Monetary Policy and the Maturity Structure of Public Debt

Michele Andreolli*

London Business School

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Abstract

This paper studies the mediating impact of the maturity of public debt in the transmission of monetary policy shocks to economic activity. A longer debt maturity attenuates greatly the effect of monetary policy: going from the average historical duration of US debt to very short term debt doubles the impact of a rise of the policy rate on output. A similar result holds in UK data. Using data on corporate debt, spreads, investment, and fiscal variables, I show that these effects can be traced back to a quantitatively important *financing channel*. A model featuring an interaction between an empirically estimated primary market friction and a standard financial accelerator is able to account for these facts.

JEL classification: E52, E62, E63, G20, H32, H63.

Key words: monetary policy transmission, public debt management, maturity structure, primary market friction.

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1 Introduction

One of the major legacies of Covid 19 is an increase in public debt. Monetary policy plays a crucial role in cushioning large shocks and in helping economies on the recovery path, balancing inflationary risks and economic activity in an environment of large balance sheets. This paper studies the effect of public indebtedness and especially its *maturity composition* on the transmission of monetary policy. Economists and policy makers intuitively feel that the maturity composition of public debt is important for macroeconomic stabilization policies. Yet it has not been analysed much. It turns out to be a lot more complex to build macroeconomic models featuring an array of debt duration than to analyse models with one-period debt contract or, at the opposite end of the spectrum, consols. Empirical work on this topic is also almost non-existent. As a result, if one asks macroeconomists how the transmission of monetary policy differs in countries in which public debt duration is high on average versus in countries in which it is low, one gets all possible answers. Some, thinking through the lenses of the fiscal theory of the price level, will argue that monetary policy is transmitted more powerfully in high duration cases. Others, seeing through the prism of heterogeneous agents models with a fraction of hands-to-mouth consumers will conjecture the opposite. Some think different effects will cancel one another. In a standard New Keynesian model the maturity structure would be irrelevant due to the Ricardian equivalence. But in a post Covid world with large balance sheets and where very different debt durations are observed - low for the US for example, high for the UK - can we afford not to know the answer to that question?

This paper is the first one to provide an answer: i) It shows empirically that monetary policy transmission to GDP is substantially *dampened* when debt is of long duration while debt maturity matters less for inflation; ii) it identifies the *financing channel* and a market friction as the main cause behind those results; iii) it uses hand collected Debt Management Office high frequency issuance data to estimate the magnitude of this market friction; iv) finally, it builds a financial accelerator model enriched with a financing channel to account for the data.

The maturity structure of debt matters for the transmission of monetary policy because of valuation effects and because debt duration affects the interest rate risk exposure of government. This is important as monetary policy is a large source of interest rate risk. As a simple example, take a country with a 100% debt to GDP ratio which has no further borrowing needs and each year covers only the interest payments. Consider two cases: in the first the whole debt is issued in 10 years nominal zero coupons bonds and in the second in bonds with overnight maturity. In the first case, the government budget is insulated from interest rate changes; in the second, a hike will push up the cost of borrowing proportionally. We can rephrase the argument in terms of market value of debt: after a hike in the first case the market value of debt will decrease as we are now discounting with a higher rate, whereas in the second the market value will remain unchanged, *ceteris paribus*. This paper argues that duration-to-GDP, which measures the changes in market value of public debt to GDP due to interest rate movements, is an important metric for fiscal authorities. As public debts are sizable for many countries, the impact of the maturity structure on fiscal balances and financial stability is high and frictions that break Ricardian equivalence matter.

In the empirical part of the paper, I construct the entire time series of the duration-to-GDP of public debt using bond by bond data in the US and in the UK since the 1970s at monthly frequency. Using local projection methods à la Jordà (2005), I show that the transmission of monetary policy is substantially dampened when debt duration is high. Going from very short term debt to the average historical duration of US debt halves the impact of a rise of the policy rate on output while having very limited effects on inflation. This result is subjected to a large array of robustness checks including regarding the possible endogeneity of the maturity structure with a novel narrative identification. By looking at the impulse responses of a broad range of variables, from corporate debt issuance, various budgetary items, corporate sector balance sheet items to investment and consumption responses, I can identify the main channel through which duration dampens the effect of monetary policy on output. When the government is insured against interest rate risk thanks to long maturity debt and the interest rate goes up, it does not need to refinance at the new higher rate. When this *insurance payout* materializes, the government borrows relatively less and this is associated to a relatively higher borrowing by the non-financial corporates, at a cheaper relative price. The non-financial sector uses these financial resources to invest more, and therefore, to increase output relatively. I therefore call this channel the*financing channel* of monetary policy[1].

The paper then provides a theoretical model of the *financing channel*. I enrich Bernanke, Gertler and Gilchrist (1999) financial accelerator model by adding a long maturity fixed rate public debt and a primary market friction for which I provide independent empirical evidence. I use institutional features of the budget process in the UK and hand collected data from the Debt Management Office to construct a series of unexpected news shocks to the supply of public debt. Importantly those shocks are orthogonal to the fiscal policy stance. This debt issuance friction reflects the the competition for funds between public and private sectors. Strikingly, even this very small friction in the issuance market, is enough to account quantitatively for all the empirical results. I can empirically and quantitatively trace the macroeconomic effects of this friction on the debt issuance market in the model and in the data. To sum up, this paper therefore establishes the existence of another transmission channel of monetary policy, the *financing channel of monetary policy* which is linked to the rollover needs of governments which themselves depend on the duration of their debt and on the path of interest rates. This channel is quantitatively important in a the current macroeconomic configuration where governments have large balance sheets and heterogeneous debt duration.

Related literature. This paper contributes to the literature on monetary policy transmission. The literature so far has analyzed how debt characteristics interact with monetary policy transmission in the context of corporate or household debt with very different channels from mine (see Ippolito, Ozdagli and Perez-Orive, 2018, Darmouni, Giesecke and Rodnyansky, 2020, Jungherr et al., 2020, Calza, Monacelli and Stracca, 2013, Garriga, Kydland and Šustek, 2017, Beraja et al., 2019, Wong, 2021). An exception is Sterk and Tenreyro (2018) which studies the role of public debt but focuses on the overall stock rather than the maturity composition, as in my paper.

A related literature studies the effect of public debt supply and its structure on asset prices (see Vayanos and Vila, 2021, Greenwood, Hanson and Stein, 2010, Greenwood and Vayanos, 2010, 2014, Greenwood, Hanson and Stein, 2015, Krishnamurthy and Vissing-Jorgensen, 2012, Bianchi and Bigio, 2021, Papoutsi, Piazzesi and Schneider, 2021). While I build on the

[1] In the paper I discuss throughly and rule out other channels.

premise of imperfect asset substitutability like these papers[2], I also analyze the causal effect of monetary policy, produce a clean high frequency identification of debt supply shocks using Debt Management Office announcements, and present a DGSE model of the economy.

The interaction between fiscal and monetary policies has been studied in the context of the fiscal theory of the price level by, among others, Leeper (1991), Cochrane (2001, 2020), also in the context of long-debt. My paper provides evidence of a new channel, *the financing channel*, through which monetary and fiscal policies interact, via segmented issuance markets.

The possibility of inflating away public debt with surprise inflation by the central bank is discussed by Hall and Sargent (2011), Giannitsarou and Scott (2008), Hilscher, Raviv and Reis (2021), Krause and Moyen (2016). I do not focus on the incentives to inflate away public debt, but I present evidence that public debt maturity matters more generally in conducting monetary policy and in shaping its transmission mechanism.

The optimal debt policy literature has studied how nominal and long maturity debt can provide insurance against exogenous shocks (see Bohn, 1988, Angeletos, 2002, Faraglia et al., 2013, 2018, Bigio, Nuño and Passadore, 2019, Bhandari et al., 2017, 2021). As an example, Bhandari et al. (2021) argue that optimal debt policy seeks to minimize interest rate risk with long maturity public debt. In this paper I focus on monetary policy as one source of interest rate risk and quantify the effects.

Finally, I contribute to the literature on the financial accelerator and find an important role for investment in the transmission of. monetary policy. I build a theoretical model incorporating financial constraints that amplify the effects of monetary policy through investment in the tradition of Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2014), Gomes, Jermann and Schmid (2016), Dmitriev and Hoddenbagh (2017).

Structure of the paper. The remainder of the paper is organized as follows. Section 2 discusses the data on public debt duration-to-GDP and its construction. In Section 3, I present the econometric methodology and the identification strategy. Section 4 presents the main results on monetary transmission in function of debt duration for the US. Section 5 examines the economic channels which may account for the results. Section 6 presents

[2] More generally, I build on the idea that asset quantities matter per se in determining asset prices (see Koijen and Yogo, 2019, Gabaix and Koijen, 2020).

the theoretical model featuring a financial accelerator and a small debt issuance friction and Section 7 shows that it can account for the empirical evidence. Section 8 concludes[3].

2 The Maturity Structure of Public Debt

Fixed-rate debt allows the issuers to be insured against interest rate risk for the duration of the contract, with longer dated debt providing insurance for a longer time period. Fiscal authorities are concerned by increases in debt servicing costs due to interest rate changes. In this paper, I propose to use *duration-to-GDP* as a metric to measure the amount of interest rate risk exposure over GDP. Duration-to-GDP measures savings and losses due to interest rate changes not in terms of units of debt but in terms of debt servicing costs over GDP, which is what matters for fiscal accounts. As an example, if we take a government with public debt equal to 1 percent of GDP, it will be immaterial for debt servicing costs if debt is overnight or in consols (perpetual) bonds. On the other hand, for a government with public debt of 100 percent of GDP, the debt maturity will have first order effects on debt servicing costs. Following a one percent permanent increase in interest rates across the yield curve, with all debt overnight, debt servicing costs on existing debt would increase by one percent of GDP in perpetuity. On the other hand, they would not move at all on existing debt under a consol strategy. Equivalently, in the first case, the market value of public debt would remain constant at 100 percent of GDP. In the second case, the market value of public debt would decline substantially, exactly by the duration amount. Following a one percent increase permanent in interest rates across the yield curve, debt duration measures both the decline in market value of this debt and the net present value of debt servicing cost savings compared with overnight debt. I show this equivalence formally in the context of the model in Proposition 1.

In order to measure duration-to-GDP precisely, I construct a monthly dataset of public nominal fixed-rate marketable debt promises held by the general public at market prices

[3]Appendix A presents the full data construction and the narrative account of the maturity choices. Appendix B provides the microfoundations of the primary market friction. The first set of online appendices expand on the empirical results. Appendix C discusses the sensitivity analysis on the main specification. Appendix D shows additional empirical results for the US and Appendix E shows the empirical results for the UK. The second set of online appendices discuss the theoretical framework. Appendix F presents the full derivation of the model, Appendix G shows further theoretical results.

for each future month; the current value of the government promises for j months ahead, following Hall and Sargent (2011) methodology. The market value of public debt for nominal fixed-rate marketable promises is

$$\sum_{j=1}^{n} q_{t,j} d_{t,j}$$

where, $q_{t,j}$ is the amount of nominal currency in period t that one needs to purchase one unit of nominal currency in period $t+j$, $d_{t,j}$ is the amount of the promises that the government has in period t to pay in period $t+j$, n is the maximum maturity of public debt[4]. *Macaulay duration* measures the percent decrease in the market value of this debt following an infinitesimal change in interest rates uniformly across the yield curve $dr = dr_{t,j}$, for $j > 0$.

$$MacDur_t = -\frac{\frac{\partial\left(\sum_{j=1}^{n} q_{t,j} b_{t,j}\right)}{\partial r}}{\sum_{j=1}^{n} q_{t,j} d_{t,j}} = \frac{\sum_{j=1}^{n} \frac{j}{12} q_{t,j} b_{t,j}}{\sum_{j=1}^{n} q_{t,j} d_{t,j}}$$

Duration-to-GDP measures the same change but in percentage points of GDP, thereby capturing the overall insurance amount relevant for debt management[5]:

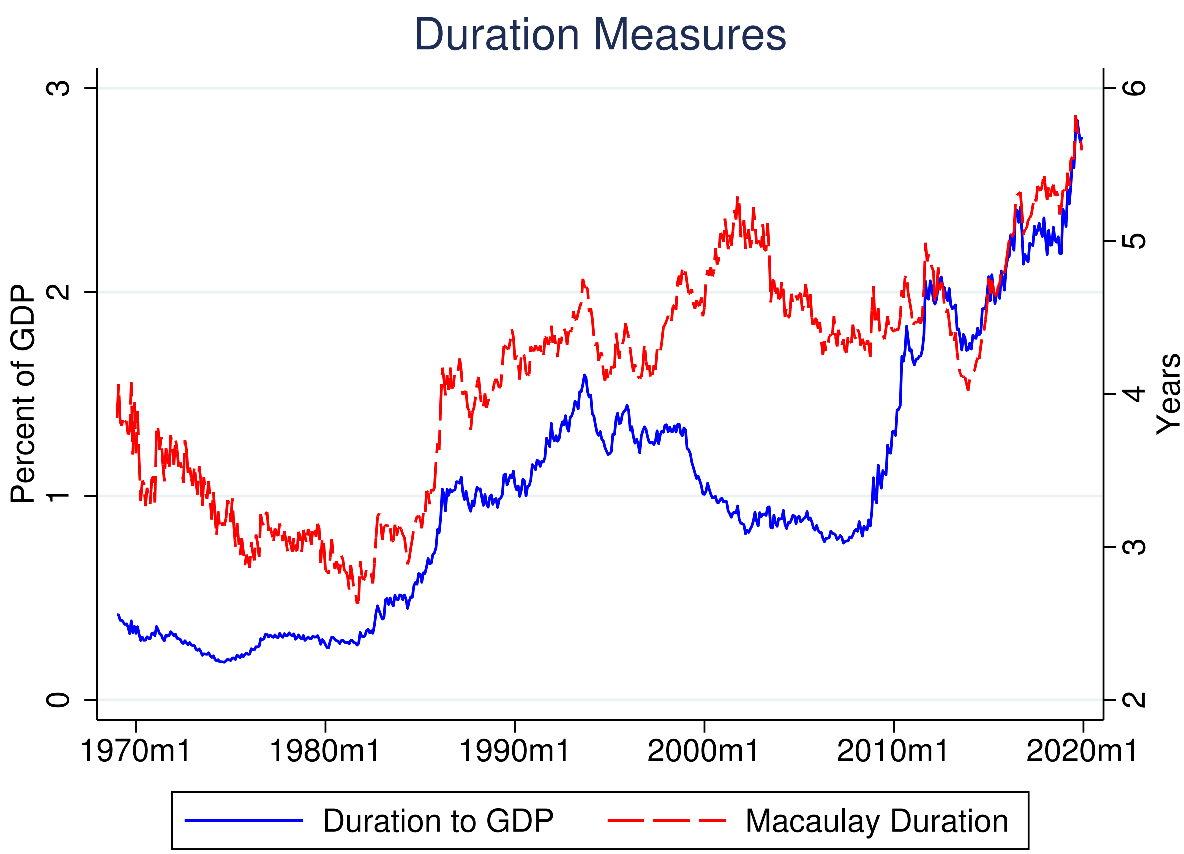
$$DurGDP_t = \frac{\sum_{j=1}^{\infty} \frac{j}{12} q_{t,j} b_{t,j}}{GDP_t} \tag{1}$$

Figure 1 presents the Macaulay duration and the duration to GDP for marketable nominal public debt held by the public for the US. It does not include the holdings of the Federal Reserve and of other public entities. A few features stand out. First, the overall level of Macaulay duration in the US has been quite low, at around 4 years on average, lower than most advanced economies. Second, up to the late seventies, Macaulay duration was declining due to a shortening of the securities issued as a law, repealed in 1975, forbade the treasury

[4]As governments issue bonds which are not zero coupon, I strip the coupons and create the equivalent series for the marketable part of $d_{t,j}$ in each month for future promises also dated monthly. With respect to the prices, I use the continuously compounded zero coupon yield curve data and compute the zero coupon prices with $q_{t,j} = e^{-\frac{j}{12} r_{t,j}}$ where j is divided by 12 to convert it to annual frequency and $r_{t,j}$ is the appropriate interest rate.

[5]The full description of the details to construct the data is in Appendix A.1. That appendix also examines how these measure behave with different assumptions: in case we use face value debt, in case we include also debt held by the FED or by other public entities, or if we include the inflation-linked debt as well. The UK data is also shown in Appendix A.1.

Figure 1: Time series of public debt Macaulay duration and duration to GDP for the US



Notes: The figure shows the time series for public debt Macaulay duration (in red dashed line) and duration to GDP (in blue solid line) for the US. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct duration to GDP each public debt discounted promise is multiplied by its maturity in years and these; then these objects are summed for each period and then divided by nominal GDP. To construct Macaulay duration each public debt discounted promise is multiplied by its maturity in years and these; then these objects are summed for each period and then divided by the sum of market value of the same bonds. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2019m12 with US data.

from issuing bonds with interest rates above 4.25, effectively pushing the treasury to lower maturities. Third, Macaulay duration and duration-to-GDP often go in tandem but not always: from the late nineties to the mid aughts duration was on a increasing path, while duration-to-GDP was declining due to lower public debt. Finally, duration-to-GDP displays decade-long waves in the US, with low levels up to the early eighties and from the late nineties and early aughts, and higher levels in the remaining periods. This contrasts with the UK with shorter cycles of around 5 years, with the two measures being negatively correlated across the two countries. A more detailed narrative account of the history of the maturity structure of public debt and its exogeneity with respect to monetary policy is in Appendix A.1.

At the end of 2019 duration to GDP was at 2.8 % of GDP. This means that if interest rates were to increase by one percentage point across the yield curve, the market value of public debt would decline by 2.8 percent of GDP. Equivalently this number would be the net present value of interest rate savings with this level of public debt and this maturity profile, relative to a scenario with the same level of debt but with overnight debt. This is not a small number and it is a lower bound as I focus only on nominal marketable bonds. If we add inflation-linked TIPS and assume a one to one correlation between nominal yield curve rates and real yield curve rates (as Nakamura and Steinsson (2018) show to be the case following monetary policy shocks), the estimate increases modestly to 3.2 %. The reason is that, although of longer maturity, TIPS are not a large share of public debt.

Two papers propose related measures to capture the value over GDP of long public debt. Greenwood and Vayanos (2014) propose to use a face value version of the duration-to-GDP measure. Specifically, they do not discount cash promises $b_{t,j}$ by $q_{t,j}$ but multiply them by 1, irrespective of maturity j: $DurGDP_t^{FaceValue} = \frac{\sum_{j=0}^{\infty} \frac{j}{12} b_{t,j}}{GDP_t}$. They do that to decrease concerns with endogeneity in prices, which is crucial for their empirical setting, where they already have debt prices in the left hand side of their regressions. However, that measure is not what the fiscal authority cares about when measuring the insurance implications for public finances of the maturity of public debt, since the level of interest rate matters.[6] Krishnamurthy and

[6] At a theoretical level, one cannot see the impact on the market value of public debt of a change in interest rates without using the yield curve. At a practical level, the face value measure tends to overstate the duration (both Macaulay and over GDP) when interest rates are high as can be seen in Figures A.2 and A.3.

Vissing-Jorgensen (2012) propose a measure at face value where they sum all debt promises above a 10 years threshold and ignore the ones below, this effectively sets $q_{t,j}$ to 1 and forces the weight on $b_{t,j}$ to be zero or one depending on whether the debt promises are due before or after the threshold: $LongDebtGDP_t^{FaceValue} = \frac{\sum_{j=10*12}^{\infty} b_{t,j}}{GDP_t}$. From the perspective of the fiscal authority, this measure captures the amount (at face value) of debt which is insured against a monetary policy rate increase, but does not give a full picture of a quantitative measure of insurance provided by public debt as duration to GDP at market value does. In Appendix C, I show how my main empirical results are robust to these alternative measures as well.

3 Empirical Methodology

The aim of this paper is to identify empirically how the impact of a monetary policy shock on economic activity depends on the current public debt structure. I interact a measure of monetary policy shocks and duration-to-GDP to see the conditional impact on economic output and prices. To identify an exogenous measure of monetary policy, in the baseline main results, I employ a narrative based monetary policy measure: the update of Romer and Romer (2004) series by Yang and Wieland (2015) for the US and the series by Cloyne and Hürtgen (2016) for the UK. The key idea it to purge the policy rate from the central bank forecasts on future economic activity. The narrative identification has the main advantage that it is available for a long sample, which is crucial in this exercise in order to have enough variation in the maturity profile of public debt[7].

Single equation local projections à la Jordà (2005) is an appropriate estimation technique to estimate differential effects. It is flexible in the dynamic response of the variable of interest and, more importantly, it allows for non-linearities (interaction between the shock and the level of duration-to-GDP). Another benefit of a non-linear local projection is that it enables to study the effect of a shock conditionally on a current state, without imposing any restriction on the evolution of the state, as argued also by Ramey and Zubairy (2018) and Tenreyro and Thwaites (2016). The main disadvantage is that it is less efficient than a VAR, which, however, would not be able to handle the non linearity. The baseline specification I run is

[7] To strengthen identification I also employ alternative identification schemes, such as a Cholesky recursive identification, presented in Appendix C.11, or a high frequency identification based on Gertler and Karadi (2015), presented in Appendix C.13.

a reduced form regression, with the narrative monetary policy shock entering directly in the regression:

$$y_{t+h} = \beta_{0,h} + \beta_{1,h}Shock_t + \beta_{2,h}Shock_t DurGDP_{t-1} + \beta_{3,h}(L)'controls_t + \varepsilon_{t+h} \quad (2)$$

For $h \geq 0$ and where y_{t+h} is the variable of interest, such as the log of industrial production, $Shock_t$ is the measure of monetary policy shock, $DurGDP_{t-1}$ is the duration-to-GDP in the month prior to the shock, divided by its own standard deviation, $controls_t$ is a vector of controls, with the associated vector of coefficients in term of the lag operator $\beta_{3,h}(L)$. As an additional estimation technique, I present the results from local projections instrumental variables, LP-IV as in Stock and Watson (2017), which controls for measurement error in the shock estimate[8]:

$$y_{t+h} = \beta_{0,h} + \beta_{1,h}\Delta i_{t,t-1} + \beta_{2,h}\Delta i_{t,t-1}DurGDP_{t-1} + \beta_{3,h}(L)'controls_t + \varepsilon_{t+h} \quad (3)$$

$$\Delta i_{t,t-1} = \gamma_{10} + \gamma_{11}Shock_t + \gamma_{12}Shock_t DurGDP_{t-1} + \gamma_{13}(L)'controls_t + \eta_{1,t}$$

$$\Delta i_{t,t-1}DurGDP_{t-1} = \gamma_{20} + \gamma_{21}Shock_t + \gamma_{22}Shock_t DurGDP_{t-1} + \gamma_{23}(L)'controls_t + \eta_{2,t}$$

for $h \geq 0$. $\Delta i_{t,t-1}$ is the first difference of the monetary policy rate. Lag controls are present to improve the fit of the IV estimate and to strengthen the identification assumption of lag exogeneity as discussed in Stock and Watson (2017). To properly identify the coefficient of interest $\beta_{2,h}$, we need four assumptions to be satisfied[9]. The first assumption is *relevance* as in standard IV and it can be tested: $Shock_t^{\perp}$ and $(Shock_t DurGDP_{t-1})^{\perp}$ must be relevant to predict $\Delta i_{t,t-1}^{\perp}$ and $(\Delta i_{t,t-1}DurGDP_{t-1})^{\perp}$. The relevant statistic is the first stage Kleibergen-Paap Wald F statistic, which allows for non iid errors. The second assumption, also present in standard IV, is *contemporaneous exogeneity*: $Shock_t^{\perp}$ and $(Shock_t DurGDP_{t-1})^{\perp}$ should be uncorrelated with ε_t. As long as $Shock_t^{\perp}$ is exogenous, than it should also be uncorrelated with $DurGDP_{t-1}^{\perp}$ and the interaction term should not be an additional hurdle for identifi-

[8]The drawback is that this does not allow for a differential effect of the shock on the Fed fund rate on impact as it normalizes the effect of the monetary policy shock to increase this rate by 1%.

[9]I follow Stock and Watson (2017) exposition and, as they do, I define $v_t^{\perp} = v_t - Proj(v_t|W_t)$ for any variable v_t and any set of controls W_t.

cation. Furthermore, $DurGDP_t$ is a stock variable dated at the last day of month t; this implies that $DurGDP_{t-1}^{\perp}$ is not yet affected by $Shock_t^{\perp}$ and it is a relevant state variable for macroeconomic decisions in period t. The third identification assumption is *lead/lag exogeneity*, that is $Shock_t^{\perp}$ and $(Shock_t DurGDP_{t-1})^{\perp}$ are uncorrelated with future and past values of $\varepsilon_{t+j}^{\perp}$, $\eta_{1,t+j}^{\perp}$, and $\eta_{2,t+j}^{\perp}$ (for $j \neq 0$). This assumption is not problematic for $j > 0$ as structural (unforecastable) shocks which have not yet realized are not likely to affect today's variables, however the past one might. As long as our instruments are correlated only with past values of $\eta_{1,t+j}^{\perp}$ and $\eta_{2,t+j}^{\perp}$ (for $j < 0$), then we can solve the potential problem by including lags of the instruments. Otherwise we need a suitable set of controls to make this assumption hold.

The final assumptions I need, is that the maturity structure of public debt is not correlated with other determinants of the effectiveness of monetary policy on output and prices. While it cannot be tested directly, the very different time series properties of debt maturity structures in the US and in the UK assuage this concern as most potential confounding factors comove across the two countries. Public debt management choices are also very slow moving, anticipated by long cycles of policy debates, and often constrained by law so that they are exogenous with respect to business cycles movements and monetary policy decisions. For this reason, possible endogeneity is less of a concern than for private debt maturity choices for corporations[10].

The estimation with local projections produces serially correlated errors even if the original data generating process does not exhibit serial correlation, for this reason all standard errors are computed with Newey and West (1987) method[11]. In all regressions, I control for the lags of the left hand side variable and the lags of economic activity, exemplified by the industrial production, the price level, commodity prices, the policy rate, and unemployment rate. An important discussion in the literature has been on whether monetary policy shocks can affect output and prices on the same month as when they happen. The assumption that

[10]In Appendix A.1.2 I provide a detailed historical narrative account of the US Treasury public debt maturity choices to highlight their independence of to the monetary policy cycle. As additional evidence, in Appendix C.14, I discuss how the most likely confounding factors could affect both the effectiveness of monetary policy and the maturity structure and show that they do not apply in my setting. I also employ an instrumental variable approach for duration-to-GDP to check robustness. The results hold as well.

[11]Montiel Olea and Plagborg-Møller (2020) argue that with lag-augmentation local projection errors are not serially correlated, my main results go through with lag-augmentation of the dependent variables and controls and heteroskedasticity robust standard errors and are available upon request.

they do not is called the recursiveness assumption by Christiano, Eichenbaum and Evans (1999). It is not necessary for the identification strategy with narrative measure of monetary policy shocks, but I still present my main finding with this assumption for comparability with Ramey (2016)[12]. Operationally, the implementation of the recursiveness assumption follows by including the contemporaneous value of all controls except the policy rate.

4 Empirical Results

As a first step, I replicate the results of Ramey (2016) for the impact of a monetary policy tightening on key macroeconomic variables as shown in Figure C.2 in Appendix C for the US.The regressions assume recursiveness and have 2 lags of the log of industrial production, the log of the price level, the unemployment rate, the effective federal funds rate, and the log of the commodity price index. Industrial production and the unemployment rate exhibit a mild expansion puzzle the first months as was previously documented in the literature. Industrial production decreases by around one percent at peak and inflation starts decreasing only after 2.5 years and reaches a decline of almost 2 percent after 4 years. The unemployment rate increases by 0.2 percentage points at the peak effect.

Those results show the average response of macroeconomic variables after a monetary policy tightening. We now turn to the main results of the paper: the transmission of monetary policy conditional on the maturity structure of government debt. Figure 2 shows the results for the US. The regressions are the same with the addition of the interaction term and controlling for a lag of duration-to-GDP. Each row shows a different left hand side variable: in the first row industrial production, in the second the price level, in the third the unemployment rate, and in the fourth the federal funds rate. The columns show different coefficients from (2). In the first column the coefficient presented is $\beta_{2,h}$, which is associated with the interaction of duration-to-GDP and the monetary policy shock; the second column is $\beta_{1,h}$, which is the coefficient associated with the shock on its own. Thus, the second column shows what is the impact of a monetary policy shock when duration-to-GDP is zero (that is, when all debt has an overnight maturity), while the first column shows how much more (or less) effective is a monetary policy shock when duration-to-GDP is one standard deviation

[12] I show in a robustness check in Appendix C.10, that my results do not hinge on this assumption

higher.

There is a positive, large and statistically significant effect of the interaction term with industrial production as a dependent variable. *This is one of the key results of the paper: when public debt to GDP has a relatively higher duration, the effect of monetary policy shocks on industrial production are muted.* At peak the differential effect reaches a value of 2 % in magnitude. This is large. A mirror way of looking at this result can be found in the second column: when all debt is overnight, the effect of a monetary policy shock reduces industrial production by as much as 2% at peak, compared with an unconditional effect of 1% reduction at peak[13].

In contrast, inspecting the second row of Figure 2, shows that the effect of a contractionary monetary policy shock on inflation does not depend on the maturity structure of public debt. This is the second key result of this paper: *there is no differential effect of inflation.* The interaction between duration-to-GDP and the shock is not statistically different from zero at any horizon. By the same token, the effect of the shock without the interaction term is very similar to the unconditional effect: there is a small price puzzle at the beginning and but the inflation effect turns negative to around -1% after 4 years. This shows that the presence of high duration debt has no inflationary effect.

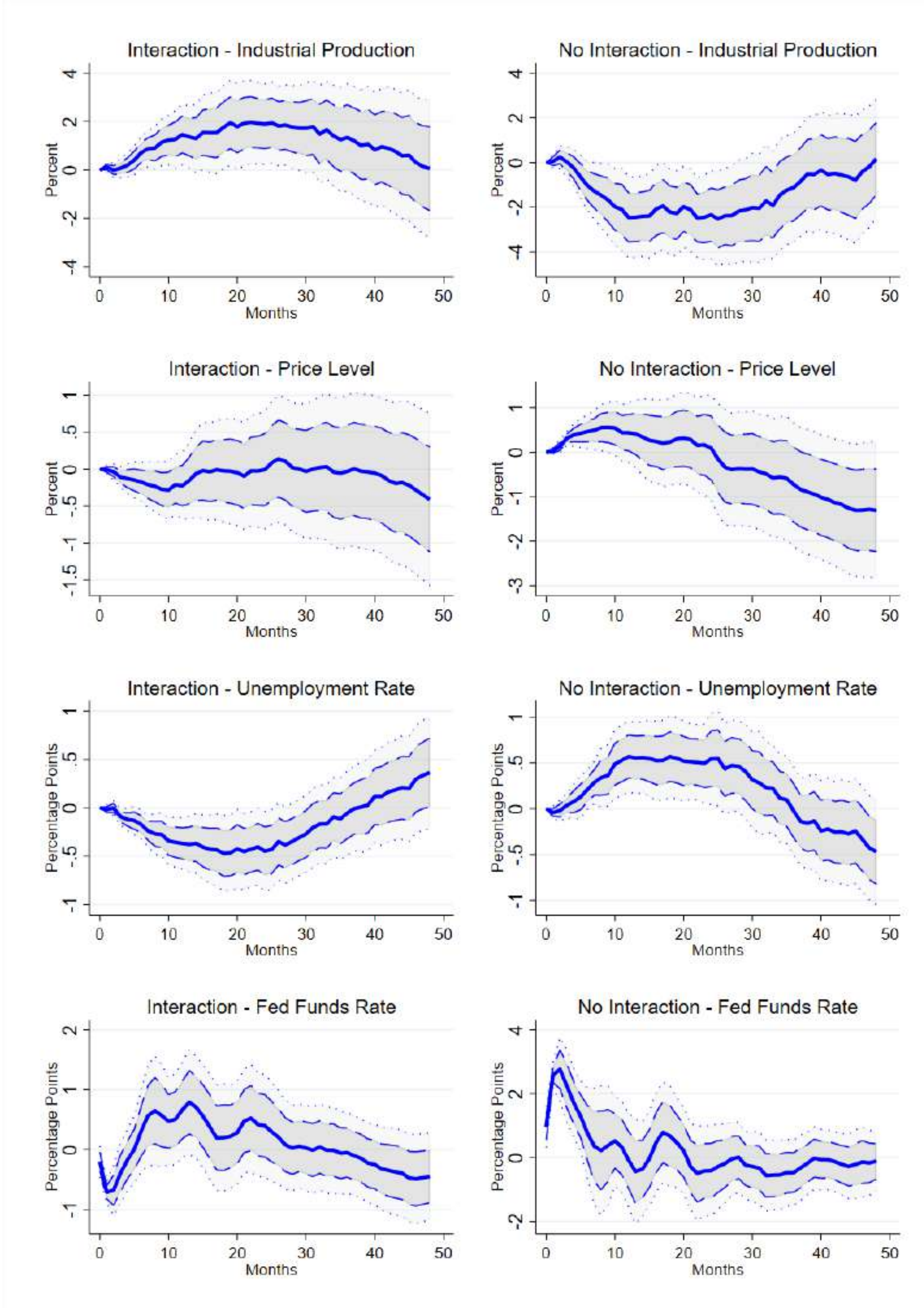
The conditional effect of the contractionary monetary policy shock on unemployment is consistent with the effect on industrial production. The interaction coefficient is negative and statistically significant. At peak, a monetary tightening increases unemployment by 0.5 percentage points less if duration-to-GDP is one standard deviation higher. Similarly, if duration-to-GDP is at zero, the effect on unemployment is magnified: unemployment increases by 0.5 percentage points rather than 0.3 unconditionally.

Finally, the last row of Figure 2 shows the effect on the Effective Federal Funds Rate. On impact, the federal funds rate increases relatively less under a high duration case, but the effect is short lived, with the interaction response turning insignificant from 5 months onward. There is relatively less overshooting following the initial shock under the longer duration case.

For robustness, I also run the regressions at quarterly frequency. The specifications are

[13] Note that the positive coefficient does not imply that when public debt duration-to-GDP is high the effect of a contractionary monetary policy on output is positive. Instead, it becomes statistically indistinguishable from zero, as shown in Appendix C.3.

Figure 2: Local projection baseline interaction regressions for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the interaction term of the shock with the Duration-to-GDP, the second column shows the shock term not interacted. Each row shows a different LHS variable.

similar. I added GDP as a dependent variable and as a control with two lags and a dummy for each quarter to control for seasonal effects. Figure C.3 shows that the unconditional effects of a contractionary monetary policy shock are similar to the monthly frequency case. GDP declines by 0.5% at peak after one year. The effect on industrial production is less precisely estimated, but it also declines at peak by 1%. Finally, the price level declines by 1.5% after 4 years.

Figure 3 presents the interaction regression results. GDP declines by 1% when all debt is short term, 1 year following the shock. Similarly, the interaction coefficient is positive and statistically significant: when public debt duration is one standard deviation higher, GDP is 0.6% relatively higher following a contractionary monetary policy shock. The results for industrial production, for the price level, and for interest rates are virtually identical to the monthly ones. *The key takeaway is that the contractionary monetary policy shocks is attenuated on industrial production when debt has longer duration-to-GDP, and there is no differential effect on inflation.*

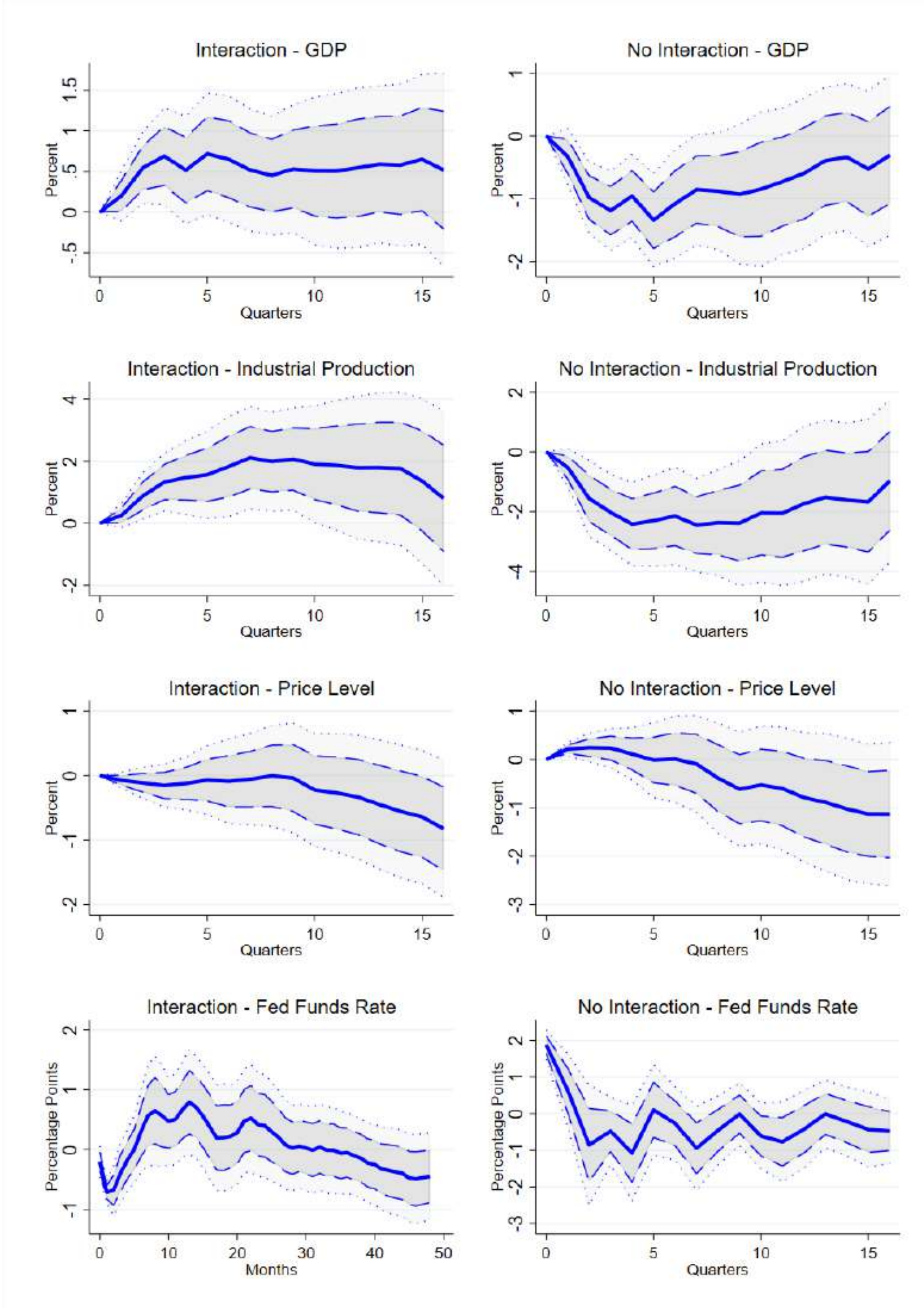
4.1 Sensitivity Analysis

Appendix C shows a vast set of robustness checks. In this section, I briefly summarize the key empirical challenges and how the sensitivity analysis addresses them.

4.1.1 Measurement Errors

The reduced form local projections are a very transparent method to estimate the dynamic effects of an identified shock, and they allow for a differential effect of the shock on the Fed funds on impact, depending on the state of the economy. However, using the shock measure directly in the local projection can lead to biased estimates if the shock metric estimates the true structural monetary policy shock with a measurement error. LP-IV methods allow to overcome the measurement error at the cost of imposing a normalization, whereby the monetary policy shock cannot affect interest rate on impact differentially depending on the state of the economy. Appendix C.2 shows that the results go through almost one-to-one with the LP-IV framework.

Figure 3: Local projection baseline quarterly interaction regressions for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969q1 to 2007q4 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, GDP, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, GDP, the price level, the commodity price index, the unemployment rate, and the Fed funds rate, one lag of duration-to-GDP, and a dummy per quarter. The first column shows the interaction term of the shock with the Duration-to-GDP, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

4.1.2 Measurement of the Maturity Structure of Public Debt

Duration-to-GDP measured with nominally fixed rate public debt held by the general public is the metric that the fiscal authority should focus on when assessing the impact of monetary policy on the government budget constraint and on financing. However, one might worry that the results hinge on the specific way duration-to-GDP was built. It does not, and we can go over a number of possible alternatives.

Macaulay duration. Duration-to-GDP is a volume measure that reflects the overall economy-wide amount of insurance long debt provides, rather than the insurance per unit of debt. In Appendix C.4, I provide the same results with the Macaulay duration of public debt to show robustness to the measure per unit of debt. The results all go through but are less precisely estimated, in line with the idea that the volume measure is the correct theoretical metric. One might wonder if we can separately identify the role of debt to GDP and Macaulay duration, but this is challenging due to the multicollinearity of the shock, the interaction of the shock with debt to GDP, and the interaction of the shock with Macaulay duration. Appendix C.5 shows the result of this exercise. Whereas most IRFs present large swings hinting to multicollinearity, overall the results point to a mediating level of duration for each level of debt to GDP.

TIPS. The main measure excludes inflation-linked debt as its value might behave differently from nominal debt following a change in nominal rates. However, Nakamura and Steinsson (2018) point out that real and nominal yield curve rates all move similarly following a monetary policy shock. Therefore, in Appendix C.6, I show that results are very similar if we include TIPS in duration-to-GDP.

Fed holdings. The duration-to-GDP variable also excludes debt held by the Fed and other public entities as the valuation gains and losses on the government bonds holdings of the Fed are matched with the opposite sign for the Treasury. One might worry that due to institutional frictions across government departments, the consolidation of public debt liabilities across the government might not be fully warranted. Therefore, in Appendix C.6 I show how the results go through if we do not net out public sector holdings of debt.

Face value. In order to compute the derivative with respect to interest rate changes of the market value of public debt [14], we need to compute duration-to-GDP at market value, with yield curve data. However, a possible worry is that interest rate levels are endogenous, so that we are actually picking up the effect of low versus high interest rates rather than the mediating effect of the public debt maturity structure. For this reason, I show in Appendix C.6 that the results go through if we construct the same measure at face value, without discounting by the yield curve (similarly to Greenwood and Vayanos (2014)).

Long debt over GDP. Duration to GDP uses all the cash flow promises public debt entails to compute the insurance amount that long maturity provides. An alternative metric could be to compute the amount of long maturity debt to GDP. This alternative metric gives equal weight to all debt above a threshold, in line with the idea that to study a monetary policy shock that affects the economy at a business cycle frequency, a 7 year debt promise provides a similar amount of insurance as a 15 year debt promise. The drawback of this measure is that we lose the direct mapping that duration-to-GDP has to changes in the market value of public debt. Appendix C.7 shows that the results are nevertheless very close if we use this metric with debt promises above a threshold (5 and 10 years) to GDP. This is the metric proposed by Krishnamurthy and Vissing-Jorgensen (2012).

Comparison with one quarter maturity debt. The baseline specification in (2) brings as a reference comparison in $\beta_{1,h}$ what would be the impact of a monetary policy shock under an overnight debt with zero duration. However, in the theoretical model presented in Section 6 the reference is one period debt, that is quarterly debt. For this reason, in Appendix C.8, I construct the difference in duration-to-GDP from an hypothetical quarterly debt to give a direct mapping to the theoretical model. The results are virtually indistinguishable and can be used to assess the performance of the theoretical exercise.

Smooth transition. I employ duration-to-GDP directly in the main empirical specification. This allows to give a cardinal value to the various levels of this variable. However, this does not allow to interpret the results as effect of monetary policy under different regimes:

[14] Or, equivalently, the insurance amount that long debt provides on the net present value of interest rate payments over GDP.

the effect under a low duration-to-GDP regime, or the effects under a long duration-to-GDP regime. Moreover, the interpretation of $\beta_{1,h}$ in the baseline specification implies a degree of extrapolation, as we never observe all debt being overnight. For this reason, in Appendix C.9, I present the results with a Smooth Transition Local Projection Method, used among other by Tenreyro and Thwaites (2016), Auerbach and Gorodnichenko (2017). I employ a logistic function transformation on the standardized duration-to-GDP, and then use this metric instead of $DurGDP$ in the regression. All the results go through: monetary policy has stronger contractionary effects under a low duration-to-GDP regime on output but not on the price level. Moving from a low to a long duration-to-GDP regime attenuates the effects on output and has no effect on inflation.

4.1.3 Identification of Monetary Policy Shocks

The updated Romer and Romer shock allows to have an identified monetary policy shock, for a long time sample. The long sample is particularly useful to identify the coefficient associated with the interaction term with duration-to-GDP, in order to have enough variation in this measure. However, one might worry that shock measure is picking up forward guidance shocks, that the recursiveness assumption is too restrictive, or in general that the shock measure is not well identified. For this reason, I show that using alternative identification schemes yield similar results. Appendix C.10, presents the results when not including the recursiveness assumption. Appendix C.11 presents the results without external instruments, when using a recursive identification only which is equivalent to a Cholesky identification (as presented by Christiano, Eichenbaum and Evans (1999)); I present results for the same sample as in the baseline and for an extended sample. In Appendix C.12, I show the IRFs using the original Romer and Romer (2004) narrative measure on their original sample. Finally, in Appendix C.13, I show the results by using a high frequency identification, from Gertler and Karadi (2015)[15]. The results from these alternative identification schemes all support the result that high duration of debt to GDP attenuates the contractionary effects of monetary policy on industrial production and unemployment but not on inflation.

[15]Results are very similar with high frequency identification schemes that control for the central bank information effect as Jarociński and Karadi (2020), Miranda-Agrippino and Ricco (2021).

4.1.4 Identification of Duration-to-GDP

The baseline specification employs directly duration-to-GDP in the local projection. The reason is that it is transparent and reverse causality is unlikely. In Appendix A.1.2, I provide a detailed narrative account of the debt maturity decisions and show how the Treasury chose the debt composition *exogenously* with respect to the actions and to the effectiveness of monetary policy on the business cycle[16].

In order to provide even stronger identification, I discuss potential confounding factors. Monetary policy could be more effective on output when duration-to-GDP is low for reasons that do not hinge on the maturity structure of public debt. In Appendix C.14, I discuss how the most likely confounding factors (whether the economy is in a recession or the slope of the yield curve) are actually likely to go in the *opposite* direction. They would imply stronger effects of monetary policy on output when debt has long duration.

Additionally, I present an instrumental variable approach for duration-to-GDP proposed by Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2014). They suggest that the overall stock of government debt at book value is a good instrument for its maturity structure as it depends on past deficits and it is independent of current market conditions. This approach yields strong instruments and IRFs very close to the baseline.

4.2 External Validity

I presented the main results for the US. However, a possible worry is that the US is special so that the results would not go through in other countries. Moreover, testing the relationship in another country can help to disentangle the mechanism at play. The US government debt has a central and unique role in the global monetary system, it plays the main role as a safe asset as surveyed in Gourinchas, Rey and Sauzet (2019). A direct implication is that US treasuries earn a convenience yield relative to similar government bonds as shows by Krishnamurthy and Lustig (2019). One might wonder if the unique situation of the US is crucial for the results or if they go through more generally.

[16] A law introduced in 1918 that capped interest rates on long bonds 4.25% constrained the Treasury up to 1988 in its desire to lengthen the maturity. From 1993 the trade-off calculation shifted toward costs pushing towards shorter maturities, and then again, in 2005 it shifted back toward rollover risk and toward longer maturities.

The UK presents an excellent laboratory to test this hypothesis. The UK government debt has relatively low default risk, but in the post war period has not been the key safe asset in the global monetary system. Very importantly for this study, the level and the time series properties of the UK duration of government debt to GDP have been markedly different from the US one. The lowest values were in the 70s and 90s, with shorter times in each regime than in the US. The correlation between duration-to-GDP across the two countries is *negative*[17]. Moreover, the UK is the country which historically had the longest maturity structure among large countries. As the US had one of of the lowest, they provide a useful contrast.

Appendix E shows the same analysis for the UK. Despite the differences in level and time series for duration-to-GDP in the two countries, the interaction coefficients, presented in Figure E.3, are remarkably similar. An increase in one standard deviation of duration-to-GDP reduces the contractionary effect of a monetary policy shock on industrial production by 2% at peak, as in the US. Moreover, the reduction of inflation is not affected by duration-to-GDP, as in the US.

That appendix also shows a number of sensitivity checks for the UK, similar to the US ones. They also highlight that the result is robust. Specifically, it shows robustness for using LP-IV (Figure E.4), Macaulay duration (Figure E.5), debt at face value (Figure E.6), inflation linked debt as well (Figure E.7), no recursiveness assumption (Figure E.8), and the smooth transition local projection method (Figure E.9).

The main takeaway from this set of results is that the effect of a monetary policy tightening on output is greatly reduced when public debt has a longer duration-to-GDP. In contrast there is no differential effect on inflation. This is very robust across specifications, identification schemes, countries, and measurement choices. In the next section, we examine the economic channels that can explain this result.

5 Inspecting the Mechanism

In the previous section we established that the maturity structure of public debt matters in the transmission of monetary policy, now we turn on why this may be the case. If public debt

[17] See Table C.4.

to GDP has a higher duration the government is insured at least partially against an interest rate rise. The government has locked-in the pre-shock lower interest rate and does not need to refinance as much at the new higher prevailing rate. The government budget constraint implies that either the government borrows relatively less or the budget surplus decreases relatively. Although the central bank controls only short interest rates[18], monetary policy shocks can affect the whole yield curve and are important for the valuation of public debt. In Appendix D.1, I show that all yields move following a monetary policy shock with short yields moving more than long yields, but bond prices proportionally move more at longer horizons due to valuation effects.

In this section, I will present a number of possible hypotheses which could explain the aggregate finding uncovered in Section 4, obtain testable implications and distinguish between them. The main three possible economic mechanisms are: (i) financing frictions "*financing channel*", (ii) heterogeneous agents models; (iii) distortionary taxation on corporates. Additionally, the fiscal theory of the price level and models that feature substitution of maturities across public and private borrowers speak directly to the phenomenon under study. I review each hypothesis in turn and present detailed evidence on fiscal policy, non financial corporations behavior and financial variables to discriminate among them (see Figures 4, 5, and 6).

In each of these figures, I present a different dependent variable in each row. The first column shows the average response of the variable of interest to a contractionary monetary policy shock. The second column presents the interaction term between duration-to-GDP and the shock. The third column presents the coefficient without the interaction term, that is the response of a monetary policy shock with overnight debt. The econometric specification is the same as in the baseline results of Figures 2 and 3, depending on the frequency of the dependent variable. In addition, the first two lags of the dependent variable are included. The full description of each variable is presented in Appendixes A.3 and A.4.

5.1 Financing Channel

The hypothesis which is in line with the empirical facts is the *financing channel*. The key idea is that when the government borrows relatively less, the non-financial corporate sector has

[18] In the sample period under study, monetary policy was mainly conducted with conventional methods.

relatively more resources at its disposal and it can borrow relatively more at a cheaper price. There is less "crowding-out". The non-financial sector will be able to use these financial resources to invest more, and therefore, to increase relatively output. In addition, the non-financial corporate sector is hit less by the rate increase, so that leverage and credit risk will be relatively lower.

Figures 4 and 5 test this hypothesis and find supporting evidence. The first row of Figure 4 presents the effects on net lending by the government (the negative of borrowing). On average, the government borrows more following an interest rate increase, with a higher borrowing by 0.5 percent of GDP after 4 years. The difference across maturity regimes is striking. When the government has a one standard deviation higher duration-to-GDP it borrows relatively less by an amount of 1 percent of GDP at peak. Similarly, when all debt is short term the government borrows more by 1 percent to GDP. This difference is large economically and very precisely estimated.

On the corporate debt quantity side, we can see that non-financial corporates issue relatively more debt (row 2, column 2 of Figure 4)[19]. As an additional check, we can see that overall bank loans increase relatively (row 1, column 2 of Figure 5).

Rows 2 to 4 of Figure 5 present various relative price measures, the spread between AAA corporate bonds and 10 years treasuries, the spread between BAA and 10 year treasuries, and the excess bond premium by Gilchrist and Zakrajšek (2012). All these measures mildly increase on average following a monetary policy shock. Importantly, all these measures decline relatively when duration-to-GDP is higher and are more strongly positive when all debt is overnight. The magnitude of the decline at peak is of a similar order of magnitude across the various measures, by about 20 basis points for the AAA-treasury spread and for the excess bond premium and by 40 basis points for the BAA-treasury spread. This is important information and even for measures with low (AAA-treasury) or no-credit risk (the excess bond premium purges the default risk from corporate bond spreads) we see a relative decline. Furthermore, the relatively higher decline in the BAA-treasury spread suggests that the credit risk also declines.

So far, we established the *financing channel* result on the debt side: when the government

[19]This debt issuance metric consolidates all debt instruments, following Greenwood, Hanson and Stein (2010) with Flow of Funds data, the detailed description is in Appendix A.4

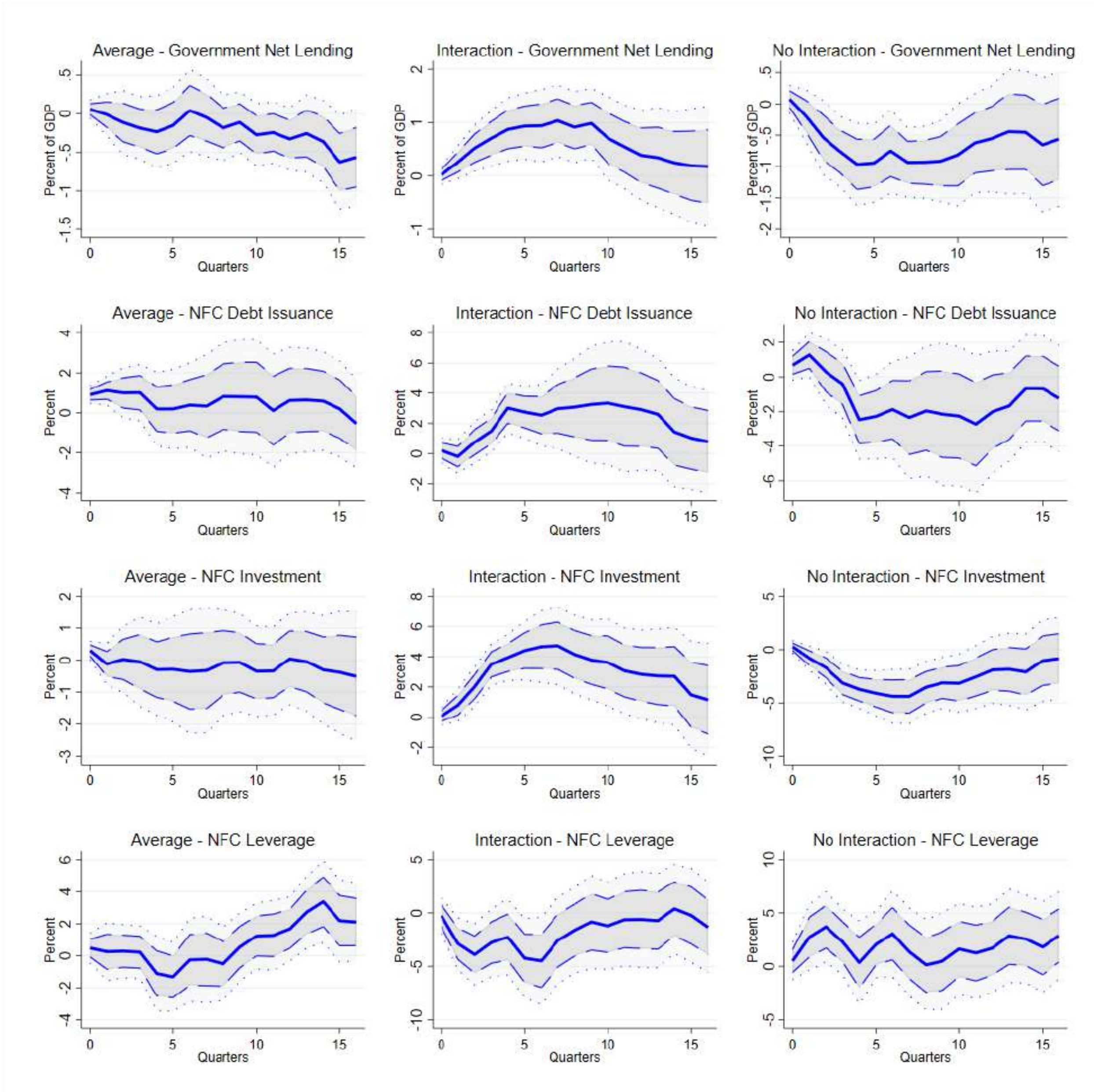
borrows less, the non-financial corporates borrow more at a cheaper price. We now turn to the real side of the *financing channel* hypothesis. The third row of Figure 4 shows that on average investment by non-financial corporates declines following a contractionary monetary policy shock, with the effect not being strongly statistically significant. However, the conditional interaction effect is positive and statistically significant, with a positive coefficient of more than 4% at peak. Similarly, if all debt is short term the contractionary monetary policy shock reduces investment by 5%. These are large numbers, but in line with the high volatility of investment.

The fourth row of Figure 4 shows the response of non-financial corporates' debt leverage. On average leverage increases by 3% after 3 years following the contractionary shock. When we turn to conditional effects, we see that leverage decreases relatively by almost 5% for a one standard deviation higher duration-to-GDP.

Direct Evidence. The *financing channel* hypothesis relies on a financial market friction that hinders the smooth function of primary markets: when the government goes less to the market, it becomes cheaper to borrow not only for the government, but also for non-financial corporates[20]. In a companion paper (Andreolli, 2021), I provide direct evidence for the salience of this friction. I estimate news shocks to the supply of marketable public debt which are orthogonal to the fiscal policy stance and other confounding factors and show how increases in the supply in public debt increase interest rates on government bonds and on corporate bonds. I estimate the shock with a high frequency approach exploiting a specific institutional feature of the UK: the Debt Management Office publishes updates on supply of marketable public securities (Gilts) just after the budget speeches of the Chancellor of the Exchequer, where he discusses the fiscal policy stance but not the supply of marketable public debt. Changes in the price of futures on long Gilts around news published by the DMO can therefore be used as instruments for marketable debt supply shocks. I provide a theoretical model of the *financing channel* in Section 6 and show that a plausible calibration is able to reproduce the empirical evidence.

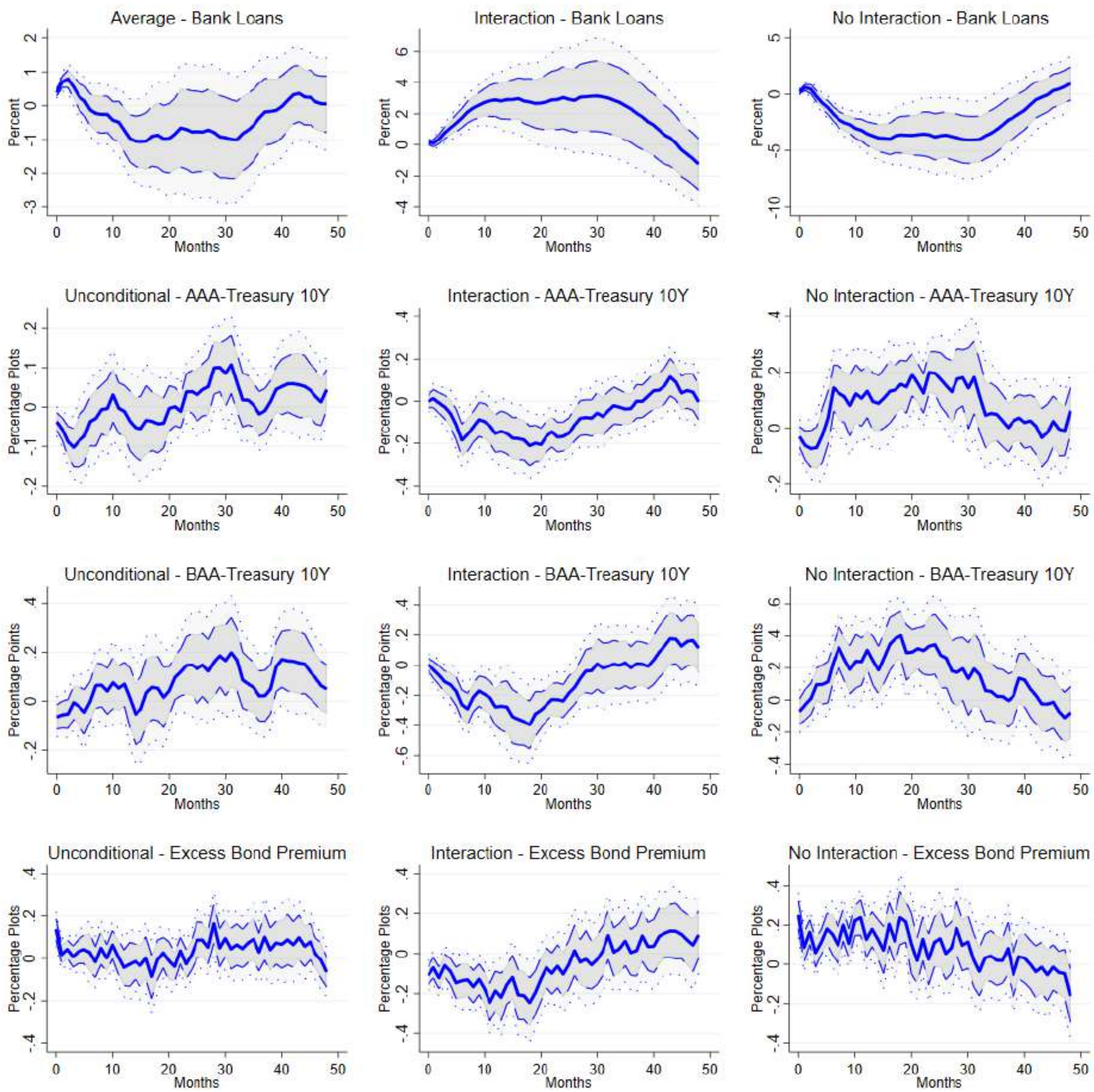
[20]This pass-through is in line with the evidence provided by Krishnamurthy and Vissing-Jorgensen (2012) and Papoutsi, Piazzesi and Schneider (2021).

Figure 4: Economic channel - competition for financing with quarterly data



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969Q1 to 2007Q4 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, GDP, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of the left hand side variable, industrial production, GDP, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the average response to a monetary policy shock. The second column shows the interaction term of the shock with the duration-to-GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

Figure 5: Economic channel - competition for financing with monthly data



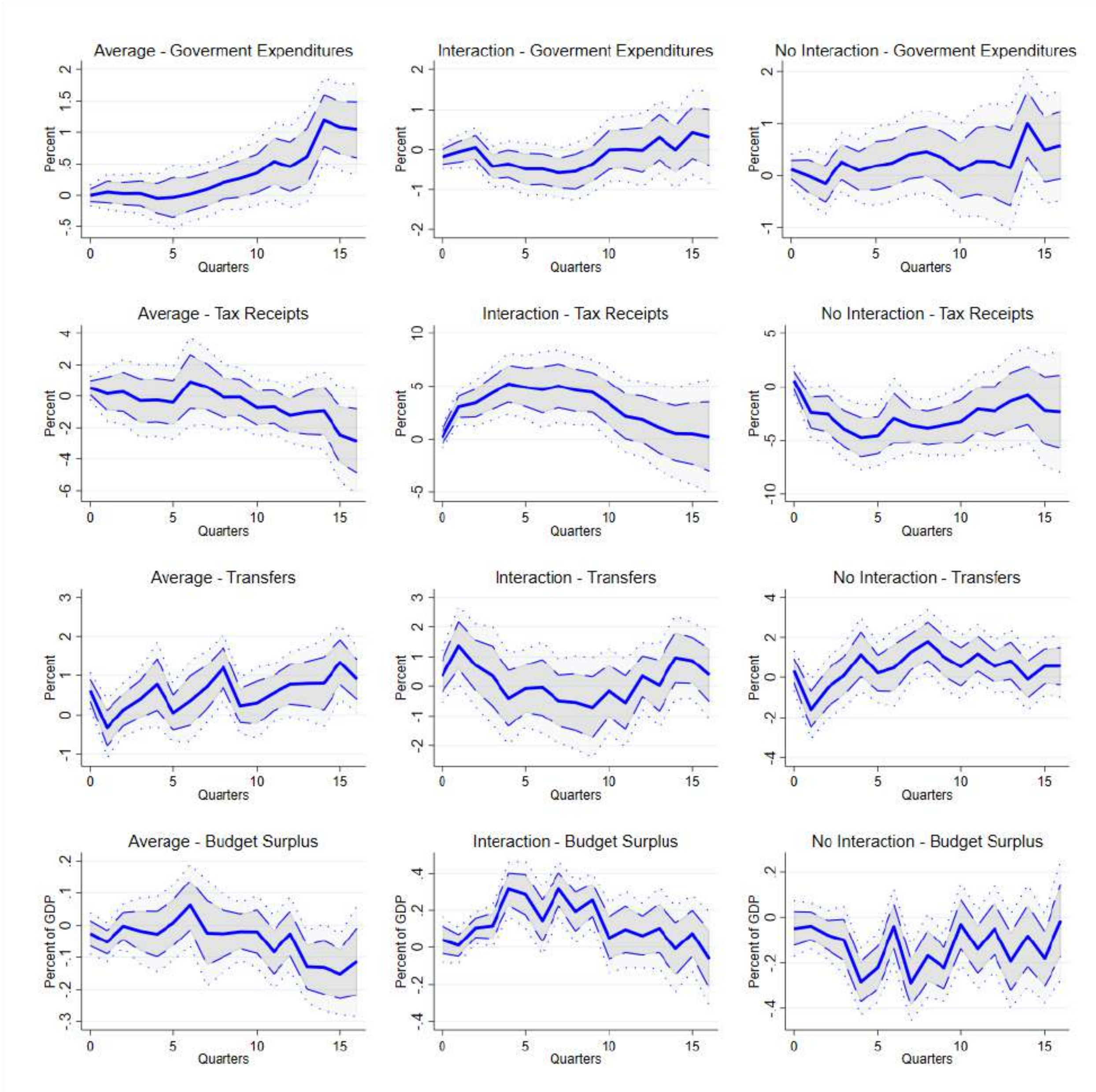
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 for all variables except for the excess bond premium, which goes from 1979m9 to 2007m12. The sample uses US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, GDP, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of the left hand side variable, industrial production, GDP, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the average response to a monetary policy shock. The second column shows the interaction term of the shock with the duration-to-GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

5.2 Heterogeneous Agents Models

The presence of hand-to-mouth agents could deliver a relatively higher output response when the government is insured without necessarily a strong effect on inflation. Following a contractionary monetary policy shock when public debt has a longer maturity, the government gains relatively due to valuation effects on public debt. The bondholders lose by the same token. If Ricardian equivalence held this would not matter. However, with heterogeneous agents it might. We should see bondholders cutting their consumption relatively, pushing the aggregate relative output response to a contractionary monetary policy shock to be negative (so that monetary policy should lower output more when duration-to-GDP is high). Therefore, in order to obtain a relatively positive effect on output as shown in the previous section, the response of the hand-to-mouths is crucial. In this framework, hand-to-mouth agents would respond positively if the government uses the relative windfall on expansionary contemporaneous fiscal policy. We should see either relatively higher government expenditures, lower taxes, higher transfers, or in general lower fiscal surplus *today*. So a testable implication of this hypothesis is that we should see a contemporaneous expansionary fiscal policy response.

