4 Things Nobody Tells You About Online News:

a Model for the New News Market

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November 2021

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Abstract:

Social media create a new type of incentives for news producers. Consumers share content, influence the visibility of articles and determine the advertisement revenues ensuing. I study the new incentives created by sharing and evaluate the potential quality of ad-funded online news. Producers rely on a subset of rational and unbiased consumers to spread news articles. The resulting news has low precision and ambiguous welfare effects. Producers' incentive to invest in news quality increases with the private knowledge of the topic; hence, when information is most needed, the generated news tends to be of lesser quality. Competition does not necessarily improve news quality – it does so only if the sharing network is *sufficiently dense.* While ad-funded online news occasionally helps consumers take better decisions, it creates welfare mostly through entertainment. Some interventions, such as flagging wrong articles, substantially improve the outcome; other approaches, such as quality certification, do not.

JEL codes: D80, D85, L82, L11, L14

Keywords: Online News, News Quality, Social Media, Competition, Networks

*This paper previously circulated as "The New News Market: a Model with Social Networks and Competition". I am grateful to Fernando Vega-Redondo for his continued support. I thank Andrea Mattozzi, Nenad Kos, Massimo Morelli, Ruben Durante, Nadia Burani, Sašo Polanec and Andrea Pasqualini for their insightful comments. I further thank discussants and participants at Bocconi University internal seminars, the 2021 ES European Summer Meetings, the 35th EEA Congress, the 14th RGS Doctoral Conference, the 2021 MPI PolEc Summer School, the 19th EEFS Conference, the 14th PEJ Meeting, the XXXV JEI doctoral meetings, the 13th VPDE PhD Workshop, the 9th Warwick Economic PhD Conference and the 9th EBR & SEB LU Doctoral Conference. I gratefully acknowledge financial support from *Fondazione Invernizzi.* Any mistake is my own.

1 Introduction

The media landscape has evolved throughout history. From the press to radio, television and the rise of the internet age, many past revolutions gave rise to concerns about news quality. Nowadays, social media are under the spotlight. The idea that the online news market may be worse than traditional media is puzzling as it arises in a highly competitive environment. Yet, in the last decade, the rise in competition was accompanied by a decrease in media trust.[1] Understanding the effect of social media on the provision of information is important as the prevalence of online news is growing; the majority of American and European adults include online outlets to their media diet.[2] Observers increasingly fear market segmentation: this could result in a two-tier market where only those paying for articles would be well-informed. Is there hope for the ad-funded outlets to provide quality news, so that even free articles would be informative? Should competition be encouraged or has social media metamorphosed the news market in a way that makes standard theory inapplicable?

While advertisement revenues and producers' reduced cost of entry date back to more than a century ago, online outlets brought something new: sharing. With social media, consumers play an active role in spreading news article, raising their visibility, thereby producers' advertisement revenues. Hence, news producers behind ad-funded online outlets respond to new incentives. Because of advertisement revenues, articles now need to be shared online. In this sense, the very presence of a news sharing network changes the effects of the previously existing market environment. In this paper, I evaluate the performance of such ad-funded online news outlets, focusing on the incentives linked to sharing behaviors. Three dimensions of the market environment are explored: the amount of private knowledge, the connectivity of the communication network and the presence of competition. After studying the effects of the environment on the provision of information, I question whether such outlets are welfare enhancing and propose possible interventions.

I explore this question by introducing a general setup to represent the online news market. The market is populated by consumers on one side and producers on the other. The agents are concerned with some state of the world, for instance, whether vaccines are effective or not. All consumers observe a private signal, e.g. whether a vaccinated friend has developed the illness. In addition, some consumers, called *seeds*, come across news articles about vaccination directly and can decide to share it on an exogenous network to other consumers, called *followers*. Seeds care about sharing true news; followers read articles that seeds share, they are not part of the strategic interaction.

An article is a signal whose realization is informative about the state of the world. Given seeds' sharing behavior, producers decide on the quality of their outlet, i.e. the precision of the signal they send. Producers choose neither the state of the world on which to report nor the news realization, only the probability for the realization to correspond to the state of the world. In

[1]See e.g. survey from Gallup, news.gallup.com/poll/321116/americans-remain-distrustful-mass-media.aspx
[2]See Pew Research Center, pewresearch.org/fact-tank/2021/01/12/; pewresearch.org/journalism/2018/10/30/

other words, producers only choose how many journalists to hire for their outlets, not what these journalists report; the more journalists, the higher the likelihood of reporting the true efficacy of vaccines. Each producer publishes one article about the same underlying state of the world, vaccine efficacy in this example, and only cares about how many consumers view their article. While the number of seeds reading a producer's outlet is exogenous, the number of followers seeing their article is endogenous. When several producers co-exist in the market, they compete *through seeds* to reach other consumers, as each consumer is restricted to see only one article.

This model brings interesting insights. Even when consumers are not behavioral, the market fails to deliver precise news. Thus, incentives created by social media do not suffice to induce high quality online news, even in a market populated by rational and unbiased agents. The business model based on advertisement revenues is flawed both because of the way it shapes producers' investment and because seeds imperfectly channel all information. The market environment then has counter intuitive effects on news quality: a lack of private knowledge is not substituted by more informative articles; the influence of competition on news quality is tied to the connectivity of the social network on which news are shared. Furthermore, the presence of news outlets has ambiguous welfare consequences, that not all interventions can overcome.

These results rely on two key mechanisms. First, the producers' incentive to invest in news quality is determined by the difference between the value of a true and a false article. Private knowledge, connectivity and competition all affect the value of true articles differently than that of false news, thus inefficiently modulating the producers' response to the market environment. Second, the market is shaped by consumers' sharing decision, which is determined by their private knowledge. Consumers' private knowledge thus bounds news quality. Below, I discuss in more detail how these two mechanisms drive all four main results.

First, ad-funded online outlets tend to fail when informative news would be the most beneficial. News quality is less valuable for a producer in an environment with low private knowledge: either because the consumers are not well-informed by their signals, or because the state of the world is *ex ante* very uncertain. As private signals get noisier, the value of a true article decreases while that of false information increases: consumers struggle distinguishing true and false news, leading them to treat any news article very similarly. As one state of the world becomes more likely, investing in news quality gets more attractive for producers, since the difference between the value of true and false information is greater when the most likely state of the world realizes. These leads producers' incentives to be misaligned with the consumers' need.

Second, competition can be detrimental; its effect depends on the network connectivity. For any market structure, high connectivity negatively affects news quality; but it does so less strongly if the market is competitive. A monopolist's incentive to invest vanishes as the network gets very dense: one single node sharing would reach almost all other consumers then. The monopolist can thus create false content and rely on a few seeds receiving an erroneous private signal to reach many followers. This intuition does not follow through in competitive markets. Producers cannot rely on these few seeds anymore; articles need to be sufficiently shared in order to survive in the network. In this sense, competition decreases the value of a false article. This force thus pushes the producers towards more investment.

Yet, the effect of competition is ambiguous. There is indeed a second opposite effect of competition. Splitting the market might be detrimental to investment, since the cost of news quality does not depend on the size of the market served. By accessing less initial seeds, the producer cannot reach as many followers, even if his articles was shared by all seeds reading it. Competition decreases the value of a true article. The strength of these forces depend on the network degree: as connectivity increases, producers have access to more and more followers while competition inside the network becomes more biting. Therefore, competition is detrimental below a connectivity threshold.

Third, the welfare value of ad-funded online news is ambiguous. Any equilibrium is Pareto inefficient. To go beyond the Pareto criterion, I consider different aspects of consumers' welfare. *Entertainment* – the utility derived from sharing – increases with news quality. To capture the value of information, I introduce an additional action, a bet, in which consumers must match the true state of the world. Agents are brought to better decision by news outlets if their expected utility from betting increases after having observed a news article.

Generally, the market fails to let seeds take better decisions. This does not rely on the presence of competition or the timing of the game; but on the central channeling role of the seeds. Producers have no incentive to publish more precise articles than the consumers' private signals, since this would suffice to being shared all the time. Therefore, the news quality is bounded by the consumers' knowledge and, for symmetric priors, seeds are always as well off by trusting their private signal for the bet. Followers, however, might take better decisions if the market is competitive: as the network tends to filter out false articles, the articles they end up seeing might be more precise than their private signal. Still, their utility from betting is bounded by their private signal.

Studying the decision to enter this bet at a cost allows to analyze whether online news pushes consumers towards action. Unsurprisingly, there exists a range of costs for which online news indeed helps agents enter the bet when it is beneficial. More surprisingly, under mild conditions, there also exists a range of costs for which news outlets are detrimental. By creating noise to consumers' private signals, news outlets too often discourage consumers to enter the bet, as more agents are wrong than right to opt out of the action. The existence a range of entry cost making the mere presence of news outlets detrimental again results from the bound placed by consumers' private knowledge on news quality.

Fourth, I analyze the effects of fact checking. I distinguish between *flagging*, fact checking articles before they are shared; and *quality certification*, fact checking past articles from news outlets in order to assess the outlets quality. The former has substantial effects on welfare by removing the bound placed on news quality; the second might marginally improve the news quality but does not remove the bound from private knowledge, hence it does not significantly affect welfare.

Flagging reduces the value of producing false information by improving the seeds' private knowledge. Interestingly, competition dilutes the effect of flagging. Actually, for any environment, there exists a level of flagging that makes competition detrimental. Indeed, flagging, like competition, reduces the value of false information; however, unlike competition, it does not decrease the value of true information. Flagging can then be seen as a substitute for competition: any outcome from competition is actually reproducible in uncompetitive markets through flagging. Certifying news outlets' quality allows producers to internalize the effects of their investment on the seeds' sharing strategy; however, the best outcome for producers is still to be shared all the time, which happens when they match consumers' private knowledge. Therefore, news quality is still bounded by the consumers' private knowledge.

The paper is organized as follows. Related literature is discussed in the remainder of this section. The general model is presented in Section 2. Section 3 analyzes the equilibrium resulting from a monopoly and a duopoly respectively; in particular, it assesses the role of the market environment and competitiveness on the outcome. Section 4 proposes a framework to assess welfare and analyzes it accordingly. Section 5 evaluates the effect of fact checking. Section 6 discusses the robustness of results and Section 7 concludes. Further extensions are provided in the Appendices A,B and C. Appendix D presents the proofs omitted in the main text.

Related literature

I contribute to several strands of the literature. I particularly relate to theoretical works on news markets, media economics and the spread of news in networks.

First, as to **news markets**, the existing theoretical literature accounts for the existence of bad quality news in a competitive but unconnected world. Allcott and Gentzkow [2017] find that uninformative news can survive if news quality is costly and if consumers cannot perfectly infer accuracy or if they enjoy partisan news. My setup is similar in that quality is costly and consumers cannot perfectly distinguish true from false articles. However, my mechanism does not fundamentally rely on outlets' quality being hidden. Furthermore, I introduce to such models an explicit network of information sharing to catalyze the spread of information.

In such unconnected news markets, the ambiguous effects of competition between news providers has been widely explored. Namely, Gentzkow and Shapiro [2008] find that competition is effective at reducing supply-driven biases, while its effects with demand-driven biases are ambiguous. Consistently with this conclusion, other authors find that competition has ambiguous effects when news consumers lack sophistication. For instance, Levy et al. [2017] study how media companies can exploit consumers' correlation neglect. They find that competition reduces the producers' ability to bias readers' beliefs, but that diversity has a cost in terms of optimal consumers' responses. Hu and Li [2018] and Perego and Yuksel [2018] study how rational inattention biases the provision of political information. Both find that competition inflates disagreement. Chen and Suen [2016] also find that competition is detrimental to the accuracy and clarity of news when readers endogenously allocate attention between outlets whose editors are biased. Interestingly, my results on competition is not motivated by biases of either side of the market.

Second, as to **media economics**, this paper relates in particular, to the influence of digitalization on media. Representative of this literature are the following papers. Anderson [2012] combines empirical and theoretical insights to offer an overview of the ad-financed business model in the internet age. Wilbur [2015] documents trends following digitalization for the mass media and how their business models has evolved. Finally, Peitz and Reisinger [2015] review various novel features resulting from new Internet media. I contribute to this literature by explicitly modeling one such new feature of online news market: shared content. I study its effects on producers incentives and equilibrium outcomes.

Note that Peitz and Reisinger [2015] briefly discuss how sharing decision might affect available content and link it to more general media biases. In this perspective, Hu [2021] studies the impact of media regulation in the digital age and finds that government regulation is rendered less effective by media biases inherent to the digital age. Because their model does not take into account any communication network, their analysis does not study interventions targeting the sharing behavior of consumers. My intervention evaluations, in contrast, only accounts for such incentives resulting from consumers' sharing decisions.

Third, as to **news in networks**, a connected world has rarely been the setup for news market models in the literature. To the best of my knowledge, only Kranton and McAdams [2020] study the effect of communication networks on the quality of information provided on the news market. My model is largely inspired by the setup they propose. While Kranton and McAdams [2020] give a compelling argument on how a network of consumer can change a producer's investment incentives, their mechanism abstracts from the role of competition. Furthermore, they do not address welfare effects of market outcomes. A key contribution of this paper is the introduction of competition and welfare considerations to the model.

Following the cascade literature,[3] the recent working paper Hsu et al. [2019] provide optimal conditions on a signal's precision for a cascade to occur when sharing is endogenous and strategic. This could, in turn, relate to a producer's objective, although no producer is featured in their setup. However, just as Kranton and McAdams [2020], Hsu et al. [2019] is set in an uncompetitive world. Finally, recent works explore the particular setup of learning on social media. Bowen et al. [2021] study learning via shared news and find that polarization emerges when agents hold misconceptions about their friends' sharing behavior. They find that news aggregators help curb polarization. Neither of these papers addresses the effects of competition between news providers in a connected world.

[3]This literature is studies learning in networks when agents learn from actions. See Bikhchandani et al. [1992], Banerjee [1992] for their seminal work.

2 Model

2.1 Environment

The market is populated by news producers and news consumers. Consumers learn about an unknown state of the world $\omega \in \{0,1\}$ through news articles and private signals.[4] There is a common prior across all agents, $\Pr(\omega = 0) = w_0$. All agents are Bayesian.

I denote the set of news consumers I, which can be finite or infinite. All consumers receive an informative binary private signal s about ω. These signals are i.i.d. among consumers with $\Pr(s = \omega|\omega) = \gamma$ for $\omega = 0, 1$. I further impose $\gamma \geq w_0$, so that consumers trust their private signal more than their prior.

In addition to private signals, the consumers can come across news articles in the following ways: they can be exposed to it directly – in such a case they are called initial *seeds* and denoted i; or they can read such news because a seed shared it. If consumers are not seeds, they are called followers and denoted f. All consumers are exposed to at most one article, but some followers might be exposed to none. When I do not want to explicitly distinguish seeds from followers, I denote the news consumers j.

The consumers are arranged on a regular network of degree d.[5] Consumers are randomly drawn to be seeds with probability b. Hence, all consumers, regardless of their role, have the same number of *random* neighbors d. The network allows followers to see articles shared by neighboring seeds. A follower sees no article if none of its neighbors shared content – either because none of the neighbors are seeds or because none of the seeds decided to share. If several neighbors shared content from different sources, the article that a follower f ends up seeing is determined stochastically. Any of f's sharing neighbor has the same probability to be seen. Therefore, the probability with which the follower sees a given source is proportional to the number of neighbors sharing this source relative to the number of neighbors having shared any article. In other words, the probability that f sees a given article k is:

$$Pr(f \text{ sees } k|A \text{ neighbors shared } k, B \text{ neighbors shared}) = \frac{A}{B}$$

For instance, say four of f's neighbors shared a piece of information, but only one of them shared k, then, f sees k with probability one fourth, although f does see *some* article with probability one.

On the other side of the market, I consider a finite set of producers K.[6] Each producer, denoted by k, publishes exactly one article.[7] Each producer reaches a seed with the same exogenous probability $\frac{b}{K}$. The producer chooses the overall quality of the news that is published.

[4] w denotes the outcome of ω. For the remainder of the paper, the distinction between random variables and their outcome is not made at long as it is clear from the context.

[5] In a regular network, all nodes have the same number of connections. Note this assumption is not fundamental to the analysis, but greatly simplifies the notation. The extension to non-regular networks is discussed in Section 6.

[6] I abuse notation by denoting K both the set of producers and its cardinality

[7] Therefore, I can abuse notation by also denoting articles by k.

However, he does not choose the article's content, which is randomly determined. The content of an article is denoted n.

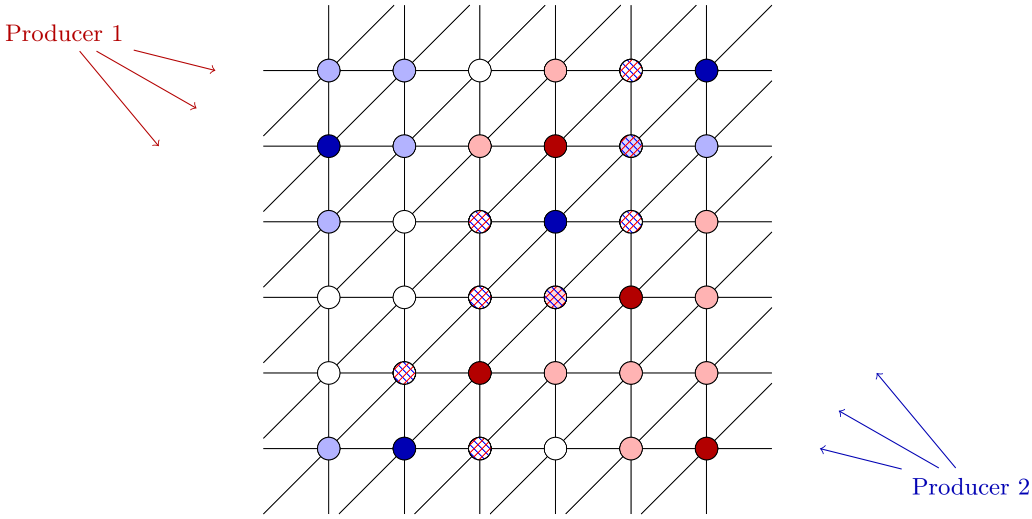

Figure 1: Illustration of a possible outcome on the news market

Figure 1 depicts the environment described with two producers. Dark colored nodes are seeds; for the illustration, assume all articles are shared. Light colored nodes are followers who see a given producer with probability one. Hatched colored nodes are followers whose source is determined at random – neutrally hatched nodes see each producer with probability $1/2$, hatched nodes with a hue see the given producer with probability $2/3$.

2.2 Timing, Objectives and Equilibrium Concept

All strategic interactions are assumed to be simultaneous.[8] The only agents active in the game are initial seeds and producers.

The producers choose the quality of their outlet to maximize their profits. The quality or precision of outlet k is defined as the probability of documenting the true state of the world, $q_k := \Pr(n = \omega|\omega)$ for $\omega = 0, 1$. Producers derive revenue from advertisement, hence from the visibility of their outlet. Their revenue is thus defined as the share of the network that sees their article [9]. Their (total) cost is determined by cost function C. C is common to all producers. I denote c the marginal cost function. I assume C increasing and strictly convex, i.e. $c(q) > 0$ and $c'(q) > 0$. Finally, I assume that without any investment in quality, the outlet produces uninformative news, that is, $q_k = 1/2$. Furthermore, $c(1/2) = 0$.

Seeds like sharing true information and dislike sharing false information. Accordingly, they choose the probability with which they share an article. This can depend on the content that

[8]Equilibria of the sequential game when $w_0 = 1/2$ are provided in the Appendix C

[9]Intuitively, the revenues are scaled for size population because they relate to advertisement revenues. One might expect advertisers to be interested in the *portion* of the population a given news outlet is able to reach. Furthermore, with this representation, the model becomes scale-free. Finally, it allows their profits to be bounded below 1

the article they read reports and whether it corresponds to the private signal they received. The article's reported content, i.e. the realization of the news signal, is denoted n; the congruence with the private signal is denoted $S = +, -$ where $S = +$ if the news content is the same as the seed's private signal, and $S = -$ otherwise.[10] The probability with which a seed shares an article from producer k whose content is n is denoted by $z_{S|n,k}$. Therefore, the seeds' strategy is a vector: $(z_{S|n,k})_{(S,n,k)\in\{+,-\}\times\{0,1\}\times K}$. As seeds want to share an article only when its content is truthful, they are assumed to receive a positive payoff from sharing true information and a negative payoff when sharing false information.[11] Seeds have the following payoff from sharing:[12]

$$u(\text{sharing article reporting } n|\omega = w) = \begin{cases} 1 & \text{if } n = w \\ -1 & \text{otherwise} \end{cases}$$

They receive payoff 0 if they do not share.

I focus on Nash Equilibria.

2.3 Best Responses

For ease of exposition, I derive the best-responses of initial seeds and producers for $w_0 = 1/2$. I then provide the best-response for general w_0 in a dedicated paragraph; details can be found in Appendix D.

2.3.1 Seeds' Problem

Take a seed who received private signal s and read a news article reporting n. Then, the seeds expected utility from sharing is:

$$p(n,s) + \Big(1 - p(n,s)\Big)(-1) = 2p(n,s) - 1$$

where $p(n,s)$ is i's posterior on the probability that the producer published a true article.

A piece of news is true if it matches the state of the world. Hence, the posterior is the probability that the state of the world is the one prescribed by the news, given what was written in the news and what the consumers themselves experienced from the world. That is, $p(n,s) := Pr(\omega = n|n,s)$. Let seeds attribute prior probability q_k to an article from k being true. Recall $w_0 = 1/2$. Using Bayes' rule:

$$\Pr(\omega = n|n,s) = \frac{\Pr(n,s|\omega=n)\Pr(\omega=n)}{\Pr(n,s)} = \frac{\Pr(\omega=n)\Pr(s|\omega=n)q_k}{\sum_w \Pr(\omega=w)\Pr(s|\omega=w)\Pr(n|\omega=w)}$$

[10]The outcome of the private signal is $s \in \{0,1\}$ while the congruence is $S \in \{+,-\}$. For instance $s = 1$ is a *positive* signal towards n being true if $n = 1$, and a negative signal towards the article being true if $n = 0$. If additionally $\omega = 1$, $s = 1$ and $S = +$ are said to be *correct* while they would be *wrong* if $\omega = 0$.

[11]This assumption can represent the interests of truth-seeking consumers. Implicitly, it also accounts for wider concerns such as reputation or attention. In fact, Appendix B assumes that seeds seek attention for themselves. Their best-response is qualitatively similar.

[12]In Appendix A, I consider more general payoffs. Most results follow through, but additional equilibria might appear.

Therefore:

$$p(0,0) = p(1,1) = \frac{\gamma q_k}{\gamma q_k + (1-\gamma)(1-q_k)} \qquad \text{and} \qquad p(0,1) = p(1,0) = \frac{(1-\gamma)q_k}{(1-\gamma)x_k + \gamma(1-q_k)}$$

As one would expect, all posteriors are increasing in q_k. A seed shares an article when its expected utility from doing so is greater than the outside option 0. Therefore, i shares news n from producer k upon receiving signal s when:

$$p(n,s) \geq 1/2$$

Since no state of the world is ex-ante more likely, any realization of the news is as likely; therefore, the reported content is only relevant in conjunction with private signals, $p(0,s) = p(1,\text{-}s)$. In other words, accounting for the possible (dis)agreement between private signal and news article is sufficient, and the subscript n can be omitted from the seeds' strategy. The identity of the news' producer being irrelevant to seeds beyond q_k, the subscript k is omitted as well. The seeds' best-response is summarized by $z = (z_+, z_-)$ and is characterized as follows:

$$(z_+^*(q_k), z_-^*(q_k)) = \begin{cases} (0,0) & \text{if } q_k < \underline{t} \\ (e,0) & \text{if } q_k = \underline{t} \\ (1,0) & \text{if } q_k \in (\underline{t}, \bar{t}) \\ (1,e) & \text{if } q_k = \bar{t} \\ (1,1) & \text{if } q_k > \bar{t} \end{cases}$$

for any $e \in [0,1]$, where $\underline{t} = (1-\gamma)$ and $\bar{t} = \gamma$.

Since $\underline{t} < \bar{t}$, the seeds' best response are weakly monotonic in q_k: $z_-^* \geq z_+^*$. In other words, one shares an article reporting the opposite of their private signal only if one would be ready to share this article, were it to report the same as their private signal. Hence, when q_k increases, the *ex-ante* probability for a node to share increases. Therefore, although the strategy z is multi-dimensional, the set of undominated z can be represented on a line.[13] Figure 2 represents how sharing decisions is affected by different news quality, and the monotone aspect of it; Figure 3 displays seeds' best-response to news' quality q_k. The same applies for any k.

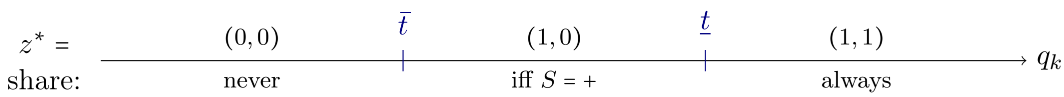

Figure 2: Sharing Decisions of Seeds for Different News Quality

[13]Formally, $z_k = (z_{+|0,k}, z_{+|1,k}, z_{-|0,k}, z_{-|1,k})$; for $w_0 = 1/2$, it is undominated to treat any news content the same way: $z_{+|0,k} = z_{+|1,k}$, $z_{-|0,k} = z_{-|1,k}$. For $q_k = \underline{t}$, any $z_{+|0,k} \neq z_{+|1,k}$ would also be undominated; however, setting $z_{+|0,k} = z_{+|1,k}$ leads to an equivalent analysis. The same applies to z_- for $q_k = \bar{t}$.

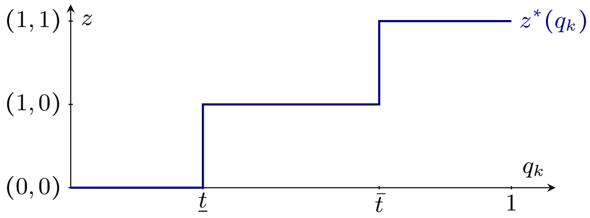

Figure 3: Best Response of Seeds as a Function of q_k

Best-response for general w_0

For $w_0 > 1/2$, the news realization n matters in the beliefs that the article is true since a news reporting the most likely state of the world is more probable to be true: $p(0,s) > p(1,\text{-}s)$. However, conditional on reading a given news content n, the seeds' best-response are as before:

$$(z^*_{+|n}(q_k), z^*_{-|n}(q_k)) = \begin{cases} (0,0) & \text{if } q_k < \underline{t}_n \\ (e,0) & \text{if } q_k = \underline{t}_n \\ (1,0) & \text{if } q_k \in (\underline{t}_n, \bar{t}_n) \\ (1,e) & \text{if } q_k = \bar{t}_n \\ (1,1) & \text{if } q_k > \bar{t}_n \end{cases}$$

for any $e \in [0,1]$, where $\underline{t}_n = \frac{(1-\gamma)\Pr(\omega \neq n)}{(1-\gamma)\Pr(\omega \neq n)+\gamma \Pr(\omega = n)}$ and $\bar{t}_n = \frac{\gamma \Pr(\omega \neq n)}{\gamma \Pr(\omega \neq n)+(1-\gamma)\Pr(\omega = n)}$.

