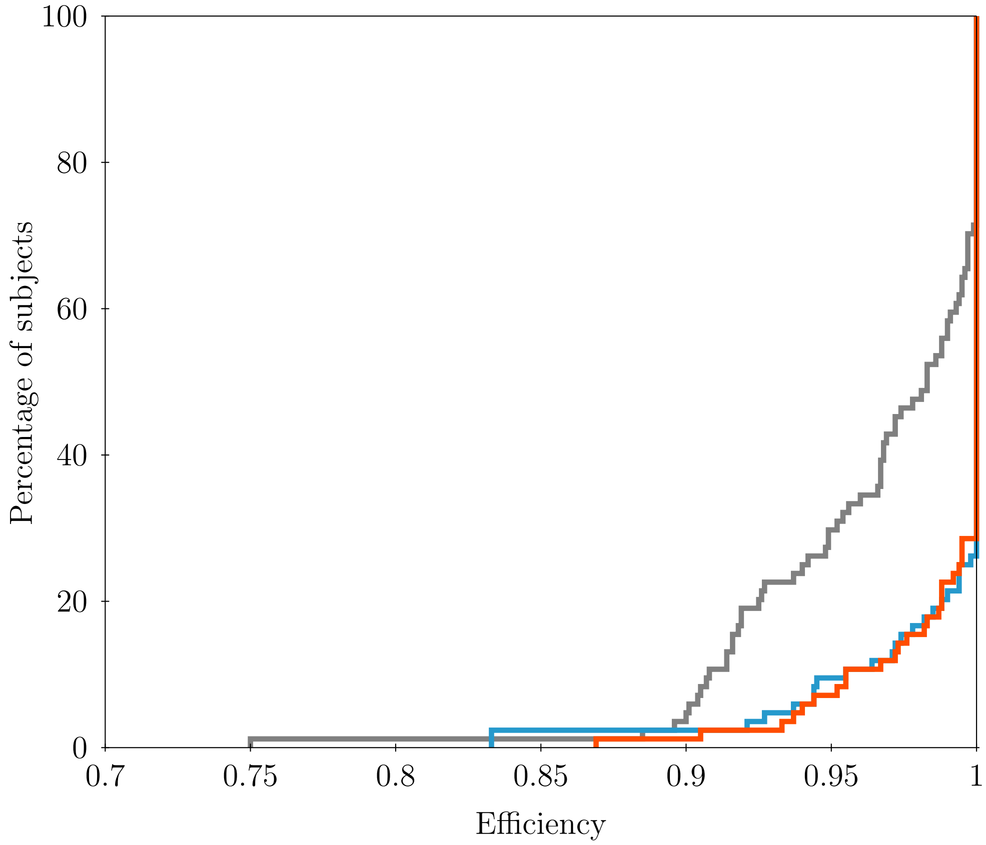


Figure 6: Empirical CDFs of CCEI (efficiency) for context G (blue), S (red) and a mixture of choices from G and S (grey).

Figure 6 shows empirical cumulative distribution functions of CCEIs for choices made in context G (in blue) and S (red). Summary statistics are presented in the second and third column of Table 2. As the Figure and Table show, the distributions in context G and S are very similar, which means subjects came equally close to utility maximisation in either treatment. A Wilcoxon signed-rank test (used throughout this section) cannot reject that the distributions are the same (p-value 0.703). CCEIs are high in both contexts, with a majority achieving the maximum possible CCEI of 1 and an average CCEI of around 0.99. This means choices are almost perfectly consistent with utility maximisation in a given context.

The grey line in Figure 6 shows CCEIs when combining choices from half the budgets

Table 2: Summary statistics of CCEI (efficiency) for context G, context S, and a mixture of choices from G and S.

	Context G	Context S	Mixture from G and S
Min	0.833	0.869	0.750
Median	1.00	1.00	0.983
Max	1.00	1.00	1.00
Mean	0.989	0.990	0.966

in context G with choices the same subject made from the remaining budgets in context S (summary statistics are presented in the fourth column in Table 2).[7] Mixing choices in this way ensures efficiencies are calculated based on the same budgets and the same number of choices, which is important because different combinations of budgets or a different number of them can make it more likely to detect more severe violations of GARP and thus effect power. By using the same budgets and the same number of choices in the mixture, power is the same when calculating CCEI in each context separately and when mixing them. The lower efficiencies observed here (p-value < 0.001 compared to either G or S) are evidence that preferences revealed in context G and S are different: because preferences differ between the two contexts, revealed preferences are contradictory, which leads to violations of GARP and low CCEIs when we treat them as if they result from maximising a single utility function.

The evidence presented in Figure 6 and Table 2 thus supports Hypothesis 1: the data is consistent with people choosing according to some preference relation in context G and choosing according to some preference relation in context S, but preferences are not the same in both contexts.

Now that it has been established that revealed preferences are different between context G and S we investigate whether these different preference relations can be connected with Agreement. In Figure 7 the empirical cumulative distribution function of Agreement efficiencies (taking the preference relation from context G as the more generous one) is

[7] There are many different ways to select these budgets. The grey line in Figure 6 is the mean of CCEIs calculated across these different combinations.

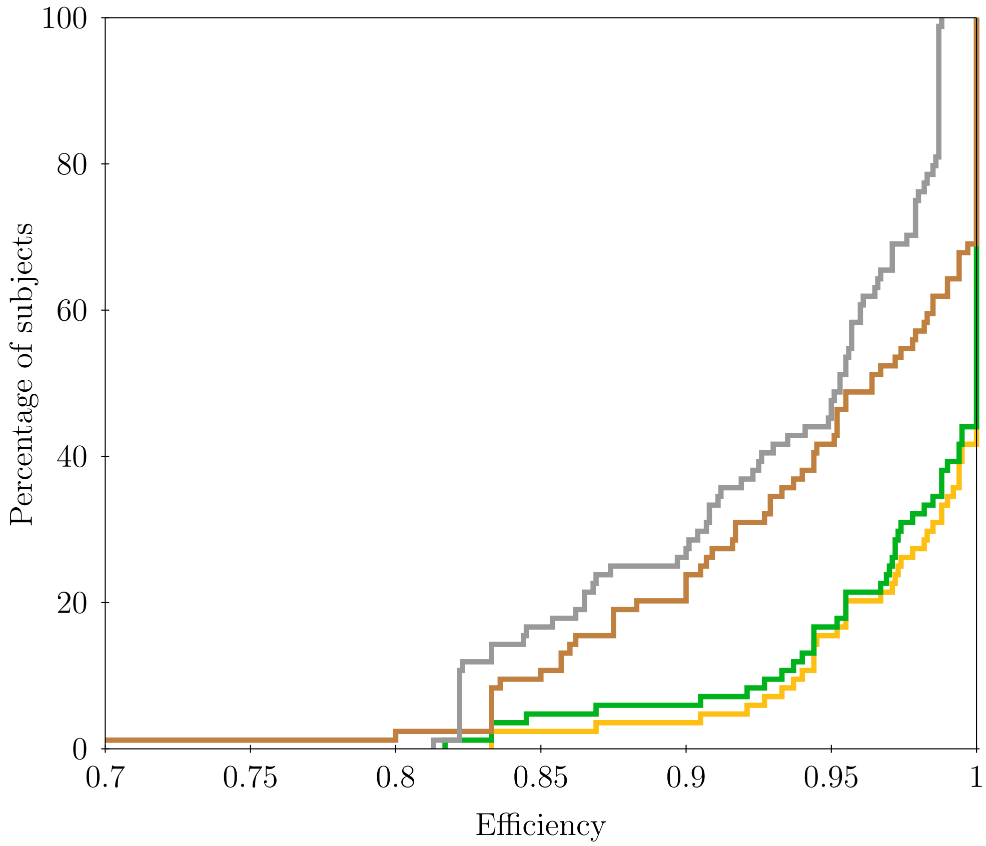


Figure 7: Empirical CDFs of the minimum CCEI from context G and S (yellow), Agreement efficiency over set G and S (green), Agreement efficiency with set G and S reversed (brown) and Agreement efficiency over observations from G of two different subjects (grey).

presented in green, together with the empirical distribution function of the minimum Afriat efficiency of context G and S (yellow). Summary statistics are presented in the second and third columns of Table 3. The minimum Afriat efficiency of context G and S (yellow in Figure 7) is an upper bound on the Agreement efficiency (green): any violation of GARP in either G or S is also a violation of AG-GARP.[8] As we can see, the distributions of Agreement efficiencies and Afriat efficiencies are very close, so Agreement fits behaviour very well.

