Overborrowing in the North and the South

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Abstract

We study the effects of the cost of credit on firm bankruptcy rates. A reduction in the cost of credit makes new firms less indebted and less prone to bankruptcy but gives mature firms stronger incentive to accumulate debt to dilute the value of past debt. The incentive to overaccumulate debt is stronger when business idiosyncratic risk is high, and the equity market is less developed as in the South. North-South differences in bankruptcy rates falls as the cost of credit increase. Credit can be excessively cheap from the point of view of maximizing steady state welfare. Maximizing steady state welfare requires a lower interest rate in the South than in the North. During the transition from a low to a high cost of credit equilibrium the failure rate of firms undershoots its long run steady state equilibrium value.

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1 Introduction

Cheaper credit stimulates business creation as well as it makes new businesses less indebted and thereby less prone to bankruptcy, which are the *traditional channel* whereby easier credit stimulates production (Cooley and Quadrini 2001, Midrigan and Xu 2014, Moll 2014, Itskhoki and Moll 2019). Cheaper credit also increases the incentives of mature firms to overborrow and to declare default. This induces a trade-off between the benefits of cheaper credit through the traditional channel and the welfare costs due to the overborrowing of mature firms. We study the effects of changes in the cost of credit on firm bankruptcy rates. A reduction in the cost of credit makes new firms less indebted and less prone to bankruptcy but gives mature firms stronger to increntive to accumulate debt to dilute the value of past debt. The incentive to overaccumulate debt is stronger when business idiosyncratic risk is high, and the market for a change of control is less developed as in the South. North-South differences in bankruptcy rates falls as the cost of credit increases. Credit can be excessively cheap from the point of view of maximizing steady state welfare. Maximizing steady state welfare requires a lower interst rate in the South than in the North.

We study the problem of a firm in a province and study the effects of a change in the cost of credit on the firm bankruptcy rate of mature firms in general equilibrium, both in steady state and in the transition to a new steady state with a different cost of credit. We calibrate the economy. We study how the steady state equilibrium of the economy changes in response to a change in the cost of credit and how the effect varies across provinces with different structural parameters. We study the transition to a high interest rate equilibrium.

There are large disparities in business failure rate across regions within a country and Italy is a good example of them. Hereafter we refer to the low GDP per capita provinces of Italy as the South and to the high GDP per capita provinces as the North. For expositional simplicity we take the difference in GDP per capita between the North and the South to be equal to two standard deviations of the province level average logged GDP per capita, which is roughly equal to 60 percent. We study the contribution of differences in firm dynamics over the 2000's and how they are related to the financial position of firms and the conditions in the credit market. The average failure rate of businesses is higher in the South than in the North and there are differences in its age profile: the failure rate of young firms (with less than 3 years of age) is similar across provinces; the failure rate of mature firms (with more than 5 years of age) is higher by roughly 1 percentage point in the South than in the North. Differences in the age profile of failure rates are related to differences in the age profile of firm debt: as they age, firms in the South accumulate more debt (relative to their productivity). Considering the period 2010-2015, an average firm in the South starts its life with a leverage ratio (total debt over value added ratio) which is 40 percentage points lower than the analogous leverage ratio in the North. After more than 15 years of life, the leverage ratio of a firm in the South is on average 20 percentage points higher than in the North. The age profiles of firm employment size and labor productivity are relatively similar in the North and the South, with a North-South gap that remains relatively constant as firms age: employment size and labor productivity increase with firm age both in the South and in the North with a relatively constant North-South percentage gap in employment size and labor productivity roughly equal to 30 percent.

To further investigate the hypothesis that mature firms in the South are more fragile and more prone to fail, we study the response of firms to idiosyncratic shocks in firm demand. We propose a novel strategy for identifying unexpected shifts in firm demand using a representative sample of mature firms with a sizeable panel component (INVIND dataset) which contains three unique pieces of information: (i) firm level information on both firm sales and prices, which allows us to identify prices and quantities; (ii) information on both expected and realized values, which allow us to recover expectation error (Wold innovations); and (iii) ex-ante information, self-reported by firms, on the expected elasticity of demand to firm prices. Combining i-iii we can then non-parametrically identify unexpected shifts in firm demand and study firm responses. We find that the variance of idiosyncratic demand shocks (a measure of idiosyncratic business risk) is higher in the South than in the North. The observed firm responses are consistent with the predictions of a canonical demand shock: both prices and quantities increase in response to a positive shift in demand while the probability of going out of business falls. There is indication that in response to an increase in firm demand, the failure probability falls substantially more in the South than in the North and that differences in the response of business failure are due to the greater debt accumulated by firms in the South than in the North. The quantitative differences are sizeable: in response to a doubling in firm demand, the failure probability falls by almost 4 percentage points in provinces with a GDP per capita equal to minus 50 percent of the national average while it barely changes in the provinces with the highest GDP per capita of Italy. Coupled with the fact that business idiosyncratic risk is higher in the South than in the North, this evidence corroborates the claim that mature firms in the South are more fragile and more likely to fail.