Figure 6 tests the hand-to-mouth hypothesis. The first row shows that on average the government does not respond contemporaneously with higher government expenditures following an interest rate increase. There is a mild increase after 4 years. More importantly, the second column shows that the interaction coefficient is not statistically different from zero. This implies that the government does not spend relatively more under a high duration-to-GDP case, as would be necessary for the hand-to-mouth hypothesis to be corroborated. The second row shows tax receipts. On average they do not respond much, with a mild decline at the end of the horizon. However, the interaction coefficient is positive, large, and statistically significant. This seems to follow the output response, but crucially, it would need to be negative, for the hand-to-mouth hypothesis to work. The third row shows transfers, where we do not see any significant response on average or conditionally to duration-to-GDP. Again, we should have seen a positive coefficient on the interaction term to corroborate the hand-to-mouth hypothesis. Finally, we can bring the sub-components together and look at the government budget surplus in the fourth row. On average, the budget surplus mildly declines following a contractionary monetary policy shock after 4 years. More importantly,

Figure 6: Economic channel - contemporaneous fiscal policy



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969Q1 to 2007Q12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, GDP, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of the left hand side variable, industrial production, GDP, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the average response to a monetary policy shock. The second column shows the interaction term of the shock with the duration-to-GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

the budget surplus relatively increases following the contractionary monetary with a longer duration-to-GDP (second column). This means that the overall relative fiscal stance goes in the *wrong direction for hand-to-mouth consumers to explain the aggregate results.*

Hence, the government does *not* give a contemporaneous fiscal impulse when it is insured against the interest rate hike. What we find instead, is a *strong differential effect on net borrowing* as shown in the first row of Figure 4.

5.3 Distortionary Corporate Taxation

Let us now turn to the distortionary corporate taxation hypothesis, which could also explain the aggregate results and the fiscal response. The main idea is that, when a government with longer dated debt borrows relatively less, following an interest rate increase, the non-financial corporates expect lower distortionary taxation in the future, they invest today and they borrow relatively more. From the perspective of debt markets, we should see an increase in demand for funds by the non-financial corporates. This implies, especially with segmented markets (see Gabaix and Koijen, 2020), that we should see an increase in the relative price of corporate debt compared to public debt. Furthermore, with relatively lower default risk and tax burden in the future, firms should be able to take on more leverage.

Figures 4 and 5 show that we find evidence for the quantity predictions of this hypothesis but not for the leverage and price predictions. The second column, fourth row of Figure 4 shows that leverage declines relatively when duration-to-GDP is higher. Rows 2 to 4 of Figure 5 show in the second column, that the various relative price measures decline when duration-to-GDP is higher. All these results on relative corporate debt prices, but especially those that purge default risks, point to the fact that *higher demand for funds by non-financial corporates due to lower future distortionary taxation cannot be the key explanation for the aggregate results.*

5.4 Fiscal Theory of the Price Level.

Finally, it is important discussing theories that feature an explicit role for the maturity of public debt and its interaction with monetary policy. First of all, the fiscal theory of the price level, in its basic formulation, would predict that a contractionary monetary policy shock would lower inflation more when duration-to-GDP is higher. The reason is that, with longer duration debt, an increase in interest rates would reduce more the market value of debt. This implies that either the price level decreases relatively more, or that the primary surpluses would relatively decline, in order to keep the budget constraint satisfied. However, we do not see a differential effect on inflation and we can see in the third row second column of Figure 6 that the government budget surplus, increases relatively with more long duration debt.

5.5 Gap filling maturity structure

Greenwood, Hanson and Stein (2010) show that when the government has relatively longer dated debt the non-financial corporate sector has relatively shorter debt and vice-versa. If this was the driving force of the results however, we should see the opposite response for investment and output: when the non-financial corporates have shorted maturity debt, they are more responsive to changes in interest rates, in line with the model by Gomes, Jermann and Schmid (2016). We see the opposite.

5.6 Taking Stock

Table 1 summarizes the predictions of the various theories discussed and how they contrast with the data. Overall, we find evidence for the *financing channel* to explain the empirical results. Following an interest rate increase, when the government has relatively longer debt, it borrows relatively less. This in turns allows for more resources to be available for the corporate borrowers (less crowding out), which borrow more at a cheaper rate. This is not to say that the other hypotheses cannot be important in the data, however, they cannot explain the overall patterns we uncovered in Sections 4 and 5.

6 Model Description

In this section, I present a model which can rationalize the empirical results. The model is a standard financial accelerator New Keynesian model in the spirit of Bernanke, Gertler and Gilchrist (1999) with two modifications. First, the government can issue fixed-rate nominal

Table 1: Economic Channel Testable Hypotheses Summary

	Financing	HANK	Dist. Tax	FTPL	Gap filling	Data
Output	+	+	+		-	+
Inflation				-		0
Primary Surplus		-		-		+
Government Consumption		+				0
Transfers		+				0
Taxes		-				+
Government Borrowing	-		-			-
Corporate Debt Quantity	+		+			+
Corporate Debt Prices	-		+		+	-
Corporate Leverage	-		+		+	-
Investment	+		+		-	+

Notes: This table shows the relative IRF response to a contractionary monetary policy shock when debt is relatively longer maturity: $\beta_{2,h}$ in regression (3). All columns except the last one indicate the predictions for the various variables from each theory. For the variables for which there is no direct prediction, none is specified. The last column presents the data counterpart. *Financing* stands for the financing channel hypothesis, HANK for the heterogeneous agent New Keynesian model, Dist Tax for the distortionary taxation on corporates, FTPL for the fiscal theory of the price level, and Gap filling for the hypothesis associating a negative relationship between public and private debt maturity. In the data column, a "+" ("-") indicates that at peak $\beta_{2,h}$ is positive (negative) and statistically significant at the 90% confidence level, a "0" indicate that it is not statistically significant at the 90% confidence level.

long term debt, which creates a history dependence on past interest rates for public debt servicing. Second, I introduce primary dealers in this economy. They face a cost to issue new debt each period. This is akin to investment adjustment costs for debt issuance. It creates a direct mapping between the quantity of government bond issued and the financing conditions the private sector faces to issue debt. The magnitude of the primary market friction is small, but can have large macroeconomic effects when variations in public debt supply across maturity regimes interacts with the financial accelerator frictions on the firm side.

6.1 Government

The government issues a bond which has geometrically decaying amortization and which pays a fixed net nominal interest rate R_t^{new} on new bonds. The principal due decays at rate δ^d, therefore if the government issues one unit of debt today it is going to repay $\delta^d + R_t^{new}$ the next period, $(\delta^d + R_t^{new})(1-\delta^d)$ in 2 periods, $(\delta^d + R_t^{new})(1-\delta^d)^2$ in 3 periods and so on, all of these in today's dollars. In each period, the government issues L_t bonds; therefore, the end of period stock of debt D_t can be written as the sum of remaining past issuances, which allows a recursive formulation:

$$D_t = L_t + (1 - \delta^d)D_{t-1}$$

Notice that all stock variables are defined as end-of-period variables, so that D_t is the stock of bonds reflecting the choices at period t. Interest rates are all defined in net terms rather than gross terms. The convenient geometric bond structure also allows for a recursive formulation for the average interest rate process on the debt stock. In each period the interest payments are:

$$R_t^{ave} = R_t^{new}\frac{L_t}{D_t} + R_{t-1}^{ave}\left(1 - \frac{L_t}{D_t}\right)$$

This debt specification allows to keep a parsimonious setting where interest rate path dependence is explicit and can have long lasting effects but the number of state variables is only two. This is close to the literature on the impact of fixed rate vs variable rate mortgages (see Kydland, Rupert and Šustek (2016)) for modeling the interest rates, and similar to Hatchondo and Martinez (2009) or Arellano and Ramanarayanan (2012) in terms of modeling long public debt as a geometric decaying process. While I model public debt with the geometric structure out of empirical relevance and theoretical parsimony, in a recent paper, Bhandari et al. (2021) show that a geometric structure for public debt is optimal in a Ramsey problem. The geometric approximation is a good fit of the data, with an average R^2 of 87%[21]. If we define F_t to be the debt payments in period t the debt dynamics system is fully determined with a third equation:

$$F_t = (R_{t-1}^{ave} + \delta^d)D_{t-1}$$

We rescale debt quantity variables in real terms with lower case letters being the real value $x_t \equiv \frac{X_t}{P_t}$ where P_t is the aggregate price level for consumption goods and inflation is defined

[21] I compute the R^2 in each period the US estimation sample by fitting the model predicted annual principal debt promises on actual promises. The average R^2 is 0.8725 and its standard deviation is low at 0.0322.

as $\pi_t \equiv \frac{P_t}{P_{t-1}}$:

$$f_t = (R_{t-1}^{ave} + \delta^d)\frac{1}{\pi_t}d_{t-1} \tag{4}$$

$$d_t = (1-\delta^d)\frac{1}{\pi_t}d_{t-1} + l_t \tag{5}$$

$$R_t^{ave} = \left(1 - \frac{l_t}{d_t}\right)R_{t-1}^{ave} + \frac{l_t}{d_t}R_t^{new} \tag{6}$$

We derive the following lemma linking the duration to δ^d and to R_t^{new}.

Lemma 1 *Assume public debt has a geometric principal structure, such that a fraction* $(1-\delta^d)$ *of the principal is repaid in each period and that interest payments on newly issued debt* L_t *are fixed in nominal terms for this bond vintage at* R_t^{new}*, then the Macaulay duration on newly issued debt is:*

$$Dur_t = \frac{1+R_t^{new}}{\delta^d + R_t^{new}} \tag{7}$$

The proof of this lemma is in Appendix F.1.1. As an illustration, if δ^d is equal to 0.05 and R_t^{new} is 0.0123 (0.05 at annual frequency) we have a duration of 16.24 quarters, of around 4 years. On the other hand, if δ^d is equal to 1 all bonds are due next quarter and duration is 1.

Fiscal policy

The government spends G_t, which moves exogenously, and receives taxes T_t. The budget constraint is:

$$F_t = P_t(T_t - G_t) + L_t$$

In real terms:

$$f_t = T_t - G_t + l_t \tag{8}$$

In order to match the empirical evidence (see Herbst and Schorfheide (2015)) , I set up a tax reaction function where taxes react slowly to changes in financing needs (passive fiscal policy) :

$$T_t = G_t + (T - G)\left(\frac{d_{t-1}}{d}\right)^{\tau_T} \tag{9}$$

With a $\tau_T \geq 0$ but low, consistent with the model presented by Eusepi and Preston (2010) and empirical evidence presented by Davig and Leeper (2007) for passive fiscal policy and Herbst and Schorfheide (2015). This means that most of the short term changes in financing needs to be absorbed by new bonds issuances. Note that this tax policy works if there is a structural primary surplus in steady state, which is the case in this model with no growth and positive steady state public debt. Government spending evolves exogenously following an AR(1) when log-linearized, with G being the steady state government consumption:

$$\frac{G_t}{G} = \left(\frac{G_{t-1}}{G}\right)^{\rho_g} \exp(\varepsilon_t^g) \tag{10}$$

The government in this model has access to a long maturity bond but behaves like an "automaton", in line with the empirical results shown in the previous sections. This could be due to high costs of adjusting its positions, as argued by Faraglia et al. (2018) for the maturity structure choice. In this paper, I abstract from the optimal choice of the government for the maturity structure of debt.

6.2 Primary Market Participants

The primary market participants buy new debt issuances of the government and of the private sector on the primary market and sell them within the period to final investors on the secondary market. These agents face a convex cost to participate in this market: when the government or the firms issue more, the funding costs increase as the primary market participants cannot absorb as easily the new funds. This creates an adjustment cost friction:

$$\bar{\Phi}\left(Total\ New\ Issuance_t\right) = \bar{\Phi}\left(\frac{B_t^{crp}}{P_t} + \frac{L_t}{P_t}\right) \tag{11}$$

With $\bar{\Phi}' \geq 0$ and $\bar{\Phi}'' \geq 0$. The functional form for $\bar{\Phi}$ is:

$$\bar{\Phi}(x) \equiv \Phi_0 \frac{x^{\Phi_1+1}}{\Phi_1+1} \exp\left(\frac{\nu_t^{\Phi}}{\Phi}\right), \; \Phi_0 \geq, \; \Phi_1 > 0$$

Where ν_t^{Φ} is a shock to the financing friction and Φ is the steady state value of the first derivative of the friction, which is there as a scaling variable. The shock follows the law of motion:

$$\nu_t^{\Phi} = \rho^{\Phi} \nu_{t-1}^{\Phi} + \varepsilon_t^{\Phi}$$

The primary market participants sell immediately on the secondary market the bonds they purchased on the primary market at price q_t^t for the government bond (secondary market price at time t for a bond issued in time t), and at price q_t^{crp} for the corporate bond. The profit maximization of these agents is given by:

$$\max_{b_t^{crp}, l_t} \; q_t^{crp} b_t^{crp} + q_t^t l_t - \left(b_t^{crp} + l_t\right) - \bar{\Phi}\left(b_t^{crp} + l_t\right) \tag{12}$$

The problem is concave, so the solution can be found by taking the first order conditions:

$$q_t^{crp} = 1 + \Phi_t$$
$$q_t^t = 1 + \Phi_t$$

where we define the primary market friction $\Phi_t \equiv \frac{\partial \bar{\Phi}\left(b_t^{crp}+l_t\right)}{\partial l_t} = \frac{\partial \bar{\Phi}\left(b_t^{crp}+l_t\right)}{\partial b_t^{crp}}$ in a compact form. In Appendix B, I propose two isomorphic microfoundations for this primary market friction, relaying on specialized investors in the primary market. The first one assumes a moral hazard problem for the primary dealer in a similar spirit to Gabaix and Maggiori (2015). The second one assumes risk averse arbitrageurs in the spirit of Vayanos and Vila (2021). I specify a primary market friction in a reduced form[22] as in equation (11) rather than with a particular microfoundation to highlight the key transmission mechanism. What is crucial is that a higher overall issuance increases interest rates, irrespective of the particular microfoundation.

[22]Examples in the literature of reduced form debt adjustment frictions in macroeconomic models include Greenwald, Krainer and Paul (2020) and Morelli, Ottonello and Perez (2019) among others.

Therefore, if the government issues more debt, interest rates for the government and the corporate sector will both increase. As I show in Appendix B, the macroeconomic implications are identical if the underlying reason comes from a moral hazard problem for primary dealers or from risk averse arbitrageurs.

6.3 Households

Households enjoy consumption goods C_t and dislike working hours H_t. They optimization problem is the following:

$$\max_{\{C_t, H_t, B_t^{crp}, B_t^{mp}, \{D_t^{t-j}\}_{j=0}^{\infty}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right] \tag{13}$$

They can invest in three separate sets of debt instruments:

- a nominally risk free monetary policy bond B_t^{mp} at rate R_t^{mp} which is in zero net supply and used only for conducting monetary policy,
- a corporate debt claim B_t^{crp} issued by corporations to finance investments, which pays a real risk free return R_t^{crp} and can be purchased on the secondary market at price q_t^{crp},
- a set of government bonds in the secondary market. D_t^{t-j} is the nominal quantity of the government bond issued $t-j$ periods ago and purchased in period t, with $j \geq 0$ and q_t^{t-j} and R_{t-j}^{new} are the corresponding price and coupon rate respectively.

The households earn wage W_t, receive profits Π_t from firm producers, and pay taxes T_t. The nominal budget constraints in a given period is:

$$P_t C_t + B_t^{mp} + q_t^{crp} B_t^{crp} + \sum_{j=0}^{\infty} q_t^{t-j} D_t^{t-j} + P_t T_t = W_t H_t + P_t \Pi_t + \tag{14}$$

$$B_{t-1}^{mp}(1 + R_{t-1}^{mp}) + B_{t-1}^{crp}(1 + R_{t-1}^{crp}) \frac{P_t}{P_{t-1}} + \sum_{j=1}^{\infty} ((1-\delta^d) q_t^{t-j} + R_{t-j}^{new} + \delta^d) D_{t-1}^{t-j}$$

As shown in Appendix F, the solution to the household problem can be characterized by four equations: a labor supply choice, an Euler equation for the monetary policy bond, an Euler equation for the corporate bond, and an Euler equation for the newly issued government bond. The convenient geometric nature of government bonds allows to have only one Euler equation for government debt rather than having to keep track of each past vintage of bonds. This simplifies the equilibrium computation substantially. Since each bond vintage affects the equilibrium due to its impact on the primary market friction, one would in principle need otherwise to keep track of all vintages of public debt. Substituting in the primary market friction from the primary market participants we obtain the following four conditions:

$$C_t^{-\sigma} w_t = \chi H_t^{\eta} \tag{15}$$

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1 + R_t^{mp})}{\pi_{t+1}} \right] \tag{16}$$

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1 + R_t^{crp})}{1 + \Phi_t} \right] \tag{17}$$

$$\frac{(1 + \Phi_t)}{(\delta^d + R_t^{new})} = (1 + R_t^{mp})^{-1} + \mathbb{E}_t \left[\beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}(\pi_{t+1})} (1 - \delta^d) \frac{(1 + \Phi_{t+1})}{(\delta^d + R_{t+1}^{new})} \right] \tag{18}$$

Equation (18) shows that the interest rate on newly issued bonds today R_t^{new} depends on the current primary market frictions, on current monetary policy rates, on the expected bond interest rates tomorrow and on tomorrows frictions in the primary market. The weight on future market conditions is directly proportional to the maturity structure. As δ^d decreases, the weight on future market conditions becomes more important. This can be seen clearly by rewriting the right-hand-side of equation (18) as a weighted average of notional nominal zero coupon bonds not affected by the primary market friction[23] where the weight is a geometrically decaying function of the duration parameter:

$$\frac{(1 + \Phi_t)}{(\delta^d + R_t^{new})} = \sum_{j=1}^{\infty} (1 - \delta^d)^{j-1} \left[1 + R_t^{zerocoupon,t,t+j} \right]^{-j}$$

[23] I define the annualized yield on these zero coupon bonds as the annualized yield equal to the expectation of the j periods ahead nominal SDF, $\left[1 + R_t^{zerocoupon,t,t+j} \right]^{-j} \equiv \mathbb{E}_t \left[\prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) \beta^j \frac{C_{t+j}^{-\sigma}}{C_t^{-\sigma}} \right]$

6.3.1 Public Debt Pricing and Duration to GDP

In this section, I combine the pricing decision by the household, the solution to the problem of the primary market participants, and the public debt structure. This allows to characterize compactly the behavior of the secondary market price of public debt and show the mapping that duration-to-GDP allows between measuring insurance against interest rate risks from a market value approach and savings in debt servicing costs.

Lemma 2 *If public debt has a geometric principal structure with fixed nominal rates and its law of motion can be described by* (4), (5), *and* (6), *primary market participants solve* (12), *and households maximize utility* (13) *subject to* (14)*; then, the secondary market price* q_t^d *of the overall debt stock* D_t *is:*

$$q_t^d = \frac{(\delta^d + R_t^{ave})}{(\delta^d + R_t^{new})}(1 + \Phi_t) \tag{19}$$

Appendix F.1.3 presents the proof by using the Euler equations for long bonds and the aggregate structure of public debt. Equation (19) shows how the secondary market price of overall debt depends on the primary market friction and on the difference between the average interest rate and interest rate on newly issued debt. Specifically, if the current rate is relatively higher than the legacy one, the price of government debt will decline. It will decline proportionally more the higher maturity public debt is. This mirrors, on the secondary market valuation side of public debt, the key channel of fixed rate long debt insurance. When the government has more long term debt and interest rates increase, investors lose, whereas the opposite happens when rates decline. If maturity is short the secondary market price of public debt becomes simply equal to the primary market friction, as for corporate debt.

The two Lemmata 1 and 2 are useful intermediate results to arrive to a key proposition of the analytical part of the model. When we have a permanent change in interest rates, a volume metric of duration, *duration-to-GDP* not only measures how much the market value of public debt will change, but also how interest servicing costs change on legacy debt compared to short debt.

Proposition 1 *Take a model where public debt has a geometric principal structure with fixed nominal rates and its law of motion can be described by* (4), (5), *and* (6), *primary market* *participants solve (12), and households maximize utility (13) subject to (14). If interest rates on newly issued debt increase permanently to R_t^{new} then; the duration of the legacy debt stock to GDP, $Dur_t D_{t-1}/Y_{t-1}^n$, is a sufficient statistic for two phenomena. First, it measures the decline in the market value of legacy public debt in GDP units. Second, it measures the net present value of interest rate savings that the current maturity of public debt allows on legacy debt compared to a one period debt maturity in GDP units.*

Appendix F.1.4 presents the proof. The equivalence result of this proposition yields a theoretical reason for which duration-to-GDP is the correct measure for the empirical exercise. Irrespective if one is interested on the response of the market value of public debt or on the interest rate servicing costs at book value, this metric is useful for both aims.

6.4 Final Good Producers

In this economy there is a perfectly competitive sector with final good producers who combine different retail varieties according to a CES aggregator:

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

This leads to the demand that the final good producers have for different varieties:

$$y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon}$$

6.5 Calvo Retailers

Retailers buy a wholesale good at price P_t^w and use it to produce the retail variety y_{it} with a linear technology that maps one to one the wholesale good to the retail variety. As each variety is differentiated they have market power and face a Calvo friction to change prices. Their real marginal cost $\mathcal{S}_t = \frac{P_t^w}{P_t} = \frac{1}{X_t}$ is the real wholesale price. The probability of not being able to reset prices is equal to θ in each period. This leads to a standard non-linear New-Keynesian Phillips Curve. The full derivation of the New-Keynesian Phillips Curve is standard, but is reproduced for completeness in Appendix F.

$$
\begin{aligned}
K_t^f &= C_t^{-\sigma} Y_t \frac{1}{X_t} \frac{\varepsilon}{\varepsilon - 1} + \theta\beta\mathbb{E}_t \pi_{t+1}^{\varepsilon} K_{t+1}^f \\
F_t^f &= C_t^{-\sigma} Y_t + \theta\beta\mathbb{E}_t \pi_{t+1}^{\varepsilon-1} F_{t+1}^f \\
\frac{K_t^f}{F_t^f} &= \left(\frac{1 - \theta\pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

6.6 Wholesalers

Wholesalers are perfectly competitive and they combine capital K_{t-1}, household labor H_t, and entrepreneurs labor H_t^e to make the wholesale goods:

$$
Y_t = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)}
$$

They sell these goods at nominal price P_t^w to retailers. They pay nominal wage W_t for each unit of household labor, nominal W_t^e for each unit of entrepreneur labor, and real risky return R_t^r to capital owners. The capital share in production is α and Ω is the share of household labor in the overall labor employed by the firm. Technology is stochastic and its process is follows an AR(1) in logs:

$$
\frac{A_t}{A} = \left(\frac{A_{t-1}}{A} \right)^{\rho_A} \exp(\varepsilon_t^A)
$$

The solution to their optimization problem is:

$$
\begin{aligned}
\frac{1}{X_t} \alpha \frac{Y_t}{K_{t-1}} &= R_t^r \\
\frac{1}{X_t} (1-\alpha)\Omega \frac{Y_t}{H_t} &= w_t \\
\frac{1}{X_t} (1-\alpha)(1-\Omega) \frac{Y_t}{H_t^e} &= w_t^e
\end{aligned}
$$

6.7 Capital Producers

Capital producers are separate from entrepreneurs and combine investment resources I_t and legacy undepreciated capital $(1-\delta)K_{t-1}$ they purchase from entrepreneurs in order to sell new capital goods with objective function:

$$\max_{\{K_t, I_t, K_{t-1}\}_{t=0}^{\infty}} Q_t K_t - I_t - Q_t^{old}(1-\delta)K_{t-1}$$

These producers face a production with capital adjustment costs:

$$K_t = I_t - \frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1} + (1-\delta)K_{t-1}$$

By substituting the law of motion/production function and taking the first order conditions we can see the solution to the capital producers problem:

$$Q_t\left(1 - \phi_K\left(\frac{I_t}{K_{t-1}} - \delta\right)\right) = 1 \tag{20}$$

$$\frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}}\right)^2 - \frac{\phi_K}{2}\delta^2 = \frac{Q_t^{old} - Q_t}{Q_t}(1-\delta) \tag{21}$$

Equation (20) is a standard equation for Tobin's real capital price Q_t. With respect to the price of legacy capital Q_t^{old} priced by equation (21), I make the same simplification as BGG. For small perturbations close to the steady state, the price of the legacy capital stock and the newly produced one are the same.

6.8 Entrepreneurs

Entrepreneurs in this economy buy capital from capital producers and invest it in the wholesale firms. They obtain a idiosyncratic return on investments, have linear (risk neutral) utility, and are protected by limited liability. In addition, they also supply inelastically labor to the wholesale firm and obtain a real wage w_t^e. Entrepreneurs exit each period with probability $1-\gamma$ and when they exit they consume the value of their firm (they only consume then). Each entrepreneur j invests capital in the wholesale firms and ex-post she obtains the aggregate return $1 + R_{t+1}^k$ multiplied by a idiosyncratic return ω_{t+1}^j. ω_{t+1}^j is distributed according to an iid log-normal with mean one, $\ln(\omega^j_{t+1}) \sim N\left(-\frac{\sigma^2_{\omega,t}}{2}, \sigma^2_{\omega,t}\right)$. I allow for the standard deviation of the idiosyncratic risk to be stochastic and to follow an AR(1) in logs:

$$\frac{\sigma_{\omega,t}}{\sigma_\omega} = \left(\frac{\sigma_{\omega,t-1}}{\sigma_\omega}\right)^{\rho_{\sigma_\omega}} \exp(\varepsilon^{\sigma_\omega}_t)$$

This is akin to the risk shock of Christiano, Motto and Rostagno (2014). What is crucial for the derivations of the entrepreneur problem with risk shocks is that the volatility of the realization for next period is known in the current period. The aggregate return is given by:

$$1 + R^k_{t+1} = \frac{R^r_{t+1} + Q_{t+1}(1-\delta)}{Q_t}$$

An entrepreneur with wealth N^j_t has to borrow B^j_t at state contingent rate Z^j_{t+1} in order to invest $Q_t K^j_t$. Her balance sheet is:

$$Q_t K^j_t = N^j_t + B^j_t$$

There is costly state verification: only entrepreneurs can observe ex-post ω^j_{t+1}. Lenders pay a monitoring cost μ proportional to the gross return on the invested capital $\mu(1 + R^k_{t+1})Q_t K_t \omega^j_{t+1}$. An entrepreneur pays back debt if the return on her investment $(1 + R^k_{t+1})Q_t K_t \omega^j_{t+1}$ is higher than the cost of servicing debt $Z^j_{t+1}B^j_t$ otherwise defaults and the lender recovers $(1-\mu)(1 + R^k_{t+1})Q_t K_t \omega^j_{t+1}$. There exists a threshold $\bar{\omega}^j_{t+1}$ above which the entrepreneur pays a fixed amount and below which there is a recovery value. The return on a loan is therefore:

$$(1 + R^j_{t+1})B^j_t = \begin{cases} Z^j_{t+1}B^j_t & if\ \omega^j_{t+1} \geq \bar{\omega}^j_{t+1} \\ (1-\mu)(1 + R^k_{t+1})Q_t K_t \omega^j_{t+1} & if\ \omega^j_{t+1} < \bar{\omega}^j_{t+1} \end{cases}$$

The return on this loan is in expected term (with respect to the idiosyncratic shock) but given a realized return R^k_{t+1}, it must be equal to the outside option of lenders R^{crp}_t (which is the same across all entrepreneurs as there is a large mass of them):

$$(1 + R^{crp}_t)(\kappa^j_t - 1) = (1 + R^k_{t+1})\kappa^j_t(\Gamma(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) - \mu G(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}))$$

Where we define leverage as $\kappa_t^j \equiv \frac{Q_t K_t^j}{N_t^j}$ and:

$$\Gamma(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}) \equiv \int_0^{\bar{\omega}_{t+1}^j} \omega f(\omega, \sigma_{\omega,t}) d\omega + \bar{\omega}_{t+1}^j (1 - F(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}))$$

$$G(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}) \equiv \int_0^{\bar{\omega}_{t+1}^j} \omega f(\omega, \sigma_{\omega,t}) d\omega$$

$$F(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}) \equiv \int_0^{\bar{\omega}_{t+1}^j} f(\omega, \sigma_{\omega,t}) d\omega$$

These functions represent the expected share of the gross value of the firm going to the lender $\Gamma(\bar{\omega}_{t+1}^j, \sigma_{\omega,t})$, the expected share of the gross value of the firm on which monitoring costs have to be paid $G(\bar{\omega}_{t+1}^j, \sigma_{\omega,t})$, and the probability of default $F(\bar{\omega}_{t+1}^j, \sigma_{\omega,t})$. These functions are characterized in Appendix F with all their derivatives. Notice that the contract with a fixed real rate as an outside option for the entrepreneurs is not necessarily the optimal one as risk neutral entrepreneurs would insure risk averse household investors who face labor income risk, as argued by Carlstrom, Fuerst and Paustian (2016) and Dmitriev and Hoddenbagh (2017) among others. I take the route proposed by Christiano, Motto and Rostagno (2014) and keep this contract out of empirical relevance, as the alternative optimal contracts would have an amount of indexation not seen in the data. [24]

Entrepeneurs maximization problem

Entrepreneurs maximize their expected wealth, protected by limited liability, subject to the participation constraint of the lenders. They choose a combination of leverage κ_t^j and default cut-off $\bar{\omega}_{t+1}^j$:

$$\max_{\{\kappa_t^j, \bar{\omega}_{t+1}^j\}} \mathbb{E}_t \max \left[(1 + R_{t+1}^k) \kappa_t^j N_t^j (\omega_{t+1}^j - \bar{\omega}_{t+1}^j), 0 \right]$$

$$s.t.$$

$$(1 + R_t^{crp})(\kappa_t^j - 1) = (1 + R_{t+1}^k) \kappa_t^j (\Gamma(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}) - \mu G(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}))$$

[24] As a robustness check, I show how the model results do not hinge on the outside option being real debt and go through with nominal debt as in Christiano, Motto and Rostagno (2014), that is, results go through when we allow for a Fisherian debt deflation channel for corporate debt, as shown in Appendix G.4.

Thanks to constant returns to scale in the production function the solution to this problem is the same irrespective of current wealth level N_t^j. This implies that the leverage and threshold choices are the same for each entrepreneur and we can aggregate to a representative entrepreneur. We define the risk spread as ratio of returns as $(1+s_{t+1}) \equiv \frac{(1+R^k_{t+1})}{(1+R^{crp}_t)}$ and derivatives of the helping functions as $\Gamma_{\omega,t+1} \equiv \Gamma_\omega(\bar{\omega}^j_{t+1}, \sigma_\omega) = \frac{\partial \Gamma(\bar{\omega}^j_{t+1}, \sigma_\omega)}{\partial \bar{\omega}^j_{t+1}}$ and $G_{\omega,t+1} \equiv G_\omega(\bar{\omega}^j_{t+1}, \sigma_\omega) = \frac{\partial G(\bar{\omega}^j_{t+1}, \sigma_\omega)}{\partial \bar{\omega}^j_{t+1}}$. In Appendix F, I show how the entrepreneur choices can be summarized with two non-linear conditions:

$$0 = \mathbb{E}_t \left[(1+s_{t+1})\kappa_t \left(1 - \Gamma_{t+1}\right) - \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}} \right] \tag{22}$$

$$\frac{\kappa_t - 1}{\kappa_t} = (1+s_{t+1})(\Gamma_{t+1} - \mu G_{t+1}) \tag{23}$$

In the first condition, entrepreneurs trade-off expected gains in terms of additional returns to increasing leverage (the first term in square brackets) against the expected cost to increasing default, and therefore increasing funding costs (the second term in square brackets). This condition holds with expectations as leverage is chosen before shocks realize. The second condition is the participation constraint for lenders who are guaranteed a certain return R_t^{crp}. It must hold ex-post, with the choice of the state-contingent threshold to enforce it. Taken together, these conditions imply a *positive monotonic relationship between leverage and the risk spread.*[25]

To finish the description of the entrepreneur sector we need to specify how their wealth and consumption behave. Equation (24) describes the aggregate law of motion of entrepreneurs wealth. Entrepreneurs wealth in the current period is the sum of the return on last period wealth for entrepreneurs who do not exit and the wage that they earn by working at the wholesale firms. Similarly, equation (25) specifies the consumption for the entrepreneurs who exit, who simply consume the current value of their firm. Finally, equation (26) specifies the return on entrepreneurs equity. This return is sensitive to the difference in corporate bond rates and return on capital invested due to their levered position. Moreover, it is lower

[25] This becomes apparent when linearizing them, as shown in Appendix F.

when monitoring costs are high.

$$N_t = \gamma(1+R_t^e)N_{t-1} + w_t^e \tag{24}$$

$$C_t^e = (1-\gamma)(1+R_t^e)N_{t-1} \tag{25}$$

$$(1+R_t^e) = \big((R_t^k - R_{t-1}^{crp})\kappa_{t-1} + (1+R_{t-1}^{crp}) - \mu(1+R_t^k)\kappa_{t-1}G(\bar{\omega}_t, \sigma_{\omega,t-1})\big) \tag{26}$$

6.9 Central Bank

The Central Bank sets monetary policy according to a Taylor rule:

$$\left(\frac{1+R_t^{mp}}{1+R^{mp}}\right)^{1+R^{mp}} = \left(\frac{1+R_{t-1}^{mp}}{1+R^{mp}}\right)^{\rho^{mp}(1+R^{mp})} \left[\left(\frac{\mathbb{E}_t\pi_{t+1}}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{(1-\rho^{mp})} \exp(\varepsilon_t^{mp}) \tag{27}$$

ρ^{mp} controls the interest rate inertia in the rule, ϕ_Y the response of interest rates to output, and ϕ_π to inflation. ε_t^{mp} is a monetary policy shock with mean zero.

6.10 Market Clearing

In order to close the model we need the goods market clearing condition

$$Y_t = C_t + C_t^e + I_t + G_t + \mu G(\bar{\omega}_t, \sigma_\omega)(1+R_t^k)N_{t-1}\kappa_{t-1}$$

Output in the economy is consumed by X and X, or X, invested or XX

We assume an inelastic labor supply for entrepreneurs:

$$H_t^e = 1$$

6.11 Equilibrium, Steady State, and Log-linearization

The equilibrium, the steady state, and the log-linearization of the equilibrium conditions around a zero inflation steady state are all presented in Appendix F. An important feature of the linearization is a convention to interpret more directly impulse response functions in light of the empirical results. Interest rate variables are linearized so that $\hat{R}_t^{crp} = R_t^{crp} - R^{crp}$ in order to interpret results as percentage point deviations. This includes the two spread variables Φ_t and s_t. Debt quantity variables are linearized over steady state GDP so that $\hat{D}_t = \frac{D_t - D}{Y}$ in order to interpret the results as changes in debt over GDP. Indeed, the main economic channel of debt supply goes through a volume effect and a standard percent deviation would not capture it. Finally, I log-linearize all other variables so that $\hat{C}_t = \frac{C_t - C}{C}$.

7 Model Calibration and Results

7.1 Calibration

The calibration of the model is presented in Table 2. Most parameters are standard. β, α, δ, ϕ_K, η, X, and Ω all come from BGG. ϕ_π and ϕ_Y are standard values for a forward looking Taylor rule, with $\phi_Y = 0.125$ implying an increase of 50 basis points in the policy rate following a 1 percent decline in output, in line with the empirical evidence provided by Smets and Wouters (2007) and Herbst and Schorfheide (2015). The monetary policy persistence ρ_{mp} and the price stickiness parameter θ are also in the empirically plausible range of Smets and Wouters (2007) and Herbst and Schorfheide (2015). The inverse of the elasticity of intertemporal substitution σ is equal to 2 which is in the range of empirical estimates surveyed by Attanasio and Weber (2010).

The primary market friction parameter ζ is a key parameter. Andreolli (2021) estimates a range between 9 and 20 basis points using exogenous variation in the supply of public debt. I pick a value of 10 basis points. This corresponds to the impact of a 1 percentage point increase on overall debt over GDP issuance on corporate and government bonds rates. Andreolli (2021) estimates news shocks to the supply of marketable public with a high frequency identification. The paper uses the high frequency government bond future price changes around announcements of the Debt Management Office following budget speeches by the UK Chancellor of the Exchequer. These announcements are orthogonal to the fiscal policy stance and other confounding factors as the Chancellor already discussed the overall fiscal stance and resulting economic activity projections during the budget speech. Interestingly, similar estimates can be found in studies employing very different methodologies. Greenwald, Krainer and Paul (2020) presents a similar friction as a holding cost of debt (it is specified on the stock of debt but they have one period bonds only), and their calibration of the

Table 2: Model Calibration

Parameter	Value	Description
β	0.99	Time Preference
α	0.35	Capital Share
δ	0.025	Depreciation Rate
ϕ_K	10	Capital Adjustment
η	1/3	Inverse of the Frish Elasticity
σ	2	Risk Aversion
θ	0.65	Calvo Degree of Price Stickiness
Ω	0.99	Share of Labor Income Accrued to Entrepreneurs
ζ	0.1	Primary Market Friction Elasticity
ϕ_π	1.5	Coefficient of the Taylor Rule on Expected Inflation
ϕ_Y	0.125	Coefficient of the Taylor Rule on Output
τ^T	2	Tax Policy Parameter
δ^d	0.05 or 1	Debt Maturity Parameter
ρ_A	0.999	Persistence of the Technology Process
s_A	0.1	Standard Deviation of the Technology Process
ρ_G	0.95	Persistence of the Government Spending Process
s_G	0.1	Standard Deviation of the Government Spending Process
ρ_{σ_ω}	0.97	Persistence of the Risk Shock Process
s_{σ_ω}	0.1	Standard Deviation of the Risk Shock Process
ρ_Φ	0.5	Persistence of the Primary Market Friction Process
s_Φ	0.1	Standard Deviation of the Primary Market Friction Process
ρ_{mp}	0.8	Smoothing of Monetary Policy Process
s_{mp}	0.1	Standard Deviation of the Monetary Policy Process
κ	2	Leverage
s	0.0025	Risk Spread
Φ	0.0025	Issuing Friction Spread
$F(\bar{\omega}, \sigma_\omega)$	0.0075	Default Rate
$\bar{D}$	1.6	Marketable Public Debt over Quarterly GDP
$\bar{G}$	0.2	Government Expenditures over GDP
X	1/1.1	Inverse of Markups
π	1	Inflation In Steady State

Notes: The first column shows the parameter or steady state value calibrated. The structural parameters for monitoring costs μ, probability of not exiting for entrepreneurs γ, and average standard deviation in idiosyncratic risk for entrepreneurs σ_ω are computed with the aid of structural parameters and calibrated steady state values for the risk spread s, primary market friction spread Φ, leverage κ, and default rate $F(\bar{\omega}, \sigma_\omega)$. Similarly, the structural parameter for the elasticicity of substitution across varieties ε is computed one to one from the steady state value for the inverse of markups X. The calibration proposes two different values for the parameter governing the maturity of public debt δ^d, either the fraction of principal that need to be refinanced each quarter is 1 (one quarter debt), or the fraction of principal that need to be refinanced each quarter is 0.05 (around four years debt, as its historical average).

exponential parameter of 25 would imply a ζ of 16 basis points. Bigio, Nuño and Passadore (2019) estimate on Spanish sovereign bond data that an increase in one percent over GDP in issuance correlates with a lowering of the primary market price relative to the secondary market price of the same security by 8 to 56 basis points. Lou, Yan and Zhang (2013) show on US treasury data that an increase in one percent over GDP in issuance is associated with 14 to 23 basis points increase in the auction price.

For the tax policy response parameter τ^T, I pick a value in the range of empirical estimates presented by Davig and Leeper (2007) and Herbst and Schorfheide (2015). This is also close to the value (1.5) that Eusepi and Preston (2010) use. Specifically, a value of 2 implies that following an increase by 1 percentage point of public debt to GDP, taxes increase by 0.0205 percentage points of GDP. Davig and Leeper (2007) estimate a value of 0.0094 in reduced form with a Markov switching process. A value of 2 implies that following an increase by 1 percent of public debt taxes increase by 0.1516 percent; Herbst and Schorfheide (2015) estimate a 90 percent confidence interval of 0.14 to 0.49 with a structural DSGE Bayesian estimation. My choice puts me slightly on the high side of the reduced form estimates and on the low side of the structural estimates.

I use long run averages for steady state value of government expenditures $\bar{G}$. For the steady state value of public debt $\bar{D}$, I use a value of 40% of annual GDP, which is close to the long-run average of marketable public debt held by the public as shown by Hall and Sargent (2011).

I compare two economies, one with public debt with a Macaulay duration close to its long run average of about 4 years, with $\delta^d = 0.05$ and one with all debt being one period ahead with $\delta^d = 1$.

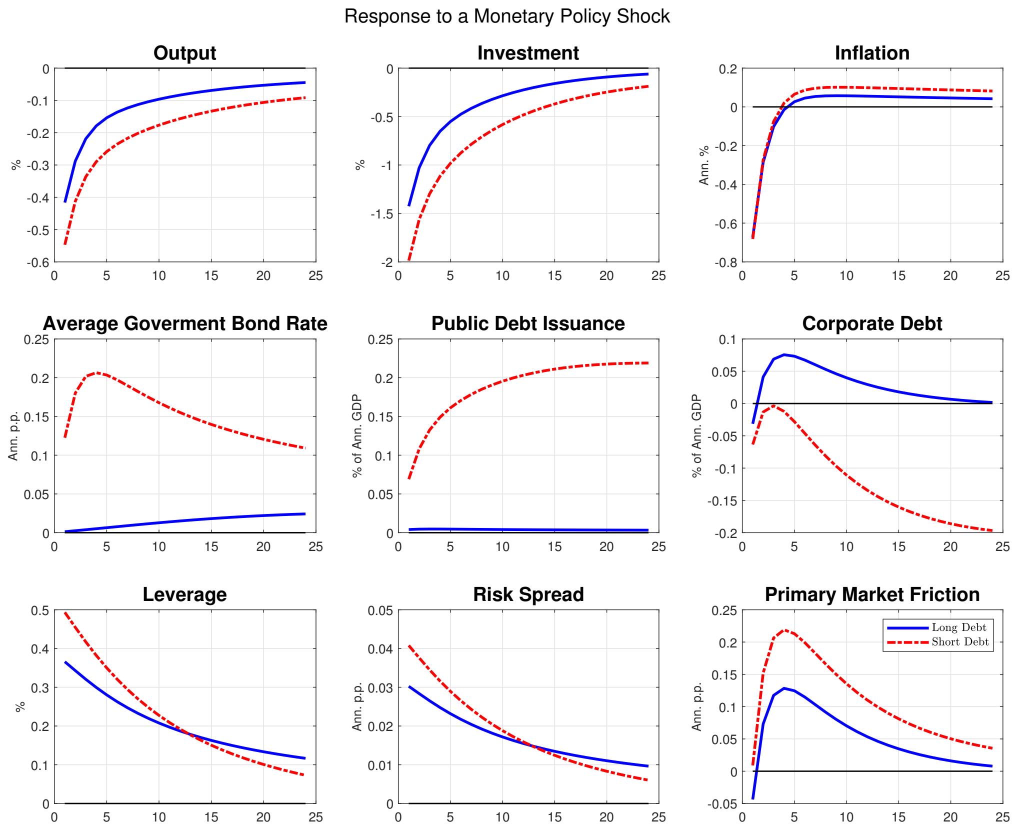
For the financial accelerator, I use the same values as BGG for leverage κ and default rate $F(\bar{\omega}, \sigma_\omega)$. For the steady state spread I assign the same overall value as BGG of 200 basis points on an annualized term, but I split this in half for the risk spread s and the primary market friction spread Φ. The resulting structural parameters are therefore very close to BGG at $\mu = 0.0593$, $\gamma = 0.9816$, and $\sigma_\omega = 0.2705$. [26]

7.2 Baseline Results

The Impulse Response Functions of key variables after a 25 basis points annualized monetary policy tightening are presented in Figure 7. The responses with public debt at its histori-

[26] Note that for the default steady states estimates vary considerably in the literature. If one uses Compustat data the average annual bankruptcy frequency (either Chapter 7 or 11) was 0.96% between 1980 to 2014 from Corbae and D'Erasmo (2017). From S&P data on corporate bonds the average default for all bonds has been 1.48% and for non-financial bonds 1.81% for the period 1981-2019. On the other hand bank lending delinquencies have been much higher than that and data from the Dallas Fed points to an average quarterly delinquency rate of 2.68% for commercial and industrial loans in the period 1987Q1-2020Q2. Glennon and Nigro (2005) estimate for small business loans an average of 17% default rate per loan across its life (and estimate a 4.18% default rate on the first year of life of the loan) in the period 1983-1998.

Figure 7: Baseline Model Impulse Response Functions



Notes: The IRFs present the response to an annualized 25 basis points monetary policy shock. The solid blue line presents the IRFs in an economy with the maturity of public debt being at its historical average of around 4 years ($\delta^d = 0.05$). The dot-dashed red line presents the IRFs in an alternative economy with the maturity of public debt being at one quarter ($\delta^d = 1$).

cal duration of around 4 years are described by the solid blue line and the responses in a counterfactual world with only one period public debt by the red dash-dot line. The average responses are similar to a baseline New Keynesian model with a financial accelerator. Output declines on impact by 40 basis points, investment by 1.4 percent, inflation by 65 basis points, the risk spread increases by about 4 basis points.

If we turn to the responses of the public debt, in the blue line case (historical duration), we see that the average interest rate on government debt increases mildly, as does public debt issuance. This happens for two reasons: the main one is that all legacy debt has its rate fixed and does not need to be refinanced, a second reason is that rates further on the yield curve respond less than short rates, so overall the interest rate on newly issued long debt respond weakly to the temporary nature of a monetary policy shock. Finally, the primary market friction increases by 0.1 percentage points. This is due to higher corporate issuance as the government does not need to refinance. This implies that the path of the primary market friction follows the sign and path of corporate debt in the long debt scenario.

I now turn to the differential impact under a counterfactual low debt maturity scenario[27] presented in the red dot-dashed line. The key difference with the long-debt case is that, in this scenario, the government borrowing profile is not insured against the interest rate increase. As the average interest rate is equal to the interest rate on newly issued government bonds, we can see that it increases by the same order of magnitude as the 25 basis points monetary policy shock. Consequently, the government needs to borrow more than 0.2 percent of GDP each quarter, a non negligible number for a small temporary monetary policy shock, which is in line with the empirical evidence presented in the first row of Figure 4. This higher issuance is what sets in motion the higher primary market friction; and from there the economic effect unfolds as in a standard financial accelerator investment channel of monetary policy. The entrepreneur is hit by higher borrowing costs which lower her net worth and increase her leverage and risk profile. This in turn makes her borrow and invest less, resulting in lower output.

A few features of this model are worth pointing out. First, this insurance channel does not have a strong impact on inflation, with inflation responding in a similar manner under the two maturity regimes, in line with the empirical results. The technical reason is that markups drive inflation in the New Keynesian Phillips Curve and, with multiple factors of production, there is not a one-to-one mapping from output to markups. The difference in output across the two regimes is offset by the change in the marginal productivity of capital. Intuitively, when public debt has a shorter maturity, the primary market works less well. Therefore, a change in interest rate hits more strongly the return on capital (because of the effect on the entrepreneurs worth) and output (because of investment), but less so markups, so that inflation behaves in a similar way in the two regimes.

The second feature pertains to the response of corporate debt under the two regimes. The model provides a testable implication on the relative response of corporate debt under the two regimes, not on the absolute response. The prediction is that corporate debt responds more negatively under a low maturity regime due to crowding-out from the primary market friction following higher issuance by the government. On the other hand, the absolute

[27] This is counterfactual in the data, not in most macroeconomic models!

response depends on the parametrization. The reason is that, following a contractionary monetary policy shock, entrepreneurs net worth declines but leverage increases, making the absolute response of corporate debt ambiguous. In the current calibration, under a long debt regime, corporate debt increases, similarly to the original BGG paper. Despite not being a discriminating prediction, the empirical results support the sign of the responses of corporate debt under the two regimes presented in Figure 7. In the fourth row of Figure 4, the unconditional response of corporate debt issuance has a positive point estimate and the response under a low duration regime is strongly negative[28].

Overall, output responds 13 basis points more under a low maturity regime, corresponding to a 31% stronger effect of monetary policy[29]. This happens with a relatively moderate increase in the spread corporate pays over the risk free rate, of less than 10 basis points. Moreover, as in the empirical results, the difference in investment response across maturity regimes is higher than the difference in output. Therefore a seemingly small primary market friction has relatively large macroeconomic effect, something we explore in more details below.

I compare the results from this theoretical exercise with the empirical ones. In Appendix C.8, I compute the difference between duration to GDP at its historical mean against a theoretical duration to GDP with an hypothetical quarterly debt. This allows to interpret the no-interaction coefficient $\beta_{1,h}$ as the response to a monetary policy shock if only quarterly maturity debt were issued. If we normalize the response of a contractionary monetary policy shock that increases the Fed funds rate by 25 basis points (by dividing by 4 the coefficient from the LP-IV), we can see that at the unconditional peak effect, monetary policy reduces industrial production by 61% more under a quarterly debt scenario, or 26 basis points. This means that the structural model can explain more than 50% of the percentage difference, highlighting the importance of the financial channel, but implying that other channels could also have a role. The model has the benefit of matching all the relative empirical responses which other channels cannot as discussed in Section 5. Specifically, following a contractionary monetary policy shock, under a longer duration, output, investment, and corporate debt are higher, inflation is about the same, and public debt issuance, leverage, and the relative price

[28] Greenwald, Krainer and Paul (2020) provide independent empirical evidence that corporate debt on average can increase following a contractionary shock.

[29] Appendix G.1 presents the results for the whole set of impulse response functions. In Appendix G.2, I show that the results are robust to varying the calibration of the parameters and I show that the change in monetary policy strength is approximately linear in the maturity parameter δ^d.

of corporate debt are lower.

7.3 Complementary between the Financial Accelerator and the Primary Market Friction

In order to generate the differential impact of monetary policy under public debt maturity regimes one needs both the financial accelerator and the primary market friction. In this subsection and in Table 3, I show how complementarity between the two frictions is key for the results. This complementarity highlights how small frictions on the primary market, can have a small direct (partial equilibrium) effect but a large indirect (general equilibrium) effect, because it interacts with the financial accelerator.

In a version of the model without the primary market friction, that is ζ is zero, the maturity structure of public debt does not matter. This is by assumption as we assumed no distortive taxation and a representative consumer in order to single out the financial market channel of transmission, this can be seen in the first row of Table 3.

Turning off the financial accelerator is more involved as the standard setting has three ingredients: limited liability, monitoring costs, and no equity issuance. If we eliminate all three we would get back to the Modigliani-Miller irrelevance of the capital structure. However, with a primary market friction on debt but not on equity we would get only equity financing, assuming that the primary market friction hits only debt issuance. For this reason, I shut off completely the entrepreneur sector. I assume that all the capital stock is financed with corporate debt which is hit by the primary market friction. Operationally, I set $\hat{\kappa}_t$, $\hat{s}_t$, $\hat{\omega}_t$, $\hat{N}_t$, and $\hat{R}^e_t$ all to zero and set $\mathbb{E}_t(\hat{R}^k_{t+1}) = \hat{R}^{crp}_t$. In this case, the risk absorption rests on the household, there is no entrepreneurial sector to absorb ex-post aggregate risk. But due to the certainty equivalence at a first order approximation it is not priced in the corporate debt rate. Note that I am stacking the cards against a complementary between the financial accelerator and the primary market friction, as here all the capital stock is subject to the primary market friction since there is no equity financing (in the baseline 50% of the capital stock is backed by equity). The resulting outcome shows the complementarity between the two frictions: the percentage difference in output across maturities is 12%, or 1.4 basis points on impact as shown in the row column of Table 3. The reason is that there is no agent with limited risk bearing capacity whose wealth is hit by higher interest costs when the government is issuing more debt[30].

Finally, in the third row of Table 3 we can see that the baseline theoretical result is higher than the sum of the two above. The financial friction increases funding costs which increases the probability of default and this feeds back into the cost of funds. Because it determines how much one can borrow, the financial accelerator friction magnifies the financing channel friction, which affects the cost of funds. The general equilibrium effects of this complementarity are non trivial.

Table 3: Complementarity Between the Financial Accelerator and the Primary Market Friction.

Percent Difference in Output Response High-Low Maturity	
No Primary Market Friction	0%
With Primary Market Friction and No Financial Accelerator	12%
With Primary Market Friction and Financial Accelerator	31%

Notes: This table shows how much more effective is monetary policy on output on a short maturity regime compared to a long maturity regime. In the first experiment, the comparison is in a case without the primary market friction. The second experiment shuts off the financial accelerator and keeps the primary market friction, this is achieved by setting $\mathbb{E}_t(\hat{R}^k_{t+1}) = \hat{R}^{crp}_t$ and setting to zero all entrepreneurs related variables as leverage, risk spread, entrepreneurs wealth, default threshold, and return on equity. The experiment presented in the third row is the baseline model with both the financial accelerator and the primary market friction present.

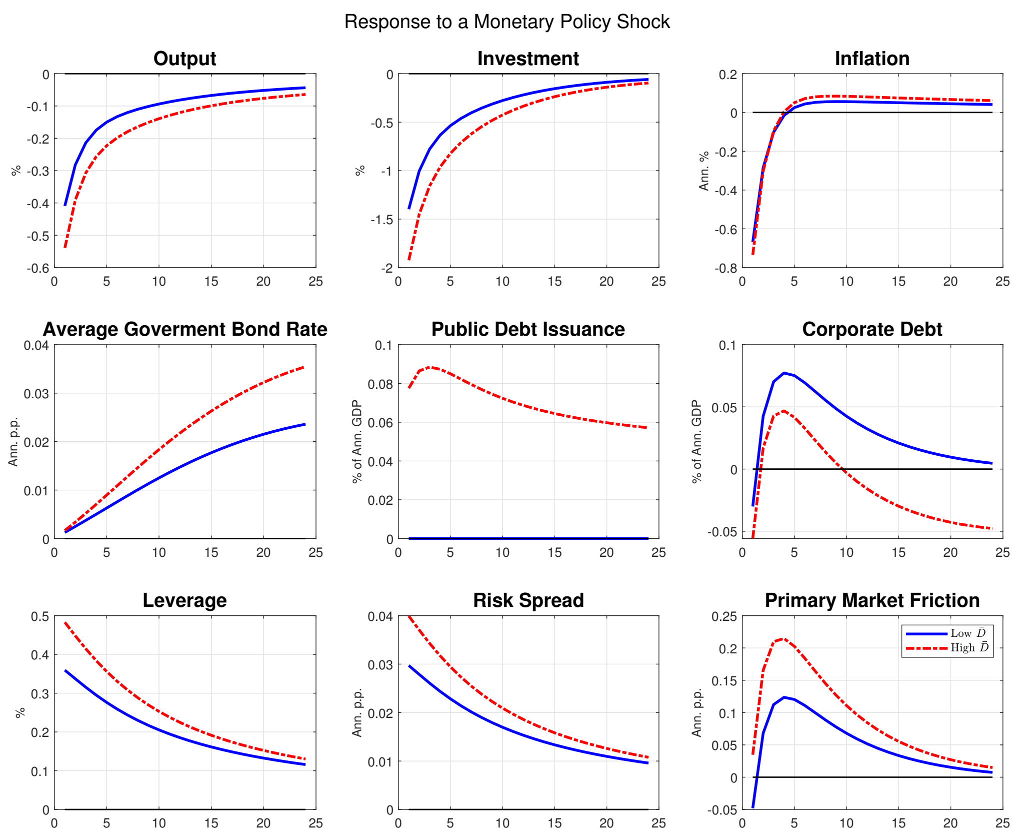
7.4 Maturity is Key

The key driving force for the results is the variation in the volume of new public debt issued following a monetary policy shock. A natural question therefore is whether the maturity structure of public debt is crucial or only a change in the stock of public debt would suffice to generate the results. At a qualitative level an increase in the stock of public debt holding maturity constant has the same effect as lowering the maturity structure holding the debt stock constant: they would both make monetary policy more effective on output. However, to reach the same quantitative effect the increase in public debt needed to match the results

[30]From this exercise, we see that the complementarity in the two frictions relies on the risk bearing capacity of the entrepreneur. As additional evidence, I show an intermediate case of financial accelerator. I keep limited liability and no equity issuance but I make bankruptcy costless, that is, I turn off monitoring costs ($\mu = 0$) and keep the same values of the other structural parameters (γ and σ_ω). The percentage difference in output is 13%, or 5 basis points on impact across maturity regimes. The absence of bankruptcy costs mutes the impact of the government additional issuance under a low maturity case as higher leverage does not feed into higher borrowing costs.

with maturity would be implausibly high: to achieve a 30% higher effectiveness of monetary policy we would need to go from 0% public debt over annual GDP to 700% public debt over annual GDP, while holding maturity at its historical average of 4 years, as shows in Figure 8. The same result can be achieved by lowering maturity from 4 years to 1 quarter holding marketable public debt at its historical average of 40%. This is an important result: *crowding out non financial firms comes not so much from high debt levels but from large roll-overs of short maturity debt*: this is the gross issuance flow that matters.

Figure 8: Model Impulse Response Functions with Different Public Debt Levels



Notes: The IRFs present the response to an annualized 25 basis points monetary policy shock. The solid blue line presents the IRFs in an economy with no public debt to GDP in steady state. The dot-dashed red line presents the IRFs in an economy with high public debt to GDP in steady state (700%). In both scenarios, the maturity of public debt is at its historical average of around 4 years ($\delta^d = 0.05$).

8 Conclusion

This paper shows that the effect of monetary policy on output is greatly attenuated when public debt has a long maturity. Going from the average historical duration of US debt to very short term debt *doubles* the impact of a rise of the policy rate on output. In contrast, the transmission mechanism on inflation is not affected much by the maturity structure. The same results hold for the United Kingdom, which has a very different maturity time profile than the United States. After providing a novel narrative account of the maturity choices, numerous robustness checks, and an exploration of different possible channels, this paper shows that these striking facts can be explained by the *financing channel* of monetary policy transmission.

Long maturity debt acts as an insurance mechanism. Following a contractionary monetary policy shock the government experiences the equivalent of an insurance payout as it does not need to refinance debt at the new higher rate. It does not react directly to this payout with more discretionary spending but instead borrows relatively less. In presence of even a small financial friction on the primary debt issuance market, this leads to higher corporate borrowing and investment and lower spreads: there is less crowding-out. This is a quantitatively important effect and the implications for monetary policy transmission are large.

A standard financial accelerator model à la Bernanke, Gertler and Gilchrist (1999) with a government issuing fixed rate long dated debt and a friction on the primary market for debt issuance can account for the results. A key parameter to calibrate is the primary market friction. I rely on Andreolli (2021) which provides estimates of the pure effect of shocks to the public debt supply using high frequency identification around Debt Management Office announcements. Despite being small in magnitude, this primary market friction matters and is at the root of the *financing channel* of monetary policy transmission.

When interest rate increases, the small primary market financial friction increases funding costs, which raises the probability of default and this feeds back into the cost of funds. Because it determines how much one can borrow, the financial accelerator friction magnifies the financing channel friction, and in turn affects the cost of funds. The general equilibrium effects of this complementarity are non trivial.

The maturity structure itself is key to explain the results. As explained, going from the average historical duration of US debt to very short term debt *doubles* the impact of a rise of the policy rate on output. To obtain the same quantitative difference in responses to a monetary policy by varying only the stock of debt, fixing its maturity at its historical mean, one would need to go from a public debt of 0 to over 700% of GDP. This is an important result: crowding out non financial firms comes not so much from high debt levels but from large roll-overs of short maturity debt. It is the gross issuance flow that matters, i.e. the *financing channel*.

The maturity of public debt can provide useful risk management tool for policy makers and can help in the coordination between the fiscal and the monetary policy makers. If governments push upward the maturity of their debt when interests are low, they can make it easier for central banks to react against sudden increases in inflation.

More generally, while I focus in this paper on variation in interest rates coming from monetary policy shocks, the role for maturity structure of public debt presented here would work for interest rate changes coming from other sources. With long maturity debt, a government will be insured from interest rate increases and will suffer from interest rate declines. In the past 30 years interest rates have been declining across the rich world. This implies that, government borrowers who borrow relatively longer, as European ones, did not benefit from interest rate declines as much as sovereign borrowers with shorter profiles, as the United States. In light of the results of this paper on the financing channel, this can spill over to the corporate sector. Similarly, this financing channel can account for some of the heterogeneity in transmission of US rate increases to emerging markets, with borrowers with shorter sovereign debt maturities being affected relatively more than those with longer dated debt.

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A Data Construction

A.1 Public Debt Data

I construct a monthly dataset of public nominal marketable debt promises at market prices for each future month, that is, the value, today, of the government promises for j months ahead from bond level data. Following Hall and Sargent (2011) methodology, the budget constraint of a government which can issue nominal and inflation linked debt at different maturities reads:

$$\frac{1}{p_t}\sum_{j=0}^{n-1} q_{t,j} d_{t-1,j+1} + \sum_{j=0}^{n-1} \bar{q}_{t,j}\bar{d}_{t-1,j+1} = PS_t + \frac{1}{p_t}\sum_{j=1}^{n} q_{t,j} d_{t,j} + \sum_{j=1}^{n} \bar{q}_{t,j}\bar{d}_{t,j}$$

Where $q_{t,j}$ is the amount of nominal currency in period t that one needs to purchase one unit of nominal currency in period $t+j$, $d_{t,j}$ is the amount of these promises that the government has in period t to pay in period $t+j$, $\bar{q}_{t,j}$ is the amount of period t goods that one needs to purchase one unit of period $t+j$ goods, $\bar{d}_{t,j}$ is the corresponding amount, n is the maximum maturity of public deb, PS_t is the primary surplus in period t goods. For both nominal and inflation-linked debt, as governments issue bonds which are not zero coupon, I strip the coupons and create the equivalent series for the marketable part of $d_{t,j}$ in each month for future promises also dated monthly. With respect to the prices, I use the continuously compounded zero coupon nominal and real yield curve data and compute the zero coupon prices with $q_{t,j} = e^{-\frac{j}{12}r_{t,j}}$ and $\bar{q}_{t,j} = e^{-\frac{j}{12}\bar{r}_{t,j}}$ for the real yield curve. j is divided by 12 to convert it to annual frequency and $r_{t,j}$ is the appropriate interest rate for the nominal yield curve and $\bar{r}_{t,j}$ is the appropriate interest rate for the real yield curve.

For the US, I use bond level CRSP data as Hall and Sargent (2011) at a monthly frequency to compute the quantity series and I use the parameters from Gürkaynak, Sack and Wright (2007) to compute the yield curve. For the UK, I use the quantity data from Ellison and Scott (2017) and official Bank of England yield curve data. As the original yield curve data is at frequencies lower than monthly in the UK data, I interpolate the series within, moreover I assume a constant rate for maturities longer than the ones present in the original data, as I need to price consols. I construct the same series for inflation linked debt from the same sources.