The brown line in Figure 7 (summary statistics are in the fourth column of Table 3) shows Agreement efficiencies when we take the data from context G as the more selfish

[8]For this reason, the difference is almost necessarily statistically significant (p-value < 0.001).

Table 3: Summary statistics of Agreement efficiencies.

	Minimum CCEI	Agreement efficiency		
	both contexts	G more generous	S more generous	Different subjects
Min	0.833	0.817	0.667	0.813
Median	1.00	1.00	0.964	0.953
Max	1.00	1.00	1.00	0.988
Mean	0.981	0.977	0.944	0.929

The second column shows the lowest CCEI a subject had in either treatment, which is an upper bound on Agreement efficiency.

data and the data from context S as the more generous data. Agreement efficiencies are then clearly lower (p-value < 0.001) than the Agreement efficiencies of the green line, which is evidence that indeed people become more generous in response to a generous action by another person.

One possible explanation for the small difference between Agreement and Afriat efficiencies is that we may have little power to reject AG-GARP. Revealed preference conditions are very general and therefore tend to be rather permissive, meaning that we may not detect violations even if the decision maker does not satisfy the conditions. The decrease in Agreement efficiencies when we reverse the generous and selfish data shows that the condition is not so weak that is unlikely to detect violations. This is a somewhat extreme case, taking data where average giving is lower as the more generous data. Therefore Figure 7 also presents (in grey) Agreement efficiencies calculated taking the choices of one subject from context G and the choices of another subject from the same context (summary statistics in the fifth column in Table 3).[9]

We do not expect the generous choices of one subject to be a more selfish version of the choices by another subject, so we expect to detect violations of AG-GARP if AG-

[9]Because the issue here is that two different conditions are tested (GARP and AG-GARP) rather than that the number of budgets differs (as in Figure 6) we cannot simply correct for the number of budgets. There are many ways to match one subject to another subject; we report the mean taken over the Agreement efficiencies calculated for every possible match.

GARP is sufficiently demanding. The Agreement efficiencies of the grey line in Figure 7 are clearly much worse (p-value < 0.001) than Agreement efficiencies from two contexts for the same subject (green), so the good performance of Agreement is not due to a lack of power, but simply because it describes behaviour well. There is a direction to how preferences change depending on the generosity of the other person, and this can be captured with Agreement.

4.3 Parametric analysis

The Agreement axiom can also be used with parametric assumptions. In this section, we perform parametric analysis to complement the revealed preference analysis of Section 4.2. We do so for constant elasticity of substitution (CES) utility functions. The Agreement axiom has an intuitive interpretation for CES utility functions, which have the following form:

$$u(z) = (\alpha z_1^\rho + (1-\alpha) z_2^\rho)^{1/\rho} \tag{1}$$

Here $z \in \mathbf{X}$ is an allocation. Parameter ρ determines the elasticity of substitution, that is, the curvature of the indifference curves. Thus ρ determines the trade-off between equality and efficiency. Parameter α is the distribution parameter, and captures how the payoff to the decision maker is traded off against the payoff of the other person. If decision makers' utility can be described by (1), that is, $u(z) = (\alpha_G z_1^{\rho_G} + (1-\alpha_G) z_2^{\rho_G})^{1/\rho_G}$ for $\succsim_G$ and $v(z) = (\alpha_S z_1^{\rho_S} + (1-\alpha_S) z_2^{\rho_S})^{1/\rho_S}$ for $\succsim_S$, the hypotheses parallel to those of our revealed preference analysis are the following (the proof is in the Appendix).

Hypothesis 3. *Player SM has different preferences in each context, which are connected by Agreement. Testable parametric implication:*

$$\alpha_G \leq \alpha_S \quad and \quad \rho_G = \rho_S.$$

Following Andreoni and Miller (2002), we estimate CES functions for each individual for both treatments. With the budget constraint $z_1 + pz_2 = m$ the demand function is

$$z_1(p, m) = \frac{[\alpha/(1-\alpha)]^{1/(1-\rho)}}{p^{-\rho/(1-\rho)} + [\alpha/(1-\alpha)]^{1/(1-\rho)}} m = \frac{D}{p^r + D} \tag{2}$$

Where $r = -\rho/(1-\rho)$ and $D = [\alpha/(1-\alpha)]^{1/(1-\rho)}$. We first estimate r and D by non-linear least squares, then back out estimations of α and ρ.

For only 59 out of the 85 player SM we can fit the CES utility function. Twelve subjects make all choices on one axis (in all cases keeping everything) in either treatment and are therefore excluded, leaving 73 subjects. Additionally, the estimation for 14 player SMs does not converge because they chose on the axis except once or twice in each treatment, or they always chose proportionally in one treatment (the ratio of the money amount player SM keeps relative to the amount they give is constant).

The median $\hat{\alpha}$ and $\hat{\rho}$ for both contexts are reported in Table 4: $\hat{\alpha}$ is bigger when player FM chooses the less generous allocation than when FM chooses the more generous allocation. A Wilcoxon signed-rank test indicates that this difference is significant (p-value 0.033). By contrast, the difference in $\hat{\rho}$ between the two contexts is not significant (p-value 0.188). This is in accordance with hypothesis 3.

Table 4: Summary of Wilcoxon Signrank Test

	Context G	Context S	p-value
Median $\hat{\alpha}$	0.85	0.98	0.033
Median $\hat{\rho}$	-0.91	-1.00	0.188

5 Discussion

Our empirical findings are largely in line with results from previous studies. For budgets with a price of 1, corresponding to the classic dictator game, we find that our player SM keeps 67.8% when FM is generous and 77.3% when FM is selfish, meaning they give away 27.4% of their endowment on average. This is close to giving in the typical dictator game at 28.3% (Engel, 2011). The share that is given away in either of our treatments separately (32.2% for context G and 22.7% for context S) is also well within the range of typical observations (cf. Engel, 2011, p. 589).

Like Andreoni and Miller (2002) we find that subjects largely satisfy utility maximisation or come very close to maximising utility when making social choices on linear budgets. We find more violations of GARP, which can be explained by the greater number of budgets we use (14 in either context rather than 8). The Afriat efficiencies we find are slightly higher than those found in the social choice experiment with linear budgets by Fisman et al. (2007), who find that only 54% of subject achieve an efficiency of 1 compared to around 74% of our subjects, which again is probably due to a difference in the number of budgets (50 in the case of Fisman et al., 2007).

Many previous studies have shown, like ours, that people reciprocate. Blount (1995) and Falk et al. (2008) find with experimental data that subjects give less to or even reduce the payoff of another player if this player rather than a randomisation device gives them a low endowment. Falk et al. (2003) find that in an ultimatum game, the same offer is more likely to be rejected if more equitable allocations could have been chosen by the proposer than if the proposer could only have offered less equitable allocations. In a lab experiment, Charness (2004) finds that employees exert more effort when they are paid a higher wage by an employer, and that they exert particularly low effort if a low wage is set by the employer rather than by a randomisation device or the experimenter. In a field experiment, Cohn et al. (2015) similarly find that paying a higher wage increased performance of workers performing a one-time job. Dohmen et al. (2009) find evidence for reciprocity in a representative survey, and find that, in line with experimental findings, reciprocity is correlated with wages and employment. Falk et al. (2018) find evidence of reciprocity in a large representative sample of 76 countries, and find that it is correlated with the frequency of armed conflict. Our main empirical contribution is that our findings support the common interpretation of such evidence as reflecting a preference for reciprocity.