The fact that firms in the South starts their life with a lower leverage ratio and end up being more indebted and relatively more likely to fail as they mature is at variance with the conventional view that firms in the South suffer because of low credit availability: the traditional model of firm dynamics with financial constraints implies that firms start financially constrained and progressively become insulated from local financial conditions thanks to the accumulation of firm internal revenue, see for example Cooley and Quadrini (2001), Clementi and Hopenhayn (2006), and Michelacci and Quadrini (2009). These models predict that in regions with more binding financial conditions, firms fail more at birth and then progressively converge to their long-run business failure rate unaffected by local financial conditions. Instead, the evidence that mature firms in the South fail more after accumulating a larger amount of debt than in the North suggests that mature firms in the South are not financially constrained. We rationalize our empirical findings by considering a model of firm dynamics where financial frictions arise because firms inefficiently accumulate new debt to dilute the value of past debt as in DeMarzo and He (2021). Financial frictions are due to the fact that firms can only borrow issuing non-stage contingent debt and firms cannot commit to refrain from increasing their debt in the future. In equilibrium, at each point in time, firms always issue some debt, according to the leverage ratchet effect emphasized by Admati, Demarzo, Hellwig, and Pfleiderer (2018).[1] The firm leverage ratio converges to a reference value which makes the firm prone to declare bankruptcy if the firm profitability unexpectedly falls. The incentive to overaccumulate debt is stronger when business idiosyncratic risk is higher: higher idiosyncratic risk increases the dispersion of firm returns and thereby increases the firm incentive to ask for more debt since the higher positive returns are appropriated by the firm in the form of equity while the more negative returns leads to bankruptcy with no or little effects on the firm equity value. The overindebtness problem is also more severe when financiers demand more debt-guarantees to protect their debt claims in case of default. Higher debt guarantees make firm debt cheaper, while the debt guarantees are paid by the entrepreneur in the distant future only in case of default and thereby give stronger incentive to firms to overaccumulate debt to dilute the value of past debt. Since in the South business idiosyncratic risk is greater and debt guarantees are stronger, both effects exacerbate the overindebtness problem of firms in the South. Quantitatively differences in business risk and debt guarantees accounts equally for the observed North-South differences in the failure rates of mature firms.

The overborrowing problem becomes more severe when credit is cheap, as in recent years. The effects of idiosyncratic risk and interest rates compound each other, implying that a fall in the cost of credit leads to more failure in the South than in the North. The data indicates that differences across provinces in failure rates have increased in recent years and that differences have increased more pronouncedly for mature firms. The model attributes the differential trends in failure rates across provinces to the recent prolonged period of cheap credit, emphasizing that a prolonged period of cheap credit tends to lead to excessive borrowing causing more business failure, particularly so in regions with high business risk. The model calls for taxing debt or better for subsidizing equity relative to debt.

We incorporate our model of firm dynamics into a fully integrated economy with a large

[1]The idea is that the firm is strictly worse off by actively reducing debt because with less debt the likelihood of default typically decreases and the value of existing debt increases which amount to a transfer of wealth from the firm to existing creditors. Conversely, if the firm raises new debt, the value of pre-existing debt falls which amount to a transfer of wealth from pre-existing creditors. This result follows from firm's inability to commit to future funding choices and it is exacerbated by past debt having equal or lower seniority than new debt.

number of provinces. In each province there is an endogenous number of firms with market power that hire workers in a competitive labor market. Firm goods are freely tradable. Workers are fully mobile across provinces and thereby the utility value of living in a province is equalized across provinces as in Rosen (1979) and Roback (1982). Provinces have a fixed endowment of immobile housing that constrains the number of workers living in the province. Provinces differ in the average productivity of firms, business idiosyncratic risk, and the amount of debt guarantees asked by financiers to protect their debt in case of default. We calibrate the model to match differences in the age profile of firms in the North and the South. A lower cost of credit increases the value of the firm at entry and promotes business creation, but, as we previously discussed, firms over-borrow, the more so the lower is the interest rate on debt, which generates a trade-off in general equilibrium: a lower cost of credit stimulates business creation but also exacerbates the overindebtness problem and thereby increases the inefficient failure of businesses. We use the model to quantify the output cost of overborrowing in environments with high and low interest rates. We also calculate the cost of credit that maximizes steady state welfare and use this benchmark to evaluate whether the cost of credit has been too low or too high in recent years. We also study the effects of subsidizing equity versus debt and whether equity should be subsidized more in Southern than in Northern Italian provinces due to the higher idiosyncratic risk present in the South.