Because $\underline{t}_0 < \underline{t}_1 < \bar{t}_0 < \bar{t}_1$, the seeds' best response are again weakly monotonic in q_k; the set of undominated strategies $z^* = (z^*_{+|0}, z^*_{+|1}, z^*_{-|0}, z^*_{-|1})$ can be represented on a line for any $w_0 < \gamma$.

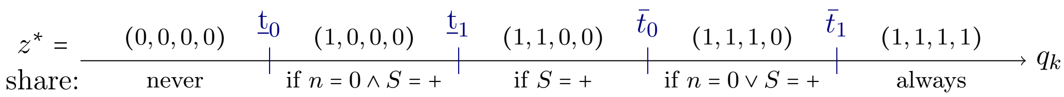

Figure 4: Sharing Decisions of Seeds for Different News Quality

Notice that $z^*_{S|0} \geq z^*_{S|1}$: one shares an article reporting the least likely state of the world only if one would be ready to share this article, were it to report the most likely state of the world, given the same (dis)agreement with private signals.

2.3.2 Producers' Problem

Consider a producer k. Let R_k take value 1 if a consumer sees producer k's article. Assume that k is facing seeds who have strategy $\mathbf{z}$, while the other producers ℓ are investing $\mathbf{q}_\ell$. Then, the expected profits for producer k who invests to reach quality q_k is:

$$\mathbb{E}(R_k | q_k; \mathbf{z}, \mathbf{q}_\ell) - C(q_k)$$

The expected share of reader as a function of k's investment in quality is found as follows.

For a random node to share the article from producer k, one needs: the consumer to be a seed – with probability b –, to come across k's article – with probability $1/K$ – and to share. The probability to share, z, depends on whether the news article corresponds to the private signal, which depends on whether k produced a true or false article. Recall that $w_0 = 1/2$ so that the news realization n is irrelevant beyond its (dis)agreement with private signals. Seeds receive a private signal corresponding to the state of the world with probability γ; this private signal corresponds to the news' content $S = +$ if the news content is true, and it is not congruent $S = -$ if the news content is false. Hence:

$$p_{T_k} = \frac{b}{K}(\gamma z_+ + (1-\gamma)z_-) \quad \text{and} \quad p_{F_k} = \frac{b}{K}(\gamma z_- + (1-\gamma)z_+)$$

The *ex ante* probability that a consumer reads k's article, which reports a true/false content – denoted $X = T, F$–,[14] represents the value of such article for producer k; it is denoted V_{X_k}. If producer k has no competitor, the probability to be read by publishing a news X is simply:

$$V_{X_k}(z) := \Pr(j \text{ seed}) + \Pr(j \text{ follower} \wedge \geq 1\ j\text{'s neigh. shared}) = b + (1-b)\big(1 - (1 - p_{X_k})^d\big)$$

However, when producer k is not alone on the market, it is not enough that a follower's neighbor shared k's article; this follower also needs to see k against all other producers ℓ's articles. Therefore:

$$V_{X_k}(z) := \Pr(j \text{ seed}) + \Pr(j \text{ follower}) \Pr(\geq 1\ j\text{'s neigh. shared}) \Pr(j \text{ sees } k \text{ against } \ell)$$

$\Pr(j \text{ sees } k \text{ against } \ell)$ depends on the number of f's neighbors having shared k against ℓ. The number of f's neighbors having shared ℓ depends on the content produced by other producers, which we denote $Y_\ell := (Y_l)_{l \neq k}$.

The *ex ante* probability that a consumer reads k's article, which is $X = T, F$, $V_{X_k}(z)$, is then:

$$V_{X_k}(z) = \sum_{Y_\ell} V_{X_k Y_\ell} \Pr(Y_\ell) = \frac{b}{K} + \sum_{Y_\ell} (1-b) \frac{p_{X_k}}{p_{X_k} + p_{Y_\ell}} \left(1 - (1 - p_{X_k} - p_{Y_\ell})^d\right) \Pr(Y_\ell) \ ^{15}$$

where $V_{X_k Y_\ell}$ is the value for k of producing an article that is $X = T, F$ when other producers have published articles which are true or false as described in Y_ℓ; and denoting $p_{Y_\ell} = \sum_{l \neq k} p_{Y_l}$, with $Y_l = T, F$.

The probability for a follower to read information k given the other articles Y_ℓ has two factors. The former, $\frac{p_{X_k}}{p_{X_k} + p_{Y_\ell}}$ represents the expected share of followers k would get, conditional on them being reached by any news, whereas the latter factor $1 - (1 - p_{X_k} - p_{Y_\ell})^d$ represents the probability of any news to reach followers. It means that sharing affects the producer's revenue through two channels: the size of the total readership and the portion of readers viewing a given producer. For instance, if the seeds of a producers' competitor start sharing more often, the total readership

[14] $-X$ is the alternative, so $X = T$ means $-X = F$ and conversely

[15] $\Pr(Y_\ell | \omega) = \prod_{l: Y_l = T} q_\ell \cdot \prod_{l: Y_l = F} (1 - q_\ell)$. For instance, with two other producers ℓ, $\Pr(T, F) = q_1(1 - q_2)$.

increases but the portion of the readership seeing that producer decreases. The relative strength of these two effects depends on the connectivity of the network d. Both factors are however increasing in p_{X_k}. Hence, as long as true articles are shared more than false articles, i.e. $z_+ \geq z_-$, true information is more visible, no matter the outcome of the competitor.

Finally, the expected portion of the network reached given an investment q_k is:

$$\mathbb{E}(R_k|q_k) = q_k V_{T_k}(z) + (1 - q_k) V_{F_k}(z)$$

Because the profits are $\mathbb{E}(R_k|q_k) - C(q_k)$, the maximization of profits implies:

$$q_k^*(z) = c^{-1}\big(V_{T_k} - V_{F_k}\big) := c^{-1}\big(\Delta V_k(z; q_\ell)\big)$$

Because $c'(q) \geq 0$, the equilibrium investment $q^*(z)$ is (weakly) increasing in $\Delta V_k(z; x_\ell)$. Thus, $\Delta V_k(z)$ denotes producer k's incentive to invest. Intuitively, it corresponds to the additional number of views the producer gets in expectation from producing a true rather than a false article. Section 3 analyzes the function shape for one and two producers.

Best-response for general w_0

For $w_0 > 1/2$, in addition to the veracity of a news article, its realization n matters, as $n = 0$ tends to be shared more $p_{X|0,k} \geq p_{X|1.k}$. In other words, the value of producing a $X = T, F$ article also depends on the state of the world. The analysis of the producers' problem is however very similar:

$$\mathbb{E}(R_k|q_k) = w_0[q_k V_{T|0,k}(z) + (1 - q_k) V_{F|1,k}(z)] + (1 - w_0)[q_k V_{T|1,k}(z) + (1 - q_k) V_{F|0,k}(z)]$$

where $V_{X|n,k}$ is the value of a $X = T, F$ article reporting content n.[16]

Finally, the best-response is:

$$q_k^*(z) = c^{-1}\big(w_0[V_{T|0,k} - V_{F|1,k}] + (1 - w_0)[V_{T|1,k} - V_{F|0,k}]\big) := c^{-1}\big(\Delta V_k(z; q_{-k})\big)$$

Intuitively, $V_{T|0,k} - V_{F|1,k}$ corresponds to the additional number of views the producer gets in expectation from producing a true rather than a false article when the most likely state of the world realizes, $\omega = 0$; while $V_{T|1,k} - V_{F|0,k}$ correspond to the same concept for $\omega = 1$.

3 Equilibrium

In this section, I characterize possible equilibria in both a non-competitive and a competitive market. I say that a market is competitive when consumers are see less articles than the amount

[16]Similarly as above, $V_{X|n,k}(z) = \frac{b}{K} + \sum_Y (1 - b) \frac{p_{X|n,k}}{p_{X|n,k} + p_{Y|m,\ell}} \big(1 - (1 - p_{X|n,k} - p_{Y|m,\ell})^d\big) \Pr(Y|\omega)$, where $m := (m_l)_{l \neq k}$ is defined implicitly by Y given X and n, e.g. $X = T, n = 0$ means $\omega = 0$, so $Y_l = T \Leftrightarrow m_l = 0$.

available on the market. In such case, producers are indeed forced to compete through seeds in order to capture followers' views. Because this setup restricts consumers to receive only one piece of information, I analyze the outcome from a monopoly and a duopoly respectively. I study the equilibrium on each market with symmetric prior $w_0 = 1/2$. For the monopoly, I furthermore characterize the equilibrium and discuss the role of the environment for general w_0.

3.1 Equilibrium without Competition

Consider a market with only one producer. For clarity purposes, I omit the k index in this section. The monopolist's incentive to invest is denoted $\Delta V_M(z)$ and can be rewritten explicitly:

$$\Delta V_M(z) = (1-b)\left[(1-p_F)^d - (1-p_T)^d\right]$$

Let us now analyze the shape of such best-response:

Lemma 1. *The monopolist's best-response to sharing $q^*(z)$ is single-peaked in z_+, with maximum $\bar{z} \in (0;1]$; it is strictly decreasing in z_-. $q^*(z)$ is continuous in z.*

Proof. See Appendix D. □

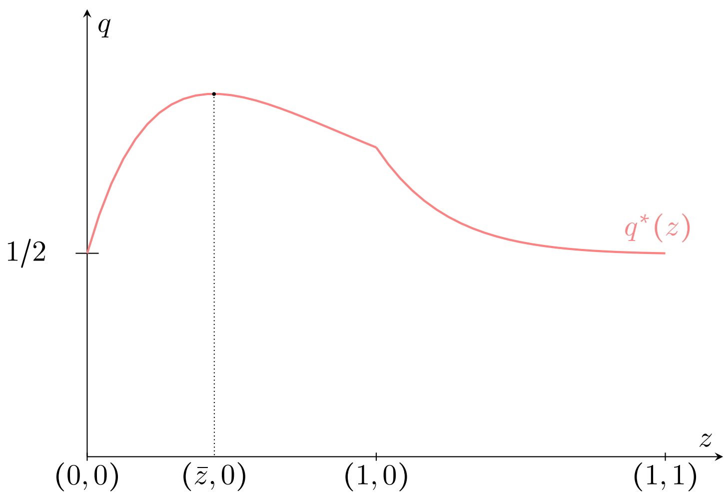

Figure 5: Producer's Best Response

Figure 5 illustrates the shape of the producer's best response. Because the seeds' strategy is not a unidimensional object, I illustrate the shape of the producer's best-response on the set of seeds' undominated strategy. As before, I represent the seeds' strategy on a line and map the corresponding image as if the argument was unidimensional. The resulting function is non-monotonic. The hump shape is explained by the effect of the network. At first, when the probability for agents to share is low, every additional node sharing reaches an almost constant number of additional followers; because the probability that this share occurs after having issued a true article is higher, true information gains much more followers than false information – the best-response is increasing. But when *enough* shares would occur, any increase in the probability of sharing would lead to shares which are likely to reach followers that would have been reached anyways; the marginal value of the probability of sharing is decreasing, because of redundant path to followers in the network. Therefore, the number of followers reached with a false article, that is rarely shared, is increasing faster with z_+ than the number of followers reached with true news; and the best-response is decreasing. Subsequently, the best response is decreasing. On the decreasing segment, agents start sharing news that does not correspond to their private signals. Therefore, the probability that this concerns a false article is higher than the probability that it applies to true information. It follows that false information accumulates views faster than true news. The difference between the value of true and false information thus decreases, making the best-response decreasing.

Now, recall that $q^* \geq 1/2$, as no investment would lead to $q = 1/2$. Furthermore, $\underline{t} < 1/2$. Therefore, we can characterize the Nash equilibrium of the monopoly.

Proposition 1. *There exists a Nash equilibrium. Any equilibrium displays positive investment and is uniquely characterized by news quality* $q_M^* = \min\{q^*(1,0), \bar{t}\}$

Proof. See Appendix D. □

The sketch of the proof is as follows. An intersection of $q^*(z)$ and $z^*(q)$ exists because both best responses are continuous and that in $z = (0,0)$ the producer's best response is above the value ensuring some sharing, while in $z = (1,1)$ the producer's best response is below the value ensuring full sharing. The intersection in the space $(q,(z_+,z_-))$ is unique because any equilibrium displays $z_+ = 1$; so that any intersection would occur on the decreasing part of the producers' best response, while the seeds' best-response is weakly increasing.[17] The point of intersection depends on the parameters; it can occur on the vertical part of the seeds' best-response, then $q^*(1,0) < \bar{t}$; or on the horizontal part of the seeds' best response, then $q^*(1,0) \geq \bar{t}$.

Figure 6 shows the equilibrium with $q^*(1,0) < \bar{t}$. Figure 7 shows the equilibrium with $q^*(1,0) > \bar{t}$. Notice that Proposition 1 implies that $q_M^* < \max_z\{q^*(z)\}$. As in Kranton and McAdams [2020], the highest investment that a producer would be willing to pay is never achieved.

Below, I derive the corresponding results for general w_0 in order to describe the role of the environment on the equilibrium outcome, in particular, connectivity and private knowledge, through both prior and precision of private signal.

Characterization for general w_0

The producer's best-response for general w_0 is very similar to that for $w_0 = 1/2$. In particular, with $\Delta V_M(z) = w_0\left[(1-p_{F|1})^d - (1-p_{T|0})^d\right] + (1-w_0)\left[(1-p_{F|0})^d - (1-p_{T|1})^d\right]$, the shape of the monopolist's best-response is as before given any realization of content n. The characterization of the equilibrium follows.

[17] Technically, for $q_M^* = \bar{t}$, any $z_{-|0} \neq z_{-|1}$ is undominated, so there would exist equilibria with $z_{-|0} \neq z_{-|1}$. I abstract from this technicality as all results follow through.

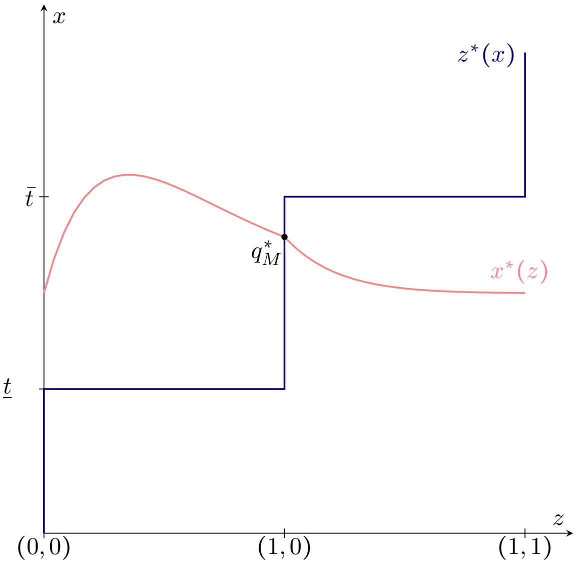

Figure 6: Equilibrium with $q_M^* = q^*(1,0)$

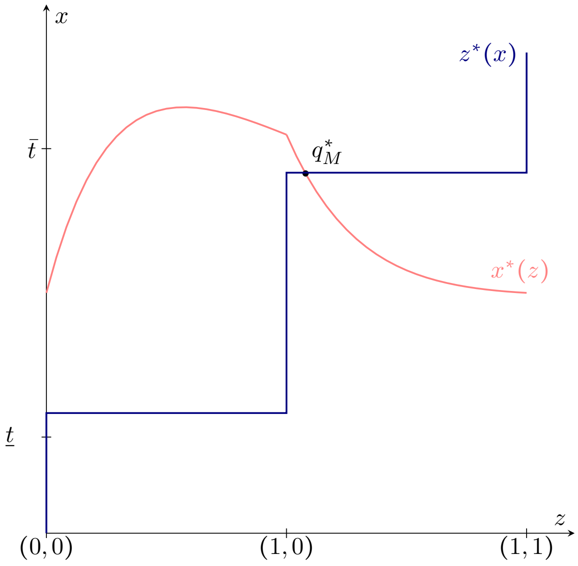

Figure 7: Equilibrium with $q_M^* = \bar{t}$

Corollary 1.

- *The monopolist's best-response $q^*(z)$ is single-peaked in $z_{+|n}$, with local maxima $\bar{z_n} \in (0;1]$; it is strictly decreasing in $z_{-|n}\cdot$; $q^*(z)$ is continuous in z.*
- *There exists a unique Nash equilibrium. It displays positive investment and is characterized by news quality $q_M^* = \max\{\min\{q^*(1,0), \bar{t}_0\}, \min\{q^*(1,1,1,0), \bar{t}_1\}\}$.*

Proof. See Appendix D. □

Figures illustrating the producers' best response and the equilibrium are provided in Appendix D. The intuition behind the characterization of the equilibrium is as before. However, when $w_0 > 1/2$, there are two horizontal and vertical parts of the seeds' best-response for a potential crossing. The inequalities describing the conditions for intersection on the different parts are summarized by the min and max operators as characterized in Corollary 1. Detailed explanations can be found in Appendix D.

Below, I explore the role of the market environment on the equilibrium outcome. The results apply for general w_0, but, when possible, intuitions are kept general for either $w_0 = 1/2$ or $w_0 > 1/2$.

3.1.1 The Role of Connectivity

High connectivity is generally detrimental to investment. In fact:

Lemma 2. *For any sharing behavior z, the monopolist's incentive to invest is single peaked in d; that is, there exists a threshold $\bar{d}$ so that $\Delta V_M(z)$ is increasing for any $d < \bar{d}$ and decreasing for any $d > \bar{d}$.*

Proof. See Appendix D □

In particular notice that as soon as $z_+ > 0$,[18] $\Delta V(z; d) \to 0$ as $d \to \infty$. This means that as the network grows more connected, the producer's incentive to invest vanishes. This insight echoes Kranton and McAdams [2020]'s Proposition 3. To illustrate this point, consider a complete network, that is a network in which every consumer is connected with every other consumer. In such a context, a monopolist would need only a single share in order to reach every single consumers on the market; therefore, the monopolist can reach as many consumers by publishing a false article, than with true information, as long as one seed receives a different private signal than the others.

3.1.2 Role of Private Knowledge

Private knowledge encompasses two parameters: the prior about the state of the world, w_0; and the precision of private signals, γ. These represent respectively how *ex-ante* uncertain the documented topic is, and how well-informed agents privately are. Through different channels, both of these parameters have a similar effect on the producer's incentive to invest.

Proposition 2. *A decrease in private knowledge implies a decrease in the producer's incentive to invest. In particular, for any undominated z, $\Delta V_M(z)$ is increasing in both γ and w_0.*

Proof. See Appendix D □

Intuitively, when γ is low, consumers are not good at distinguishing true from false articles; hence, false information tends to be shared almost as often as true information. The value of a true article is low while that of a false article is high. Therefore, the incentive for the producer to invest is low, as publishing a true article would not raise his visibility by a lot.

The channel through which *ex-ante* uncertainty affects the incentive to invest is different. Because seeds share more often if the article content corresponds to the most likely state of the world, the difference of value between true and false news is greater when the most likely state of the world realizes. Hence, investment is more beneficial to the producer when $\omega = 0$. The expected profits from any given investment thus increases when the most beneficial state becomes more likely.

Proposition 2 does not specify the effect of private knowledge on the equilibrium outcome. Indeed, γ and w_0 also affect the seeds' best-response. However, the equilibrium outcome is generally affected by a change in private knowledge in the same way as the producer's incentive to invest.

Corollary 2. *Generally, a decrease in private knowledge implies a decrease in the equilibrium investment. In particular:*

(i) q^*_M increases with γ.

(ii) q^*_M increases with w_0 if and only if $q^*_M \neq \bar{t}_0$.

[18] Technically, $z_{+|n} > 0$ for both $n = 0, 1$ is required.

Proof. See Appendix D □

Interestingly, a lack of private knowledge tends not to be compensated for by the market. Indeed, a decrease in private knowledge generally leads to worse information provision. Hence, the online news market fails exactly when it is the most needed: when the state of the world is very uncertain, either because of little prior knowledge, or because of poorly informative private signals. This seems to indicate that the inefficiencies from the market structure, and in particular from the fact that online outlets derive revenues from advertisement, can be very problematic.

Another source of inefficiency linked to private knowledge appears from the strategic interaction, and in particular, from the seeds' imperfect knowledge.

Remark 1. In equilibrium, $q_M^* \leq \bar{t}_1$. Therefore, news quality is bounded by agent's private knowledge w_0 and γ.

A proper setup to formally study such inefficiencies is introduced in Section 4.

3.2 Equilibrium with Competition

I now assume that two producers coexist on the market. Because of tractability concerns, results are provided only for $w_0 = 1/2$. Let the two competitors be denoted by k and ℓ. The producer k's best response given ℓ's investment and sharing strategy z can be rewritten:

$$\Delta V_k(z; q_\ell) = (1-b)\Big[V_{T_k} - V_{F_k}\Big] = (1-b)\Big[q_\ell\Big(V_{T_k T_\ell} - V_{F_k T_\ell}\Big) + (1-q_\ell)\Big(V_{T_k F_\ell} - V_{F_k F_\ell}\Big)\Big]$$

Where:

$$V_{X_k Y_\ell} = \frac{p_{X_k}}{p_{X_k} + p_{Y_\ell}}\Big(1 - (1 - p_{X_k} - p_{Y_\ell})^d\Big)$$

For any article published by ℓ, whether true or false $Y = T, F$, the visibility of k is higher for true articles than for false news, i.e. $V_{T_k Y_\ell} \geq V_{F_k Y_\ell}$. Therefore, $\Delta V_k(z_k; z_\ell, q_\ell^*) \geq 0$. In particular, the incentive to invest is strictly positive as long as true news is shared more often than false news, i.e. for any $z_{T_k} > z_{F_k}$; it is null for $z_{T_k} = z_{F_k}$.

The shape of $\Delta V_k(z_k; z_\ell, q_\ell)$ in z_k is similar to the monopoly case; however, k's best-response also depends on the sharing behavior of seeds reached be ℓ, as well as ℓ's investment.

Lemma 3. *Duopolist k's best-response is as follows:*

(i) $q_k^*(z_k; z_\ell, q_\ell)$ *is single-peaked in* z_{T_k} *with maximum* $\bar{z}_k \in (0; 1]$*; it is strictly decreasing in* z_{F_k}*.*

(ii) $q_k^*(z_k; z_\ell, q_\ell)$ *relation with* z_ℓ *depends on* d*. For small* d*, it is decreasing in* z_ℓ*. For large* d*, it is decreasing in* z_ℓ *for* $p_{F_\ell}^2 > p_{T_k} p_{F_k}$*.*

(iii) $q_k^*(z_k; z_\ell, q_\ell)$ *is decreasing in* x_ℓ *for any* $z_{X_\ell} \geq z_{X_k}$*.*

(iv) $q_k^*(z_k; z_\ell, q_\ell)$ *is continuous in* z_k*,* z_ℓ *and* x_ℓ*.*

Proof. See Appendix D □

As before, the best-response of producer k in non-monotonic to his own seeds' sharing z_k. Interestingly, if the sharing pattern is the same for either producer, their investment are strategic substitutes.

I can now characterize the NE. I call *symmetric equilibria* any equilibrium in which $z_k = z_\ell$ and $q_k = q_\ell$. In this case, $\Delta V_k = \Delta V_\ell$. I denote this common function $\Delta V_D((z_T, z_F), q)$, and omit the producers' indices on the seeds' best response z.

Proposition 3. *The only symmetric equilibrium features positive investment and is characterized by news precision* $q_D^* = \arg\min_{q\in[1/2,\gamma]} \left|\Delta V_D\big((1,0);q\big) - c(q)\right|$.

Proof. See Appendix D □

The proof of existence is similar to that of the monopoly case. There are two cases to distinguish, as illustrated on Figure 8 and Figure 9.

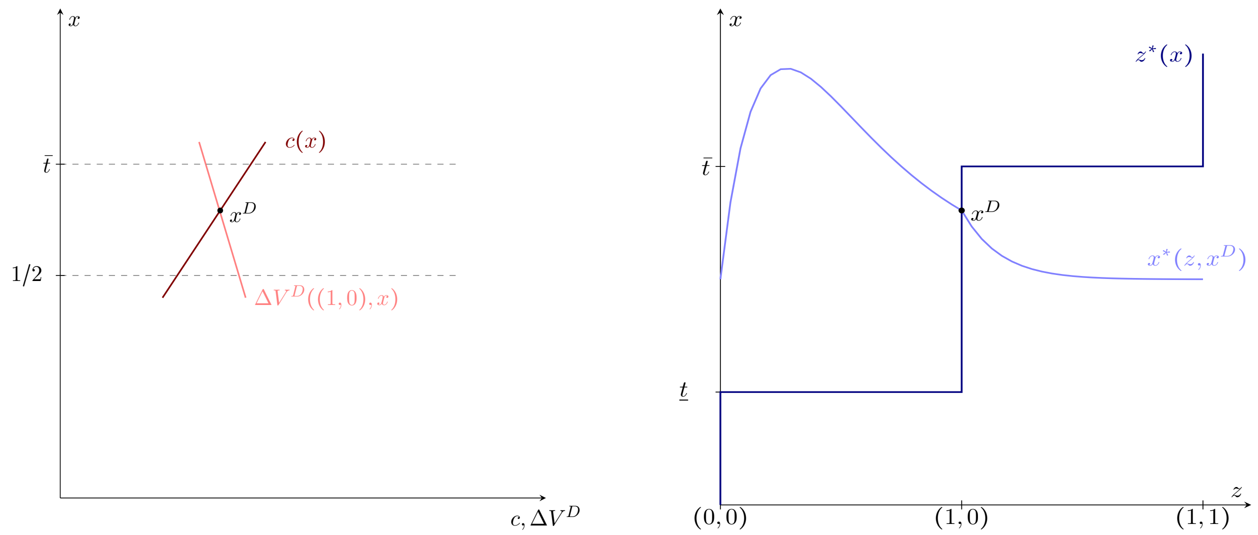

Figure 8: Illustration of a case for which $q_D^* \in (1/2, \bar t)$

When $c(\bar t) \geq \Delta V_D((1,0),\bar t)$, as in Figure 8, there exists an intersection between $c(q)$ and $\Delta V_D((1,0),q)$ in the interval $(1/2,\bar t]$ (left panel). Given that ℓ invests q_D^*, k's best response crosses the seeds' best response in $((1,0),q_D^*))$. Therefore, this intersection is a NE.