Macaulay duration measures the percent decrease in the market value of this debt following an infinitesimal change in interest rates uniformly across the yield curve $dr = dr_{t,j}$, for $j > 0$. On the other hand, duration-to-GDP measures the same change but not in percent terms but in percentage points of GDP, thereby capturing the overall insurance amount relevant for debt management.

$$MacDur_t = -\frac{\frac{\partial\left(\sum_{j=1}^{n} q_{t,j} b_{t,j}\right)}{\partial r}}{\sum_{j=1}^{n} q_{t,j} d_{t,j}} = \frac{\sum_{j=1}^{n} \frac{j}{12} q_{t,j} b_{t,j}}{\sum_{j=1}^{n} q_{t,j} d_{t,j}}$$

$$DurGDP_t = \frac{\sum_{j=0}^{\infty} \frac{j}{12} q_{t,j} b_{t,j}}{GDP_t}$$

The analysis is based on marketable debt, but this should not be a big concern as Hall and Sargent (2011), Ellison and Scott (2017) show that in the period I consider, most of US and UK public debt was marketable, the last periods in which non marketable debt plaid an important role as during the World Wars and during the Korean War for the US. Moreover, I exclude treasury bills which are not present in the data sources I use. This is not problematic as these are very short maturity debt which would get a small weight in the duration-to-GDP measure, as an example a 3 month promise is weighted by 0.25 in duration-to-GDP, whereas a 10 years promise is weighted by 10. These caveats show another advantage of the duration-to-GDP measure compared with Macaulay duration, whereas they are likely to be minor issues for duration-to-GDP and we know that all time series points are lower bonds on the true duration-to-GDP, the same cannot be said for the Macaulay duration. In duration-to-GDP we divide by GDP, which is not affected by issues with measurement of debt, on the other hand Macaulay duration is, as the denominator is computed with debt data as well, so that the exclusion of treasury bills would become problematic.

In quarterly regressions, I divide nominal debt by nominal GDP for that quarter, however this is not feasible for the monthly regressions. There, I need to have a measure of GDP which does not depend on future GDP in order to not create spurious regressions, so interpolation is not an implementable path. In the baseline specification, the nominal income measure is a random walk forecast for nominal GDP, that is for the last month in a quarter I use nominal GDP of that quarter[31], for the other two months I use the nominal GDP for the last quarter[32].

A.1.1 US Data Description

Figure 1 in the main text presents the Macaulay duration and the duration-to-GDP for marketable nominal public debt held by the public for the US. In this section, I present similar data with alternative assumptions and with different cuts of the data.

First of all, Figure A.1 presents the nominal marketable public debt over GDP held by the public. This figure can be seen as the ratio between duration-to-GDP over Macaulay duration presented in Figure 1. If we compare the behavior of public debt over GDP with duration-to-GDP, we can see how they correlate, but not one to one, duration-to-GDP increased much faster in the mid eighties and declined much less than public debt in the early aughts. Following we the increase in public debt due to the financial crisis we see a flattening of public debt, but duration-to-GDP keeps raising due to the lengthening in maturities (this holds even after netting out FED holdings).

Figure A.2 present how duration-to-GDP looks with alternative measures. The baseline blue solid line shows the data as Figure 1, it is the measure constructed from marketable public debt held by the public at market value. The red long-dashed line *face value* shows the same measure with debt at face value, that is, each debt promise $b_{t,j}$ is not deflated with the yield curve data $q_{t,j}$, but is simply multiplied by one. This is the measure used in Greenwood and Vayanos (2014). We can see how this measures overvalues periods with relatively high interest rates due to discounting, but the overall path is smoother than when we deflated with yield curve data. The green short-dashed line *also FED holdings* plots the same market value measure, but with the overall outstanding amount and not only the amount held by the public. The additional debt includes intra-agency holdings and especially FED holding of public debt. We can see how the green line was always higher than than the blue one, but before QE2, the amount was constant. Following the start of QE2, the FED held a significant amount of long public debt. We can quantify the amount of interest rate risk held by the

[31] This is appropriate as all debt data is dated at the last day of the month.

[32] In a robustness check on the main empirical results presented on Figure 2 and available upon request, I employ an alternative nominal income measure which is present at monthly frequency: industrial production multiplied by the CPI index. This specification also yields similar results.

FED with duration-to-GDP of government held debt, to almost one percent of GDP in the mid twenty tens. Finally, the sienna dashed line *also TIPS* includes to the baseline series also inflation-linked bonds (TIPS). As duration-to-GDP measures how much would the value of public debt decrease following a one percent increase in interest rate across the nominal yield curve, we need to make an additional assumption on how changes in the nominal yield curve translates into the real yield curve. As this would put an additional assumption on the baseline measures and as TIPS are a relatively small share of public debt, especially in the estimation period, I did not include them in the baseline measure. However, for completeness I add them here and show in Appendix C that results are robust to their inclusion. Consistent with yield curve responses following monetary policy shocks highlighted by Nakamura and Steinsson (2018), I assume that the real yield curve rates move one to one with their nominal counterparts so that the new measure is computed simply by summing TIPS to nominal debt $\frac{\sum_{j=0}^{\infty} \frac{j}{12}(q_{t,j}b_{t,j}+\bar{q}_{t,j}\bar{b}_{t,j})}{GDP_t}$. The overall behavior is similar, with the TIPS mattering mildly only in the final period.

Figure A.3 presents the same figure for Macaulay duration. The overall pattern and differences across measures are the same as for duration-to-GDP. I would like to highlight only the difference between the baseline and the measure which includes FED holdings. Before QE, the government was holding similar bonds as the general public, so that the two duration measures line up together. However, from 2010 they started to diverge, with the treasury extending maturities, but the FED effectively reducing the supply of long term debt available to the general public. Notice however, that this decline was not sufficient to lower the insurance mechanism of long debt for the treasury, as duration-to-GDP was still increasing in this period. This shows how looking only at Macaulay duration can be misleading, as one would conclude that the insurance mechanism was declining relative to the pre-QE period by looking at Macaulay duration.

Figure A.4 presents the underlying data before being aggregated at the baseline duration-to-GDP measure, nominal marketable public debt promises held by the public excluding t-bills. It shows for each period and maturity, both at monthly intervals, the amount over GDP of the market value of each debt promises over GDP: $\frac{q_{t,j}b_{t,j}}{GDP_t}$. These numbers may look small as they represent the share of GDP owed at each future month. Moreover, debt issuers have tried in recent year to span the yield curve, leading to a high number of relatively small issuances. We can also notice how between the late seventies and the early nineties the Treasury was not issuing long debt at all.

Figure A.5 presents the share of public debt at different maturities. The underlying data is as in the baseline, nominal marketable public debt promises held by the public excluding t-bills. It presents public debt promises shares in four bins with thresholds at 5, 10, and 20 years. We can see how the largest share of public debt is debt below 5 years, even if we exclude t-bills. The reason is that the Treasury issues short dated debt and the fact that bonds carry coupons creases a number of cash flows early on even for long dated debt.

A.1.2 A Brief History of Public Debt Maturity Choices in the US

Objectives. The Treasury has chosen the maturity of public debt with two generally conflicting objectives in mind. On one hand, the Treasury tries to keep funding costs as low as possible, and this generally pushes the debt toward shorter maturities as the yield curve is generally upward sloping. On the other hand, the treasury tries to minimize some notion of rollover and interest rate risk, thereby pushing towards a higher maturity. The official objective of the US Office of Debt Management is: "Fund the government at the least cost to the taxpayer over time", and among the strategies we find: "Maintain manageable rollovers and changes in interest expense" (see Office of Debt Management, 2020).[33]

The relative importance of these competing aims has shifted, giving rise to a time series variation in the maturity structure of public debt. But Importantly for this paper, the changes in the maturity regimes were taken for reasons that do not pertain to monetary policy choices or to factors determing the strength of monetary policy. **The determinants of maturity choice are exogenous with respect to monetray policy, they are slow moving and for a large part of the sample are determined by legal restrictions.**

I now review the main historical developments relevant for the maturity of public debt in the US. We can see clearly the slow moving debt management choices with the Macaulay duration at Face Value in Figure A.3.

[33]During our entire sample, officials express a dual objective function as documented by Greenwood et al. (2014). In 1998 the Assistant Secretary of the Treasury for Financial Markets Gary Gensler argued at the House committee on Ways and Means that the objective of debt management is "achieving the lowest cost financing for the taxpayers", and "a balanced maturity structure also mitigates refunding risks". In 2008, Director of the Office of Debt Management Karthik Ramanathan also argued for achieving the "lowest cost of financing over time" and to "spread debt across maturities to reduce risk".

The modern debt management framework in the US started before the estimation sample, following the Treasury-Fed accords of 1951, when the Fed became independent in setting monetary policy and the Treasury took charge of debt management. After a brief period of coordination between the Treasury and the Fed with operation Twist, where the Fed intervened in long dated debts to flatten the yield curve, the Fed moved to a policy of intervention only on the short rate in the mid 60s. This followed the consensus that the policy did not work as expected as argued by Modigliani and Sutch (1966). This sealed the independence of the two institutions in the period under study.

The interest rate ceiling law period. In the earlier part of the sample, up to the mid 70s, we can see a constant decline in the maturity of debt. This was happening due to a shortening of the securities issued as a law forbade the Treasury from issuing bonds with interest rates above 4.25%. This law was instituted in 1918 (see p. 635 Friedman and Schwartz, 1963) and was not binding for a long time as interest rates were low. In the early 60s, Friedman and Schwartz (1963) argued that the ceiling might constraint the maturity choices of the Treasury and that a possible repeal was a point of political debate. The ceiling became binding from the mid 60s on longer date bonds, effectively pushing the Treasury to lower maturities (the law applied to bonds but not to medium term notes). In a testimony at the Ways and Means House Committee, Secretary of the Treasury John B. Connally, in 1971, expressed how this was a binding constraint for debt management and his desire to lengthen the existing maturity: "Because of the interest rate ceiling, the Treasury has been unable to sell a security maturing in more than seven years since mid-1965. The result has been a substantial and serious piling up of the debt in the short-term area" (see Ways and Means Committee Hearing, 1971).

This law was repealed in steps. Congress approved an initial allowance for $10 billions issuances above the ceiling for bonds maturing in more then 7 years following the 1971 testimony. It then allowed an extension to bonds only maturing in more than 10 years and the increased the allowance to $12 billions in March 1976 (see HR11893, 1976); this coincided with the lengthening on the Face Value duration that we can see in Figure A.3 in the late 70s. The allowance was increased several times again, in June 1976 to $17 billion, in 1977 to $27 billion, in 1978 to $32 billion, in April 1979 to $40 billion, in 19 September 1979 to $50 billion, and in 1980 to $70 billion (see references in Garbade, 2015). We can find evidence that the ceiling was binding in political debates throughout the period. In 1982, the Treasury asked, unsuccessfully, to Congress to the repeal the ceiling in order to increase the maturity of public debt: "Treasury has exhausted its $70 billion authority to issue long term bonds and was forced to cancel its regular quarterly issues of 20-year bonds in April and 30-year bonds in May. Treasury believes it must continue to issue bonds to maintain a presence in all maturity sectors of the bond market and to resist shortening the maturity of the public debt" (see Committee on Finance Hearing, 1982).

The ceiling was not repealed, but it raised again following the Treasury request in 1982 to $110 billion; and then again in 1983 to $150 billion, in 1984 to $200 billion, in 1985 to $250 billion, and in 1987 to $270 billion (see references in Garbade, 2015). Finally, the law was repealed in 1988 (see US Treasury, 2021). Throughout the 60s, 70s, and 80s we can see a desire of the Treasury to lengthen the maturity to minimize refinancing risk, restrained by Congress which was more afraid of the high cost of longer maturity debt[34]. The debate and the law ceilings were *exogenous* with respect to the monetary policy actions and its strength in affecting output.

Focus on costs. Until the interest rate ceiling law was in place, the focus of the Treasury had been increasing the average maturity with the aim of reducing rollover risk. However, the repeal allowed the Treasury to reassess its maturity choices. The push toward longer maturities ended soon after the repeal, with a stronger emphasis on lowering average debt costs in the Treasury. Wessel (1993) argues that the Clinton administration would want to save on debt servicing costs with the reasoning: "the case for the change is straightforward: The interest rate on a 30-year Treasury bond is substantially higher than the rate on a three-month Treasury bill. If the Treasury borrowed less at 30-year rates and more at three-month rates, taxpayers could save billions of dollars". Following a decision to lower the maturity of debt the Secretary of the Treasury Lloyd Bentsen argued: "We have considered this issue very carefully and believe the restructuring of our debt mix, over the long run, is in the best interests of American taxpayers. This action to shorten the maturity of Treasury borrowing

[34] As reported by Shanahan (1971), the chairman of the House Banking Committee, Representative Wright Patman of Texas argued at the Ways and Means Committee "the existence of the 4-1/4 percent interest rate ceiling was the only thing that had kept the Treasury from selling bonds with interest rates as high as 8 per cent and maturities of 30, 40 or even 50 years, during the recent period of tight money".

will produce real savings on interest costs over time" (see references in Garbade, 2015)[35].

The case for shortening of the average duration to lower borrowing costs kept going until the mid-aughts, with a mechanical increase in Macaulay duration, but not in duration-to-GDP in the late 90s. The strong fiscal surpluses and declining levels of public debt pushed down duration-to-GDP, but initially increased Macaulay duration as the Treasury simply issued lower amounts of new debt, without retiring existing long bonds. The desire the shorten debt maturity was the reason why the Treasury undertook the buyback operations in 2000 and 2001, together with wanting to concentrate issuance in a smaller number of larger issues to maintain liquidity in the Treasuries market (see Garbade and Rutherford, 2007). The Secretary of the Treasury Lawrence Summers made this argument in 1999 for the advantages of the buyback operation: "first, by prepaying the debt we would be able to maintain larger auction sizes than would otherwise be possible. Enhancing the liquidity of Treasuries benchmark securities should lower the governments interest costs over time and promote overall market liquidity. Second, by paying off debt that has substantial remaining maturity, we would be able to prevent what would otherwise be a potentially costly and unjustified increase in the average maturity of our debt" (see Summers, 1999). As part of the commitment to lower the average maturity the Treasury decided to stop issuing 30 years bonds in 2001 (see US Treasury, 2021)[36].

Long maturity strikes back. The intellectual environment shifted again in the mid aughts, this time in favor of long bonds, with a stronger emphasis on rollover risk. The decision to lengthen the maturity started before the financial crisis and the subsequent increase in debt, in a period of relatively strong tax receipts and low federal deficit. The Treasury reintroducing the 30 years bond in 2006[37] and discontinuing the 3 years note in 2007.

The estimation sample ends at the beginning of the financial crisis, but for completeness,

[35] Note that the yield curve was not particularly steep in the early 90s, in December 1993 the difference between a 10 year and a 3 month Treasury rates was 2.76%. In September 1982, when the Treasury asked Congress to be able to issue longer maturity debt, this spread was 3.85%.

[36] This can be seen graphically in Figure A.4 with the 30 years promises disappearing after 2001.

[37] The Treasury Borrowing Advisory Committee, the advisory committee of private institutions who buy government debt, remarked in their quarterly report in April 2005: "reintroducing 30-year bonds would serve to mitigate rollover risk given large maturities in coming years" (see Treasury Borrowing Advisory Committee, 2005).

I show how the maturity choices of the Treasury continued in the same path. The increase in maturity of Treasury issuances kept increasing following the financial crisis until today. The risk of rollover was emphasized more often. As an example, the Treasury Borrowing Advisory Committee suggested in 2009: "The conclusions were that the potential for inflation, higher interest rates, and roll over risk should be of material concern. In most economic scenarios, lengthening the average maturity of debt from 53 months to 74-90 months was recommended. Committee members commented that while real progress has been made in terms of lengthening the average maturity of US Treasury debt to 53 months [...], more needs to be done in this regard" (see Treasury Borrowing Advisory Committee, 2009). Then again in 2010: "further lengthening of the average maturity should take precedence" (see Treasury Borrowing Advisory Committee, 2010). The push toward higher maturities took shape in a period when the yield curve was relatively steep, with short rates at the zero lower bound, but longer rates still well in positive territory. The Treasury was buying relatively expensive insurance.

Between the 1951 Treasury accords and the QE2 period, there was a strong separation of roles between the Fed and the Treasury. We can directly see this by noticing how close the baseline value and the value for *Also Fed Holdings* are for Macaulay duration and duration-to-GDP in Figures A.2 and A.3. However, that changed with QE2, when the Fed bought long dated Treasury bonds. In this last period, the two agencies have conflicting objectives (see Greenwood et al., 2014): the Treasury tried to increase the maturity of public debt to minimize rollover risk and the Fed tried to lower the maturity to lower the term premium. The overall duration-to-GDP increases in line with the Treasury objective. However, I exclude this last period also to avoid any potential contamination.

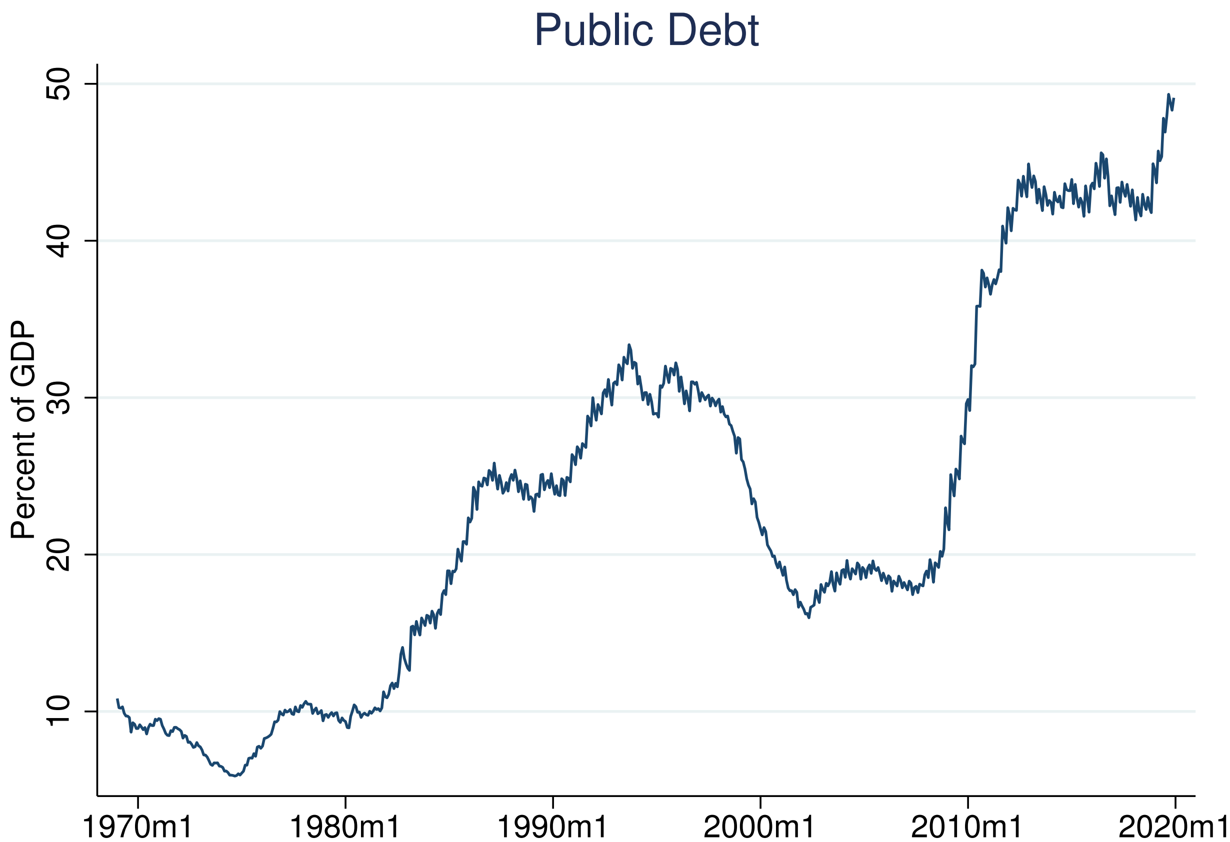
To summarize, this narrative account of the maturity choices shows how the Treasury chose the debt composition *exogenously* with respect to the monetary policy cycle. In the period up to 1988, the Treasury was directly constrained by the 4.25% interest rate ceiling law in its maturity choices. From 1993 the trade-off calculation shifted toward costs pushing towards shorter maturities, and then again, in 2005 it shifted back toward rollover risk and toward longer maturities.

A.1.3 UK Data Description

This completes the discussion of the US data. Figure A.6 presents duration-to-GDP and Macaulay duration for the UK and this figure is the counterpart to Figure 1 for the US. It shows nominal marketable debt. As for the US it excludes inflation-linked bonds, t-bills, and non-marketable debt. The key difference with the US data is that for the UK we have only outstanding debt and cannot net out Bank of England or public sector holdings of Gilts. This is not an issue for the main estimation period which ends in December 2007, as substantial holdings of Gilts by the Bank of England started with QE, that is from 2009. The only caveat comes from inspecting Figure A.6, the numbers from 2009 onward are likely to be overestimated. To give an order of magnitude, in September 2019 the Bank of England held 23% of all gilts, so we might have to lower the numbers from the QE period by roughly a fourth (assuming that the Bank of England held the market in terms of maturity split). By inspecting Macaulay duration, we can see how duration at face value from Figure A.9, has been declining constantly from 1969 up to the early nineties. However, at market value, we can see much smaller swings in this period due to the high interest rates. Macaulay duration declined substantially in the mid seventies with the increase in interest rates and it has been increasing since despite the decline in face value. From the mid nineties onward we can see an increase in both Macaulay duration at market value and at face value, with the increase at market value being steeper due to the declining interest rates. With respect to duration-to-GDP, we can see in the pre-2009 period the behavior going in phases. Duration to GDP was relatively high before 1974, between the late seventies and the late eighties, and then in the late nineties and late aughts. On the other hand, it was low in the mid seventies, in and the early nineties. From the mid nineties there was an upward trend, with lower values relative to trend in the mid aughts. This behavior is quite distinct from the US one, where phases of low and high values happened at lower frequency and different times. The fact that the empirical results are present for both countries with these different time series behaviors is reassuring and give weights to the idea that the maturity of public debt matters for the transmission of monetary policy.

Figures A.7, A.8, A.9, A.11, and A.10 presents the same cuts of the data for the UK as for the US. Overall a few points stand out, first of all inflation linked debt has a bigger role for the UK than for the US to compute duration-to-GDP, the reason is that this type

Figure A.1: Time series of nominal marketable public debt over GDP held by the public for the US



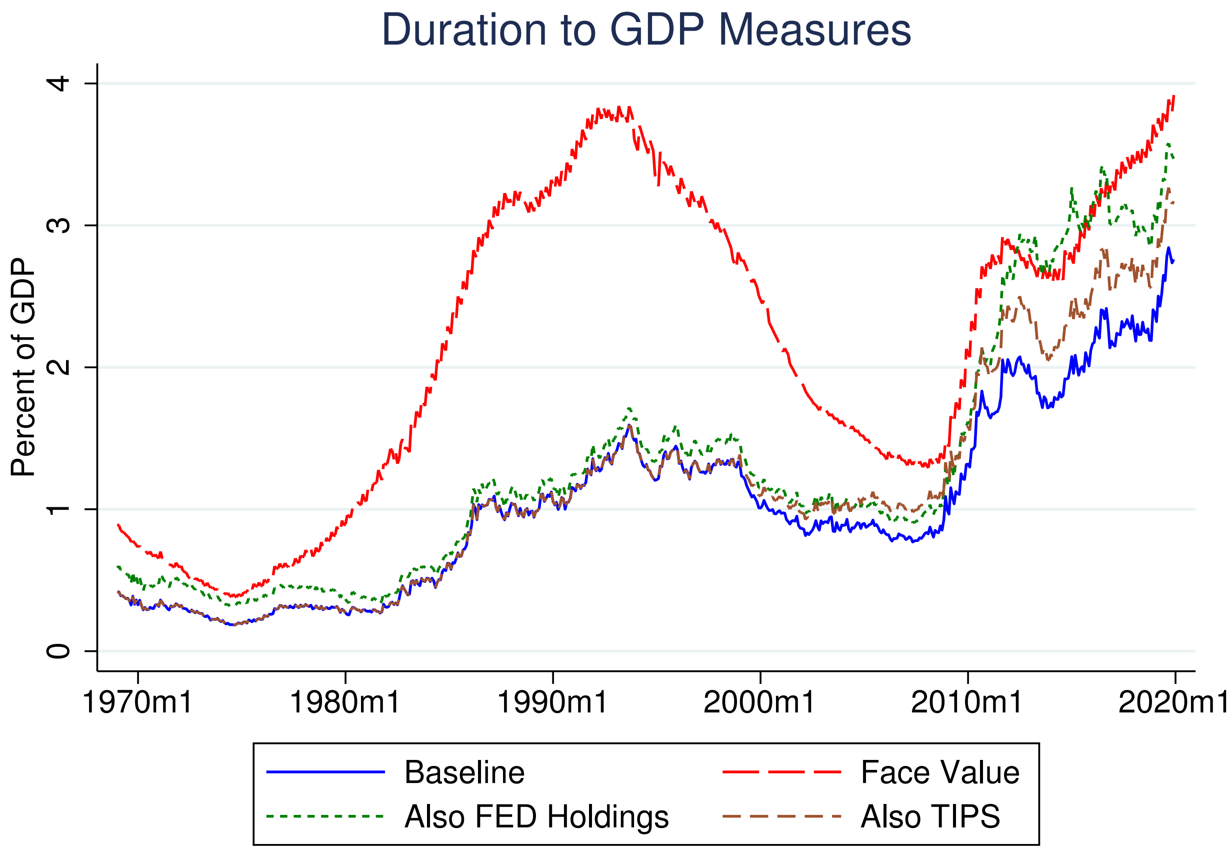
Notes: The figure shows the time series for public debt over GDP for the US. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2019m12 with US data.

of debt tends to have a longer dated life and it has been used extensively by the UK debt management office. We can also see from the baseline Macaulay duration and the shares of debt that the UK issues relatively longer debt than the US on average. This is also true across other countries, with the UK having longer debt than most European countries.

A.2 Monetary Policy Shock Data

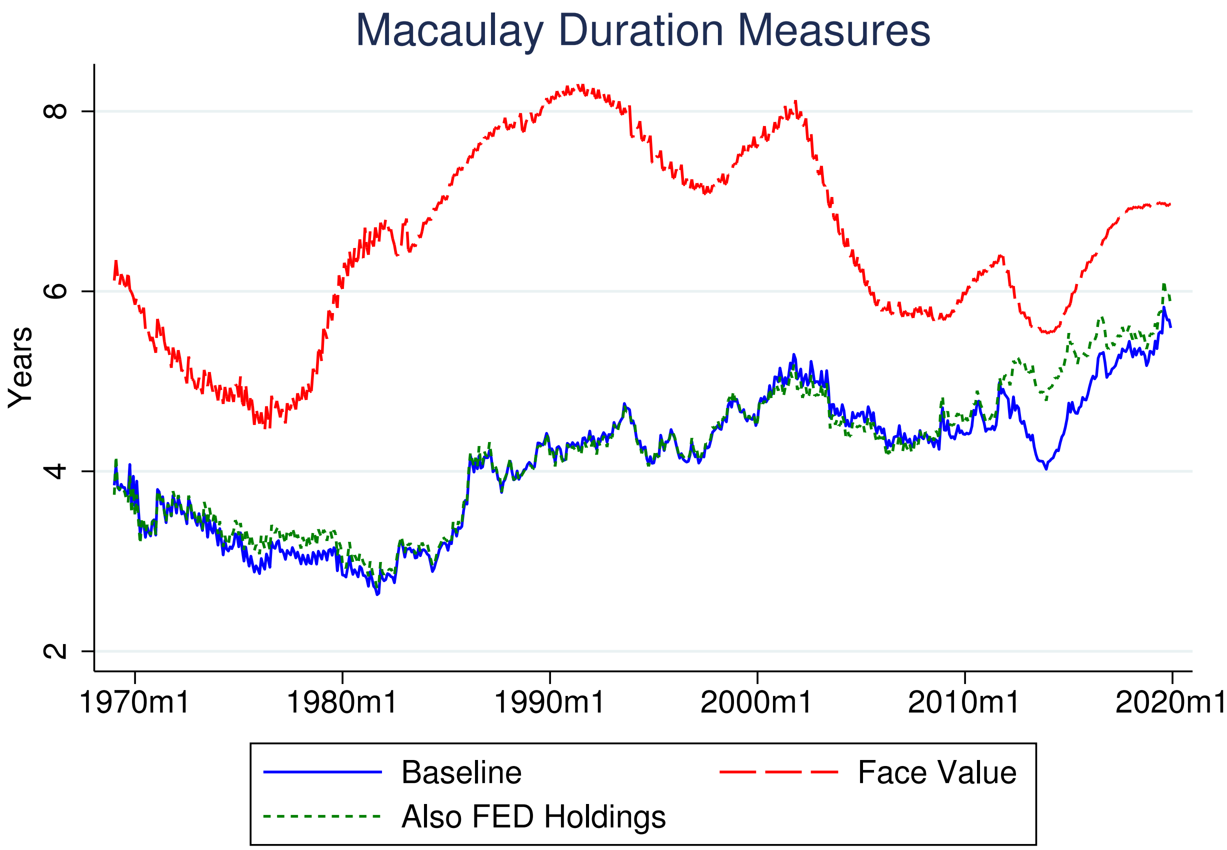
The narrative measure of monetary policy shock are in the spirit of Romer and Romer (2004). For the US, the series is the updated Romer and Romer (2004) measure by Yang and Wieland (2015) that spans the period from January 1969 to December 2007. The UK series is the one

Figure A.2: Time series of public debt duration-to-GDP for the US with different measures



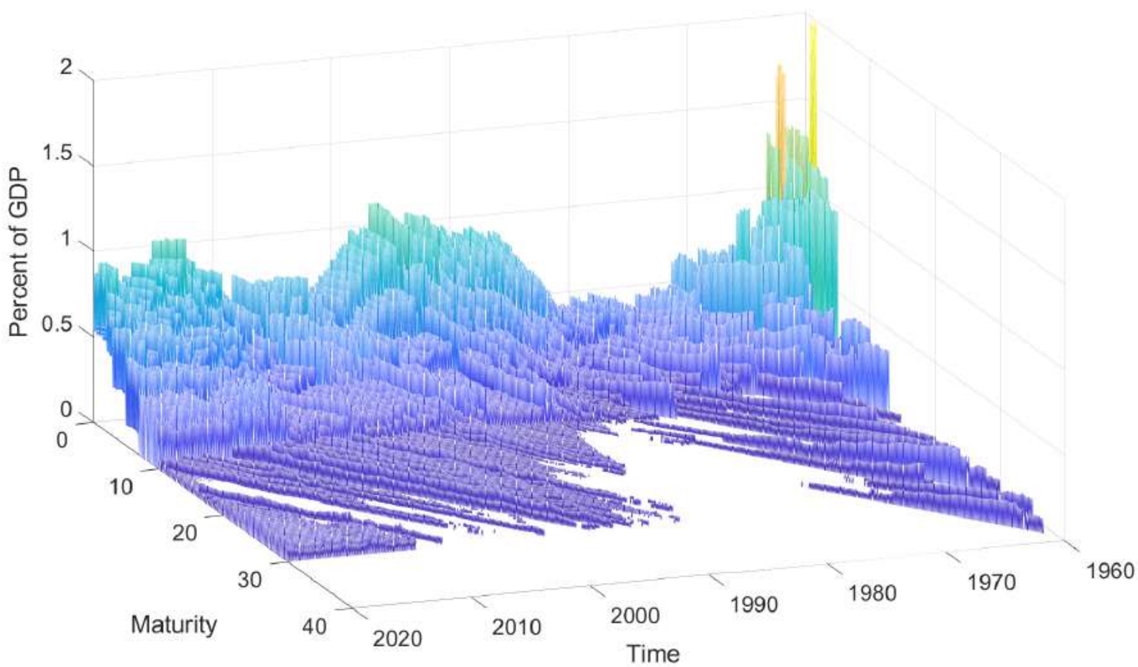
Notes: The figure shows the time series for public debt duration-to-GDP for the US. For the baseline series, the public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct duration-to-GDP each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by nominal GDP. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). For the face value series each promise is not discounted at market value with yield curve data but is multiplied by one. For the also FED holdings series, I use the overall amount outstanding per each bond and do not net out FED or intra-agencies holdings. For the Also TIPS series, I sum the TIPS as well, this implies that we assume a one to one correlation between nominal yield curve rates and real yield curve rates. The sample goes from 1969m1 to 2019m12 with US data.

Figure A.3: Time series of public debt Macaulay duration for the US with different measures



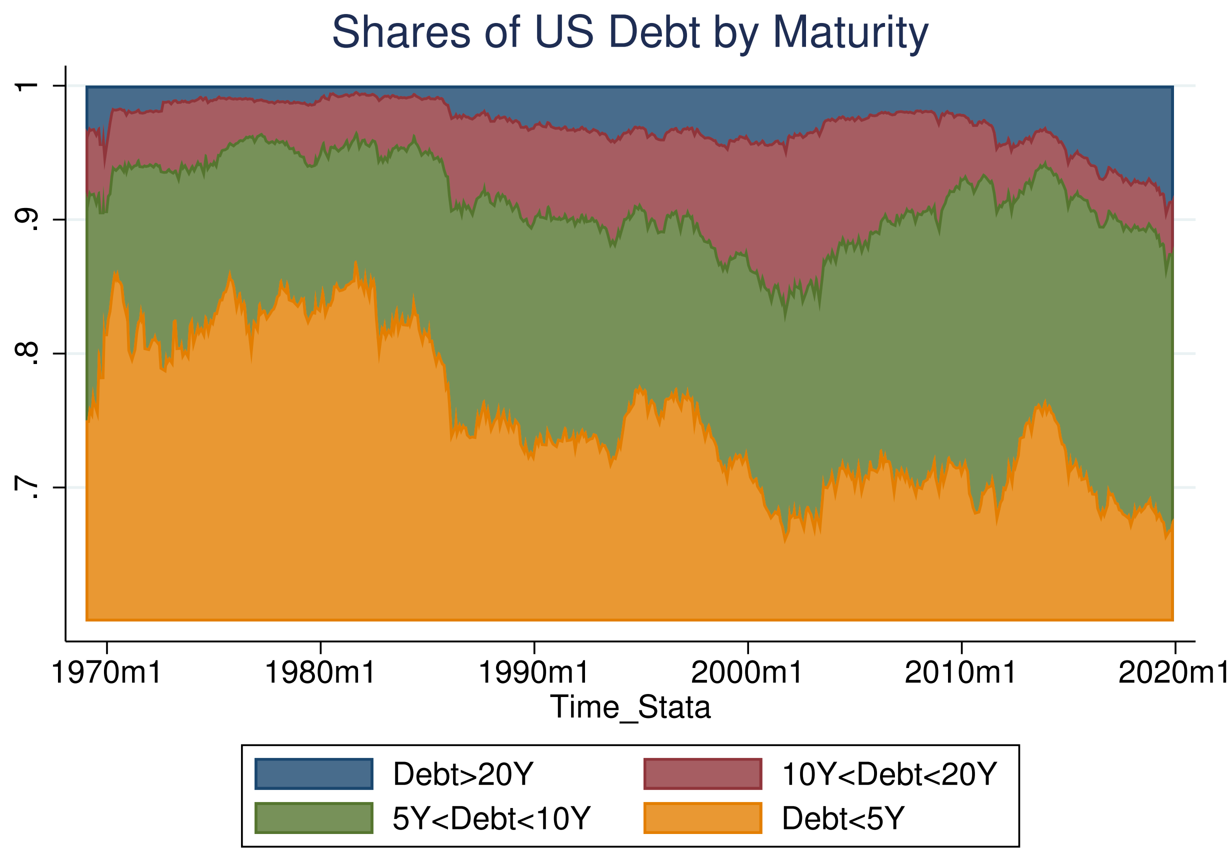
Notes: The figure shows the time series for public debt Macaulay duration for the US. For the baseline series, the public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct Macaulay duration each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by their market value (the sum without multiplying by maturity). For the face value series each promise is not discounted at market value with yield curve data but is multiplied by one. For the also FED holdings series, I use the overall amount outstanding per each bond and do not net out FED or intra-agencies holdings. The sample goes from 1969m1 to 2019m12 with US data.

Figure A.4: Public debt promises over GDP at various maturities for the US



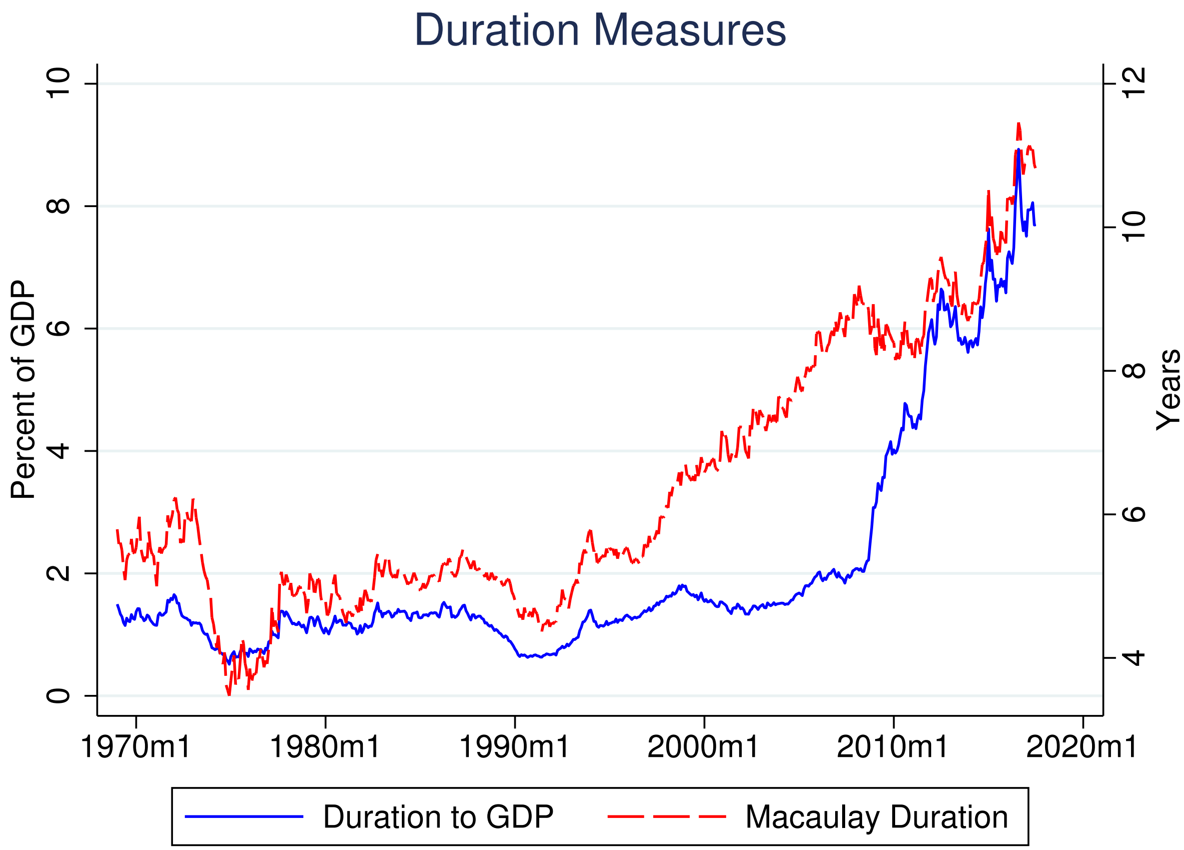
Notes: The figure shows the distribution of public debt promises for the US. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The market value of each promise is deflated by nominal GDP. The time and maturity dimensions are both at monthly frequency. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2019m12 with US data.

Figure A.5: Public debt shares at various maturities for the US



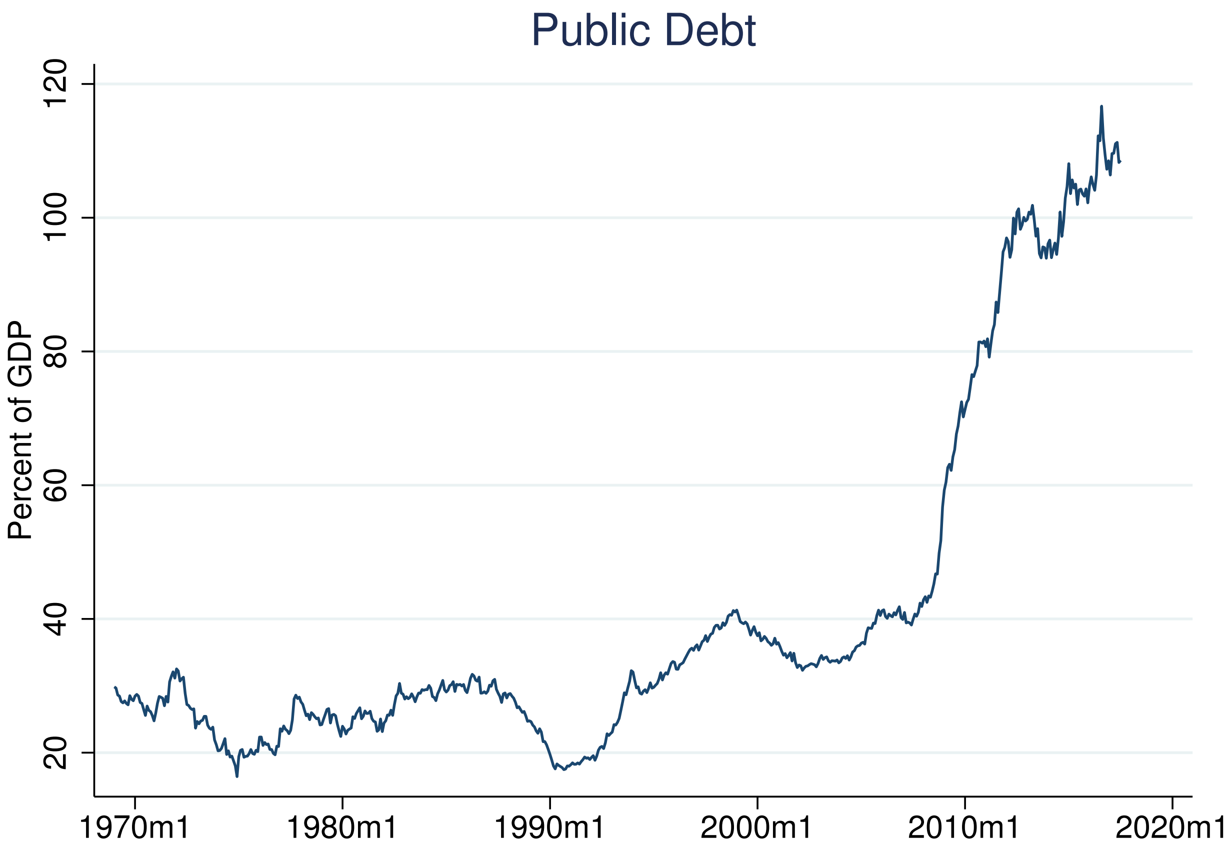
Notes: The figure shows the time series for the shares of debt over GDP for the US within 4 bins: debt below 5 years, debt between 5 and 10 years, debt between 10 and 20 years, and debt above 20 years. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The sum of public debt promises within the bins is divided by the sum of public debt promises across all the bins. The sample goes from 1969m1 to 2019m12 with US data.

Figure A.6: Time series of public debt Macaulay duration and duration-to-GDP for the UK



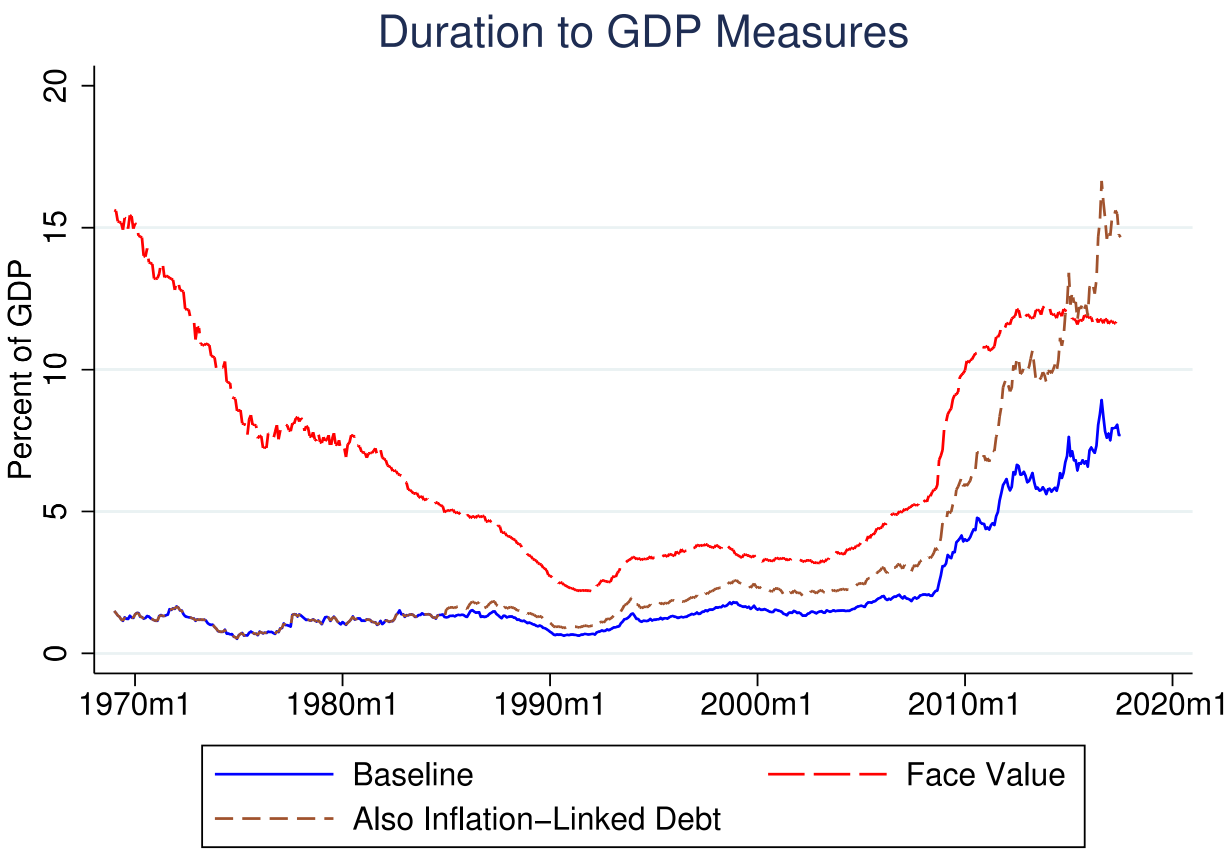
Notes: The figure shows the time series for public debt Macaulay duration (in red dashed line) and duration-to-GDP (in blue solid line) for the UK. The public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct duration-to-GDP each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by nominal GDP. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2017m7 with UK data.

Figure A.7: Time series of nominal marketable public debt over GDP for the UK



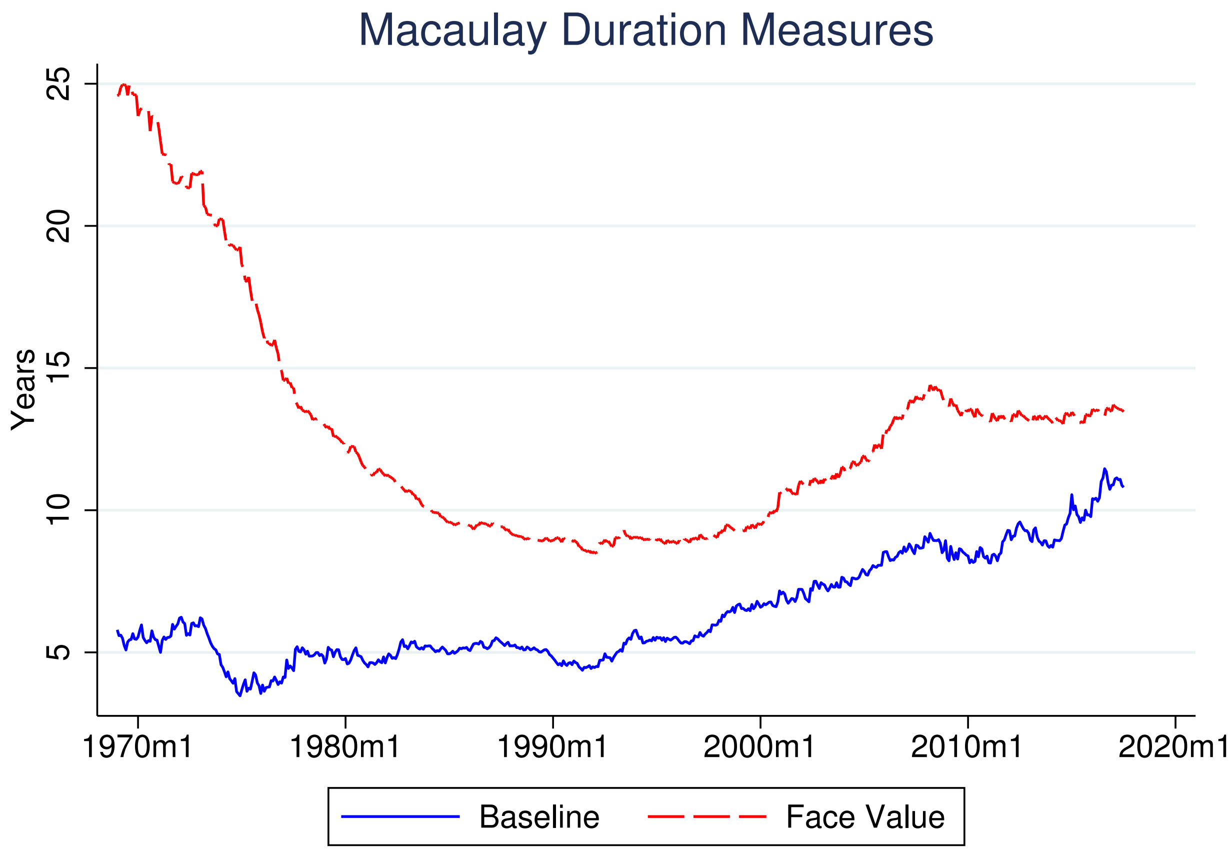
Notes: The figure shows the time series for public debt over GDP for the UK. The public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2017m7 with UK data.

Figure A.8: Time series of public debt duration-to-GDP for the UK with different measures



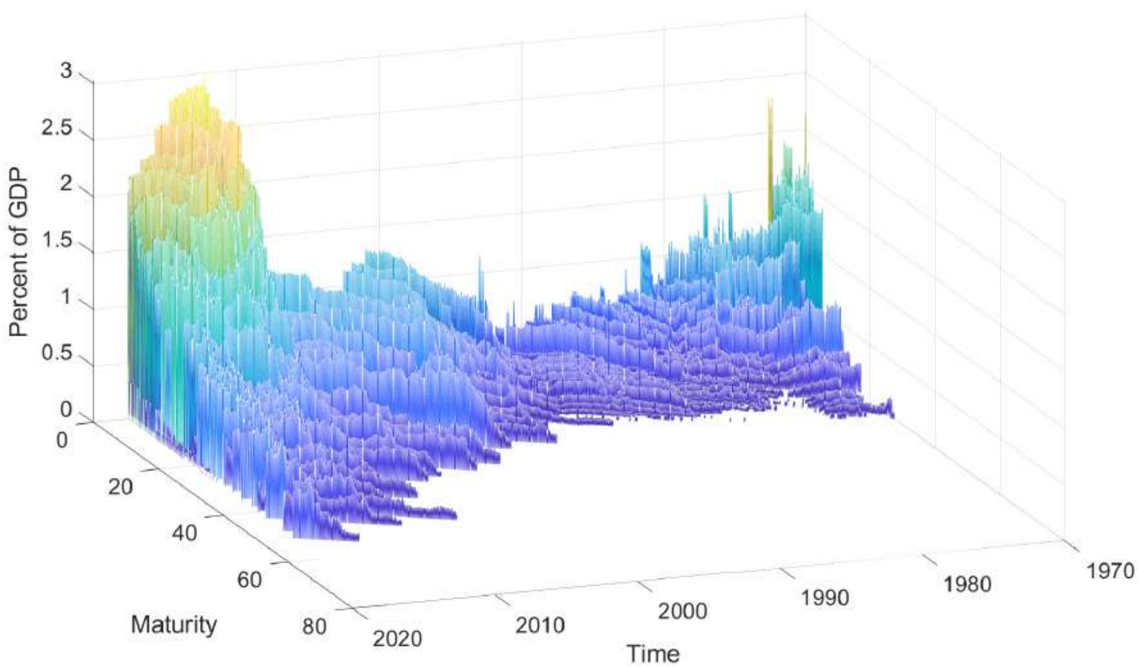
Notes: The figure shows the time series for public debt duration-to-GDP for the UK. For the baseline series, the public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct duration-to-GDP each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by nominal GDP. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). For the face value series each promise is not discounted at market value with yield curve data but is multiplied by one. For the also inflation linked debt series, I sum the inflation linked debt as well, this implies that we assume a one to one correlation between nominal yield curve rates and real yield curve rates. The sample goes from 1969m1 to 2017m7 with UK data.

Figure A.9: Time series of public debt Macaulay duration for the UK with different measures



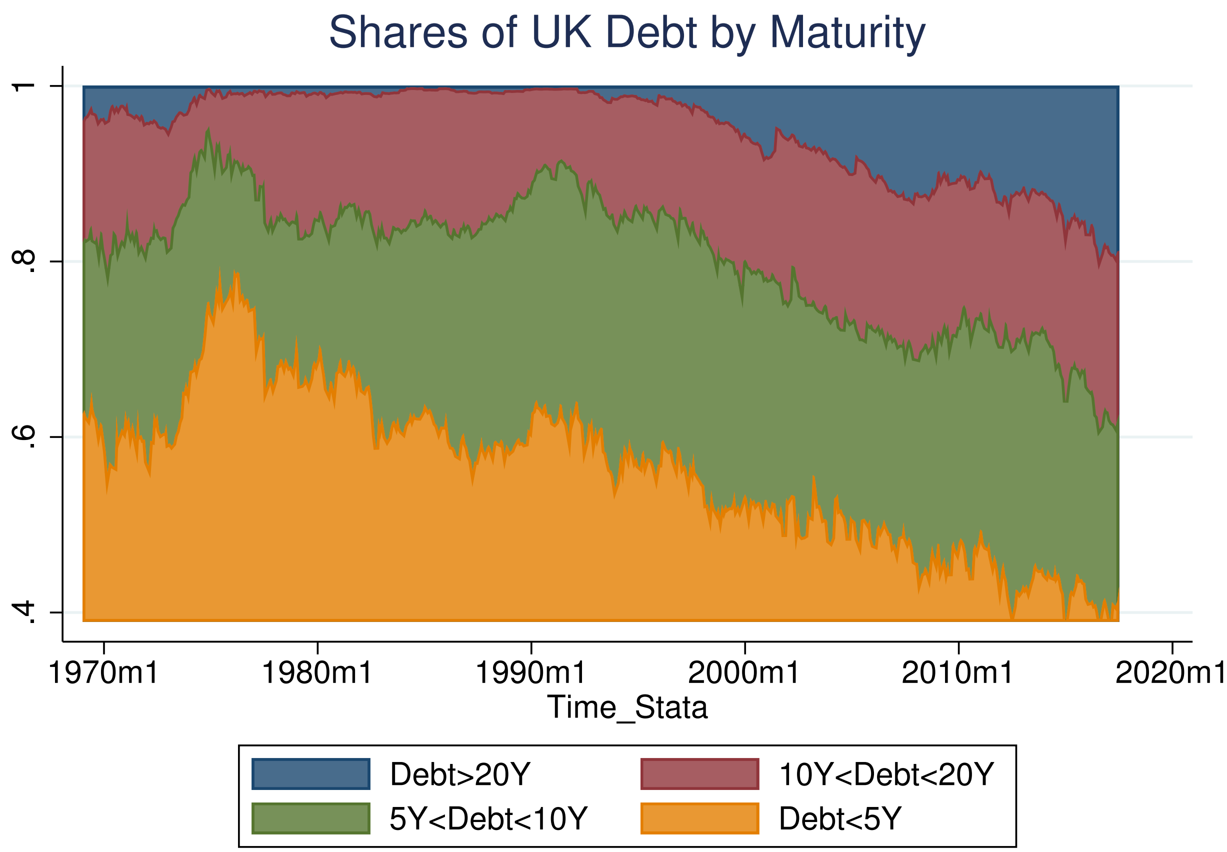
Notes: The figure shows the time series for public debt Macaulay duration for the UK. For the baseline series, the public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct Macaulay duration each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by their market value (the sum without multiplying by maturity). For the face value series each promise is not discounted at market value with yield curve data but is multiplied by one. The sample goes from 1969m1 to 2017m7 with UK data.

Figure A.10: Public debt promises over GDP at various maturities for the UK



Notes: The figure shows the distribution of public debt promises for the UK. The public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The market value of each promise is deflated by nominal GDP. The time and maturity dimensions are both at monthly frequency. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2017m7 with UK data.

Figure A.11: Public debt shares at various maturities for the UK



Notes: The figure shows the time series for the shares of debt over GDP for the UK within 4 bins: debt below 5 years, debt between 5 and 10 years, debt between 10 and 20 years, and debt above 20 years. The public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The sum of public debt promises within the bins is divided by the sum of public debt promises across all the bins. The sample goes from 1969m1 to 2017m7 with UK data.

Table A.1: Macroeconomic US Data

Variable	Transformation	Source	Code
Industrial Production	log times 100	FRED	INDPRO
CPI Price Level	log times 100	FRED	CPIAUCSL
Unemployment Rare	as is	FRED	UNRATE
Effective Federal Funds Rate	as is	FRED	FEDFUNDS
GDP	log times 100	FRED	GDPC1
Commodity Price Index	log times 100	CRB Commodity Price Index, downloaded from Ramey (2016)	LPCOM
Government Consumption Expenditures and Gross Investment	log times 100 less lCPI	FRED	GCE
Government current transfer payments	log times 100 less lCPI	FRED	A084RC1Q027SBEA
Federal Tax Receipts	log times 100 less lCPI	FRED	W006RC1Q027SBEA
Federal Budget Surplus	percent over GDP times 100	FRED	M318501Q027NBEA
Bank Loans	log times 100 less lCPI	FRED	BUSLOANS
AAA Spread over 10 Year Treasury	as is	FRED	AAA10YM
BAA Spread over 10 Year Treasury	as is	FRED	BAA10YM
Nonfinancial Corporate Business, Gross Fixed Investment, Flow.	log times 100 less lCPI	FRED	BOGZ1FA105019005Q
Private Nonresidential Fixed Investment	log times 100 less lCPI	FRED	PNFI
Excess Bond Premium	as is	From Gilchrist and Zakrajšek (2012), downloaded from Gertler and Karadi (2015)	EBP
Commercial Paper Spread	as is	Downloaded from Gertler and Karadi (2015)	CP3M_SPREAD
Mortgage Spread	as is	Downloaded from Gertler and Karadi (2015)	MORTG_SPREAD

Notes: The first column shows the name of the variable. The second column shows which transformation has been applied to be used in the empirical analysis. The third column discusses where the variable was retrieved. Finally, the fourth column shows the code of the variable in the source. For variables from FRED this is the code which can be used to retrieve the variable. For the variables from a previous study the code is the name of the variable in the replication files.

produced by Cloyne and Hürtgen (2016) from January 1975 to December 2007. These series represent the main constraint in term of sample which I can use in the regressions. The high frequency measure comes from the Proxy-VAR ran by Gertler and Karadi (2015), there, the structural shock is present from July 1980 to June 2012.

A.3 Macroeconomic Data

The macroeconomic series used in the various regressions are specified in Tables A.1 and A.2.

A.4 Flow of Funds Data

In this section, I discuss how I use the flow of funds data for the US to construct the measures of corporate debt issuance and leverage for the non-financial corporate sector. I follow Green-

Table A.2: Macroeconomic UK Data

Variable	Transformation	Source	Code
Industrial Production	log times 100	BoE: A Millennium of Macroeconomic Data	Index of Industrial Production
RPIX Inflation	12 months inflation of the retail price index excluding mortgage payments	ONS, downloaded from Cloyne and Hürtgen (2016)	RPIX12m
CPI Price Level	log times 100	BoE: A Millennium of Macroeconomic Data	Spliced monthly Consumer Price index, 1914-2015
Unemployment Rare	as is	BoE: A Millennium of Macroeconomic Data	Monthly unemployment rate
Bank Rate	as is	BoE: A Millennium of Macroeconomic Data	Bank Rate
GDP	log times 100	BoE: A Millennium of Macroeconomic Data	GDP at market prices. Chained volume measure, £mn, 2013 reference year prices
Commodity Price Index	log	IMF, downloaded from Cloyne and Hürtgen (2016)	CommodityPriceIndex (log)

Notes: The first column shows the name of the variable. The second column shows which transformation has been applied to be used in the empirical analysis. The third column discusses where the variable was retrieved. Finally, the fourth column shows the code of the variable in the source. For variables from "BoE: A Millennium of Macroeconomic Data" this is the name of the variable in the excel file. For the variables from a previous study the code is the name of the variable in the replication files.

wood, Hanson and Stein (2010) on corporate debt issuance, who build on Baker, Greenwood and Wurgler (2003). They assume that all short debt is refinanced each period and that the new issuance of long debt is equal to the change in the stock of long debt plus 0.025 times the stock of long debt in the previous quarter, which implies an average maturity of 10 years for long non-financial corporate debt. This measure allows to focus on gross issuance, the sum of short debt and long debt issuance, as this is the relevant metric for the theoretical model. The measure is in log real terms times 100. The data source is Table L.103 Nonfinancial Corporate Business of the flow of funds. Short debt is defined as the sum of "Commercial Paper" (FL103169100.Q), "Depository Institution Loans not elsewhere classified" (FL103168005.Q), and bank loans not elsewhere classified," and "Other loans and advances"[38] (FL103169005.Q). Long debt is defined as the sum of "Municipal Securities[39]" (FL103162000.Q), "Corporate Bonds" (FL103163003.Q), and "Mortgages" (FL103165005.Q). Notice that the flow of funds data changes the codes and definitions across iterations. This description is accurate on the Q4 2019 publication (published on March 2020).

With respect to leverage for the non-financial corporate sector I build a measure relating to the debt leverage, in line with the spirit of the model. Leverage is defined as 100 times

[38] These are loans from rest of the world, U.S. government, and non-bank financial institutions.

[39] In the context of nonfinancial corporate businesses these are industrial revenue bonds. They are issued by state and local governments to finance private investment and secured in interest and principal by the industrial user of the funds.

"Debt Securities and Loans" (FRED code BCNSDODNS) over the sum of "Total Liabilities" (FRED code: TLBSNNCB) and "Corporate Equities" (FRED code: NCBCEL). The measure is in log terms times 100.

B Microfoundation of the Primary Market Friction

In this section I propose two possible microfoundations to the primary market friction. The key idea underlying both microfoundations is based upon some degree of market segmentation. The primary market for government and corporate debt can be accessed only by specialized players. In the real world, they would be primary dealers and desk in investment banks dealing with underwriting and placing bonds.

B.1 Risk-Averse Arbitrageurs

The first microfoundation is based on risk averse primary market participants. These agents are risk averse and live for one initial sub-period. They are selected out of the households, buy newly issued government debt l and newly issued corporate debt crp on the primary market with either equity e_t or riskless debt b_t and they subsequently sell it to the secondary market at price $q^i_{t+\Delta}$ for $i = \{l, crp\}$. There is a technological constraint that forces them to bid in advance of knowing the final secondary market price.

The consequence of these assumptions is that the primary market price is generally lower than the secondary market price. This is a reward for risk taking for these specialized investors. One can find evidence of this outcome empirically. Primary dealers both for government and private bonds must use their own balance sheet to provide liquidity in the secondary market for such securities because of contractual obligations. This is a well known phenomenon in issuance for most market based financial instruments even for very liquid government bond auctions as discussed in Duffie (2010). Lou, Yan and Zhang (2013) document this *auction cycle* for US treasuries and Eisl et al. (2019), Beetsma et al. (2016), Sigaux (2018), Bigio, Nuño and Passadore (2019) for Eurozone sovereign issuers. All these papers relate this empirical finding to segmented primary markets and limited risk bearing capacity of these investors.