Lowering the cost of credit through a monetary policy loosing typically expands economic activity, but as emphasized by Brunnermeier and Koby (2018) there could be a reversal interest rate below which further reductions in the rate becomes contractionary. In Brunnermeier and Koby (2018) a reversal rate arises because excessively low policy rates erode the equity value of bank capital, which in the presence of a bank capital constraint could lead to lower lending, and thereby to a contraction in economic activity, see Repullo (2020) for further discussion of the mechanism. We provide an alternative reason for why an excessively low cost of credit could be contractionary on economic activity and welfare that does not require sticky prices nor a capital constraint on the supply of credit: in our model a reversal rate arises because cheap credit inefficiently increases firm bankruptcies. The reversal rate is around 1 percent in real terms. Differently from Brunnermeier and Koby (2018), the credit supply keeps increasing as the cost of credit keeps falling below the reversal rate.

In our model there is overinvestment due to overborrowing, which leads to a fall in steady state consumption when the cost of credit becomes excessively low. This happens even if capital income (the difference between value added and labor income) remains greater than investment expenditures, which as proposed by Abel, Mankiw, Summers, and Zeckhauser (1989) is enough to identify dynamic efficiency in the neoclassical growth model, see Geerolf (2018) for evidence that the Abel et al. condition might have failed to hold in recent years for advanced economies.

In our model there is overinvestment because fims inefficiently declares bankruptcy that leads to excessive business turnover. Capital income remains greater than investment expenditures because there is a fixed cost at entry, future income is capitalized with a spositive rate, entrepreneurs have an outside option. So capital income (equal to total value added minus labor income) is always higher than investment. The over-investment problem arises at the margin as the result of the iefficinet detsruction of valuable businesses.

Relate to literature on misallocation and capital inflows Benigno, Converse, and Fornaro (2015), Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez (2017) García-Santana, Moral-Benito, Pijoan-Mas, and Ramos (2020) Calligaris, Del Gatto, Hassan, Ottaviano, and Schivardi (2016), Cingano and Hassan (2022) qualifies the evidence that bank credit increases misallocation in response to a rise in capital inflows. Cingano and Hassan (2022) shows that capital inflows were channeled through the banking sector. Reis (2013), Varela (2018). Noone of these papers focus on the risk of bankruptcy of cheap and abundant credit and we are first in trying to characterize the optimal cost of credit studying transitional dynamics.

2 Model

Time t is continuous and there is a measure one of provinces $i \in [0,1]$.

Agents and preferences In province i at time t, there is a large (exogenous) mass $\mathcal{E}_i$ of local entrepreneurs and an (endogenous) mass ℓ_{it} of workers. Agents have a discount rate r and maximize the present value of their instantaneous utility. Entrepreneurs die with poisson arrival rate δ_i. Dead entrepreneurs are replaced by a newly born cohort of entrepreneurs. Each entrepreneur owns a tree that yields income ς_i per period until it becomes a useless deadwood with Poisson arrival rate ϱ_i.[2] Entrepreneurs have a linear in consumption instantaneous utility function. Workers are infinitely lived, mobile and inelastically supplies one unit of labor in the province i of current residence, earning the wage w_{it}. A fraction ψ of workers has the option to leave the province of residence and emigrate to their favorite province. As in Rosen (1979) and Roback (1982), to restrict the workforce of each province, we assume that the living cost for workers is increasing in the province workforce, due to local (unmodelled) congestions in the use of housing, infrastructure or amenities. Workers instantaneous utility in province i at t, u_{it}, is increasing in consumption c (equal to wages) and decreasing in local congestions $h_i(\ell_{it})$

$$u_{it} = w_{it} - h_i(\ell_{it}), \tag{1}$$

where $h_i(\ell) = \bar{h}_i \ell^\eta$, with $\bar{h}_i, \eta > 0$. The aggregate labor supply is normalized to one so that

$$\int_0^1 \ell_{it} = 1. \tag{2}$$

Firm technology The (endogenous) mass of firms in the economy is equal to

$$M_t = \int_0^1 m_{it} di \tag{3}$$

where m_{it} is the number of firms in province i at t. The output of firm $j \in [0, M_t]$ is freely tradable and final output is produced by a representative firm with CES production function

$$Y_t = \left[\int_0^{M_t} (Z_{jt})^{\frac{1}{\nu}} (q_{jt})^{\frac{\nu-1}{\nu}} dj\right]^{\frac{\nu}{\nu-1}} \tag{4}$$

with $\nu > 2$.[3] q_{jt} is firm j output and Z_{jt} is an idiosyncratic demand shifter which evolves according to the geometric Brownian motion

$$dZ_j = \sigma_i Z_j d\omega_{jt} \tag{5}$$

[2] Upon the entrepreneur death, if the tree is still productive and free from claims by third parties, its income is rebated lump-sum to local entrepreneurs.