When $c(\bar t) < \Delta V_D((1,0),\bar t)$, as in in Figure 9, $c(q)$ lies completely on the left of $\Delta V_D((1,0),q)$, without ever intersecting in the interval $q \in (1/2,\bar t)$ (left panel). Equivalently, k's best response to z given that ℓ invests $q_\ell < \bar t$ intersects the seeds' best response z above $\bar t$ (right panel). This means that there does not exist a symmetric NE in which $z^* = (1,0)$. However, if $z_F > 0$, $\Delta V_D(z,q)$ is shifted to the left in the space $(\Delta V_D, q)$, so that it now crosses $c(q)$ (left panel). Furthermore, because q_ℓ increases to $\bar t$, the curve $q_k^*(z,q_\ell)$ is shifted downwards in the space (z,q_k) (right panel). For some $z_F > 0$, $\Delta V_D((1,z_F),\bar t) = c(\bar t)$, so that $q_D^* = \bar t$.

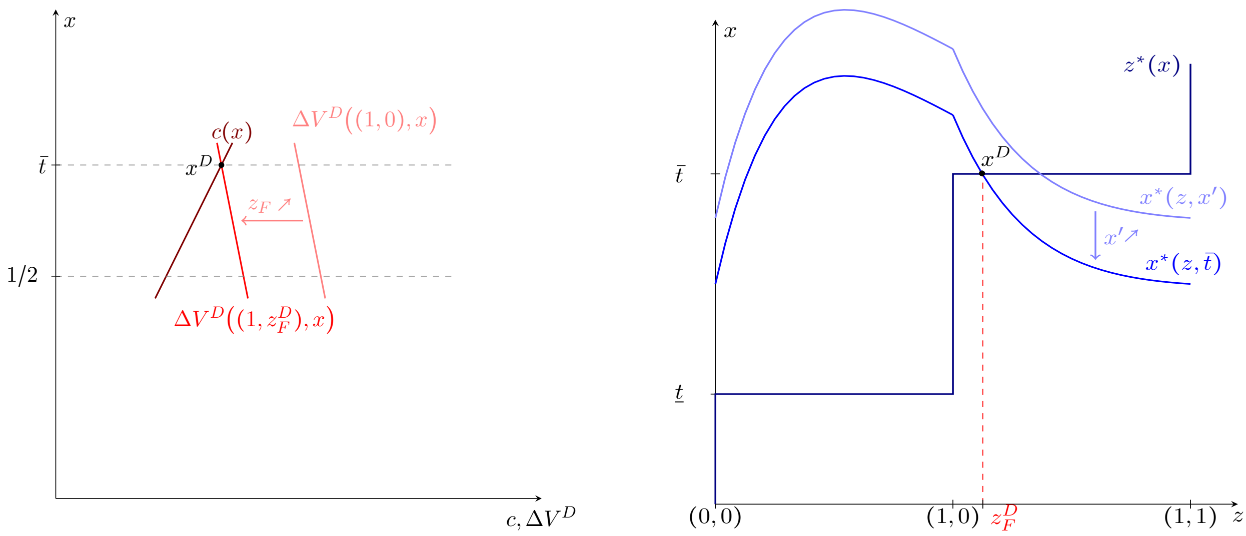

Figure 9: Illustration of a case for which $q_D^* = \bar{t}$

While the symmetric equilibrium is unique, asymmetric equilibria generally exist and are not unique.

Remark 2. Let $S := \frac{1}{2}(1-b\gamma)^d + \frac{1}{2}(1-b(1-\gamma))^d - (1-\frac{1}{2}b)^d$. If the marginal cost function is linear with slope different than S, there are no equilibria with $q_k \neq q_\ell$ and $(q_k, q_\ell) \in (1/2, \gamma)$.

Proof. Assume $q_D^* \in (1/2, \gamma)$. Assume that there exists an $q_k > q_\ell$, with $(q_k, q_\ell) \in (1/2, \gamma)$. Then, $c(q_k) = \Delta V_k((1,0),(1,0),q_\ell)$ and $c(q_\ell) = \Delta V_\ell((1,0),(1,0),q_k)$, so that $c(q_k) - c(q_\ell) = S(q_k - q_\ell)$, which is impossible if c has a slope different from S. □

3.3 Effects of Competition

The symmetric equilibrium q_D^* is compared to q_M^*. For cases to be comparable, let $w_0 = 1/2$. I confront the two types of markets in terms of connectivity and signal precision. Furthermore, $q_M^* \geq q_D^*$ only if $\Delta V_M(z) > \Delta V_D(z, q_D^*)$; therefore, I focus on $\Delta V_M(z)$ and $\Delta V_D(z, q_D^*)$ in this section.

3.3.1 The Role of Connectivity

The comparison between monopoly and duopoly depends on the connectivity of the network. Indeed, the presence of a competitor has an ambiguous effect on a producer's incentive to invest: on the one hand, investment might increase because each follower is harder to reach, so that the producer needs to be *sufficiently* shared; on the other hand, news quality might decrease because each producer reaches fewer seeds, so that fewer followers can be reached. In other words, by making the number of shares more important, competition decreases the value of false information; by reducing the producers' potential readership, it decreases the value of true information.

The strength of both of these forces depends on the connectivity. In a very connected network, each seed is connected to most followers, so that a producer *can* reach almost all followers even when a competitor is present. In a sparsely connected network, each follower is unlikely to be connected to several seeds, so that the probability to reach a follower is almost independent from the other competitor's outcome. Therefore, competition should lead to lower investment in sparse network, but would be beneficial to news quality in dense networks. Theorem 1 formalizes this; in particular, there is a unique threshold for a network connectivity that determines which of the two forces dominate.

Theorem 1. *There exists a unique threshold* $\bar{d}$ *such that* $q_M^*(d) \geq q_D^*(d)$ *for all* $d < \bar{d}$ *and* $q_M^*(d) \leq q_D^*(d)$ *for all* $d > \bar{d}$

Proof. Define $DV(d) := \frac{\Delta V_M(z;d) - \Delta V_D(z,q;d)}{1-b}$. First, notice it that for $d = 1$, $DV(d) > 0$; however for $d \to \infty$, $DV(d) < 0$. Therefore, there must exist some d_0 such that $DV(d_0) \geq 0 > DV(d_0 + 1)$. All that is left to do is to show that such d_0 is unique. This is the case because if $DV(d_1) > DV(d_1 + 1)$ for some d_1, then DV is decreasing for all subsequent $d > d_1$. See Appendix D for technical details. □

To further illustrate the two mechanisms at hand, I consider a few specific instances. Take $d = 1$. Figure 10 depicts such a network with 12 nodes. On the left panel, the producer, was his investment sufficient to make all seeds share, would reach four additional nodes; on the right panel, the same producer who now shares the market with a competitor can, at best, reach only two followers. Therefore, his incentive to invest when a competitor is present is half that of the case with no competition.

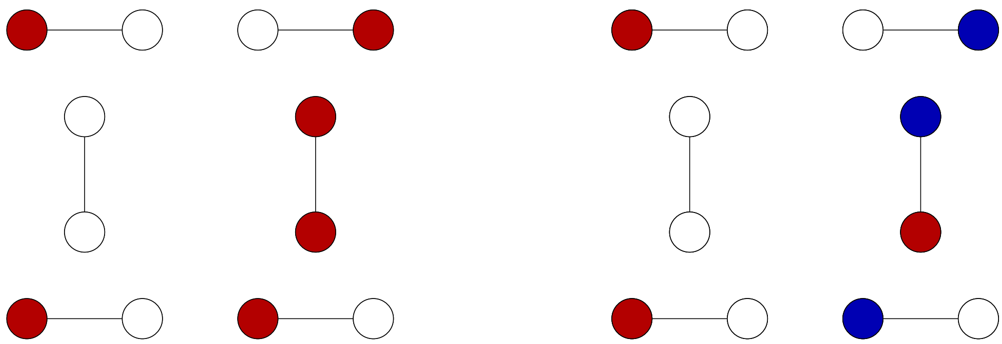

Figure 10: Possible followers reached with one vs. two producer(s) on the market

This insight applies in any network with $d = 1$. Indeed, the number of followers a produce can reach is linearly proportional to the number of seeds who share content. Therefore, the producers' incentive to invest is linearly proportional to the additional number of seeds who share when publishing a true article: for each additional share, the producers expect one additional view from a follower. Because a monopolist exogenously reaches twice as many seeds as each duopolist, a duopolist's incentive to invest is half that of a monopolist:

$$\Delta V_M(z;1) = (1-b)(2\gamma-1)(z_+ - z_-) > (1-b)\tfrac{1}{2}(2\gamma-1)(z_+ - z_-) = \Delta V_D(z,q;1) \quad \forall z_+ > z_-$$

As the network's connectivity grows, this force vanishes, while the intensity of the competition increases. Figure 11 underlines how the strength of competition is made greater by a denser network. In the part of the network depicted, the node in the center happens to be a follower surrounded by seeds reading different articles. Most nodes reached by producer 1 (in red) do not share, as represented by a grey circle. In a network where $d = 4$, producer 1 still has one chance out of two to reach the central node; with $d = 8$, his chances are only 1 out of 4 given the same sharing pattern.

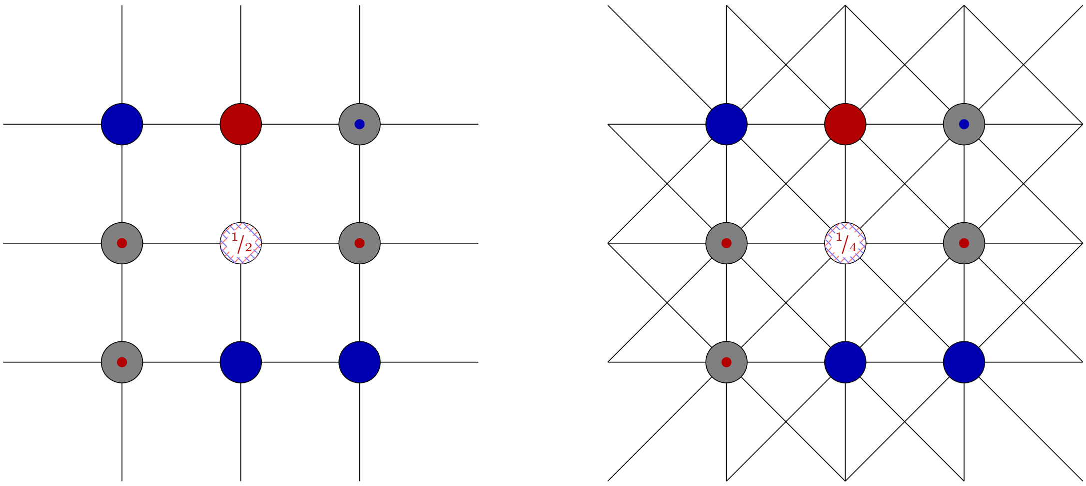

Figure 11: Probability of reaching a follower with $d = 4$ vs. $d = 8$

This insight continues to apply as d grows. Take $d \to \infty$ such that that all nodes are connected to each other.[19] All followers will be reached by an article as soon as the probability of sharing is not null, however arbitrarily small. For a monopolist, the incentive to invest vanishes, since only one seed sharing suffices to reach the entirety of the network. It is quite the opposite in a duopoly. The probability to reach a follower depends on the decision of every seed. Hence, the incentive to invest is proportional to the ratio of additional seeds of his sharing, to all sharing seeds:

$$\Delta V_M(z;\infty) = 0 < (1-b)\frac{(2\gamma-1)(z_+ - z_-)}{z_+ + z_-} = \Delta V_D(z,q;\infty) \quad \text{since } (1-\varepsilon)^d \to 0 \ \forall \varepsilon > 0$$

Remark 3. A further increase in competition, beyond two producers, is either always detrimental, or detrimental for sparser networks.

Proof. Take any competition with K competitors and a symmetric incentive to invest. Add one producer k'. The difference in the incentive to invest between the K and $K+1$ is positive in $d = 1$ and has a sign which depends on the parameters for $d \to \infty$. See Appendix D for the

[19] This requires the network to be infinite. Because the finiteness of the network is not essential to the specification, let us assume, for convenience, that $|I| \to \infty$, and that d grows as fast as $|I|$.

computation. □

3.3.2 The Role of Signal Precision

Remark 4.

(i) When the seeds get perfectly informative private signal, monopoly yields higher investments than duopoly.

(ii) When the seeds get perfectly uninformative private signal, the incentive to invest vanishes for both a monopoly and a duopoly.

Proof. It suffices to derive the value of $\Delta V_M(z) - \Delta V_D(z)$ for $\gamma \to 1$ and $\gamma \to \frac{1}{2}$. Computations can be found in Appendix D. □

When the signal is perfectly informative, seeds only share true information. Then, the monopolist has the highest possible incentive to invest: false information is worthless; with true information, he reaches all the followers the network allows him to reach. For the duopolist, false information is also worthless, but true information is less beneficial. Indeed, if the competitor released true information, they together reach the same portion of followers as the monopolist would have, but they split this audience in two; if the competitor released false information, the duopolist gets the whole share of followers reached, but he reaches fewer followers than the monopolist would have since he is read by fewer seeds.

When the signal is perfectly uninformative, the result is very intuitive: as the private signal is noisy, the agents are not able to tell true from false information, so that they treat both type of news without accounting for their private signal. Because the game is simultaneous, the producer does not internalize the effect of his investment on the consumers' prior, so that no investment is featured in equilibrium.

Numerically, for b and d high enough, there exists a threshold for γ such that duopoly is yielding a higher investment than monopoly for any private signal with lower precision. It indicates that a higher signal precision has stronger effects in a monopoly than in a duopoly. Intuitively, when the signal precision is high, consumers are relatively good at distinguishing true from false articles; therefore, very few seeds are sharing false news, and the monopolist cannot rely on them to reach enough followers. Therefore, the positive effect of competition, that making followers harder to reach, is marginal; while its negative effect, that reducing the number of followers who are reachable, is still significant. Overall, competition is then detrimental.

The example below details some numerical applications.

Example. Consider the sign of $\Delta V_M - \Delta V_D$ as a function of γ. In the competitive case, the incentive for one producer to invest is influenced by the competitor's investment. In a symmetric equilibrium, a change in γ indirectly influences ΔV_D through q_D^*. In this example, I only consider the direct effects.[20] In particular, throughout the example, I consider $q_\ell^* = 0.6$ and study k's best

[20]Considering indirect effects would require to specify a marginal cost function.

response. Furthermore, I assume $z^* = (1,0)$ for both producers. I report the threshold for γ above which $\Delta V_M - \Delta V_D > 0$. For any γ greater than the threshold reported, the incentive to invest in a monopoly is greater. I consider three level of broadcast reach and connectivity. The thresholds rounded to three digits are reported in Table 1.

	$b = 0.25$	$b = 0.5$	$b = 0.75$
$d = 5$	–	–	0.612
$d = 10$	–	0.820	0.914
$d = 20$	0.811	0.941	0.964

Table 1: Private signal precision thresholds with $q_D^* = 0.6$

Note that the thresholds reported are all above $q_D^* = 0.6$, which is consistent with $z^* = (1,0)$. No threshold reported means that no matter the signal precision, monopoly creates a bigger incentive to invest than competition. Echoing Theorem 1, Table 1 illustrates how duopoly leads to a higher incentive to invest for a bigger range of signal precision as d and b increase.

4 Welfare

So far, I have only been interested in the market outcomes, as measured by investment. Welfare has not been addressed.

First, I note that the market outcome is inefficient.

Proposition 4. *Any equilibrium outcome on the news market with revenues derived from ads is Pareto inefficient*

Proof. Take the case of a monopoly, with equilibrium $e^* = (q^*, z^*)$. Define $q^c(z; e^*)$ as the level of news quality that makes a consumer whose sharing decision is z indifferent between (q^c, z) and e^*. Likewise, define $q^p(z; e^*)$ as the level of news quality that ensures to the producer faced with sharing decision z the same revenue as e^*. If $\frac{\partial q^c}{\partial z} < \frac{\partial q^p}{\partial z}$ [21], there is room for Pareto improvement since consumers require less investment to marginally increase their sharing than the producer is ready to offer for the same marginal increase in sharing. Now, the FOC of equilibrium imply $0 \leq \frac{\partial q^c}{\partial z} < \infty$ while $\frac{\partial q^p}{\partial z} \to \infty$. The same reasoning applies to duopolists. □

To analyze the welfare resulting from the production of news, I propose two approaches. The first one relates to the *entertainment* purpose of possible sharing behavior. In this sense, only seeds and producers are part of the analysis. The seeds' decision to share an article depends on the utility of sharing as defined above. However, this does not capture how *informative* the article is. In particular, it does not allow to judge whether agents are making, on average, better

[21] We abuse notation here in order to keep the intuition as clear as possible. While z is a vector, recall that, when q increases, the consumers would first share the most likely congruent news, then any congruent news, then the most likely news anyways, and then any news. Therefore, with ∂z, I mean to designate a marginal change in the sharing probability **in the relevant dimension**. So for instance if z=(1,0,0,0), ∂z is actually $\partial z_{+|1}$; if $z = (0.5, 0, 0, 0)$, then we mean $\partial z_{+|0}$.

choices. To address this question, I introduce an additional action to be taken by all news' consumers after the strategic interactions have unfolded. This permits to analyze whether, on average, agents are able to take better decision; as well as whether the information contained in articles published in online outlets that derive revenues from advertisement can motivate agents to take actions they would have opted out from, were they informed only privately. Furthermore, this measure of welfare accounts for followers' well-being.

4.1 Framework of Analysis

Once the game is played out, I assume that a further action takes place: all consumers can chose $a \in \{0,1\}$ to match the state of the world. I think of this as a financial bet, but it can capture a wider range of utility derived from information. This action can depend on the private signal they receive and on the content of the article they read (if any). I assume that this bet has entry price r, and that consumers might decide to opt out of the bet after having observed their signal and the news article (if any). I represent the case in which the consumers cannot opt out of this action with $r = 0$. The benefits from matching the state of the world is assumed to be the same as their loss from a mismatch:

$$u_j(a|\omega = w) = \begin{cases} 1 & \text{if } a_j = w \\ -1 & \text{otherwise} \end{cases}$$

From the consumer perspective, I study three different utilities: the utility derived by seeds from sharing, the utility derived by any consumer from betting and the utility derived by any consumer from choosing to enter the bet. I refer to the utility from sharing as $u_i(z)$ and to the utility from betting as $u_j(a)$, while the utility from entering the bet also depends on the betting price r.

Lemma 4. *The relevant expected utilities are as follows:*

- *Seeds' expected utility from sharing is:*

$$\mathbb{E}(u_i(z)) = \sum_{S,n,k} z_{S|n,k}\Big[q_k \Pr(X, \omega = n) - (1-q_k)\Pr(\neg X, \omega \neq n)\Big]\frac{1}{K}$$

- *Seeds betting $a(n,s) = n$ with probability $z_{S|n,k}$ have expected utility:*

$$\mathbb{E}(u_i(a)) = \sum_{S,n,k} (2z_{S|n,k} - 1)\Big[q_k \Pr(X, \omega = n) - (1-q_k)\Pr(\neg X, \omega \neq n)\Big]\frac{1}{K}$$

Upon reading some news, followers betting n with probability $z^f_{S|n}$ have expected utility:

$$\sum_{m,S,n,k} \left(2z^f_{S|n,k} - 1\right)\left[q_k \frac{p_{T|n,k}}{p_{T|n,k} + p_{Y|m,-k}} \Pr(m, S, \omega = n) - (1-q_k)\frac{p_{F|n,k}}{p_{F|n,k} + p_{\neg Y|m,-k}} \Pr(m, \neg S, \omega \neq n)\right]$$

- *Consumers' expected utility from entering the bet is:* $\mathbb{E}\left(\max\{u_j(a) - r; 0\}\right)$

Proof. See Appendix D. □

Recall that, $S = +$ iff $n = s$ and $S = -$ otherwise. Therefore, $\Pr(+, \omega = n) = \gamma \Pr(\omega = n)$ and $\Pr(-, \omega = n) = (1 - \gamma) \Pr(\omega = n)$.[22]

The seeds' expected utility from sharing news reporting n after private signal s is $2p(n, s) - 1$, as shown in Section 2.3.1. To find their expected utility from sharing, it suffices to incorporate their decision to share or not, $z_{S|n,k}$, and the probability for the news of producer k to report n after private signal s. Note that the utility from sharing of a random consumer is $b\,\mathbb{E}(u_i(z))$.

Because of the similarity in payoff structure, the seeds' betting decision follows the same threshold rule as their sharing decision: they bet that the state of the world is the one reported in the article if the probability for the true state of the world to correspond to the news, $p(n, s)$, is greater than 1/2. Therefore $a(n, s) = n$ is played with probability $z_{S|n}$.[23] By a slight abuse of notation, I refer to this betting strategy as $z_{S|n}$ as well.

Remark 5. For any strategy with $z_{S|n,k} > 0$ for some (S, n, k), the expected utility from sharing is strictly increasing in q_k; the expected utility from betting is not.

Despite the similarity in strategies, the expected utility from sharing and betting are different. This occurs because seeds are constraint not to share if they do not believe the news content, while they can still bet their private signal rather than the news in such a case. For instance, an outlet that would systematically report an erroneous content, i.e. $x_k = 0$, would lead to no sharing and so a null utility from sharing; but it would be perfectly informative: betting the opposite than the article reports would always ensure to correctly match the state of the world, thus granting the maximal possible utility from betting.

Notice that followers can bet according to a different strategy than seeds. All consumers have the same preferences and priors; however, in competitive markets, the precision of articles received by followers is higher than that of the outlets which issue them. Indeed, the network filters out false articles: true articles are shared more, so they reach followers with higher probability. Internalizing this effect, followers require less precision from the outlet in order to start betting what the news reports. The decision rule can be defined implicitly as $z^f_{X|n,k} = 1$ when the expected utility from betting the news content is higher than that from betting the private signal.[24]

The expected utility from betting of a consumer taken at random can be defined by accounting for the probability for the consumer to assume the role of either a seed or a follower; and, upon

[22]Likewise, Y is implicitly determined by m and ω. So for instance, take $n = \omega = 0$, then $m = 0$ would lead to $Y = T$ and $\text{-}Y = F$, while $m = 1$ would mean $Y = F$ and $\text{-}Y = T$.

[23]Formally, $\mathbb{E}(\mathbb{1}_{a=n|n,s}) = z_{S|n}$. When $p(n, s) = 1/2$, the seeds are indifferent between betting the news content or its opposite. The tie rule was chosen in order to keep consistency. Because when indifferent between several strategies, by definition, their utility is equal among all strategies, this assumption does not influence the welfare analysis.

[24]I.e. a follower bets article content n by producer k after receiving a private signal $X = T, F$ with probability:

$$z^f_{X|n,k} = 1 \Leftrightarrow \Sigma_m \left[q_k \Pr(X, m|\omega = n) \frac{p_{T|n,k}}{p_{T|n,k} + p_{Y|m,-k}} \Pr(\omega = n) - (1 - q_k) \Pr(X, \text{-}Y|\omega \neq n) \frac{p_{F|n,k}}{p_{F|n,k} + p_{\text{-}Y|m,-k}} \Pr(\omega \neq n) \right] > 0$$

being a follower, for the endogenous probability to read an article.[25]

To understand whether news drives agent to take a bet from which they would have otherwise opted out, I allow consumers to chose whether to enter the bet after observing (n, s). They opt out from it if its expecting value, as described by $u_j(a)$, is lower than the cost of entry $r > 0$. Again, this depends on the content reported in the news they read (if any) and its congruence with their private signal.

4.2 Welfare for symmetric priors

Throughout this section, I assume $w_0 = 1/2$. Accordingly, I evaluate whether the presence of news outlets has welfare benefits for consumers and I discuss the effect of competition on total welfare.

Theorem 2. *When no state of the world is ex ante more likely, the existence of news outlets has ambiguous effects on consumers' welfare:*

(i) For seeds, the presence of news outlets does not improve their betting decision and is detrimental to their capacity to enter the bet for a non-zero measure set of r. It is beneficial to their expected utility from sharing.

(ii) For followers, the presence of news outlets does not improve the betting decision and is detrimental to their capacity to enter the bet for a non-zero measure set of r if the market is not competitive or if the competitive symmetric investment $q_D^* \leq \frac{\gamma - \sqrt{\gamma(1-\gamma)}}{2\gamma - 1}$

Proof. See Appendix D. □

As seeds cannot derive utility from sharing without any articles to share, the presence of news outlets has a positive effect on seeds' sharing utility. This relates to a concept of entertainment: news consumers might be entertained by ad-funded online news outlets. The other results from Theorem 2 show how consumers might however not be informed by such outlets.

With $w_0 = 1/2$, the news quality in any equilibrium is such that $q^* \leq \gamma$. Seeds are thus either better off betting their private signal, or indifferent between betting their signal or the news content. Therefore, news' outlets are not improving on their choice. In other words, the presence of news outlets does not bring seeds to better decisions.

Followers, however, might benefit from competition. In a competitive market, true news is more visible to followers as the network filters out false articles. This raises the quality of the articles perceived by followers, which might become more precise than the followers' private signal. The quality of news reaching followers is exactly γ for $q_D^* = \frac{\gamma - \sqrt{\gamma(1-\gamma)}}{2\gamma - 1}$.

Note that, although articles reaching are true more often than q^*, the quality of news perceived by followers is still bounded by γ.

[25] Formally, conditional on being a follower, the expected utility from betting is:

$$\sum_{m,X,n,k} \left\{ (2\gamma - 1)(1 - p_{X|n,k} - p_{Y|m,-k})^d Pr(m, n, \omega) + \left(2z^f_{X|n,k} - 1\right) \left[q_k V_{T_k Y_{-k}} \Pr(X, m, \omega = n) - (1 - q_k) V_{F_k Y_{-k}} \Pr(X, m, \omega \neq n) \right] \right\}$$

Lemma 5. *In a competitive market, the utility from betting for followers upon receive some news is bounded by the precision of their private signal. In particular,* $\mathbb{E}(u_f(a)|\textit{seeing some article}) \in \left[2\gamma - 1; \frac{3}{2}(2\gamma - 1)\right]$.

Proof. See Appendix D. □

The upper bound is found by computing $\mathbb{E}(u_f(a))$ for $q_D^* = \gamma$ and $z^* = (1, 0)$. Indeed, the network is the best at filtering out false information for $z_F = 0$.

Theorem 2 also shows how the presence of news outlets can be detrimental to news consumers, even if they are fully Bayesian. Intuitively, for low to moderate entry costs r, consumers would enter the bet without the presence of any news outlets, as their private signal is informative enough to justify the cost r. However, upon reading a news article whose content disagrees with their private signal, consumers are too uncertain about the state of the world to enter the bet. Now, because the news outlets are more noisy than the private signals, consumers are more often wrongly than rightly dissuaded.