These agents have mean-variance utility on their terminal value of wealth $e_{t+\Delta}$ with absolute risk aversion a.

$$\max_{b_t, b_t^{crp}, l_t} \mathbb{E}_t \left[e_{t+\Delta}\right] - \frac{a}{2} \mathbb{V}ar_t \left[e_{t+\Delta}\right]$$

Their two sub-periods budget constraints are:

$$e_t = b_t + b_t^{crp} + l_t$$
$$e_{t+\Delta} = b_t + q_{t+\Delta}^{crp} b_{t+\Delta}^{crp} + q_{t+\Delta}^{l} l_{t+\Delta}$$

Where I assume for simplicity that the sub-period is short enough for the riskless asset to not earn any interest. All the results go through with a non-zero interest as well, at the cost of added notation without any further insight. We can consolidate the two budget constraint into one by substituting in for the riskless debt:

$$e_{t+\Delta} = e_t + (q_{t+\Delta}^{l} - 1) l_{t+\Delta} + (q_{t+\Delta}^{crp} - 1) b_{t+\Delta}^{crp}$$

Which has the intuitive interpretation that the investor can trade-off risky capital gains in the primary markets for the riskless debt. Before solving the problem, we can define the joint second moments of the capital gains as $\sigma_{l,crp} = \mathbb{C}ov_t(q_{t+\Delta}^{l} - 1, q_{t+\Delta}^{crp} - 1)$ for the covariance between the government and corporate debt capital gains, and similarly for the variances. We can rewrite the variance of terminal wealth: $\mathbb{V}ar_t \left[e_{t+\Delta}\right] = \sigma_l^2 l_t^2 + \sigma_{crp}^2 (b_t^{crp})^2 + 2\sigma_{l,crp} l_t b_t^{crp}$. With this expression, we can plug it into the objective function and solve for the optimal allocation:

$$\frac{\partial}{\partial l_t} : \mathbb{E}_t \left[q_{t+\Delta}^{l} - 1\right] = a \left[\sigma_l^2 l_t + \sigma_{l,crp} b_t^{crp}\right]$$
$$\frac{\partial}{\partial b_t^{crp}} : \mathbb{E}_t \left[q_{t+\Delta}^{crp} - 1\right] = a \left[\sigma_{l,crp} l_t + \sigma_{crp}^2 b_t^{crp}\right]$$

For a given volatility process of the returns to the capital gains this links the quantity of debt to the risk premia associated with participating in the primary market. Note that this creates a mapping from public debt quantity supplied to the corporate debt price and this mapping is $a\sigma_{l,crp}$. With the additional assumption that $\sigma_{crp}^2 = \sigma_l^2 \sigma_{l,crp}$, that is, perfect correlation of returns and same variance, then the two risk premia are identical and are exactly equal to $\hat{\Phi}_t$ in the linearized model. Moreover, $a\sigma_{l,crp}$ becomes equal to ζ.

This microfoundation shows how we can rationalize the primary market friction with the widely used framework of risk averse arbitrageurs in segmented market. However, this is only one such possibility to achieve it and the results do not hinge on its specifics. In order to show the generality of the primary market friction, in the next subsection, I propose another microfoundation isomorphic to this one.

B.2 Moral Hazard

As an alternative microfoundation I employ the framework presented by Gabaix and Maggiori (2015): a moral hazard problem whereby the primary market dealers can abscond a fraction Γb_t of the total borrowing from the lender. This is similar to Kiyotaki and Moore (1997) with the addition that the fraction absconded can depend on the size of the balance sheet of the dealers, to highlight the role of financial complexity.

In this framework, the setting is as in the first microfoundation with the simplifying assumptions of linear utility and no initial equity. We can proceed by splitting the problem in two steps. The first step is the choice of the balance sheet b_t. The second step is the choice of what type of debt to buy. To this aim, I define the return on total assets $q^A_{t+\Delta} \equiv \frac{q^l_{t+\Delta} l_t + q^{crp}_{t+\Delta} b^{crp}_t}{b_t}$. The first step of the problem of the primary market participant can be written as a choice of the borrowing level:

$$\max_{b_t} \mathbb{E}_t \left[(q^A_{t+\Delta} - 1) b_t \right]$$
$$s.t. \; b_t \leq (1 - \Gamma b_t) \mathbb{E}_t \left[q^A_{t+\Delta} b_t \right]$$

That is, the objective of the agent is to maximize the expected value of the net return on assets. The problem is subject to total borrowing being at most the fraction not absconded of the expected terminal value of the firm. As the objective function is linear and the constraint is concave in b_t the constraint binds and we can find the optimal size by solving the constraint for the positive solution of b_t:

$$
\begin{aligned}
\frac{1}{\mathbb{E}_t\left[q^A_{t+\Delta}\right]} &= 1 - \Gamma b_t \\
b_t &= \frac{1}{\Gamma}\frac{\mathbb{E}_t\left[q^A_{t+\Delta}\right] - 1}{\mathbb{E}_t\left[q^A_{t+\Delta}\right]}
\end{aligned}
$$

This expression has intuitive sense. The size of the balance sheet will be lower the higher the fraction of divertable output and the lower the expected return on investing in the primary market. The overall return is linear in the returns in the corporate and government debt, therefore, to have investment in both the return must be the same in equilibrium. In addition, an increase in government debt quantity has the same impact on the price of both government and corporate debt, and vice-versa.

$$
\begin{aligned}
\frac{\mathbb{E}_t\left[q^l_{t+\Delta}\right] - 1}{\mathbb{E}_t\left[q^l_{t+\Delta}\right]} &= \Gamma(b^{crp}_t + l_t) \\
\frac{\mathbb{E}_t\left[q^{crp}_{t+\Delta}\right] - 1}{\mathbb{E}_t\left[q^{crp}_{t+\Delta}\right]} &= \Gamma(b^{crp}_t + l_t)
\end{aligned}
$$

These expressions again map directly to the primary market friction presented in the model. The message from the two microfoundations is that the relationship between debt quantities and prices in the primary market arise from a variety of standard models in macroeconomics and finance. Moreover, the exact nature of the friction does not matter per se.

Online Appendixes for Monetary Policy and the Maturity Structure of Public Debt

Michele Andreolli
London Business School

C Sensitivity on Main Empirical Results

C.1 Baseline LP results without interaction term

The main empirical results of this paper highlight conditional effects of monetary policy. As a baseline comparison, it is important to show the underlying average effect results, not conditional on debt maturity. For this reason, this subsection presents the LP and LP-IV regression results for the narrative identification from Ramey (2016) for the US.

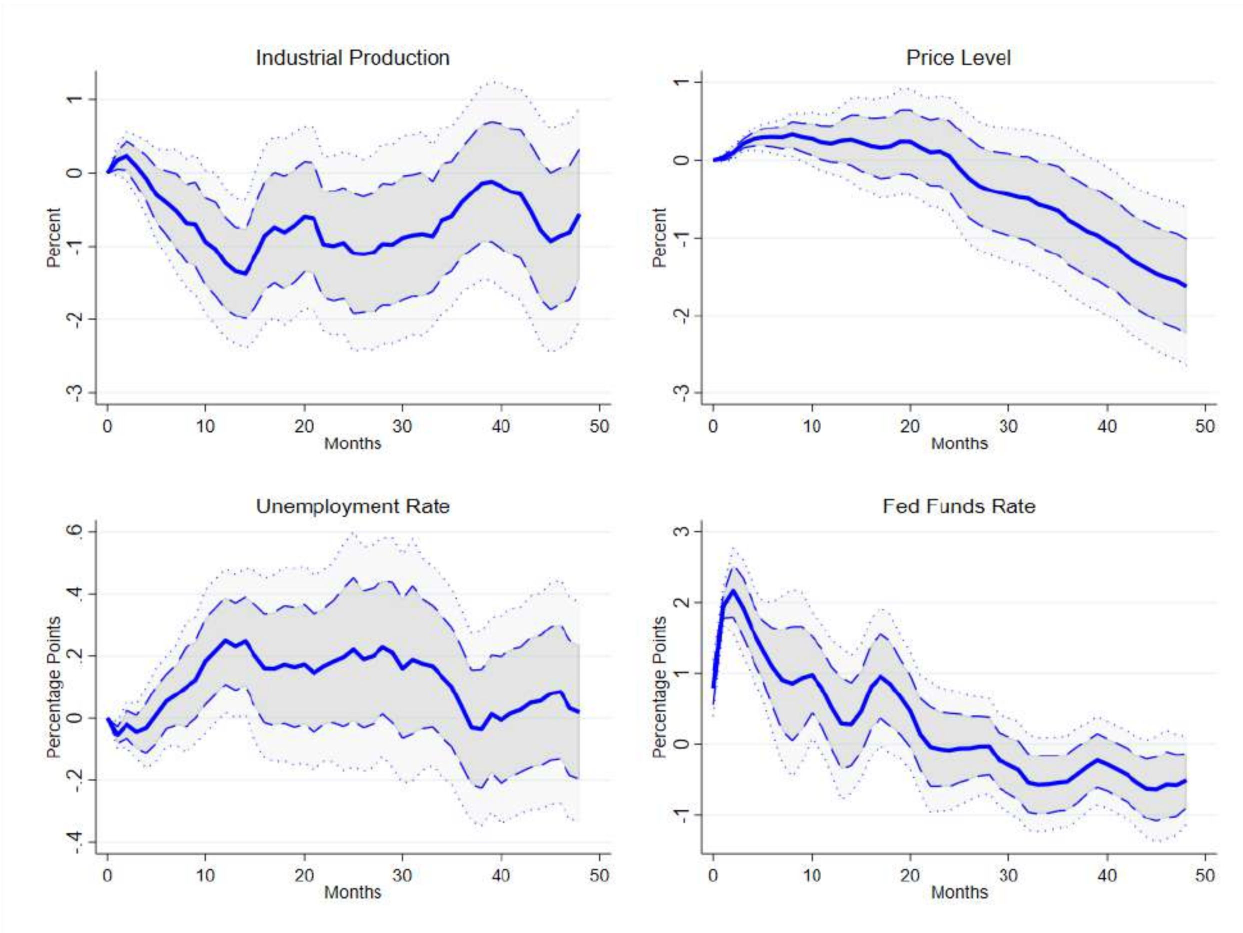
Figure C.1 presents the replication of the results of Ramey (2016) for the impact of the monetary policy shock on key macroeconomic variables. The regressions incorporate the recursiveness assumption and for the US have 2 lags of the log of industrial production, the log of the price level, the unemployment rate, the effective federal funds rate, and the log of the commodity price index. We can see that industrial production and the unemployment rate exhibit a mild expansion puzzle at the first months as it was previously documented in the literature. Industrial production decreases by around one percent at peak and inflation starts decreasing only after 2.5 years and reaches a decline of almost 2 percent after 4 years. The unemployment rate increases by 0.2 percentage points at the peak effect.

The IRFs are remarkably similar when we use a local projection instrumental variable (LP-IV) framework. Figure C.2 displays the IRFs when we do not use directly the monetary policy shock proxy in the regression, but when we use it as an instrument for the structural monetary policy shock. The implementation is to instrument the change in the federal funds rate with the narrative shock measure. The regressions present the same controls and recursiveness assumption. The results could be different if the shock proxy measures the true shock with noise. This does not seem to be a concern in this framework as the IRFs are similar across figures C.1 and C.2. Furthermore, the first stage robust F-statistic is high at 39.04, so there does not seem to be any weak instrument problem. We can interpret these

IRFs as what is the impact of a monetary policy shock that raises in the current month the effective federal funds rate by 1%. Industrial production declines by at most 1.6 percent at around one year, the price level declines by 2% after 4 years, and the unemployment rate increases by 0.7 percentage points, again at around one year.

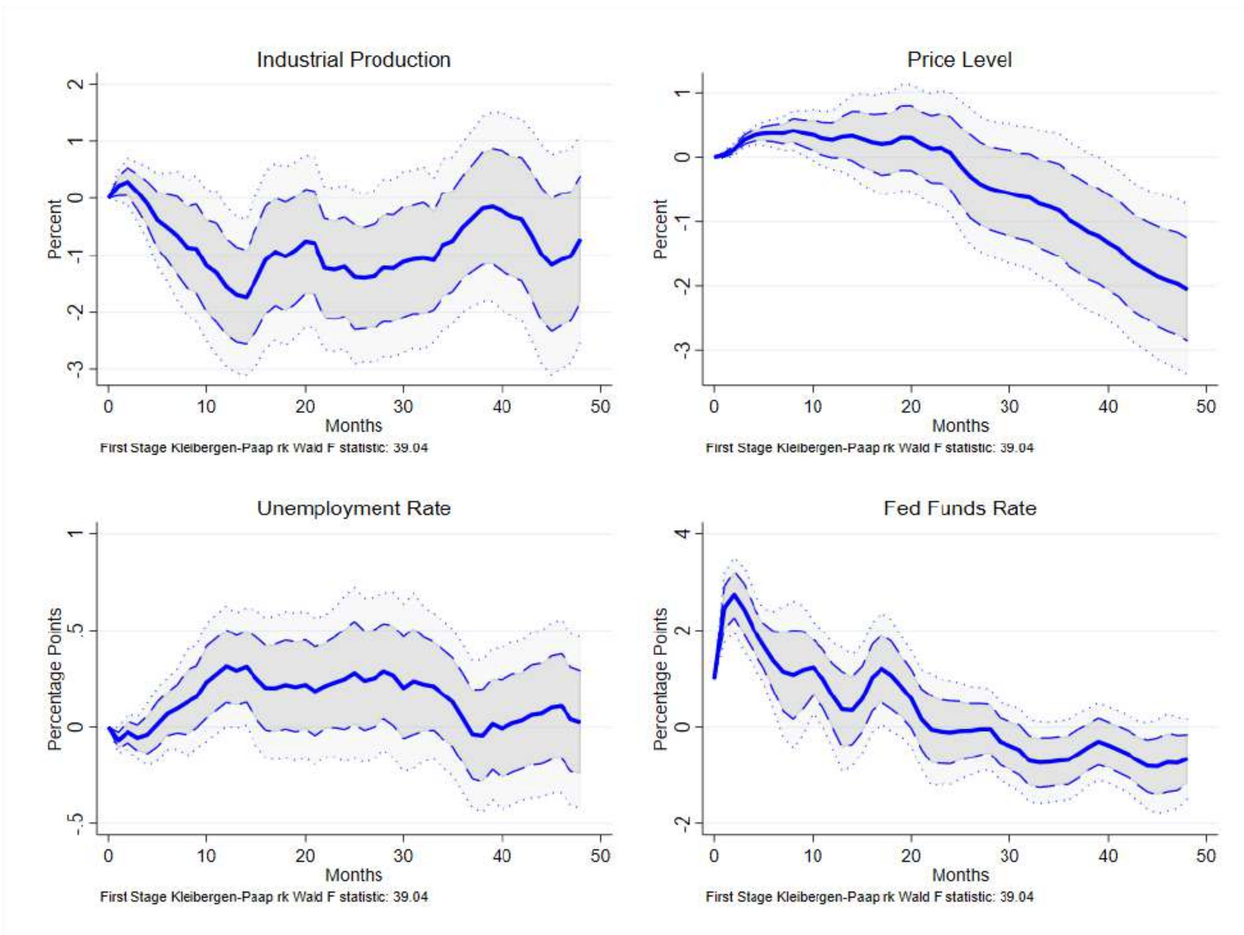
Figures C.3 and C.4 show the same regressions for the local projection without any interaction term at quarterly frequency. Figure C.3 presents the reduced form regressions and Figure C.4 presents the results in the LP-IV framework. In these regression I added GDP as a dependent variable and as a control with two lags and the recursiveness assumption. Additionally, I add a dummy for each quarter to control for seasonal effects. If we start with LP results from Figure C.3 we can see that industrial production declines at a peak of 1 percent, whereas GDP declines by -0.5 percent. For both variables the peak decline happens after around one year. The price level declines by about 1.5 percent after two years. Finally, the Federal Funds rate increases by more than 1.5 percentage points. The overall results are similar to the monthly ones. If we turn to the LP-IV results in Figure C.4, we can see how the pattern is th same as in the LP regressions, with magnitudes being lower as we normalize the monetary policy shock to have a one percentage point effect on the Federal Funds Rate on impact.

Figure C.1: Unconditional local projection regressions for the US



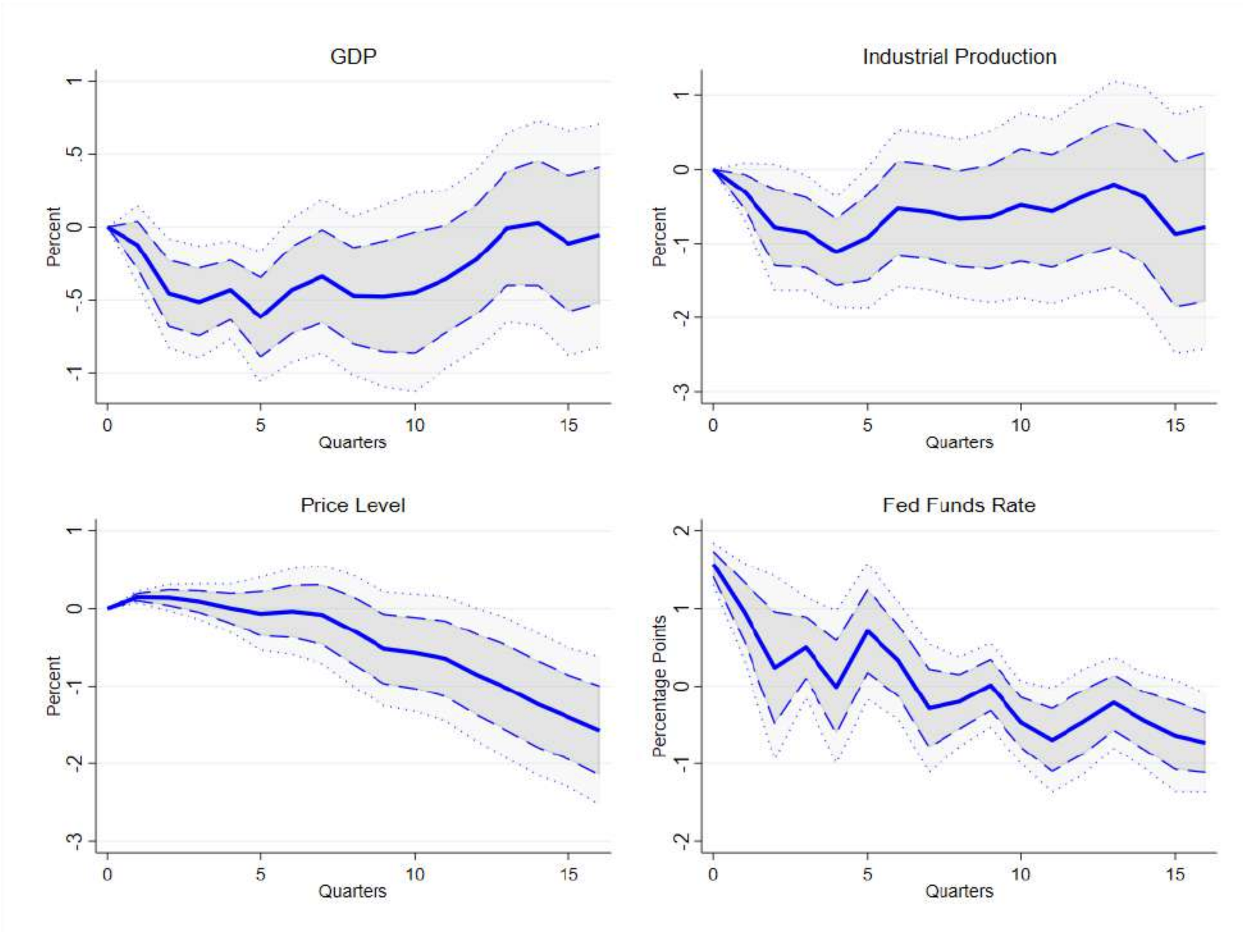
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate.

Figure C.2: Unconditional local projection instrumental variable regressions for the US



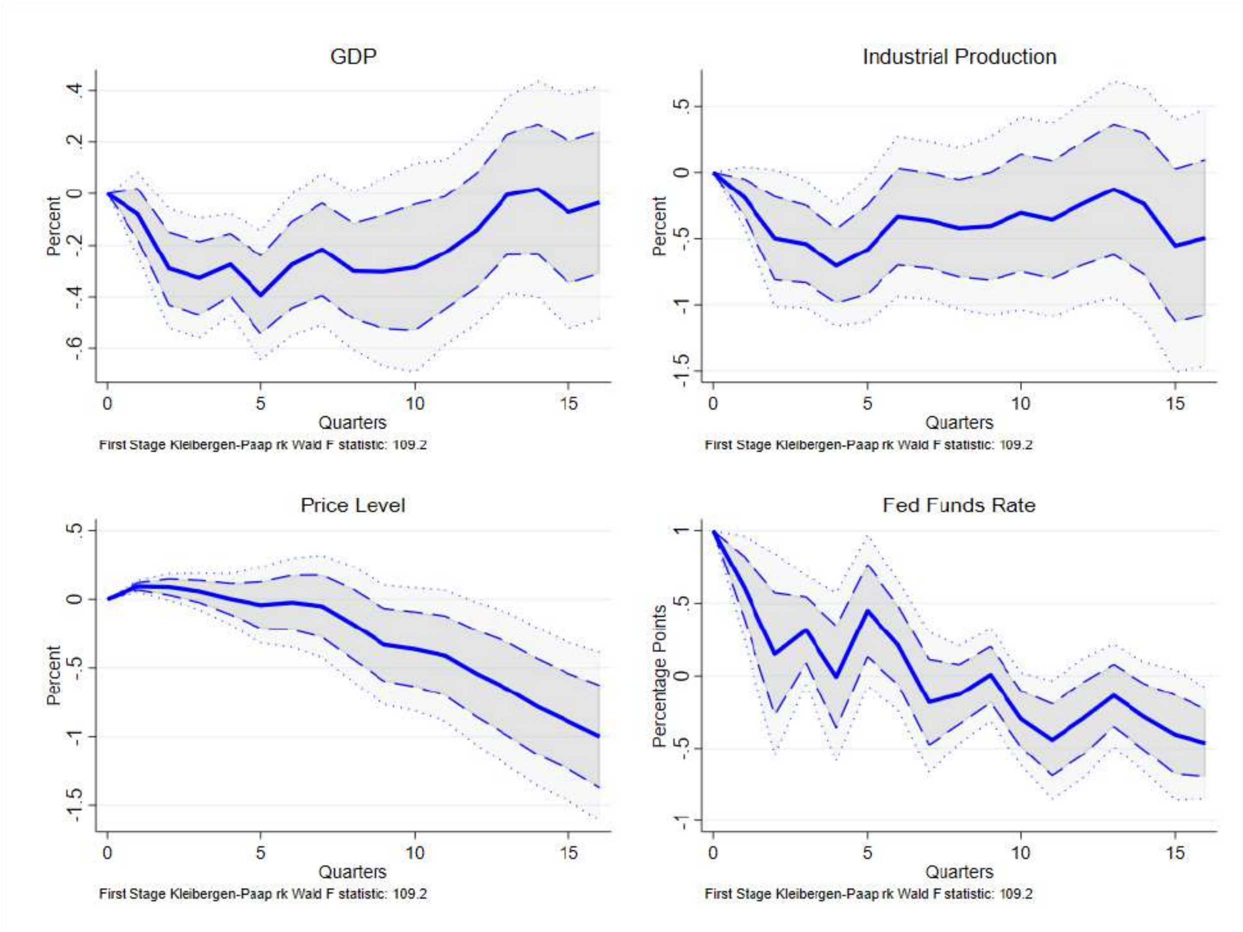
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. The instrumented variable is the change in the Fed funds rate. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate.

Figure C.3: Unconditional local projection regressions for the US at quarterly frequency



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969q1 to 2007q4 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, GDP, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, GDP, the price level, the commodity price index, the unemployment rate, and the Fed funds rate.

Figure C.4: Unconditional local projection instrumental variable regressions for the US at quarterly frequency



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969q1 to 2007q4 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. The instrumented variable is the change in the Fed funds rate. Regressions performed with the recursiveness assumption on industrial production, GDP, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, GDP, the price level, the commodity price index, the unemployment rate, and the Fed funds rate.

C.2 Local Projection Instrumental Variables Results

Local projection regressions using the shock measure directly have the benefit of being very transparent and of not imposing any normalization on the effect of the monetary policy shock contemporaneously on interest rates. On the other hand, with an instrumental variable approach we normalize the monetary policy shock to have the impact of increasing interest rates by one percent on impact, this implies in the current framework that a monetary policy shock cannot have a differential effect on interest rates on impact depending on the level of duration to GDP. In Figure 2, we could see that interest rates increase mildly less if there is relatively longer duration to GDP. For this reason I present the baseline results with reduced form local projections, however, LP-IV have a number of advantages. First of all, if the instrument is measured with noise, inference is valid under LP-IV but might be biased under LP. Moreover, LP-IV provides a test of instrument strength, which is particularly useful for the interaction term I am proposing.

Figure C.5 presents the results for the interaction term coefficients of (3), $\beta_{2,h}$, and Table C.1 presents the first stage regression coefficients of (3): γ_{11}, γ_{12}, γ_{21}, and γ_{22}. The LP-IV presented here follow the same specification as the baseline LP presented in Figure 2: monetary policy shocks are estimated with the updated Romer and Romer method. Each regression incorporates the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. Finally, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP.

From Figure C.5, we can see how the interaction coefficients are very close in magnitude, path and significance to the reduced form coefficients presented in Figure 2. An increase by one standard deviation of duration to GDP attenuates the effect of a contractionary monetary policy shock by almost 3% at peak after 2 years. There is no statistically significant differential effect of public debt duration for the effect of monetary policy on the price level. The mediating effect on industrial production is similar on unemployment, whereby at peak the interaction coefficient reaches -0.7 percentage points. The small increase in magnitude in the LP-IV compared to the LP results in the interaction regressions is in line with the difference in magnitude we can see in the linear average regressions.

The first stage Kleibergen-Paap robust F-stat is high at 48.02, which is well above the 10

Table C.1: First Stage Regressions

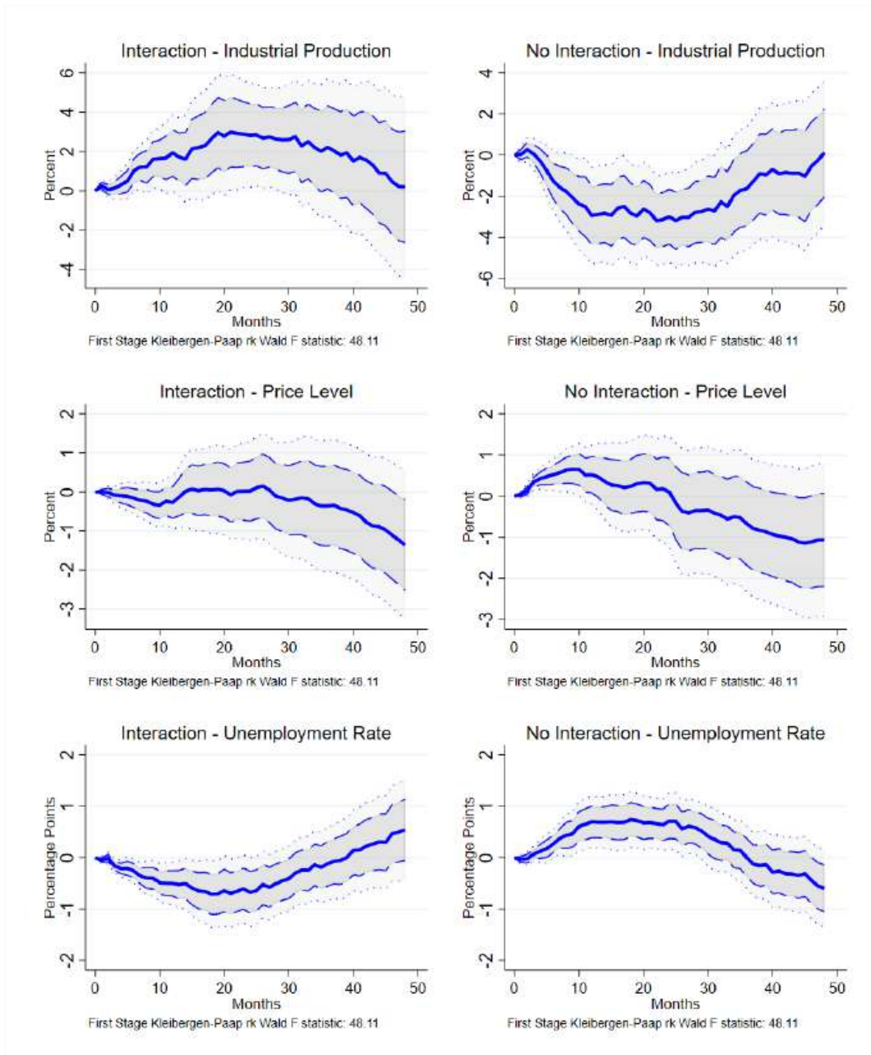
VARIABLES	(1) $\Delta i_{t;t-1}$	(2) $\Delta i_{t;t-1}DurGDP_{t-1}$	(3) $\Delta i_{t;t-1}$	(4) $\Delta i_{t;t-1}DurGDP_{t-1}$
$Shock_t$	0.963***	0.203	1.074***	0.349*
	(0.219)	(0.152)	(0.299)	(0.202)
$Shock_t DurGDP_{t-1}$	-0.198*	0.455***	-0.172	0.454***
	(0.101)	(0.101)	(0.132)	(0.121)
Observations	467	467	467	467
Controls	Recursive	Recursive	Minimal	Minimal

Notes: Newey-West standard errors in parentheses. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. The depended variables are the instrumented variables in the LP-IV framework; they are the change in the Fed funds rate and the interaction between the Fed funds rate change and the lagged duration to GDP. The first two columns show the first stage regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, these two regressions include the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The second two sets of regressions show the same first stage regressions with minimal controls, that is, controlling only for one lag of duration to GDP.

rule of thumb for weak instruments. Notice that this F statistic tests jointly if $Shock_t$ and $Shock_t DurGDP_{t-1}$ are a strong set of instruments for $\Delta i_{t,t-1}$ and $\Delta i_{t,t-1}DurGDP_{t-1}$. This is the relevant statistic for the object of interest, however, it is still informative to examine the first stage more in detail. The first 2 columns of Table C.1 do that, they show γ_{11}, γ_{12}, γ_{21}, and γ_{22} with the same controls as in Figure C.5. In addition, columns (3) and (4) show the same first stage regressions while controlling only for $DurGDP_{t-1}$ as a robustness check. From examining the first two columns we can see how the instrument on its own has a strong, almost one to one, effect on the change in the federal funds rate. Moreover, when duration to GDP is higher, the effect of a monetary policy shock on impact is lower[40]. Similarly, the main effect on $\Delta i_{t;t-1}DurGDP_{t-1}$ is due to $Shock_t DurGDP_{t-1}$, with a positive, and statistically significant impact. The results columns (3) and (4) show that the results are robust in excluding the macroeconomic controls.

[40]This mirrors the last row of Figure 2.

Figure C.5: Local projection instrumental variable baseline interaction regressions for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. The instrumented variables are the change in the Fed funds rate and the interaction between the Fed funds rate change and the lagged duration to GDP. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. Each panel shows the interaction coefficient between the instrumented change in the Fed funds rate and duration to GDP.

C.3 Test on Monetary Policy Effects when Duration to GDP is High

From Figures 2 and C.5 we can conclude that monetary policy has stronger contractionary effects when duration to GDP is low, and that increasing duration to GDP lowers the contractionary monetary policy effects on industrial production. However, as the coefficient estimated on the interaction is high, one might wonder if the effect of an increase in interest rates due to a monetary policy shock turns positive on economic activity when duration to GDP is high. In this section, I present evidence that this is not the case. Specifically, when duration to GDP is high the effect of an increase in interest rates turns insignificant on industrial production.

The hypothesis tested is whether at each horizon h the effect of a monetary policy shock is different from zero when duration to GDP is one standard deviation above its historical mean. I conduct the test both with the reduced form specification in (2) and the LP-IV in (3). The empirical specification is the same as in Figures 2 and C.5 for LP and LP-IV respectively, with Newey and West (1987) standard errors, the recursiveness assumption, and two lags of the macroeconomic controls. If we re-run the specification with duration to GDP standardized the test is simply: $H_0 : \beta_{1,h}^{std} + \beta_{2,h}^{std} = 0$ for each horizon $h = 0, \cdots, H$.

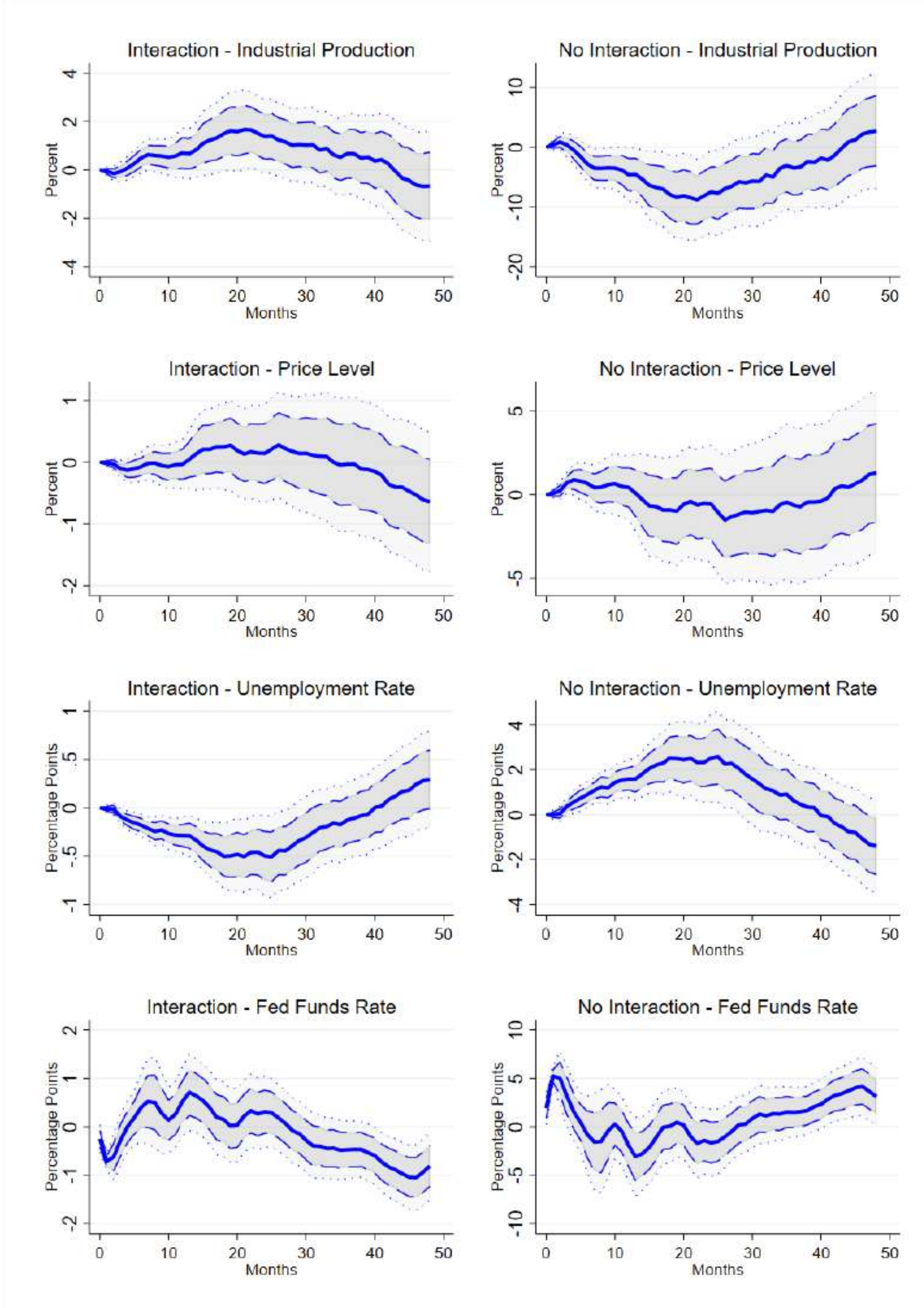
The results are economically the same across the LP and LP-IV specifications. The p-value associated with the test is smaller than 0.05 only for the first month following the shock, at $h = 1$. This is due to the activity puzzle that we can see also on the linear average regressions in C.1 and C.2. For each following horizon ($h = 2, \cdots, 48$) we have p-values all above 0.10, indicating that we cannot reject the hypothesis that monetary policy effects on economic activity are not statistically different from zero at the 90% confidence level.

Overall, we can conclude that monetary policy has relatively lower contractionary effects on industrial production when duration to GDP is higher, and for high levels of duration to GDP the effects turn insignificant.

C.4 Results with Macaulay Duration

Figure C.6 shows a robustness check where the standard Macaulay duration replaces the Macaulay duration to GDP. This duration is measured in years and the interaction coefficients still show the Macaulay duration divided by its own standard deviation. Overall results point in the same direction as the those with the new Macaulay duration to GDP but are generally less precisely estimated. This is not surprising, as what matters for the insurance mechanism of the maturity structure is the overall amount of insurance: the amount of insurance over GDP, and not per unit of debt. The coefficients on the first column represent how much more (or less) is monetary policy effective on the left hand side variable when public debt has a one standard deviation longer duration. The no interaction term in the second column refers to the impact of monetary policy on a left hand side variable when the government has a zero years maturity of public debt. The closest empirical counterpart would be to have all public debt that needs to be refinanced overnight. On the first row, we can see the impact on industrial production. At peak having a one standard deviation longer debt Macaulay duration implies having a monetary policy which is more than 2% less effective on output. This coefficient is less precisely estimated than the coefficient on Figure C.1. If all government debt were overnight the impact on industrial production would be massive at almost -10% at peak, but this coefficient is badly estimated. We can find similar results for all remaining variables, the direction of each IRF is the same as in Figure C.1, but the estimates are much less precise.

Figure C.6: local projection regressions with Macaulay duration for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the interaction term of the shock with the Macaulay Duration, the second column shows the shock term not interacted. Each row shows a different LHS variable.

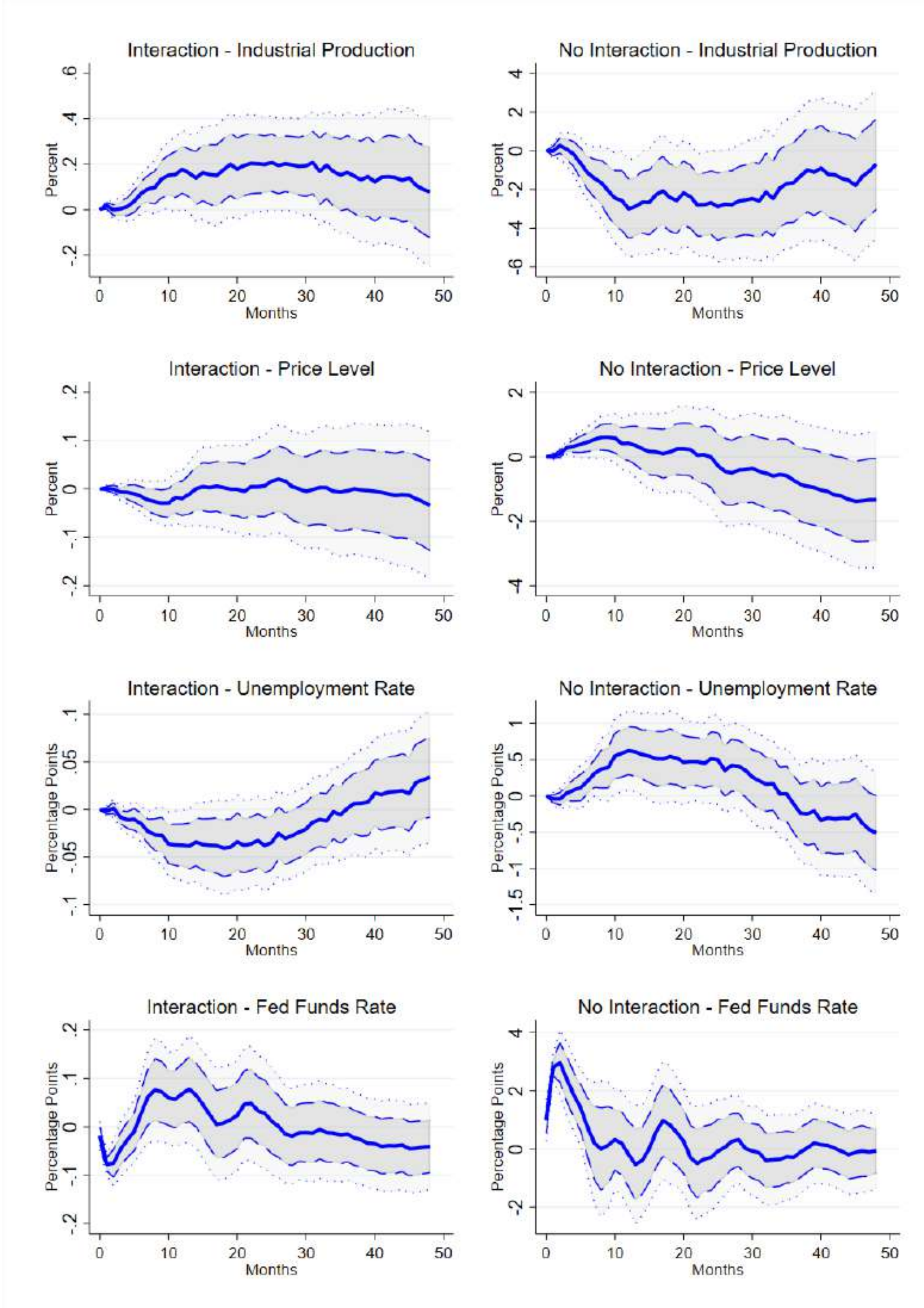
C.5 Results with Public Debt

Duration to GDP is the measure that correctly captures the insurance mechanism of fixed rate long maturity debt from the perspective of the fiscal authority. The reason is that Macaulay duration itself captures this insurance mechanism per unit of debt, that is, by how much the market value of one unit of public debt would increase following a one percent decline in interest rates. By scaling the measure to the overall amount of public debt to GDP in the economy we can find the relevant metric for the fiscal authority, which cares about the insurance on interest payments over GDP. An interesting question that arises is whether it is possible to separately identify the roles of Macaulay duration and debt to GDP. Unfortunately, this is quite challenging with the current strategy. The reason is that we would need to use three highly collinear variables in the local projection regressions: the monetary policy shock, the interaction of the monetary policy shock with Macaulay duration, and the interaction of the monetary policy shock with public debt to GDP.

Figure C.8 presents out of completeness the results of this exercise. We can see that IRFs show large swings generally associated with multicollinearity. The first column shows the interaction term of the monetary policy shock with Macaulay duration, the second column the interaction term the monetary policy shock with public debt to GDP, and the third column the coefficients associated with the monetary policy shock alone. The first row shows industrial production as a outcome variable, the second the price level, the third unemployment, and the last the Fed funds rate. We cannot see much for industrial production, as there are large swings at the end of the sample. Moreover, the price level regressions are not significant across the board, and the Fed funds rate regressions swing around due to multicollinearity. The only interesting results can be seen in the initial periods (up to the third year) of the unemployment regressions. There, we can see how higher duration of public debt leads to a lower effect of monetary policy on unemployment, there does not seem to be an effect of debt to GDP, and the no-interaction regression points to a contractionary effect of monetary policy on unemployment when there is debt tends to zero and any such debt is overnight. The unemployment results point to the mediating role of maturity once we control for debt levels. However, the results are quite unstable due to multicollinearity so should only be taken as suggestive.

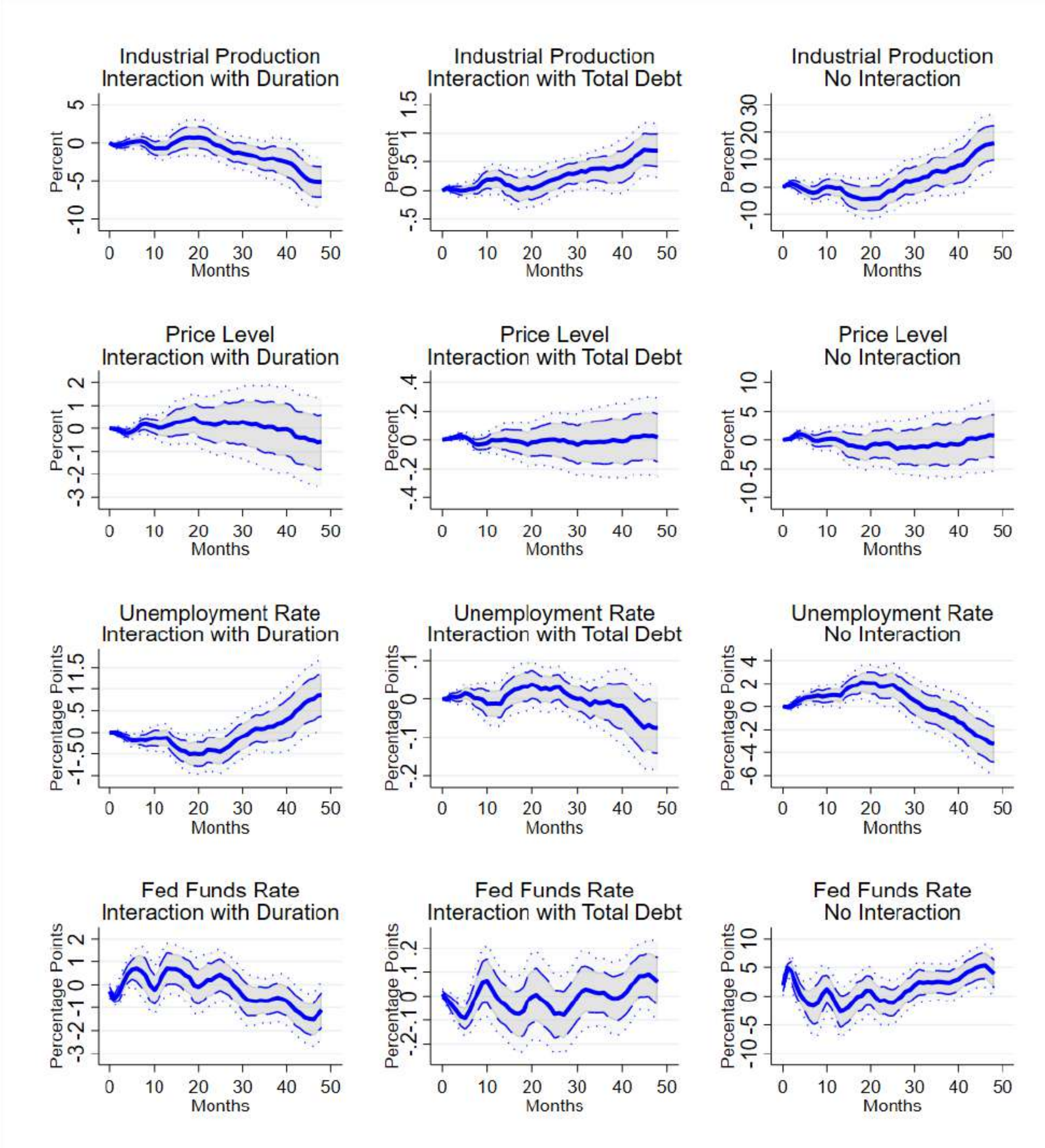
As an additional robustness check, I present results also for public debt over GDP on its own. Public debt is constructed from the same data as the duration measures for consistency, I only use marketable nominally-fixed rate bonds held by the general public. As public debt over GDP is correlated with the Macaulay duration, it is not possible to identify separately the effect of public debt. However, a hint that what matters is the insurance of public debt measured as duration to GDP is that we can identify more precisely the coefficients of the baseline regressions with duration to GDP in Figure 2 rather then those with public debt in Figure C.7. Overall, we can see all IRFs pointing to the same direction, but the interaction coefficients for industrial production and unemployment (both indicating less contractionary monetary policy with more debt) are less precisely estimated in Figure C.7.

Figure C.7: local projection regressions with Public Debt for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of public debt to GDP. The first column shows the interaction term of the shock with the debt to GDP computed from nominally fixed rate marketable public bonds held by the general public, the second column shows the shock term not interacted. Each row shows a different LHS variable.

Figure C.8: local projection regressions with Public Debt and Macaulay Duration for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of Macaulay duration and public debt to GDP. The first column shows the interaction term of the shock with the Macaulay Duration, the second column shows interaction term with debt to GDP computed from nominally fixed rate marketable public bonds held by the general public, and the third column shows the shock term not interacted. Each row shows a different LHS variable.

C.6 Results with Duration to GDP Computed from Alternative Debt Definition

In the baseline specifications, I presented the results of duration to GDP for a subset of public debt: nominally fixed rate marketable bonds at market value held by the general public. The reason for this choice is a mix of data availability and that this measure is the most suited for the problem at study.

I use marketable bonds as there is data that allows to compute for each of these bond the principal and coupon payments. The exclusion of non marketable debt should not be a concern as, in the period I consider, most of US and UK public debt was marketable, the last periods in which non marketable debt paid an important role as during World Wars and during the Korean War for the US. Moreover, I exclude treasury bills which are not present in the data I use. This is not very problematic as my main statistic of duration over GDP is only mildly affected by securities with very short maturity as treasury bills. Existing treasury bills prices are not strongly affected by changes in interest rates. We can see this mathematically in (1), as the securities with short maturity j are weighted by a low value j. Furthermore, I divide (1) by GDP, which is not affected by treasury bills[41].

Duration to GDP is computed with the market value of public debt as this measures how much would public debt to GDP increase with a one percent decline in interest rates across the yield curve. Furthermore, I use the lagged value of duration to GDP and a plausibly exogenous monetary policy shock, which under the identification assumption, cannot be forecasted with prior information. Consequently, the use of market prices should not weaken identification. In a different context, Greenwood and Vayanos (2014) propose to use a similar metric with face value debt promises:

$$DurGDPFaceValue_t = \frac{\sum_{j=0}^{\infty} j b_{t,j}}{GDP_t}$$

This metric does not have a direct interpretation[42] as the baseline one but has the benefit

[41]Notice that, Macaulay duration suffers more from the exclusion of treasury bills. The reason is that, with Macaulay duration, one needs to divide by the market value of public debt, which is affected by the inclusion of treasury bills.

[42]Specifically, it gives too much weight to long debt, for a given interest rate the price of long debt much

being more stable in time. For this reason, Figure C.9 presents the same regression results as 2 with the alternative metric of duration to GDP at face value. The results point to the same direction as in the baseline: a contractionary monetary shock reduces output less the higher duration to GDP at face value. Similarly, a contractionary monetary attenuated the increases in unemployment and does not have an effect on the transmission to the price level. The difference with the regressions with duration to GDP at market value is that the coefficients on the interactions on industrial production and unemployment are less precisely estimated.

The baseline statistic of duration to GDP uses only nominally-fixed rate treasury bonds and excludes inflation-linked TIPS. The reason is that we need an additional assumption to interpret the results with TIPS debt included. Specifically, duration to GDP with nominal debt can be interpreted as how much nominal public debt over GDP increases following one percent decrease in interest rates across the yield curve. In order to add TIPS we need to make an additional assumption on how much the real yield curve (on TIPS) decreases across maturities following a one percent decrease in interest rates across the nominal yield curve. In the exercise that follows I assume a one to one increase, in line with the findings of Nakamura and Steinsson (2018) following a monetary policy shock. The resulting formula for duration to GDP with TIPS included is:

$$DurGDPwithTIPS_t = \frac{\sum_{j=0}^{\infty} \left[j p_{t,j} b_{t,j} + j \bar{p}_{t,j} \bar{b}_{t,j} \right]}{GDP_t}$$

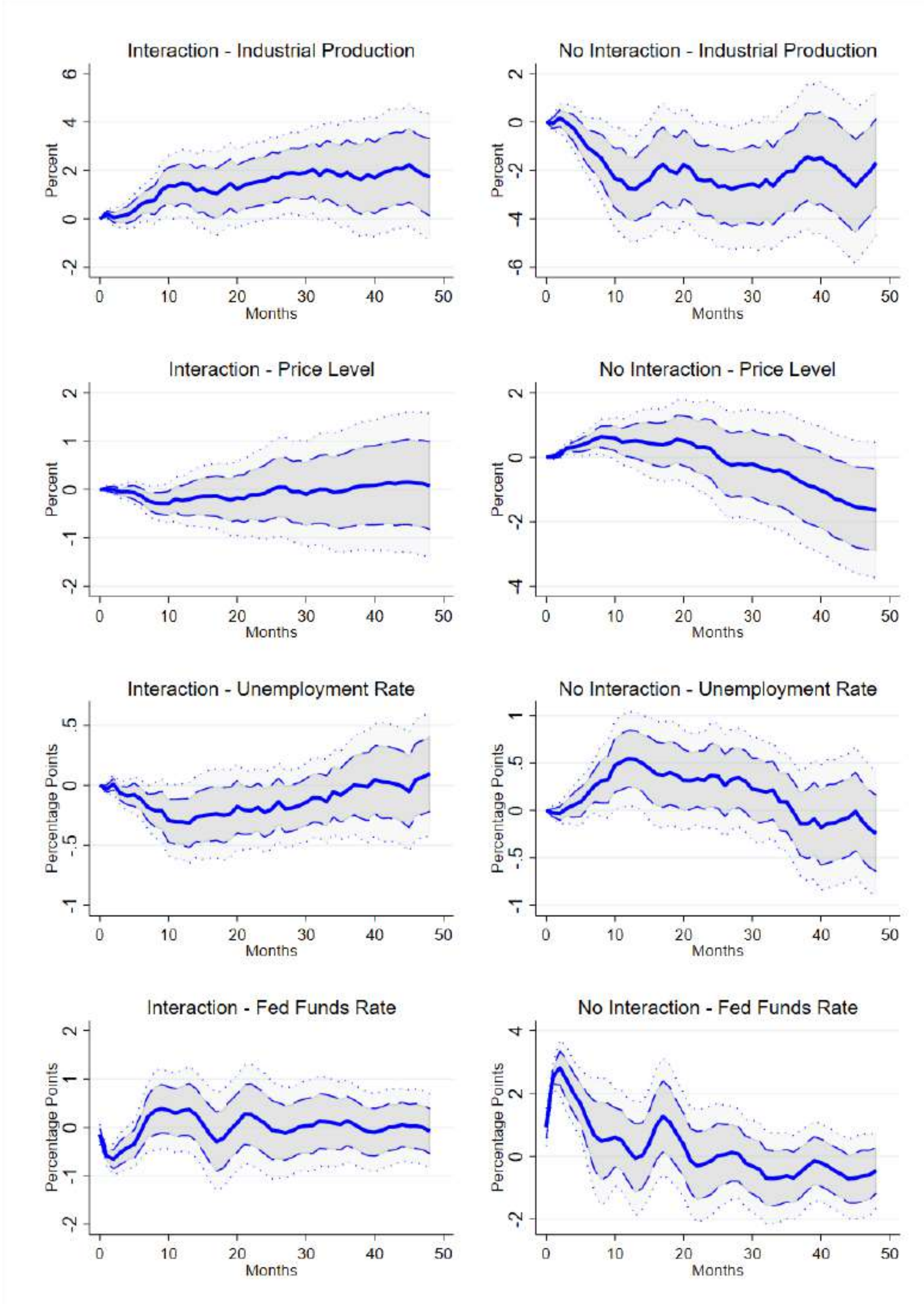
Notice that, this measure is quite close to the baseline one as the US treasury issues mainly nominal bonds and the issuance of TIPS started only in the last part of the sample, in 1999. Figure C.10 presents the results of this exercise. As expected, the results in sign, magnitude, and significance of all IRFs mirror closely the ones in Figure 2. Monetary policy is less effective on industrial production and unemployment when there is more long term debt but the effect does not go through the price level.

Public debt held by the government sector (e.g. social security or FED) should not matter to explain the results. The reason is that an increase in valuation of debt is a negative news for the treasury as they could have borrowed at a cheaper rate if debt was all overnight, but

lower than short debt even for moderately positive levels of interest rates.

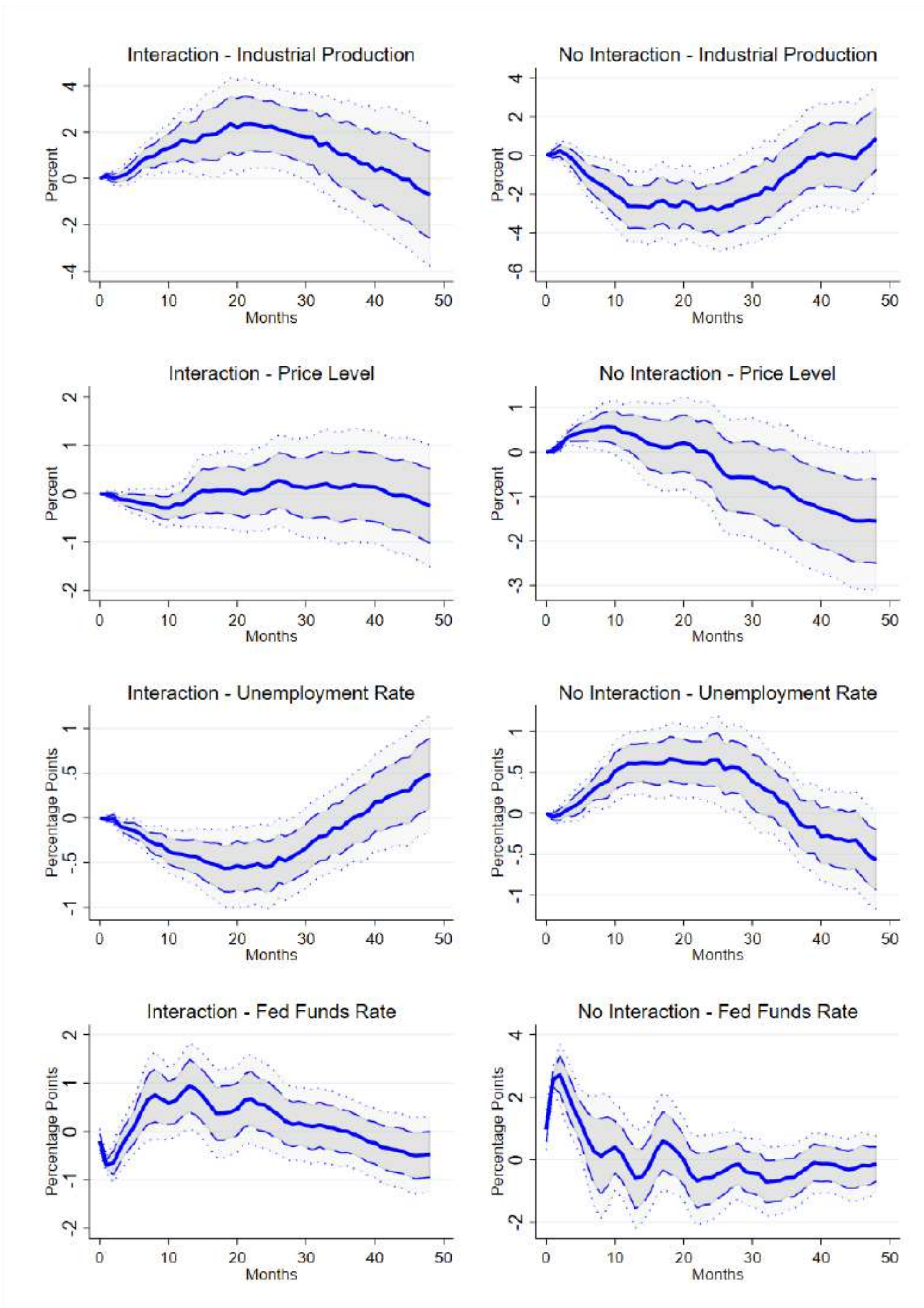
it is a positive news for bond holders. For any bond held by a government owned entity, the net effect is zero, the loss of one branch is the gain of the other. This is why I exclude the debt held by government entities and focus only on debt held by the general public from my baseline metric. However, one might argue that frictions within the government sector do not allow this consolidation as the gains or losses from government entities are not transmitted to the treasury. To assuage risks associated to this Figure C.11 displays the IRFs where duration to GDP is constructed from all outstanding nominal fixed rate marketable bonds. Again, results are remarkably close to Figure 2 in sign, magnitude, and significance across all IRFs. This is not surprising as, prior to the QE era duration to GDP of outstanding debt tracked quite closely duration to GDP of debt held by the general public. In the post QE era the FED started to intervene heavily in specific market segments, thereby lowering duration to GDP for debt held by the general public relatively to duration to GDP for debt held only by the general public.

Figure C.9: Local projection regressions with duration to GDP constructed from face value debt



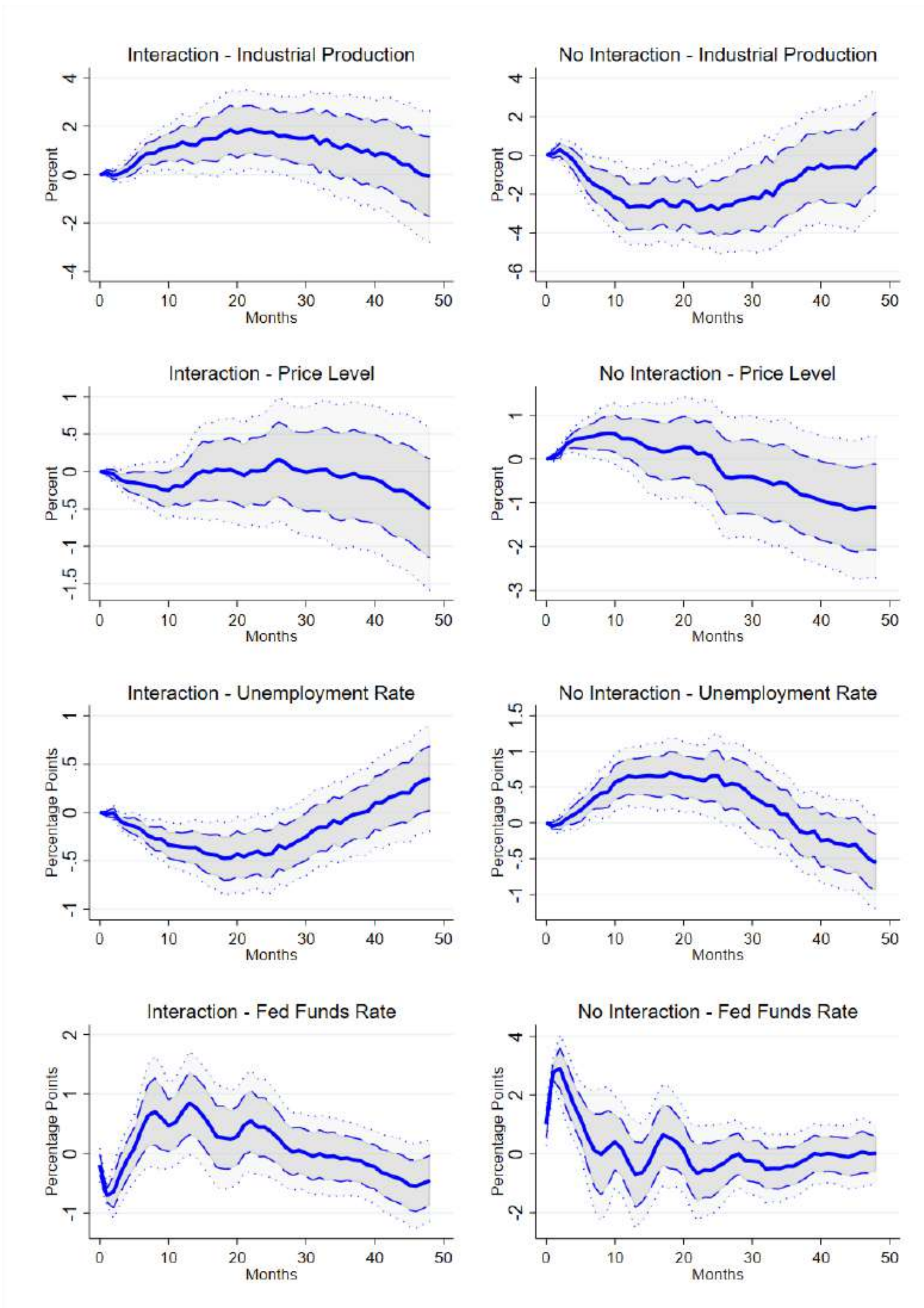
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the interaction term of the shock with the duration over GDP constructed with face value debt, the second column shows the shock term not interacted. Each row shows a different LHS variable.

Figure C.10: Local projection regressions with duration to GDP constructed from both nominal treasury bonds and inflation linked TIPS



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the interaction term of the shock with the duration over GDP constructed with both nominally fixed rate bonds and TIPS, the second column shows the shock term not interacted. Each row shows a different LHS variable.

Figure C.11: Local projection regressions with duration to GDP constructed from debt held both by the general public and the government sector



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the interaction term of the shock with the duration over GDP constructed with debt held both by the general public and the government sector, the second column shows the shock term not interacted. Each row shows a different LHS variable.