Some studies (e.g. Reuben and Suetens, 2012; Cabral et al., 2014) have looked into the frequency of behaviour consistent with a strategic motive relative to the frequency of behaviour without a strategic motive. However, none of these studies test whether their findings are consistent with a preference for reciprocity, which requires that concerns for reciprocity can be expressed as a well-behaved preference ordering. Testing this hypothesis is only possible with intersecting budgets such as the ones presented in Figure 4. Essentially, without such intersecting budgets, it is not possible to distinguish between intrinsic reciprocity (a preference) and *strong reciprocity*, a tendency to reward generous behaviour and punish selfish behaviour (Fehr et al., 2002). Strong reciprocity may reflect a preference (intrinsic), but it may also be the consequence of following a social norm or a heuristic (as pointed out by Reuben and Suetens, 2018), because of the salience of actions taken by others, or for some other reason.

A decision maker having different preferences depending on the behaviour of others means there are multiple preference relations according to which a person may choose. Our theoretical approach is in that sense similar to random utility models. As our Agreement axiom is essentially a single-crossing condition, the work on random utility models most closely related to our paper is Apesteguia et al. (2017). Whereas they focus on the stochastic choices function for random utility models with single-crossing utility, our result is on the revealed preference implications and utility representation of single-crossing utility as captured by our Agreement axiom. Adams et al. (2017) have a similar focus on the revealed preference implications of different preference relations, but their model imposes no restrictions on the data ('anything goes'). Because our method allows for classifying different revealed preference relations, it is also related to Crawford and Pendakur (2013) and Castillo and Freer (2018). They focus on classifying groups of different preferences relations, whereas we also have results on how different preferences relate to each other.

Cox et al. (2008) consider the nonparametric implications of different social preference relations, under the assumption of the existence of differentiable utility functions that represent preferences. If one preference relation is MAT (more altruistic than) the other preference relation according to Cox et al.'s (2008) definition, then the two preference relations satisfy our Agreement axiom. We do not assume the existence of utility functions (differentiable or otherwise), but instead start from a behavioural axiom and derive a revealed preference condition which is equivalent to the existence of utility functions that satisfy our axiom.

6 Further Discussion: Beyond Reciprocity

Agreement is essentially an single crossing condition that captures how the preferences change from one to another. Thus, the agreement GARP can be applied broadly to test the change of preferences, which is not limited to the same person in different contexts or even in the social preferences context.

Next we give an example about how to apply AG-GARP to rationalise the choices of subjects in generous context of our experiment. We first apply GARP to choices of subjects. We use CCEI=0.95 as a benchmark, i.e. if CCEI of the choices of two subjects is greater than 0.95, then they have the same preference relation, otherwise different preference relations. Pairwise comparison gives us the result as shown in Figure 8. We then test the choices of subjects by AG-GARP, and it is as in the Figure 9.

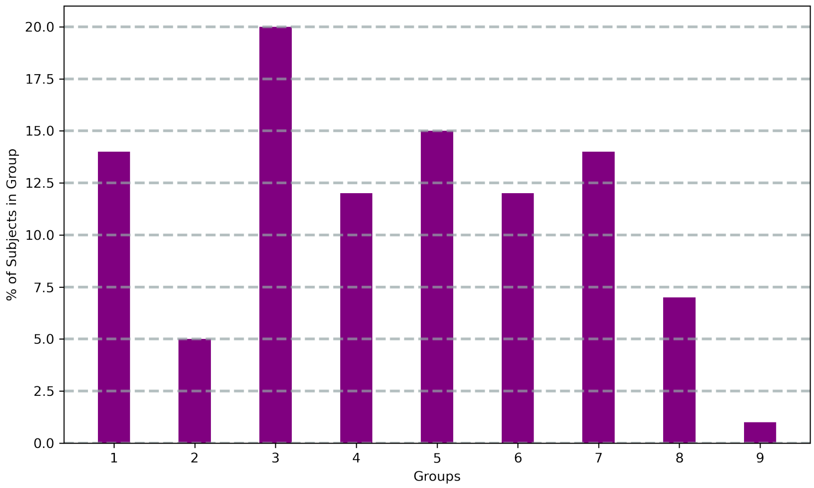


Figure 8: Subjects grouped by GARP

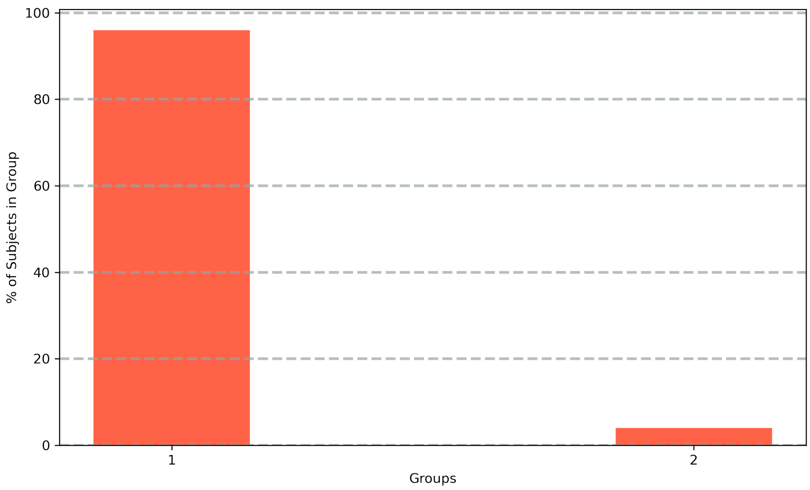


Figure 9: Subjects grouped by AG-GARP

Figure 8 describes that our subjects have night different preference relations according GARP. However, compared to GARP, 97% of subjects can be rationalised by AG-GARP. This example shows that AG-GARP can largely capture the heterogeneity of preferences among subjects.

7 Conclusion

We introduce a new axiom which we call Agreement to give empirical meaning to the idea that one preference relation is more generous (or more selfish) than another preference relation. The revealed preference condition we derive from this axiom generalises GARP when preferences are context-specific, but where preferences in one context are informative about preferences in the another context. We show that if and only if data satisfies our revealed preference condition, there exist utility functions, one of which is more selfish than the other in line with our axiom, that represent the preferences revealed by someone's choices. The Agreement axiom and the revealed preference implications we derive can be used to model changing preferences whenever someone likes some particular good better in one context than another, not only social preferences. Our revealed preference method allows for predicting choices in one context based on choices observed in a different context and for drawing conclusions about welfare.

Applying our revealed preference results to a social choice experiment, where the context was generated by the relative selfishness of another player, we find that people have consistent preferences for a given level of selfishness of the other player, but that their preferences depend on the selfishness of the other payer. Furthermore, we find that choices are largely consistent with Agreement. Our findings provide support for the interpretation of reciprocal behaviour as reflecting a preference and for modelling reciprocal behaviour as being the result of people maximising reciprocal preferences.

Appendix: Proofs

Proof of Proposition 1. First, we show Agreement implies the alternate version. Proof by contradiction. Suppose there exist x', y' such that $x' \succsim_G y'$ and $x'_1 \geq y'_1$ but $y' \succ_S x'$. Then by Agreement $y' \succ_G x'$, a contradiction of the assumed preference relation $x' \succsim_G y'$. The proof that the alternative formulation implies Agreement works analogously. □

Proof of Proposition 2. Since Ω satisfies GARP, by Varian (1982), there exist U^i and $\lambda^i > 0$, $i = 1, ..., n$ such that for all $x \in \mathbb{R}^2_+$,

$$U(x) = \min_{i \leq n}(U^i + \lambda^i p^i (x - x^i))$$

Given $x \in \mathbb{R}^2_+$, $u : \mathbb{R}^2_+ \to \mathbb{R}$, such that $U(u(x), u(x)) = U(x)$. Then u is the certainty equivalent (CE) function that represents the same preference as U, moreover, for every $x \in \mathbb{R}^2_+$, $\min\{x_1, x_2\} \leq u(x) \leq \max\{x_1, x_2\}$. Now we show that u is continuous, strictly increasing and quasiconcave, which is equivalent to showing that $\succsim$ represented by U is continuous, strictly increasing and convex.