Note that for moderate to high entry costs r, the presence of news outlets can be beneficial. In Lemma 6, I further characterize the cases in which news outlets are beneficial or detrimental to consumers' ability to enter the bet.

Lemma 6.

(i) News' outlets are for seeds' capacity to enter the bet: beneficial for $r \in [r_s, \bar{r}]$*, detrimental for* $r \in [\underline{r}, r_s)$*, and neutral otherwise. These effects are strict for* $q^* < \gamma$.

(ii) For followers: the same applies for uncompetitive market; similar thresholds $\underline{r}'$ *and* $\bar{r}'$ *exist if the market is competitive with symmetric investment* $q_D^* < \frac{\gamma - \sqrt{\gamma(1-\gamma)}}{2\gamma - 1}$*; and news outlets are never detrimental otherwise.*

With $\underline{r} = 2\frac{\gamma(1-q)}{\gamma(1-q)+(1-\gamma)q} - 1; r_s = 2\gamma - 1; \bar{r} = 2\frac{\gamma q}{\gamma q+(1-\gamma)(1-q)} - 1$

Proof. See Appendix D. □

The proof compares the behavior of consumers with and without the presence of news for any r. In particular, for $r \in [\underline{r}, \bar{r}]$, seeds only participate to the bet if $n = s$. If, without an article, they would not have participated to the bet, the information transmitted thanks to news outlets is beneficial, as most seeds being prompted to participate are placing the right bet. However, if without an article, they would have participated to the bet, then the information transmitted thanks to news outlets is detrimental. Indeed, in such a case, the article is wrong more often than the private signal, so most seeds who opted out should have placed a bet. Figure 12 illustrates this intuition.

The same reasoning applies to followers. However, in a competitive market, the quality of information perceived by followers can be greater than the precision of the individual news outlets. Therefore, $q^* \leq \gamma$ is not sufficient for the articles read by followers to be more noisy than

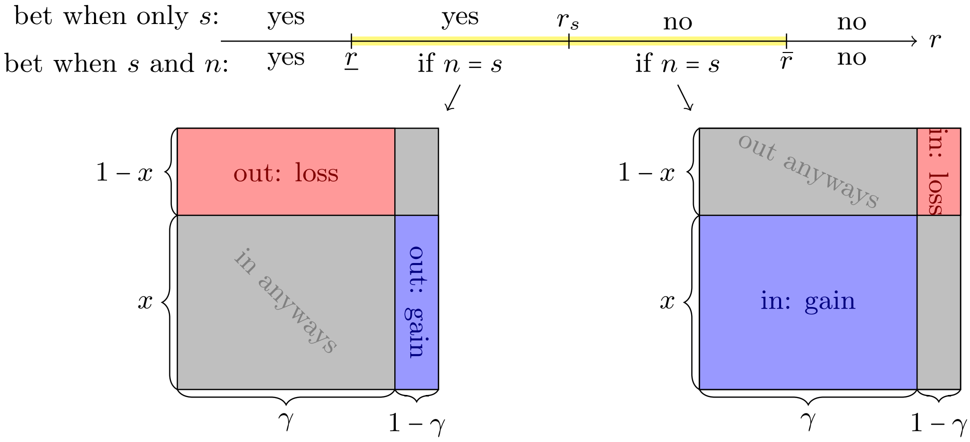

Figure 12: Illustration of Lemma 6's proof

their private signal. Furthermore, the values of r that make followers change their behavior after having received a news article has to account for the quality of the news they perceive.[26]

The welfare consequences of competition can now be assessed more carefully, by comparing consumers' expected gains from betting in a monopoly and a duopoly.

Proposition 5. *Irrespective of the aspect of consumer welfare considered, competition can hinder total welfare even if* $q_D^* > q_M^*$.

Proof. See Appendix D □

When considering the total welfare effect of competition, both sides of the markets have to be considered. Seeds are not made better off and their quantity does not change in expectation. Followers might be better off if q_D^* is close enough to $\bar{t}$; however, the number of followers encountering an article might be affected by competition. As seeds share more types of news, more followers come across possibly informative news, but the least informative becomes the news, as the network fails to filter out wrong articles. Producers split their readership while the total production cost doubles. Therefore, the effect of competition on welfare depends on the level of news quality, the connectivity of the network and the ratio of seeds in the population.

4.3 Welfare for asymmetric priors

When the prior about the state of the world w_0 is different from 1/2, the quality of the news is bounded by private *knowledge*. It means that the article published by news outlets might be more precise than private signals. However, generally, the insights from Section 4.2 are still applicable: the presence of news outlets have ambiguous effects on consumers' welfare.

[26]In particular, $\underline{r}' = 2\frac{\sum_Y \gamma(1-q)V_{FY}}{\gamma(1-q)V_{FY}+(1-\gamma)qV_{T\text{-}Y}} - 1$ and $\bar{r}' = 2\frac{\sum_Y \gamma q V_{TY}}{\gamma q V_{TY}+(1-\gamma)qV_{F\text{-}Y}} - 1$

Corollary 3. *In uncompetitive markets, for any prior on the state of the world $w_0 < \gamma$:*

(i) Consumers are brought to better decisions iff $q^* > \gamma$.

(ii) The gains from betting are still bounded by the precision of the private signal. In particular, $\mathbb{E}(u_j(a)) \in \left[2\gamma - 1; \frac{2\gamma-1}{1-2\gamma(1-\gamma)}\right]$

Proof. (i) follows from Theorem 2; for (ii), see Appendix D. □

This results echoes Theorem 2: the expected gains from the bet are restricted by the consumers' private knowledge. The bound is derived by considering a prior so strong that agents are indifferent between betting their private signal and their prior.

If the bet has an entry cost r, as in the symmetric prior case, the presence of news outlets has ambiguous effects on the capacity for consumers to enter the bet.

Corollary 4. *In uncompetitive markets, for any prior on the state of the world $w_0 < \gamma$, and any equilibrium outcome $q_M^* < \max\left\{\gamma, \frac{{w_0}^2}{{w_0}^2+(1-w_0)^2}\right\}$:*

(i) There exists a non-zero measure interval for r for which news outlets reduce consumers' capacity to decide to enter the bet.

(ii) The outlets effect on consumers' capacity to enter the bet can be non-monotonic in r.

Proof. The analysis is similar to Lemma 6. See Appendix D for the computations. □

5 Fact Checking

In this section, I study the effect of fact checking when applied to articles or to outlets. Applied to articles, I study how flagging false information affects welfare and its differential effect on news quality in non-competitive and competitive markets. Applied to news outlets, I question how much can quality certification improve producers investment.

5.1 Flagging

I wonder how flagging false information helps the provision of information on the market. In particular, let us assume that with some probability ρ, an information that does not correspond to the state of the world would be flagged by the platform on which seeds share before they decide whether to share. Because they care about truth only, such flagged information will never be shared. Hence, we can see flagging as perfectly informative signals, substituting the need for private signal. Therefore, one would expect this intervention to improve the outcome by decreasing the value of false information.

Remark 6. The presence of flagging removes the bound placed on news quality from the precision of private information.

Another interesting feature of flagging is that the marginal benefit of increasing the probability of being flagged depends on the market structure; in particular, when there is competition, there are strategic consideration to take into accounts. On one hand, an increased ρ makes false information relatively less valuable than true information; on the other hand, an increased ρ might make one's competitor more prone to being flagged, which in term decreases one's incentive to invest, as false information might be enough to survived faced to a flagged competitor.

To see this, let us rewrite the producers' best responses in a monopoly and duopoly when facing a probability ρ that false information is flagged. For the monopolist it is proportional to:

$$V_T - (1-\rho)V_F$$

For duopolists that behave symmetrically, it is proportional to:

$$qV_{TT} + (1-q)\big[(1-\rho)V_{TF} + \rho V_{T\varnothing}\big] - q(1-\rho)V_{FT} - (1-q)(1-\rho)\big[(1-\rho)V_{FF} + \rho V_{F\varnothing}\big]$$

with $V_{X\varnothing}$ denoting the value of publishing a $X = T, F$ article when the competitor has been flagged, that is, $V_{X\varnothing} = 1 - (1 - \frac{p_X}{2})^d$.

To analyze the tradeoff described above, I study how $\Delta V_M(z, q; \rho) - \Delta V_D(z, q; \rho)$ evolves with ρ. I find that flagging is more efficient in a monopoly.

Proposition 6.

(i) Flagging has a stronger effect in monopolies than in duopolies; i.e. $\frac{\partial(\Delta V_M(z;\rho) - \Delta V_D(z,q;\rho))}{\partial \rho} > 0$

(ii) For any environment, there exists a level of flagging that makes competition detrimental; i.e. $\forall(\gamma, b, d), \exists \rho' : \Delta V_M(z; \rho') \geq \Delta V_D(z, q_D^*; \rho')$, *where the inequality is strict for any positive probability of sharing,* $z_T > 0$.

Proof. See Appendix D. □

Proposition 6 underlines how intervening is more difficult in competitive news markets. First, the same intervention has a stronger effect on a monopolist than on duopolists. Intuitively, competition dilutes the effect of flagging because of the strategic interaction between producers' investment. If flagging occurs more often, the value for any producer of publishing false information decreases; however, for a duopolist, it is more likely that the competitor has been flagged, and thus not to have to compete in the network in order to reach followers.

To make this intuition more tangible, consider the two forces discussed to put into perspective Theorem 1. Recall that the effect of competition on news quality depends on the connectivity of the network because of the trade-off between two forces: on the one hand, followers are harder to reach; on the other hand, the potential readership is reduced. Now, flagging false articles makes followers harder to reach anyways, with or without competition. Therefore, the benefits of competition are less and less relevant as flagging increases; the negative effect of competition however remains, since the maximum number of readers that can be reached is independent of the flagging probability.

The second result from Proposition 6 shows that competition is always detrimental to the incentive to invest if flagging occurs often enough. Therefore, one can think of this intervention as a substitute for encouraging a change in the market structure towards more competitive markets. In fact, any market outcome from competition is reproducible though flagging.

Corollary 5. *Any outcome $q_D^* > q_M^*$ is reproducible in a monopoly with some level of flagging ρ'; i.e. $\exists \rho' : \Delta V_M(z;\rho') = \Delta V_D(z, q_D^*; 0)$.*

Proof. Follows from Proposition 6; see Appendix D for details. □

Proposition 6 and Corollary 5 both use the following element: if all false articles are flagged, $\rho = 1$, monopoly yields higher incentive to invest than duopoly. This echoes Remark 4 as both $\rho = 1$ and $\gamma = 1$ make false information worthless. As explained above, competition can be positive in that it worsens the value of false information; but if false information is useless anyways, only the reduction of the readership remains. More generally, flagging can be seen as a substitute for consumers' private signal. Interestingly, flagging forces outlets to provide news that goes beyond consumers private knowledge, and thus to create informative content.

Remark 7. When false articles are flagged, news quality is not bounded by private knowledge anymore.

While these conclusions rely on a setup that ignores any type of partisanship and distrust of the flagging institutions, they still underline the importance of flagging to counteract the weak incentives created by the business model of ad-funded online news outlets.

5.2 Quality Certification

I now wonder how welfare could be improved upon if the consumers were observing the actual quality of information. In terms of policy, this could for instance correspond to the role of a third party institution in charge of certifying the average quality of a news source, or an average *fact checking* score to be displayed on the online outlet.[27]

To understand the implication of such a policy, we need to assume a sequential move game. Appendix C presents the SPE of the monopoly when $w_0 = 1/2$. The outcome depends on the shape of the total cost function $C(q)$.

However, in a sequential move game, the seeds' best-responses would not change. Therefore, the threshold on news quality for which they would share any type of information, regardless of their private signal, does not change. This threshold is also the maximum achievable quality in a sequential game, which is set by the consumers' private knowledge.

Remark 8. Even when observable, news quality is bounded by private knowledge: $q^* < \bar{t}_1$. Therefore, the presence of news outlets still has ambiguous effects on consumers welfare.

[27]Such initiatives already exist, such as The Trust Project or Media Bias/Fact Check.

As emphasized in Remark 8, most results from Section 4 still apply when the quality of news outlets is observable. In particular, Theorem 2, Lemma 5 and 6, and Corollary 3 and 4 all apply.

Interestingly, both flagging and quality certification rely on the same type of policy: fact checking; yet, they have very different implications. This indicates that a major barrier to high quality online news is consumers' limited private knowledge; improving consumers' trust in news outlets is not sufficient to correct for the inefficiency generated by a business model in which revenues are generated from visibility.

6 Discussion

Most of the results exposed in this paper rely on the two following insights: the producers' incentive to invest is determined by the difference between the value of true and false articles and the consumers' private knowledge bounds news quality. These insights are robust to many extensions of the model.

6.1 Different setup

The setup studied so far analyzes a simplistic market in which all consumers are identical. All have the same number of neighbors, the same probability of being a seed, the same amount of private knowledge. These assumptions make my analysis more transparent; below, I show that it does not drive any of my results.

6.1.1 Irregular Networks & Seeds' Selection

The setup assumes a regular network, in which every node has exactly d neighbors and has the same probability b to be a seeds. Relaxing these assumptions would be inconsequential to the results as long as seeds' are not targeted, that is, as long as there is no strategic component to the seeds' identity. Denote $V_X(d_j)$ the probability that a node j with degree d_j is reached by the producer producing a $X = T, F$ article. This would be defined as before. Given any network with degree distribution $\delta(d_j)$, the producer's incentive to invest is simply $\sum_{d_j} \delta(d_j) \left[V_T(d_j) - V_F(d_j)\right]$. This allows for the probability of being a seed to be degree-specific; denoting this probability $b(d_j)$, and taking a monopolist best-response for exposition ease, $V_X(d_j) = \frac{b(d_j)}{K} + \left(1 - b(d_j)\right)\left(1 - (1 - p_X)^{d_j}\right)$. All results would follow through. Details are provided in Appendix D.

6.1.2 Heterogenous signal precision

Consider that consumers receive private signals according to different signal precision γ_j. Most results would directly apply. The seeds' problem would not significantly change: it would be precision-specific – possibly seed-specific. For producers, the probability to be shared after publishing a $X = T, F$ article would incorporate the heterogeneity of signal precision; denoting

$\psi(\gamma_i)$ the proportion of seeds with signal precision γ_i, the probability to be shared becomes: $p_X = \frac{b}{K}\sum_i \psi(\gamma_i)\big(\gamma_i z_S + (1-\gamma_i) z_{\text{-}S}\big)$ where $S = +$ for $Y = T$. The analysis would then be directly applicable.

Note that the news quality in such a context would be bounded by the highest signal precision. Therefore, nodes whose signal precision is noisier would benefit from the presence of news outlets; results from Section 4 would then be mitigated to account for the distribution ψ. In particular, a low proportion of nodes with maximal signal precision would make the presence of news outlets more beneficial if the bound is reached; however, the bound would be less likely to be reached as the producers' could rely on the large proportion of nodes with noisier private signals to be shared.

There would not be such a trade-off if signal precision was to differ between seeds and followers. In particular, if the seeds' private information was more precise than that of followers, the presence of news outlets would more often be beneficial to followers. In the opposite case, news outlets would likely be uninformative to both seeds and followers. In any case, the benefits from news outlets are still bounded by agents private knowledge.

6.2 Different Objectives

The model I analyze considers agents whose objectives are straightforward and, potentially, simplistic. While such assumptions offer tractability and clarity, one might wonder to which extend the results are robust to further considerations. Below, I explore how the main mechanisms at play would carry through in richer context.

6.2.1 Seeds' Problem

Strategic Considerations

In the appendix, I propose two extensions for the seeds' objectives. Appendix A studies the market outcomes for more general payoff structure; while more equilibria might exist, most insights carry through. In particular, I assume that sharing false news entails a different loss than the benefits of sharing true news. When the loss from sharing a false article is greater than the benefits from sharing true information, seeds are more demanding in terms of news quality in order to share; therefore, the producers has to be more precise than private knowledge. Consumers are more then brought to better decision more easily; news quality is still bounded, and the bound is still a function of private knowledge. Furthermore, all results pertaining to producers' incentive to invest directly follow through since producers' best-response is unaffected by the seeds' problem.

Appendix B characterizes the best-response of seeds seeking *likes* rather than truth. The problem is more complex and less tractable; however, seeds' best-response have a similar shape: it is weakly monotonic in news' quality; true information is still shared more often than false articles. In particular, I assume that seeds share if they expected the number of *likes* from their post to be higher than an exogenous threshold τ; followers are behavioral, in that they like the article they see if its content correspond to their private signal. Then, true articles bring about more likes than false articles, so that seeds might want to share news only if it correspond to their private signal. This depends on how many likes they require to share, and how likely it is that their neighbor will see their post over that of another seed. However, for reasonable τ, there still exists a level of news quality that would induce them to always share, which depends on the precision of their private knowledge. Therefore, private knowledge still bounds news quality. Again, all results about producers' incentive to invest directly apply.

Behavioral Biases & Partisanship

Consumers' behavioral biases are expected to worsen the market outcome. Consider for instance confirmation bias as modeled in Rabin and Schrag [1999]: with some probability consumers misinterpret the news content if it disagrees with their private signal; they would then share it as if it was congruent with their private signal. Confirmation bias would not affect the seeds' best-response but would lead both true and false information to be shared more often. The value of false information would then increase faster than that of true information: more agents receive contradicting information when an article is false, increasing the probability for false articles to be shared faster. Eventually, for any environment and sharing pattern, confirmation bias would lower the producers' incentive to invest.

Another cognitive bias that could worsen the outcome is a taste for sensationalism. If consumers' payoff is affected by both the veracity and the sensationalism of the news, seeds are expected to be less demanding in terms of news quality to be willing to share news that is not congruent with their private signal. Sharing a false article in such a case would indeed be perceived less damageable because of their taste for sensationalism. This would reduce the value of the upper bound placed on news quality, while leaving the producers' incentive to invest unchanged. Appendix D proposes a payoff structure to model such a mechanism.

Finally, partisan consumers' could be modeled as seeds sharing a given news content regardless of the realization of their private signal. The expected effects of this type of partisanship would be similar to the effects of confirmation bias. Instead of any node having a probability to misinterpret the news and share when they should not have, there is a probability for any node to be partisan and sharing when others would not have. From the producers' perspective, the value of false information increases more than the value of true information and investment is less attractive.

6.2.2 Producers' Problem

Beyond Visibility

How would the producers' problem be affected if news quality mattered beyond visibility? Producers might indeed get further benefits from being a reputable source of information. Such a setup would not affect the effect that the environment and competitiveness of the market would have on producers' incentives. The bound placed on news quality could be removed, although this would not necessarily occur.

In Appendix D, I propose two frameworks to consider these effects. The underlying intuition is as follows. Assume that these benefits from reputation continuously depend on the level of news quality. The producers' incentive to invest would be shifted upwards; News quality would thus increase in equilibrium. Whether the bound is removed depends on the marginal value of reputation benefits and the marginal cost function: if the marginal value of reputation is lower than the marginal cost of the news quality at the bound, then all results follow through. Otherwise, producers invest until the marginal cost of news quality equated marginal benefits from reputation. Assume that these benefits from reputation discretely depend on the level of news quality: either they occur or they do not. As the producers' reputation benefits would not be affected by an increase in news' quality at the margin, producers' incentive to invest remain unaffected. Whether news quality increases in equilibrium depends on the level of quality required to benefit from reputation, as well as the total profits.

Subscription-Based Revenues

Would a business model that allows the producers to internalize the value of information of consumers decrease inefficiencies? While the game studied in this paper is not set to explore this question in details, it can still underline an interesting tradeoff. Let producers derive their revenues from subscription rather than advertisement. One could see the betting gains from reading news as opposed to relying on private signals as consumers' willingness to pay for information, i.e. willingness to pay for subscription. With subscription, the marginal cost of investment for the producer equates the marginal value of information for the consumer, in equilibrium; therefore, there are no Pareto inefficiencies on this front. However, this comes at the cost of losing advertisement revenues. Assuming that such advertisement creates a surplus for the society, it is not clear which business model should be preferred.

7 Conclusion

In this paper, I evaluate the performance of ad-funded online news outlets. I find that, without any intervention, they tend to be highly inefficient. First, news quality is bounded by the amount of private knowledge existing on the topic. The market does not compensate for a lack of private knowledge. High news quality is thus achievable only when the topic documented is already well-known: either because the outcome about this topic is rather certain; or because consumers are privately informed about it. The incentive created by sharing behaviors are a first cause for this result. Producers only care about being shared; as seeds rely on their knowledge to judge whether a content is worth sharing, having them share is not demanding when they are ill-informed. The second cause is the higher value of investment when the more likely state of the world realizes. Indeed, seeds are then more ready to share news documenting an expected state of the world. Thus, uncertain topics generate a lesser incentive to invest than topics for which information is less needed.

I additionally show that competition does not necessarily lead to better news quality. By comparing the outcomes of a monopoly and a duopoly, I conclude that monopoly is preferable in sparser networks populated by well-informed agents. This result puts into light two important forces appearing with competition. On the one hand, followers are harder to reach. This reduces the value of false information, as false articles would barely survive in the network when competing with true news. On the other hand, fewer followers can be reached. This reduces the value of true information, as an article shared by all seeds reading it would still reach few followers. When the network is sparse, the latter force dominates, making competition detrimental to news quality. This shows the limits of competition as a mean towards efficiency.

Furthermore, any online news market based on advertisement revenue is Pareto inefficient. I provide a framework to study welfare and find that online news outlets create value from entertainment but are not necessarily informative. In particular, the existence of online news rarely bring news consumers to take better decision; even when it does, their gain from it are still bounded by the precision of their private information. Furthermore, the presence of online news can be detrimental to Bayesian consumers, as it might discourage them from taking a costly and risky action that would have actually been beneficial to them. A range of entry cost that makes online news detrimental generally exist.

Finally, I discuss how fact checking could improve news informativeness. Flagging false articles reduces their value, thus incentivizing producers to publish true articles more often. Because flagging substitutes private information, news quality is not bounded by private knowledge anymore. However, flagging is less efficient in competitive markets; actually, if false articles are flagged sufficiently often, competition is detrimental to news quality in any market environment. Therefore, one can substitute the positive effects of competition with flagging. To the contrary, allowing consumers to observe the quality of news outlets, for instance through a certification from an external institution, would not remove the bound placed on news quality by private knowledge.

This analysis is attractive because it gives consumers an endogenous control over information flow but not over news content. Furthermore, distortions that are inherent to a social network should be essential in underlining the differences between social media and other historical instances of ad-based business models for news. The central role of competition in this paper is reflected by its predominance in online outlets, as well as online networks. My analysis is robust to many extensions and puts into perspective the limits of the business model of ad-funded online news outlets; under such business models, the information provided online cannot be reliable, even when all news consumers are rational and unbiased.

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Appendix

A Asymmetric Loss From Sharing

A.1 Best Response

In this section of the appendix, I characterize the results derived in Section 3 for more general payoffs from sharing. In particular, while I restrict the benefit from sharing true news to 1, I consider a loss λ when false news is shared. The seeds' payoff thus becomes:

$$u(\text{sharing article with content } n|\omega = w) = \begin{cases} 1 & \text{if } n = w \\ -\lambda & \text{otherwise} \end{cases}$$

This changes the seeds' best-response. In particular, it modifies the thresholds according to which they start sharing different news content after their private signal. We can redefine:

$$\underline{t}_0^\lambda = \frac{\lambda(1-\gamma)(1-w_0)}{\lambda(1-\gamma)(1-w_0)+\gamma w_0} \qquad \bar{t}_0^\lambda = \frac{\lambda\gamma(1-w_0)}{\lambda\gamma(1-w_0)+(1-\gamma)w_0}$$
$$\underline{t}_1^\lambda = \frac{\lambda(1-\gamma)w_0}{\lambda(1-\gamma)w_0+\gamma(1-w_0)} \qquad \bar{t}_1^\lambda = \frac{\lambda\gamma w_0}{\lambda\gamma w_0+(1-\gamma)(1-w_0)}$$

The producers' best response does not change.

A.2 Equilibrium without Competition

The Nash equilibria might change. In particular, because the thresholds can now be all above or all below the no-investment quality 1/2, the best-responses of seeds and producers might cross in many ways. We define $\bar{q_0} := q^*(z^-_{+|0}, 0, 0, 0)$; $\bar{q_F} = q^*(1, z^-_{+|1}, 0, 0)\}$ and $\tilde{q_0} := q^*(1, 0, 0, 0)$; $\tilde{q_1} := q^*(1, 1, 0, 0)$.

Proposition 1.A. *If either* $1/2 \geq \bar{t}_1^\lambda$*; or both* $\bar{q_0} < \underline{t}_0^\lambda$ *and* $\bar{q_1} < \underline{t}_1^\lambda$*, then there is a unique equilibrium with zero investment and* $q_M^* = 1/2$*. Otherwise, an equilibrium with positive investment exists, which is determined as follows:*

- $q_M^* = \max\{\tilde{q_0}, \underline{t}_0^\lambda\}$ *if* $\bar{q_1} < \underline{t}_1^\lambda$*,*
- $q_M^* = \max\{\tilde{q_1}, \underline{t}_1^\lambda\}\}$ *if* $\underline{t}_1^\lambda \leq \bar{x_1}$ *and* $\tilde{x_1} \leq \bar{t}_0^\lambda$
- $q_M^* = \max\{\bar{t}_0^\lambda, \min\{q^*(1,1,1,0), \bar{t}_1^\lambda\}\}$ *otherwise.*

In essence, the proof considers all possible crossing given the shape of the respective best-responses. Intuitively, the seeds' best-response is not assumed to be above the producer's best-response in $z = (0, 0, 0, 0)$ anymore, which allows for more possible crossings.

Remark 1.A. *In equilibrium, $q^*_M \leq \bar{t}^\lambda_1$. Therefore, news quality is still bounded by agent's private knowledge w_0 and γ.*

The consequences on the comparative statistics are overall the same. In particular, all results pertaining to the effect of a parameter on the producer's incentive to invest can directly be applied as the producer's best-response is identical in this extension. Note the following change:

Corollary 2.A. *Take any increase in w_0.*

- *For a marginal increase, the inequalities detailed in Proposition 1. A do not change, so that the maximal equilibrium investment q^*_M increases iff $q^*_M \neq \underline{t}_0$ and $q^*_M \neq \bar{t}_0$*
- *For bigger increases, the maximal equilibrium investment q^*_M increases iff $q^*_M \neq \underline{t}_0$, $q^*_M \neq \bar{t}_0$ and c^{-1} is steep enough, i.e. c^{-1} is such that, for any $w'_0 > w_0$, $q^* > \underline{t}_1$ implies $q^{*\prime} > \underline{t}'_1$.*

Proof. See Appendix D □

A.3 Equilibrium with Competition

As in the monopoly, new equilibria might appear when seeds' best-response depends on λ. Let $\bar{q}_k := \max_{z_{T_k}} q^*_k((z_{T_k}, 0); 0)$ and $\tilde{q}_m = q^*((1,0),(0,0);0)$

Remark (Additional). *There might exist other Nash Equilibria.*

(i) For $\underline{t}^\lambda > 1/2$ always exist a set of equilibria with zero investment $x^*_k \in [0, 1/2] \forall k \in K$.