C.7 Results with the Share of Long Debt over GDP

In this section I propose an alternative measure to gauge the size of insurance provided by long debt: the share of debt promises above a threshold over GDP. I construct this measure in each period by summing over all debt promises discounted at the yield curve rate above a threshold, and then dividing this by current nominal GDP. For a threshold ι we can define the measure on monthly data as:

$$LongDebt_{t,\iota} = \frac{\sum_{j=0}^{\infty} \mathbf{1}_{\iota,j} q_{t,j} b_{t,j}}{GDP_t}$$

$$\mathbf{1}_{\iota,j} = \begin{cases} 1 & \text{if } j \geq \iota/12 \\ 0 & \text{otherwise} \end{cases}$$

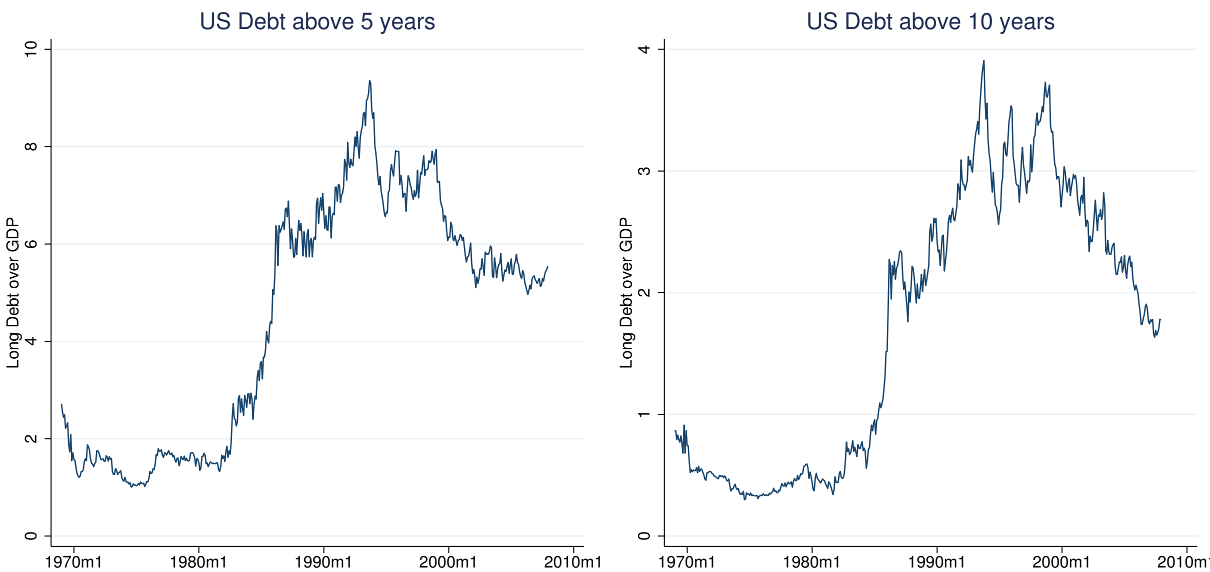
This is similar to duration over GDP with the difference that the indicator function is substituted with $j/12$ in the duration to GDP measures. Both measures give more weight to long debt than they do to short debt, with the indicator function giving only zeros and ones. This new measure has a few advantages and disadvantages. The main advantage is that it is not sensitive to mismeasurement of short debt. This is relevant in the current context as treasury bills are not present in the data I use. The disadvantages of this measure are that the thresholds are arbitrary and it does not have a direct interpretation as with duration to GDP. Duration to GDP measures the increase in market value of public debt to GDP following a one percent decrease in interest rates across the yield curve. Equivalently, it measures what is the present discounted value of interest rate costs over GDP relative increase compared to overnight debt following the same rate change. Long debt above a threshold ι over GDP measures how much debt does not need to be refinanced, or is insured, in the next ι years.

Figure C.12 presents the time series for this measure for two thresholds: 5 and 10 years. We can see that the time series properties are similar to Figure 1. In the first part of the sample the US had very little long debt outstanding, with a large increase in the mid eighties, followed by a slower increase up to the mid nineties. In the last part of this sample there was a gentle decline up to the financial crisis.

Figures C.13 and C.14 report the same specification as Figure 2 with the new interaction terms. Figure C.13 shows the results with a 5 years threshold and C.14 with a 10 years threshold. The results of this exercise are strikingly similar to the baseline. Monetary policy is less effective on output the higher the amount of long debt in the economy. Being in an economy with one percent more long debt above 5 years over GDP lowers the impact of monetary policy by one percent at its peak. When there is not debt above 5 years monetary policy has stronger effects on industrial production, up to almost 3%. As in the baseline, unemployment behaves as a mirror to industrial production and the amount of long debt does not seem to influence the transmission of monetary policy on inflation. Interest rates increase mildly less than under long debt, especially in months 2 to 4. Results with a 10 years threshold point to the same results with the interaction terms being of a higher magnitude due to a lower magnitude of long debt above 10 years.

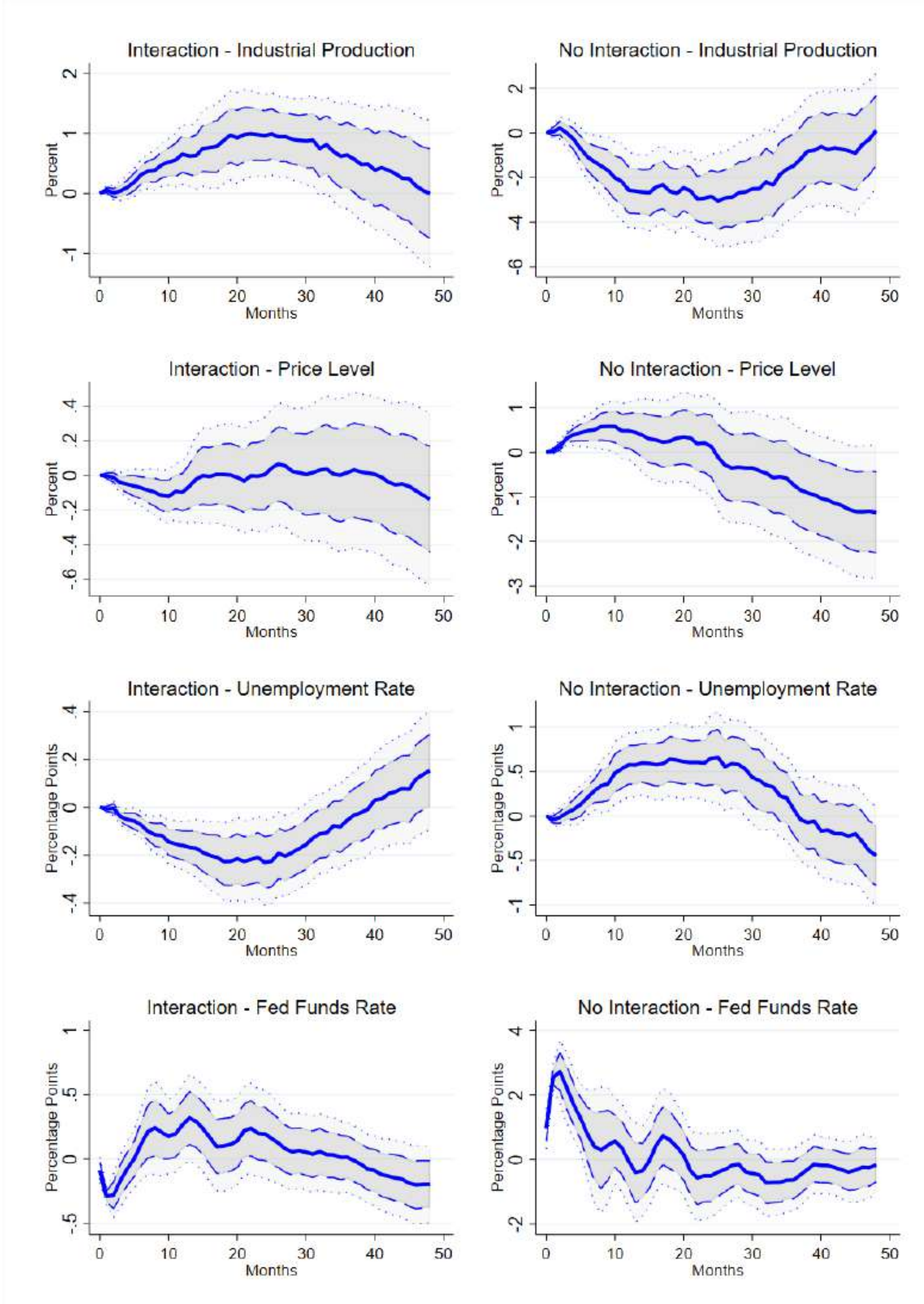
Overall, measuring the insurance mechanism provided by long fixed rate debt with a more immediate measure as long debt over GDP yields similar results as measuring it with duration over GDP. Longer maturity public debt lowers the effect of monetary policy on output but does not affect its transmission to inflation.

Figure C.12: Time series of long debt over GDP for the US



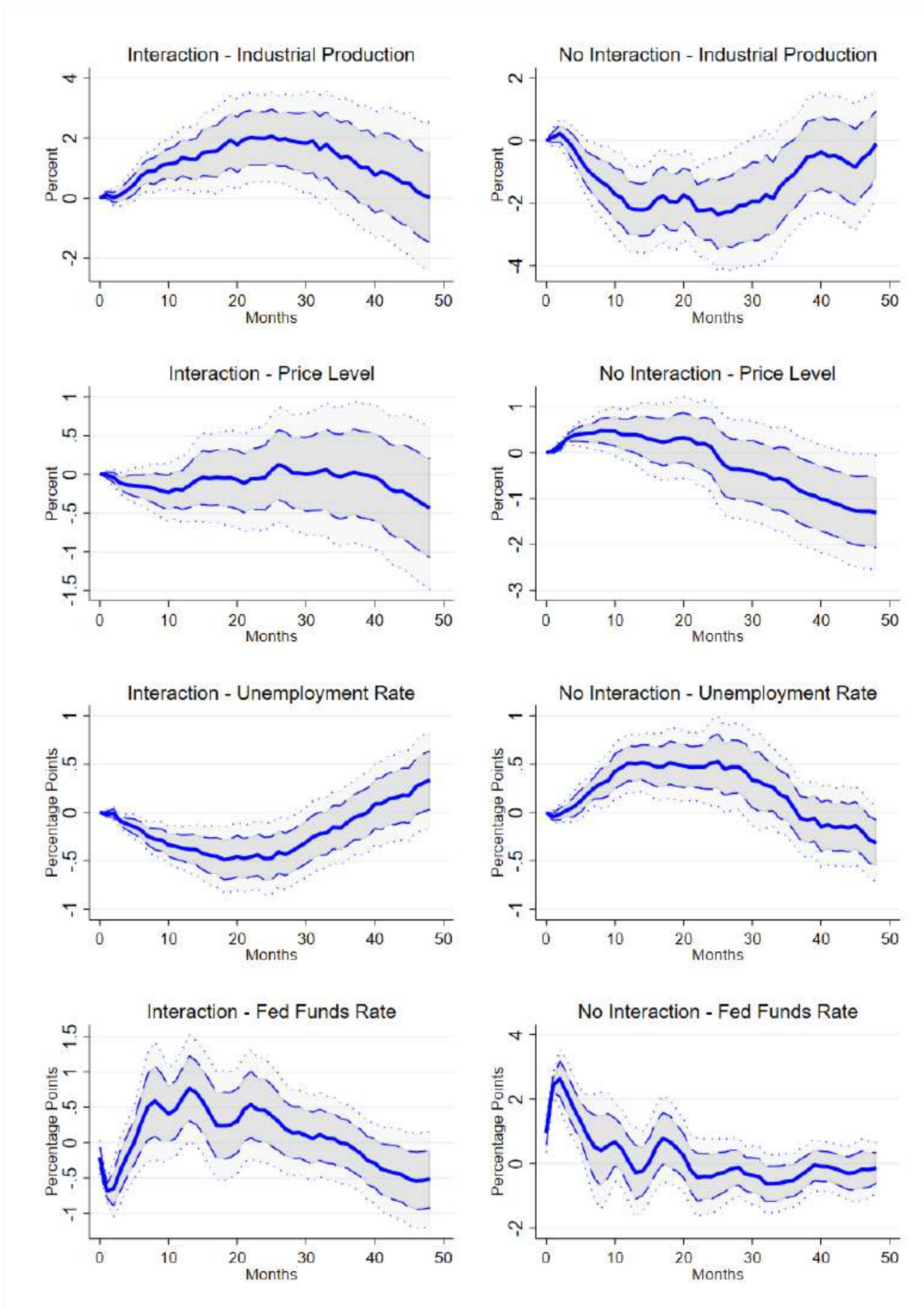
Notes: The figures show the time series for long debt over GDP for the US above two thresholds: 5 and 10 years. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The sum of public debt promises above one of the two thresholds is divided by nominal GDP. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2007m12 with US data.

Figure C.13: Local projection regressions with long debt above 5 years over GDP for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of long debt above 5 years GDP. The first column shows the interaction term of the shock with long debt above 5 years over GDP, the second column shows the shock term not interacted. Each row shows a different LHS variable.

Figure C.14: Local projection regressions with long debt above 10 years over GDP for the US

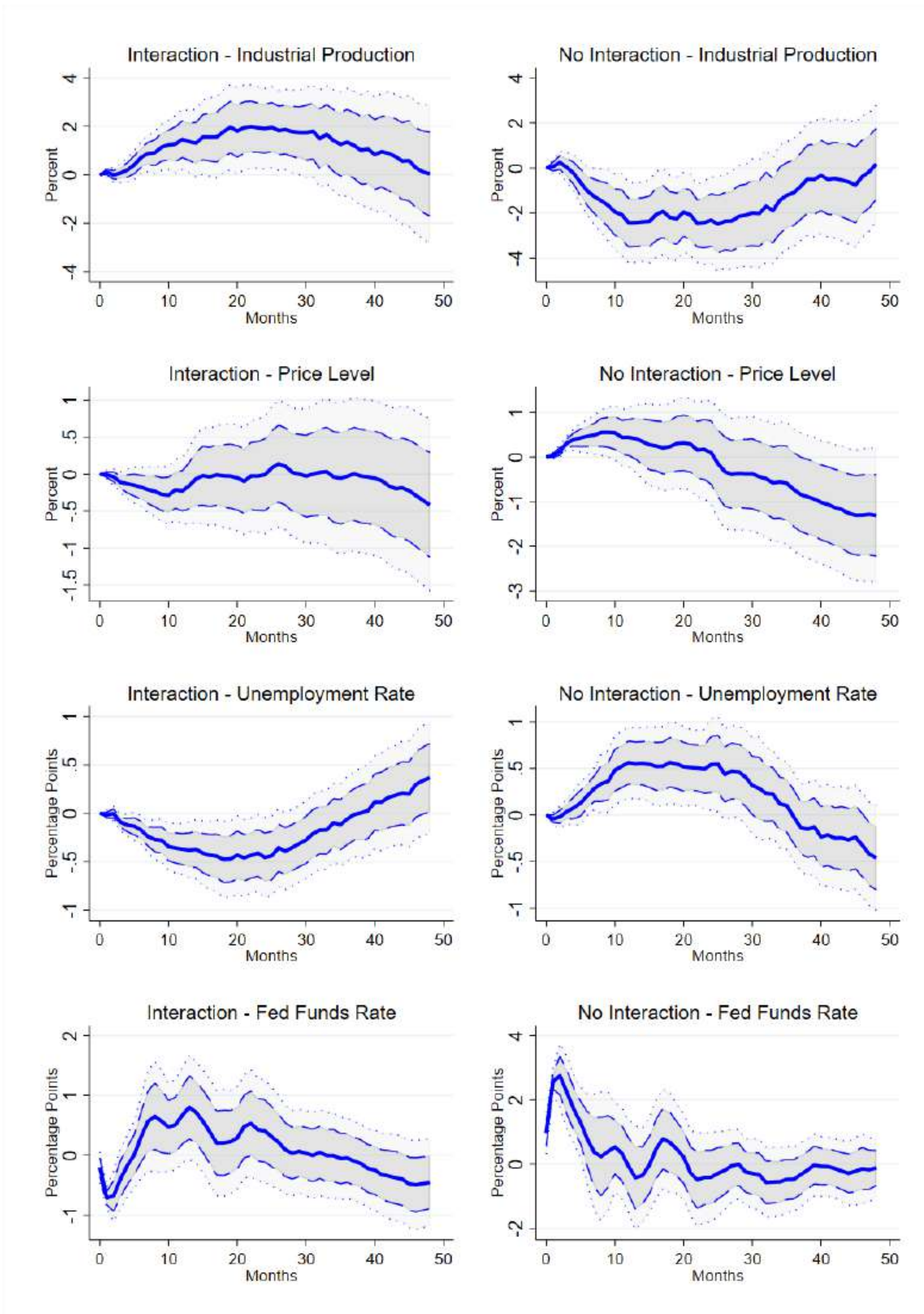


Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of long debt above 10 years GDP. The first column shows the interaction term of the shock with long debt above 10 years over GDP, the second column shows the shock term not interacted. Each row shows a different LHS variable.

C.8 Results with Deviation of Duration to GDP from Hypothetical one Period Duration to GDP

Figure C.15 presents the baseline results with the interaction term being the deviation of duration to GDP from the theoretical duration of public debt, if all debt was issued as a one quarter debt. This allows to interpret the non-interaction regression directly as in the model, where I compare the actual average duration of public debt to a one period (quarter) debt case. The results are virtually identical to the baseline ones.

Figure C.15: Local projection regressions with deviation of duration to GDP from theoretical duration of one quarter debt



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the interaction term of the shock with the deviation of actual Duration over GDP from the theoretical duration of all debt being one quarter, the second column shows the shock term not interacted. Each row shows a different LHS variable.

C.9 Results with Smooth Transition Method

In the main specification, I interacted the monetary policy shock with duration to GDP divided by its standard deviation directly. This is appropriate in this context as duration to GDP has a well specified meaning as the amount of insurance public debt maturity is providing to the government. However, one might be interested in checking the effects across regimes, across a high and a low duration to GDP regimes. In order to do this, in this section, I apply the smooth transition local projection method to my setting. This method was used to estimate the effects of fiscal policy (Auerbach and Gorodnichenko, 2017, Gorodnichenko and Auerbach, 2013, Ramey and Zubairy, 2018) and monetary policy (Tenreyro and Thwaites, 2016) depending on whether the economy is in a recession or expansion. The benefits of a non-linear local projection approach are the same as in the baseline, whereby we can test the effect of a shock in a given state/regime, without restricting the regime to stay constant. This method employs a smooth increasing transformation of the state variable of interest, duration to GDP in the previous month in this paper, as an interaction term. I follow Granger and Terasvirta (1993) as the aforementioned papers, and employ a logistic function on the standardized variable:

$$F(Z_t) = \frac{\exp\left(\theta \frac{Z_t - \bar{Z}}{std(Z)}\right)}{1 + \exp\left(\theta \frac{Z_t - \bar{Z}}{std(Z)}\right)}$$

Where θ controls the speed of transition from one regimes to the other. The reduced form specification with duration to GDP becomes:

$$y_{t+h} = \beta_{0,h}^{STLPM} + \beta_{1,h}^{STLPM} Shock_t + \beta_{2,h}^{STLPM} Shock_t F(DurGDP_{t-1}) + \beta_{3,h}^{STLPM}(L)' controls_t + \varepsilon_{t+h} \tag{C.1}$$

As $F(DurGDP_{t-1})$ is increasing and bounded between zero and one, we can interpret $\beta_{1,h}^{STLPM}$ as response to a contractionary monetary shock on y_{t+h} in a low duration to GDP regime, and $\beta_{2,h}^{STLPM}$ as the differential impact when we move from a low to a high duration to GDP regime. I follow Tenreyro and Thwaites (2016) and set $\theta = 3$[43].

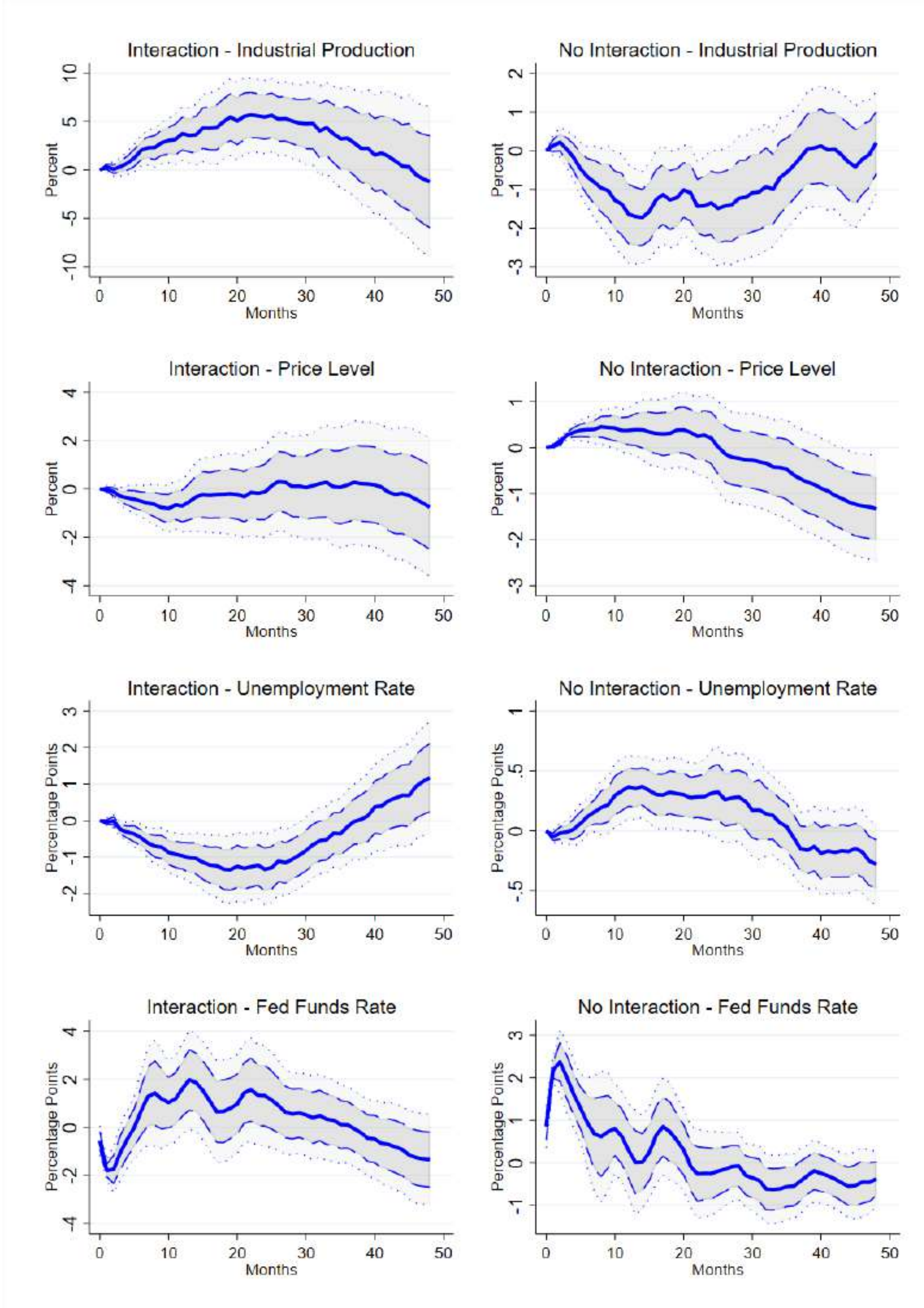
Figure C.16 presents the results of this experiment. Under a low maturity regimes we

[43] If we use $\theta = 1.5$ as Auerbach and Gorodnichenko (2017) the results are very close.

still find the same results as in the baseline, monetary policy is more contractionary than on average, at peak the shock reduces industrial production by 1.7 percent. This coefficient has a lower magnitude than the baseline response under a hypothetical overnight debt. The reason is that in this in the baseline we are extrapolating to a hypothetical overnight debt but we never observe it. On the other hand, in this exercise we are looking at the lowest observed values. If we turn to the interaction term, that is, if we move from the low duration to GDP regime to the high duration to GDP regime we see a coefficient at peak of 5%, which implies that a substantially lower effect of monetary policy on industrial production. This does not imply that the overall effect turns positive under the high duration to GDP regime, if we perform the same test as in section C.3 we find a p-value below 0.05 only for the first month, due to the activity puzzle.

The other results are remarkably similar to the baseline as well. The effect on prices is the same across maturity regimes with a reduction in line with the average linear results. Unemployment mirrors industrial production, being relatively lower under the high duration regime to GDP. Finally, also the Fed funds rate responds similarly, with a smaller effect on the first few months if we move from the low to the high duration to GDP regime, with the later response being quite similar to the low duration to GDP regime.

Figure C.16: Smooth transition local projection interaction regressions for the US



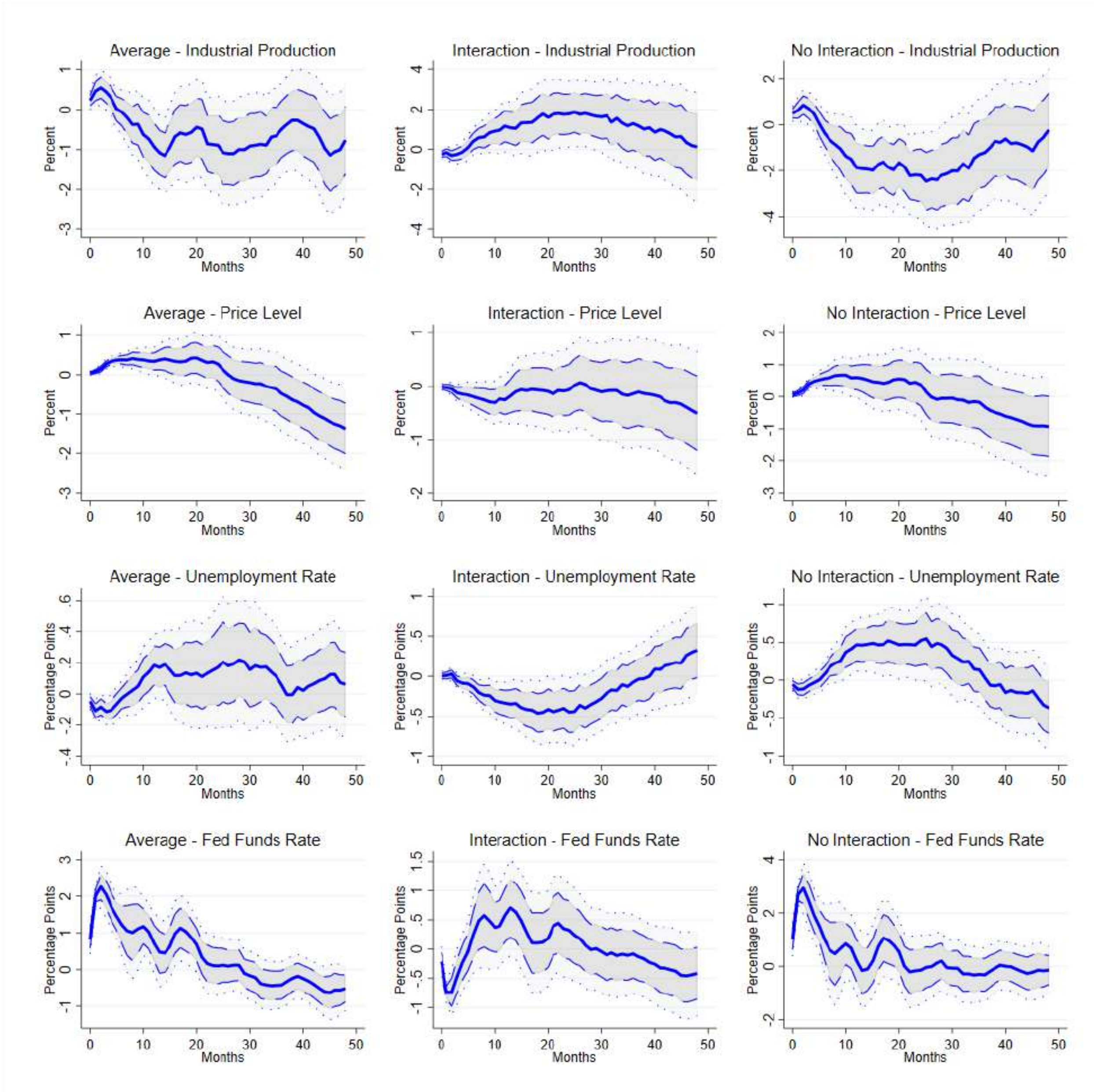
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and $F(DurGDP_{t-1})$, the smooth logistic transformation of the lag of duration to GDP. The speed of transition parameter θ is equal to 3. The first column shows the interaction term of the shock with $F(DurGDP_{t-1})$, the second column shows the shock term not interacted. Each row shows a different LHS variable.

C.10 Results without Recursiveness Assumption

The recursiveness assumption is the assumption that monetary policy cannot affect contemporaneously real variables as industrial production or monetary variables as inflation due to stickiness and lag in action by economic agents. In a univariate local projection setting, the recursiveness assumption is implemented by adding the contemporaneous variables for all the variables which cannot be affected in the same period by the monetary policy shock; in my specification these are industrial production, the price level, and the unemployment rate. In the case without external instruments, it is equivalent to identifying monetary policy shocks with a Cholesky decomposition in a VAR as Christiano, Eichenbaum and Evans (1999) do. When external instruments, such as a narrative or a high frequency instrument, are present, this assumption is not necessary to identify monetary policy shocks but it used to sharpen the identification, as Romer and Romer (2004). Ramey (2016) presents the baseline results with the narrative instrument with the recursiveness assumption and discusses the impacts of not imposing it. In her local projections without interaction terms, she finds that not imposing the recursiveness assumption yields an activity puzzle on industrial production, that is industrial production increases on impact following a monetary policy shock. Furthermore, the price puzzle becomes more pronounced without the recursiveness assumption.

We now move to the specifications with the interaction term of the monetary policy shock with duration to GDP presented in Figure C.17. This figure shows the same results as in Figures C.1 and 2 without the recursiveness assumption. The regression is the same, with the exception that contemporaneous controls for industrial production, the price level, and the unemployment rate are not present anymore. In the first and third columns, we can see the same phenomena described by Ramey (2016); there is an activity puzzle for industrial production and unemployment and a marked price puzzle. However, the conditional effect of having long vs short debt are almost the same as in the baseline. When duration to GDP is higher monetary policy lowers output less (first row, second column) and increases unemployment less (third row, second column). Moreover, there does not seem to be any differential effect on inflation.

Figure C.17: Local projection regressions without recursiveness assumption for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed without the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the average effect. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

C.11 Results with Recursive Identification

In this subsection I add an additional identification strategy for the monetary policy shock in the spirit of Christiano, Eichenbaum and Evans (1999). The identifying assumption is that monetary policy cannot have an impact on real variables and prices on the same period as the shock happens, but only on the following months. In the context of a univariate local projection this is achieved by including the contemporaneous control for industrial production, the price level, and the unemployment rate. Notice that, the interaction is between the measure of the monetary policy and the lagged value for duration to GDP. This implies that the identification hurdles specific to the interaction term are the same as in the case with external instruments. Consequently, if one is willing to believe the Cholesky-like recursive identification we interpret the results as in Figure 2. I implement this identification by running the un-instrumented version of (3):

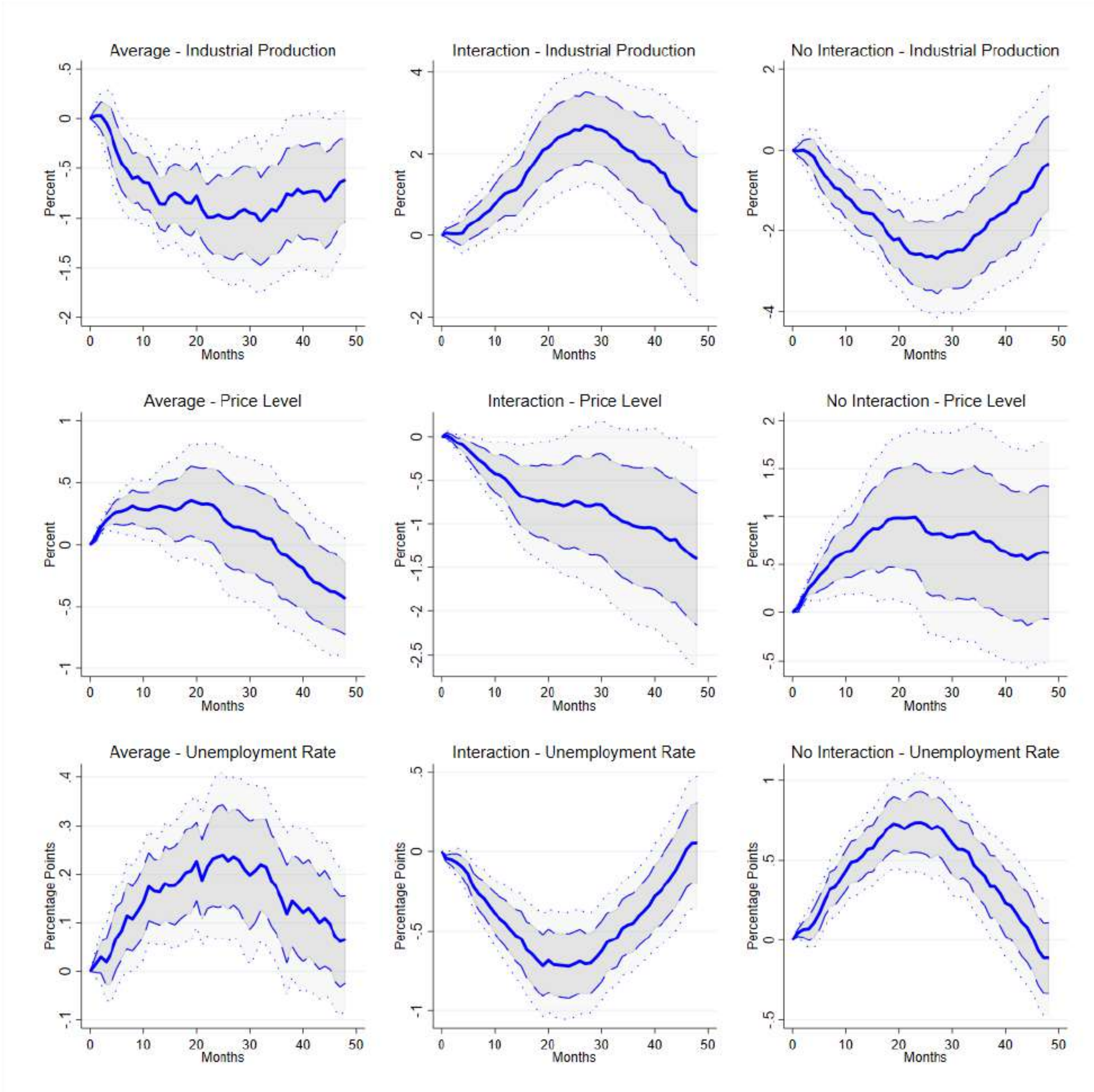
$$y_{t+h} = \beta_{0,h} + \beta_{1,h}\Delta i_{t,t-1} + \beta_{2,h}\Delta i_{t,t-1}DurGDP_{t-1} + \beta_{3,h}(L)'controls_t + \varepsilon_{t+h}$$

Where controls include the lagged value of duration to GDP, the first two lags of the Fed funds rate, and the first two lags and the contemporaneous value for industrial production, the price level, the commodity price index, the unemployment rate.

Figure C.18 presents the IRFs with the recursive identification in the same sample as in Figure 2. Even with this identification we can see a strong mediating effect of the maturity structure on the transmission of monetary policy. The higher the duration to GDP, the lower the contractionary effect of monetary policy on industrial production and unemployment. At peak, having a standard deviation of duration to GDP more lowers the impact of monetary policy by 2.5%. If all debt was overnight, monetary policy would be stronger and would reduce industrial production at almost -3%. Both the interaction term and the no-interaction terms are precisely estimated for industrial production and unemployment, even more than in the external instrument case. The results for the price level are counterintuitive, both for the conditional results and the average linear ones shown in the first row. On average, and in the case of short debt in the conditional case, we see a very long lasting price puzzle, that is, a contractionary monetary policy shock increases prices significantly for 2 years and only then starts to decline. For this reason, I would refrain to interpret the results also on the interaction. There we see a decline, although it is almost never significant.

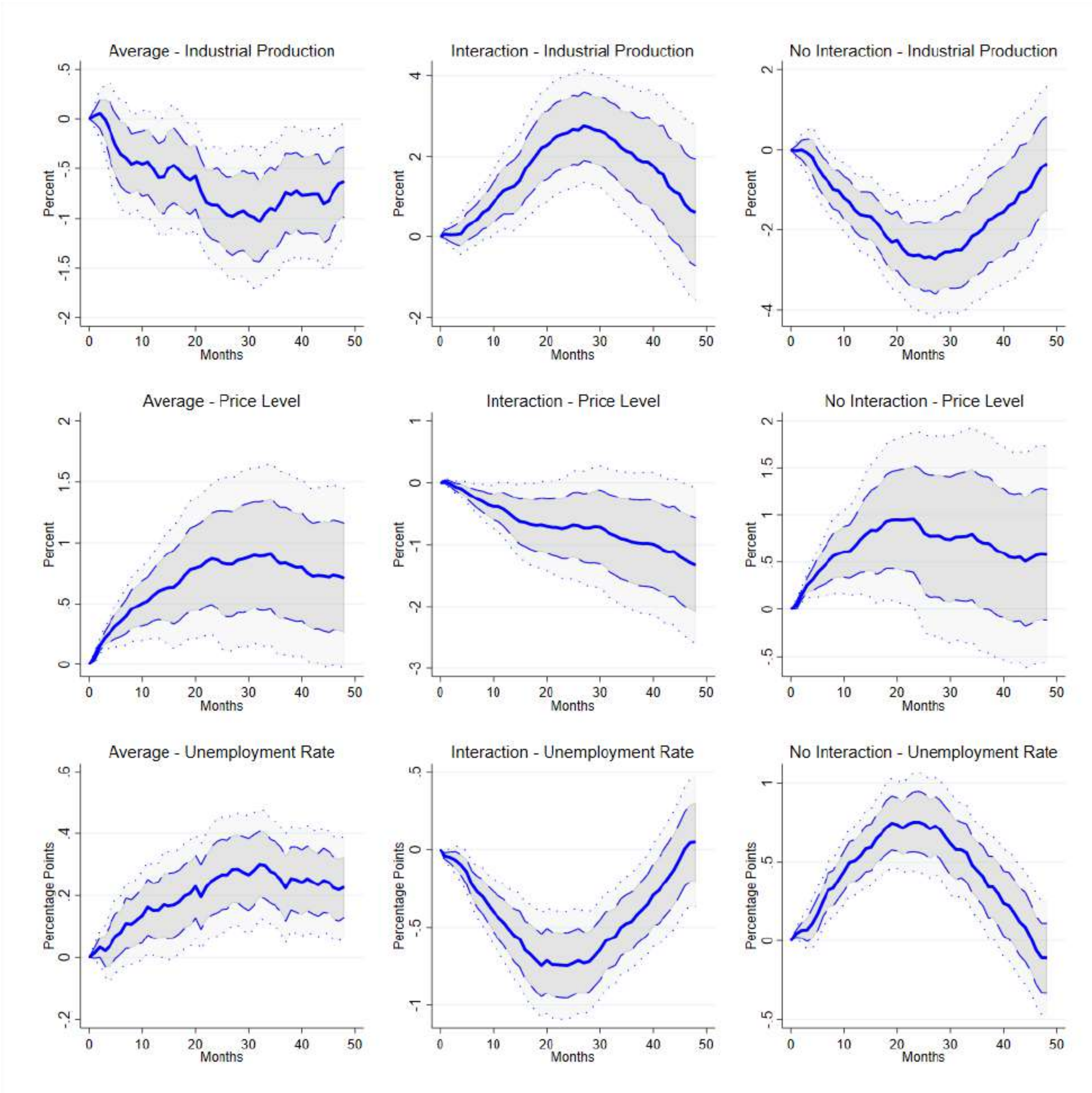
Figure C.19 presents the same IRFs on a longer sample that goes from 1959m7 to 2013m1. We can perform the regressions on a longer sample as well as we are not limited anymore by the monetary policy shock sample. We can see that the results are virtually unchanged from the restricted sample. We find a strong mediating effect of monetary policy on real variables but find a counterintuitive price puzzle for inflation which does not end even as the horizon ends. Notice that this prize puzzle persists if we include more lags of dependent variables. In an additional exercise I re-run the same specification with 6 lags, with results virtually unchanged. These results are available upon request.

Figure C.18: Local projection regressions with recursive identification for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the recursive identification. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. The interaction regression include also and one lag of duration to GDP. The first column shows the average linear regression. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

Figure C.19: Local projection regressions with recursive identification for the US with a longer sample



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1959m7 to 2013m1 with US data. Identification of the monetary policy shock is achieved with the recursive identification. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. The interaction regression include also and one lag of duration to GDP. The first column shows the average linear regression. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

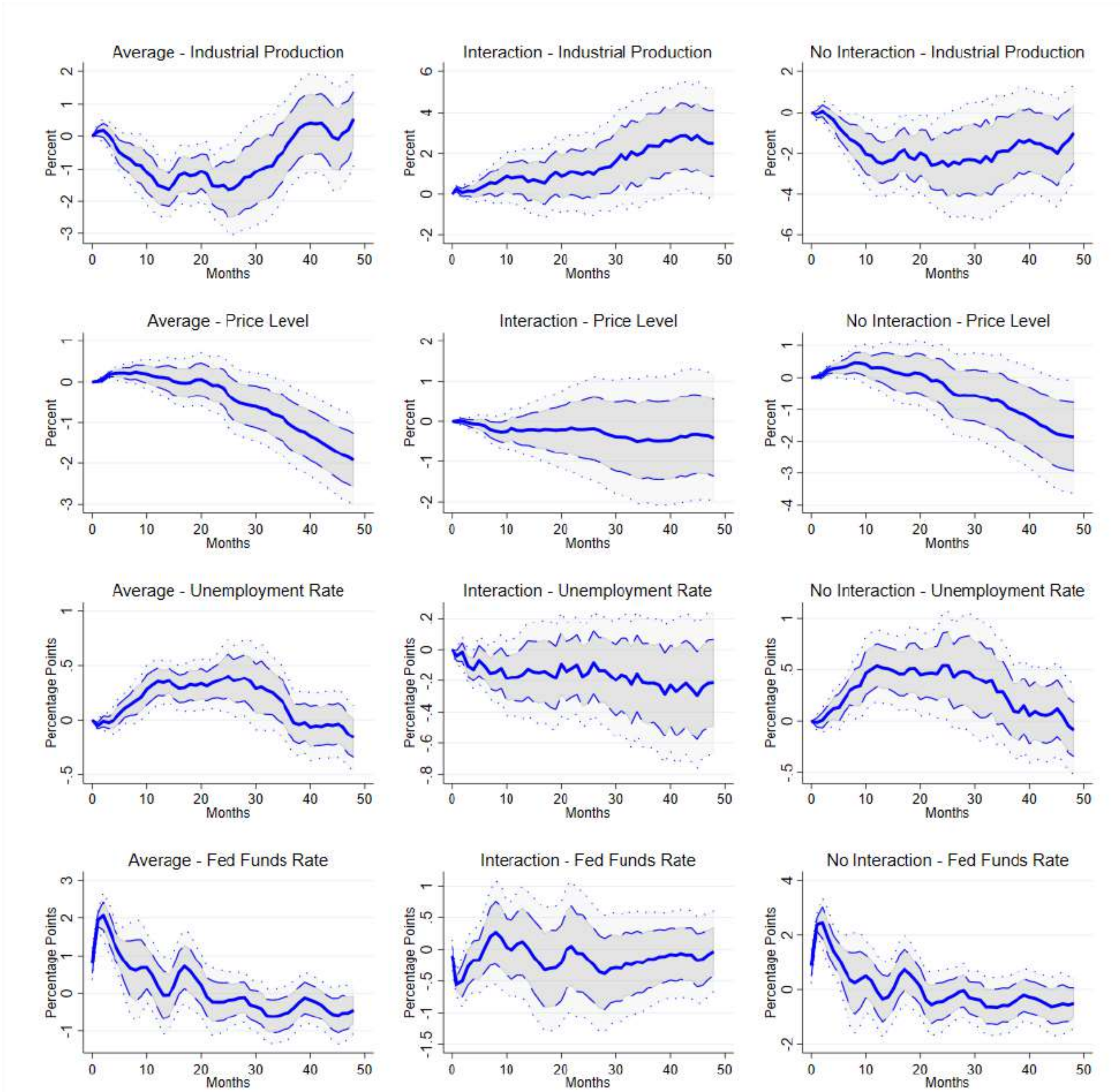
C.12 Results with Original Romer and Romer Shock

The baseline specification shows the results with the extended sample for the Romer and Romer (2004) by Yang and Wieland (2015) in order to exploit a longer time series variation. In this subsection, I show the results with the original monetary policy shock measure by Romer and Romer (2004). The sample goes from 1969m1 to 1996m12.

Figure C.20 presents the results of this experiment. The overall patterns are the same as in 2, simply estimated with lower precision, possibly due to the lower sample size. In the first row we can see how monetary policy is less effective on industrial production the higher the duration to GDP. At peak, monetary policy is about 2% less effective on industrial production when public debt has a one standard deviation higher duration to GDP. This number is similar to the 2% peak in 2. Interestingly, the peak in the baseline specification appears at around two years after the shock, but here it appears at the fourth year. Similarly, the response of inflation does not appear to differ depending on whether debt has a higher or lower duration. When debt is more long term, unemployment is relatively lower following a monetary policy shock and; on impact, interest rate increase by less, although the impact is short lived and quantitatively small.

Overall, using the original Romer and Romer narrative measure to identify the monetary policy shock brings similar results as to the updated measure.

Figure C.20: Local projection regressions with original Romer and Romer shock for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 1996m12 with US data. Identification of the monetary policy shock is achieved with the original narrative method by Romer and Romer (2004). Regressions performed without the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. The interaction regression include also and one lag of duration to GDP. The first column shows the average linear regression. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

C.13 Results with High Frequency Identification

High frequency identification has been used extensively in recent work to identify monetary policy shocks. The key idea is to use changes in federal funds future prices around policy announcements to identify monetary policy shocks that are orthogonal to the information set of financial market participants. The benefit over the narrative method is that, when the FED pursues forward guidance, there are changes in Fed funds futures which are orthogonal to past Greenbook forecast but that might be already anticipated by economic agents. These changes would not be picked up by the high frequency identification scheme. This advantage, together with the exploration of the information channel of monetary policy has contributed to the wide usage of this method in recent research, as Kuttner (2001), Gurkaynak, Sack and Swanson (2004), Gertler and Karadi (2015), Gerko and Rey (2017), Jarociński and Karadi (2020), Miranda-Agrippino and Ricco (2021).

However, this method has a big disadvantage in the context of this study: the sample for which we have data on high frequency future prices is too short, as they are available only from the January 1991. This is problematic on its own, especially in a local projection framework, as Ramey (2016) showed that Fed funds futures do not work on their own as instruments on linear average local projections. This is a manifestation of the "power problem" discussed in Nakamura and Steinsson (2018). Secondly, the small time series sample does not allow to have enough variation on duration to GDP, which is crucial to estimate the interaction coefficient.

I try to overcome the problems arsing from the small sample by adding additional structure to these shocks. I extract the high frequency shocks directly from the proxy-VAR ran by Gertler and Karadi (2015), in this way, the estimated structural shock is present from July 1980 to June 2012. The reason for which the sample is longer comes from having a longer estimation sample and a shorter identification sample in the proxy-VAR. The estimation sample is the sample under which the reduced form VAR is ran (e.g. $Y_t = AY_{t-1} + u_t$) which is longer and goes from July 1979 to June 2012. This yields a set of estimated residuals ($\hat{u}_t$) July 1980 to June 2012. In the identification sample, from January 1991 to June 2012, one recovers the mapping from the reduced form residuals to the structural shock by using the high frequency proxy in a set of IV regressions. This mapping can be used on all the sample for which we have the estimated residuals to obtain an estimate for the structural monetary policy shock[44].

This structural shock extracted from a VAR is identified up to a scaling I run these specifications only with the LP-IV specification that easily solve this issue, similarly to Cloyne et al. (2018). Moreover, using the extracted shock as an instrument is beneficial as generated instruments do not suffer from the problems related to generated regressors (Pagan, 1984) as discussed by Wooldridge (2010).

In order to be comparable with the other regressions of this paper and with the Gertler and Karadi (2015) paper I use the following specification for the results presented in Figure C.21. All the regressions have the first two lags of: industrial production, the price level, the unemployment rate, the commodity price index, the unemployment rate, the one year government bond rate, the excess bond premium, the mortgage spread, and the 3 month commercial paper spread. I employ the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. The regression include one lag of duration to GDP. The instrumented variables are the change in the one year government bond rate and the interaction between the one year government bond rate change and the lagged duration to GDP. The instruments are use the structural shock discussed above.

Figure C.21 presents the results for this exercise. The first column presents the linear average results, without duration to GDP, and we can see that the monetary policy shock extracted from the proxy-VAR can replicate the proxy-VAR results for industrial production and unemployment. On average, a monetary policy shock that increases the one year government bond rate lowers industrial production by 1% and increases unemployment by more than 0.5 percentage points. On the other hand, it still has problem to replicate the results on the price level. The price level is reduced by -0.25%, but the effect is never statistically significant. These results indicate that for real variables, the reason for the discrepancy in results between the proxy-VAR and the LP Ramey (2016) found is likely due to the shorter sample employed with the LP-IV with the futures used directly.

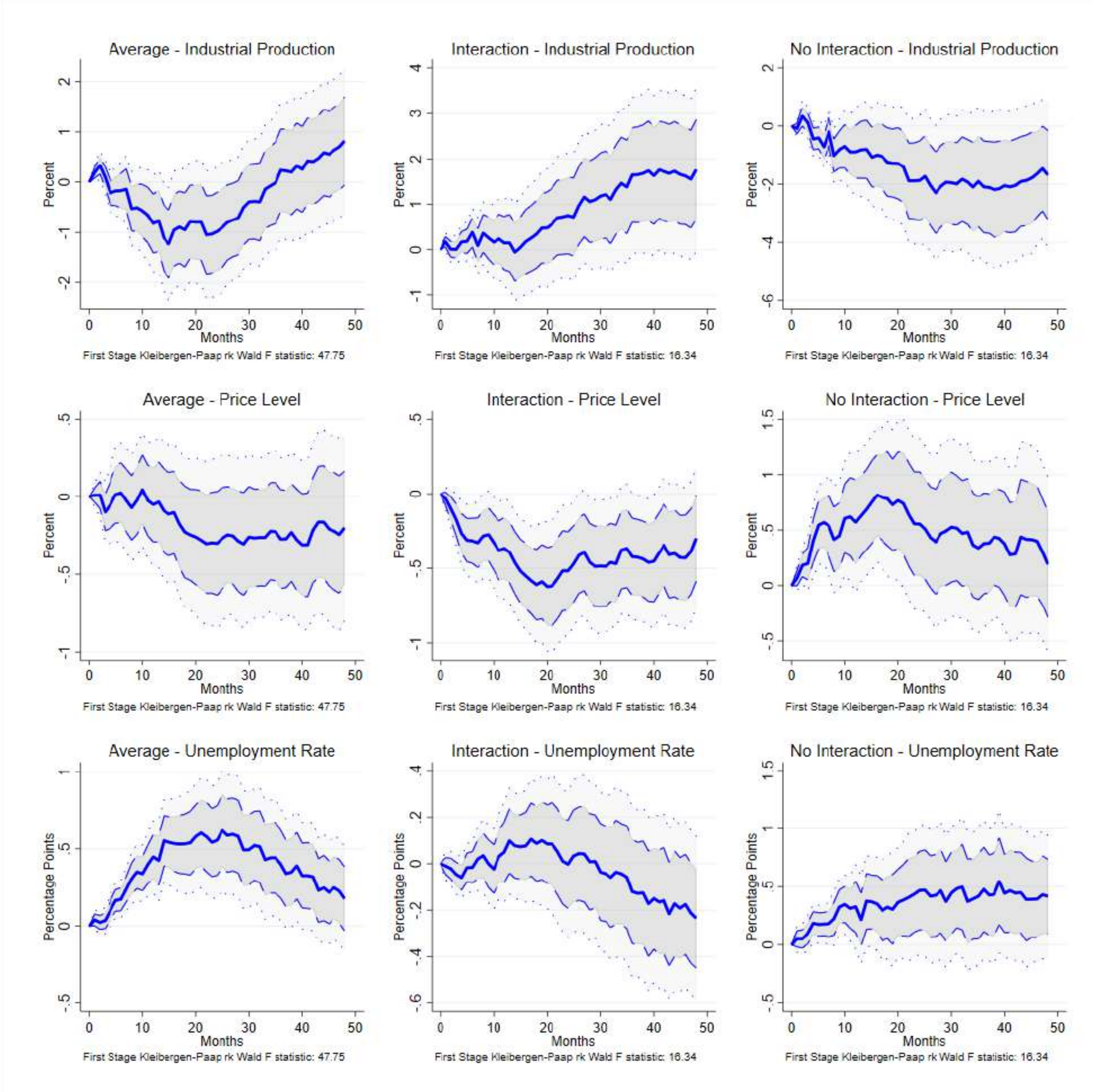
The second and third column of Figure C.21 present the conditional effects. On the first row we can see that monetary policy is less effective on reducing industrial production when

[44] IRFs are very similar if we apply the same methodology with high frequency instruments that control for the central bank information effect as Jarociński and Karadi (2020) or Miranda-Agrippino and Ricco (2021), instead of the Gertler and Karadi (2015) instrument. Results are available upon request.

duration to GDP is higher. At peak, the effect is attenuated by 2% when duration to GDP is one standard deviation higher. This coefficient is remarkably close to the baseline coefficient identified with the narrative method. Interestingly, the peak happens at the end of the horizon rather than at the middle as with the narrative method. The effect of a monetary policy shock when all debt is overnight tends to -2%, indicating a stronger contractionary effect with short debt. Again, the peak happens later than under narrative identification, but the magnitude is the same. The second row shows the impact on prices. Here a word of caution is warranted, this method with the proxy-VAR shocks did not work well in the linear average results, so we have to interpret the conditional results with a pinch of salt. These would point to a relatively lower inflation response under a higher duration to GDP and even a positive response under a overnight debt scenario. Finally, in the third row we can see how unemployment is specular to industrial production. Unemployment increases relatively less when duration to GDP is higher, by 20 basis points. As with industrial production the peak happens relatively later in the horizon.

Overall, the high frequency estimation points to a similar role for the duration of debt to GDP on the transmission of monetary policy. Monetary policy is less effective on reducing output when duration to GDP is relatively higher.

Figure C.21: Local projection regressions with high frequency identification for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1980m7 to 2012m6 with US data. Identification of the monetary policy shock is achieved with the structural monetary policy shock extracted from the Proxy-VAR of Gertler and Karadi (2015). The instrumented variables are the change in the one year government bond rate and the interaction between the one year government bond rate change and the lagged duration to GDP. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, the one year government bond rate, the excess bond premium, the mortgage spread, the 3 month commercial paper spread, and one lag of duration to GDP. The first column shows the linear average response to a monetary policy shock. This specification presents the same regressions without the interaction terms and the lag of duration to GDP. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

C.14 Results on Possible Endogeneity of the Maturity Structure

A possible concern with the results is that the maturity structure is endogenous and we are picking up a spurious relationship. This could be the case if monetary policy is more effective on output when duration to GDP is low for reasons that do not hinge on the maturity structure of public debt. In this section, I show that this is not likely to be a problem by examining reverse causality, possible confounding factors, and by using an instrumental variable approach.

C.14.1 Reverse Causality

Reverse causality, i.e. the debt management authority chooses a lower maturity when monetary policy is more effective on output, is unlikely to be a concern. The debt management office would be choosing to increase the interest rate risk in public debt exactly when interest rate changes have stronger effects on output and tax receipts. It is hard to find a rationale for this: if anything, they should be pushing to lower interest rate risk by lengthening the duration of debt in these periods. Moreover, and very importantly reverse causality is unlikely to be a concern due to the institutional details of debt management: debt management authorities take their maturity decisions at a frequency which is much lower than the one at which monetary policy can affect the economy. In Appendix A.1.2, I provide a detailed narrative analysis of this claim for the US Treasury.

C.14.2 Confounding Factors

Alternatively, there could be an omitted variable that drives both a low duration of public debt and a high effectiveness of monetary policy on output. Possible candidates include whether the economy is in a recession, the default risk in the economy, the level and slope of the yield curve, and demographic trends. In Tables C.2 and C.3, I present how these measures correlate with the baseline measure of duration to GDP at market value and with Macaulay duration at face value which highlights the maturity structure and it is not mechanically correlated with interest rates.

Tenreyro and Thwaites (2016) show that in recessions monetary policy is less effective on output, so if public debt was longer maturity in recessions, this could confound the results.

In columns 3 and 7 of Table C.2, I show that the opposite is true, public debt has shorter maturity during recession, meaning that this phenomenon cannot be the source of the results of this paper.

Another possibility is that when there is higher default risk in the economy the government cannot issue longer debt and monetary policy can affect the economy more as the balance sheet of financial intermediaries is strained. However, in columns 4 and 8 of Table C.2, I find that the opposite is true, a higher default spread in corporate bond markets is associated with longer maturity debt, so that also this hypothesis cannot explain the results.

The level and slope of the yield curve could affect the Treasury's choice of maturity and the effectiveness of monetary policy. When the level of interest rate is low, the maturity of public debt is generally higher, as it can be seen in columns 1 and 5 of Table C.2. However, it is unclear a priori if monetary policy should be more or less effective in periods of low interest rates. On the one hand, low interest rates can be associated with liquidity traps when monetary policy is less effective; on the other hand, when interest rates are at a low level, a given change change in interest rates has stronger effects on collateral values implying potentially stronger effects of monetary policy. We check directly if monetary policy is more or less effective on output when interest rates are low, by running the baseline local projection regression (2), with the level of interest rates in the previous month as the interaction term. This regression shows there are no strong conditional effects of monetary policy depending on the interest rate levels, the figure is available upon request.

The slope of the yield curve could also be a confounding factor. A flat yield curve is generally associated with recessions, which are associated with lower effectiveness of monetary policy (Tenreyro and Thwaites, 2016). If the treasury takes advantage of the flat yield curve to increase the maturity structure, then a longer duration would be associated to weak effects of monetary policy that are independent of the proposed channel. However, in columns 2 and 6 of Table C.2, we see the opposite pattern: a flat yield curve is actually associated with high duration of public debt! This implies that this is not likely to be a confounding factor. [45]

[45] Note that the positive correlation of slope and maturity is of independent interest: it implies that in equilibrium the treasury does not tilt its maturity position to take advantage of the flat yield curve. When the treasury has issued more long term debt, its relative price, compared to short term debt is higher, in line with the preferred habitat theory of Vayanos and Vila (2021).

Table C.2: Duration measures regressions

VARIABLES	(1) $DurGDP$	(2) $DurGDP$	(3) $DurGDP$	(4) $DurGDP$	(5) Dur^{FV}	(6) Dur^{FV}	(7) Dur^{FV}	(8) Dur^{FV}
YC Level	-0.0674***				-0.0520***			
	(0.00361)				(0.0114)			
YC Slope		0.127***				0.339***		
		(0.0105)				(0.0312)		
Recession			-0.342***				-0.343**	
			(0.0435)				(0.150)	
Default Spread				0.303***				0.468***
				(0.0251)				(0.0774)
Observations	468	468	468	420	468	468	468	420
R-squared	0.221	0.148	0.091	0.188	0.017	0.134	0.012	0.056

Notes: Robust standard errors in parentheses. The sample goes from 1969m1 to 2007m12 for columns 1, 2, 3, 5, 6, and 7 and from 1973m1 to 2007m12 for columns 4 and 8. The regressions are on US data. The depended variables are duration to GDP at market value for columns 1 to 4 and the Macaulay duration at face value for columns 5 to 8. "YC Level" is the yield curve level measured with the 3 month rate on government debt, "YC Slope" is the yield curve slope measured with the difference between the 10 years and 3 month rates on government debt, "Recession" is a dummy for the NBER based Recession Indicator (Fred code: USRECM), and the "Default Spread" measures the default premium on corporate bonds estimated with the Gilchrist and Zakrajšek (2012) methodology. Each regression also contains a constant.

Demographic trends could be driving both the strength of monetary policy and the maturity structure of public debt. As societies grow older the effectiveness of monetary policy could be altered, but it is ex-ante ambiguous how. Monetary policy could become less strong as argued by Wong (2021) and Cloyne, Ferreira and Surico (2019) as older households do not hold mortgages and are less likely to be liquidity constrained. On the other hand, monetary policy could become stronger as argued by Berg et al. (2019) if wealth effects are quantitatively important or if older households consume relatively more in sectors with sticky prices. In the first case, if an older population is associated with longer maturity debt, as people demand longer debt for retirement, this could be a possible confounding factor that could explain the results. However, in the data we see opposite patterns in the US and in the UK. In Table C.3, I show how the age dependency ratio is positively correlated with duration to GDP in the US but negatively in the UK. This implies that demographic trends are unlikely to confound the results.

Finally, the external validity of the US results with UK data can help to shed light on potential confounding factors through a direct comparison. Many real and financial macroeconomic variables are strongly correlated across countries (see Miranda-Agrippino and Rey, 2020), therefore, the correlation of duration to GDP across the two countries is useful evidence. If the correlation was positive, one could worry that a common factor was driving both variables. Table C.4 presents the correlation between the US and the UK duration

Table C.3: Duration to GDP in US and UK and Demographic Trends

VARIABLES	(1) $DurGDP_{US}$	(2) Dur^{FV}_{US}	(3) $DurGDP_{UK}$	(4) Dur^{FV}_{UK}
US Dependency Ratio	0.585***	0.941***		
	(0.0102)	(0.0312)		
UK Dependency Ratio			-0.138***	-1.727***
			(0.0307)	(0.0480)
Observations	348	348	348	348
R-squared	0.891	0.465	0.055	0.449

Notes: Robust standard errors in parentheses. The sample goes from 1979m1 to 2007m12. The first two columns are the measures on US data, columns 3 and 4 are the measures on UK data. The first and third columns show the correlation for duration to GDP at market value, the second column and fourth show the correlation for Macaulay duration at face value. The Dependency Ratio variable measures the "Age Dependency Ratio: Older Dependents to Working-Age Population" from the World Bank data (Fred codes: SPPOPDPNDOLUSA and SPPOPDPNDOLGBR) and it is interpolated from annual to monthly frequency. Each regression also contains a constant.

to GDP at market (face) value on the first (second) column. The relationship is negative and statistically significant for both measures, implying that it is unlikely that a common confounding factor drives them both.

C.14.3 Instrumental Variable Approach

As an additional check that higher effectiveness of monetary policy under a low maturity regime is not due to confounding factors, I employ an instrumental variable approach for duration to GDP. Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2014) suggest that the overall stock of government debt is a good instrument for the maturity structure of public debt, with metrics similar to duration over GDP. The argument is that the overall stock of public debt is a good instrument as it is orthogonal to current market conditions since it is due to past government deficits. One can test the relevance assumption with the first stage F-statistic. I follow Greenwood and Vayanos (2014) and use the stock of government bonds, including TIPS, held by the general public at face value, in order to purge the measure from price movements. I employ the baseline LP-IV specification presented in equation (3) and instrument the change in the federal funds rate, the lag of duration to GDP, and their interaction, with the narrative monetary shock, the lag of debt to GDP, and their interaction. Columns 1 to 3 of Table C.5 presents the first stage, and Figure C.22 present the IRFs.

Table C.4: Duration to GDP in US and UK Comparison

VARIABLES	(1) $DurGDP_{US}$	(2) $DurGDP_{US}^{FV}$
$DurGDP_{UK}$	-0.0877**	
	(0.0411)	
$DurGDP_{UK}^{FV}$		-0.478***
		(0.0142)
Observations	348	348
R-squared	0.007	0.544

Notes: Robust standard errors in parentheses. The sample goes from 1979m1 to 2007m12. The left hand side variables are the measures on US data, the right hand side ones are the measures on UK data. The first column shows the correlation for duration to GDP at market value, the second column shows the correlation for duration to GDP at face value. Each regression also contains a constant.

The first stage is strong with a robust F-Stat at 37.87. Moreover, we can see that the strongest effects, with the highest level of significance, can be found on the diagonal. This implies that the highest predictive power for each variable can be found in its direct instrument counterpart, e.g. for the interaction term in column 2 the only significant instrument is the iteration terms between the monetary policy instrument and debt to GDP.

As a robustness check, columns 4 to 6 of Table C.5 show the same first stage regressions without using any macro control. The magnitude, sign, and significance of the coefficients are all very similar and the robust F-Stat is still high ag 29.45. Table C.6 presents another sensitivity check, highlighting the strength of the debt to GDP instrument, abstracting from the instrument strength of the narrative monetary policy instrument. It shows a first stage where I instrument the lag of duration to GDP and the interaction between the lag of duration to GDP with the narrative instrument. In columns 1 and 2, I use the macro controls and the narrative instrument, and in columns 3 and 4 I only control for the narrative instrument. Here again the instrument is strong with very high robust F-Stats.

Figure C.22 presents the IRFs from this experiment. The results are very similar in direction, magnitude, and significance to the baseline LP-IV results presented in Figure C.5. A contractionary monetary policy shock is attenuated on its impact on industrial production and unemployment when public debt has a longer duration and there is no differential effects on inflation[46].