(1) U is continuous, so $\succsim$ is continuous.

(2) Let $x = (x_1, x_2) \in \mathbb{R}^2_+$ and $\epsilon > 0$, then for all i

$$U^i + \lambda^i p^i((x_1 + \epsilon, x_2) - x^i) = U^i + \lambda^i p^i (x - x^i) + \lambda p^i(\epsilon, 0) > U^i + \lambda p^i (x - x^i)$$

Then,

$$\min_{i \leq n}(U^i + \lambda p^i((x_1 + \epsilon, x_2) - x^i)) > \min_{i \leq n}(U^i + \lambda p^i(x - x^i))$$

That is, $x + (\epsilon, 0) \succ x$. Similarly, we have $x + (0, \epsilon) \succ x$ for all $\epsilon > 0$.

(3) Given $x, y \in \mathbb{R}^2_+$ with $U(x) = U(y)$ and $\alpha \in (0, 1)$, then

$$\begin{aligned} U(\alpha x + (1-\alpha)y) &= \min_{i \leq n}(U^i + \lambda p^i((\alpha x + (1-\alpha)y) - x^i)) \\ &\geq \alpha \min_{i \leq n}(U^i + \lambda p^i(x - x^i)) + (1-\alpha) \min_{i \leq n}(U^i + \lambda p^i(y - x^i)) \\ &= U(x) = U(y) \end{aligned}$$

Thus, $\succsim$ is convex.

Therefore, u is continuous, strictly increasing and quasiconcave.

□

In our model, Ω is two-dimensional. Then GARP can be simplified by the following Theorem.

Theorem 2 (Theorem 1 of Banerjee and Murphy 2006). *Given* $\mathbf{X} \subset \mathbb{R}^2_+$, *GARP is equivalent to that* $x^i \boldsymbol{R}^0 x^j$ *not* $x^j \boldsymbol{P}^0 x^i$.

Let $N_\varepsilon(x) = \{a \in \mathbf{X} : d(a, x) < \varepsilon\}$ be the open neighbourhood of x, and let d be the Euclidean distance. The interior of a set S, int S, is the set of all $a \in S$, such that there exists ε_a such that $N_{\varepsilon_a}(a) \subset S$. The boundary of a set S is $\mathrm{d}S = \{a \in S : a \notin \mathrm{int}S\}$. For a finite set $\{s^1, \cdots, s^n\} = S \in \mathbf{X}$, the *convex hull* of S is

$$CH(S) = \{a \in \mathbf{X} : a = \sum_{i=1}^{n} \lambda^i s^i, \sum_{i=1}^{n} \lambda^i = 1 \quad \lambda^i \in [0, 1] \quad \text{for all } i\}.$$

The *convex monotonic hull* of S is,

$$CMH(S) = CH(\{a \in \mathbf{X} : a \geq s^i \quad \text{for some } i = 1, \cdots, n\}).$$

Let $Z = \{z^i\}_{i=1}^k = \{x_i\}_{i=1}^n \cup \{y_j\}_{j=1}^m$, and $P = \{r^i\}_{i=1}^k$ be the set of corresponding prices to Z and $\mathbf{B}(r^i) = \{a \in \mathbf{X} : a \cdot r^i < 1\}$. We define $RW(\tilde{\boldsymbol{R}}_S^0, z^m)$ to be the revealed worse set of observation (z^m, r^m) according to the revealed preference $\tilde{R}_S$. Specifically, if $(z^m, r^m) \in \Omega_S$, $RW(\tilde{\boldsymbol{R}}_S^0, z^m) = \{a \in \mathbf{X} : a \cdot r^i < 1\}$ and if $(z^m, r^m) \in \Omega_G$, $RW(\tilde{\boldsymbol{R}}_S^0, z^m) = \{a \in \mathbf{X} : a \cdot r^i < 1, \ a_1 \leq z_1^m\}$. Before proving Theorem 1, we first present two Lemmas.

Lemma 1. *If* Ω *satisfies AG-GARP, then for all* $z^0 \in Z$, $z^0 \in \mathrm{d}CMH(\{z^i \in Z : z^i \tilde{\boldsymbol{R}}_G z^0\})$ *and* $z^0 \in \mathrm{d}CMH(\{z^i \in Z : z^i \tilde{\boldsymbol{R}}_S z^0\})$.

Proof. We prove by contradiction. Assume that $z^0 \notin \mathrm{d}CMH(\{z^i \in Z : z^i \tilde{\boldsymbol{R}}_S z^0\})$. Let $C = CMH(\{z^i \in Z : z^i \tilde{\boldsymbol{R}}_S z^0\})$, $Z^* = \{z^i \in Z : z^i \tilde{\boldsymbol{R}}_S z^0\}$ and $\bar{Z} = Z \cap \mathrm{d}C$. W.l.o.g., we relabel indices of $\bar{Z} = \{\bar{z}^i\}_{i=1}^l$ such that $\bar{z}_1^{i+1} < \bar{z}_1^i$ for $i = 1, \cdots, l-1$.

The Lemma holds trivially when $Z = \emptyset$. Suppose that $Z \neq \emptyset$. If $z^0 \in \mathrm{int}C$, then there exist $\bar{z}^{i^*} \in \bar{Z}$, and $z^m \in (Z \cap \mathrm{int}C)$ such that $\bar{z}^{i^*} \tilde{\boldsymbol{R}}_S^0 z^m$. Next we prove the following two claims.

(i) $\bar{z}^i \tilde{\boldsymbol{R}}_S^0 \bar{z}^{i+1}$ for $i \in \{1, \cdots, l-1\}$;

(ii) $\bar{z}^i \tilde{\boldsymbol{R}}_S^0 \bar{z}^{i-1}$ for $i \in \{2, \cdots, l\}$.

Proof of Claim 1. We start from $\bar{z}^1$. Since $\bar{z}^1 \in \mathrm{d}C$ and $\bar{z}^1 \tilde{\boldsymbol{R}}_S \bar{z}^0$, we must have $\bar{z}^1 \tilde{\boldsymbol{R}}_S^0 \bar{z}^j$ for some $\bar{z}^j \in C$. Then $\bar{z}^j \in \mathbf{B}(r^1)$, which implies $\mathbf{B}(r^1) \cap C \neq \{\bar{z}^1\}$. Also $\bar{z}_1^1 > \bar{z}_1^2$, $\bar{z}^1 \tilde{\boldsymbol{R}}_S^0 \bar{z}^2$.

Consider $\bar{z}^2$. By AG-GARP, we cannot have $\bar{z}^2 \tilde{\boldsymbol{P}}_S^0 \bar{z}^1$. If $\bar{z}^1 \tilde{\boldsymbol{R}}_S^0 \bar{z}^2 \tilde{\boldsymbol{R}}_S^0 \bar{z}^1$, then $\{\bar{z}^1, \bar{z}^2\} \in \mathrm{d}\mathbf{B}(r^1)$ and $\mathbf{B}(r^1) = \mathbf{B}(r^2)$. Additionally, $\{\bar{z}^1, \bar{z}^1\} \in \mathrm{d}C$, $\mathbf{B}(r^1), \mathbf{B}(r^1)$ are supporting hyperplanes of C. Then both $\bar{z}^1$ and $\bar{z}^2$ can only be preferred to other choices in C if both are preferred to $\bar{z}^3$ and $\bar{z}^3 \in \mathrm{d}\mathbf{B}(r^2)$. Since $\bar{z}_1^2 > \bar{z}_1^3$, then $\bar{z}^2 \tilde{\boldsymbol{R}}^0 \bar{z}^3$. If $\bar{z}^1 \tilde{\boldsymbol{P}}_S^0 \bar{z}^2$, then $\bar{z}^2 \tilde{\boldsymbol{R}}_S^0 \bar{z}^3$ holds the same as we derive $\bar{z}^1 \tilde{\boldsymbol{R}}_S^0 \bar{z}^2$. Therefore, by induction, $\bar{z}^i \tilde{\boldsymbol{R}}_S^0 \bar{z}^{i+1}$ for $i \in \{1, \cdots, l-1\}$.