(ii) If $1/2 < \bar{t}^\lambda$ and $\min_k \bar{q}_k \geq (\underline{t}^\lambda)$, there exists a set of equilibria in which exactly one producer invests $q_m = \max\{\underline{t}^\lambda), \min\{\tilde{q}_m, \bar{t}^\lambda\}$, and the other does not invest.

Define $\bar{z}_D := \arg\max_{z_T} \left\{\Delta V\big((z_T, 0), \underline{t}^\lambda\big)\right\}$ and $\bar{q}_D = \Delta V(\bar{z}^D_T, \underline{t}^\lambda)$.

Proposition 3.A. *If $1/2 < \bar{t}^\lambda$ and $\underline{t}^\lambda \leq \bar{q}_D$, there exists a symmetric equilibrium that features positive investment and $q^*_D = \arg\min_{q \in [\underline{t}^\lambda, \bar{t}^\lambda]} \left|\Delta V_D\big((1,0); q\big) - c(q)\right|$.*

Furthermore, we can distinguish equilibria with respect to their stability.

Corollary (Additional). *q^*_D is the only equilibrium with symmetric positive investment that is stable for the interaction between seeds and producers .*

Finally, I wonder about other asymmetric equilibria and find:

Remark 2.A. *(i) If $\bar{q}_m < \underline{t}^\lambda$, the unique equilibrium is that featuring no investment.*
(ii) If $\bar{q}_D < \underline{t}^\lambda \leq \bar{q}_m$, the only equilibria with positive investment have one producer investing q_m while the other does not invest.
(iii) If $\bar{q}_D \geq \underline{t}^\lambda = q_m$, the only equilibria with positive investment for both producers feature $q^*_D = \underline{t}^\lambda$.
(iv) If the cost function is linear, there are no equilibrium with $q_k \neq q_\ell$ and $(x_k, x_\ell) \in (\underline{t}^\lambda, \bar{t}^\lambda)$ as long as $c(q)$'s slope is different from S.

Because the following comparison between monopoly and duopoly focuses on the producers' best-responses, all results follow through.

B Attention-Seeking Seeds

In this appendix, I explore an extension of the model with $w_0 = 1/2$ and symmetric behavior for all seeds' $z_k = z_\ell = z$. I assume that seeds do not intrinsically care whether the news they share is true or false; but they do care about receiving good feedback about it, e.g. a lot of *likes*. I characterize the best response of attention-seeking seeds to news quality q.

B.1 The Attention-Seeker Problem

I assume that seeds, contrary to producers, cannot observe the actual number of followers they reach; however, they can observe how many followers reacted to their shared post, as, typically, social media feature some sort of feedbacks, be it comments, likes, or re-shares. I focus on positive reactions, that I call *likes*, and assume that followers like a post if they receive a private signal consistent with it. In the context exposed previously, it means that followers receive a binary signal that can agree or disagree with the news, and like only if their private signal is congruent with the news – regardless of the prior probability for news to be true.

As before, seeds simultaneously choose whether to share the piece of news issued by ℓ, given their private signal s. Seeds decide to share if the amount of likes they expect to collect with their post exceeds a threshold $\tau \leq d$. It can be interpreted as the value of an outside option – e.g. posting another type of article would yield τ likes – or, simply, the cost of sharing.

For consistency, I still denote R_{fi} the random variable which is one if f sees the post from i. As before, a follower sees only one post. If more than one neighbor shared a post, the follower sees the post from one random sharing neighbor, with uniform probability, that is:

$$Pr(R_{fi} = 1 | \; s \text{ neighbors of } f \text{ shared}) = \frac{1}{s}$$

where s is the outcome of the random variable S counting the number of f's neighbors who shared.

Define the random variable L_{fi} which is one if f likes the post shared by i. Recall that s is the random private signal that a follower receives. Then:

$$Pr(L_{fi} = 1) = Pr(L_{fi} = 1 | R_{fi} = 1) Pr(R_{fi} = 1) = Pr(S = +) Pr(R_{fi} = 1)$$

An seed expects a different amount of likes for true and false information because, if read, true news gets more likes than false information. The expected number of likes also depends on the visibility of the news, which in turn depends on the sharing decisions of all neighbors of each followers. Define n as the random variable counting the number of shares from f's neighbors, excluding i. The expected number of likes i gets from sharing a piece of information which is

$X \in \{T, F\}$ is thus:

$$\mathbb{E}\left(\sum_{f\in\mathcal{N}_i} L_{fi} = 1 \Big| X\right) = dPr(f \text{ is a follower})Pr(S = +|X)\mathbb{E}\left(\frac{1}{P+1}\Big|X\right)$$

Now recall, upon reading a piece of news, seed i, too, gets a private signal about the truthfulness of the news, whose precision is γ. As before, all seeds have a common prior x_k about the probability for producer k to release true information. Let $p(n, s)$ denote i's posterior upon receiving signal s and reading news n. Then, a seed decides to share a piece of information if and only if:

$$p(n,s)d(1-b)\gamma\mathbb{E}\left(\frac{1}{P+1}\Big|T\right) + (1-p(n,s))d(1-b)(1-\gamma)\mathbb{E}\left(\frac{1}{P+1}\Big|F\right) \geq \tau$$

Notice that the seeds' utility now depends on more than the producers' investment; it also depends on the behavior of other seeds. In particular, because seeds compete for likes, which occur only upon being seen, they would prefer a situation in which they are the only sharer. If true information is shared more, then this coordination concern would make them less prone to share true news; however, true information also brings more likes. Thus, there is a trade-off between visibility and veracity.

B.2 Seeds' Best Response

In this section, I focus on symmetric strategies $z_i = z \ \forall i$ and, by a slight misuse of language, I call best-response the pair of functions $(z^*_+(q), z^*_-(q))$ which maps q into $[0,1]$ such that $z^*(q, \mathbf{z}^*(\mathbf{q})) = z^*(q)$.[28] Hence, given any investment q, I look at the subset of strategies which can be consistent with a symmetric equilibrium on the seeds' side.

As usual, p_X denotes the probability that a $X = T, F$ news gets shared. Then, $n \sim \mathcal{B}(p_X, d-1)$ We can rewrite:

$$\mathbb{E}\left(\sum_{f\in\mathcal{N}_i} L_{fi} = 1|X\right) = d(1-b)Pr(S = +|X)\frac{1}{dp_X}\left(1-(1-p_X)^d\right)$$

Thus, the expected number of like is:

$$p(n,s)\gamma\frac{1-b}{p_T}\left(1-(1-p_T)^d\right) + (1-p(n,s))(1-\gamma)\frac{1-b}{p_F}\left(1-(1-p_F)^d\right)$$

Lemma 7. *For any q, $z^*_+(q) \geq z^*_-(q)$.*

Corollary 6. $\mathbb{E}($ *# likes* $|X)$ *is increasing in* $p(n,s)$*, for any* $S \in \{+,-\}, X \in \{T, F\}$.

Proof. It is enough to notice that, since $p_T > p_F$: $\frac{p_T}{(1-(1-p_T)^d)}\frac{(1-(1-p_F)^d)}{p_F} > 1$ so that the coefficient

[28] Technically, each seed's best response would be a pair of $(I+1)$-dimensional function, that each maps q and $\mathbf{z_{-i}}$ into $[0,1]$, with I the random variable counting the number of seeds, and whose expectation is bI.

of $p(n,s)$ is positive. □

I can now characterize the symmetric best-response of attention-seeking seeds

Proposition 7.

(i) *For any* $\tau \leq \gamma\delta$, $z_+^*(q;\tau) = z_-^*(q;\tau) = 1$ *if and only if* $q \geq \hat{q}(\tau)$.

(ii) *For any* $\tau \geq (1-\gamma)d(1-b)$, $z_+^*(q;\tau) = z_-^*(q;\tau) = 0$ *if and only if* $q \leq \underset{\smile}{q}(\tau)$.

(iii) *For any* $\tau \in [\tau_1, \tau_2]$, $z_+^*(q;\tau) = 1, z_-^*(q;\tau) = 0$ *if only if* $q \in [q_1(\tau), q_2(\tau)]$.

Where:

$$\delta(b) = \tfrac{1-b}{b}[1-(1-b)^d], \quad \tau_1(b) = \tfrac{1-b}{b}[1-(1-b(1-\gamma))^d], \quad \tau_2(b) = \tfrac{1-b}{b}[1-(1-b\gamma)^d]$$

And, given $Q = \frac{\frac{b\tau}{1-b} - 1 + (1-b(1-\gamma))^d}{(1-b(1-\gamma))^d - (1-b\gamma)^d}$,

$$\hat{q}(\tau) = \tfrac{\gamma}{2\gamma-1}\tfrac{\tau-(1-\gamma)\delta}{\tau}, \quad \underset{\smile}{q}(\tau) = \tfrac{1-\gamma}{2\gamma-1}\tfrac{\tau-(1-\gamma)d(1-b)}{d(1-b)-\tau}, \quad q_1(\tau) = \tfrac{(1-\gamma)Q}{(1-\gamma)Q+\gamma(1-Q)}, \quad q_2(\tau) = \tfrac{\gamma Q}{\gamma Q+(1-\gamma)(1-Q)}$$

Corollary 7.

(i) *For any* $\tau \leq \gamma\delta$, *if* $q \geq \hat{q}(\tau)$, $z_+(q, \mathbf{z_{-i}}; \tau) = z_-(q, \mathbf{z_{-i}}; \tau) = 1$ *is the only best response for any (non symmetric) vector of seeds* $-i \neq i$*'s actions.*

(ii) *For any* $\tau \geq (1-\gamma)d$, *if* $q \leq \underset{\smile}{q}(\tau)$, $z_+(q, \mathbf{z_{-i}}; \tau) = z_-(q, \mathbf{z_{-i}}; \tau) = 0$ *is the only best response for any (non symmetric) vector of seeds* $-i \neq i$*'s actions.*

Proof. Again, it is enough to recall that the number of likes is decreasing in the probability for another seed to share □

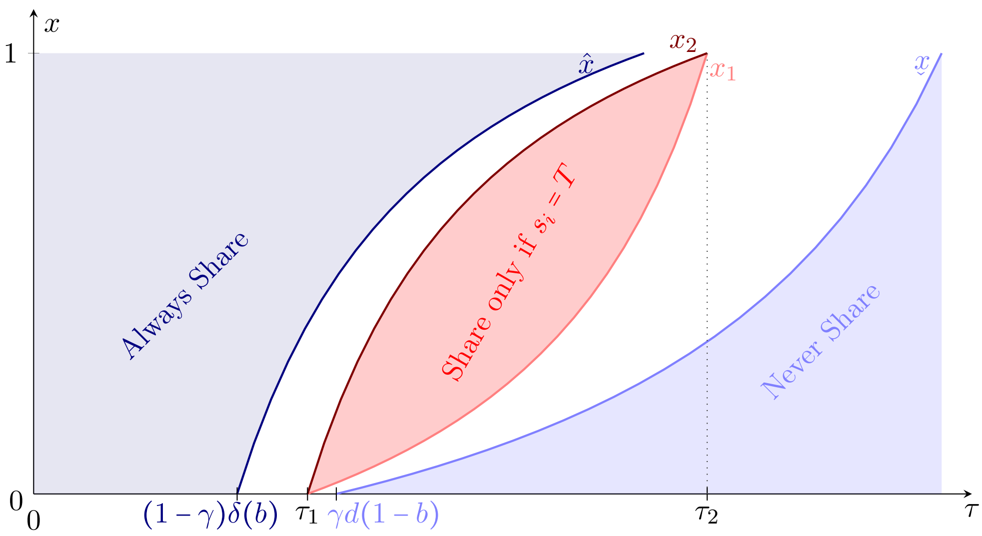

Figure 13: Illustration of $z_{ps}^*(q;\tau)$ with $b = 0.2, \gamma = 0.75, d = 5$

Corollary 8. *Define* z_{ps} *as the restriction of* z *to pure strategies. For any* (q,τ), $z_{ps}^*(q;\tau)$ *either does not exist or is unique.*

Proof. Consider the parameter space (τ, q). Theorem 7 describes three subsets of best-responses that do not intersect. No other pure strategies is sustainable, as, by proposition **??**, $(0,1)$ is never a best-response. $\square$

Figure 13 illustrates the different region of pure strategy best-responses in space (τ, q). First, one can notice that for some values of τ, the investment of the producer has no effect on the sharing decision of seeds. If τ is *too* low, seeds are not very demanding in terms of likes, so that they are always willing to share. If τ is *too* high, seeds are too demanding in terms of likes, and they never share any information.

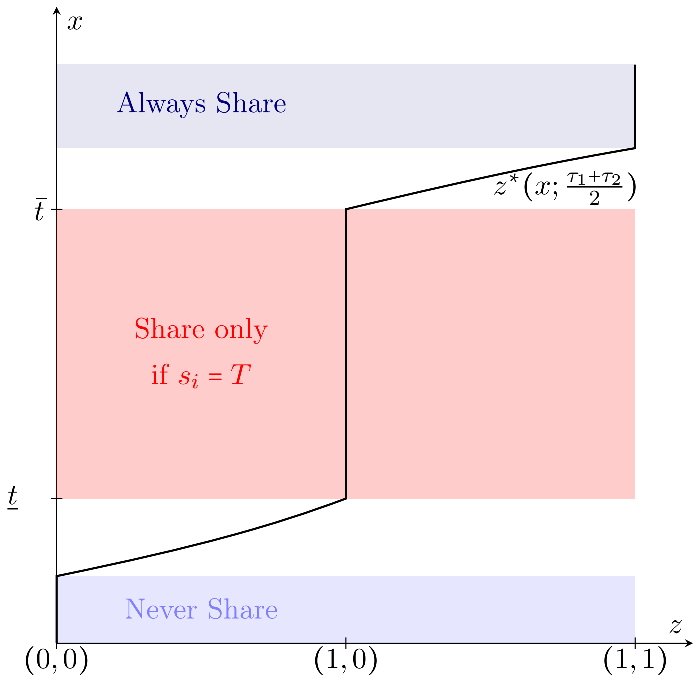

Figure 14: Illustration of $z^*(q;\tau)$ in $\tau = \frac{\tau_1+\tau_2}{2}$ with $b = 0.2, \gamma = 0.75, d = 5$

For intermediate values of τ, however, the symmetric best-response of seeds is fairly similar to that studied in the benchmark model. To understand so, let us fix a particular value for τ; we want to understand z^* as a function of q. This means fixing one value of τ on Figure 13 and translating the different areas in term of z. This results in Figure 14, which illustrates the symmetric best-response $z^*(q)$ for $\tau = \frac{\tau_1+\tau_2}{2}$. Notice that, for this particular τ, $q_1 = 1 - \gamma$ and $q_2 = \gamma$. It means that, for q between $1-\gamma$ and γ, the symmetric best-response of attention seeking seeds exactly corresponds to that of naive seeds in the benchmark model.

However, for $q \notin [1-\gamma, \gamma]$, attention-seekers' best response changes. Say the producer invests exactly $1 - \gamma$. In the benchmark model, upon receiving a positive private signal, seeds were indifferent between sharing or not, as the probability the news was true in such a case was exactly one half. But now, attention seekers' strategies are substitutes; therefore, upon receiving a positive private signal, they can be indifferent between sharing or not only for one particular sharing strategies of the other seeds. This latter strategy is the unique only symmetric best-response to q. For $\underset{\smile}{q}\left(\frac{\tau_1+\tau_2}{2}\right) < q < 1-\gamma$, z_+^* is strictly increasing in q;[29] for $\hat{q}\left(\frac{\tau_1+\tau_2}{2}\right) > q > \gamma$, z_-^* is strictly increasing in q [30]

The best-response of attention-seeking seed is thus fairly similar to that of naive seeds for the right value of τ. The problem of seeds as studied in the main text can thus be thought of as a simplification of more complex preferences.

[29] z_+^* is implicitly determined by:

$$\frac{\gamma q}{(1-\gamma)(1-q)} = -\frac{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b(1-\gamma)z_+)^d}{z_+}}{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b\gamma z_+)^d}{z_+}}$$

[30] z_-^* is implicitly determined by:

$$\frac{(1-\gamma)q}{\gamma(1-q)} = -\frac{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b(1-\gamma)-b\gamma z_-)^d}{1+\frac{\gamma}{1-\gamma}z_-}}{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b\gamma-b(1-\gamma)z_-)^d}{1+\frac{1-\gamma}{\gamma}z_-}}$$

C Equilibria with Sequential Moves

In this appendix, I solve the model presented in Section 2 with $w_0 = 1/2$, $K = 1$ and $z_{+|0} = z_{+|1} := z_T$, $z_{-|0} = z_{-|1} := z_F$, as a sequential game. In particular, I assume the following timing:

t=1 Producers k simultaneously choose their precision level $Pr(\text{news } k \text{ is T}) = x_k$.

* Network is formed. One piece of news per producer is issued. Consumers receive a private signal.

t=2 Seeds i simultaneously choose whether to share the article they read.

Note that, as the seeds play last, their problem does not change. q is now the actual investment and not their prior about it; and the best-response is now their contingent strategy. Nothing else changes. Thus, I will only analyze the choice of the producer in the first period.

Now, the producer is internalizing his effect on seeds' action. Because their strategy is not smooth, the producer's consider different cases. Recall that the producer wants to maximize:

$$q\Delta V(z(q)) + V_F(z(q)) - C(q)$$

Because the existence of some SPE might rely on the particular tie rule chosen when the seeds' are indifferent between sharing or not, I always take the tie rule the most advantageous to investment.

Any other level of news quality than $1/2, x'_M, \gamma$ where $x'_M = c^{-1}(\Delta V(1,0))$ is suboptimal. Now for $x'_M \neq 1/2$, $1/2$ cannot be part of a SPE. Indeed, because c is increasing, we know that:

$$q'_M \Delta V(1,0) - C(q'_M) = \int_{1/2}^{q'_M} \Delta V(1,0) - c(q)dq \geq 0$$

Furthermore, if $q'_M > \gamma$, then q'_M cannot be part of any SPE. Otherwise, the total profits have to be compared in q'_M and γ.

For clarity concerns, I only characterize the producer's investment prescribed in the SPE:

- If $q'_M < \gamma$:
  - If $V(1,1) - V(1,0) > C(\gamma) - C(q'_M)$, then the SPE prescribe such that the producer invests γ.
  - If $V(1,1) - V(1,0) < C(\gamma) - C(q'_M)$, then the SPE prescribe such that the producer invests q'_M.
  - If $V(1,1) - V(1,0) = C(\gamma) - C(q'_M)$ then both investments described above are part of an SPE.
- If $q'_M \geq \gamma$, then the SPE prescribe that the producer invests γ.

D Proofs and computations

D 2.3.2 The Producers' Problem

Multinomial: the Distribution of an Outcome conditional on a Sum of Outcomes

Consider a random vector $X \sim Multi(n,p)$ of dimension k. By definition, we have:

$$Pr(X_1 = a, X_2 = b) = {p_1}^a {p_2}^b (1 - p_1 - p_2)^{n-a-b} \frac{n!}{a!b!(n-a-b)!}$$

Now, because each trial is independent, we have that $X_1 + X_2 \sim \mathcal{B}(p_1 + p_2, n)$. Hence:

$$Pr(X_1 + X_2 = s) = (p_1 + p_2)^s (1 - p_1 - p_2)^{n-s} \frac{n!}{s!(n-s)!}$$

Therefore, we find the following conditional distribution:

$$Pr(X_1 = a | X_1 + X_2 = s) = \frac{Pr(X_1 = a, X_2 = s - a)}{Pr(X_1 + X_2 = s)}$$

$$= \frac{{p_1}^a {p_2}^{s-a} (1 - p_1 - p_2)^{n-s} \frac{n!}{a!(s-a)!(n-s)!}}{(p_1 + p_2)^s (1 - p_1 - p_2)^{n-s} \frac{n!}{s!(n-s)!}} = \frac{{p_1}^a {p_2}^{s-a}}{(p_1 + p_2)^s} \frac{s!}{a!(s-a)!}$$

Note that it can be rewritten as:

$$Pr(X_1 = a | X_1 + X_2 = s) = \frac{{p_1}^a {p_2}^{s-a}}{(p_1 + p_2)^{s-a+a}} \frac{s!}{a!(s-a)!} = \left(\frac{p_1}{p_1 + p_2}\right)^a \left(\frac{p_2}{p_1 + p_2}\right)^{s-a} \frac{s!}{a!(s-a)!}$$

Hence, the conditional random variable $X_1|_{X_1+X_2} \sim \mathcal{B}\left(n, \frac{p_1}{p_1+p_2}\right)$.

The Probability of Being Read by a Follower (as a Producer)

First, note that:

$$Pr(\text{follower sees } k) = \sum_{s=0}^{d} Pr(\text{follower sees } k \text{ and } s \text{ neighbors shared})$$

$$= \sum_{s=0}^{d} Pr(\text{follower sees } k \mid s \text{ shares}) Pr(s \text{ shares})$$

Now, we also have:

$$Pr(\text{follower sees } k \mid s \text{ shares}) = \sum_{\nu=s}^{d} Pr(\text{follower sees } k \text{ and } \nu \text{ neighbors shared } k \mid s \text{ shares})$$
$$= \sum_{e=s}^{d} Pr(\text{follower sees } k \mid \nu \text{ and } s) Pr(\nu \text{ shares of } k \mid s \text{ shares})$$

Finally, using the conditional probability derived above, we rewrite the probability for a follower to see a piece of news n from producer k which is X, given ℓ produced m which is Y, as:

$$\sum_{s=1}^{d}\sum_{\nu=0}^{s}\frac{\nu}{s}\left(\frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}}\right)^{\nu}\left(\frac{p_{Y|m,\ell}}{p_{X|n,k}+p_{Y|m,\ell}}\right)^{s-\nu}\frac{s!}{\nu!(s-\nu)!}(p_{X|n,k}+p_{Y|m,\ell})^{s}(1-p_{X|n,k}-p_{Y|m,\ell})^{d-s}\frac{d!}{s!(d-s)!}$$

where $p_{X|n,k}$ and $p_{Y|m,\ell}$ are the probability that a neighbor shares a piece of news from k, ℓ, given it is true/false. Defining $f(\nu)$ as the pmf of a $\mathcal{B}(s, \frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}})$, we simplify the latter expression by:

$$\sum_{s=1}^{d}\frac{1}{s}(p_{X|n,k}+p_{Y|m,\ell})^{s}(1-p_{X|n,k}-p_{Y|m,\ell})^{d-s}\frac{d!}{s!(d-s)!}\sum_{\nu=0}^{s}\nu f(\nu)$$
$$=\sum_{s=1}^{d}\frac{1}{s}(p_{X|n,k}+p_{Y|m,\ell})^{s}(1-p_{X|n,k}-p_{Y|m,\ell})^{d-s}\frac{d!}{s!(d-s)!}s\frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}}$$
$$=\frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}}\sum_{s=1}^{d}(p_{X|n,k}+p_{Y|m,\ell})^{s}(1-p_{X|n,k}-p_{Y|m,\ell})^{d-s}\frac{d!}{s!(d-s)!}$$
$$=\frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}}\left(\sum_{s=0}^{d}(p_{X|n,k}+p_{Y|m,\ell})^{s}(1-p_{X|n,k}-p_{Y|m,\ell})^{d-s}\frac{d!}{s!(d-s)!}\right.$$
$$\left.-(p_{X|n,k}+p_{Y|m,\ell})^{0}(1-p_{X|n,k}-p_{Y|m,\ell})^{d-0}\frac{d!}{0!(d-0)!}\right)$$
$$=\frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}}\left(1-(1-p_{X|n,k}-p_{Y|m,\ell})^{d}\right)$$

We conclude by writing:

$$Pr(R_k=1|\omega,n,m)=\frac{b}{K}+\frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}}\left(1-(1-p_{X|n,k}-p_{Y|m,\ell})^{d}\right)$$

D 3 Equilibrium

D 3.1 Equilibrium without Competition

Lemma 1: shape of monopolist best-response & interpretation

- Because $c^{-1}(\cdot)$ is, by assumption, increasing in its argument, $q^*(z)$ is increasing (resp. decreasing) in $z_{S|n}$ iff $\Delta V(z)$ is increasing (resp. decreasing) in $z_{S|n}$. Now, we have:
  - I call single peaked a function which admits a single maximum point; therefore, any non-constant concave function $f(x)$ defined on a closed interval is single-peaked in x. Hence, it suffices to show that $\Delta V(z)$'s first derivative is decreasing in $z_{+|n}$.

    For $z_{+|0}$, we have:

    $$\frac{\partial \Delta V(z)}{\partial z_{+|0}} \frac{1}{1-b} = -d(1-\gamma)(1-w_0)\big(1-b(1-\gamma)z_{0,0}\big)\big)^{d-1} + dw_0\gamma\big(1-b\gamma z_{0,0}\big)^{d-1}$$

    Whose sign is ambiguous. It is positive for $z_{+|0} = 0$ and decreasing in $z_{+|0}$ since $\frac{1-b\gamma z_{0,0}}{1-b(1-\gamma)z_{0,0}} \leq 1$ and decreases with $z_{+|0}$

    The derivation is similar for $z_{+|1}$.
  - $\Delta V(z)$'s first derivative w.r.t. to $z_{-|0}$, is negative. Indeed:

    $$\frac{\partial \Delta V(z)}{\partial z_{0,1}} \frac{1}{1-b} = -d\gamma(1-w_0)\big(1-b(\gamma z_{0,1} + (1-\gamma))\big)^{d-1} + dw_0\gamma\big(1-b(\gamma + (1-\gamma)z_{0,1})\big)^{d-1} < 0$$

    Where the last inequality comes from $\frac{\gamma}{1-\gamma} > \frac{w_0}{1-w_0} \geq 1$. The derivation is similar for $z_{-|1}$.
- For any given $z_{S|n}$, $\Delta V(z)$ is a polynomial function of $z_{S|n}$, so it is continuous within each segment $z_{S|n} \in (0,1)$. The function is also continuous between segments. Indeed, $\lim_{z_{+|0}\to 1} \Delta V(z) = \lim_{z_{+|1}\to 0} \Delta V(z)$ and $\lim_{z_{+|1}\to 1} \Delta V(z) = \lim_{z_{-|0}\to 0} \Delta V(z)$.