[46]Results are very similar also if the instrument for duration to GDP is measured with only nominal debt,

Table C.5: First stage regressions with instruments for interest rates and for duration to GDP

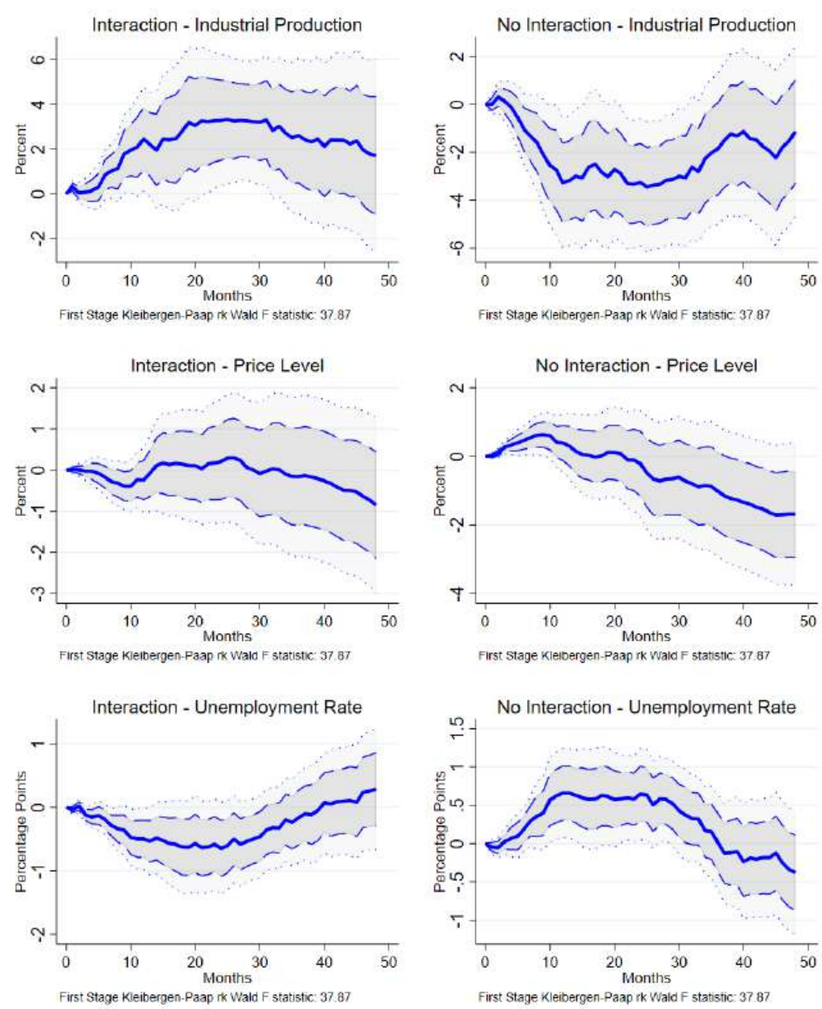
VARIABLES	(1) $\Delta i_{t;t-1}$	(2) $\Delta i_{t;t-1} DurGDP_{t-1}$	(3) $DurGDP_{t-1}$	(4) $\Delta i_{t;t-1}$	(5) $\Delta i_{t;t-1} DurGDP_{t-1}$	(6) $DurGDP_{t-1}$
$Shock_t$	1.019***	-0.160	0.143**	1.165***	0.00616	0.107
	(0.242)	(0.193)	(0.0718)	(0.332)	(0.252)	(0.0897)
$Shock_t DebtGDP_{t-1}$	-0.152*	0.493***	-0.0948**	-0.156	0.476***	-0.101*
	(0.0807)	(0.0884)	(0.0414)	(0.103)	(0.107)	(0.0584)
$DebtGDP_{t-1}$	-0.193**	-0.0985	0.969***	-0.0308	-0.0451	0.978***
	(0.0812)	(0.0837)	(0.0526)	(0.0292)	(0.0307)	(0.0184)
Observations	467	467	467	467	467	467
Robust F-Stat	37.87	37.87	37.87	29.45	29.45	29.45
Controls	Recursive	Recursive	Recursive	Minimal	Minimal	Minimal

Notes: Newey-West standard errors in parentheses. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Identification of duration to GDP is achieved with public debt to GDP. Public debt is measured with the stock of marketable nominal and inflation linked debt at face value. The depended variables are the instrumented variables in the LP-IV framework; they are the change in the Fed funds rate, the interaction between the Fed funds rate change and the lagged duration to GDP, and the lagged duration to GDP. Duration to GDP is measured with the marketable nominal debt at market value held by the general public. The first three columns show the first stage regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, these two regressions include the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. Columns 4 to 6 show the same first stage regressions with minimal controls, that is, controlling only for a constant.

The analysis of this subsection shows that the endogeneity of the maturity structure is not likely to pose a problem in the interpretation of the empirical results.

with debt at market value, or with debt also held by the government. These IRFs are available upon request.

Figure C.22: Local projection instrumental variable with instruments for interest rates and for duration to GDP



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Identification of duration to GDP is achieved with public debt to GDP. Public debt is measured with the stock of marketable nominal and inflation linked debt at face value. The instrumented variables are the change in the Fed funds rate, the lagged duration to GDP, and their interaction. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. The first column shows the interaction term of the change in the Fed funds rate with the Duration over GDP, the second column shows the Fed funds rate term not interacted. Each row shows a different LHS variable.

Table C.6: First stage regressions with and instrument for duration to GDP

VARIABLES	(1) $Shock_t DurGDP_{t-1}$	(2) $DurGDP_{t-1}$	(3) $Shock_t DurGDP_{t-1}$	(4) $DurGDP_{t-1}$
$Shock_t DebtGDP_{t-1}$	0.977***	-0.0948**	0.982***	-0.101*
	(0.0371)	(0.0414)	(0.0358)	(0.0584)
$DebtGDP_{t-1}$	0.0166**	0.969***	0.00324	0.978***
	(0.00803)	(0.0526)	(0.00278)	(0.0184)
Observations	467	467	467	467
Robust F-Stat	171.17	171.17	451.14	451.14
Controls	Recursive	Recursive	Minimal	Minimal

Notes: Newey-West standard errors in parentheses. The sample goes from 1969m1 to 2007m12 with US data. Identification of duration to GDP is achieved with public debt to GDP. Public debt is measured with the stock of marketable nominal and inflation linked debt at face value. The depended variables are the interaction between the narrative monetary policy shock and the lagged duration to GDP, and the lagged duration to GDP on its own. Duration to GDP is measured with the marketable nominal debt at market value held by the general public. The first two columns show the first stage regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, these two regressions include the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and a control for the narrative monetary policy shock. Columns 3 and 4 show the same first stage regressions with minimal controls, that is, controlling only for the narrative monetary policy shock.

D Further Empirical Results

D.1 Results on Bond Yields and Prices

Monetary policy has strong effects not only on the short end of the yield curve, but also can have effects on the long end of the curve, as argued by Nakamura and Steinsson (2018). This provides a rationale for why a shock to the short rate can matter for valuation of long-debt and how long-debt can provide insurance against such shock. Even if monetary policy affects more strongly short than long rates, it may have large effects on valuation of long debt because debt prices at longer horizons react more than one to one to variations in the interest rate. Take a 10 year zero coupon bond with a yield continuously compounded, its price is: $p_{10} = e^{-10y_{10}}$. Therefore, the derivative of the log of the price to a change in the interest rate is such that an increase in one percentage point in that yield leads to a decrease of 10 percent in the bond price: $\frac{\partial log(p_{10})}{\partial y_{10}} = -10$, which is much larger than the impact on a short bond price!

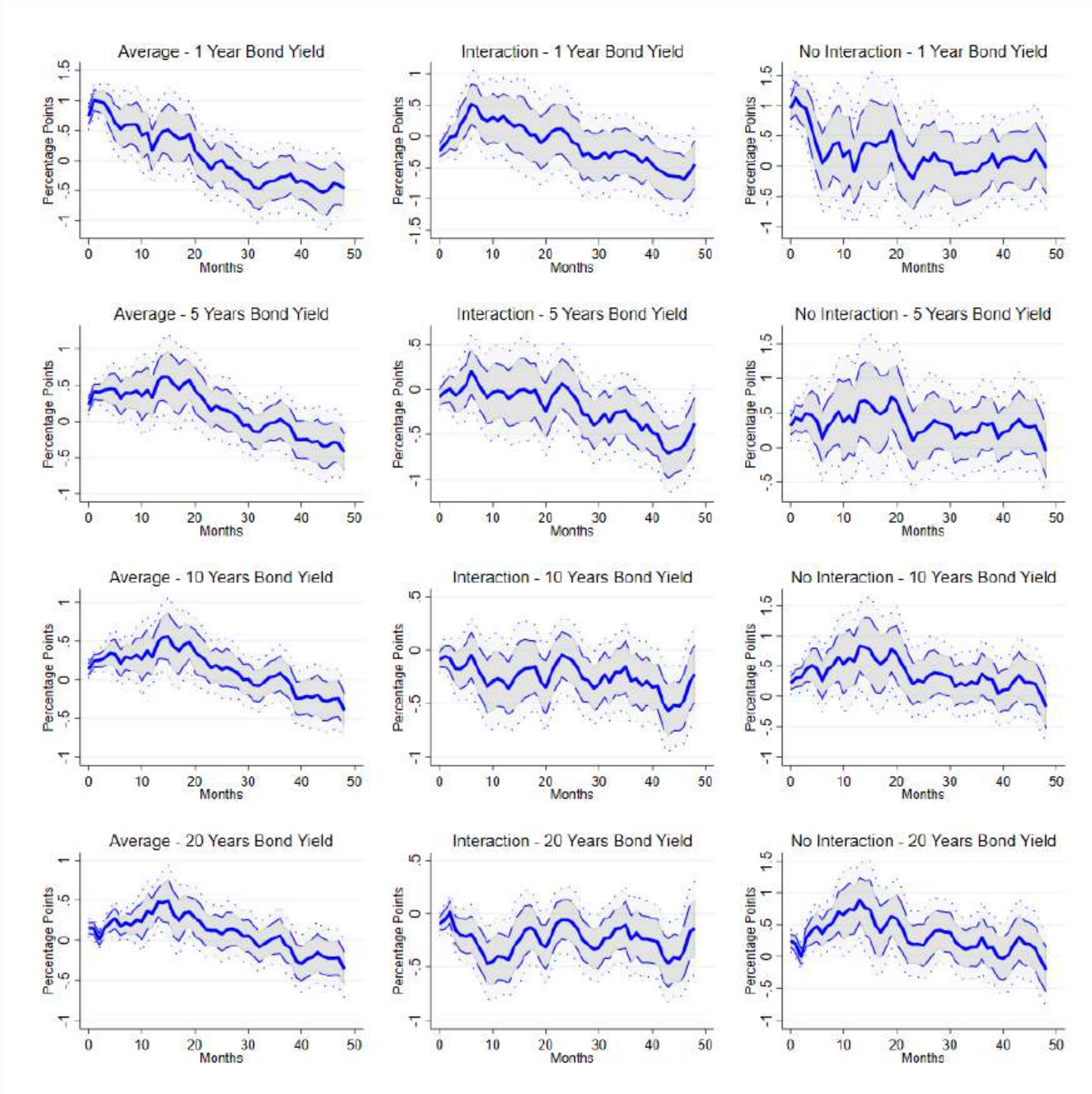
Column 1 of Table D.1 presents the impact on yields at different maturities to a contractionary monetary policy shock.[47] The first row presents the response of the 1 year bond yield, the second of a 5 year bond, the third of a 10 year bond, and the fourth of a 20 year bond. All yields increase and, unsurprisingly, the shorter term yield respond relatively more to a monetary policy shock.

In the first column of Table D.2 we can now see the impact on the log of the corresponding bond prices multiplied by 100. Despite the fact that long maturity bond yields moved less than the short ones, the prices decline a substantial amount. Whereas the price of one year bonds declines by about 1 percent, the price of 20 years bonds declines by 10 percent.

Furthermore, the effects on bond yield and prices seems to be the same irrespective of the whether public debt duration to GDP is low or high. Columns 2 and 3 of figures D.1 and D.2 present the interaction regressions with duration to GDP. Column 2 shows how the interactions terms are not statistically different from zero. This result gives weight to the idea that what matters is the insurance mechanism that long debt provides, but it is not affected by the transmission through the yield curve.

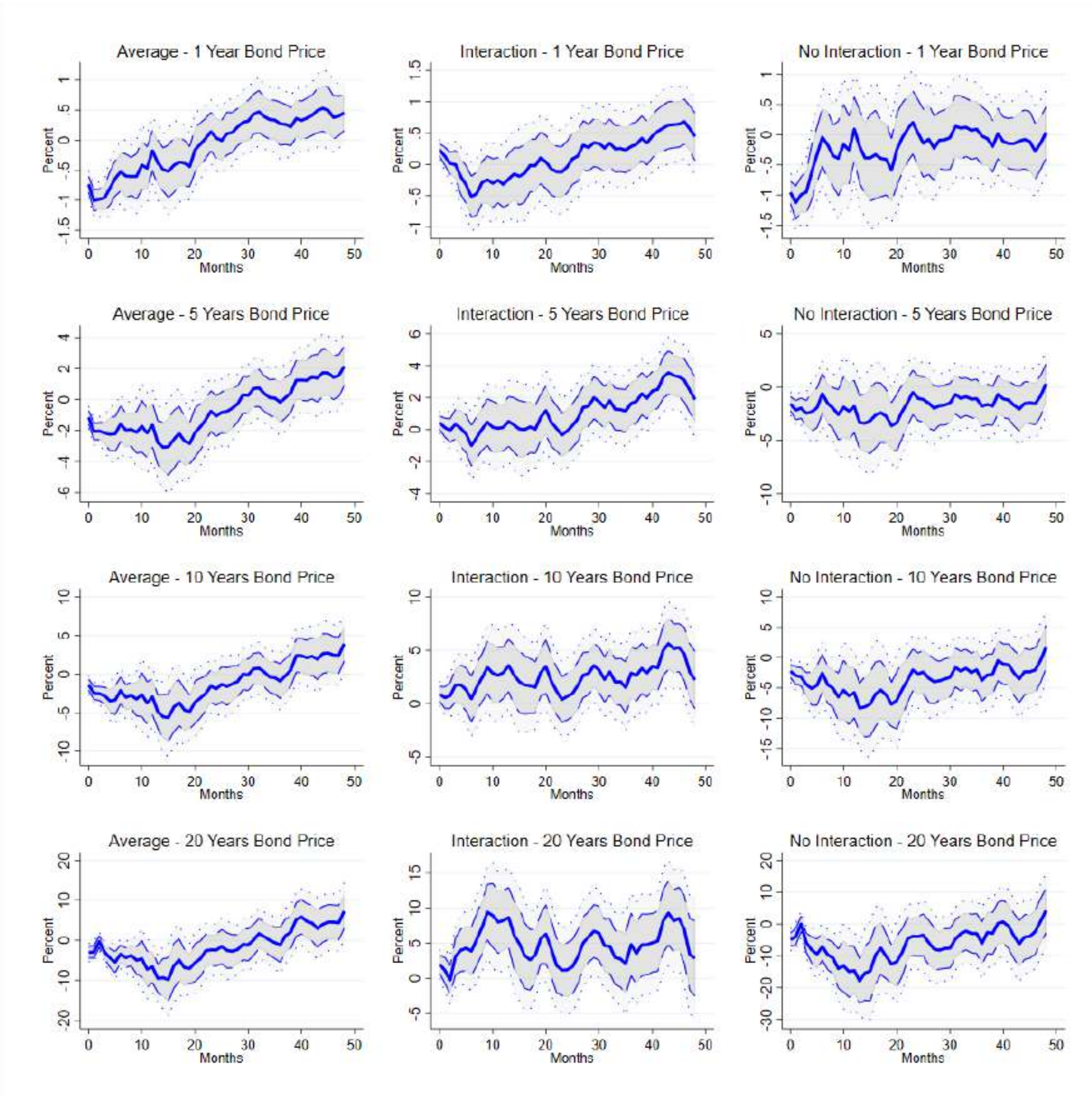
[47] All the tables in this section display the results with controls as in the baseline results with the recursiveness assumption on macroeconomic variables and with two lags of bond yields. Alternative specifications produce similar IRFs and are available upon request.

Figure D.1: Local projection regressions with interaction of duration to GDP for bond yields for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the unconditional response to a monetary policy shock. This specification presents the same regressions without the interaction terms and the lag of duration to GDP. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

Figure D.2: Local projection regressions with interaction of duration to GDP for bond prices for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the unconditional response to a monetary policy shock. This specification presents the same regressions without the interaction terms and the lag of duration to GDP. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

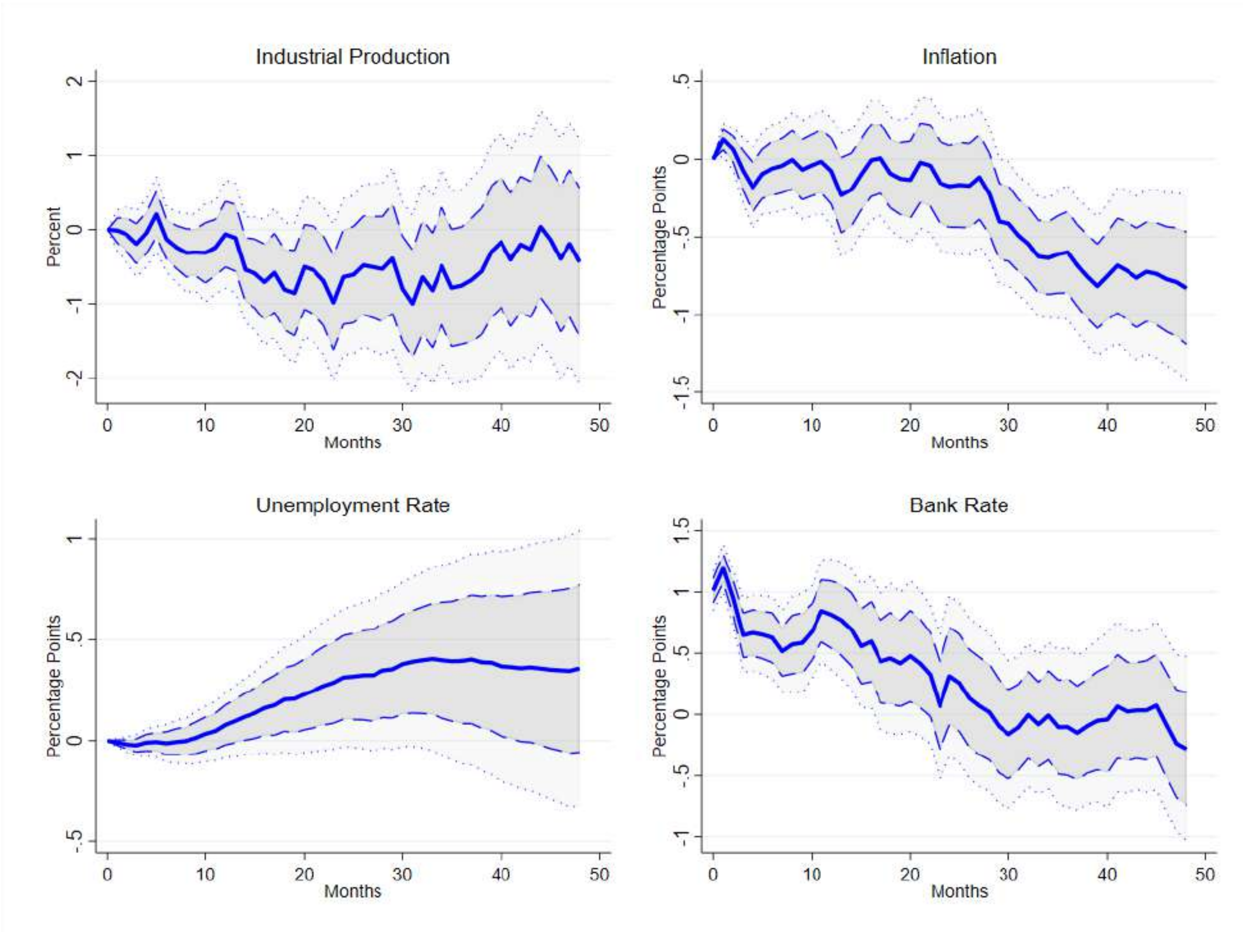
E Results with UK Data

E.1 Baseline LP Results without Interaction Term

As in the case of the US, the first step is to present the average effects of monetary policy. For this reason, this subsection presents the LP and LP-IV regression results for the narrative identification from Cloyne and Hürtgen (2016) for the UK.

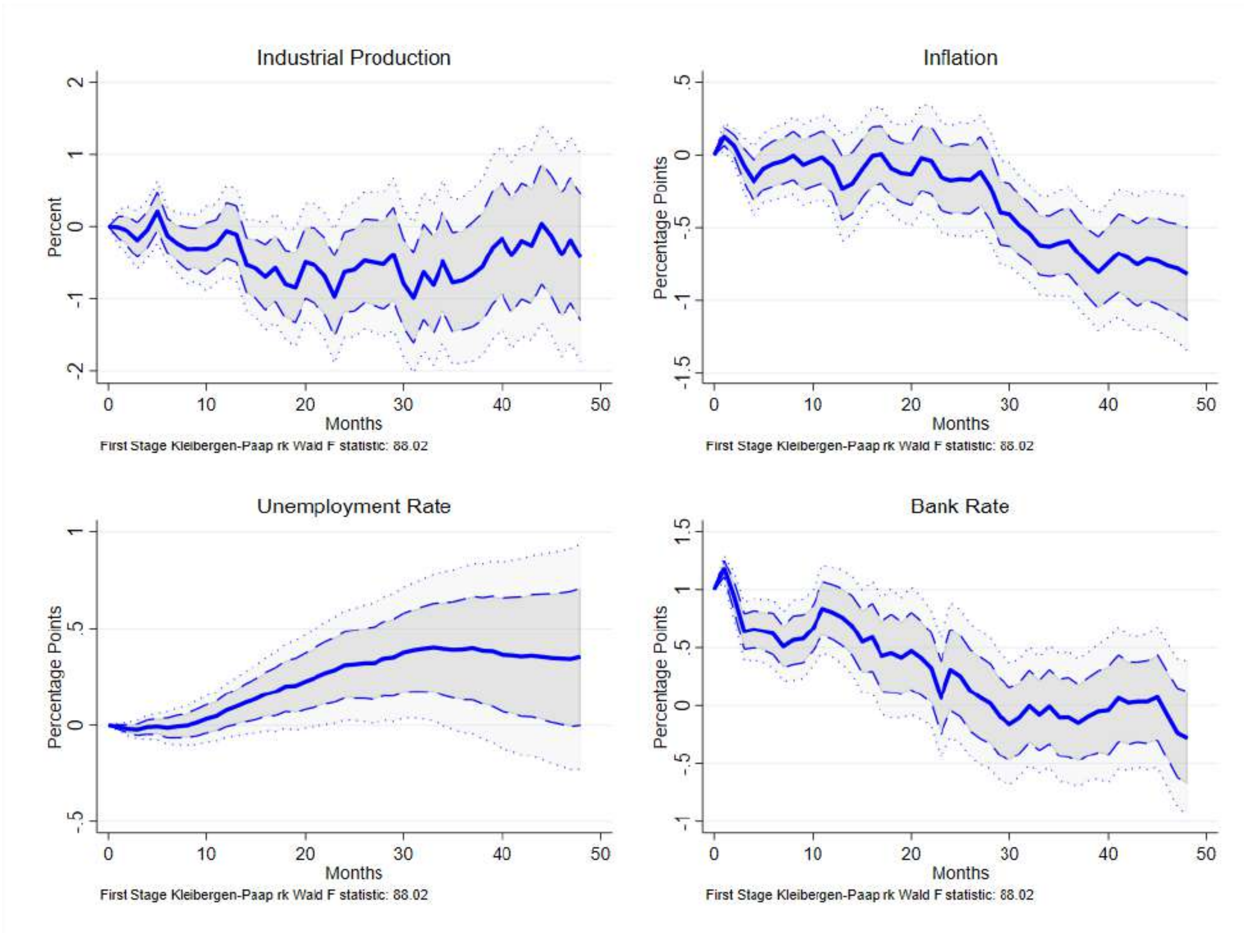
Figure E.1 presents the replication of the results of Cloyne and Hürtgen (2016) for the impact of the monetary policy shock on key macroeconomic variables with a reduced form local projection. The regressions incorporate the recursiveness assumption and have 4 lags of first differences of the log of industrial production, of the year-on-year RPIX inflation (that excludes mortgage payments), of the unemployment rate, of the Bank rate, and of the log of the commodity price index. In addition, each regression adds 48 lags of the monetary policy shock as in Cloyne and Hürtgen (2016). As in that paper, I run the specification in first differences for the the macroeconomic controls and with h-steps ahead differences for the dependent variable ($y_{t+h} - y_{t-1}$). The estimation sample goes from 1979m1 to 2007m12. In the first panel we can see the response of industrial production. The impulse response functions are not very precisely estimated, but we can see how industrial production declines by around one percent 2 years after the monetary policy shock and it reverts slowly to zero at then end of the 4 years window. For inflation, we can can see in the second panels, that the monetary policy shock does not have a strong effect in the first 2 years, but then turns negative and reaches almost one percentage point reduction at the end of the sample. Unemployment, shown in the third panel, behaves more smoothly and it increases by almost half a percentage point 3 years after the shock. Finally, the bank rate increases by one percentage point on impact and then reverts back to the baseline in less than 2 years. Overall, the results point to strong effects of monetary policy, with a delayed impact, especially on inflation. As the US case, the results are quite similar with a local projection instrumental variable (LP-IV) estimation. The results are show in Figure E.2 and represent the impact of a monetary policy shock that raises the Bank rate by one percentage point.

Figure E.1: Unconditional local projection regressions for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable.

Figure E.2: Unconditional local projection instrumental variable regressions for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. The instrumented variable is the change in the Bank rate. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable.

E.2 Results with Duration to GDP

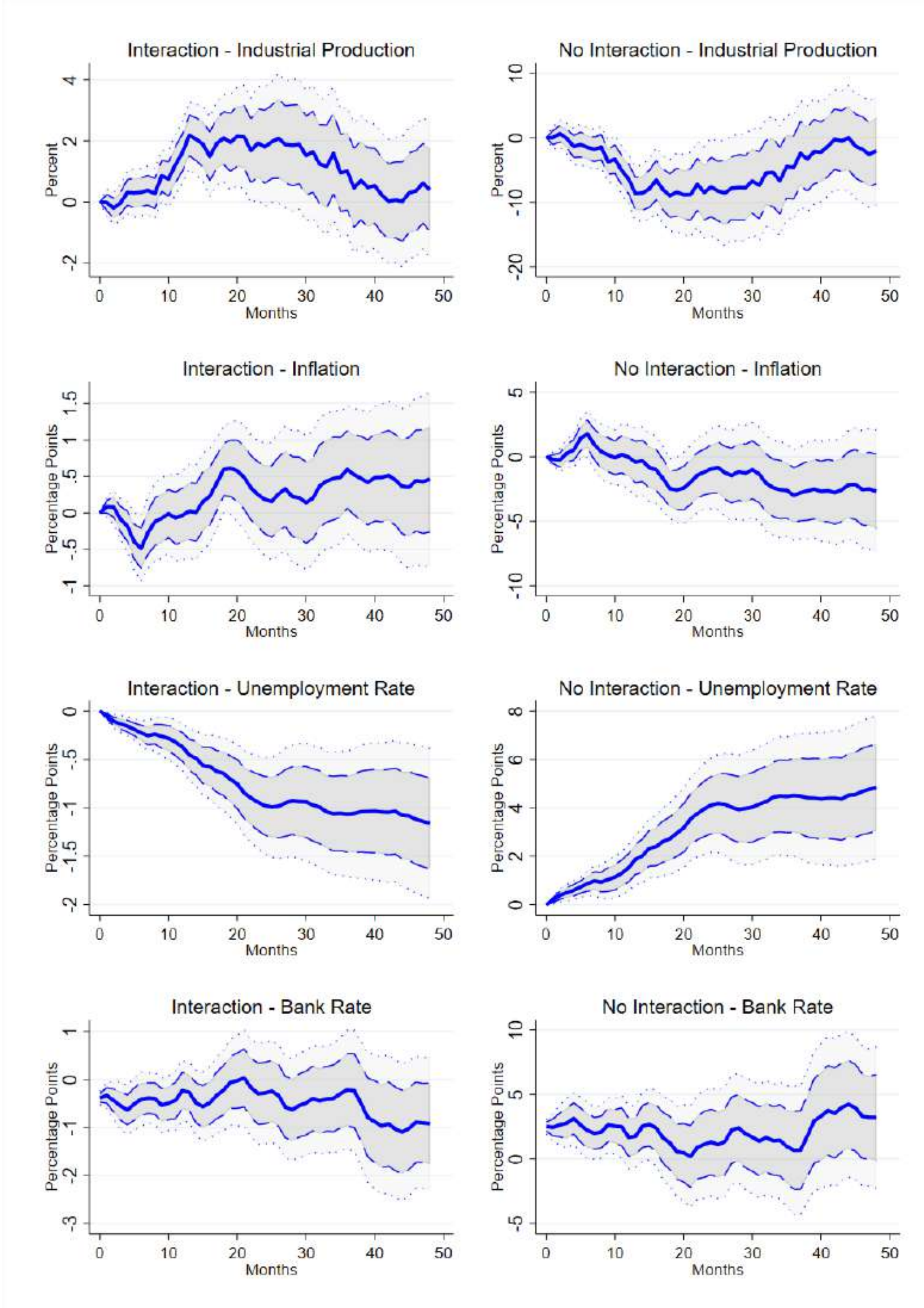
This section presents the baseline interaction results for the UK with a reduced form. Figure E.3 mirrors Figure 2 for the US.

Each regression follows the same specification as in the unconditional regressions with in addition the interaction of the lag of duration to GDP with the measure of the shock and one lag of duration to GDP on its own. In each row of E.3 we can see a different dependent variable. The second column presents the impact of a monetary policy shock in the hypothetical situation when all debt is overnight. The first column presents the impact on increasing duration to GDP by one standard deviation on the effect of a monetary policy shock on the dependent variable. Notice that, the UK has had high average level for duration to GDP throughout the sample as we can see by comparing Figures A.6 for the UK, with Figures 1 for the US, in line with the idea that the UK has the longest maturity of public debt among large economies. This implies that extrapolating the effect of a monetary policy under overnight duration is less informative for the UK than for the US. This implies that for these results it makes the most sense to focus on the interaction term only[48].

The first row of Figure E.3 presents the response of industrial production to a contractionary monetary policy shock. The key result can be found in the first column, monetary policy shocks are attenuated when duration to GDP is higher. An increase of one standard deviation of duration to GDP reduced the contractionary impact of monetary policy by 2% at peak. This effect is economically large, statistically significant, and remarkably close to the US result. Notice that the path of duration to GDP of the UK and of the US is not positively correlated, which gives credit to the idea that we are not picking up something driving both results but a feature of the maturity stricture of public debt. On the UK estimation sample, from 1979m1 to 2007m12 the correlation between the duration to GDP across the two countries was -0.0830. If we look at the effect of a contractionary monetary policy shock under overnight debt on the second column, we can see how the effect is stronger than in the baseline. However, as overnight debt is not a reasonable comparison for the UK the magnitude of the coefficient is too large.

[48] To overcome the difference in levels, we can compare regimes of historically low duration to GDP with regimes with historically high duration to GDP with a smooth transition local projection method. The UK results are presented in Section E.4 and the US ones in C.9. With that specification the overall magnitudes also under a low maturity regime are very similar.

Figure E.3: Local projection baseline interaction regressions for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of duration to GDP The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with the Duration over GDP, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

We now turn to the effects in inflation presented in the second row of Figure E.3. Having a lower or higher level of duration to GDP does not seem to alter the transmission of monetary policy to inflation. The interaction coefficient is not statistically different from zero in any horizon except for a short blip in months 5 and 6 of a small magnitude. If we move to the no-interaction term we see the same pattern as in the unconditional response, being not precisely estimated for the same reason as for industrial production. This result also chimes with the US one.

Unemployment, shown in the third row of figure E.3, behaves specularly to industrial production. An increase of one standard deviation in duration to GDP lowers the effect of a contractionary monetary policy shock on unemployment by one percentage point. The effect is stronger than in the US, but is consistent with a higher response of unemployment also unconditionally in the UK. An interesting feature is that we do not seem to converge back to zero for unemployment at the end of the sample, possibly indicating that the effects tend to be longer lived.

Finally, the fourth row of Figure E.3 presents the results for the response of the Bank Rate. In the interaction IRF we can see that there is a mild lower impact of the shock when debt to GDP has a longer duration. This effect is small and short lived, in line with the predictions of the structural model. If we look at the non-interaction effects we can see stronger response, however, the magnitude is relatively high due to the no-interaction coefficient being the comparison with a unlikely overnight debt.

The key take away of this exercise is that, the interaction results are similar in sign and magnitude to the US ones. A higher level of duration to GDP attenuates the contractionary effect of monetary policy on reducing output but does not have an effect on prices. This happens despite the fact that the US and the UK are countries with vastly different duration to GDP, both in level and in time series properties.

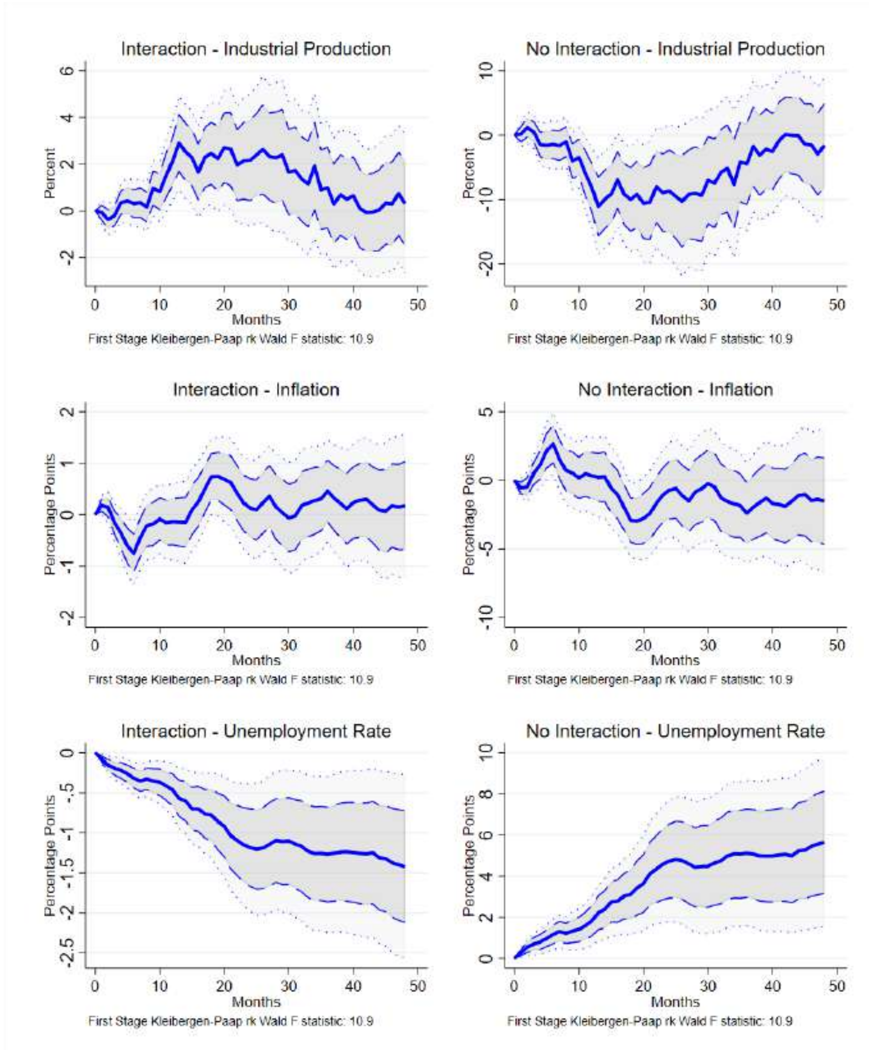
Notice that also the UK results are robust to different specifications and debt construction mechanism. The next section shows the same robustness checks as in the US that are feasible in the UK data. The only results to highlight particularly is the one presented in Section E.4 as it allows a more appropriate comparison to the US results.

E.3 Additional Empirical Results

This section presents the robustness checks on the baseline specification for the UK. Each result mirrors the US ones presented in Appendix C, which contains a more detailed discussion of each of these specification. Figure E.4 mirrors Figure C.5 for the instrumental variables results. Figure E.5 mirrors Figure C.6 for using the Macaulay duration instead of duration to GDP. Figure E.6 mirrors Figure C.9 as it shows the results for duration to GDP computed at a book value rather than market value. Figure E.7 mirrors Figure C.10 by including inflation linked debt in the duration to GDP measure. Finally, Figure E.8 mirrors Figure **??** and presents the results without the recursiveness assumption.

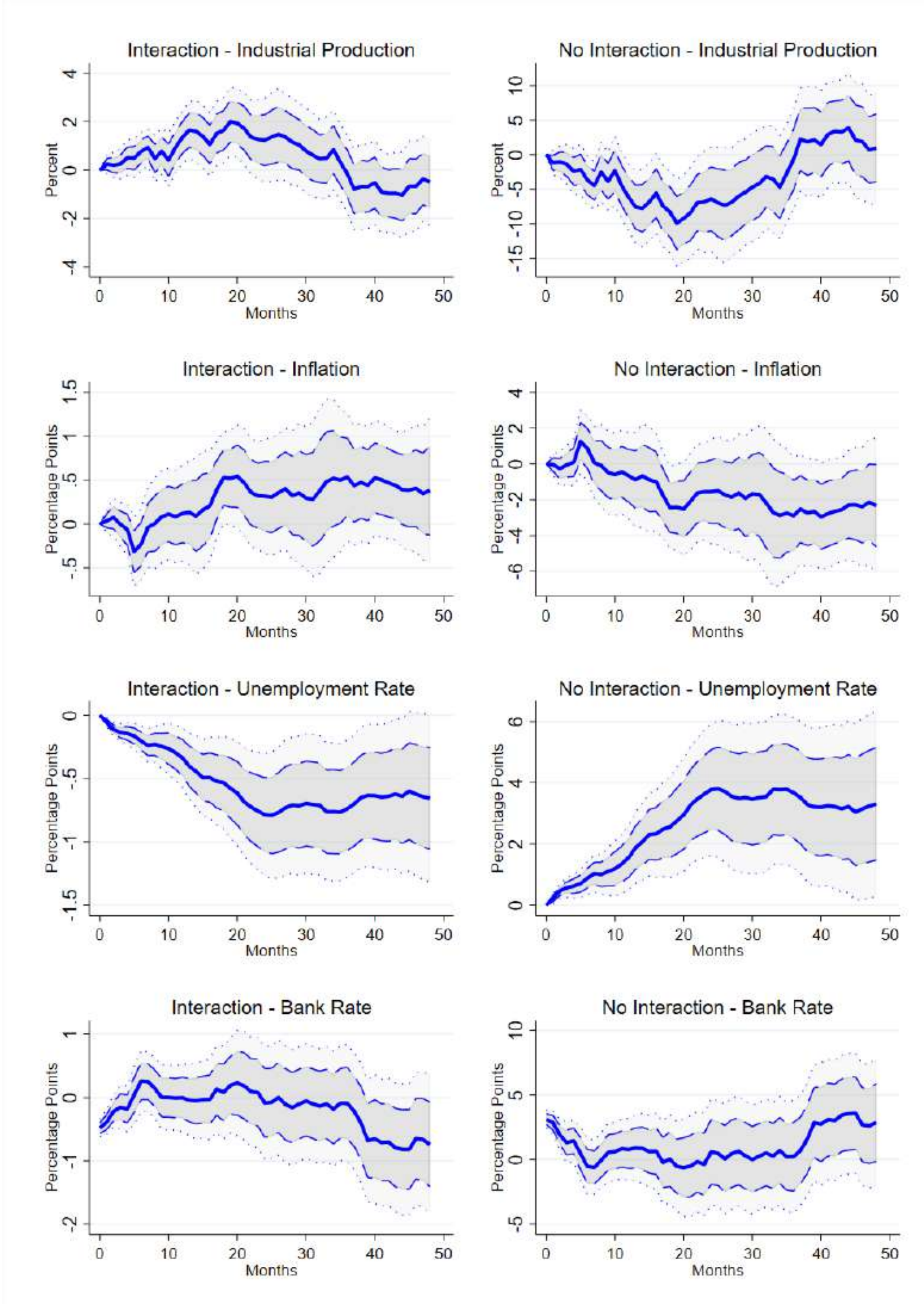
Overall, all of these specifications support the main finding that long duration debt to GDP attenuates the impact of monetary policy on output, but does not have an effect on prices. A few results are worth mentioning. First of all, the results at face value are less precisely estimated, most likely that for a country as the UK where the maturity is high, computing duration at face value gives too much weight to high interest rates periods, as can be seen in Figure A.9. Second of all, we cannot construct the series with public holding of government debt by public entities as the Bank of England, so we cannot distinguish the effect with and without these holdings, as I did for the US; however, this is unlikely to be a problem as the estimation sample ends on December 2007 and the Bank of England started to buy large amounts of public debt only from QE in 2009. Finally, it is particularly reassuring that the results go through when including inflation linked debt, as they are a larger share of the UK debt.

Figure E.4: Local projection instrumental variable baseline interaction regressions for the UK



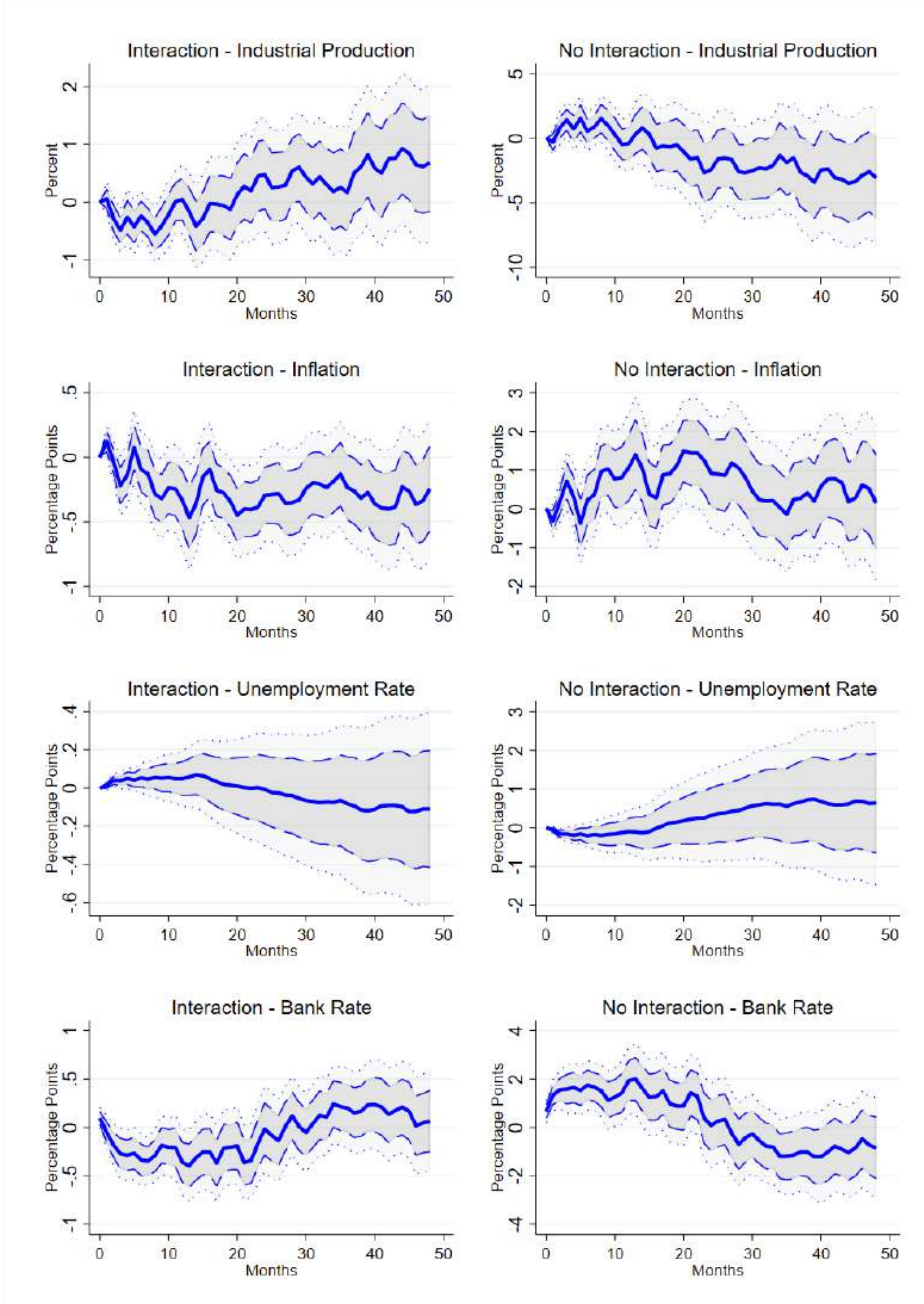
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. The instrumented variables are the change in the Bank rate and the interaction between the change in the Bank rate and the lagged duration to GDP. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of duration to GDP. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. Each panel shows the interaction coefficient between the instrumented change in Bank rate and duration to GDP.

Figure E.5: local projection regressions with Macaulay duration for the UK



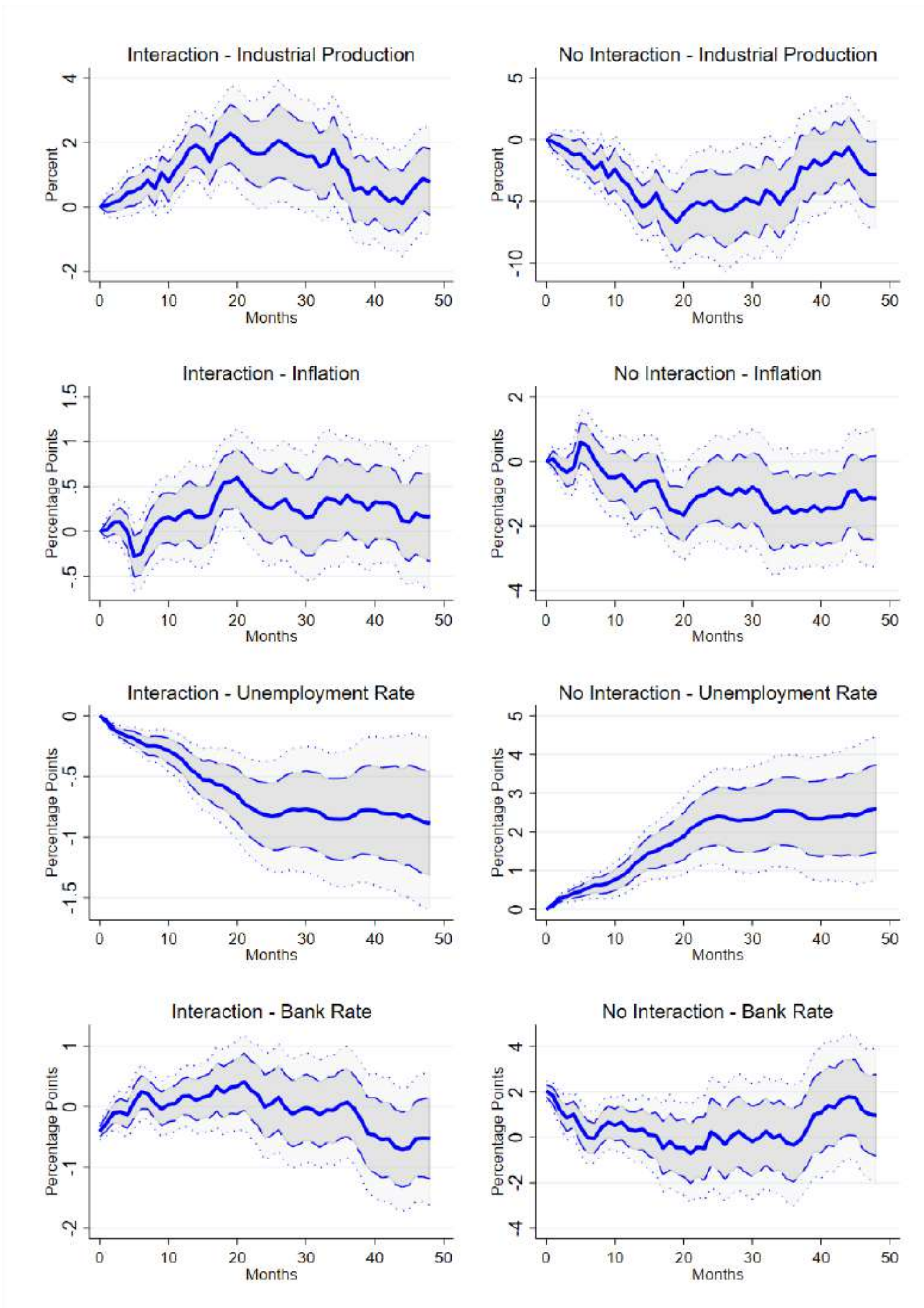
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of Macaulay duration. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with the Macaulay duration, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

Figure E.6: Local projection regressions with duration to GDP constructed from face value debt for the UK



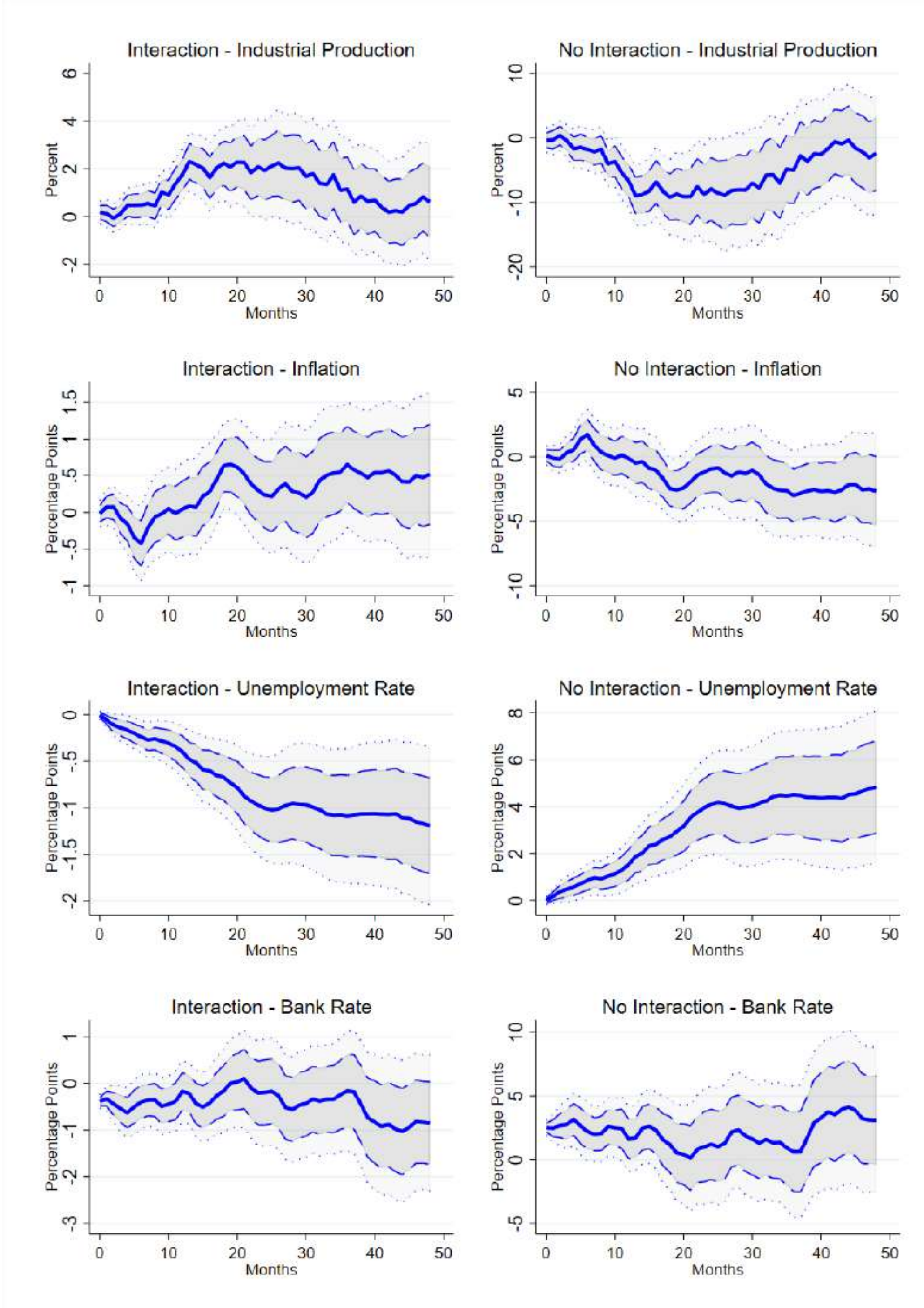
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed without the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of duration to GDP constructed with face value debt. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with the Duration over GDP constructed with face value debt, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

Figure E.7: Local projection regressions with duration to GDP constructed from both nominal treasury bonds and inflation linked bonds for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed without the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of duration to GDP constructed with both nominally fixed rate bonds and inflation linked bonds. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with the Duration over GDP constructed with both nominally fixed rate bonds and inflation linked bonds, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

Figure E.8: Local projection regressions without recursiveness assumption for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed without the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of duration to GDP. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with the Duration over GDP, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

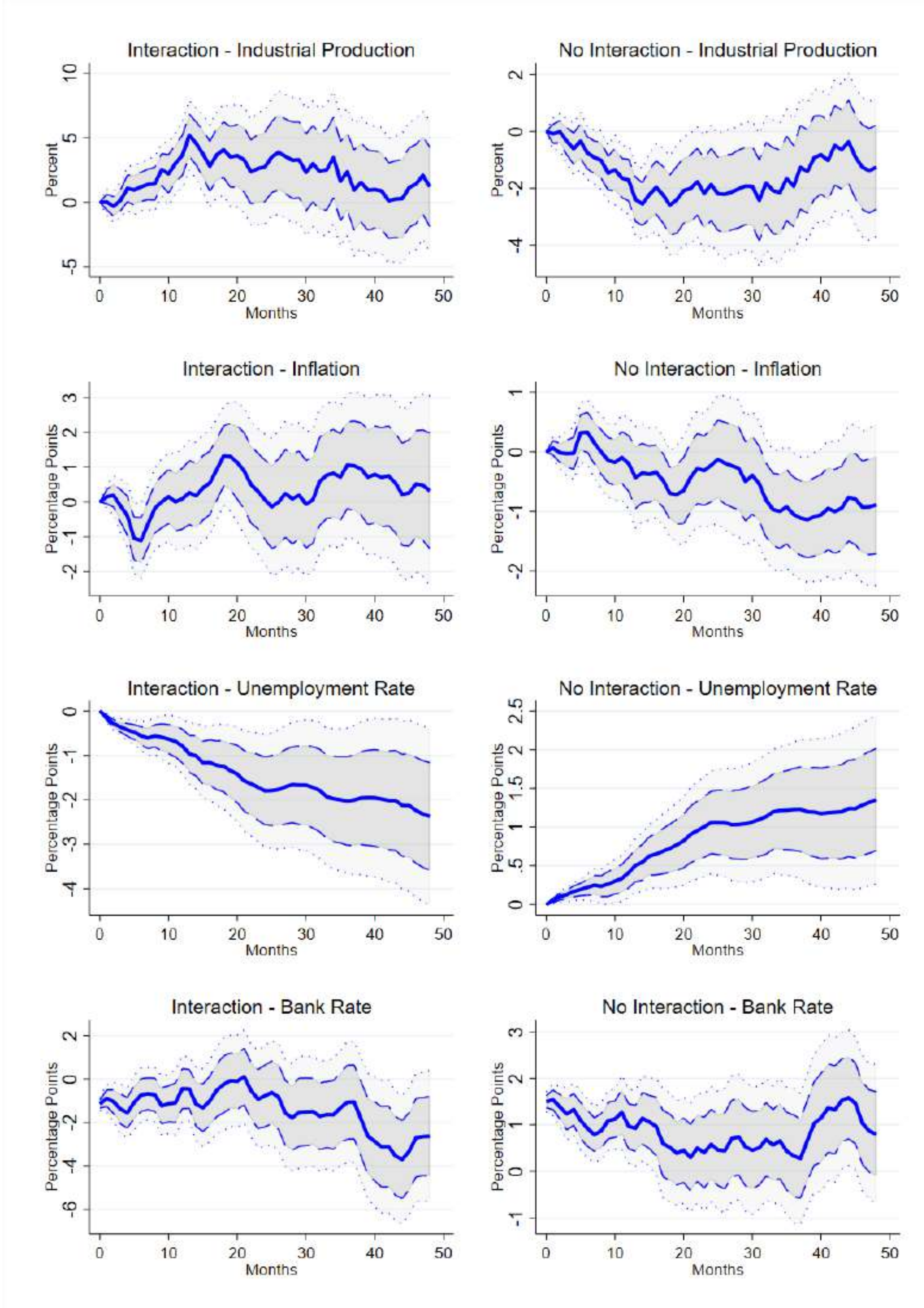
E.4 Results with Smooth Transition Method

Finally, I replicate the smooth transition local projection method for the UK data as well. This method is particularly well suited for the UK data as, due to the historically high duration to GDP in the UK, the overnight debt comparison I employ in the baseline is less intuitive than in the US. In the smooth transition method, we study the effects of a monetary policy shock under a low duration to GDP regimes and we compare it with the high duration to GDP regimes as shown in equation (C.1) and discussed in section C.9.

Figure E.9 presents the results from this exercise with the baseline specification. If we examine the effect of a monetary policy on industrial production under a low duration to GDP regime we see results which are quite close to the US results: the reduction is about 2% at peak. This shows how using the regime comparison is particularly useful for the UK. The results on the iteration term, that is the difference in effect on output when we move from a low to a high duration to GDP regime, are also in line with the US ones, with a positive coefficient of about 5%. Monetary policy is much less effective on output when the economy is in a high duration to GDP regime.

When we turn to prices we can still a find similar result to the US. Under the low duration to GDP the effect of a monetary policy reduced inflation by about 1%. If we move to a high duration to GDP regime we still find no statistically significant difference. Unemployment mirrors industrial production, and the Bank rate increases relatively less during a high duration regime, in line with the US results and with the theoretical model.

Figure E.9: Smooth transition local projection interaction regressions for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and $F(DurGDP_{t-1})$, the smooth logistic transformation of the lag of duration to GDP. The speed of transition parameter θ is equal to 3. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with $F(DurGDP_{t-1})$, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

F Model Derivations

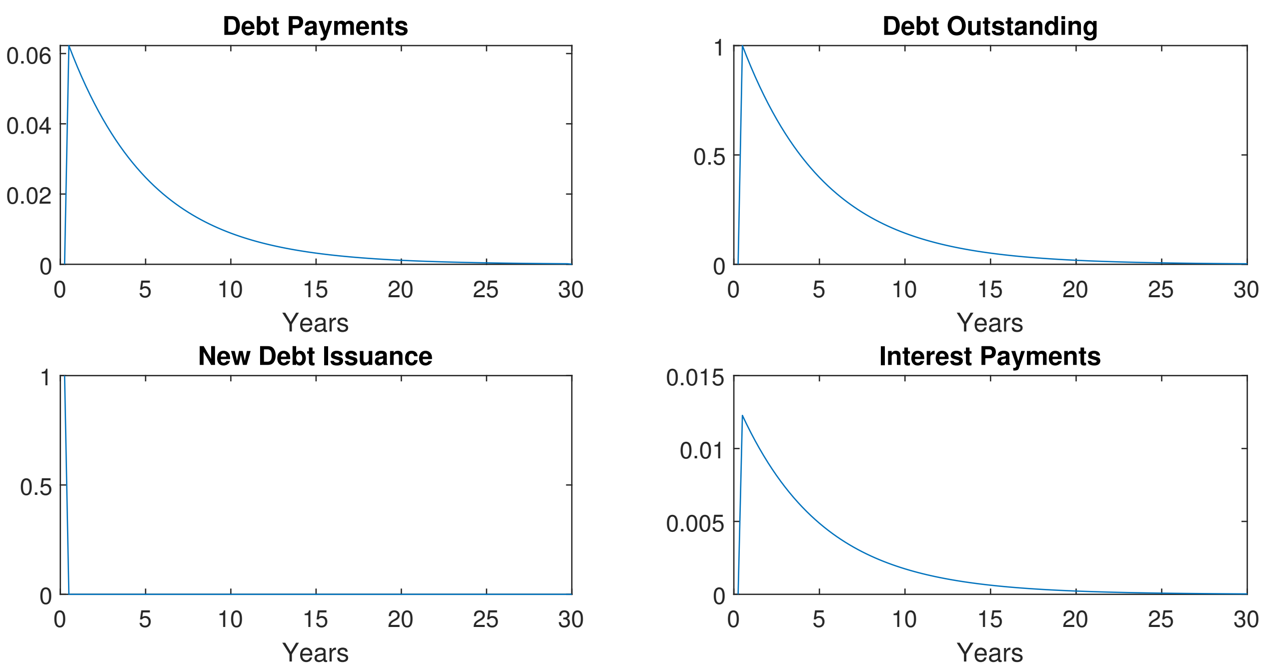
F.1 Model Extended Derivations

In this appendix, I provide further derivations for theoretical model.

F.1.1 Government Debt

As a first illustrative step, Figure F.1 shows an example of the debt schedule following an issuance of one unit of debt on the various debt variables. Then, I show that the debt structure allows a parsimonious formulation for duration by providing the proof to Lemma 1.

Figure F.1: Debt Repayment Schedule



Notes: these panels show the impact of an increase of one unit of new debt starting from no debt, that is $L_t = 1$ and $D_{t-1} = 0$, on debt dynamics. Time is quarterly, the interest rate on new debt R_t^{new} is 0.0123 (0.05 at annual frequency), the maturity parameter δ^d is equal to 0.05; therefore, duration is equal to 16.24 quarters. The first panel shows the overall debt payments in each future quarter F_{t+j}. The second panel shows the principal outstanding in each future quarter D_{t+j}. The third panel shows the flow of new issuances L_{t+j}, which is equal to 1 only in the first period. Finally, the fourth panel shows the interest payments $D_{t+j}R_{t+j}^{ave}$.

Proof of Lemma 1.