Proof of Claim 2. We start from $\bar{z}^l$. Since $\bar{z}^l \in \mathrm{d}C$, we must have $\bar{z}^l \tilde{\boldsymbol{R}}_S^0 \bar{z}^j$ for some $\bar{z}^j \in C$. Because $\bar{z}_1^l < \bar{z}_1^i$ for all $\bar{z}^i \in C$, we cannot have $\bar{z}^l \tilde{\boldsymbol{R}}_G^0 \bar{z}^j$. Thus, $\bar{z}^l \in \{x_i\}_{i=1}^n$ and $\bar{z}^l \boldsymbol{R}_G^0 \bar{z}^j$, then $\bar{z}^l \boldsymbol{R}_G^0 \bar{z}^{l-1}$. Similarly to the proof of Claim 1, we have $\bar{z}^i \tilde{\boldsymbol{R}}_S^0 \bar{z}^{i-1}$ for $i \in \{2, \cdots, l\}$.

(1) The boundary case: if $i^* = 1$, then $\bar{z}^1 \tilde{\boldsymbol{R}}_S^0 z^m$. Since $z^m \in \mathrm{int}C$, there is $\bar{z}^k \in \bar{Z}$, such that $z^m \in \mathrm{int}(CMH\{\bar{z}^k\})$, that is $z^m \tilde{\boldsymbol{P}}_S^0 \bar{z}^2$. Then we have $\bar{z}^1 \tilde{\boldsymbol{P}}_S^0 \bar{z}^k$. However by claim (i), $\bar{z}^1 \tilde{\boldsymbol{R}}_S^0 \bar{z}^k$, and we have a contradiction. A symmetric arguments applies for $i^* = l$.

(2) If $1 < i^* < l$, then if $z_1^m < \bar{z}_1^l$, the contradiction is the same as for the boundary case $i^* = l$; otherwise it is the same as for $i^* = 1$.

This finishes the proof of Lemma 1. □

Lemma 2. *Let $C_G = CMH(\{z^i \in Z : z^i \tilde{\boldsymbol{R}}_G z^0\})$ and $C_S = CMH(\{z^i \in Z : z^i \tilde{\boldsymbol{R}}_S z^0\})$. Let $\{z^i\}_{i=1}^l$ be the set of observed choices on $\mathrm{d}C_G$ or $\mathrm{d}C_S$ such that $z_1^i > z_1^{i+1}$ for all $i = 1, \cdots, l-1$. If Ω satisfies AG-GARP then $z^1 \tilde{\boldsymbol{R}}_G^0 z^2, z^2 \tilde{\boldsymbol{R}}_G^0 z^3, \cdots, z^j \tilde{\boldsymbol{R}}_G^0 z^0$ and $z^l \tilde{\boldsymbol{R}}_G^0 z^{l-1}, z^{l-1} \tilde{\boldsymbol{R}}_G^0 z^{l-2}, \cdots, z^{j+1} \tilde{\boldsymbol{R}}_G^0 z^0$ for some $j \in \{1, \cdots, l\}$; and $z^1 \tilde{\boldsymbol{R}}_G^0 z^2, z^2 \tilde{\boldsymbol{R}}_G^0 z^3, \cdots, z^k$ $\tilde{\boldsymbol{R}}_G^0 z^0$ and $z^l \tilde{\boldsymbol{R}}_S^0 z^{l-1} \tilde{\boldsymbol{R}}_S^0, \cdots, \tilde{\boldsymbol{R}}_S^0 z^{k+1} \tilde{\boldsymbol{R}}_S^0 z^0$ for some $k \in \{1, \cdots, l\}$.*

Proof. By Lemma 1, $z^0 \in \mathrm{d}C_G$. There is $z^j \in \{z^i\}_{i=1}^l$, such that (z^j, z^0) and (z^0, z^{j+1}) are on the same supporting hyperplane of C respectively. Following the proof of Lemma 1, we have $z^1 \tilde{\boldsymbol{R}}_G^0 z^2, z^2 \tilde{\boldsymbol{R}}_G^0 z^3, \cdots, z^j \tilde{\boldsymbol{R}}_G^0 z^0$ and $z^l \tilde{\boldsymbol{R}}_G^0 z^{l-1}, z^{l-1} \tilde{\boldsymbol{R}}_G^0 z^{l-2}, \cdots, z^{j+1} \tilde{\boldsymbol{R}}_G^0 z^0$ for some $j \in \{1, \cdots, l\}$. □

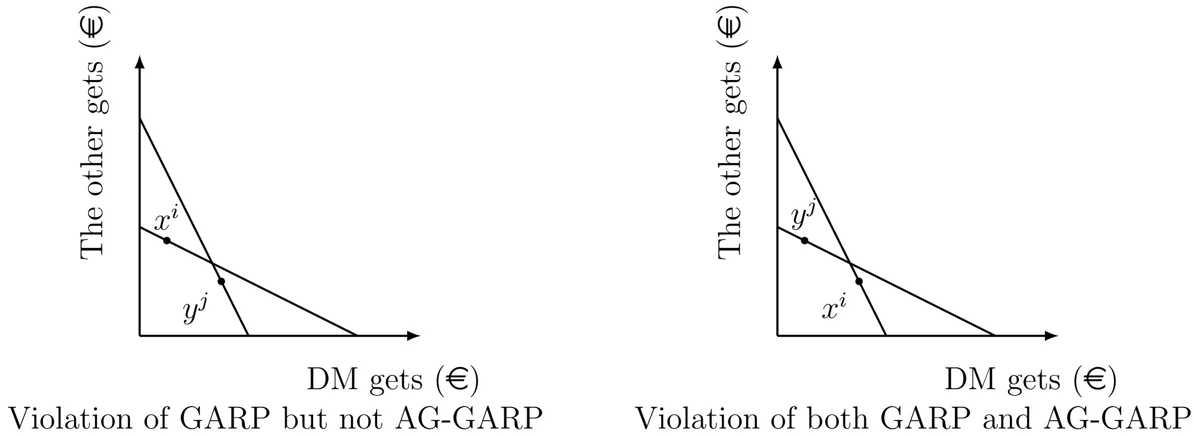


Figure 10: The relation between GARP and AG-GARP, where x^i is chosen in context G and y^i is chosen in context S.

The Agreement axiom only allows one direction of violations of GARP (see Figure 10). Observations in both figures violate GARP. However, if x^i is chosen in context G and y^i is chosen in context S, Agreement allows such choices in the left figure but does not admit those in the right figure. Lemma 1 shows that Agreement is still enough to conclude that observations on the boundary of monotonic convex hull are revealed indifferent. Lemma 2 explains in detail the relation of observations on the boundary of monotonic convex hull. Next we give the proof of Theorem 1.

Proof of Theorem 1. We only prove the sufficiency of AG-GARP to get the representation in our Theorem.

Sketch of the proof. First we construct virtual budgets for every allocation in Ω_S such that these new bundles together with Ω_G satisfy GARP and the extended revealed relation is embedded in the new relation. Then by Proposition 2, we have a continuous, monotonic and quasiconcave generous utility rationalising the data. Secondly, based on the constructed generous utility we recover the budget for countable dense bundles in $\mathbb{R}^2$. Lastly, we add these countable allocations with corresponding virtual budgets to those observations in context S sequentially, and the limit is our desired selfish utility.