In addition, note that the global maximum is in $z = (1, \bar{z}_1, 0, 0)$ for some priors, and in $z = \{(\bar{z}_0, 0, 0, 0)\}$ for all other priors. Indeed, for $w_0 = \gamma$, $\bar{q}_0 > \bar{q}_1$, while for $w_0 = 1/2$, $\bar{q}_0 < \bar{q}_1$. Figure 15 and 16 illustrate the shape of the producer's best response. Again, I represent the seeds' strategy on a line and map the corresponding image as if the argument was unidimensional. The resulting function is non-monotonic. Each hump shaped segment is explained by the effect of the network as in the main text. The two humps follow from the same mechanism applying in two different cases: when news 0 is produced first, and then when news 1 is published.

Furthermore, Figure 17 and 18 illustrate two cases. Figure 6 shows the equilibrium with $q^*(1,1,0,0) > \bar{t}_0 > q^*(1,1,1,0)$. Figure 7 shows the equilibrium with $\bar{t}_1 > q^*(1,1,1,0) > \bar{t}_0$.

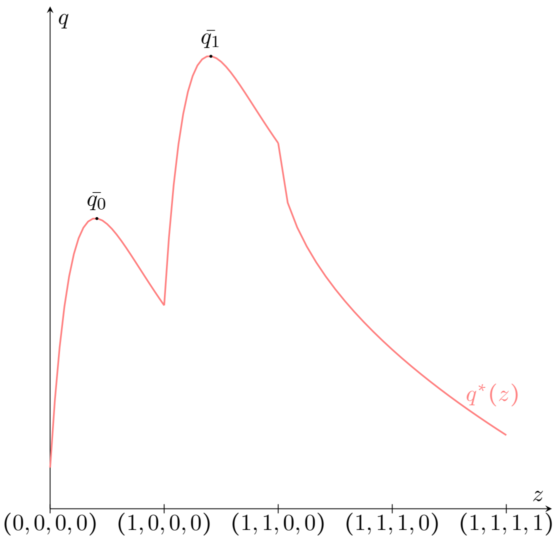

Figure 15: Producer's Best Response, $\bar{q}_0 < \bar{q}_1$

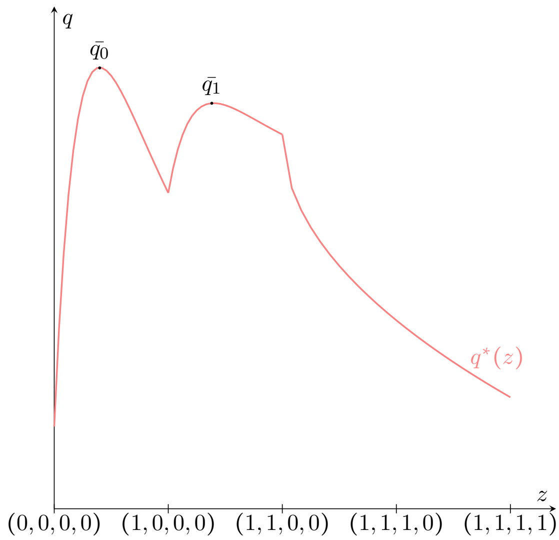

Figure 16: Producer's Best Response, $\bar{q}_0 > \bar{q}_1$

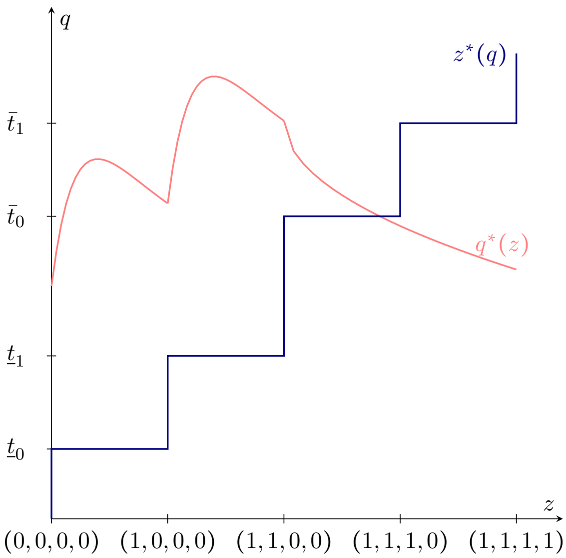

Figure 17: Equilibrium with $q_M^* = q^*(1,0)$

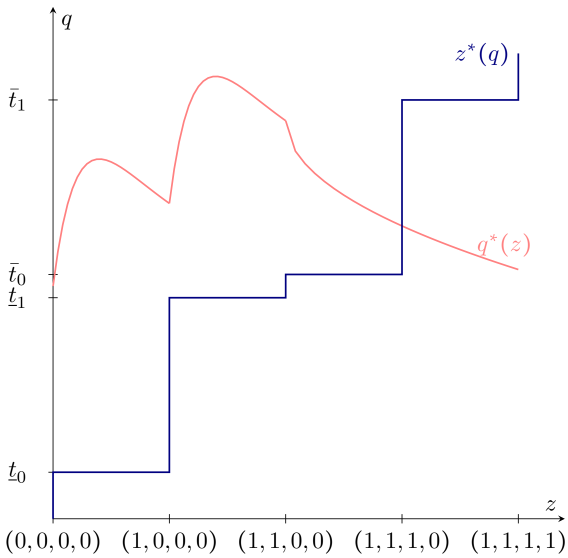

Figure 18: Equilibrium with $q_M^* = \bar{t}$

Proposition 1: characterization of the monopoly equilibrium

First, notice that any positive equilibrium investment has to achieve $q \leq \bar{t}_1$. Indeed, $q = \bar{t}_1$ is enough to insure that the producer's news is always share, so that any additional investment would increase costs without increasing benefits. Furthermore, note that even if no investment occurs, sharing can occur. Indeed, faced to completely uninformative news' outlet, agents will still share an article whose content matches their private signal, because the private signal is informative. Therefore, any equilibrium displays $z_{0,0} = z_{1,1} = 1$; and the equlibrium will occur on the decreasing part of the producers' best response. Furthermore, note that $q^*(1,1,1,1) = 1/2$. Indeed, if news gets systematically shared, the producer has no incentive to invest since true news is treated as false news. Because the relevant portion of $q^*(z)$ is strictly decreasing, while $z^*(q)$ is weakly increasing, any intersection has to be unique. Because both best responses are continuous and that in $z = (0,0,0,0)$ the producer's best response is above the value ensuring

some sharing, while in $z = (1, 1, 1, 1)$, the producer's best response is below the value ensuring full sharing, the intersection must exist. Therefore, a NE must exist and is unique.
Because the cost function will determine different levels for $q^*(1, 1, 0, 0)$ and $q^*(1, 1, 1, 0)$, we need to understand how these values compare to $\bar{t}_0 < \bar{t}_1$. If $q^*(1, 1, 0, 0) < \bar{t}_0$, from the shapes of the best responses, we have $q^*(1, 1, 1, 0) < q^*(1, 1, 0, 0) < \bar{t}_0 < \bar{t}_1$ so that $q^*_M = q^*(1, 1, 0, 0)$. Indeed, in such a case, because $q^*_M < \bar{t}_0$, the seeds will share an article only if its content matches their private signal: $z^*(q^*_M) = (1, 1, 0, 0)$. This is also optimal for the producer, as, by definition $c(q^*_M) = c(\Delta V(1, 1, 0, 0))$. Furthermore, no other investment is optimal as c is strictly increasing. The same reasoning applies for $\bar{t}_0 < q^*(1, 1, 1, 0) < \bar{t}_1$. Now, consider $\bar{t}_0 < q^*(1, 1, 0, 0)$ but $q^*(1, 1, 1, 0) < \bar{t}_0$. Then, $q^*_M = \bar{t}_0$. Indeed, as $q^*(1, 1, 1, 0) < \bar{t}_0 < q^*(1, 1, 0, 0)$, and because $q^*(z)$ continuous, there must exist some $z^*_{0,1}$ such that $c^{-1}(\Delta V(1, 1, z^*_{0,1}, 0)) = \bar{t}_0$. It is easy to verify that this constitutes a NE. The same reasoning applies for $\bar{t}_1 < q^*(1, 1, 1, 0)$.

When $w_0 = 1/2$, $\bar{t}_0 = \bar{t}_1$, so that the characterization simplifies to $q^*_M = \min\{q^*(1, 0), \bar{t}\}$.

Lemma 2: the role of connectivity

$\Delta V(z)$ is single-peaked in d because it is the weighted sum of two hump-shaped single-peaked function of d. Indeed, $f(d) := (1 - p_{F|1})^d - (1 - p_{T|0})^d$ is single peaked as $f(d + 1) - f(d) = -p_{F|1}(1 - p_{F|1})^d + p_{T|1}(1 - p_{T|1})^d$ whose sign depends on d. It is positive for $d = 0$ and negative for d big enough. Furthermore, $f(d + 1) - f(d) < 0 \Rightarrow f(d + 2) - f(d + 1) < 0$. The same applies to $(1 - p_{F|0})^d - (1 - p_{T|1})^d$.

Proposition 2 and Corollary 2: the effects of private knowledge

Recall that $\Delta V(z) = w_0\left((1 - p_{F|1})^d - (1 - p_{T|0})^d\right) - (1 - w_0)\left((1 - p_{F|0})^d - (1 - p_{T|1})^d\right)$, with $p_{X|n} = b\left(\gamma z_{S|n} + (1 - \gamma) z_{\text{-}X|n}\right)$. Therefore:

- $\frac{\partial \Delta V(z)}{\partial \gamma} \geq 0$ as $z_{+|0} - z_{-|1} \geq 0$ and $z_{+|1} - z_{-|0} \geq 0$.
- $\frac{\partial \Delta V(z)}{\partial w_0} \geq 0$ as $(1 - p_{F|1})^d - (1 - p_{F|0})^d \geq 0$ and $(1 - p_{T|1})^d - (1 - p_{T|0})^d \geq 0$.

Where the derivative is null only for $z = (0, 0, 0, 0)$ and $z = (1, 1, 1, 1)$.

Furthermore, γ and w_0 have the following effects on the seeds' best-response:

- $\frac{\partial \underline{t}_0}{\partial \gamma} < 0$, $\frac{\partial \underline{t}_1}{\partial \gamma} < 0, \frac{\partial \bar{t}_0}{\partial \gamma} > 0, \frac{\partial \bar{t}_1}{\partial \gamma} > 0$
- $\frac{\partial \underline{t}_0}{\partial w_0} < 0$, $\frac{\partial \bar{t}_0}{\partial w_0} < 0, \frac{\partial \bar{t}_1}{\partial w_0} > 0, \frac{\partial \bar{t}_1}{\partial w_0} > 0$.

Therefore, q^*_M unambiguously increases with γ. For w_0, as $q^*(1, 1, 0, 0)$ and $q^*(1, 1, 1, 0)$ are weakly increasing in w_0; no increase in w_0 would change the inequalities detailed in Proposition 1. Therefore, q^*_M increases iff $q^*_M \neq \bar{t}_0$.

D 3.2 Equilibrium with Competition

Lemma 3: shape of duopolist best-response

(i) Because $c^{-1}(\cdot)$ is, by assumption, increasing in its argument, $q^*(z)$ is increasing (resp. decreasing) in z_{X_k} iff $\Delta V(z_k; z_\ell, q_\ell)$ is increasing (resp. decreasing) in z_{X_k}. Now, we have:

– I show that $V_{T_k Y_\ell} - V_{F_k Y_\ell}$ is concave in z_{T_k} for $z_{T_k} \in [0,1]$ and any $Y = T, F$. We have:

$$\begin{aligned} V_{T_k Y_\ell} - V_{F_k Y_\ell} &= \frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}}\left(1 - (1 - p_{T_k} - p_{Y_\ell})^d\right) - \frac{p_{F_k}}{p_{F_k} + p_{Y_\ell}}\left(1 - (1 - p_{F_k} - p_{Y_\ell})^d\right) \\ &= \frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}}\left((1 - p_{F_k} - p_{Y_\ell})^d - (1 - p_{T_k} - p_{Y_\ell})^d\right) \\ &\quad + \left(\frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k} + p_{Y_\ell}}\right)\left(1 - (1 - p_{F_k} - p_{Y_\ell})^d\right) \end{aligned}$$

We know that $\frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}}$ and $\left(1-(1-p_{F_k}-p_{Y_\ell})^d\right)$ are both strictly increasing and weakly concave in z_{T_k}. From the analysis of the monopolist's best response, we also know that $\left((1 - p_{F_k} - p_{Y_\ell})^d - (1 - p_{T_k} - p_{Y_\ell})^d\right)$ is single-peaked. As the product of weakly concave functions is weakly concave, all that is left to do is to show that $\frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k}+p_{Y_\ell}}$ is single peaked. We have:

$$\begin{aligned} \frac{\partial \frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k}+p_{Y_\ell}}}{\partial z_{T_k}} &= \frac{\partial \frac{p_{Y_\ell}(p_{T_k}-p_{F_k})}{(p_{T_k}+p_{Y_\ell})(p_{F_k}+p_{Y_\ell})}}{\partial z_{T_k}} = \frac{\partial \frac{p_{Y_\ell}(\frac{1}{2}b(2\gamma-1)z_{T_k})}{(\frac{1}{2}b\gamma z_{T_k}+p_{Y_\ell})(\frac{1}{2}b(1-\gamma)z_{T_k}+p_{Y_\ell})}}{\partial z_{T_k}} \\ &= \frac{p_{Y_\ell}\left[\frac{1}{2}b(2\gamma-1)(\frac{1}{4}b^2\gamma(1-\gamma){z_{T_k}}^2 + \frac{1}{2}bp_{Y_\ell}z_{T_k} + {p_{Y_\ell}}^2) - \left(\frac{1}{2}b^2\gamma(1-\gamma)z_{T_k} + \frac{1}{2}bp_{Y_\ell}\right)\frac{1}{2}b(2\gamma-1)z_{T_k}\right]}{(p_{T_k} + p_{T_\ell})^2(p_{F_k} + p_{Y_\ell})^2} \\ &= \frac{p_{Y_\ell}\frac{1}{2}b(2\gamma-1)\left[p_{Y_\ell} - \frac{1}{4}b^2\gamma(1-\gamma){z_{T_k}}^2\right]}{2(p_{T_k} + p_{Y_\ell})^2(p_{F_k} + p_{T_\ell})^2} \end{aligned}$$

Which is positive in $z_{T_k} = 0$ and decreases with z_{T_k}.

– For z_{F_k}, we have:

$$\frac{\partial \Delta V(z_k; z_\ell, q_\ell)}{\partial z_{F_k}} = (1-b)\left(\Pr(Y_\ell)\frac{\partial V_{T_k Y_\ell} - V_{F_k Y_\ell}}{\partial z_{F_k}}\right) < 0$$

Indeed,

$$\begin{aligned} \frac{\partial V_{T_k Y_\ell} - V_{F_k Y_\ell}}{\partial z_{F_k}} = \frac{1}{2}b\Bigg[p_{Y_\ell}\left(\frac{(1-\gamma)}{(p_{T_k} + p_{Y_\ell})^2}\left(1 - (1 - p_{T_k} - p_{Y_\ell})\right)^d - \frac{\gamma}{(p_{F_k} + p_{Y_\ell})^2}\left(1 - (1 - p_{F_k} - p_{Y_\ell})^d\right)\right) \\ + d\frac{(1-\gamma)p_{T_k}}{p_{T_k} + p_{Y_\ell}}(1 - p_{T_k} - p_{Y_\ell})^{d-1} - d\frac{\gamma p_{F_k}}{p_{F_k} + p_{Y_\ell}}(1 - p_{F_k} - p_{Y_\ell})^{d-1}\Bigg] \end{aligned}$$

Which is a sum of negative terms. Indeed, the first term is negative because $\frac{1-(1-x)^d}{x^2}$

is decreasing in x and $p_{T_k} \geq p_{F_k}$. We know that $\frac{1-(1-x)^d}{x^2}$ is decreasing in x because:

$$\frac{\partial \frac{1-(1-x)^d}{x^2}}{\partial x} x^4 = d(1-x)^{d-1}x^2 - 2x(1-(1-x)^d) = (1-x)^{d-1}x\big(dx+2(1-x)\big) - 2x < x(-x+2) - 2x < 0$$

where the first inequality follows from $(1-x)^{d-1}\big((d-2)x+2\big)$ being decreasing in d so that among all d, $d=1$ maximizes the expression.

The second term is negative as $(1-p_{T_k}-p_{Y_\ell})^{d-1} < (1-p_{F_k}-p_{Y_\ell})^{d-1}$; and $\frac{(1-\gamma)p_{T_k}}{p_{T_k}+p_{Y_\ell}} < \frac{\gamma p_{F_k}}{p_{F_k}+p_{Y_\ell}}$. The last inequality holds because:

$$(1-\gamma)p_{T_k}(p_{F_k}+p_{Y_\ell}) - \gamma p_{F_k}(p_{T_k}+p_{Y_\ell}) = p_{F_k}p_{T_k}(1-2\gamma) + p_{Y_\ell}\big((1-\gamma)p_{T_k} - \gamma p_{F_k}\big)$$

is the sum of two negative terms; indeed: $1-2\gamma < 0$ and

$$(1-\gamma)p_{T_k} - \gamma p_{F_k} = \frac{1}{2}b\big[(1-\gamma)(\gamma+(1-\gamma)z_{F_k}) - \gamma(1-\gamma+\gamma z_{F_k})\big] = \frac{1}{2}b\big[(1-\gamma)^2-\gamma^2\big]z_{F_k} < 0$$

(ii) For $d=2$, $\frac{\partial V_{T_kY_\ell} - V_{F_kY_\ell}}{\partial z_{Y_\ell}}$ is decreasing in z_{Y_ℓ} for $Y = T, F$. We have:

$$\begin{aligned}\frac{\partial V_{T_kY_\ell} - V_{F_kY_\ell}}{\partial p_{Y_\ell}} = &-\frac{p_{T_k}}{(p_{T_k}+p_{Y_\ell})^2}\Big[1-(1-p_{T_k}-p_{Y_\ell})^{d-1}\big(1+(d-1)(p_{T_k}+p_{Y_\ell})\big)\Big]\\ &+\frac{p_{F_k}}{(p_{F_k}+p_{Y_\ell})^2}\Big[1-(1-p_{F_k}-p_{Y_\ell})^{d-1}\big(1+(d-1)(p_{F_k}+p_{Y_\ell})\big)\Big] < 0\end{aligned}$$

If $-\frac{p_{T_k}}{(p_{T_k}+p_{Y_\ell})^2}\big(-(p_{T_k}+p_{Y_\ell})^2\big) > \frac{p_{F_k}}{(p_{F_k}+p_{Y_\ell})^2}\big(-(p_{F_k}+p_{Y_\ell})^2\big)$ which is ensured by $p_{T_k} \geq p_{F_k}$.[31]

When $d \to \infty$, the expression is determined by the sign of $-\frac{p_{T_k}}{(p_{T_k}+p_{Y_\ell})^2} + \frac{p_{F_k}}{(p_{F_k}+p_{Y_\ell})^2}$ which is negative for $p_{Y_\ell}^2 > p_{F_k}p_{T_k}$.

(iii) It is enough to prove that $\Big(V_{T_kT_\ell} - V_{F_kT_\ell}\Big) - \Big(V_{T_kF_\ell} - V_{F_kF_\ell}\Big) \leq 0$. This expression can be rewritten:

$$\begin{aligned}&\tfrac{p_{T_k}}{p_{T_k}+p_{T_\ell}}\left(1-(1-p_{T_k}-p_{T_\ell})^d\right) - \tfrac{p_{F_k}}{p_{F_k}+p_{T_\ell}}\left(1-(1-p_{F_k}-p_{T_\ell})^d\right)\\ &\quad - \tfrac{p_{T_k}}{p_{T_k}+p_{F_\ell}}\left(1-(1-p_{T_k}-p_{F_\ell})^d\right) + \tfrac{p_{F_k}}{p_{F_k}+p_{F}}\left(1-(1-p_{F_k}-p_{F_\ell})^d\right)\\ =&\tfrac{p_{T_k}}{(p_{T_k}+p_{T_\ell})(p_{T_k}+p_{F_\ell})}\left(p_{F_\ell}-p_{T_\ell}-(1-p_{T_k}-p_{T_\ell})^d(p_{T_k}+p_{F_\ell}) + (1-p_{T_k}-p_{F_\ell})^d(p_{T_k}+p_{T_\ell})\right)\\ &\quad - \tfrac{p_{F_k}}{(p_{F_k}+p_{T_\ell})(p_{F_k}+p_{F_\ell})}\left(p_{F_\ell}-p_{T_\ell}-(1-p_{F_k}-p_{T_\ell})^d(p_{F_k}+p_{F_\ell}) + (1-p_{F_k}-p_{F_\ell})^d(p_{F_k}+p_{T_\ell})\right)\end{aligned}$$

Let us define α such that $p_{T_k}+p_{F_\ell} = \alpha(p_{T_k}+p_{T_\ell}) + (1-\alpha)(p_{F_k}+p_{F_\ell})$; therefore $p_{F_k}+p_{T_\ell} =$

[31] Notice that a similar inequality holds for $d=3$. From numerical insights, the difference is expected to be increasing then decreasing in d.

$(1-\alpha)(p_{T_k}+p_{T_\ell})+\alpha(p_{F_k}+p_{F_\ell})$. Because $(1-x)^d$ is convex, we have:

$$\begin{aligned}
&(1-p_{T_k}-p_{F_\ell})^d(p_{T_k}+p_{T_\ell})+(1-p_{F_k}-p_{T_\ell})^d(p_{F_k}+p_{F_\ell})\\
&\quad-(1-p_{T_k}-p_{T_\ell})^d(p_{T_k}+p_{F_\ell})-(1-p_{F_k}-p_{F_\ell})^d(p_{F_k}+p_{T_\ell})\\
&<\alpha(1-p_{T_k}-p_{T_\ell})^d(p_{T_k}+p_{T_\ell})+(1-\alpha)(1-p_{F_k}-p_{F_\ell})^d(p_{T_k}+p_{T_\ell})\\
&\quad+(1-\alpha)(1-p_{T_k}-p_{T_\ell})^d(p_{F_k}+p_{F_\ell})+\alpha(1-p_{F_k}-p_{F_\ell})^d(p_{F_k}+p_{F_\ell})\\
&\quad-(1-p_{T_k}-p_{T_\ell})^d(p_{T_k}+p_{F_\ell})-(1-p_{F_k}-p_{F_\ell})^d(p_{F_k}+p_{T_\ell}) \qquad =0
\end{aligned}$$

Therefore the second factor of the first term of the sum is lower than the second factor of the second term of the sum, which is itself negative. If the first factor of the first term is greater than the first factor of the second term, we are done. And indeed, if $p_{uX}<p_{vX}$, we have:

$$\frac{p_{T_k}}{p_{T_k}^2+p_{T_k}(p_{T_\ell}+p_{F_\ell})+p_{T_\ell}p_{F_\ell}}>\frac{p_{F_k}}{p_{F_k}^2+p_{F_k}(p_{T_\ell}+p_{F_\ell})+p_{T_\ell}p_{F_\ell}}$$

$$p_{T_\ell}p_{F_\ell}(p_{T_k}-p_{F_k})>p_{T_k}p_{F_k}(p_{T_k}-p_{F_k})=p_{T_k}^2p_{F_k}-p_{T_k}p_{F_k}^2$$

(iv) As in the monopoly case, for any given z_{X_k}, $\Delta V(z_k;z_\ell,q_\ell)$ is a polynomial function of z_{X_n} and is also continuous between segments, as $\lim_{z_{T_k}\to 1}\Delta V(z)=\lim_{z_{F_k}\to 0}\Delta V(z_k;z_\ell,q_\ell$. Furthermore, $\Delta V(z_k;z_\ell,q_\ell)$ is also polynomial function of z_ℓ and x_ℓ, so it is continuous.

Proposition 3: characterization of the duopoly symmetric equilibrium

First note that any equilibrium news quality lies in $[1/2,\bar{t}]$. Indeed, recall that $\Delta V_D((0,0),q)=\Delta V_D((1,1),q)=0$. Clearly, for any $q>\bar{t}$, $c(q)>0=\Delta V_D(z^*(q),q)$, which would be suboptimal for the producer.

First I prove that a symmetric equilibrium exists. Then, I show it is unique.

Consider two cases:

1. If $c(\bar{t})\geq\Delta V_D((1,0),\bar{t})$, then $\exists\tilde{q}\in[1/2,\bar{t}]$: $c(\tilde{q})=\Delta V((1,0),\tilde{q})$. Indeed, recall that c is weakly increasing in q and $\Delta V_D((1,0),q)$ strictly decreasing in q. We also notice that $c(\tilde{q})=\Delta V((1,0),\tilde{q})$ while $c(\tilde{q})=\Delta V((1,0),\tilde{q})$. Because both c and ΔV are continuous in q, they must intersect on $[1/2,\bar{t}]$. Clearly, $(\tilde{q},(1,0))$ is a NE.
   This equilibrium is unique. First notice that for $z=(1,0)$, the intersection must be unique given the shape of the respective best responses. Let us show that no other undominated sharing rule can be consistent with an equilibrium in this case. A sharing rule $(z,0)$ with $z<1$ would require $q<1/2$, which is impossible. A sharing rule $(1,z)$ with $z>0$ would require $q\geq\bar{t}$. This cannot occur in equilibrium since, for any $z\in[0,1]$, $\Delta V_D((1,z),\bar{t})<\Delta V_D((1,z),q_D^*)=c(q_D^*)<c(\bar{t})$. Hence, $c(\bar{t})>\Delta V_D((1,z),\bar{t})$, so that $q^*((1,z),\bar{t})<\bar{t}$ for any z.
2. If $c(\bar{t})<\Delta V_D((1,0),\bar{t})$, then $\exists\tilde{z}_F\in[0,1]$: $c(\bar{t})=\Delta V((1,\tilde{z}_F),\bar{t})$. Indeed, by assumption

$c(\bar{t}) < \Delta V_D((1,0);\bar{t})$ and we know that $c(\bar{t}) > 0 = \Delta V_D((1,1);\bar{t})$. Because $\Delta V_D(z;q)$ is continuous in z_F, there must exist such $\tilde{z}_F$. Because $V_D(z;q)$ is strictly decreasing in z_F, this equilibrium is unique.

D 3.3 Effects of Competition

Theorem 1: shape of $\Delta V_M(z) - \Delta V_D(z,q)$ ***in*** d

Given $DV(d) := \frac{\Delta V_M(z;d) - \Delta V_D(z,q;d)}{1-b}$, we want to show that $DV(d) > DV(d+1) \Rightarrow DV(d+1) > DV(d+2)$. For readability, let us define for this proof:

$$c_1 = 1 - \frac{q}{2} \qquad c_2 = \frac{1+q}{2} \qquad c_3 = \frac{p_T}{p_T + p_F} - q$$

Note that $c_1 > 0$, $c_2 > 0$ and c_3's sign depends on z and q.