Macaulay duration weights each cash flow of a debt instrument by its maturity and divides it by the net present value of these cash flows. In case of L_t new debt issued at prevailing new rate R_t^{new}:

$$Dur_t = \frac{\sum_{j=1}^{\infty} j \frac{(\delta^d + R_t^{new})(1-\delta^d)^{j-1}}{(1+R_t^{new})^j} L_t}{\sum_{j=1}^{\infty} \frac{(\delta^d + R_t^{new})(1-\delta^d)^{j-1}}{(1+R_t^{new})^j} L_t}$$

If we simplify and use the formula for geometric series we get,

$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \sum_{j=1}^{\infty} j \left(\frac{1 - \delta^d}{1 + R_t^{new}} \right)^{j-1}$$

Take the sum from $j = 1$ to $j = 0$ due to the presence of j in the sum, recognize that we can express the expression inside the sum as a derivative,

$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \sum_{j=0}^{\infty} j \left(\frac{1 - \delta^d}{1 + R_t^{new}} \right)^{j-1}$$

$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \sum_{j=0}^{\infty} \frac{d}{d\left(\frac{1-\delta^d}{1+R_t^{new}}\right)} \left(\frac{1 - \delta^d}{1 + R_t^{new}} \right)^{j}$$

Use the formula for geometric series, retake the derivative, and simplify to obtain (7),

$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \frac{d}{d\left(\frac{1-\delta^d}{1+R_t^{new}}\right)} \frac{1}{1 - \left(\frac{1-\delta^d}{1+R_t^{new}}\right)}$$

$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \frac{1}{\left(1 - \frac{1-\delta^d}{1+R_t^{new}}\right)^2}$$

$$Dur_t = \frac{1 + R_t^{new}}{\delta^d + R_t^{new}}$$

■

F.1.2 Households

The overall problem of the households

$$\max_{\{C_t,H_t,B_t^{crp},B_t^{mp},\{D_t^{t-j}\}_{j=0}^{\infty}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta}\right]$$

$$s.t. \quad P_t C_t + B_t^{mp} + q_t^{crp} B_t^{crp} + \sum_{j=0}^{\infty} q_t^{t-j} D_t^{t-j} + P_t T_t = W_t H_t + P_t \Pi_t +$$

$$B_{t-1}^{mp}(1+R_{t-1}^{mp}) + B_{t-1}^{crp}(1+R_{t-1}^{crp})\frac{P_t}{P_{t-1}} + \sum_{j=1}^{\infty}((1-\delta^d)q_t^{t-j} + R_{t-j}^{new} + \delta^d)D_{t-1}^{t-j}$$

As a first step, I make the budget constraint real:

$$C_t + b_t^{mp} + q_t^{crp} b_t^{crp} + \sum_{j=0}^{\infty} q_t^{t-j} d_t^{t-j} + T_t = w_t H_t + \Pi_t +$$

$$b_{t-1}^{mp}\frac{1+R_{t-1}^{mp}}{\pi_t} + b_{t-1}^{crp}(1+R_{t-1}^{crp}) + \sum_{j=1}^{\infty}((1-\delta^d)q_t^{t-j} + R_{t-j}^{new} + \delta^d)\frac{d_{t-1}^{t-j}}{\pi_t}$$

Write the Lagrangian and take the FOCs:

$$\begin{aligned}
\frac{\partial L}{\partial C_t} &: \ C_t^{-\sigma} = \lambda_t \\
\frac{\partial L}{\partial H_t} &: \ \chi H_t^{\eta} = \lambda_t w_t \\
\frac{\partial L}{\partial b_t^{mp}} &: \ \lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{(1+R_t^{mp})}{\pi_{t+1}}\right] \\
\frac{\partial L}{\partial b_t^{crp}} &: \ \lambda_t q_t^{crp} = \beta \mathbb{E}_t \left[\lambda_{t+1}(1+R_t^{crp})\right] \\
\frac{\partial L}{\partial d_t^{t-j}} &: \ \lambda_t q_t^{t-j} = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}} \left((1-\delta^d) q_{t+1}^{t-j} + R_{t-j}^{new} + \delta^d\right)\right]
\end{aligned}$$

We get a standard labor supply choice, a standard Euler for the monetary policy bond, the

Euler for the corporate bond and the government bonds with the secondary market prices:

$$
\begin{aligned}
C_t^{-\sigma} w_t &= \chi H_t^{\eta} \\
1 &= \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1 + R_t^{mp})}{\pi_{t+1}} \right] \\
1 &= \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1 + R_t^{crp})}{q_t^{crp}} \right] \\
q_t^{t-j} &= \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left((1 - \delta^d) q_{t+1}^{t-j} + R_{t-j}^{new} + \delta^d \right) \right]
\end{aligned}
$$

The last condition is a standard asset pricing equation with the price of the bond today being equal to the capital gain on the non-matured portion of the bond $(1 - \delta^d) q_{t+1}^{t-j}$ in addition to the payout being the promised fixed interest rate R_{t-j}^{new} and the repayment of the principal δ^d, all discounted by the SDF for nominal assets $\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}}$. As a next step, we substitute out the secondary market price with the friction Φ_t we can write the Euler equation for newly issued public debt in terms of tomorrows rate on new bonds as. Take the equation at $j = 0$:

$$
q_t^t = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left((1 - \delta^d) q_{t+1}^t + R_t^{new} + \delta^d \right) \right]
$$

Expand the recursive term to be an infinite sum:

$$
\begin{aligned}
q_t^t &= \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left(R_t^{new} + \delta^d \right) \right] + \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} (1 - \delta^d) q_{t+1}^t \right] \\
q_t^t &= \mathbb{E}_t \sum_{j=1}^{\infty} \left[\beta^j \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) (1 - \delta^d)^{j-1} \left(R_t^{new} + \delta^d \right) \right]
\end{aligned}
$$

Take to the left side terms at time t:

$$
C_t^{-\sigma} \frac{q_t^t}{R_t^{new} + \delta^d} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\beta^j \left(C_{t+j} \right)^{-\sigma} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) (1 - \delta^d)^{j-1} \right]
$$

Expand the first term and notice that the expression admits a recursive representation in term of the bonds issued in the following period.

$$
\begin{aligned}
C_t^{-\sigma}\frac{q_t^t}{R_t^{new}+\delta^d} &= \mathbb{E}_t\Bigg[\beta\left(C_{t+1}\right)^{-\sigma}\frac{1}{\pi_{t+1}}+ \\
&\quad +(1-\delta^d)\beta\frac{1}{\pi_{t+1}}\sum_{j=1}^{\infty}\beta^j\left(C_{t+1+j}\right)^{-\sigma}\prod_{k=1}^{j}\left(\frac{1}{\pi_{t+1+k}}\right)(1-\delta^d)^{j-1}\Bigg] \\
C_t^{-\sigma}\frac{q_t^t}{R_t^{new}+\delta^d} &= \mathbb{E}_t\Bigg[\beta\left(C_{t+1}\right)^{-\sigma}\frac{1}{\pi_{t+1}}+(1-\delta^d)\beta\frac{1}{\pi_{t+1}}C_{t+1}^{-\sigma}\frac{q_{t+1}^{t+1}}{R_{t+1}^{new}+\delta^d}\Bigg]
\end{aligned}
$$

Rearrange, substitute out the monetary policy rate Euler equation, and the primary market friction

$$
\begin{aligned}
\frac{q_t^t}{R_t^{new}+\delta^d} &= (1+R_t^{mp})^{-1}+\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\pi_{t+1}}(1-\delta^d)\frac{q_{t+1}^{t+1}}{R_{t+1}^{new}+\delta^d}\right] \\
\frac{1+\Phi_t}{R_t^{new}+\delta^d} &= (1+R_t^{mp})^{-1}+\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\pi_{t+1}}(1-\delta^d)\frac{1+\Phi_{t+1}}{R_{t+1}^{new}+\delta^d}\right]
\end{aligned}
$$

We do not need to carry around the prices of all other government bonds, but only of the currently issued one and the expected price on government bonds issued tomorrow. The interest rate on newly issued bonds today R_t^{new} depends on the current primary market frictions, on current monetary policy rates, and on the expected bond interest rates tomorrow and on tomorrows frictions in the primary market. Furthermore, the higher maturity is the higher the weight of futures rates compared to current short ones in determining the rate on newly issued government bonds. Furthermore, we can see that the Euler for public debt can be rewritten to equate the convolution of primary market friction and interest rates on newly issued bonds as a decaying average of a nominal yield curve of zero-coupon bonds:

$$\lambda_t \left(1+\Phi_t\right) = \mathbb{E}_t \left[(\delta^d + R_t^{new}) \sum_{j=1}^{\infty} (1-\delta^d)^{j-1} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) \beta^j \lambda_{t+j} \right]$$

$$\frac{(1+\Phi_t)}{(\delta^d + R_t^{new})} = \sum_{j=1}^{\infty} (1-\delta^d)^{j-1} \mathbb{E}_t \left[\prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) \beta^j \frac{\lambda_{t+j}}{\lambda_t} \right]$$

$$\frac{(1+\Phi_t)}{(\delta^d + R_t^{new})} = \sum_{j=1}^{\infty} (1-\delta^d)^{j-1} \left[1 + R_t^{zerocoupon,t,t+j} \right]^{-j}$$

F.1.3 Public Debt Pricing

Proof of Lemma 2.

This subsection presents the derivation for the secondary market value of public debt. To this aim, we price separately each vintage of government bonds and then aggregate the price to the whole stock of debt. Take the Euler equation for a generic bond issued j periods ago:

$$q_t^{t-j} = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left((1-\delta^d) q_{t+1}^{t-j} + R_{t-j}^{new} + \delta^d \right) \right]$$

Notice that we can proceed as in the $j=0$ case and find:

$$C_t^{-\sigma} \frac{q_t^{t-j}}{R_{t-j}^{new} + \delta^d} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\beta^j \left(C_{t+j} \right)^{-\sigma} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) (1-\delta^d)^{j-1} \right]$$

This expression on the right is the same for all j as it is for $j=0$, therefore we can equate them and write:

$$q_t^{t-j} = \frac{R_{t-j}^{new} + \delta^d}{R_t^{new} + \delta^d} (1+\Phi_t)$$

Without the friction the price of a bond issued j periods ago is simply the ratio of the cashflows per period per unit of bond, with a higher interest rate legacy bond commanding a higher price. Notice also that the difference in interest rate has a higher impact in the case of longer maturity bonds (lower δ^d). As in the case of bonds issued in the current period and purchased on the secondary market, the price is higher with a higher primary market friction due to no arbitrage: if purchasing bonds on the primary market is more expensive their return will be higher to compensate, but this would push the price upward in the frictionless secondary market. Finally, we can define the secondary market value of public debt with an average price q_t^d:

$$
\begin{aligned}
D_t q_t^d &= \sum_{j=0}^{\infty} (1-\delta^d)^j L_{t-j} q_t^{t-j} \\
D_t q_t^d &= \sum_{j=0}^{\infty} (1-\delta^d)^j L_{t-j} \frac{(\delta^d + R_{t-j}^{new})}{(\delta^d + R_t^{new})} (1+\Phi_t) \\
D_t q_t^d &= \frac{(1+\Phi_t)}{(\delta^d + R_t^{new})} D_t (\delta^d + R_t^{ave}) \\
q_t^d &= \frac{(\delta^d + R_t^{ave})}{(\delta^d + R_t^{new})} (1+\Phi_t)
\end{aligned}
$$

Which is the same result as for each single bond, except with the average interest rate, this is due to the convenient geometric repayment formula. Notice that the price of government debt would increase following an increase in the primary market friction when we hold interest rates constant. When there is an increase in the primary market friction, the interest rate on newly issued bonds would increase, and depending on the shock and the maturity of the bond the impact can be higher than the friction itself and the value of public debt would decline. This is the case in my calibration following a monetary policy shock the price of debt and the primary market friction move in opposite directions when debt is long, but in the same one when debt is one period (mechanically as there $R_t^{ave} = R_t^{new}$).

■

F.1.4 Duration and Debt Servicing Costs

Macaulay duration measures how much the value of debt changes following a one percent increase in interest rates across the yield curve. In this section, I prove Proposition 1 and I show how this is also a measure of how much insurance long debt provides against interest rate risk in terms of debt servicing costs. Specifically, I show how, following a one percent permanent increase in interest rates across the yield curve, duration measures the net present value of interest rate savings with long debt relative to short debt on existing debt.

Proof of Proposition 1.

Take the public debt structure presented in the model, with geometrically decaying principal. At period $t-1$ we have a stock of debt D_{t-1} with average interest rate R_{t-1}^{ave}. We will focus on changes on interest payments on existing debt, not on the whole stock of debt in equilibrium. For this reason, we consider new issuances to roll-over the existing stock of debt $L^{to\ roll-over_t}$ which is simply equal to the maturing fraction of the principal $\delta^d D_{t-1}$. From the law of motion of the principal of public debt we can see how this keeps this stock constant at D_{t-1}. Following an interest rate change to R_t^{new} we learn about at the end of period $t-1$, we can see how interest payments on rolled over legacy debt will simply be a weighted average of the new interest rate and the legacy one. This holds even if we go further in the future $j \geq 0$ as the change in R_t^{new} is permanent. The weight on legacy rates declines as we go further in the future as this debt is slowly maturing.

$$
\begin{aligned}
R_t^{ave,legacy} &= R_t^{new}\delta^d + R_{t-1}^{ave}\left(1-\delta^d\right) \\
R_{t+j}^{ave,legacy} &= R_t^{new}\delta^d \sum_{k=0}^{j}\left(1-\delta^d\right)^j + R_{t-1}^{ave}\left(1-\delta^d\right)^{j+1} \\
R_{t+j}^{ave,legacy} &= R_t^{new}\left[1-\left(1-\delta^d\right)^{j+1}\right] + R_{t-1}^{ave}\left(1-\delta^d\right)^{j+1}
\end{aligned}
$$

We discount the difference in interest payments between a short debt ($\delta^d = 1$),profile so that $R_{t+j}^{ave,legacy}|_{\delta^d=1} = R_t^{new}$ for $j \geq 0$ and the observed maturity with $\delta^d \leq 1$.

$$
\begin{aligned}
\sum_{j=0}^{\infty} \frac{R_t^{new}D_{t-1} - R_{t+j}^{ave,legacy}D_{t-1}}{(1+R_t^{new})^{j+1}} &= \sum_{j=0}^{\infty}\left(\frac{1-\delta^d}{1+R_t^{new}}\right)^{j+1} D_{t-1}(R_t^{new} - R_{t-1}^{ave}) \\
&= \left(\frac{1+R_t^{new}}{\delta^d + R_t^{new}} - 1\right) D_{t-1}(R_t^{new} - R_{t-1}^{ave}) \\
&= \left(Dur_t - 1\right) D_{t-1}(R_t^{new} - R_{t-1}^{ave})
\end{aligned}
$$

We can see how the change in debt servicing costs following an interest rate change of $(R_t^{new} - R_{t-1}^{ave})$ is exactly equal to the difference in duration, where I used the result from

Lemma 1, between debt with maturity parameter δ^d and debt with one period duration. This implies that the net present value of interest rate savings coming from long debt is correctly captured by its duration. Moreover, this maps as well to the changes in the market value of public debt. We take $\frac{q_t^d}{1+\Phi_t}$ as the overall market value (or the market value that accounts for the primary market friction) of legacy public debt from Lemma 2 and notice the same pattern:

$$
\begin{aligned}
\frac{q_t^d}{1+\Phi_t} D_{t-1} &= \frac{(\delta^d + R_t^{ave,legacy})}{(\delta^d + R_t^{new})} D_{t-1} \\
&= \frac{(\delta^d + R_t^{new}\delta^d + R_{t-1}^{ave}\left(1-\delta^d\right))}{(\delta^d + R_t^{new})} D_{t-1} \\
&= \left(1 - \frac{1-\delta^d}{\delta^d + R_t^{new}}(R_t^{new} - R_{t-1}^{ave})\right) D_{t-1} \\
&= \left(1 - \left(\frac{1+R_t^{new}}{\delta^d + R_t^{new}} - 1\right)(R_t^{new} - R_{t-1}^{ave})\right) D_{t-1} \\
&= \left(1 - (Dur_t - 1)\left(R_t^{new} - R_{t-1}^{ave}\right)\right) D_{t-1} \\
&= D_{t-1} - (Dur_t - 1) D_{t-1}(R_t^{new} - R_{t-1}^{ave})
\end{aligned}
$$

Following the same rate increase, the market value of public debt decreases exactly by the difference in duration between debt with maturity parameter δ^d and debt with one period duration. If we divide both terms nominal GDP ($Y_{t-1}^n \equiv P_{t-1}Y_{t-1}$) we obtain duration-to-GDP of legacy debt ($Dur_t D_{t-1}/Y_{t-1}^n$). This terminates the proof and establishes how we can use debt duration to measure how much insurance against interest rate changes the maturity profile gives both in term in debt servicing costs and valuation effects of the market value of public debt.

■

F.2 Calvo Retailers

In this section, I solve a standard retailers problem to obtain the non linear New-Keynesian Phillips Curve. Retailers buy a wholesale good at price P_t^w and use it to produce the retail variety y_{it} with a linear technology that maps one to one the wholesale good to the retail variety. As each variety is differentiated they have market power and face a Calvo friction to change prices. Their real marginal cost $\mathcal{S}_t = \frac{P_t^w}{P_t} = \frac{1}{X_t}$ is the real wholesale price. The probability of not being able to reset prices is equal to θ in each period. The discounted present value of profits:

$$\mathbb{E}_t \sum_{j=0}^{\infty} SDF_{t,t+j} \left(P_{i,t+j}Y_{i,t+j} - P_{t+j}\mathcal{S}_{t+j}Y_{i,t+j}\right)$$

All firms that in period t can reset their price face the same problem (this happens with probability $1-\theta$), therefore will choose the same price $\tilde{P}_t$, that maximizes profits as long as it remains in place:

$$\mathbb{E}_t \sum_{j=0}^{\infty} SDF_{t,t+j}\theta^j \left(\tilde{P}_t Y_{i,t+j} - P_{t+j}\mathcal{S}_{t+j}Y_{i,t+j}\right)$$

Substitute the demand equation:

$$\mathbb{E}_t \sum_{j=0}^{\infty} SDF_{t,t+j}\theta^j \left(\tilde{P}_t - P_{t+j}\mathcal{S}_{t+j}\right) Y_{t+j} \left(\frac{\tilde{P}_t}{P_{t+j}}\right)^{-\varepsilon}$$

Take FOC:

$$\mathbb{E}_t \sum_{j=0}^{\infty} SDF_{t,t+j}\theta^j P_{t+j}^{\varepsilon} \left((1-\varepsilon)(\tilde{P}_t)^{-\varepsilon} + \varepsilon(\tilde{P}_t)^{-\varepsilon-1}P_{t+j}\mathcal{S}_{t+j}\right) Y_{t+j} = 0$$

$$\mathbb{E}_t \sum_{j=0}^{\infty} SDF_{t,t+j}\theta^j P_{t+j}^{\varepsilon+1} \left(\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1}\mathcal{S}_{t+j}\right) Y_{t+j} = 0$$

Substitute the household's SDF for nominal profits ($SDF_{t,t+k} = \beta^j \frac{C_{t+j}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+j}}$, as retailers are owned by the households and not by the entrepreneurs), with the ratio of prices being there as we are discounting a nominal cash flow:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+j}} \theta^j P_{t+j}^{\varepsilon+1} \left(\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1}\mathcal{S}_{t+j}\right) Y_{t+j} = 0$$

$$\mathbb{E}_t \sum_{j=0}^{\infty} C_{t+j}^{-\sigma} Y_{t+j} (\theta\beta)^j P_{t+j}^{\varepsilon} \left(\frac{\tilde{P}_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon-1}\mathcal{S}_{t+j}\right) = 0$$

Define the inverse of accumulated inflation as $x_{t,t+j} \equiv (\pi_{t+1} \cdots \pi_{t+j})^{-1} = \frac{P_t}{P_{t+j}}$:

$$\mathbb{E}_t \sum_{j=0}^{\infty} C_{t+j}^{-\sigma} Y_{t+j} (\theta\beta)^j x_{t,t+j}^{-\varepsilon} \left(\frac{\tilde{P}_t}{P_t} x_{t,t+j} - \frac{\varepsilon}{\varepsilon - 1} \mathcal{S}_{t+j} \right) = 0$$

We can now solve explicitly for the price ratio:

$$\frac{\tilde{P}_t}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} C_{t+j}^{-\sigma} Y_{t+j} (\theta\beta)^j x_{t,t+j}^{-\varepsilon} \left(\mathcal{S}_{t+j} \right)}{\mathbb{E}_t \sum_{j=0}^{\infty} C_{t+j}^{-\sigma} Y_{t+j} (\theta\beta)^j x_{t,t+j}^{1-\varepsilon}}$$

Define the terms of the fraction as K_t^f and F_t^f and notice that they have a recursive representation:

$$\begin{aligned}
\frac{\tilde{P}_t}{P_t} &= \frac{\mathbb{E}_t \sum_{j=0}^{\infty} C_{t+j}^{-\sigma} Y_{t+j} (\theta\beta)^j x_{t,t+j}^{-\varepsilon} \left(\mathcal{S}_{t+j} \right) \frac{\varepsilon}{\varepsilon-1}}{\mathbb{E}_t \sum_{j=0}^{\infty} C_{t+j}^{-\sigma} Y_{t+j} (\theta\beta)^j x_{t,t+j}^{1-\varepsilon}} \\
\frac{\tilde{P}_t}{P_t} &= \frac{K_t^f}{F_t^f} \\
K_t^f &\equiv \mathbb{E}_t \sum_{j=0}^{\infty} C_{t+j}^{-\sigma} Y_{t+j} (\theta\beta)^j x_{t,t+j}^{-\varepsilon} \left(\mathcal{S}_{t+j} \right) \frac{\varepsilon}{\varepsilon - 1} \\
K_t^f &= C_t^{-\sigma} Y_t \mathcal{S}_t \frac{\varepsilon}{\varepsilon - 1} + \theta\beta \mathbb{E}_t \pi_{t+1}^{\varepsilon} K_{t+1}^f \\
F_t^f &\equiv \mathbb{E}_t \sum_{j=0}^{\infty} C_{t+j}^{-\sigma} Y_{t+j} (\theta\beta)^j x_{t,t+j}^{1-\varepsilon} \\
F_t^f &= C_t^{-\sigma} Y_t + \theta\beta \mathbb{E}_t \pi_{t+1}^{\varepsilon-1} F_{t+1}^f
\end{aligned}$$

Note that the price index is the combination of the firms that could re-optimize and of those who could not.

$$
\begin{aligned}
P_t &= \left(\int_0^1 P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \\
P_t &= \left(\int_{reoptimzers} (\tilde{P}_t)^{1-\varepsilon} di + \int_{non-reoptimzers} (P_{i,t-1})^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \\
P_t &= \left((1-\theta)(\tilde{P}_t)^{1-\varepsilon} + \theta \int_0^1 (P_{i,t-1})^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \\
P_t &= \left((1-\theta)(\tilde{P}_t)^{1-\varepsilon} + \theta (P_{t-1})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

Rearrange:

$$
\begin{aligned}
P_t &= \left((1-\theta)(\tilde{P}_t)^{1-\varepsilon} + \theta (P_{t-1})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \\
\pi_t^{1-\varepsilon} &= (1-\theta) \left(\frac{\tilde{P}_t}{P_t} \right)^{1-\varepsilon} \pi_t^{1-\varepsilon} + \theta \\
\frac{\tilde{P}_t}{P_t} &= \left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

The non-linear Phillips Curve can be expressed by equating the two expressions:

$$
\frac{K_t^f}{F_t^f} = \left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} \tag{F.1}
$$

We can also write these equations with instead of the marginal cost $\mathcal{S}_t$ the relative price of wholesale goods made explicit as a mark-up:

$$
\begin{aligned}
K_t^f &= C_t^{-\sigma} Y_t \frac{1}{X_t} \frac{\varepsilon}{\varepsilon-1} + \theta\beta\mathbb{E}_t \pi_{t+1}^{\varepsilon} K_{t+1}^f \\
F_t^f &= C_t^{-\sigma} Y_t + \theta\beta\mathbb{E}_t \pi_{t+1}^{\varepsilon-1} F_{t+1}^f \\
\frac{K_t^f}{F_t^f} &= \left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

F.3 Entrepreneurs

This section provides the detailed entrepreneurs problem solution. The first step is to show how to arrive to the participation constraint for the lenders. As discussed in the main text, an entrepreneur pays back debt if the return on her investment $(1 + R^k_{t+1})Q_t K_t \omega^j_{t+1}$ is higher than the cost of servicing debt $Z^j_{t+1}B^j_t$ otherwise defaults and the lender recovers $(1-\mu)(1+R^k_{t+1})Q_t K_t \omega^j_{t+1}$. There exists a threshold $\bar{\omega}^j_{t+1}$ above which the entrepreneur pays a fixed amount and below the recovery value. From the perspective of the lender, the return on a single loan is therefore:

$$(1 + R^j_{t+1})B^j_t = \begin{cases} Z^j_{t+1}B^j_t & if\ \omega^j_{t+1} \geq \bar{\omega}^j_{t+1} \\ (1-\mu)(1+R^k_{t+1})Q_t K_t \omega^j_{t+1} & if\ \omega^j_{t+1} < \bar{\omega}^j_{t+1} \end{cases}$$

The return on this loan in expected term (with respect to the idiosyncratic shock) but given a realized return R^k_{t+1}, must be equal to the outside option of lenders R^{crp}_t. This loan return is guaranteed for lenders as there is a large mass of entrepreneurs. We can take the expectation with respect to ω:

$$\begin{aligned}
(1 + R^{crp}_t)B^j_t &= \int_0^{\bar{\omega}^j_{t+1}} (1-\mu)(1+R^k_{t+1})Q_t K^j_t \omega f(\omega, \sigma_{\omega,t}) d\omega + \\
&+ \int_{\bar{\omega}^j_{t+1}}^{\infty} \bar{\omega}^j_{t+1}(1+R^k_{t+1})Q_t K^j_t f(\omega, \sigma_{\omega,t}) d\omega \\
(1 + R^{crp}_t)B^j_t &= (1+R^k_{t+1})Q_t K^j_t \Bigg[(1-\mu)\int_0^{\bar{\omega}^j_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega \\
&+ \bar{\omega}^j_{t+1}(1 - F(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}))\Bigg] \\
(1 + R^{crp}_t)B^j_t &= (1+R^k_{t+1})Q_t K^j_t (\Gamma(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) - \mu G(\bar{\omega}^j_{t+1}, \sigma_{\omega,t})) \\
(1 + R^{crp}_t)(\kappa^j_t - 1) &= (1+R^k_{t+1})\kappa^j_t (\Gamma(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) - \mu G(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}))
\end{aligned}$$

Where, as in the main text, we define leverage as $\kappa^j_t \equiv \frac{Q_t K^j_t}{N^j_t}$, and the helping functions $\Gamma(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) = \int_0^{\bar{\omega}^j_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega + \bar{\omega}^j_{t+1}(1 - F(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}))$, $G(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) = \int_0^{\bar{\omega}^j_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega$, and $F(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) = \int_0^{\bar{\omega}^j_{t+1}} f(\omega, \sigma_{\omega,t}) d\omega$. The entrepreneurs problem is to maximize expected wealth (so that the objective is linear), where they are protected by limited liability (so that the objective has the max operator), subject to the participation constraint of the lenders. They choose a combination of leverage κ_t^j before uncertainty is realized and default cut-off $\bar{\omega}_{t+1}^j$ contingent on shock realization:

$$\max_{\{\kappa_t^j, \bar{\omega}_{t+1}^j\}} \mathbb{E}_t \max\left[(1+R_{t+1}^k)\kappa_t^j N_t^j(\omega_{t+1}^j - \bar{\omega}_{t+1}^j), 0\right]$$

$$s.t.$$

$$(1+R_t^{crp})(\kappa_t^j - 1) = (1+R_{t+1}^k)\kappa_t^j(\Gamma(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}) - \mu G(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}))$$

This equivalent to:

$$\max_{\{\kappa_t^j, \bar{\omega}_{t+1}^j\}} L = \mathbb{E}_t\Bigg[\frac{(1+R_{t+1}^k)}{(1+R_t^{crp})}\kappa_t^j \left(\int_{\bar{\omega}_{t+1}^j}^{\infty} \omega f(\omega, \sigma_{\omega,t}) d\omega - \bar{\omega}_{t+1}^j(1 - F(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}))\right) +$$
$$-\lambda_{t+1}\left[(\kappa_t^j - 1) - \frac{(1+R_{t+1}^k)}{(1+R_t^{crp})}\kappa_t^j(\Gamma(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}) - \mu G(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}))\right]\Bigg]$$

We can define the risk spread as the ratio of returns, $(1+s_{t+1}) \equiv \frac{(1+R_{t+1}^k)}{(1+R_t^{crp})}$, and notice that the objective function can be rewritten by taking advantage of the fact that the expected value of ω_{t+1} is 1: $1 - \Gamma(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}) = 1 - \int_0^{\bar{\omega}_{t+1}^j} \omega f(\omega, \sigma_{\omega,t}) d\omega - \bar{\omega}_{t+1}^j(1 - F(\bar{\omega}_{t+1}^j, \sigma_{\omega,t})) = \int_0^{\bar{\omega}_{t+1}^j} \omega f(\omega, \sigma_{\omega,t}) d\omega + \int_{\bar{\omega}_{t+1}^j}^{\infty} \omega f(\omega, \sigma_{\omega,t}) d\omega - \int_0^{\bar{\omega}_{t+1}^j} \omega f(\omega, \sigma_{\omega,t}) d\omega - \bar{\omega}_{t+1}^j(1 - F(\bar{\omega}_{t+1}^j, \sigma_{\omega,t})) = \int_{\bar{\omega}_{t+1}^j}^{\infty} \omega f(\omega, \sigma_{\omega,t}) d\omega - \bar{\omega}_{t+1}^j(1 - F(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}))$:

$$\max_{\{\kappa_t^j, \bar{\omega}_{t+1}^j\}} L = \mathbb{E}_t\Bigg[(1+s_{t+1})\kappa_t^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j, \sigma_{\omega,t})\right) +$$
$$-\lambda_{t+1}\left[1 - (1+s_{t+1})\frac{\kappa_t^j}{\kappa_t^j - 1}(\Gamma(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}) - \mu G(\bar{\omega}_{t+1}^j, \sigma_{\omega,t}))\right]\Bigg]$$

We can solve this problem by taking first order conditions:

$$\begin{aligned}
\frac{\partial L}{\partial \kappa_t^j} &: \ \mathbb{E}_t\Bigg[(1+s_{t+1})\left(1-\Gamma(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})\right) \\
&\quad - \lambda_{t+1}(1+s_{t+1})\frac{1}{(\kappa_t^j-1)^2}(\Gamma(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})-\mu G(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\Bigg]=0 \\
\frac{\partial L}{\partial \bar{\omega}_{t+1}^j} &: \ -(1+s_{t+1})\kappa_t^j\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}) \\
&\quad + \lambda_{t+1}\Bigg[(1+s_{t+1})\frac{\kappa_t^j}{\kappa_t^j-1}(\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})-\mu G_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\Bigg]=0
\end{aligned}$$

To solve the problem, first simplify the first order condition with respect to the threshold:

$$-\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})+\lambda_{t+1}\left[\frac{1}{\kappa_t^j-1}(\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})-\mu G_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\right]=0$$

This equation defines the Lagrange multiplier λ_{t+1} in terms of $\bar{\omega}_{t+1}^j$ and, once we substitute this in the first first order condition we have a $\bar{\omega}_{t+1}^j$ in terms of the ratio $(1+s_{t+1})$. We can use the participation constraint to find the mapping from the ratio $(1+s_{t+1})$ to leverage κ_t^j. As the first order conditions are the same for all entrepreneurs irrespective of their equity level, the leverage and threshold choices are the same for all: κ_t and $\bar{\omega}_{t+1}$. To ease notation let $\Gamma_{\omega,t+1}\equiv\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})$ and similarly for G and other derivatives.

$$\begin{aligned}
\lambda_{t+1} &= \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1}-\mu G_{\omega,t+1}}(\kappa_t-1) \\
0 &= \mathbb{E}_t\left[(1+s_{t+1})\left(1-\Gamma_{t+1}\right)-\lambda_{t+1}(1+s_{t+1})\frac{1}{(\kappa_t-1)^2}(\Gamma_{t+1}-\mu G_{t+1})\right] \\
\frac{\kappa_t-1}{\kappa_t} &= (1+s_{t+1})(\Gamma_{t+1}-\mu G_{t+1})
\end{aligned}$$

Plug the Lagrangian multiplier in the second equation to reduce the system to 2 equations:

$$\begin{aligned}
0 &= \mathbb{E}_t\left[(1+s_{t+1})\left(1-\Gamma_{t+1}\right)-\frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1}-\mu G_{\omega,t+1}}(1+s_{t+1})\frac{1}{(\kappa_t-1)}(\Gamma_{t+1}-\mu G_{t+1})\right] \\
\frac{\kappa_t-1}{\kappa_t} &= (1+s_{t+1})(\Gamma_{t+1}-\mu G_{t+1})
\end{aligned}$$

Use participation constraint to simplify the first equation. These two equations summarize the non-linear system for the entrepreneur choice:

$$
\begin{aligned}
0 &= \mathbb{E}_t \left[(1+s_{t+1})\kappa_t \left(1-\Gamma_{t+1}\right) - \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}} \right] \\
\frac{\kappa_t - 1}{\kappa_t} &= (1+s_{t+1})(\Gamma_{t+1} - \mu G_{t+1})
\end{aligned}
$$

To substitute out the threshold choice we have to log-linearize the system as $\bar{\omega}_{t+1}$ is now defined implicitly, as done in the linearization section. To close the description of the entrepreneur sector we need to specify how the wealth and consumption of entrepreneurs behave. As a first step, I show the value of equity today is the return on capital invested less the amount paid to the lender from last period:

$$
\begin{aligned}
V_t &= (1+R_t^k) Q_{t-1} K_{t-1} \\
&\quad - \left((1+R_{t-1}^{crp}) + \frac{\mu (1+R_t^k) Q_{t-1} K_{t-1}}{Q_{t-1}K_{t-1} - N_{t-1}} \int_0^{\bar{\omega}_t} \omega dF(\omega, \sigma_{\omega,t-1}) \right) (Q_{t-1}K_{t-1} - N_{t-1})
\end{aligned}
$$

We can expand this to describe it in terms of equity amount in the previous period and leverage:

$$
\begin{aligned}
V_t &= \left((1+R_t^k)\kappa_{t-1} - \left((1+R_{t-1}^{crp}) + \mu \frac{(1+R_t^k)\kappa_{t-1}}{\kappa_{t-1} - 1} G(\bar{\omega}_t, \sigma_{\omega,t-1}) \right) (\kappa_{t-1} - 1) \right) N_{t-1} \\
V_t &= \left((R_t^k - R_{t-1}^{crp})\kappa_{t-1} + (1+R_{t-1}^{crp}) - \mu (1+R_t^k)\kappa_{t-1} G(\bar{\omega}_t, \sigma_{\omega,t-1}) \right) N_{t-1}
\end{aligned}
$$

Based on this result we can define the return on equity as what maps the equity quantity last period into the equity value today:

$$
(1+R_t^e) = \left((R_t^k - R_{t-1}^{crp})\kappa_{t-1} + (1+R_{t-1}^{crp}) - \mu (1+R_t^k)\kappa_{t-1} G(\bar{\omega}_t, \sigma_{\omega,t-1}) \right)
$$

From there the last two equations presented in the main text follow directly. The new equity is equal to the return on last period equity for the entrepreneurs who do not exit in addition to labor income. Entrepreneurs who exit consume the value of the firm:

$$N_t = \gamma(1 + R_t^e)N_{t-1} + w_t^e$$

$$C_t^e = (1 - \gamma)(1 + R_t^e)N_{t-1}$$

F.4 All Equilibrium Condition

The competitive equilibrium consists of 16 endogenous allocations $\{C_t,\ C_t^e,\ I_t,\ Y_t,\ K_t^f,\ F_t^f, \kappa_t,\ \bar{\omega}_t,\ N_t,\ K_t,\ H_t,\ H_t^e,\ l_t,\ f_t,\ d_t,\ T_t\}$, 15 prices $\{w_t,\ w_t^e,\ \pi_t,\ R_t,\ R_t^{crp},\ R_t^k,\ R_t^r,\ R_t^e,\ s_t,\ X_t,\ Q_t,\ \Phi_t,\ R_t^{new},\ R_t^{ave},\ R_t^{mp}\}$, and 4 exogenous processes $\{G_t,\ \sigma_{\omega,t},\ A_t,\ \nu_t^{\Phi}\}$; such that households, primary market participants, final good producers, retailers, wholesalers, capital producers, and entrepreneurs optimize, the central bank follows a Taylore rule, the treasury follows the tax rule, and markets clear. The equilibrium is characterized by the following equations and processes:

$$
\begin{aligned}
C_t^{-\sigma} w_t &= \chi H_t^{\eta} \\
C_t^{-\sigma} &= \mathbb{E}_t \left[\beta C_{t+1}^{-\sigma} \frac{(1+R_t^{mp})}{\pi_{t+1}} \right] \\
K_t^f &= C_t^{-\sigma} Y_t \frac{1}{X_t} \frac{\varepsilon}{\varepsilon - 1} + \theta \beta \mathbb{E}_t \pi_{t+1}^{\varepsilon} K_{t+1}^f \\
F_t^f &= C_t^{-\sigma} Y_t + \theta \beta \mathbb{E}_t \pi_{t+1}^{\varepsilon-1} F_{t+1}^f \\
\frac{K_t^f}{F_t^f} &= \left(\frac{1 - \theta \pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} \\
Y_t &= A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)} \\
R_t^r &= \frac{1}{X_t} \alpha \frac{Y_t}{K_{t-1}} \\
w_t &= \frac{1}{X_t} (1-\alpha) \Omega \frac{Y_t}{H_t} \\
w_t^e &= \frac{1}{X_t} (1-\alpha)(1-\Omega) \frac{Y_t}{H_t^e} \\
K_t &= I_t - \frac{\phi_K}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} + (1-\delta) K_{t-1} \\
Q_t & \left(1 - \phi_K \left(\frac{I_t}{K_{t-1}} - \delta \right) \right) = 1 \\
1 + R_{t+1}^k &= \frac{\frac{1}{X_{t+1}} \alpha \frac{Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t} \\
\kappa_t &= \frac{Q_t K_t}{N_t} \\
1 + s_t &= \frac{1 + R_t^k}{1 + R_{t-1}^{crp}} \\
0 &= \mathbb{E}_t \left[(1+s_{t+1}) \kappa_t \left(1 - \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t})\right) - \frac{\Gamma_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\Gamma_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - \mu G_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})} \right] \\
\frac{\kappa_t - 1}{\kappa_t} &= (1+s_{t+1})(\Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - \mu G(\bar{\omega}_{t+1}, \sigma_{\omega,t})) \\
N_t &= \gamma (1 + R_t^e) N_{t-1} + w_t^e \\
C_t^e &= (1-\gamma)(1+R_t^e) N_{t-1} \\
1 + R_t^e &= \left((R_t^k - R_{t-1}^{crp}) \kappa_{t-1} + (1 + R_{t-1}^{crp}) - \mu (1 + R_t^k) \kappa_{t-1} G(\bar{\omega}_t, \sigma_{\omega,t-1}) \right) \\
Y_t &= C_t + C_t^e + I_t + G_t + \mu G(\bar{\omega}_t, \sigma_{\omega,t-1})(1+R_t^k) N_{t-1} \kappa_{t-1} \\
H_t^e &= 1
\end{aligned}
$$

$$f_t = (R^{ave}_{t-1} + \delta^d)\frac{1}{\pi_t}d_{t-1}$$

$$d_t = (1-\delta^d)\frac{1}{\pi_t}d_{t-1} + l_t$$

$$R^{ave}_t = \left(1 - \frac{l_t}{d_t}\right)R^{ave}_{t-1} + \frac{l_t}{d_t}R^{new}_t$$

$$f_t = T_t - G_t + l_t$$

$$T_t = G_t + (T-G)\left(\frac{d_{t-1}}{d}\right)^{\tau_T}$$

$$\hat{\Phi}_t = \zeta(\hat{b}^{crp}_t + \hat{l}_t) + \nu^{\Phi}_t$$

$$\frac{(1+\Phi_t)}{(\delta^d + R^{new}_t)} = (1+R^{mp}_t)^{-1} + \mathbb{E}_t\left[\beta\frac{C^{-\sigma}_{t+1}}{C^{-\sigma}_t(\pi_{t+1})}(1-\delta^d)\frac{(1+\Phi_{t+1})}{(\delta^d + R^{new}_{t+1})}\right]$$

$$C^{-\sigma}_t = \mathbb{E}_t\left[\beta C^{-\sigma}_{t+1}\frac{(1+R^{crp}_t)}{1+\Phi_t}\right]$$

$$1 + R_t = \frac{(1+R^{crp}_t)}{1+\Phi_t}$$

$$\left(\frac{1+R^{mp}_t}{1+R^{mp}}\right)^{1+R^{mp}} = \left(\frac{1+R^{mp}_{t-1}}{1+R^{mp}}\right)^{\rho^{mp}(1+R^{mp})}\left[\left(\frac{\mathbb{E}_t\pi_{t+1}}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{(1-\rho^{mp})}\exp(\varepsilon^{mp}_t)$$

$$\frac{G_t}{G} = \left(\frac{G_{t-1}}{G}\right)^{\rho_g}\exp(\varepsilon^g_t)$$

$$\frac{A_t}{A} = \left(\frac{\hat{A}_{t-1}}{A}\right)^{\rho_A}\exp(\varepsilon^A_t)$$

$$\frac{\sigma_{\omega,t}}{\sigma_\omega} = \left(\frac{\sigma_{\omega,t-1}}{\sigma_\omega}\right)^{\rho_{\sigma_\omega}}\exp(\varepsilon^{\sigma_\omega}_t)$$

$$\nu^{\Phi}_t = \rho_\Phi \nu^{\Phi}_{t-1} + \varepsilon^{\Phi}_t$$

F.5 Steady State

In this section I derive the steady state around a zero inflation steady state. Steady state variables are written simply without the time subscript. We assume an exogenous level of government consumption to GDP $\bar{G} \equiv G/Y$. I follow BGG and, for the financial accelerator, I target the steady state level of the financial accelerator friction s and the leverage level κ, and the average default rate $F(\bar{\omega}, \sigma_\omega)$ and obtain the resulting default threshold $\bar{\omega}$, monitoring cost μ, and volatility of idiosyncratic productivity shocks σ_ω. I take a similar route for the primary marker friction and I target the steady state friction Φ and the impact of an increase in debt by one percent of GDP to rates ζ. I find the implied parameter of the financial accelerator by solving non-linearly:

$$
\begin{aligned}
0 &= \left[(1+s)\kappa\left(1-\Gamma(\bar{\omega},\sigma_{\omega,t})\right) - \frac{\Gamma_{\omega}(\bar{\omega},\sigma_{\omega})}{\Gamma_{\omega}(\bar{\omega},\sigma_{\omega}) - \mu G_{\omega}(\bar{\omega},\sigma_{\omega,t})}\right] \\
\frac{\kappa-1}{\kappa} &= (1+s)(\Gamma(\bar{\omega},\sigma_{\omega}) - \mu G(\bar{\omega},\sigma_{\omega})) \\
F(\bar{\omega},\sigma_{\omega,t}) &= Default\ Rate
\end{aligned}
$$

With:

$$
\begin{aligned}
G(\bar{\omega},\sigma_{\omega}) &= \Phi\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}}\right) \\
F(\bar{\omega},\sigma_{\omega}) &= \Phi\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_{\omega}^2}{2}}{\sigma_{\omega}}\right) \\
\Gamma(\bar{\omega},\sigma_{\omega}) &= G(\bar{\omega},\sigma_{\omega}) + \bar{\omega}(1 - F(\bar{\omega},\sigma_{\omega})) \\
G_{\omega}(\bar{\omega},\sigma_{\omega}) &= \bar{\omega} f(\bar{\omega},\sigma_{\omega}) \\
\Gamma_{\omega}(\bar{\omega},\sigma_{\omega}) &= (1 - F(\bar{\omega},\sigma_{\omega}))
\end{aligned}
$$

Notice that these conditions also restrict the value of the death rate of entrepreneurs γ as shown below. The price of capital and the investment rate:

$$
\begin{aligned}
I &= \delta K \\
Q &= 1
\end{aligned}
$$

Then solve the interest rates:

$$
\begin{aligned}
1 + R^n &= \frac{1}{\beta} \\
1 + R^{crp} &= (1 + R^n)(1 + \Phi) \\
1 + R^k &= (1 + R^{crp})(1 + s) \\
1 + R^r &= R^k - (1 + \delta)
\end{aligned}
$$

Find the steady state values of the Phillips curve variables and markup:

$$
\begin{aligned}
K^f &= \frac{1}{1-\theta\beta} C^{-\sigma} Y \\
F^f &= \frac{1}{1-\theta\beta} C^{-\sigma} Y \\
\frac{1}{X} &= \frac{\varepsilon}{\varepsilon - 1}
\end{aligned}
$$

Let's solve for the average return on invested entrepreneur wealth:

$$
1 + R^e = \big((R^k - R^{crp})\kappa + (1 + R^{crp}) - \mu(1 + R^k)\kappa G(\bar{\omega}, \sigma_\omega)\big)
$$

That can be simplified with steady states relations in terms of explicitly chosen values:

$$
\begin{aligned}
1 + R^e &= \big((R^k - R^{crp})\kappa + (1 + R^{crp}) - \mu(1 + R^k)\kappa G(\bar{\omega}, \sigma_\omega)\big) \\
1 + R^e &= (1 + R^{crp})(s + 1)\kappa \left(1 - \Gamma(\bar{\omega}, \sigma_\omega)\right)
\end{aligned}
$$

This allows to write entrepreneur wealth in term of the entrepreneur wage:

$$
N = \frac{1}{1 - \gamma(1 + R^e)} w^e
$$

Notice that, we can express the capital stock in terms of hours, net return on capital, and parameters from the first order condition of the firm with respect to capital:

$$
\begin{aligned}
R^r &= \alpha \frac{A}{X} K^{\alpha - 1} H^{(1-\alpha)\Omega} \\
K &= \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{1}{1-\alpha}} H^{\Omega}
\end{aligned}
$$

From here we can also express entrepreneurs wage by substituting out capital:

$$
\begin{aligned}
w^e &= (1-\alpha)(1-\Omega) \frac{A}{X} K^{\alpha} H^{(1-\alpha)\Omega} \\
w^e &= (1-\alpha)(1-\Omega) \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{\alpha}{1-\alpha}} H^{\Omega}
\end{aligned}
$$

We can now solve for the implied death rate of entrepreneurs γ by equating the expression of entrepreneur wealth in term of the entrepreneur wage and the in term of capital and leverage:

$$
\begin{aligned}
N &= N \\
\frac{1}{1-\gamma(1+R^e)} w^e &= \frac{1}{\kappa} K \\
\frac{1}{1-\gamma(1+R^e)} (1-\alpha)(1-\Omega) \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{\alpha}{1-\alpha}} H^{\Omega} &= \frac{1}{\kappa} \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{1}{1-\alpha}} H^{\Omega} \\
1-\gamma(1+R^e) &= \frac{(1-\alpha)(1-\Omega)}{\alpha} R^r \kappa \\
\gamma &= \frac{1-\frac{(1-\alpha)(1-\Omega)}{\alpha} R^r \kappa}{(1+R^e)}
\end{aligned}
$$

From here we can set hours to one in steady state and find all resulting steady state values. Notice that this will imply a value for χ.

$$
\begin{aligned}
H &= 1 \\
K &= \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{1}{1-\alpha}} \\
N &= \frac{1}{\kappa} K \\
Y &= A K^{\alpha} \\
w^e &= (1-\alpha)(1-\Omega) \frac{Y}{X} \\
G &= \bar{G} Y \\
I &= \delta K \\
C^e &= (1-\gamma)(1+R^e) N \\
C &= Y - G - I - C^e - \mu G(\bar{\omega}, \sigma_{\omega})(1+R^k)\kappa N \\
\chi &= W H^{-\eta} C^{-\sigma}
\end{aligned}
$$

The long government bond Euler in steady state:

$$\frac{(1+\Phi)}{(\delta^d + R^{new})} = (1+R^{mp})^{-1} + \left[\beta \frac{C^{-\sigma}}{C^{-\sigma}\pi}(1-\delta^d)\frac{(1+\Phi)}{(\delta^d + R^{new})}\right]$$

$$\frac{(1+\Phi)}{(\delta^d + R^{new})}(1-\beta(1-\delta^d)) = (1+R^{mp})^{-1}$$

$$\frac{(1+\Phi)}{(\delta^d + R^{new})} = \frac{1}{1+R^{mp}-(1-\delta^d)}$$

$$R^{new} = R^{mp}\,(1+\Phi) + \delta^d\Phi$$

We are targeting a level of public debt to GDP in steady state: $\bar{d} = \frac{d}{Y}$, this implies we can find the values for the government variables, start with new issuances:

$$l = \delta^d d$$

$$l = \delta^d \bar{d} Y$$

Repayments:

$$n = (R^{new} + \delta^d)d$$

Now to find the tax in steady state combine these results with the government budget constraint:

$$n = T - G + l$$

$$(R^{new} + \delta^d)d = T - G + \delta^d d$$

$$R^{new} d = T - G$$

$$T = R^{new} d + G$$

$$T = (R^{new}\bar{d} + \bar{G})Y$$

F.6 Log-linearization

Here I linearize all equilibrium conditions around a zero inflation steady state. Interest rate variables are linearized so that $\hat{R}_t^{crp} = R_t^{crp} - R^{crp}$ in order to interpret results as percentage point deviations, this includes the two spread friction variables Φ_t and s_t. Debt quantity variables are linearized over steady state GDP so that $\hat{D}_t = \frac{D_t - D}{Y}$ in order to interpret the results as changes in debt over GDP, I do this as the main economic channel of debt supply goes through a volume effect and a standard percent deviation would not capture it. Finally, I log-linearize all other variables so that $\hat{C}_t = \frac{C_t - C}{C}$[49].

F.6.1 Government Sector

Start with repayments:

$$
\begin{aligned}
n_t &= (R^{ave}_{t-1} + \delta)\frac{1}{\pi_t} d_{t-1} \\
Y\hat{n}_t &= d\hat{R}^{ave}_{t-1} + (R^{ave} + \delta)Y\hat{d}_{t-1} - (R^{ave} + \delta)d\hat{\pi}_t \\
\hat{n}_t &= \bar{d}\hat{R}^{ave}_{t-1} + (R^{new} + \delta)\hat{d}_{t-1} - (R^{new} + \delta)\bar{d}\hat{\pi}_t
\end{aligned}
$$

The law of motion of public debt:

$$
\begin{aligned}
d_t &= (1 - \delta)\frac{1}{\pi_t} d_{t-1} + l_t \\
\hat{d}_t &= (1 - \delta)\hat{d}_{t-1} - (1 - \delta)\bar{d}\hat{\pi}_t + \hat{l}_t
\end{aligned}
$$

The law of motion of average interest rates on public debt:

$$
\begin{aligned}
R^{ave}_t &= \left(1 - \frac{l_t}{d_t}\right) R^{ave}_{t-1} + \frac{l_t}{d_t} R^{new}_t \\
\hat{R}^{ave}_t &= \left(1 - \frac{l}{d}\right) \hat{R}^{ave}_{t-1} - \frac{1}{d} R^{new} \frac{d}{\bar{d}} \hat{l}_t + \frac{l}{(d)^2} R^{new} \frac{d}{\bar{d}} \hat{d}_t + \frac{l}{d} \hat{R}^{new}_t + \frac{1}{d} R^{new} \frac{d}{\bar{d}} \hat{l}_t - \frac{l}{(d)^2} R^{new} \frac{d}{\bar{d}} \hat{d}_t \\
\hat{R}^{ave}_t &= (1 - \delta)\, \hat{R}^{ave}_{t-1} + \delta \hat{R}^{new}_t
\end{aligned}
$$

The government budget constraint:

$$
\begin{aligned}
n_t &= T_t - G_t + l_t \\
\hat{n}_t &= (R^{new}\bar{d} + \bar{G})\hat{T}_t - \bar{G}\hat{G}_t + \hat{l}_t
\end{aligned}
$$

[49] The MPK, R^r is log-linearized

The tax policy:

$$T_t = G_t + (T - G)\left(\frac{d_{t-1}}{d}\right)^{\tau_T}$$

$$T\hat{T}_t = G\hat{G}_t + \tau_T(T - G)\left(\frac{d}{d}\right)^{\tau_T - 1}\frac{1}{d}Y\hat{d}_{t-1}$$

$$(R^{new}\bar{d} + \bar{G})\hat{T}_t = \bar{G}\hat{G}_t + \tau_T R^{new}\hat{d}_{t-1}$$

F.6.2 Primary Dealers Financial Friction

The primary market friction:

$$\Phi_t = \Phi_0\left(b_t^{crp} + l_t\right)^{\Phi_1}\exp\left(\frac{\nu_t^{\Phi}}{\Phi}\right)$$

In steady state the friction is:

$$\Phi = \Phi_0\left(b^{crp} + l\right)^{\Phi_1}$$

As Φ_t represents a spread it is already in percentage points, therefore we linearize it instead of log-linearize it, moreover, for all debt variables we take deviations over gdp rather than over its steady state:

$$\hat{\Phi}_t = \Phi\Phi_1\left(\frac{Y}{b^{crp} + l}\frac{db_t^{crp}}{Y} + \frac{Y}{b^{crp} + l}\frac{dl_t}{Y}\right) + \Phi\frac{1}{\Phi}\exp\left(\frac{0}{\Phi}\right)\nu_t^{\Phi}$$

$$\hat{\Phi}_t = \Phi\Phi_1\left(\frac{Y}{b^{crp} + l}\hat{b}_t^{crp} + \frac{Y}{b^{crp} + l}\hat{l}_t\right) + \nu_t^{\Phi}$$

$$\hat{\Phi}_t = \zeta\left(\hat{b}_t^{crp} + \hat{l}_t\right) + \nu_t^{\Phi}$$

F.6.3 Household FOCs

Start with household FOCs:

$$
\begin{aligned}
-\sigma\hat{C}_t + \hat{w}_t &= \eta\hat{H}_t \\
-\sigma\hat{C}_t &= -\sigma\mathbb{E}_t[\hat{C}_{t+1}] + \frac{1}{1+R}\mathbb{E}_t[\hat{R}_t] \\
-\sigma\hat{C}_t &= -\sigma\mathbb{E}_t[\hat{C}_{t+1}] + \frac{1}{1+R}\hat{R}_t^{mp} - \mathbb{E}_t[\hat{\pi}_{t+1}] \\
\frac{1}{1+R}\hat{R}_t &= \frac{1}{1+R^{crp}}\hat{R}_t^{crp} - \frac{1}{1+\Phi}\hat{\Phi}_t
\end{aligned}
$$

Recall we can write the steady state for the Euler for government debt as:

$$
\begin{aligned}
\frac{(1+\Phi)}{(\delta^d + R^{new})} &= \frac{1}{R^{mp} + \delta^d} \\
R^{new} &= R^{mp}(1+\Phi) + \delta^d\Phi
\end{aligned}
$$

The Euler for government debt is:

$$
\frac{(1+\Phi_t)}{(\delta^d + R_t^{new})} = (1+R_t^{mp})^{-1} + \mathbb{E}_t\left[\beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}\pi_{t+1}}(1-\delta^d)\frac{(1+\Phi_{t+1})}{(\delta^d + R_{t+1}^{new})}\right]
$$

To log linearize this equation use a trick to simply the expression. Define a helping variable $\Xi_t \equiv \frac{(1+\Phi_t)}{(\delta^d + R_t^{new})}$ and log-linearize the Euler with this expression where for interest rates we use the linearization:

$$
\Xi_t = (1+R_t^{mp})^{-1} + \mathbb{E}_t\left[\beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}\pi_{t+1}}(1-\delta^d)\Xi_{t+1}\right]
$$

Take the Taylor expansion:

$$
\begin{aligned}
\Xi\hat{\Xi}_t &= -(1+R^{mp})^{-2}\hat{R}_t^{mp} + \beta(1-\delta^d)\Xi\mathbb{E}_t\left[-\sigma\hat{C}_{t+1} + \sigma\hat{C}_t - \hat{\pi}_{t+1} + \hat{\Xi}_{t+1}\right] \\
\Xi\hat{\Xi}_t &= -\beta^2\hat{R}_t^{mp} + \beta(1-\delta^d)\Xi\mathbb{E}_t\left[-\sigma\hat{C}_{t+1} + \sigma\hat{C}_t - \hat{\pi}_{t+1} + \hat{\Xi}_{t+1}\right]
\end{aligned}
$$

From the steady state relationships we know $\Xi = \frac{1}{1+R^{mp}}\frac{1}{1-\beta(1-\delta^d)} = \beta\frac{1}{1-\beta(1-\delta^d)} = \frac{1}{R^{mp}+\delta^d}$:

$$\begin{aligned}
\beta\frac{1}{1-\beta(1-\delta^d)}\hat{\Xi}_t &= -\beta^2\hat{R}_t^{mp} + \beta(1-\delta^d)\beta\frac{1}{1-\beta(1-\delta^d)}\mathbb{E}_t\left[-\sigma\hat{C}_{t+1} + \sigma\hat{C}_t - \hat{\pi}_{t+1} + \hat{\Xi}_{t+1}\right] \\
\frac{1}{1-\beta(1-\delta^d)}\hat{\Xi}_t &= -\beta\hat{R}_t^{mp} + (1-\delta^d)\beta\frac{1}{1-\beta(1-\delta^d)}\mathbb{E}_t\left[-\sigma\hat{C}_{t+1} + \sigma\hat{C}_t - \hat{\pi}_{t+1} + \hat{\Xi}_{t+1}\right] \\
\hat{\Xi}_t &= -(1-\beta(1-\delta^d))\beta\hat{R}_t^{mp} + (1-\delta^d)\beta\mathbb{E}_t\left[-\sigma\hat{C}_{t+1} + \sigma\hat{C}_t - \hat{\pi}_{t+1} + \hat{\Xi}_{t+1}\right]
\end{aligned}$$

Notice that from the Euler for the monetary policy bond we know
$\mathbb{E}_t\left[-\sigma\hat{C}_{t+1} + \sigma\hat{C}_t + \left(\beta\hat{R}_t^{mp} - \hat{\pi}_{t+1}\right)\right] = 0$. This allows to simplify:

$$\begin{aligned}
\hat{\Xi}_t &= -(1-\beta(1-\delta^d))\beta\hat{R}_t^{mp} + (1-\delta^d)\beta\mathbb{E}_t\left[-\sigma\hat{C}_{t+1} + \sigma\hat{C}_t - \hat{\pi}_{t+1} + \hat{\Xi}_{t+1}\right] \\
\hat{\Xi}_t &= -\beta\hat{R}_t^{mp} + (1-\delta^d)\beta\mathbb{E}_t\left[\hat{\Xi}_{t+1}\right]
\end{aligned}$$

Now apply the Taylor expansion on Ξ_t:

$$\begin{aligned}
\Xi_t &= \frac{(1+\Phi_t)}{(\delta^d + R_t^{new})} \\
\Xi\hat{\Xi}_t &= \frac{1}{(\delta^d + R^{new})}\hat{\Phi}_t - \frac{(1+\Phi)}{(\delta^d + R^{new})^2}\hat{R}_t^{new} \\
\frac{(1+\Phi)}{(\delta^d + R^{new})}\hat{\Xi}_t &= \frac{(1)}{(\delta^d + R^{new})}\hat{\Phi}_t - \frac{(1+\Phi)}{(\delta^d + R^{new})^2}\hat{R}_t^{new} \\
\hat{\Xi}_t &= \frac{1}{1+\Phi}\hat{\Phi}_t - \frac{1}{(\delta^d + R^{new})}\hat{R}_t^{new}
\end{aligned}$$

Combine this with the previous expression:

$$\begin{aligned}
\hat{\Xi}_t &= -\frac{1}{1+R^{mp}}\hat{R}_t^{mp} + (1-\delta^d)\beta\mathbb{E}_t\left[\hat{\Xi}_{t+1}\right] \\
\frac{1}{1+\Phi}\hat{\Phi}_t - \frac{1}{(\delta^d + R^{new})}\hat{R}_t^{new} &= -\frac{1}{1+R^{mp}}\hat{R}_t^{mp} + (1-\delta^d)\beta\mathbb{E}_t\left[\frac{1}{1+\Phi}\hat{\Phi}_{t+1} - \frac{1}{(\delta^d + R^{new})}\hat{R}_{t+1}^{new}\right] \\
-\frac{1}{1+\Phi}\hat{\Phi}_t + \frac{1}{(\delta^d + R^{new})}\hat{R}_t^{new} &= \frac{1}{1+R^{mp}}\hat{R}_t^{mp} + (1-\delta^d)\beta\mathbb{E}_t\left[-\frac{1}{1+\Phi}\hat{\Phi}_{t+1} + \frac{1}{(\delta^d + R^{new})}\hat{R}_{t+1}^{new}\right]
\end{aligned}$$

F.6.4 Secondary Market Value of Public Debt

We can linearize the formula (with q_t^d being log-linearized), in steady state $q = 1 + \Phi$

$$
\begin{aligned}
q\hat{q}_t^d &= \hat{\Phi}_t + \frac{1+\Phi}{\delta^d + R^{new}}(\hat{R}_t^{ave} - \hat{R}_t^{new}) \\
q\hat{q}_t^d &= \hat{\Phi}_t + \frac{1+\Phi}{\delta^d + \frac{1-\beta}{\beta}(1+\Phi) + \delta^d\Phi}(\hat{R}_t^{ave} - \hat{R}_t^{new}) \\
q\hat{q}_t^d &= \hat{\Phi}_t + \frac{\beta}{1-\beta(1-\delta^d)}(\hat{R}_t^{ave} - \hat{R}_t^{new})
\end{aligned}
$$

F.6.5 Calvo Retailers

Here we find the linear New Keynesian Phillips Curve.

$$
\begin{aligned}
K_t^f &= C_t^{-\sigma} Y_t \frac{1}{X_t}\frac{\epsilon}{\epsilon-1} + \theta\beta\mathbb{E}_t \pi_{t+1}^{\epsilon} K_{t+1}^f \\
K^f \hat{K}_t^f &= -\sigma C^{-\sigma} Y \frac{1}{X}\frac{\epsilon}{\epsilon-1}\hat{C}_t + C^{-\sigma} Y \frac{1}{X}\frac{\epsilon}{\epsilon-1}\hat{Y}_t - C^{-\sigma} Y \frac{1}{X}\frac{\epsilon}{\epsilon-1}\hat{X}_t \\
&\quad + \theta\beta\mathbb{E}_t \pi^{\epsilon} K^f \hat{K}_{t+1}^f + \theta\beta\mathbb{E}_t \pi^{\epsilon-1} K^f \epsilon\hat{\pi}_{t+1} \\
\frac{C^{-\sigma}Y}{1-\theta\beta}\hat{K}_t^f &= -\sigma C^{-\sigma} Y \hat{C}_t + C^{-\sigma} Y \hat{Y}_t - C^{-\sigma} Y \hat{X}_t \\
&\quad + \theta\beta\frac{C^{-\sigma}Y}{1-\theta\beta}\mathbb{E}_t \hat{K}_{t+1}^f + \theta\beta\frac{C^{-\sigma}Y}{1-\theta\beta}\mathbb{E}_t \epsilon\hat{\pi}_{t+1} \\
\hat{K}_t^f &= -\sigma(1-\theta\beta)\hat{C}_t + (1-\theta\beta)\hat{Y}_t - (1-\theta\beta)\hat{X}_t \\
&\quad + \theta\beta\mathbb{E}_t \hat{K}_{t+1}^f + \theta\beta\mathbb{E}_t \epsilon\hat{\pi}_{t+1} \\
F_t^f &= C_t^{-\sigma} Y_t + \theta\beta\mathbb{E}_t \pi_{t+1}^{\epsilon-1} F_{t+1}^f \\
\hat{F}_t^f &= -\sigma(1-\theta\beta)\hat{C}_t + (1-\theta\beta)\hat{Y}_t \\
&\quad + \theta\beta\mathbb{E}_t \hat{F}_{t+1}^f + \theta\beta\mathbb{E}_t (\epsilon-1)\hat{\pi}_{t+1}
\end{aligned}
$$

The NKPC:

$$
\begin{aligned}
\frac{K_t^f}{F_t^f} &= \left(\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}} \\
\frac{K^f}{F^f}\hat{K}_t^f - \frac{K^f}{F^f}\hat{F}_t^f &= -\frac{1}{\epsilon-1}\left(\frac{1-\theta\pi^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}-1}\left(-\frac{\theta}{1-\theta}\pi^{\epsilon-1-1}\right)(\epsilon-1)\hat{\pi}_t \\
\hat{K}_t^f - \hat{F}_t^f &= \frac{\theta}{1-\theta}\hat{\pi}_t
\end{aligned}
$$

Notice that this holds in each period. Now substitute in the two expressions for the recursive variables:

$$
\begin{aligned}
\frac{\theta}{1-\theta}\hat{\pi}_t &= \hat{K}_t^f - \hat{F}_t^f \\
\frac{\theta}{1-\theta}\hat{\pi}_t &= \left(-\sigma(1-\theta\beta)\hat{C}_t + (1-\theta\beta)\hat{Y}_t - (1-\theta\beta)\hat{X}_t + \theta\beta\mathbb{E}_t\hat{K}_{t+1}^f + \theta\beta\mathbb{E}_t\epsilon\hat{\pi}_{t+1}\right) \\
&\quad - \left(-\sigma(1-\theta\beta)\hat{C}_t + (1-\theta\beta)\hat{Y}_t + \theta\beta\mathbb{E}_t\hat{F}_{t+1}^f + \theta\beta\mathbb{E}_t(\epsilon-1)\hat{\pi}_{t+1}\right) \\
\frac{\theta}{1-\theta}\hat{\pi}_t &= -(1-\theta\beta)\hat{X}_t + \theta\beta\mathbb{E}_t\left(\hat{K}_{t+1}^f - \hat{F}_{t+1}^f\right) + \theta\beta\mathbb{E}_t\hat{\pi}_{t+1} \\
\frac{\theta}{1-\theta}\hat{\pi}_t &= -(1-\theta\beta)\hat{X}_t + \theta\beta\mathbb{E}_t\left(\frac{\theta}{1-\theta}\hat{\pi}_{t+1}\right) + \theta\beta\mathbb{E}_t\hat{\pi}_{t+1} \\
\hat{\pi}_t &= -\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{X}_t + \beta\mathbb{E}_t\left(\hat{\pi}_{t+1}\right)
\end{aligned}
$$

F.6.6 Wholesalers

The production function:

$$
\hat{Y}_t = \hat{A}_t + \alpha\hat{K}_{t-1} + (1-\alpha)\Omega\hat{H}_t
$$

The first order conditions:

$$
\begin{aligned}
\hat{R}_t^r &= -\hat{X}_t + \hat{Y}_t - \hat{K}_{t-1} \\
\hat{w}_t &= -\hat{X}_t + \hat{Y}_t - \hat{H}_t \\
\hat{w}_t^e &= -\hat{X}_t + \hat{Y}_t
\end{aligned}
$$

F.6.7 Capital Producers

Start with the new capital production function:

$$
\begin{aligned}
K_t &= I_t - \frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1} + (1-\delta)K_{t-1} \\
\hat{K}_t &= \delta \hat{I}_t + (1-\delta)\hat{K}_{t-1}
\end{aligned}
$$

The price of capital equation:

$$
\begin{aligned}
Q_t\left(1 - \phi_K\left(\frac{I_t}{K_{t-1}} - \delta\right)\right) &= 1 \\
\hat{Q}_t =& \delta\phi_K(\hat{I}_t - \hat{K}_{t-1})
\end{aligned}
$$

The return on capital:

$$
\begin{aligned}
1 + R^k_{t+1} &= \frac{\frac{1}{X_{t+1}}\alpha\frac{Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t} \\
(1+R^k)Q\hat{Q}_t + Q\hat{R}^k_{t+1} &= R^r\hat{R}^r_{t+1} + Q\hat{Q}_{t+1}(1-\delta) \\
\hat{R}^k_{t+1} &= R^r\hat{R}^r_{t+1} + (1-\delta)\hat{Q}_{t+1} - (R^r + (1-\delta))\hat{Q}_t
\end{aligned}
$$

F.6.8 Entrepreneurs

F.6.9 Leverage - Spread Definitions

Start with definitions of leverage and spreads:

$$
\begin{aligned}
\kappa_t &= \frac{Q_t K_t}{N_t} \\
\hat{\kappa}_t &= \hat{Q}_t + \hat{K}_t - \hat{N}_t
\end{aligned}
$$

$$
\begin{aligned}
1 + s_t &= \frac{1 + R^k_t}{1 + R^{crp}_{t-1}} \\
\frac{1}{1+s}\hat{s}_t &= \frac{1}{1+R^k}\hat{R}^k_t - \frac{1}{1+R^{crp}}\hat{R}^{crp}_{t-1}
\end{aligned}
$$