Suppose that AG-GARP is satisfied. We show the existence of the selfish utility function. Start by adding allocation x^1 into Ω_S. That is, we construct a virtual budget, p^{1*}, associated with x^1 such that $\Omega_S \cup (x^1, p^{1*})$ satisfies GARP and $\tilde{R}_S$ is embedded in the revealed relation from $\Omega_S \cup (x^1, p^{1*})$. We discuss the following two cases:

(1) If $\Omega_S \cup (x^1, p^1)$ satisfies GARP, then we let $p^{1*} = p^1$ and $\Omega_S^1 = \Omega_S \cup (x^1, p^1)$;

(2) Otherwise, Let $Z^1 = \{y_j\}_{j=1}^m \cup x^1$ and $C = CMH(\{z^i \in Z^0 : z^i \tilde{\boldsymbol{R}}_S x^1\})$, then $x^1 \in \mathrm{d}C$ by Lemma 1. Lemma 2 implies that if $z^i \in Z^1$ and x^1 adjacent on $\mathrm{d}C$, then $z^i \tilde{\boldsymbol{R}}_S^0 x^1$. AG-GARP implies that not $x^1 \tilde{\boldsymbol{R}}_S^0 z^i$. Thus $\mathbf{B}(p^1) \cap intC = \emptyset$. Then $RW(\tilde{\boldsymbol{R}}_S^0, x^1) \cap C = \emptyset$. Both $RW(\tilde{\boldsymbol{R}}_S^0, x^1)$ and C are convex, and by the separating hyperplane theorem, the hyperplane separating $RW(\tilde{\boldsymbol{R}}_S^0, x^1)$ and C has the form of $\{a \in \mathbf{X} : (p_1^1 + \theta^1, p_2^1)(a - x^1) = 0\}$ with some $\theta^1 \geq 0$. Denote $p^{1^*} = (p_1^1 + \theta^1, p_2^1)$. Note that for all $x \in \mathbf{X}$, $x^1 \tilde{P}_S x$ implies that $x \in (\tilde{\boldsymbol{R}}_S^0, x^1)$, that is $x^1 \cdot p^{1*} > x \cdot p^{1*}$. Thus, the new bundle (x^1, p^{1*}) keeps the information from context G.

We repeat this for (x^2, p^2). We add (x^2, p^{2^*}) into Ω_S^1. By induction we have $\Omega_S^n = \Omega_S \cup \{x^i, p^{i^*}\}_{i=1}^n$ and Ω_S^* satisfies GARP. In the same way, we have $\Omega_G^m = \Omega_G \cup \{y^j, q^{j^*}\}_{j=1}^m$ and Ω_G^* satisfies GARP. Let $F_0 = \{x^i\}_{i=1}^n \cup \{y^j\}_{j=1}^m$.

Because Ω_G^* and Ω_S^* satisfy GARP, by proposition 2 there exist continuous, strictly increasing and quasiconcave utility functions u and v_0 that rationalise these sets of (virtual) observations. Moreover, because the revealed preference relations on Ω_G^* and Ω_S^* satisfy Agreement by construction, for all $x, y \in F_0$, $u(x) \geq (>)\, u(y)$ with $x_1 \geq y_1$ implies that $v_0(x) \geq (>)\, v_0(y)$.

Let $\{\alpha^i\}_{i=1}^\infty$ be the countable rational dense of $\mathbb{R}_+^2$, $F_k = \{\alpha^1, \cdots, \alpha^k\}$ and $F_\infty = \{\alpha^1, \cdots, \alpha^k, \cdots\}$.

Lemma 3. *Given F_k, there exist two sets of prices $\{l^i\}_{i=1}^k$ and $\{l'^i\}_{i=1}^k$ such that $\Omega_G^* \cup \{\alpha^i, l^i\}_{i=1}^k$ and $\Omega_S^* \cup \{\alpha^i, l'^i\}_{i=1}^k$ satisfy GARP and jointly satisfy Agreement.*

Proof. We adopt the idea of the proof of Lemma 2 in Reny (2015), based on u, there are virtual prices l_i corresponding to α^i for all i, such that $\Omega_G^* \cup \{\alpha^i, l^i\}_{i=1}^k$ satisfies GARP and u rationalises $\Omega_G^* \cup \{\alpha^i, l^i\}_{i=1}^k$.

Then we show that $\Omega_G^* \cup \{\alpha^i, l^i\}_{i=1}^k$ and Ω_S^* satisfies Agreement. Assume for contradiction that Agreement is violated, then there is some (α^t, l^t), $(\beta, q') \in \Omega_G^*$and $(\beta, q^*) \in \Omega_S^*$, such that at least one of the following is true:

(a) $l^t \alpha^t \geq l^t \beta$ and $q^* \beta > q^* \alpha^t$ with $\alpha_1^t \geq \beta_1$;

(b) $l^t\alpha^t > l^t\beta$ and $q^*\beta \geq q^*\alpha^t$ with $\alpha_1^t \geq \beta_1$;

Assume (a) holds, that is, $l^t\alpha^t \geq l^t\beta$ and $q^*\beta > q^*\alpha^t$ with $\alpha_1^t \geq \beta_1$. Since $q^*\beta > q^*\alpha^t$, then $q'\beta > q'\alpha^t$ since $\alpha_1^t \geq \beta_1$ and q^* put more weight on first coordinate than q'. Together with $l^t\alpha^t \geq l^t\beta$, GARP is violated in Ω_G^*, a contradiction. Similarly, we can obtain a contradiction of statement (b). Therefore, $\Omega_G^* \cup \{\alpha^i, l^i\}_{i=1}^k$ and Ω_S^* satisfies Agreement.

Next, we apply the techniques we used before to construct virtual budgets so that there are l_i' corresponding to α_i for all i, so $\Omega_S^* \cup \{\alpha^i, l_i'\}_{i=1}^k$ satisfies GARP, and the construction guarantees that Agreement is satisfied. □

We denote $\Omega_G^k = \Omega_G^* \cup \{\alpha^i, l_i\}_{i=1}^k$ and $\Omega_S^k = \Omega_S^* \cup \{\alpha^i, l_i'\}_{i=1}^k$. Both Ω_G^k and Ω_S^k satisfy GARP, and they jointly satisfy Agreement.

Lemma 4. *There exists a continuous, strictly increasing and quasiconcave v_k that rationalises Ω_S^k such that for all $x, y \in F_0 \cup F_k$ with $x_1 \geq y_1$, $u(x) \geq (>)\, u(y)$ implies $v_k(x) \geq (>)\, v_k(y)$, and for all $x, y \in F_0 \cup F_k$ with $x_1 \leq y_1$, $v_k(x) \geq (>)\, v_k(y)$ implies $u(x) \geq (>)\, u(y)$*

Proof. By Lemma 3, the virtual budgets $\{\alpha^i, l_i\}_{i=1}^k$ are constructed according to u, thus u rationalises Ω_G^k. Since Ω_S^k satisfies GARP, Proposition 2 implies that there is a continuous, strictly increasing and quasiconcave v_k that rationalises Ω_S^k. The relation of the two utilities can be derived in the same way as before. □

Define the selfish utility as

$$v := \sup_N \inf_{k \geq N} (v_k),$$

where N is a natural number and the operator works in the following way: given a natural number N, take the infimum over all v_k for $k \geq N$, and then take the supremum for all natural numbers N. Every v_k is a certainty equivalent function, so $\{v_k\}_{k \in N}$ are uniformly bounded and so is v, for all $x \in \mathbf{X}$, $\min\{x_1, x_2\} \leq v(x) \leq \max\{x_1, x_2\}$. We now show some properties of v.

(i) v is continuous.