We begin by rewriting the assumption $DV(d) - DV(d+1) > 0$ as:

$$-c_1\left((1-p_T)^d - (1-p_T)^{d+1}\right) + c_2\left((1-p_F)^d - (1-p_F)^{d+1}\right) + c_3\left(\left(1 - \tfrac{p_T+p_F}{2}\right)^d - \left(1 - \tfrac{p_T+p_F}{2}\right)^{d+1}\right) > 0$$

$$-c_1\left((1-p_T)^d p_T\right) + c_2\left((1-p_F)^d p_F\right) + c_3\left(\left(1 - \frac{p_T+p_F}{2}\right)^d \frac{p_T+p_F}{2}\right) > 0$$

Therefore, defining for readability again:

$$A := c_1\left((1-p_T)^d p_T\right) - \tfrac{1}{2}c_3\left(\left(1 - \tfrac{p_T+p_F}{2}\right)^d \tfrac{p_T+p_F}{2}\right)$$

$$B := c_2\left((1-p_F)^d p_F\right) + \tfrac{1}{2}c_3\left(\left(1 - \tfrac{p_T+p_F}{2}\right)^d \tfrac{p_T+p_F}{2}\right)$$

$DV(d+1) - DV(d) < 0$ is equivalent to $B > A$. Notice that $B > 0$ because when $c_3 > 0$ makes B a sum of positive term, and when $c_3 < 0$ A is a sum of positive term so that $B > A > 0$.

Likewise we develop $DV(d+2) - DV(d+1)$ as:

$$-c_1\left((1-p_T)^d p_T(1-p_T)\right) + c_2\left((1-p_F)^d p_F(1-p_F)\right) + c_3\left(\left(1 - \frac{p_T+p_F}{2}\right)^d \frac{p_T+p_F}{2}\frac{1}{2}(1 - p_T + 1 - p_F)\right)$$

Therefore:

$$DV(d+1) - DV(d+2) = -(1-p_T)A + (1-p_F)B > 0$$

where the last inequality follows from $p_T > p_F$

Remark 3: beyond two competitors

The difference in the incentive to invest between competition with K and $K+1$ producers is proportional to:

$$\begin{aligned}\textstyle\sum_Y \Pr(Y)\Bigg\{&\Big[\tfrac{p_{T_k}}{p_{T_k}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{Y_{-k}}}{K})^d\big)-\tfrac{p_{F_k}}{p_{F_k}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{Y_{-k}}}{K})^d\big)\Big]\\ &-q\Big[q\tfrac{p_{T_k}}{p_{T_k}+p_{k'T}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{T_{k'}}+p_{Y_{-k}}}{K+1})^d\big)-\tfrac{p_{F_k}}{p_{F_k}+p_{T_{k'}}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{u'T}+p_{Y_{-k}}}{K+1})^d\big)\Big]\\ &-(1-q)\Big[\tfrac{p_{T_k}}{p_{T_k}+p_{F_{k'}}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{F_{k'}}+p_{Y_{-k}}}{K+1})^d\big)-\tfrac{p_{F_k}}{p_{F_k}+p_{F_{k'}}+p_{Y_{-k}}}\big(1-(1-\tfrac{p_{T_k}+p_{F_{k'}}+p_{Y_{-k}}}{K+1})^d\big)\Big]\Bigg\}\end{aligned}$$

Whose sign is positive in $d=1$, meaning that the incentive to invest with $K+1$ producer is smaller in the symmetric case than the incentive to invest with K producers. The expression's sign depends on the parameters for $d \to \infty$.

Remark 4: the role of signal precision

(i) When $\gamma \to 1$, note that the set of the seeds' best response reduces to $\{(1,0)\}$. Then: $\Delta V_M(z) = (1-b)(1-(1-b)^d) > (1-b)\frac{1}{2}q(1-(1-b)^d) + (1-q)(1-(1-\frac{1}{2}b)^d) = \Delta V_D(z,q)$ Because $\frac{1-(1-b)^d}{1-(1-\frac{1}{b})^d} > \frac{1-q}{1-\frac{1}{2}q}$ $\forall q \in [0,1]$.

(ii) When $\gamma \to \frac{1}{2}$, $p_T = p_F$ for any z, so that the incentive to invest vanishes on both types of market: $\Delta V_M(z) = 0 = \Delta V_D(z;q)$

D 4 Welfare

D 4.2 Framework of Analysis

Lemma 4: Consumers' expected utility

- Conditional on receiving news n after private signal s, accounting for the optimal decision whether to share or not, the utility from sharing is $\max\{2p(n,s)-1;0\}$; now $2p(n,s)-1>0 \Rightarrow z_{S|n,k}>0$ where $X=T$ iff $n=s$. Therefore, $\max\{2p(n,s)-1;0\} = z_{S|n,k}\big(2p(n,s)-1\big)$. The expected utility from sharing is thus: $\sum_k \frac{1}{K}\sum_{s,n} z_{S|n,k}\big(2p(n,s)-1\big)\Pr(n,s)$. Note that suming over possible s is equivalent to suming over possible X as X,n and s,n are isomorphic. The final expression is fund by plugging the expression for $p(n,s)$ and $\Pr(n,s)$ in the sum.

- Conditional on receiving news n after private signal s, accounting for the optimal decision to bet, the utility from sharing is $\max\{2p(n,s)-1;1-2p(n,s)\}$ where the former argument expresses the expected gain from betting $a=n$ and the latter, the expected gain from betting $a \neq n$. As before, $2p(n,s)-1>0 \Rightarrow z_{S|n,k}=1$; therefore, it is optimal for seeds to bet $a=n$ after (n,s) with the same probability as they share n after (n,s). If $z_{S|n,k}=0$, they bet $a \neq n$ and get $1-2p(n,s)$; when $1 > z_{S|n,k} > 0$ they are indifferent as $p(n,s) = 1/2$. Therefore, $\max\{2p(n,s)-1; 1-2p(n,s)\} = \left(2z_{S|n,k}-1\right)\left(2p(n,s)-1\right)$. As before, the final expression for the expected utility of seeds from betting is found by plugging he expression for $p(n,s)$ and $\Pr(n,s)$ into: $\sum_k \frac{1}{K}\sum_{s,n}\left(2z_{S|n,k}-1\right)\left(2p(n,s)-1\right)\Pr(n,s)$.

Followers, conditional on receiving some news, account for the higher probability to be reached by a true news because of the filtering effect of the network. If there is no competition, the expression for their expected utility from betting is the same as seeds. Otherwise,

$$Pr(\omega = n|n,s,k) = \frac{\Pr(n,s|\omega=n)\Pr(\text{see k over -k}|\omega=n)Pr(\omega=n)}{Pr(n,s,\text{sees } k)} = \frac{q_k \Pr(T)\sum_Y \Pr(\text{see } T \text{ over } Y\ )\Pr(\omega=n)}{\sum_w \Pr(n|\omega=w)\Pr(X)\sum_Y \Pr(\text{sees } X \text{ over } Y\ )\Pr(\omega=w)}$$

and their utility is found as $\max\{2Pr(\omega = n|n,s,k)-1; 1-2Pr(\omega = n|n,s,k)\}$.

Note that upon receiving no news, followers simply bet their private signal and get $2\gamma-1$ in expectation.

- Upon each possible outcome (n,s), consumers do not enter if $u_j(a|n,s) < r$.

D 4.2 Welfare for symmetric priors

Theorem 2, Lemma 5 and Corollary 3: outlets' influence on betting decisions

Consider $w_0 = 1/2$.

(i) The expected utility of a seed who would always follow the news article is: $\sum_s q\Pr(s=n) - (1-q)\Pr(s \neq n)$, which is smaller or equal to $(2\gamma-1)$ for any $q \leq \gamma$. If the influence follows the news article only when $n = s$, then it is equivalent to always following the private signal. $q^* \leq \gamma$ so seeds are always as well off following their private signal.

(ii) Conditional on receiving a news article n, without competition, the expected utility from a follower is the same as the expected utility from the seed; hence follower are always. as well of betting their private signal in a market without competition.
In a competitive market, consider a follower receiving a signal different than the article they read. If the follower is better off following the news in this case, then the presence of news outlets allows him to take better decision. A follower's expected utility from betting the content of a news article conditional on receiving one with $n \neq s$ is:

$$\sum_m \left[q(1-\gamma)\Pr(m|\omega=n)\frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}} - (1-q)\gamma\Pr(m|\omega\neq n)\frac{p_{F_k}}{p_{F_k}+p_{Y_\ell}} \right]$$

where $Y = T$ when $m = \omega$ and $Y = F$ for $m \neq \omega$ Because the equilibrium is symmetric, $p_{T_k} = p_{T_\ell} = \frac{p_T}{2}$ and $p_{F_k} = p_{F_\ell} = \frac{p_F}{2}$. Therefore, the value for the expected utility of a follower receiving $n \neq s$ when the news quality is q is:

$$\mathbb{E}(u_j(a)|n\neq s) = \left[q^2(1-\gamma)\frac{1}{2} - (1-q)q\gamma\frac{p_F}{p_T+p_F}\right] + \left[q(1-q)(1-\gamma)\frac{p_T}{p_T+p_F} - (1-q)^2\gamma\frac{1}{2}\right]$$

Which is maximized, given any q, at z^* is such that $p_T = b\gamma$; $p_F = b(1-\gamma)$. Then:

$$\mathbb{E}(u_j(a)|n \neq s) = \frac{(q^2(1-\gamma)-(1-q)^2\gamma)}{2} + q(1-q)\frac{(1-\gamma)\big(\gamma b\big)-\gamma\big((1-\gamma)b\big)}{\gamma b+(1-\gamma)b} = \frac{(q^2(1-\gamma)-(1-q)^2\gamma)}{2}$$

The follower is better off following the article rather than his private signal when this expected utility is greater than 0, which requires $q\frac{\gamma-\sqrt{\gamma(1-\gamma)}}{2\gamma-1}$.
The follower's maximal utility is still bounded by γ because $q \leq \gamma$. In particular, for $q = \gamma$ and $z^* = (1,0)$, $z^f = (1,1)$ so that the expected utility of a follower conditional on receiving some news is:

$$q\left(\frac{1}{2} + \frac{p_T}{p_T+p_F}\right) - (1-q)\left(\frac{p_F}{p_F+p_T} + \frac{1}{2}\right) = \frac{3}{2}(2\gamma-1)$$

Consider any w_0 in an uncompetitive environment.

(i) As in equilibrium, q^* can be greater than γ, consumers can be made better off by betting the news article content: $\sum_w \left(\sum_s \left(q\Pr(s=n) - (1-q)\Pr(s \neq n)\right)\Pr(\omega = w)\right) > 2\gamma - 1$ iff $q > \gamma$.

(ii) Consider $q^* \in (\gamma, \bar{t}_1]$. Then, $z_{+|0} = z_{+|1} = z_{-|0} = 1$. The expected utility of a seed – or of a follower conditional on seeing a news article – is:

$$\mathbb{E}(u_i) \sum_{X,n}(2z_{S|n} - 1)\big[qPr(X)\Pr(\omega = n) - (1-q)Pr(-X)\Pr(\omega \neq n)\big]$$
$$= \big[q\gamma w_0 - (1-q)(1-\gamma)(1-w_0)\big] + \big[q\gamma(1-w_0) - (1-q)(1-\gamma)w_0\big] + \big[q(1-\gamma)w_0 - (1-q)(1-\gamma)(1-$$

Note that this equality is also valid for $z_{+|1} > 0$; Indeed $z_{+|1} > 0 \Rightarrow q^* = \bar{t}_1$ but $\bar{t}_1 Pr(F)\Pr(\omega = n) - (1-\bar{t}_1)Pr(T)\Pr(\omega \neq n) = 0$. Therefore,

$$\begin{aligned}
\mathbb{E}(u_i) &= 2\gamma - 1 + 2(q(1-\gamma)w_0 - (1-q)\gamma(1-w_0)) \\
&\leq 2\gamma - 1 + 2(\bar{t}_1(1-\gamma)w_0 - (1-\bar{t}_1)\gamma(1-w_0)) \\
&= 2\gamma - 1 + \frac{2\gamma(1-\gamma)}{\gamma w_0 + (1-\gamma)(1-w_0)}(2w_0 - 1) \\
&\leq 2\gamma - 1 + \frac{2\gamma(1-\gamma)}{\gamma^2 + (1-\gamma)^2}(2\gamma - 1) \\
&= 2\gamma - 1\left(1 + \frac{2\gamma(1-\gamma)}{1 - 2\gamma(1-\gamma)}\right)
\end{aligned}$$

Where the last inequality follows from

$$\frac{\partial \frac{2w_0-1}{\gamma w_0+(1-\gamma)(1-w_0)}}{\partial w_0} = \frac{2\big(\gamma w_0+(1-\gamma)(1-w_0)\big)-(2w_0-1)(2\gamma-1)}{\big(\gamma w_0+(1-\gamma)(1-w_0)\big)^2} = \frac{1}{\big(\gamma w_0+(1-\gamma)(1-w_0)\big)^2} > 0$$

Theorem 2 and Corollary 4: outlets' influence on entering the bet

Consider again $w_0 = 1/2$. Let us compare the decision to enter the bet with and without news. Without news, all consumers take the same action: they opt out of the bet if $r > r_s$ and enter the bet for $r \leq r_s$. With news, seeds would opt out for $r > \bar{r}$, enter following any news with $r \leq \underline{r}$ and enter only for $n = s$ with $\underline{r} < r \leq \bar{r}$. Their behavior changes only in the interval $[\underline{r}, \bar{r}]$.

- For $r \in (r_s, \bar{r}]$, news articles push agents to enter the bet. All agents with $n = s$ place a bet. Given any state of the world, there are γq agents receiving $n = s$ corresponding to the right state of the world, i.e. who win the bet; and $(1-\gamma)(1-q)$ seeds who lose the bet. As $\gamma q > (1-\gamma)(1-q)$, there are more winners than losers.
- For $r \in [\underline{r}, r_s]$, news articles discourage agents to enter the bet. All agents with $n \neq s$ opt out. Given any state of the world, there are $(1-\gamma)q$ agents receiving $n \neq s$ who had the wrong private signal so who are better off opting out; and $\gamma(1-q)$ seeds who are worse off. As $\gamma(1-q) > (1-\gamma)q$, there are more losers than winners.

Consider any w_0 in an uncompetitive environment. Consumers might decide to enter the bet conditional on the private signal content's they receive. Let r_s be the bet price that makes consumers indifferent between betting or not, were they to only observe their private signal s. Then:

$$r_0 = 2\frac{\gamma w_0}{\gamma w_0 + (1-\gamma)(1-w_0)} - 1 > 2\frac{\gamma(1-w_0)}{\gamma(1-w_0) + (1-\gamma)w_0} - 1 = r_1$$

Furthermore, let $\underline{r}_s$ be the bet price that makes consumers indifferent between betting or not when receiving signal s and news content $n \neq s$. Then:

$$\underline{r}_0 = 2\frac{\gamma(1-q)w_0}{\gamma(1-q)w_0 + (1-\gamma)q(1-w_0)} - 1 > 2\frac{\max\{\gamma(1-q)(1-w_0); (1-\gamma)qw_0\}}{\gamma(1-q)(1-w_0) + (1-\gamma)qw_0} - 1 = \underline{r}_1$$

Finally, let $\bar{r}_s$ be the bet price that makes consumers indifferent between betting or not when receiving signal s and news content $n = s$. Then:

$$\bar{r}_0 = 2\frac{\gamma q w_0}{\gamma q w_0 + (1-\gamma)(1-q)(1-w_0)} - 1 > 2\frac{\gamma q(1-w_0)}{\gamma q(1-w_0) + (1-\gamma)(1-q)w_0} - 1 = \bar{r}_1$$

Now, $\underline{r}_s < r_s < \bar{r}_s$ for both $s = 0, 1$; furthermore, as in the proof of Lemma 6, conditional on receiving a signal s, consumers are expected to be worse at deciding whether to enter the bet for $r \in [\underline{r}_s, r_s]$ for $s = 0$ and for $s = 1$ and $q_M^* \leq \underline{t}_0$; and better for $r \in [r_s, \bar{r}_s]$.

One must thus determine the relative order of the different thresholds, and compare the gains and losses from the presence of outlets for each case.

- If $q < w_0$, then $\underline{r}_1 < r_1 < \bar{r}_1 < \underline{r}_0 < r_0 < \bar{r}_0$. Then, for $r \in [\underline{r}_0; r_0]$, the presence of news outlets does not change the consumers decision to enter the bet if $s = 1$ but dissuades them to enter for $s = 0 \neq 1 = n$. This dissuasion happens with probability $\gamma(1-q)$ for $\omega = 0$ and with probability $(1-\gamma)q$ for $\omega = 1$. The total expected effect is thus $-\gamma(1-q)w_0 + (1-\gamma)q(1-w_0) < 0$ for any equilibrium $q < \underline{t}_1$.
- If $q \in \left[w_0; \frac{{w_0}^2}{{w_0}^2 + (1-w_0)^2}\right]$, then $\underline{r}_1 < r_1 < \underline{r}_0 < \bar{r}_1 < r_0 < \bar{r}_0$. Then, for $r \in [\bar{r}_1; r_0]$, the presence of news outlets does not change the consumers decision to enter the bet if $s = 1$ but as before dissuades to many of them to enter for $s = 0 \neq 1 = n$.

- If $q > \frac{{w_0}^2}{{w_0}^2+(1-w_0)^2}$, then $\underline{r}_1 < \underline{r}_0 < r_1 < r_0 < \bar{r}_1 < \bar{r}_0$. Then, for $r \in [\underline{r}_0; r_1]$, the presence of news outlets dissuades consumers to enter for any $s \neq n$. This creates a loss for $s = w$, which happens with probability $\gamma(1-q)$ for either $\omega = w$; and a gain for $s \neq w$, which happens with probability $(1-\gamma)q$ for either $\omega = w$. The total expected effect is thus negative as long as $q < \gamma$.

Finally notice how, for instance for $q < w_0$, the presence of news outlets is positive for $r \in [r_1, \bar{r}_1]$ and $r \in [r_0, \bar{r}_0]$ but negative for $r \in [\underline{r}_0; r_0]$, while $\bar{r}_1 < \underline{r}_0 < r_0$.

Proposition 5: effect of competition on total welfare

- About the expected utility from sharing: taking $d \to \infty$, the difference in profits is $-2C(q_D)$ while the difference in expected utility from sharing is $q_D\gamma - (1-q_D)\gamma$. There exists a cost function $C(q)$ such that $2C(q_D) > q_D\gamma - (1-q_D)\gamma$, for instance $C(q) = \frac{q^2}{2}$.

- About the expected utility from betting: Consider a cost function such that $q^*_M < q^*_D < \frac{\gamma-\sqrt{\gamma(1-\gamma)}}{2\gamma-1}$. Then, by Theorem 2, neither seeds nor followers are made better off by the presence of a second news outlet. The difference in total revenues from producers is:

$$q(1-q)\left[(1-p_T)^d + (1-p_F)^d - 2\left(1 - \frac{p_T+p_F}{2}\right)\right]$$

  The total cost of production doubles. For $d \to \infty$ the revenues are the same, so that any cost function $C(q)$ would surpass that the total gain in revenues.

  The same applies to the utility from entering the bet for $r < \underline{r}$ and $r > \bar{r}$.

D 5 Evaluation of Intervention

D 5.1 Flagging

Proposition 6 and Corollary 5: effect of flagging with or without competition

Let the difference of incentive to invest with flagging be $FDV(z,q;\rho) := \Delta V_M(z;\rho) - \Delta V_D(z,q;\rho)$. Let us first rewrite:

$$\frac{\partial FDV(z,q;\rho)}{\partial \rho} = V_F + (1-q)V_{TF} - (1-q)V_{T\varnothing} - qV_{FT} - 2(1-q)(1-q)V_{FF} + (1-q)(1-2\rho)V_{F\varnothing}$$

To prove that this derivative is positive, I show that $\frac{\partial^2 FDV(z,q;\rho)}{\partial\rho\partial q} \geq 0$, so that $\frac{\partial FDV(z,q;\rho)}{\partial\rho} \geq \left.\frac{\partial FDV(z,q;\rho)}{\partial\rho}\right|_{q=1/2}$ I then move to show that $\left.\frac{\partial FDV(z,q;\rho)}{\partial\rho}\right|_{q=1/2} > 0$.

To show that $\frac{\partial^2 FDV(z,q;\rho)}{\partial\rho\partial q} \geq 0$, let us rewrite:

$$
\begin{aligned}
\frac{\partial^2 FDV(z,q;\rho)}{\partial\rho\partial q} &= -V_{TF} + V_{T\varnothing} - V_{FT} + 2(1-\rho)V_{FF} - (1-2\rho)V_{F\varnothing} \\
&= -\tfrac{p_T}{p_T+p_F}\Big(1-(1-\tfrac{p_T+p_F}{2})^d\Big) + \Big(1-(1-\tfrac{p_T}{2})^d\Big) - \tfrac{p_F}{p_T+p_F}\Big(1-(1-\tfrac{p_T+p_F}{2})^d\Big) \\
&\quad + (1-\rho)\Big(1-(1-p_F)^d\Big) - (1-2\rho)\Big(1-(1-\frac{p_F}{2})^d\Big) \\
&= \rho + (1-\tfrac{p_T+p_F}{2})^d - (1-\tfrac{p_T}{2})^d - (1-\rho)(1-p_F)^d + (1-2\rho)(1-\tfrac{p_F}{2})^d \\
&= \rho\Big[1+(1-p_F)^d - 2(1-\tfrac{p_F}{2})^d\Big] + \Big[(1-\tfrac{p_T+p_F}{2})^d - (1-\tfrac{p_T}{2})^d - (1-p_F)^d + (1-\tfrac{p_F}{2})^d\Big]
\end{aligned}
$$

Now, this expression is the sum of two positive terms. Indeed:

- the first term is increasing in p_F so that $1+(1-p_F)^d-2(1-\frac{p_F}{2})^d \geq 1+(1-p_F)^d-2(1-\frac{p_F}{2})^d\Big|_{p_F=0} = 0$
- the second term is increasing in p_T so that $(1-\frac{p_T+p_F}{2})^d - (1-\frac{p_T}{2})^d - (1-p_F)^d + (1-\frac{p_F}{2})^d \geq$ $(1-\frac{p_T+p_F}{2})^d - (1-\frac{p_T}{2})^d - (1-p_F)^d + (1-\frac{p_F}{2})^d\Big|_{p_T=p_F} = 0$

We can thus conclude that $\frac{\partial^2\left(\Delta V_M(z;\rho)-\Delta V_D(z,q;\rho)\right)}{\partial\rho\partial q} \geq 0$

Let us now show that $\frac{\partial FDV(z,q;\rho)}{\partial\rho}\Big|_{q=1/2} > 0$. We can rewrite:

$$
\frac{\partial FDV(z,q;\rho)}{\partial\rho}\Big|_{q=1/2} = V_F + \frac{1}{2}V_{TF} - \frac{1}{2}V_{T\varnothing} - \frac{1}{2}V_{FT} - (1-\rho)V_{FF} + \frac{1}{2}(1-2\rho)V_{F\varnothing}
$$

Noting that $V_{FF} = \frac{1}{2}V_F$, we get:

$$
\begin{aligned}
\frac{\partial FDV(z,q;\rho)}{\partial\rho}\Big|_{q=1/2} &= \left[\frac{(1+\rho)}{2}V_F - \rho V_{F\varnothing}\right] + \frac{1}{2}\left[V_{TF} - V_{FT} - V_{T\varnothing} + V_{F\varnothing}\right] \\
&= \left[\tfrac{1+\rho}{2}V_F - \rho V_{F\varnothing}\right] + \tfrac{1}{2}\left[\tfrac{p_T-p_F}{p_T+p_F}\Big(1-(1-\tfrac{p_T+p_F}{2})^d\Big) + (1-\tfrac{p_T}{2})^d - (1-\tfrac{p_F}{2})^d\right]
\end{aligned}
$$

Again, this is the sum of two positive terms.

- The first term is positive as $\frac{1+\rho}{2} > \rho$ and $V_F \geq V_{F\varnothing}$. Note that the term is strictly positive for $p_F > 0$.
- It is more cumbersome to show that the second term is positive. We show that it is non-decreasing in d and then show it is weakly positive for $d = 1$.To show that it is non-decreasing in d, we proceed by induction. For ease of notation, let us define for this proof:

$$
E(d) := \tfrac{p_T-p_F}{p_T+p_F}\Big(1-(1-\tfrac{p_T+p_F}{2})^d\Big) + (1-\tfrac{p_T}{2})^d - (1-\tfrac{p_F}{2})^d
$$

Then,

$$E(d) - E(d+1) = -\frac{p_T - p_F}{p_T + p_F}\frac{p_T + p_F}{2}(1 - \frac{p_T + p_F}{2})^d + \frac{p_T}{2}(1 - \frac{p_T}{2})^d - \frac{p_F}{2}(1 - \frac{p_F}{2})^d$$
$$= \underbrace{\frac{p_T}{2}\left[(1 - \frac{p_T}{2})^d - (1 - \frac{p_T + p_F}{2})^d\right]}_{:=A} - \underbrace{\frac{p_F}{2}\left[(1 - \frac{p_F}{2})^d - (1 - \frac{p_T + p_F}{2})^d\right]}_{:=B}$$

Therefore $E(d) - E(d+1) < 0$ for $A < B$. We want to show that if $E(d)$ it is non-decreasing at some d', then it is non-decreasing for all subsequent $d > d'$. The inductive step requires us to show that for $A \leq B$, $E(d+1) - E(d+2) \leq 0$. This is indeed the case as:

$$E(d+1) - E(d+2) = \frac{p_T}{2}\left[(1 - \frac{p_T}{2})^d(1 - \frac{p_T}{2}) - (1 - \frac{p_T + p_F}{2})^d(1 - \frac{p_T}{2}) + (1 - \frac{p_T + p_F}{2})^d(-\frac{p_F}{2})\right]$$
$$- \frac{p_F}{2}\left[(1 - \frac{p_F}{2})^d(1 - \frac{p_F}{2}) - (1 - \frac{p_T + p_F}{2})^d(1 - \frac{p_F}{2}) + (1 - \frac{p_T + p_F}{2})^d(-\frac{p_T}{2})\right]$$
$$= (1 - \frac{p_T}{2})A - (1 - \frac{p_T}{2})B$$

Because $(1 - \frac{p_T}{2}) \leq (1 - \frac{p_F}{2})$ and $A \geq 0$, we do have: $A \leq B \Rightarrow (1 - \frac{p_T}{2})A \leq (1 - \frac{p_T}{2})B$. Finally, it is easy to verify that for $d = 1$, $A = B$, so that $E(1) - E(2) = 0$. [32]

We can thus conclude $\frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2} \geq 0$ for any $p_F \geq 0$ and $\frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2} > 0$ for any $p_F > 0$, which concludes the proof of the stronger effect of flagging in a monopoly.