F.6.10 Leverage - Spread Relation

Now move to the two non-linear equilibrium conditions:

$$
\begin{aligned}
0 &= \mathbb{E}_t \left[(1+s_{t+1})\kappa_t \left(1-\Gamma_{t+1}\right) - \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}} \right] \\
\frac{\kappa_t - 1}{\kappa_t} &= (1+s_{t+1})(\Gamma_{t+1} - \mu G_{t+1})
\end{aligned}
$$

To substitute out the threshold choice we have to log-linearize the system as $\bar{\omega}_{t+1}$ is now defined implicitly. Start with the participation constraint. To ease notation let variables without subscripts being steady state values $\Gamma_\omega \equiv \Gamma_\omega(\bar{\omega}_{ss}, \sigma_{\omega,ss})$ and similarly for G and other derivatives.

$$
\frac{1}{\kappa}\hat{\kappa}_t = (1+s)(\Gamma - \mu G)\frac{\hat{s}_{t+1}}{1+s} + (1+s)(\Gamma_\omega - \mu G_\omega)\bar{\omega}\hat{\omega}_{t+1} + (1+s)(\Gamma_\sigma - \mu G_\sigma)\sigma_\omega \hat{\sigma}_{\omega,t}
$$

Simplify with steady state relationship:

$$
\frac{1}{\kappa - 1}\hat{\kappa}_t = \frac{\hat{s}_{t+1}}{1+s} + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}\bar{\omega}\hat{\omega}_{t+1} + \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega \hat{\sigma}_{\omega,t}
$$

Make explicit $\bar{\omega}\hat{\omega}_{t+1}$:

$$
\bar{\omega}\hat{\omega}_{t+1} = \frac{\Gamma - \mu G}{\Gamma_\omega - \mu G_\omega}\left[\frac{1}{\kappa - 1}\hat{\kappa}_t - \frac{\hat{s}_{t+1}}{1+s} - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega \hat{\sigma}_{\omega,t} \right]
$$

Now log-linearize the optimal choice:

$$
\begin{aligned}
0 = \mathbb{E}_t \Bigg[& (1+s)\kappa \left(1-\Gamma\right)\hat{\kappa}_t + (1+s)\kappa\left(1-\Gamma\right)\frac{\hat{s}_{t+1}}{1+s} - (1+s)\kappa\Gamma_\omega \hat{\omega}_{t+1} - (1+s)\kappa\Gamma_\sigma \hat{\sigma}_{\omega,t} \\
& + \mu \frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{(\Gamma_\omega - \mu G_\omega)^2}\bar{\omega}\hat{\omega}_{t+1} + \mu \frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{(\Gamma_\omega - \mu G_\omega)^2}\sigma_\omega \hat{\sigma}_{\omega,t} \Bigg]
\end{aligned}
$$

Use steady state relationship:

$$0 = \mathbb{E}_t\Bigg[\hat{\kappa}_t + \frac{\hat{s}_{t+1}}{1+s} - \frac{\Gamma_\omega}{(1-\Gamma)}\bar{\omega}\hat{\omega}_{t+1} - \frac{\Gamma_\sigma}{(1-\Gamma)}\sigma_\omega\hat{\sigma}_{\omega,t} \\ + \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\bar{\omega}\hat{\omega}_{t+1} + \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\sigma_\omega\hat{\sigma}_{\omega,t}\Bigg]$$

Group variables:

$$0 = \mathbb{E}_t\Bigg[\hat{\kappa}_t + \frac{\hat{s}_{t+1}}{1+s} \\ + \left[\mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)} - \frac{\Gamma_\omega}{(1-\Gamma)}\right]\bar{\omega}\hat{\omega}_{t+1} + \left[\mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)} - \frac{\Gamma_\sigma}{(1-\Gamma)}\right]\sigma_\omega\hat{\sigma}_{\omega,t}\Bigg]$$

Make explicit $\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}$:

$$\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1} = \frac{1}{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]}\left[\hat{\kappa}_t + \mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s} - \left[\frac{\Gamma_\sigma}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$

In this way we can eliminate $\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}$ by taking expectations of the log-linearized participation constraint and equating it with the equation we just derived:

$$\frac{\Gamma - \mu G}{\Gamma_\omega - \mu G_\omega}\left[\frac{1}{\kappa - 1}\hat{\kappa}_t - \mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s} - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega\hat{\sigma}_{\omega,t}\right] \\ = \frac{1}{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]}\left[\hat{\kappa}_t + \mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s} - \left[\frac{\Gamma_\sigma}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$

Some algebra:

$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\left[\frac{1}{\kappa - 1}\hat{\kappa}_t - \mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s} - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega\hat{\sigma}_{\omega,t}\right] \\ = \left[\hat{\kappa}_t + \mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s} - \left[\frac{\Gamma_\sigma}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$

$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}} + 1 = \frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right] + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}$$

$$\begin{aligned}
\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\frac{1}{\kappa - 1} - 1 &= \frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\frac{1}{\kappa-1} - \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}} \\
&= \frac{\left[\frac{\Gamma_\omega - \mu G_\omega}{\kappa s} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\frac{1}{\kappa-1} - \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}} \\
&= \frac{\left[\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}(\kappa - 1) - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\frac{1}{\kappa-1} - \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}} \\
&= -\frac{\left[\mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\frac{1}{\kappa-1}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}
\end{aligned}$$

$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G} - \left[\frac{\Gamma_\sigma}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right] =$$
$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G} - \left[\frac{\Gamma_\sigma}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}$$

Use these results to make explicit the spread relationship with leverage and risk:

$$\begin{aligned}
\frac{\mathbb{E}_t s_{t+1}}{1+s} = &-\frac{\mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}}{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right] + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\frac{1}{\kappa - 1}\hat{\kappa}_t + \\
&+ \frac{-\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G} + \left[\frac{\Gamma_\sigma}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right] + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\sigma_\omega\hat{\sigma}_{\omega,t}
\end{aligned}$$

F.6.11 Law of Motion of N

To find the law of motion of entrepreneur's wealth we are going to use a number of steady state relationships:

$$
\begin{aligned}
1 + R^e &= (1 + R^{crp})(1 + s)\kappa \left(1 - \Gamma(\bar{\omega}, \sigma_\omega)\right) \\
1 - \gamma(1 + R^e) &= \frac{(1-\alpha)(1-\Omega)}{\alpha} R^r \kappa
\end{aligned}
$$

Start with the average return on entrepreneur's wealth:

$$
\begin{aligned}
1 + R^e_t &= \left((R^k_t - R^{crp}_{t-1})\kappa_{t-1} + (1 + R^{crp}_{t-1}) - \mu(1 + R^k_t)\kappa_{t-1} G(\bar{\omega}_t, \sigma_{\omega,t-1})\right) \\
\hat{R}^e_t &= \kappa \left(\hat{R}^k_t - \hat{R}^{crp}_{t-1}\right) + \kappa(1 + R^{crp})(s)\hat{\kappa}_{t-1} + \hat{R}^{crp}_{t-1} \\
&\quad - \mu G(\bar{\omega}, \sigma_\omega)(1 + R^{crp})(1 + s)\kappa \left(\frac{\hat{R}^k_t}{1 + R^k} + \hat{\kappa}_{t-1} + \frac{G_\omega \bar{\omega}}{G(\bar{\omega}, \sigma_\omega)}\hat{\omega}_t + \frac{G_\sigma \sigma_\omega}{G(\bar{\omega}, \sigma_\omega)}\hat{\sigma}_{\omega,t-1}\right)
\end{aligned}
$$

Notice that, as monitoring costs $\mu G(\bar{\omega}, \sigma_\omega)$ are small in the proposed calibration, the terms in the second line will negligible. Now move to the law of motion of entrepreneur's wealth:

$$
\begin{aligned}
N_t &= \gamma(1 + R^e_t)N_{t-1} + w^e_t \\
N\hat{N}_t &= \gamma(1 + R^e)N \left(\frac{\hat{R}^e_t}{1 + R^e} + \hat{N}_{t-1}\right) + w^e \hat{w}^e_t \\
N\hat{N}_t &= \gamma(1 + R^e)N \left(\frac{\hat{R}^e_t}{1 + R^e} + \hat{N}_{t-1}\right) + N\left(1 - \gamma R^e\right)\hat{w}^e_t \\
\hat{N}_t &= \gamma(1 + R^e) \left(\frac{\hat{R}^e_t}{1 + R^e} + \hat{N}_{t-1}\right) + (1 - \gamma(1 + R^e))\hat{w}^e_t \\
\hat{N}_t &= \gamma(1 + R^e) \left(\frac{\hat{R}^e_t}{1 + R^e} + \hat{N}_{t-1}\right) + \frac{(1-\alpha)(1-\Omega)}{\alpha} R^r \kappa \hat{w}^e_t
\end{aligned}
$$

F.6.12 Entrepreneur's Consumption

Finally, let's log-linarize the entrepreneur's consumption in a similar way as for the law of motion of N:

$$C_t^e = (1-\gamma)(1+R_t^e)N_{t-1}$$
$$\hat{C}_t^e = \hat{N}_{t-1} + \frac{\hat{R}_t^e}{1+R^e}$$

F.6.13 Resource Constraints

Now move to the resource constraint:

$$Y_t = C_t + C_t^e + I_t + G_t + \mu G(\bar{\omega}_t, \sigma_{\omega,t-1})(1+R_t^k)N_{t-1}\kappa_{t-1}$$
$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{C^e}{Y}\hat{C}_t^e + \frac{I}{Y}\hat{I}_t + \bar{G}\hat{G}_t$$
$$+ \mu G(\bar{\omega}, \sigma_\omega)(1+R^k)\frac{K}{Y}\left(\hat{\kappa}_{t-1} + \hat{N}_{t-1} + \frac{\hat{R}_t^k}{1+R^k} + \frac{G_\omega \bar{\omega}}{G(\bar{\omega}, \sigma_\omega)}\hat{\omega}_t + \frac{G_\sigma \sigma_\omega}{G(\bar{\omega}, \sigma_\omega)}\hat{\sigma}_{\omega,t-1}\right)$$

Where in the second line of the last equation one could approximate to zero as monitoring costs are low in percent of GDP.

F.6.14 Taylor Rule and Exogenous Processes

The Taylor rule and exogenous processes were written to have a convenient representation following the Taylor expansion:

$$\hat{R}_t^{mp} = \rho^{mp}\hat{R}_{t-1}^{mp} + (1-\rho^{mp})[\phi_\pi \mathbb{E}_t \hat{\pi}_{t+1} + \phi_Y \hat{Y}_t] + \varepsilon_t^{mp}$$
$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \varepsilon_t^G$$
$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_t^A$$
$$\hat{\sigma}_{\omega,t} = \rho_{\sigma_\omega}\hat{\sigma}_{\omega,t-1} + \varepsilon_t^{\sigma_\omega}$$
$$\hat{\nu}_t^\Phi = \rho_\Phi \hat{\nu}_{t-1}^\Phi + \varepsilon_t^\Phi$$

F.6.15 All Equilibrium Conditions

The linearized competitive equilibrium consists of 13 endogenous allocations $\{\hat{C}_t$, $\hat{C}^e_t$, $\hat{I}_t$, $\hat{Y}_t$,$\hat{\kappa}_t$, $\hat{\bar{\omega}}_t$, $\hat{N}_t$, $\hat{K}_t$, $\hat{H}_t$,$\hat{l}_t$, $\hat{f}_t$, $\hat{d}_t$, $\hat{T}_t\}$, 15 prices $\{\hat{w}_t$, $\hat{w}^e_t$, $\hat{\pi}_t$, $\hat{R}_t$, $\hat{R}^{crp}_t$, $\hat{R}^k_t$, $\hat{R}^r_t$, $\hat{R}^e_t$, $\hat{s}_t$,$\hat{X}_t$, $\hat{Q}_t$, $\hat{\Phi}_t$, $\hat{R}^{new}_t$, $\hat{R}^{ave}_t$, $\hat{R}^{mp}_t\}$, and 4 exogenous processes $\{\hat{G}_t$, $\hat{\sigma}_{\omega,t}$, $\hat{A}_t$, $\hat{\nu}^{\Phi}_t\}$; such that households, primary market participants, final good producers, retailers, wholesalers, capital producers, and entrepreneurs optimize, the central bank follows a Taylore rule, the treasury follows the tax rule, and markets clear. The equilibrium is characterized by the following equations and processes:

$$
\begin{aligned}
-\sigma\hat{C}_t + \hat{w}_t &= \eta\hat{H}_t \\
-\sigma\hat{C}_t &= -\sigma\mathbb{E}_t[\hat{C}_{t+1}] + \frac{1}{1+R}\hat{R}_t \\
-\sigma\hat{C}_t &= -\sigma\mathbb{E}_t[\hat{C}_{t+1}] + \frac{1}{1+R^{mp}}\hat{R}^{mp}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] \\
\hat{\pi}_t &= -\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{X}_t + \beta\mathbb{E}_t\left(\hat{\pi}_{t+1}\right) \\
\hat{Y}_t &= \hat{A}_t + \alpha\hat{K}_{t-1} + (1-\alpha)\Omega\hat{H}_t \\
\hat{R}^r_t &= -\hat{X}_t + \hat{Y}_t - \hat{K}_{t-1} \\
\hat{w}_t &= -\hat{X}_t + \hat{Y}_t - \hat{H}_t \\
\hat{w}^e_t &= -\hat{X}_t + \hat{Y}_t \\
\hat{K}_t &= \delta\hat{I}_t + (1-\delta)\hat{K}_{t-1} \\
\hat{Q}_t &= \delta\phi_K(\hat{I}_t - \hat{K}_{t-1}) \\
\hat{R}^k_{t+1} &= R^r\hat{R}^r_{t+1} + (1-\delta)\hat{Q}_{t+1} - (R^r + (1-\delta))\hat{Q}_t \\
\hat{\kappa}_t &= \hat{Q}_t + \hat{K}_t - \hat{N}_t \\
\frac{\mathbb{E}_t\hat{s}_{t+1}}{1+s} &= \psi_{s,\kappa}\hat{\kappa}_t + \psi_{s,\sigma_\omega}\hat{\sigma}_{\omega,t} \\
\hat{\omega}_{t+1} &= \psi_{\omega,\kappa}\hat{\kappa}_t + \psi_{\omega,s}\frac{\hat{s}_{t+1}}{1+s} + \psi_{\omega,\sigma_\omega}\hat{\sigma}_{\omega,t} \\
\hat{R}^e_t &= \kappa\left(\hat{R}^k_t - \hat{R}^{crp}_{t-1}\right) + \kappa(1+R^{crp})(s)\hat{\kappa}_{t-1} + \hat{R}^{crp}_{t-1} \\
&\quad - \mu G(\bar{\omega},\sigma_\omega)(1+R^{crp})(1+s)\kappa\left(\frac{\hat{R}^k_t}{1+R^k} + \hat{\kappa}_{t-1} + \frac{G_\omega\bar{\omega}}{G(\bar{\omega},\sigma_\omega)}\hat{\omega}_t + \frac{G_\sigma\sigma_\omega}{G(\bar{\omega},\sigma_\omega)}\hat{\sigma}_{\omega,t-1}\right) \\
\hat{N}_t &= \gamma(1+R^e)\left(\frac{\hat{R}^e_t}{1+R^e} + \hat{N}_{t-1}\right) + \frac{(1-\alpha)(1-\Omega)}{\alpha}R^r\kappa\hat{w}^e_t \\
\hat{C}^e_t &= \hat{N}_{t-1} + \frac{\hat{R}^e_t}{1+R^e} \\
\hat{Y}_t &= \frac{C}{Y}\hat{C}_t + \frac{C^e}{Y}\hat{C}^e_t + \frac{I}{Y}\hat{I}_t + \bar{G}\hat{G}_t \\
&\quad + \mu G(\bar{\omega},\sigma_\omega)(1+R^k)\frac{K}{Y}\left(\hat{\kappa}_{t-1} + \hat{N}_{t-1} + \frac{\hat{R}^k_t}{1+R^k} + \frac{G_\omega\bar{\omega}}{G(\bar{\omega},\sigma_\omega)}\hat{\omega}_t + \frac{G_\sigma\sigma_\omega}{G(\bar{\omega},\sigma_\omega)}\hat{\sigma}_{\omega,t-1}\right)
\end{aligned}
$$

$$\frac{1}{1+s}\hat{s}_t = \frac{1}{1+R^k}\hat{R}^k_t - \frac{1}{1+R^{crp}}\hat{R}^{crp}_{t-1}$$

$$\frac{1}{1+R^{crp}}\hat{R}^{crp}_t = \frac{1}{1+R}\hat{R}_t + \frac{1}{1+\Phi}\hat{\Phi}_t$$

$$\hat{\Phi}_t = \zeta\left(\hat{b}^{crp}_t + \hat{l}_t\right) + \nu^{\Phi}_t$$

$$-\frac{1}{1+\Phi}\hat{\Phi}_t + \frac{1}{(\delta^d + R^{new})}\hat{R}^{new}_t = \frac{1}{1+R^{mp}}\hat{R}^{mp}_t + (1-\delta^d)\beta\mathbb{E}_t\left[-\frac{1}{1+\Phi}\hat{\Phi}_{t+1} + \frac{1}{(\delta^d + R^{new})}\hat{R}^{new}_{t+1}\right]$$

$$\hat{n}_t = \bar{d}\hat{R}^{ave}_{t-1} + (R^{new} + \delta^d)\hat{d}_{t-1} - (R^{new} + \delta^d)\bar{d}\hat{\pi}_t$$

$$\hat{d}_t = (1-\delta^d)\hat{d}_{t-1} - (1-\delta^d)\bar{d}\hat{\pi}_t + \hat{l}_t$$

$$\hat{R}^{ave}_t = \left(1-\delta^d\right)\hat{R}^{ave}_{t-1} + \delta^d\hat{R}^{new}_t$$

$$\hat{n}_t = (R^{new}\bar{d} + \bar{G})\hat{T}_t - \bar{G}\hat{G}_t + \hat{l}_t$$

$$(R^{new}\bar{d} + \bar{G})\hat{T}_t = \bar{G}\hat{G}_t + \tau_T R^{new}\hat{d}_{t-1}$$

$$\hat{R}^{mp}_t = \rho^{mp}\hat{R}^{mp}_{t-1} + (1-\rho^{mp})[\phi_\pi\mathbb{E}_t\hat{\pi}_{t+1} + \phi_Y\hat{Y}_t] + \varepsilon^{mp}_t$$

$$\hat{G}_t = \rho_G\hat{G}_{t-1} + \varepsilon^G_t$$

$$\hat{A}_t = \rho_A\hat{A}_{t-1} + \varepsilon^A_t$$

$$\hat{\sigma}_{\omega,t} = \rho_{\sigma_\omega}\hat{\sigma}_{\omega,t-1} + \varepsilon^{\sigma_\omega}_t$$

$$\hat{\nu}^{\Phi}_t = \rho_\Phi\hat{\nu}^{\Phi}_{t-1} + \varepsilon^{\Phi}_t$$

F.7 Derivations for Entrepreneurs Helping Functions

In this appendix I present all the derivatives of the entrepreneurs functions $\Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t})$ and $G(\bar{\omega}_{t+1}, \sigma_{\omega,t})$ which are used in the log-linearized equilibrium. Let me start with the definition of these two functions, the CFD, and the PFD of the log-normal distribution:

$$
\begin{aligned}
\Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t}) &\equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega + \bar{\omega}_{t+1}(1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t})) \\
G(\bar{\omega}_{t+1}, \sigma_{\omega,t}) &\equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega \\
F(\bar{\omega}_{t+1}, \sigma_{\omega,t}) &\equiv \int_0^{\bar{\omega}_{t+1}} f(\omega, \sigma_{\omega,t}) d\omega \\
f(\omega, \sigma_{\omega,t}) &\equiv \frac{1}{\omega \sigma_{\omega,t}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{\ln(\omega) + \frac{\sigma_{\omega,t}^2}{2}}{\sigma_{\omega,t}} \right)^2 \right)
\end{aligned}
$$

The first two derivatives which are going to be useful later, are the first derivatives of the PDF (f) with respect to ω and σ:

$$
\begin{aligned}
\frac{\partial f(\omega, \sigma_{\omega,t})}{\partial \omega} &\equiv f_\omega(\omega, \sigma_{\omega,t}) \\
&= -\frac{1}{\omega} f(\omega, \sigma_{\omega,t}) + f(\omega, \sigma_{\omega,t}) \left(-\frac{1}{2}\right) 2 \left(\frac{\ln(\omega) + \frac{\sigma_{\omega,t}^2}{2}}{\sigma_{\omega,t}} \right) \frac{1}{\sigma_{\omega,t}} \frac{1}{\omega} \\
&= -\frac{1}{\omega} f(\omega, \sigma_{\omega,t}) \left(\frac{\ln(\omega)}{\sigma_{\omega,t}^2} + \frac{3}{2} \right) \\
\frac{\partial f(\omega, \sigma_{\omega,t})}{\partial \sigma_{\omega,t}} &\equiv f_\sigma(\omega, \sigma_{\omega,t}) \\
&= -\frac{1}{\sigma_{\omega,t}} f(\omega, \sigma_{\omega,t}) + f(\omega, \sigma_{\omega,t}) \left(-\frac{1}{2}\right) 2 \left(\frac{\ln(\omega) + \frac{\sigma_{\omega,t}^2}{2}}{\sigma_{\omega,t}} \right) \left(\frac{\sigma_{\omega,t}\sigma_{\omega,t} - \left[\ln(\omega) + \frac{\sigma_{\omega,t}^2}{2}\right]}{\sigma_{\omega,t}^2} \right) \\
&= -\frac{1}{\sigma_{\omega,t}} f(\omega, \sigma_{\omega,t}) + \frac{1}{\sigma_{\omega,t}} f(\omega, \sigma_{\omega,t}) \left(\frac{\ln(\omega)^2}{\sigma_{\omega,t}^2} - \frac{\sigma_{\omega,t}^2}{4} \right)
\end{aligned}
$$

Now, notice how one can rewrite $G(\bar{\omega}_{t+1}, \sigma_{\omega,t})$ with a simple trick:

$$
\begin{aligned}
G(\bar{\omega}_{t+1}, \sigma_{\omega,t}) &= \int_0^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega \\
&= \int_0^{\bar{\omega}_{t+1}} \omega \left(\frac{1}{\omega \sigma_{\omega,t} \sqrt{2\pi}} \exp \left[-\frac{\left(\ln(\omega) + \frac{\sigma^2_{\omega,t}}{2} \right)^2}{2\sigma^2_{\omega,t}} \right] \right) d\omega \\
&= \int_0^{\bar{\omega}_{t+1}} \left(\frac{1}{\sigma_{\omega,t} \sqrt{2\pi}} \exp \left[-\frac{\left(\ln(\omega) + \frac{\sigma^2_{\omega,t}}{2} \right)^2}{2\sigma^2_{\omega,t}} \right] \right) d\omega \\
&= \int_0^{\bar{\omega}_{t+1}} \left(\frac{1}{\sigma_{\omega,t} \sqrt{2\pi}} \exp \left[-\frac{\left(\ln(\omega)^2 + \left[\frac{\sigma^2_{\omega,t}}{2} \right]^2 + \ln(\omega) \sigma^2_{\omega,t} \right)}{2\sigma^2_{\omega,t}} \right] \right) d\omega \\
&= \int_0^{\bar{\omega}_{t+1}} \left(\frac{1}{\sigma_{\omega,t} \sqrt{2\pi}} \exp \left[-\frac{\left(\ln(\omega)^2 + \left[\frac{\sigma^2_{\omega,t}}{2} \right]^2 - \ln(\omega) \sigma^2_{\omega,t} + 2\ln(\omega) \sigma^2_{\omega,t} \right)}{2\sigma^2_{\omega,t}} \right] \right) d\omega \\
&= \int_0^{\bar{\omega}_{t+1}} \left(\frac{1}{\sigma_{\omega,t} \sqrt{2\pi}} \exp \left[-\frac{\left(\ln(\omega)^2 + \left[\frac{\sigma^2_{\omega,t}}{2} \right]^2 - \ln(\omega) \sigma^2_{\omega,t} \right)}{2\sigma^2_{\omega,t}} - \ln(\omega) \right] \right) d\omega \\
&= \int_0^{\bar{\omega}_{t+1}} \left(\frac{1}{\omega \sigma_{\omega,t} \sqrt{2\pi}} \exp \left[-\frac{\left(\ln(\omega)^2 + \left[\frac{\sigma^2_{\omega,t}}{2} \right]^2 - \ln(\omega) \sigma^2_{\omega,t} \right)}{2\sigma^2_{\omega,t}} \right] \right) d\omega \\
&= \int_0^{\bar{\omega}_{t+1}} \left(\frac{1}{\omega \sigma_{\omega,t} \sqrt{2\pi}} \exp \left[-\frac{\left(\ln(\omega) - \frac{\sigma^2_{\omega,t}}{2} \right)^2}{2\sigma^2_{\omega,t}} \right] \right) d\omega
\end{aligned}
$$

Now, this looks like the CDF of a log-normal with centrality $\frac{\sigma^2_{\omega,t}}{2}$ rather than $-\frac{\sigma^2_{\omega,t}}{2}$ as in our case. We can transform this into a standard normal CDF, out of completeness of the derivation let's do it step by step with integration by substitution, first do a change of variable of $x = \ln(\omega)$ such that $dx = d\omega \frac{1}{\omega}$:

$$
G(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = \int_{-\infty}^{\ln(\bar{\omega}_{t+1})} \left(\frac{1}{\sigma_{\omega,t} \sqrt{2\pi}} \exp \left[-\frac{\left(x - \frac{\sigma^2_{\omega,t}}{2} \right)^2}{2\sigma^2_{\omega,t}} \right] \right) dx
$$

This is the CDF of a non-standard normal, to get there let's do another change in variable with $v = v(x) = \frac{x - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}$, with $dv = dx\frac{1}{\sigma_{\omega,t}}$:

$$
\begin{aligned}
G(\bar{\omega}_{t+1}, \sigma_{\omega,t}) &= \int_{-\infty}^{\frac{\ln(\bar{\omega}_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}} \left(\frac{1}{\sqrt{2\pi}} \exp\left[-\frac{v^2}{2} \right] \right) dx \\
G(\bar{\omega}_{t+1}, \sigma_{\omega,t}) &= \Phi\left(\frac{\ln(\bar{\omega}_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}} \right)
\end{aligned}
$$

Where Φ is the CDF of a standard normal. Now do the same steps for the cumulative of the log-normal $F(\bar{\omega}_{t+1}, \sigma_{\omega,t})$:

$$
\begin{aligned}
F(\bar{\omega}_{t+1}, \sigma_{\omega,t}) &= \int_0^{\bar{\omega}_{t+1}} f(\omega, \sigma_{\omega,t}) d\omega \\
&= \int_0^{\bar{\omega}_{t+1}} \left(\frac{1}{\omega \sigma_{\omega,t} \sqrt{2\pi}} \exp\left[-\frac{\left(\ln(\omega) + \frac{\sigma^2_{\omega,t}}{2} \right)^2}{2\sigma^2_{\omega,t}} \right] \right) d\omega \\
&= \int_{-\infty}^{\frac{\ln(\bar{\omega}_{t+1}) + \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}} \left(\frac{1}{\sqrt{2\pi}} \exp\left[-\frac{v^2}{2} \right] \right) dx \\
F(\bar{\omega}_{t+1}, \sigma_{\omega,t}) &= \Phi\left(\frac{\ln(\bar{\omega}_{t+1}) + \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}} \right)
\end{aligned}
$$

With these, let's do the last preliminary step by computing the derivatives of the CFD with respect to the threshold $\bar{\omega}_{t+1}$ and the variance $\sigma_{\omega,t}$, there ϕ is the PDF of a standard normal:

$$
\begin{aligned}
\frac{\partial F(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\partial \bar{\omega}_{t+1}} &\equiv F_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \\
&= f(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \\
\frac{\partial F(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\partial \sigma_{\omega,t}} &\equiv F_\sigma(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \\
&= \left(-\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}} + \frac{1}{2} \right) \phi\left(\frac{\ln(\bar{\omega}_{t+1}) + \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}} \right) \\
&= -\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}} - \frac{1}{2} \right) \bar{\omega}_{t+1} \sigma_{\omega,t} f(\bar{\omega}_{t+1}, \sigma_{\omega,t})
\end{aligned}
$$

As the preliminary steps are finished, let's start with the first derivatives of interest, first the first derivatives of G with respect to the threshold $\bar{\omega}_{t+1}$ and the variance $\sigma_{\omega,t}$:

$$\begin{aligned}
\frac{\partial G(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial \bar{\omega}_{t+1}} &\equiv G_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= \bar{\omega}_{t+1} f(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
\frac{\partial G(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial \sigma_{\omega,t}} &\equiv G_{\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= \left(-\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}} - \frac{1}{2}\right) \phi\left(\frac{\ln(\bar{\omega}_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right) \\
&= -\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}} + \frac{1}{2}\right) \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(\bar{\omega}_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)^2\right] \\
&= -\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}} + \frac{1}{2}\right) \bar{\omega}_{t+1} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(\bar{\omega}_{t+1}) + \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)^2\right] \\
&= -\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}} + \frac{1}{2}\right) \bar{\omega}^2_{t+1} \sigma_{\omega,t} f(\bar{\omega}_{t+1},\sigma_{\omega,t})
\end{aligned}$$

Second, the first derivatives of Γ with respect to the threshold $\bar{\omega}_{t+1}$ and the variance $\sigma_{\omega,t}$:

$$\begin{aligned}
\frac{\partial \Gamma(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial \bar{\omega}_{t+1}} &\equiv \Gamma_{\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= \bar{\omega}_{t+1} f(\bar{\omega}_{t+1},\sigma_{\omega,t}) + (1 - F(\bar{\omega}_{t+1},\sigma_{\omega,t})) - \bar{\omega}_{t+1} f(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= 1 - F(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
\frac{\partial \Gamma(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial \sigma_{\omega,t}} &\equiv \Gamma_{\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= G_{\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) - \bar{\omega}_{t+1} F_{\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= -\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}} + \frac{1}{2}\right) \bar{\omega}^2_{t+1} \sigma_{\omega,t} f(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&+ \bar{\omega}_{t+1} \left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}} - \frac{1}{2}\right) \bar{\omega}_{t+1} \sigma_{\omega,t} f(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= -\bar{\omega}^2_{t+1} \sigma_{\omega,t} f(\bar{\omega}_{t+1},\sigma_{\omega,t})
\end{aligned}$$

The second derivatives of G starting from G_ω:

$$
\begin{aligned}
\frac{\partial^2 G(\bar\omega_{t+1},\sigma_{\omega,t})}{\partial\bar\omega_{t+1}\partial\bar\omega_{t+1}} &\equiv G_{\omega\omega}(\bar\omega_{t+1},\sigma_{\omega,t})\\
&= f(\bar\omega_{t+1},\sigma_{\omega,t}) + \bar\omega_{t+1} f_\omega(\bar\omega_{t+1},\sigma_{\omega,t})\\
&= f(\bar\omega_{t+1},\sigma_{\omega,t}) - \bar\omega_{t+1}\frac{1}{\bar\omega_{t+1}} f(\bar\omega_{t+1},\sigma_{\omega,t})\left(\frac{\ln(\bar\omega_{t+1})}{\sigma^2_{\omega,t}} + \frac{3}{2}\right)\\
&= -f(\bar\omega_{t+1},\sigma_{\omega,t})\left(\frac{\ln(\bar\omega_{t+1})}{\sigma^2_{\omega,t}} + \frac{1}{2}\right)\\
\frac{\partial^2 G(\bar\omega_{t+1},\sigma_{\omega,t})}{\partial\bar\omega_{t+1}\partial\sigma_{\omega,t}} &\equiv G_{\omega\sigma}(\bar\omega_{t+1},\sigma_{\omega,t})\\
&= \bar\omega_{t+1} f_\sigma(\bar\omega_{t+1},\sigma_{\omega,t})\\
&= -\frac{1}{\sigma_{\omega,t}}\bar\omega_{t+1} f(\omega,\sigma_{\omega,t}) + \frac{1}{\sigma_{\omega,t}}\bar\omega_{t+1} f(\omega,\sigma_{\omega,t})\left(\frac{\ln(\omega)^2}{\sigma^2_{\omega,t}} - \frac{\sigma^2_{\omega,t}}{4}\right)\\
&= \frac{1}{\sigma_{\omega,t}}\bar\omega_{t+1} f(\omega,\sigma_{\omega,t})\left(-1 + \frac{\ln(\omega)^2}{\sigma^2_{\omega,t}} - \frac{\sigma^2_{\omega,t}}{4}\right)
\end{aligned}
$$

Now starting from G_σ for G_σ:

$$
\begin{aligned}
\frac{\partial^2 G(\bar\omega_{t+1},\sigma_{\omega,t})}{\partial\sigma_{\omega,t}\partial\sigma_{\omega,t}} &\equiv G_{\sigma\sigma}(\bar\omega_{t+1},\sigma_{\omega,t})\\
&= 2\frac{\ln(\bar\omega_{t+1})}{\sigma^3_{\omega,t}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(\bar\omega_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)^2\right]\\
&- \left(\frac{\ln(\bar\omega_{t+1})}{\sigma^2_{\omega,t}} + \frac{1}{2}\right)\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(\bar\omega_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)^2\right]\left(-\frac{1}{2}\right)2*\\
&* \left(\frac{\ln(\bar\omega_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)\left(\frac{-\ln(\bar\omega_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma^2_{\omega,t}}\right)\\
&= 2\frac{\ln(\bar\omega_{t+1})}{\sigma^3_{\omega,t}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(\bar\omega_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)^2\right]\\
&- \left(\frac{\ln(\bar\omega_{t+1})}{\sigma^2_{\omega,t}} + \frac{1}{2}\right)^2\left(\frac{\ln(\bar\omega_{t+1})}{\sigma^2_{\omega,t}} - \frac{1}{2}\right)\frac{1}{\sigma_{\omega,t}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(\bar\omega_{t+1}) - \frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)^2\right]
\end{aligned}
$$

The second derivatives of Γ starting from Γ_ω:

$$\begin{aligned}
\frac{\partial^2\Gamma(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\bar{\omega}_{t+1}\partial\bar{\omega}_{t+1}} &\equiv \Gamma_{\omega\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= -f(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
\frac{\partial^2\Gamma(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\bar{\omega}_{t+1}\partial\sigma_{\omega,t}} &\equiv \Gamma_{\omega\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= -F_\sigma(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= \left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma_{\omega,t}^2}-\frac{1}{2}\right)\bar{\omega}_{t+1}\sigma_{\omega,t}f(\bar{\omega}_{t+1},\sigma_{\omega,t})
\end{aligned}$$

Now from Γ_σ for $\Gamma_{\sigma\sigma}$:

$$\begin{aligned}
\frac{\partial^2\Gamma(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\sigma_{\omega,t}\partial\sigma_{\omega,t}} &\equiv \Gamma_{\sigma\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= -\bar{\omega}_{t+1}^2 f(\bar{\omega}_{t+1},\sigma_{\omega,t}) - \bar{\omega}_{t+1}^2\sigma_{\omega,t}f_\sigma(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&= -\bar{\omega}_{t+1}^2 f(\bar{\omega}_{t+1},\sigma_{\omega,t}) \\
&- \bar{\omega}_{t+1}^2\sigma_{\omega,t}\left[-\frac{1}{\sigma_{\omega,t}}f(\omega,\sigma_{\omega,t})+\frac{1}{\sigma_{\omega,t}}f(\omega,\sigma_{\omega,t})\left(\frac{\ln(\omega)^2}{\sigma_{\omega,t}^2}-\frac{\sigma_{\omega,t}^2}{4}\right)\right] \\
&= -\bar{\omega}_{t+1}^2 f(\omega,\sigma_{\omega,t})\left(\frac{\ln(\omega)^2}{\sigma_{\omega,t}^2}-\frac{\sigma_{\omega,t}^2}{4}\right)
\end{aligned}$$

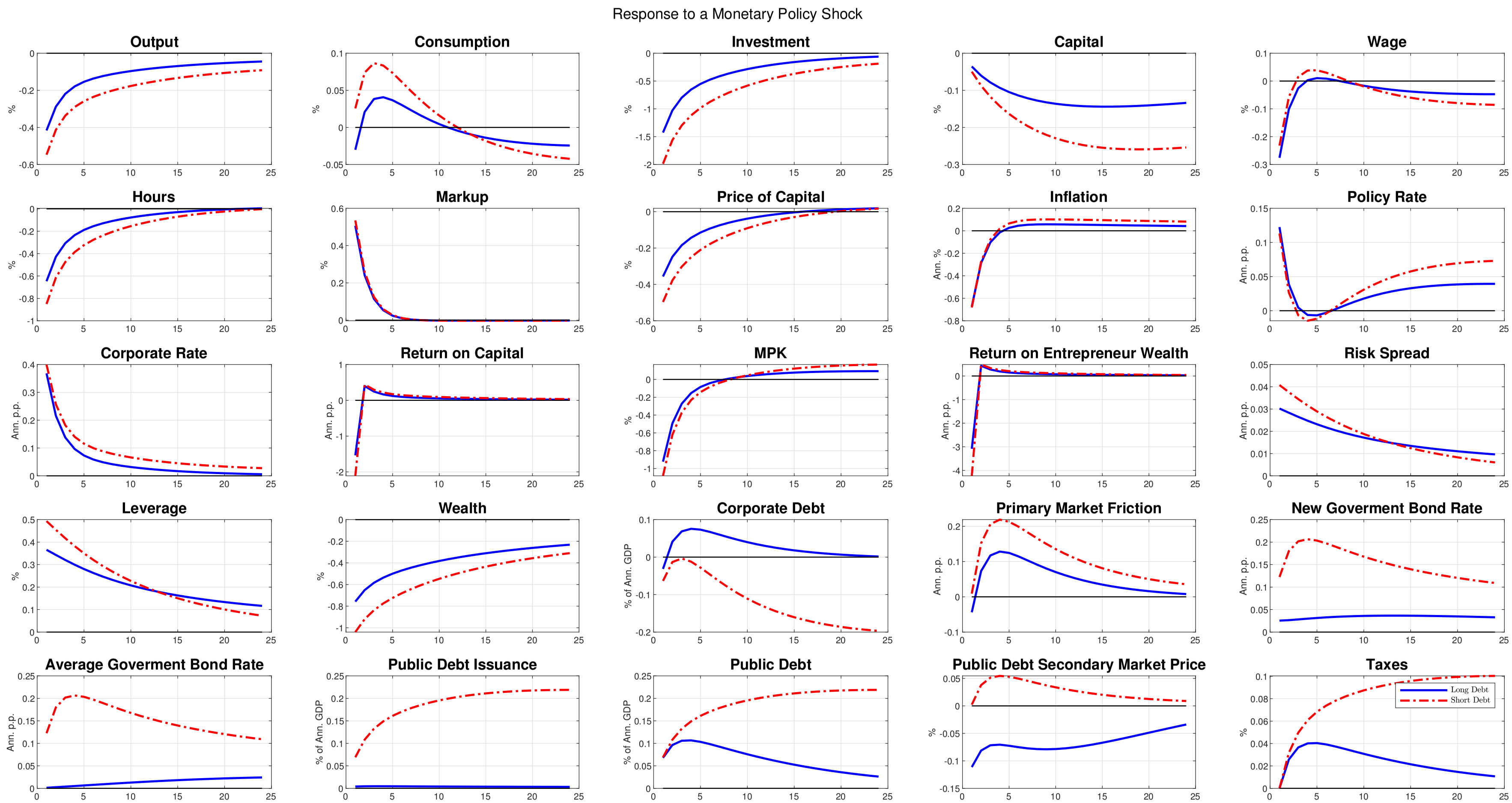
G Further Model Results

G.1 All Impulse Response Functions

Figure G.1 presents the whole set of impulse response functions to a contractionary 25 basis points monetary policy shocks in the baseline model, it is the more complete version of Figure 7. We can see the responses with public debt at its historical duration of around 4 years on the solid blue line and the responses in a counterfactual world with only one period public debt with the red dash-dot line.

In the baseline long debt case, output declines by about 40 basis points and investment by 1.4 percent, inflation by 0.6 percent in annualized terms. Leverage increases by 0.4 percent and the risk spread by 3 basis points. These are all standard results for a financial accelerator model and are the ones discussed in the main text in Section 7.2. On the additional IRFs a few stand out, first of all, the consumption response is muted, being an order of magnitude lower than output and investment. At most consumption declines by 3 basis points. This is in line with the empirical results. The interest rate in newly issued government debt increases only mildly. The reason is that, this is a long rate and the monetary policy shock is temporary in nature. Public debt jumps up in real terms as inflation declines have a Fisherian effect of increasing the debt burden. On the other hand, the secondary market value of public debt declines, as the existing debt rate as a lower average rate than the newly issued debt. This effect is not overturned by the higher primary market friction, which pushes upward the value of public debt. Taxes adjust slowly to respond to higher funding costs. The remaining part of the IRFs are standard for a financial accelerator New Keynesian model. Capital, wages, hours worked, the price of capital, the ex-post return on capital, the marginal productivity of capital, the ex-post return on entrepreneurs wealth and entrepreneurs wealth all decline on impact following a contractionary monetary policy shock. On the other hand markups increase.

Figure G.1: Baseline Model Impulse Response Functions For All Variables



Notes: The IRFs present the response to an annualized 25 basis points monetary policy shock. The solid blue line presents the IRFs in an economy with the maturity of public debt being at its historical average of around 4 years ($\delta^d = 0.05$). The dot-dashed red line presents the IRFs in an alternative economy with the maturity of public debt being at one quarter ($\delta^d = 1$).

Table G.1: Parameters Varied in Sensitivity Analysis

Parameter	Mean	Standard Deviation	Distribution
κ	2	1	Shifted Gamma (from 1)
s	0.0025	0.0025	Beta
$F(\omega, \sigma_\omega)$	0.0075	0.0075	Beta
ϕ_π	1.5	0.1	Normal
ϕ_Y	0.125	0.03	Normal
ρ_{mp}	0.8	0.05	Beta
θ	0.65	0.05	Beta
ζ	0.1	0.05	Beta
Φ	0.0025	0.0025	Beta
σ	2	0.5	Gamma
τ_T	2	0.5	Gamma
$\bar{D}$	1.6	0.5	Gamma

Notes: The first column shows which parameter is being varies. The second column shows the mean of the chosen distribution, this is equal to the baseline model calibration presented in Table 2. The third column shows the standard deviation of the sensitivity distribution. Finally, the fourth column specifies the distribution the parameter is being drawn from. With respect to leverage κ, the draws come from a shifted Gamma. That is, I draw from a Gamma with mean 1 and standard deviation 1 and then add 1 to each draw. The reason is that, in this model, leverage can go from 1 to infinity, and the shifted distribution allows for this while being centered at its calibrated value 2.

If we turn to the responses in the one period debt presented with the dot-dashed red line, we can see how the government must increase issuance as the average rate on public debt shoots up. As discussed in Section 7.2, the primary market friction increases and this creates a further amplification mechanism with the financial accelerator. Output and investment decline relatively more, and leverage and the risk spread increase relatively more. A few points come from the additional set of IRFs presented here. First of all, consumption still does not respond much, it is even positive on impact, but its magnitude is low at 3 basis points. Interestingly, the policy rate responds relatively less in this scenario, but the difference is very minor, as in the data. If we analyze public debt, we can see that the initial response is the same as in the long-debt case due to the Fisherian effect of inflation on nominal debt. On the other hand, in the following periods public debt keeps rising due to the higher issuance costs. By the same token taxes keep rising to avoid an explosive path for public debt. Finally, we can see how the secondary market price of public debt jumps due to the higher friction; in this case, the primary market price of public debt is always one as the coupons adjust to ensure it.

G.2 Sensitivity

It is important to check that the results do not hinge on any specific calibration, so in this section I perform a sensitivity analysis similar in spirit to a prior predictive analysis. The distribution of a few selected parameters are presented in table G.1. In this table, I use the posterior percentiles from either Smets and Wouters (2007) or Herbst and Schorfheide (2015) for ρ_{mp}, θ, and ϕ_Y to inform their distributions.

In Table G.2, we can see the outcome of this sensitivity analysis. In the first set of results, I show how some key metrics vary when all the parameters discussed above change, in the subsequent ones, I let one parameter vary at the time. For each case, I present a comparison between the high debt maturity case and the low debt maturity case. The first metric chosen is the percent difference in monetary policy strength on impact, how much more strongly output responds in a low maturity world. The other metrics show the highest difference (in absolute value) for a number of variables, all annualized.

The result is that leverage κ has an impact on the difference between high and low maturity on leverage and wealth, risk spread s only on spreads, and default rate $F(\omega, \sigma_\omega)$ as well only on spreads. The steady state value for the primary market friction Φ does not have a big impact, reassuringly. The tax parameter tau_T only has an impact on taxes, but not on much else. The steady state level of debt $\bar{D}$ has an impact on the difference between high and low maturity on corporate debt, output, the primary market friction, investment, wealth, and taxes. This is consistent with the idea that the impact of maturity of public debt matters when it can insure a big amount of debt to interest rate changes (it matters more for a high debt country as Japan than for a low debt one as Luxembourg). Interestingly, it matters mainly for the low maturity world, consistent with the idea that the primary market friction would hit more if all debt must be renewed each period. On the other hand in the baseline model with high maturity the only thing that moves with debt are taxes and debt itself. The primary market friction elasticity ζ has a big impact on the difference of output, corporate debt,the primary market friction, investment, wealth, and leverage. However, on absolute does not matter much as we can see in the sensitivity on high maturity case. Most results are again driven by changes in the low maturity case. The parameters relating to the Taylor rule and price stickiness can have a large impact on the absolute results as in a standard small size New-Keynesian model and this one is no different, however, in general the relative magnitudes of my finding remain unchanged and they do not hit particularly the primary market friction, that is no parameter kills it.

When we draw from all parameter distributions together we obtain a 90% confidence interval for how much more effective would have been monetary policy on output under a short debt scenario that ranges from 8% to 92% more effective, that is in basis points a difference that goes from 4 to 72 basis points difference following a 25 basis points increase in interest rates. Also in this case, all relative statements go through, with the most variation seen in the response of investment, corporate debt, public debt but not much for inflation and the policy rate. When drawing for each parameter alone I did 500 draws, when all parameters together 50000.

G.3 Sensitivity of the Maturity Structure

In Section 7.4 we could see how maturity is the key to find the quantitative results on the strength of monetary policy and that changes in steady state public debt would have needed to be too high to deliver the same quantitative result. Therefore, a natural question that comes is how the results vary with the maturity structure. In the baseline experiment, I compared a one period (quarter) debt with debt duration of about 4 years ($\delta^d = 0.05$), so that 5% of the debt needs to be refinanced every quarter, in line with historical averages for the United States. In order to answer this question I plot in Figure G.2 the peak response of output to a monetary policy shock for different levels of δ^d. I allow δ^d to vary from 1 (1 quarter) to 0.0066 (15 years). The figure clearly shows a linear relationship, with more debt needed to be refinanced leading to stronger monetary policy responses. It is worth noting that Macaulay duration has an hyperbola-like formula $\frac{1+R^{new}}{\delta^d+R^{new}}$, so that increasing debt duration by one year from 1 quarter debt will have a stronger effect on the output response than from 10 years.

G.4 Fisherian Channel for Corporate Debt

In the baseline model, I followed BGG and let the outside option for lenders to the entrepreneurs to be fixed in real terms. This would have been the optimal contract coming from trading between risk neutral entrepreneurs and risk averse households in the absence of

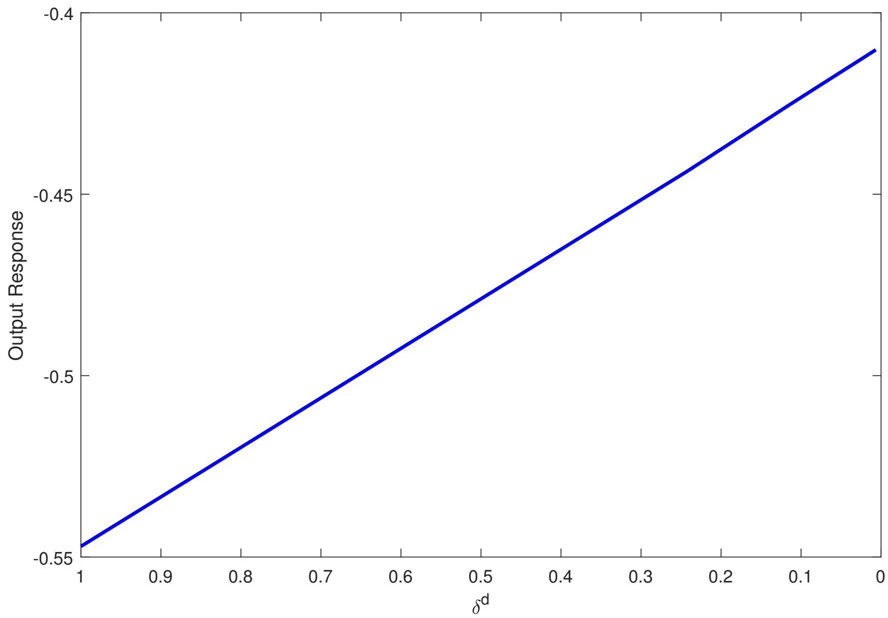
Table G.2: Sensitivity Analysis for Difference in Response to a Monetary Policy Shock under High and Low Maturity Models

	Metric	Mean	Median	5th Percentile	95th Percentile
All Parameters	Percent Difference in Output	-0.3684	-0.2735	-0.9145	-0.0778
	Difference in Output	0.1728	0.1064	0.0330	0.4551
	Difference in Risk Spread	-0.0209	-0.0029	-0.0576	0.0000
	Difference in Primary Market Friction	-0.1150	-0.0775	-0.2897	-0.0240
	Difference in Investment	0.7182	0.4482	0.1431	1.8094
	Difference in Corporate Debt	0.2101	0.1688	0.0637	0.4865
	Difference in Public Debt Issuance	-0.2260	-0.1867	-0.5178	-0.0778
	Difference in Inflation	0.1249	0.0203	-0.0397	0.4256
κ	Percent Difference in Output	-0.3577	-0.3379	-0.4711	-0.2894
	Difference in Output	0.1409	0.1386	0.1296	0.1570
	Difference in Risk Spread	-0.0114	-0.0090	-0.0293	-0.0023
	Difference in Primary Market Friction	-0.1003	-0.0972	-0.1240	-0.0851
	Difference in Investment	0.5918	0.5864	0.5582	0.6630
	Difference in Corporate Debt	0.2008	0.2041	0.1838	0.2101
	Difference in Public Debt Issuance	-0.2162	-0.2187	-0.2255	-0.2019
	Difference in Inflation	0.0339	0.0243	0.0031	0.0875
s	Percent Difference in Output	-0.3061	-0.3010	-0.3498	-0.2775
	Difference in Output	0.1263	0.1231	0.1098	0.1524
	Difference in Risk Spread	-0.0091	-0.0063	-0.0269	-0.0005
	Difference in Primary Market Friction	-0.0877	-0.0858	-0.1035	-0.0773
	Difference in Investment	0.5408	0.5243	0.4624	0.6703
	Difference in Corporate Debt	0.2039	0.2046	0.1929	0.2128
	Difference in Public Debt Issuance	-0.2174	-0.2176	-0.2224	-0.2117
	Difference in Inflation	0.0057	0.0023	-0.0098	0.0315
$F(\omega, \sigma_\omega)$	Percent Difference in Output	-0.3199	-0.3178	-0.3430	-0.3036
	Difference in Output	0.1350	0.1334	0.1235	0.1518
	Difference in Risk Spread	-0.0129	-0.0118	-0.0220	-0.0071
	Difference in Primary Market Friction	-0.0926	-0.0919	-0.1007	-0.0869
	Difference in Investment	0.5768	0.5706	0.5331	0.6402
	Difference in Corporate Debt	0.1993	0.1997	0.1923	0.2052
	Difference in Public Debt Issuance	-0.2147	-0.2149	-0.2186	-0.2102
	Difference in Inflation	0.0139	0.0122	0.0024	0.0310
ζ	Percent Difference in Output	-0.3005	-0.3042	-0.4258	-0.1754
	Difference in Output	0.1238	0.1258	0.0633	0.1835
	Difference in Risk Spread	-0.0100	-0.0101	-0.0147	-0.0052
	Difference in Primary Market Friction	-0.0856	-0.0871	-0.1268	-0.0435
	Difference in Investment	0.5310	0.5398	0.2766	0.7797
	Difference in Corporate Debt	0.1912	0.1965	0.1297	0.2403
	Difference in Public Debt Issuance	-0.2079	-0.2119	-0.2495	-0.1567
	Difference in Inflation	0.0088	0.0081	-0.0064	0.0283
Φ	Percent Difference in Output	-0.3141	-0.3111	-0.3324	-0.3048
	Difference in Output	0.1308	0.1308	0.1307	0.1309
	Difference in Risk Spread	-0.0105	-0.0104	-0.0113	-0.0102
	Difference in Primary Market Friction	-0.0907	-0.0896	-0.0970	-0.0875
	Difference in Investment	0.5606	0.5541	0.5398	0.6008
	Difference in Corporate Debt	0.2012	0.2027	0.1915	0.2065
	Difference in Public Debt Issuance	-0.2159	-0.2167	-0.2189	-0.2109
	Difference in Inflation	0.0096	0.0081	0.0050	0.0188
$\bar{D}$	Percent Difference in Output	-0.3035	-0.3017	-0.4337	-0.1740
	Difference in Output	0.1264	0.1255	0.0719	0.1819
	Difference in Risk Spread	-0.0102	-0.0101	-0.0147	-0.0058
	Difference in Primary Market Friction	-0.0875	-0.0870	-0.1247	-0.0504
	Difference in Investment	0.5428	0.5376	0.3053	0.7879
	Difference in Corporate Debt	0.1999	0.1895	0.0918	0.3401
	Difference in Public Debt Issuance	-0.2142	-0.2038	-0.3585	-0.1011
	Difference in Inflation	0.0078	0.0103	-0.0051	0.0125
τ_T	Percent Difference in Output	-0.3179	-0.3162	-0.3376	-0.3050
	Difference in Output	0.1323	0.1316	0.1271	0.1403
	Difference in Risk Spread	-0.0107	-0.0106	-0.0112	-0.0103
	Difference in Primary Market Friction	-0.0918	-0.0913	-0.0982	-0.0878
	Difference in Investment	0.5661	0.5637	0.5477	0.5938
	Difference in Corporate Debt	0.1959	0.1968	0.1633	0.2247
	Difference in Public Debt Issuance	-0.2123	-0.2123	-0.2364	-0.1875
	Difference in Inflation	0.0128	0.0113	0.0021	0.0292

	Metric	Mean	Median	5th Percentile	95th Percentile
σ	Percent Difference in Output	-0.3094	-0.3120	-0.3487	-0.2604
	Difference in Output	0.1290	0.1300	0.1104	0.1441
	Difference in Risk Spread	-0.0105	-0.0105	-0.0111	-0.0098
	Difference in Primary Market Friction	-0.0900	-0.0903	-0.0954	-0.0834
	Difference in Investment	0.5579	0.5594	0.5266	0.5837
	Difference in Corporate Debt	0.2007	0.2008	0.1972	0.2039
	Difference in Public Debt Issuance	-0.2154	-0.2156	-0.2187	-0.2117
	Difference in Inflation	0.0084	0.0091	-0.0037	0.0181
θ	Percent Difference in Output	-0.3276	-0.3178	-0.4936	-0.1898
	Difference in Output	0.1374	0.1324	0.0772	0.2126
	Difference in Risk Spread	-0.0111	-0.0107	-0.0170	-0.0062
	Difference in Primary Market Friction	-0.0956	-0.0918	-0.1507	-0.0531
	Difference in Investment	0.5879	0.5677	0.3349	0.8995
	Difference in Corporate Debt	0.2063	0.2024	0.1537	0.2699
	Difference in Public Debt Issuance	-0.2214	-0.2173	-0.2902	-0.1647
	Difference in Inflation	0.0203	0.0107	-0.0147	0.0867
ϕ_π	Percent Difference in Output	-0.3052	-0.3129	-0.3592	-0.2279
	Difference in Output	0.1266	0.1303	0.0961	0.1459
	Difference in Risk Spread	-0.0102	-0.0105	-0.0117	-0.0078
	Difference in Primary Market Friction	-0.0884	-0.0904	-0.0968	-0.0739
	Difference in Investment	0.5440	0.5591	0.4237	0.6151
	Difference in Corporate Debt	0.1982	0.2008	0.1840	0.2040
	Difference in Public Debt Issuance	-0.2127	-0.2155	-0.2190	-0.1973
	Difference in Inflation	0.0052	0.0088	-0.0419	0.0395
ϕ_Y	Percent Difference in Output	-0.3532	-0.3177	-0.6281	-0.1768
	Difference in Output	0.1617	0.1328	0.0664	0.3377
	Difference in Risk Spread	-0.0128	-0.0107	-0.0263	-0.0051
	Difference in Primary Market Friction	-0.1039	-0.0916	-0.1881	-0.0537
	Difference in Investment	0.6743	0.5687	0.2982	1.3457
	Difference in Corporate Debt	0.2169	0.2025	0.1446	0.3292
	Difference in Public Debt Issuance	-0.2328	-0.2173	-0.3536	-0.1551
	Difference in Inflation	0.0566	0.0119	-0.0313	0.2751
ρ_{mp}	Percent Difference in Output	-0.3060	-0.3120	-0.3156	-0.2739
	Difference in Output	0.1330	0.1330	0.0942	0.1727
	Difference in Risk Spread	-0.0108	-0.0107	-0.0141	-0.0076
	Difference in Primary Market Friction	-0.0934	-0.0922	-0.1256	-0.0664
	Difference in Investment	0.5735	0.5705	0.4045	0.7573
	Difference in Corporate Debt	0.2101	0.2048	0.1454	0.2961
	Difference in Public Debt Issuance	-0.2255	-0.2198	-0.3178	-0.1561
	Difference in Inflation	0.0025	0.0089	-0.0345	0.0125

Notes: The table shows a sensitivity analysis on the how high debt maturity case and the low debt maturity case respond differently following a 25 basis points contractionary monetary policy shock. The first column, shown sideways, display which scenario is being considered. In the first case (All Parameters), I allow all parameters to vary according to the distribution discussed in Table G.1. All other set of results show the same information when we allow to vary only one parameter at the time, where this parameter is shown sideways in the first column. In the first parameter is being varied. The second column shows the metric displayed. The remaining columns show for that metric, the average, the median, the 5th percentile, and the 95th percentiles of the draws that produced a model solution. In each scenario, I present the eight metrics, all pertaining to the different behavior under the two maturity cases. The first metric show is the percent difference in monetary policy strength on impact, how much more strongly output responds in a low maturity world. The other seven metrics show the highest difference (in absolute value) between the high debt maturity case and the low debt maturity case, for a number of variables, all annualized. As an example, a positive value for the difference in investment implies that investments response is relatively higher under the high maturity case than under the low maturity case. The first experiment with all parameters has 50000 draws, the other experiments, when I vary one parameter at the time has 500 draws. I keep all draws which for which both models (low and high maturity) admit a solution.

Figure G.2: Maximum Output Response Varying Public Debt Maturity



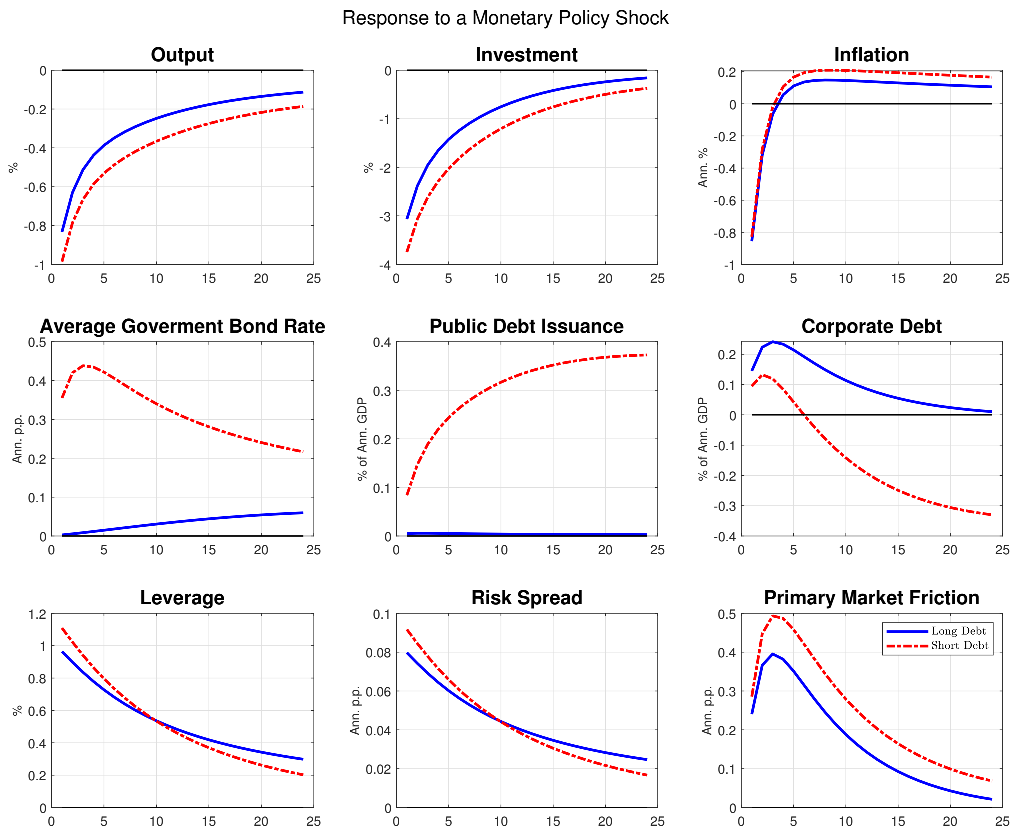
Notes: This figure presents the response on output on impact to an annualized 25 basis points monetary policy shock, for different values of the maturity structure δ^d. This parameter represents the fraction of the debt principal which must be refinanced each period, it goes on the horizontal axis from 1 (1 quarter) to 0.0066 (15 years). As we go to the right the maturity of public debt is increasing.

labor risk. However, in presence of labor risk for risk averse households, the optimal contract would have entrepreneurs would insuring this risk by providing an indexed contract. This argument has been proposed by Carlstrom, Fuerst and Paustian (2016) and Dmitriev and Hoddenbagh (2017) among others. I kept that contract out of comparability with BGG and higher empirical relevance than a corporate bond with an interest rate with an amount of indexation not seen in the data. However, an even more realistic contract (even if even harder to argue in terms of optimal contracts with this set-up) would be one with the rate being fixed in nominal terms rather than real terms. This is the route proposed by Christiano, Motto and Rostagno (2014).

In the model, having a nominally fixed rate corporate debt at rate $R_t^{crp,nom}$, changes the Euler equation for corporate debt and the lenders participation constraints that entrepreneurs face when choosing leverage and default threshold. Specifically, the outside option is now risky ex-post as it varies with realized inflation, so that the realized risk spread is the ratio of the real return on capital investments over the realized real return on the nominal corporate debt: $1+s_{t+1} = \frac{1+R^k_{t+1}}{\frac{1+R^{crp,nom}_t}{\pi_{t+1}}}$. After solving the new equilibrium and taking the Taylor expansion around the zero inflation steady state, the changes are to substitute $\hat{R}^{crp}_{t-1}$ with $\hat{R}^{crp,nom}_{t-1} - \hat{\pi}_t$ and $\hat{R}^{crp}_t$ with $\hat{R}^{crp,nom}_t - \mathbb{E}(\hat{\pi}_{t+1})$ everywhere in the linearized equilibrium.

Figure G.3 presents the results from this experiment. Allowing for fixed nominal rate debt for corporate bonds creates a Fisherian debt deflation channel that makes the effects of monetary policy stronger. Following a contractionary monetary policy inflation declines, which increases the ex-post real return entrepreneurs must pay. The higher rate on liabilities lowers entrepreneurs wealth, thereby increasing leverage and the risk spread. This in turn lowers investments and output. By comparing the results with the baseline case of real debt from Figure 7, we can see how the monetary shock has stronger effects both under the high and low maturity scenarios. The peak effect on output, under the long maturity case, moves from -40 basis points in the real debt case to -80 basis points in the nominal debt case. If we move to the comparison across maturities, the difference is not really affected by the nature of corporate debt. The difference in output at impact between the long maturity case and the short maturity case increases from 13 basis points to 15, a negligible difference. The outcome of this experiment is to show how allowing for nominally fixed rate debt does not alter the conclusions on the differential effectiveness of monetary policy under the two maturity profiles.

Figure G.3: Model Impulse Response Functions with Nominal Fixed Rate Corporate Debt



Notes: The IRFs present the response to an annualized 25 basis points monetary policy shock. The solid blue line presents the IRFs in an economy with the maturity of public debt being at its historical average of around 4 years ($\delta^d = 0.05$). The dot-dashed red line presents the IRFs in an alternative economy with the maturity of public debt being at one quarter ($\delta^d = 1$). The corporate bond rate is specified as a one period debt with a fixed rate in nominal term.