Assume *per contra* that there is a convergent $\{x^n\} \subset X$ with $x^n \to x \in X$, such that $\sup_N \inf_{k\geq N}(v_k(x)) > \limsup_N \inf_{k\geq N}(v_k(x^n))$ and so there is a subsequence of $\{v_{k^*}\}$ such that

$$v_{k^*}(x) > v_{k^*}(x^n) + \varepsilon_n \quad \text{for all} \quad n \geq N.$$

First, the case where $x_1^n \geq x_1$ for all n. By monotonicity, there are $\{y^n\} \subset \bigcup F_k$ and $y \in \bigcup F_k$ such that $y_1^n \geq y_1$ for all n, $y^n \to y$ and

$$v_{k^*}(y) > v_{k^*}(y^n) + \varepsilon_n \quad \text{for all} \quad n \geq N$$

By Lemma 4, this implies that $u(y) > u(y^n)$ for all $n \geq N$. Moreover since u is continuous, there exists a $y^* \in \bigcup F_k$ with $y_1^* < y_1$ such that $u(y^*) < u(y)$ and $0 < v_k(y^*) - v_k(y^n) < \varepsilon_n$ for all $k \geq N$. Thus, $v_k(y^*) > v_k(y)$ for all $k \geq N$. However, using Lemma 4 again, $u(y^*) \leq u(y)$ with $y_1^* < y_1$ implies $v_{k^*}(y^*) \leq v_{y^*}(y)$ for $y^*, y \in F_{k^*}$, a contradiction.

Now we assume $x_1^n < x_1$ for all n and define a double-indexed sequence $\{x_k^n\}$ such that given k, $v_k(x_k^n) = v_k(x^n)$ and $(x_k^n)_1 \geq x_1$. Thus, given k, $x_k^n \to x$ as $n \to \infty$. Since $\inf_{k\geq N}(v_k(x^n)) \geq \inf_{k\geq N}(v_k(x_k^n))$, $\sup_N \inf_{k\geq N}(v_k(x^n)) \geq \sup_N \inf_{k\geq N}(v_k(x_k^n))$ and so $\lim_n \sup_N \inf_{k\geq N}(v_k(x^n)) \geq \limsup_n(\sup_N \inf_{k\geq N}(v_k(x_k^n)))$. By a similar argument as before, we reach a contraction.

(ii) u and v AG-rationalises Ω.

For all k, generous utility u and selfish utility v_k AG-rationalises $\Omega = \Omega_G \cup \Omega_S$, then for every $x, y \in \Omega$, $x\tilde{\mathbf{R}}_s y$ implies that $v_k(x) \geq v_k(y)$. That is, given $x, y \in \Omega$, $x\tilde{\mathbf{R}}_s y$ implies that the inequality

$$v_k(x) \geq v_k(y)$$

holds for all k. Taking the supinf on both side of the equality, we have

$$\sup_N \inf_{k\geq N}(v_k(x)) \geq \sup_N \inf_{k\geq N}(v_k(y)).$$

Therefore, u and $\sup_N \inf_{k\geq N}(v_k)$ AG-rationalises Ω.

(iii) v is quasiconcave .

Given $x, y \in \mathbf{X}$ and $\lambda \in (0,1)$,

$$\begin{aligned}
& v_k(\lambda x + (1-\lambda)y) \geq \min\{v_k(x), v_k(y)\} \quad \text{holds for all natural} \quad k \\
& \Rightarrow \inf_{k \geq N} v_k(\lambda x + (1-\lambda)y) \geq \inf_{k \geq N} \min\{v_k(x), v_k(y)\} \\
& \Rightarrow \inf_{k \geq N} v_k(\lambda x + (1-\lambda)y) \geq \min\{\inf_{k \geq N} v_k(x), \inf_{k \geq N} v_k(y)\} \\
& \Rightarrow \sup_N \inf_{k \geq N} v_k(\lambda x + (1-\lambda)y) \geq \sup_N \min\{\inf_{k \geq N} v_k(x), \inf_{k \geq N} v_k(y)\} \\
& \Rightarrow \sup_N \inf_{k \geq N} v_k(\lambda x + (1-\lambda)y) \geq \min\{\sup_N \inf_{k \geq N} v_k(x), \sup_N \inf_{k \geq N} v_k(y)\} \\
& \Rightarrow v(\lambda x + (1-\lambda)y) \geq \min\{v(x), v(y)\}.
\end{aligned}$$

Next we show the following Lemma, which extends Lemma 4 to an infinite number of virtual observations.

Lemma 5. *For all $x, y \in \mathbf{X}$, $u(x)\ (\geq) > u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq (>)\ v(y)$*

Proof. The proof is done in two steps.

(a) First we show for all $x, y \in F_\infty$, $u(x) \geq (>)\, u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq (>)$ $v(y)$. Given $x, y \in \{\alpha^i\}_{i=1}^{\infty}$, $u(x) \geq u(y)$ with $x_1 \geq y_1$, there exists N^* such that for all $N \geq N^*$, $x, y \in F_N$, we have

$$v_N(x) \geq v_N(y) \quad \text{for all} \quad N \geq N^*$$

$$\Rightarrow \inf_{k \geq N} v_k(x) \geq \inf_{k \geq N} v_k(y) \quad \text{for all} \quad N \geq N^*.$$

Naturally, we have

$$\sup_N \inf_{k \geq N} v_{F_k}(x) \geq \sup_N \inf_{k \geq N} v_{F_k}(y), \quad \text{that is,} \quad v(x) \geq v(y).$$

Given $x, y \in \{\alpha^i\}_{i=1}^{\infty}$, $v(y) \geq v(x)$ with $x_1 \geq y_1$, there exists N^* such that for all $N \geq N^*$, $x, y \in F_N$. Assume for contradiction that there is $K \geq N^*$, $v_K(y) < v_K(x)$, then for all $N \geq K$, $v_N(y) < v_N(x)$, that is $v(y) < v(x)$. Thus, for all $K \geq N^*$,$v_K(y) \geq v_K(x)$. By Lemma 4, $u(y) \geq u(x)$.

Assume for contradiction that there are $x, y \in F_\infty$, such that $u(x) > u(y)$ with $x_1 \geq y_1$ and $v(x) \leq v(y)$. By the previous argument, $v(x) \leq v(y)$ and $x_1 \geq y_1$ implies that $u(y) \geq u(x)$, we have contradiction.

(b) The relation between u and v holds for F_∞, so what is left is to show that for all $x, y \in \mathbf{X}$ and not $x, y \in F_\infty$, $u(x) \geq (>)\, u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq (>)\, v(y)$. Given irrational $x, y \in \mathbf{X}$, $u(x) \geq u(y)$ with $x_1 \geq y_1$, there exist a rational decreasing sequence $\{a^n\}_{n=1}^\infty$ and increasing sequence $\{b^n\}_{n=1}^\infty$ that satisfy $\lim_{n\to\infty} a^n = x$ and $\lim_{n\to\infty} b^n = y$. Then we have $u(a^n) \geq u(x) \geq u(y) \geq u(b^n)$ with $a_1^n \geq b_1^n$ for all n, so $v(a^n) \geq v(b^n)$ for all n. The continuity of v implies

$$v(x) = v(\lim_{n\to\infty} a^n) = \lim_{n\to\infty} v(a^n) \geq \lim_{n\to\infty} v(b^n) = v(\lim_{n\to\infty} b^n) = v(y).$$

Given irrational $x, y \in \mathbf{X}$, $u(x) > u(y)$ with $x_1 \geq y_1$, there exist a rational increasing sequence $\{a^n\}_{n=1}^\infty$ and decreasing sequence $\{b^n\}_{n=1}^\infty$ that satisfy $u(a^1) > u(b^1)$, $\lim_{n\to\infty} a^n = x$ and $\lim_{n\to\infty} b^n = y$. Then we have $u(a^n) > u(b^n)$ with $a_1^n \geq b_1^n$ for all n, so $v(a^n) > v(b^n)$ for all n. The continuity of v implies

$$v(x) = v(\lim_{n\to\infty} a^n) = \lim_{n\to\infty} v(a^n) \geq v(a^1) > v(b^1) \geq \lim_{n\to\infty} v(b^n) = v(\lim_{n\to\infty} b^n) = v(y).$$