To show that there exists a level of flagging that makes competition detrimental to the producers' incentive to invest, it is enough to note that $\Delta V_M(z;\rho)$ is continuous in ρ and that with $\rho = 1$, $\Delta V_M(z;1) > \Delta V_D(z,q;1)$ since $\Delta V_M(z;1) - \Delta V_D(z,q;1) = V_T - qV_{TT} - (1-q)V_{T\varnothing} > 0$. To show that any outcome $q_D^* > q_M^*$ is reproducible in a monopoly, notice $\Delta V_D(z,q;1) > \Delta V_D(z,q;0)$.

D 6 Discussion

Irregular Networks and Seeds' Selection

Denote $\Delta V(d_j)$ the producer's incentive to invest in a regular network of degree d_j as derived in the main text. Let $\Delta V(\delta)$ be the producer's incentive to invest in a network with degree distribution δ. $\Delta V(d_j)$ is continuous in d_j; hence, there exists a representative degree $\tilde{d}$ such that $\Delta V(\tilde{d}) = \sum_{d_j} \delta(d_j)\Delta V(d_j)$. The equilibria can be characterized applying Proposition 1 and 3 with $d = \tilde{d}$. The role of private knowledge is qualitatively the same for every $\Delta V(d_j)$, hence for $\Delta V(\delta)$: Proposition 2 and Corollary 2 apply. The role of connectivity on the producer's incentive to invest can be assessed in terms of $\tilde{d}$. The effects of competition through connectivity also carry through as $\Delta V_M(d_j) - \Delta V_D(d_j)$, is continuous in d_j; hence, there exists a representative degree $\check{d}$ such that $\Delta V_M(\check{d}) - \Delta V_M(\check{d}) = \sum_{d_j} \delta(d_j)\Big(\Delta V_M(d_j) - \Delta V_D V(d_j)\Big)$. All other results directly apply.

[32]Note that if $A > 0$ and $p_T > p_F$, $A \leq B \Rightarrow (1 - \frac{p_T}{2})A > (1 - \frac{p_T}{2})B$. Therefore, the term is strictly increasing for any $d \geq 2$, $p_T > p_F$.

Behavioral Biases and Partisanship

Consider confirmation bias. When $S = -$, with probability ϵ, seeds misinterpret the news content and believe it corresponds to their private signal. Then, the probability for an article to be shared becomes: $p_T = \frac{b}{K}\Big[\Big(\gamma + (1-\gamma)\epsilon\Big)z_+ + (1-\gamma)(1-\epsilon)z_-\Big]$ and $p_F = \frac{b}{K}\Big[\Big(\gamma\epsilon + (1-\gamma)\Big)z_+ + \gamma(1-\epsilon)z_-\Big]$. The analysis would then be directly applicable. For instance, take a monopoly. $\frac{\partial \Delta V(z)}{\partial \epsilon} = -d(z_+ - z_-)\Big(\gamma(1-p_F)^{d-1} - 1 - \gamma(1-p_T)^{d-1}\Big) \leq 0$ as $z_+ - z_- \geq 0$, $\gamma < 1-\gamma$ and $(1-p_F)^{d-1} \leq (1-p_T)^{d-1}$. The same applies to the duopoly case.

Consider sensationalism. Seeds are assumed to enjoy sharing an article that is not congruent with their private signal because of their taste for sensationalism. In particular, assume that they get a utility premium from such a share of ϵ. Their payoff from sharing is then:

$$u(\text{sharing article } n|\omega = w, S) = \begin{cases} 1 + \epsilon \mathbb{1}_{S=-} & \text{if } n = w \\ -1 + \epsilon \mathbb{1}_{S=-} & \text{otherwise} \end{cases}$$

It follows that their expected utility from sharing when $S = -$ is $2p(n,s) - 1 + \epsilon$, so that $z_{-|n} > 0$ if $q > \tilde{t}_n = \bar{t}_n = \frac{(1-\epsilon)\gamma \Pr(\omega \neq n)}{(1-\epsilon)\gamma \Pr(\omega \neq n) + \epsilon(1-\gamma)\Pr(\omega = n)}$. Therefore, the news quality is now bounded by $\tilde{t}_1$ and $\tilde{t}_1 < \bar{t}_1$.

Beyond Visibility

Consider that news quality affects reputation benefits continuously. In particular, assume that the producers revenues can be written $\mathbb{E}(R_k|q) + \nu q$. Then the best-response of the producer would be: $\tilde{q}^*(z) = c^{-1}(\Delta V(z) + \nu) > c^{-1}(\Delta V(z)) = q^*(z)$. However, to understand whether news quality could surpass $\bar{t}_1$, one needs to understand whether the producers' best-response might lie completely above the seeds' best-response. This would occur if $c^{-1}(\nu) = c^{-1}(\Delta V((1,1)) + \nu) > \bar{t}_1$. When this is the case, the equilibrium news quality is $c^{-1}(\nu)$; otherwise, one can apply Proposition 1 and 3 with $\tilde{q}^*(z) = c^{-1}(\Delta V(z) + \nu)$.

Consider that news quality affects reputation benefits discretely. In particular, assume that the producers revenues can be written $\mathbb{E}(R_k|q) + \nu \mathbb{1}_{q > \bar{q}}$. Then the best-response of neither side of the market would be affected. However, if $q^* < \bar{q}$, the producer would invest $\bar{q}$ iff $\mathbb{E}(R_k|\bar{q}) + \nu - C(\bar{q}) > \mathbb{E}(R_k|q^*) + \nu - C(q^*)$.

D A Asymmetric Loss From Sharing

Proposition 1.A: characterization of the equilibrium without competition

First notice that any positive equilibrium investment has to lie within $[\underline{t}_0^\lambda, \bar{t}_1^\lambda]$. Indeed, it is easy to see that for any $q < \underline{t}_0^\lambda$, no news is ever shared so that the producer has no incentive to invest; likewise, $q = \bar{t}_1^\lambda$ is enough to insure that the producer's news is always share, so that investing more than this does not increase the producer's benefit.

It follows that, if $1/2 \geq \bar{t}_1^\lambda$, the producer will never want to invest more than 1/2 – intuitively, the producer's best response lies above the seeds' best response. If $\bar{q}_0 < \underline{t}_0^\lambda$ and $\bar{q}_1 < \underline{t}_1^\lambda$, it is too costly for the producer to invest more than 1/2, as for any sharing strategy z, the marginal benefit from investing $q > 1/2$ is lower than its marginal cost – intuitively, the producer's best response lies below the seeds' best response. Indeed, we know that $q < \underline{t}_0^\lambda$ cannot be an equilibrium. For any $q \in [\underline{t}_0^\lambda, \underline{t}_1^\lambda)$, by definition $\bar{q}_0 = c^{-1}(\Delta V(z_{\bar{0},0}, 0, 0, 0)) \geq c(\Delta V(z^*(q))$ so that $c(q) \geq c(\underline{t}_0^\lambda) > c(\bar{q}_0) \geq c(\Delta V(z^*(q))$. Likewise, for $q \in [\underline{t}_1^\lambda, \bar{t}_0^\lambda)$, as $\bar{q}_1 < \underline{t}_1^\lambda$, we have $c(q) \geq c(\underline{t}_1^\lambda) > c(\bar{q}_1) \geq c(\Delta V(z^*(q))$. For any $q \geq \bar{t}_0^\lambda$, $c(q) \geq c(\bar{t}_0^\lambda) > c(\max\{\bar{q}_0, \bar{q}_1\}) \geq c(\Delta V(z^*(q))$ Now, let us understand what happens if positive investment is possible. If $\bar{q}_1 < \underline{t}_1^\lambda$, as argued above, the investment has to be such that $q \in [\underline{t}_0^\lambda, \underline{t}_1^\lambda)$. Because $\bar{q}_0 > \underline{t}_0^\lambda$, and $q^*(z)$ continuous, there there must exist some $z^*_{T|0}$ such that $c^{-1}(\Delta V(z^*_{T|0}, 0, 0, 0)) = \underline{t}_0^\lambda$. If $\tilde{q}_0 < \underline{t}_0^\lambda$, the maximal investment equilibrium is thus $\underline{t}_0^\lambda$; otherwise, $\tilde{q}_0$ is an equilibrium as $x_{\tilde{0}0} \in [\underline{t}_0^\lambda, \underline{t}_1^\lambda)$ and by definition, $c(\tilde{q}_0) = \Delta V(1, 0, 0, 0)$, and leads to more investment. A similar reasoning applies to $\bar{q}_1 \geq \underline{t}_1^\lambda$ and $\tilde{q}_1 \leq \bar{t}_0^\lambda$.
Finally, if $\bar{q}_1 \geq \underline{t}_1^\lambda$ and $\tilde{q}_1 > \bar{t}_0^\lambda$, because $q^*(z)$ is decreasing in $z_{-|0}$ and $z_{-|1}$, and continuous, there must exist a $q' \geq \bar{t}_0^\lambda$ and a $z' = (1, 1, z_{-|0}, z_{-|1})$ such that $c(q') = \Delta V(z')$. It is easy to verify that $\max\{\bar{t}_0^\lambda, \min\{q^*(1,1,1,0), \bar{t}_1^\lambda\}\}$ yield the highest q on $[\underline{t}_0^\lambda, \underline{t}_1^\lambda]$ such that $c(q') = \Delta V(z')$.

Additional Remark: existence of other equilibria

(i) $x_k^*((0,0), z_{-k}, q_{-k}) \in [0, \min\{1/2, \underline{t}^\lambda\}]$ and $z_{T_k}(q_k) = z_{F_k}(q_k) = 0$ for $q_k \in [0, \min\{1/2, \underline{t}^\lambda\}]$.

(ii) Notice that for $p_{X-k} = q_{-k} = 0$, $\Delta V_k(z, q_{-k}) = (1-b)\left[(1 - 1/2b\underline{t}^\lambda)^d - (1 - 1/2b\bar{t}^\lambda)^d\right]$, which corresponds to the monopoly case up to 1/2, which is accounted for when defining $\bar{q}_k$. Furthermore, it is a best response for $-k$ to not invest if $z_{-k} = (0,0)$, which is a best response if $q_k \in [0, \min\{1/2, \underline{t}^\lambda\}]$.

Proposition 3.A: characterization of the equilibrium with competition

First note that any equilibrium investment bigger than 1/2 has to lie in $[\underline{t}^\lambda, \bar{t}^\lambda]$. Indeed, recall that $\Delta V_D((0,0), q) = \Delta V_D((1,1), q) = 0$. Hence, clearly, for any $q < \min\{\underline{t}^\lambda, \bar{q}\}$ or $q > \max\{\bar{t}^\lambda, 1/2\}$, $c(q) > 0 = \Delta V_D(z^*(q), q)$, which would be suboptimal for the producer.

Given $1/2 < \bar{t}^\lambda$ and $\underline{t}^\lambda \leq \bar{q}_D$, different parameters allow for two cases:

1. If $c(\underline{t}^\lambda) \leq \Delta V_D((1,0), \underline{t}^\lambda)$ and $\Delta V_D((1,0), \bar{t}^\lambda) < c(\bar{t}^\lambda)$, then $\exists \tilde{q} \in [\underline{t}^\lambda, \bar{t}^\lambda]$: $c(\tilde{q}) = \Delta V((1,0), \tilde{q})$. Indeed, recall that c is weakly increasing in q and $\Delta V_D((1,0), q)$ strictly decreasing in q. Clearly, $(\tilde{q}, (1,0))$ is a NE.
   It is the symmetric NE which leads to the highest investment. Indeed, assume there exists another symmetric equilibrium with investment $q' > q_D^*$. As argued above, $q' \in \{\underline{t}^\lambda, \bar{t}^\lambda\}$.

   For $q' = \bar{t}^\lambda > q_D^*$ to be part of an equilibrium, there must exist a $z' = (1, z'_F)$ with $z'_F > 0$ such that $V_D(z', q') = c(\bar{t}^\lambda)$. It is impossible, because $c(\bar{t}^\lambda) > c(q_D^*) = \Delta V_D((1,0), q_D^*) >$

$\Delta V_D((1,0),\bar{t}^\lambda) > \Delta V_D((1,z_F),\bar{t}^\lambda)\ \forall z_F > 0$, where the last inequality uses that $\Delta V_D(z;q)$ is decreasing in z_F.

2. If $c(\underline{t}^\lambda) > \Delta V_D((1,0),\underline{t}^\lambda)$, then $\exists \tilde{z}_T \in [\bar{z}_T^D, 1]$: $c(\underline{t}^\lambda) = \Delta V(\tilde{z}_T, \underline{t}^\lambda)$. Indeed, by assumption $\Delta V_D((\bar{z}_T^D,0);\underline{t}^\lambda) > c(\underline{t}^\lambda) > \Delta V_D((1,0);\underline{t}^\lambda)$, and $\Delta V_D(z;q)$ is continuous and decreasing on $[\bar{z}_T^D, 1]$. Clearly, $(\underline{t}^\lambda, (\tilde{z}_T, 0))$ is a NE.

3. If $c(\bar{t}^\lambda) < \Delta V_D((1,0),\bar{t}^\lambda)$, then $\exists \tilde{z}_F \in [0,1]$: $c(\bar{t}^\lambda) = \Delta V_D((1,\tilde{z}_F),\underline{t}^\lambda)$. Indeed, by assumption $\Delta V_D((0,0);\bar{t}^\lambda) = 0 < c(\bar{t}^\lambda) < \Delta V_D((1,0);\underline{t}^\lambda)$, and $\Delta V_D(z;q)$ is continuous and decreasing in z_F. Clearly, $(\bar{t}^\lambda, (1, \tilde{z}_F))$ is a NE.

Additional Corollary and Remark 2.A: discussion of other equilibria

About the equilibria's stability:
First if $1/2 > \underline{t}^\ell$, $\Delta V_D(z, \underline{t}^\ell) > \underline{t}^\ell$ for any z. Because $\Delta V_D(\bar{z}_T, q)$ is continuous and increasing on $[0, \bar{z}_T]$, we know that, given any q, $c^{-1}(\Delta V_D(z, \underline{t}^\ell))$ crosses $z_v^*(q)$ only once. So for any q_0, there is a unique $q', z^*(q')$. We pick the q_0 that leads to equilibrium $q_0, z^*(q_0)$, which must be unique. If $\bar{q}_D \geq \underline{t}^\lambda > 1/2$, then given any $q = \underline{t}^\lambda$, $c^{-1}(\Delta V_D(z,q))$ crosses $z_v^*(q)$ twice: once for some $z'_T < \bar{z}_T$ with $\Delta V_D(z'_T, q)) = c(\underline{t}^\lambda)$; and once afterwards. The slope of $\Delta V_D(z,q))$ in $z'_T < \bar{z}_T < 1$ is strictly increasing. The investment required for seeds to share upon receiving congruent private signal with probability z'_T is equal to $\underline{t}^\lambda$ with slope 0. Therefore, the equilibrium $(\underline{t}^\lambda, (z'_T, 0))$ cannot be stable. In particular, any stable equilibrium must have $z_k, z_\ell > z'_T$.
Finally, we prove that q_D^* is the only stable equilibrium with symmetric investment by noting that $q_k = q_\ell = q^*$ implies $z_k = z_\ell > z'_T$. Indeed, any equilibrium investment q^* requires $\Delta V_u(z_k, z_\ell; q^*) = \Delta V_v(z_k, z_\ell; q^*)$. Now, because $\frac{\partial \Delta V_k}{\partial z_k} \neq -\frac{\partial \Delta V_k}{\partial z_v}$ for every $z_k, z_\ell > z'_T$, the unique z_k, z_ℓ supporting q_D^* must be defined by $\Delta V(z,q)$; therefore, $z_u = z_v$.

About other asymmetric equilibria (i) $\bar{q}_m < \underline{t}^\lambda$ means that, even if the network is free of competition, there is no sharing rule that could convince a producer to invest. Therefore, no investment can occur.
(ii) $\bar{q}_D < \underline{t}^\lambda$ means that there does not exist a symmetric equilibrium with positive investment, i.e. $\forall z_k, \underline{t}^\lambda > \Delta V_k(z_k, z_k; \underline{t}^\lambda)$. Furthermore, no other equilibrium with positive investment can exist. By using the proof of the Additional Corollary above, $z_k = z_\ell$ if $q_k = q_\ell$, so $z_k \neq z_\ell$ is inconsistent with $q_k < q_\ell$. Finally, $z_k < z_\ell$ and $q_k = \underline{t}^\lambda < q_\ell$ cannot be an equilibrium as $\underline{t}^\lambda > \Delta V_k(z_k, z_k; \underline{t}^\lambda) > \Delta V_k(z_k, z_\lambda; q_\ell)$ for $z_\lambda > z_k$ $q_\lambda > \bar{t}^\lambda$.
(iii) $\underline{t}^\lambda = q_m$ implies that $\Delta V_k(z_k, z_\lambda; q_\ell) < \Delta V_k(z_k, (0,0); 0) < \underline{t}^\lambda$ for any z_k. Hence x_k cannot exceed $\underline{t}^\lambda$. As the same applies to q_ℓ, both producers must be investing the minimum $\underline{t}^\lambda$ if they do invest. Furthermore, $x_k = q_\ell$ implies $z_k = z_\ell$. (iv) Assume $q_D^* \in (\underline{t}^\lambda, \bar{t}^\lambda)$. Assume that there exists an $q_k > q_\ell$, with $(q_k, 1_\ell) \in (\underline{t}^\lambda, \bar{t}^\lambda)$. Then, $c(q_k) = \Delta V_k((1,0),(1,0),q_\ell)$ and $c(q_\ell) = \Delta V_\ell((1,0),(1,0),q_k)$, so that $c(q_k) - c(q_\ell) = S(q_k - q_\ell)$, which is impossible if c has a slope different from S.
If $q_D^* \in \{\underline{t}^\lambda, \bar{t}^\lambda\}$, then for any q_ℓ, $c(q_k) \neq \Delta V_k((1,0),(1,0),q_\ell)$ so there cannot be any equilibrium where both producer invest away from the minimum.

D B Attention-Seeking Seeds

B.2 Seeds' Best Response

The Probability of Being Read by a Follower (as an Seed)

Because we take the perspective of a given seed i, we now define the random variable $S \sim \mathcal{B}(d-1, p_X)$ as the number of times i's followers' neighbors' have shared, **in addition to** i.

$$
\begin{aligned}
\mathbb{E}\left(\frac{1}{S+1}\right) &= \sum_{s=0}^{d-1} \frac{1}{s+1}(p_X)^s(1-p_X)^{d-1-s}\frac{(d-1)!}{s!(d-1-s)!} \\
&= \frac{1}{dp_X}\sum_{s=0}^{d-1}(p_X)^{s+1}(1-p_X)^{d-s-1}\frac{d!}{(s+1)!(d-s-1)!} \\
&= \frac{1}{dp_X}\sum_{\tilde{s}=1}^{d}(p_X)^{\tilde{s}}(1-p_X)^{d-\tilde{s}}\frac{d!}{\tilde{s}!(d-\tilde{s})!} \\
&= \frac{1}{dp_X}\left[\sum_{\tilde{s}=0}^{d}(p_X)^{\tilde{s}}(1-p_X)^{d-\tilde{s}}\frac{d!}{\tilde{s}!(d-\tilde{s})!} - (p_X)^0(1-p_X)^{d-0}\frac{d!}{0!d!}\right] \\
&= \frac{1}{dp_X}\left[1-(1-p_X)^d\right]
\end{aligned}
$$

where $\tilde{s} = s + 1$.

Lemma 7: true news are shared more

By contradiction, suppose that $z_F^*(q) > z_T^*(q)$, so that $p_F > p_T$. For this to be sustainable, we need $\mathbb{E}(\#\text{likes}|s_i = T) \leq \tau \leq \mathbb{E}(\#\text{likes}|s_i = F)$. However, this happens only when:

$$
\frac{\gamma}{1-\gamma} < \frac{p_T}{(1-(1-p_T)^d)}\frac{(1-(1-p_F)^d)}{p_F}
$$

Indeed, we have:

$$
p_\ell(T;x_\ell)\gamma\frac{1-b}{p_T}\left(1-(1-p_T)^d\right)+(1-p_\ell(T;x_\ell))(1-\gamma)\frac{1-b}{p_F}\left(1-(1-p_F)^d\right) < p_\ell(F;x_\ell)\gamma\frac{1}{p_T}\left(1-(1-p_T)^d\right)+(1-p_\ell(F;x_\ell))(
$$

$$
[p_\ell(T;x_\ell)-p_\ell(F;x_\ell)]\gamma\frac{1}{p_T}\left(1-(1-p_T)^d\right) < [p_\ell(T;x_\ell)-p_\ell(F;x_\ell)](1-\gamma)\frac{1}{p_F}\left(1-(1-p_F)^d\right)
$$

$$
\gamma\frac{1}{p_T}\left(1-(1-p_T)^d\right) < (1-\gamma)\frac{1}{p_F}\left(1-(1-p_F)^d\right)
$$

Because $\frac{\gamma}{1-\gamma} > 1$, we need $\frac{p_T}{(1-(1-p_T)^d)}\frac{(1-(1-p_F)^d)}{p_F} > 1$. Now $f(x) = \frac{x}{(1-(1-x)^d)}$ is an increasing function; indeed, we have:

$$
\text{sign}\left(\frac{\partial f}{\partial x}\bigg|_{x\in(0,1)}\right) = \text{sign}\left(\frac{1-(1-x)^d - xd(1-x)^{d-1}}{(1-(1-x)^d)^2}\right) = \text{sign}\left(1-(1-x)^d - xd(1-x)^{d-1}\right)
$$

Now, $g(x) := (1-x)^d - xd(1-x)^{d-1} < 1$ over $x \in [0,1]$. Indeed, $g(0) = 1, g(1) = 0$, and g strictly decreasing in-between, since:

$$\left.\frac{\partial g}{\partial x}\right|_{x\in[0,1]} = (d-1)(1-x)^{d-2}\Big[(1-x) - \big(1 + x(d-1)\big)\Big] = (d-1)(1-x)^{d-2}[-xd] \leq 0$$

As f is continuous on $[0,1]$ with $f(0) = \frac{1}{d}$ and $f(1) = 1$, f is indeed increasing.

Therefore, we conclude that $p_T > p_F$, a contradiction.

Proposition 7: characterization of the seeds' best-response

(i) Given $\tau \leq \gamma\delta$, if $q \geq \hat{q}(\tau)$, it is easy to verify that always sharing is a best response, i.e. $\mathbb{E}(\#\text{ likes } | T, \mathbf{z_{-i}} = (\mathbf{1},\mathbf{1})) > \mathbb{E}(\#\text{ likes } | F, \mathbf{z_{-i}} = (\mathbf{1},\mathbf{1})) \geq \tau$. Indeed, if every other seeds always share, $p_T = p_F = b$, then the expected number of likes upon receiving a false signal is:

$$[p(F;q)\gamma + (1-p(F,q))(1-\gamma)]\frac{1-b}{b}(1-(1-b)^d)$$

Which is higher than τ iff: $p(F;q) \geq \frac{\frac{\tau}{\delta(b)} - (1-\gamma)}{2\gamma - 1}$. Given that $p(F;q) = \frac{(1-\gamma)q}{(1-\gamma)q + \gamma q}$, this happens iff $q \geq \frac{\gamma}{2\gamma-1}\frac{\tau - (1-\gamma)\delta}{\tau} = \hat{q}(\tau)$. Because $\tau \leq \gamma\delta$, $\hat{q}(\tau) \leq 1$; for $\tau < (1-\gamma)\delta$, $\hat{q}(\tau) < 0$, the condition is always fulfilled.

For proving the converse, recall that $\frac{1-(1-p)^d}{p}$ is decreasing in p. Suppose there exists another $p' < b$ that is sustained in equilibrium. Then, $\mathbb{E}(\#\text{ likes } | F, p' < b) > \mathbb{E}(\#\text{ likes } | F, p = b) \geq \tau$ so that i would have an incentive to deviate towards $p_i = 1$.

(ii) Likewise, given $\tau \geq (1-\gamma)d(1-b)$, if $q \leq \underset{\smile}{q}(\tau)$, then even $d(1-b)[p(T,q)\gamma + (1-p(T,q))(1-\gamma)]$ likes are not enough for anyone to share, so that $(0,0)$ is a the best response to any p given q and τ. Indeed, if every other seeds never share, $p_T = p_F = 0$. Then, the expected number of likes upon receiving a true signal is:

$$[p(T;q)\gamma + (1-p(T,q))(1-\gamma)]d(1-b)$$

Which is lower than τ iff: $p(T;q) \leq \frac{\frac{\tau}{d(1-b)} - (1-\rho)}{2\rho - 1}$. Given that $p(T;q) = \frac{\gamma q}{\gamma q + (1-\gamma)q}$, this happens iff $q \leq \frac{1-\gamma}{2\gamma-1}\frac{\tau - (1-\gamma)d(1-b)}{d(1-b) - \tau} = \underset{\smile}{q}(\tau)$. Because $\tau \geq (1-\gamma)d(1-b)$, $\underset{\smile}{q}(\tau) \geq 0$; for $\tau > \gamma d(1-b)$, $\hat{q}(\tau) > 1$, the condition is always fulfilled.

(iii) Again, we can simply verify that, given $\tau \in [\tau_1, \tau_2]$, if $q \in [q_1(\tau), q_2(\tau)]$ and every $-i$ seed is sharing only when they receive a positive signal, $\mathbb{E}(\#\text{ likes } | T) \geq \tau \geq \mathbb{E}(\#\text{ likes } | F)$. Any $z_{-i,F} > 0$ would lower the $\mathbb{E}(\#\text{ likes } | F)$ further away from τ, making i set $z_{i,F} = 0$; any $z_{-i,T} < 1$ would increase the $\mathbb{E}(\#\text{ likes } | T)$ further away from τ, making i set $z_{i,T} = 1$.

Indeed, if every other seeds share only upon receiving a positive signal, $p_T = 1, p_F = 0$. Then, i also only shares upon receiving a positive signal iff:

$$p(T;q)\frac{1-(1-b\gamma)^d}{b} + (1-p(T,q))\frac{1-(1-b(1-\gamma))^d}{b}) > \tau > p(F;q)\frac{1-(1-b\gamma)^d}{b} + (1-p(F,q))\frac{1-(1-b(1-\gamma))^d}{b}$$

Which is possible only if $\tau \in [\tau_1, \tau_2]$. Note that if $\tau \in \{\tau_1, \tau_2\}$, $q_1 = q_2 \in \{0, 1\}$.

Replace $p(T; q)$ and $p(F; q)$ by the adequate expression to find the range q_1, q_2.