□

Let $x, y \in \mathbf{X}$ with $x > y$ ($x \geq y$ and [either $x_1 > y_1$ or $x_2 > y_2$]), then $u(x) > u(y)$. By Lemma 5, we have $v(x) > v(y)$, that is, v is strictly increasing. Therefore, there exist continuous, strictly increasing and quasiconcave generous utility $u(x)$ and selfish utility $v(x)$ that AG-rationalises Ω. Moreover, for all $x, y \in \mathbf{X}$, $u(x) \geq u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq v(y)$ and for all $x, y \in \mathbf{X}$, $v(x) \geq v(y)$ with $x_1 \leq y_1$ implies that $u(x) \geq u(y)$. This finishes the proof. □

The following proposition states that two preferences $\succsim_1$ is MAT $\succsim_2$ implies that preferences $\succsim_1$ and $\succsim_2$ satisfies Agreement.

Proposition 3. *Given two preference relations $\succsim_1$ and $\succsim_2$ on $\mathbf{X}$, represented by differentiable and monotonic utility functions u and v respectively, if $\succsim_1$ MAT $\succsim_2$ then $\succsim_1$ and $\succsim_2$ satisfy agreement.*

Proof. If $\succsim_1$ and $\succsim_2$ satisfy agreement and $\succsim_1$ is more generous, $x \succsim_1 y$ with $x_1 \geq y_1$ implies that $x \succsim_2 y$ for all $x, y \in \mathbf{X}$. Both $\succsim_1$ and $\succsim_2$ have utility representations u and v respectively, thus it is equivalent to show that if $\succsim_1$ MAT $\succsim_2$ then $u(x) \geq (>) u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq (>) v(y)$ for all $x, y \in \mathbf{X}$.

Given $x \in \mathbf{X}$, let $f_x : \mathbb{R}_+ \to \mathbb{R}_+$, $f_x(x_1) = x_2$ and $u(a, f_x(a)) = k$ for some constant $k > 0$ for all $a \in \mathbb{R}_+$. That is, f_x denote the indifference curve of preference $\succsim_1$ through $x \in \mathbf{X}$. Similarly let g_x be the indifference curve of preference $\succsim_2$ through $x \in \mathbf{X}$. By the definition of MAT, $\succsim_1$ MAT $\succsim_2$ implies that $f'_x(a) \geq g'_x(a)$ for all $a \in \mathbb{R}_+$ and $x \in \mathbf{X}$.

Choose any $x, y \in \mathbf{X}$ with $x_1 \geq y_1$ and $u(x) \geq u(y)$. We have $f_x(y_1) \geq y_2$. Define $F_x(a) = f_x(a) - g_x(a)$, then $F'_x(a) = f'_x(a) - g'_x(a) \geq 0$ for all $a \in \mathbb{R}_+$, so F_x is non-decreasing. Moreover, since $F_x(x_1) = f_x(x_1) - g_x(x_1) = x_2 - x_2 = 0$ and $x_1 \geq y_1$, $F_x(y_1) = f_x(y_1) - g_x(y_1) \leq 0$, that is, $f_x(y_1) \leq g_x(y_1)$. Thus we have $g_x(y_1) \geq y_2$, and so $v(x) \geq v(y)$. The proof is similar if we replace the weak inequalities by strict ones.

□

Theorem 3. *Suppose $\succsim_G$ and $\succsim_S$ are represented by CES functions, that is, $u(z) = (\alpha_G z_1^{\rho_G} + (1-\alpha_G) z_2^{\rho_G})^{1/\rho_G}$ for $\succsim_G$ and $v(z) = (\alpha_S z_1^{\rho_S} + (1-\alpha_S) z_2^{\rho_S})^{1/\rho_S}$ for $\succsim_S$. Then Agreement is satisfied if and only if $\alpha_G \leq \alpha_S$ and $\rho_G = \rho_S$.*

Proof. $\Leftarrow$ Assume that $\alpha_G \leq \alpha_S$ and $\rho_G = \rho_S = \rho$. Given $x, y \in \mathbf{X}$, if $x \succsim_G y$ and $x_1 > y_1$, then

$$u(x) = (\alpha_G x_1^\rho + (1-\alpha_G) x_2^\rho)^{1/\rho} \geq u(y) = (\alpha_G y_1^\rho + (1-\alpha_G) y_2^\rho)^{1/\rho}.$$

Since $x_1 > y_1$ and $\alpha_G \leq \alpha_S$, then

$$v(x) = (\alpha_S x_1^\rho + (1-\alpha_S) x_2^\rho)^{1/\rho} \geq v(y) = (\alpha_S y_1^\rho + (1-\alpha_S) y_2^\rho)^{1/\rho},$$

Thus, we have $x \succsim_S y$.

$\Rightarrow$ Assume that Agreement holds. First we show that Agreement implies that marginal rate of substitution (MRS) of the generous preference is smaller than that of selfish preference. Given $\varepsilon > 0$, $\lambda(\varepsilon)$ is the value such that the following indifference holds:

$$(x + \varepsilon, y) \sim_G (x, y + \lambda(\varepsilon)) \tag{3}$$

By Agreement, 3 implies that

$$(x + \varepsilon, y) \succsim_S (x, y + \lambda(\varepsilon)) \tag{4}$$

Writing 3 and 4 in utility functions,

$$u(x+\varepsilon, y) = u(x, y+\lambda(\varepsilon)) \Rightarrow v(x+\varepsilon, y) \geq v(x, y+\lambda(\varepsilon)),$$

which is equivalent to,

$$\frac{u(x+\varepsilon, y) - u(x, y)}{\varepsilon} = \frac{u(x, y+\lambda(\varepsilon)) - u(x, y)}{\lambda(\varepsilon)}$$

$$\Rightarrow \frac{v(x+\varepsilon, y) - v(x, y)}{\varepsilon} \geq \frac{v(x, y+\lambda(\varepsilon)) - v(x, y)}{\lambda(\varepsilon)},$$

That is,

$$\frac{\frac{u(x+\varepsilon,y)-u(x,y)}{\varepsilon}}{\frac{u(x,y+\lambda(\varepsilon))-u(x,y)}{\varepsilon}} = \frac{\varepsilon}{\lambda(\varepsilon)}$$

$$\Rightarrow \frac{\frac{v(x+\varepsilon,y)-v(x,y)}{\varepsilon}}{\frac{v(x,y+\lambda(\varepsilon))-v(x,y)}{\varepsilon}} \geq \frac{\varepsilon}{\lambda(\varepsilon)}.$$

Since u and v are CES functions which are smooth, let $\varepsilon \downarrow 0$ and $K = \lim_{\varepsilon\downarrow 0} \frac{\varepsilon}{\lambda(\varepsilon)}$, then we have

$$-\frac{\frac{\partial u}{\partial a_1}}{\frac{\partial u}{\partial a_2}} = MRS_u = K \Rightarrow -\frac{\frac{\partial v}{\partial a_1}}{\frac{\partial v}{\partial a_2}} = MRS_v \geq K.$$

Therefore, $MRS_v \geq MRS_u$.

Substituting the $MRS_v \geq MRS_u$, with CES functions, we have for all $a = (a_1, a_2) \in$ **X**,

$$\frac{\alpha_S}{1-\alpha_S}(\frac{a_1}{a_2})^{\rho_S} \geq \frac{\alpha_G}{1-\alpha_G}(\frac{a_1}{a_2})^{\rho_G}$$

Then $\alpha_S \geq \alpha_G$ and $\rho_G = \rho_S$.

□

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