[3] The requirement $\nu > 2$ guarantee the exhistence of an insulating equilibrium discussed in Section XXXX.

where ω_{jt} is a standard Brownian motion (zero mean and unit variance) idiosyncratic across firms. Hereafter, the index i always refers to the province where firm j operates and parameters indexed by i are province specific—in this case σ_i. We refer to Z_{jt} as firm j *technology.* Firm j has access to a linear in labor production function $q_j = n_j$ where n_j is firm j employment. Labor is hired in a competitive local labor market at the wage w_{it}. Running the firm involves a leisure cost $\chi_i Z_{jt}$ to the entrepreneur.

Two useful statistics We denote by p_{jt} the price set by firm j at t and take final output as the numeraire so that

$$1 = \left[\int_0^{M_t} Z_j \left(p_{jt}\right)^{1-\nu} dj \right]^{\frac{1}{1-\nu}} \tag{6}$$

Since the representative firm in (4) takes prices as given, firm j faces the demand

$$q_{jt} = Z_{jt} \left(p_{jt}\right)^{-\nu} Y_t \tag{7}$$

Given the production technology $q_{jt} = n_{jt}$ and the demand in (7), firm j optimally chooses

$$n_{jt} = \left(1 - \frac{1}{\nu}\right)^{\nu} w_{it}^{-\nu} \, Y_t \, Z_{jt}, \tag{8}$$

Firm j revenue net of labour costs and the leisure cost of the entrepreneur is $\mathcal{R}_{it} Z_{jt}$ where

$$\mathcal{R}_{it} \equiv \frac{\mathcal{A}_{it}}{\nu} - \chi_i, \tag{9}$$

is the *net value* per technology unit in province i at time t and $\mathcal{A}_{it}$ is the corresponding market *value added* per technology unit (function of local wages w_{it} and aggregate output Y_t), equal to

$$\mathcal{A}_{it} = \left(\frac{\nu}{\nu - 1} \, w_{it} \right)^{1-\nu} Y_t, \tag{10}$$

which is decreasing in wages w_{it} and increasing in aggregate output Y_t.

Debt There is a perfectly competitive financial sector that lends to entrepreneurs with a business at interest rate $r_c < r$. A government authority controlls r_c with lending subsidies, financed through lump sum taxes on local entrepreneurs.[4] The rate r_c is common across provinces, in line with the data (see below). The net profits of the financial sector in a province are rebated as lump-sum payments to entrepreneurs in the province.[5] The debt of a firm producing in province i is modelled as a bond with coupon $\varkappa_i$ that expires with Poisson arrival rate ρ_i. A fraction φ_i of the debt is guaranteed.

[4]We focus the exposition on lending subsidies, but in practice the risk-free cost of credit r_c, can be controlled by a monetary or fiscal authority: by the monetary authority through a discount window lending facility, by the fiscal authority through lending subsidies or taxes.

[5]As a result, the local demand of housing is determined only by net output in the province. In the integrated equilibrium this assumption implies that taxes used to finance the financial sector do not have adirect effect on the allocation of resources, but only through the equilibrium cost of credit r_c.

Default and exit The firm declares bankruptcy when its equity value falls below the expected value of default. Upon default, with probability $\phi_i \in [0,1)$ the firm renegotiates its debt B with all creditors obtaining an haircut $1-\alpha$ to B, and thereafter restarts production with new debt αB; with probability $1-\phi_i$, the renegotiation fails, the firm pays the debt guarantees $\varphi_i B$ and exits forever from the market. The firm also exits if the entrepreneur exogenously dies. Upon death, debt guarantees are void. The bond price of a firm in province i at time t with debt B, technology Z and net value $\mathcal{R}$ is equal to the expected present value of the future debt repayments to financiers discounted at the financiers rate r_c, which is equal to

$$X_{it}(\mathbf{S}) = E\left[\int_0^\tau e^{-(r_c+\delta_i+\rho_i)s}\left(\varkappa_i+\rho_i\right)ds + e^{-r_c\tau}\left[\varphi_i+\phi_i\alpha X_{it+\tau}\left(\alpha B_{t+\tau}, Z_{t+\tau}, \mathcal{R}_{t+\tau}\right)\right]\right] \tag{11}$$

where $\mathbf{S} = (B, Z, \mathcal{R})$ and τ is the stochastic endogenous time to bankruptcy: $B_{t+\tau}$, $Z_{t+\tau}$, and $\mathcal{R}_{t+\tau}$ denote firm debt, technology and net value at the time of bankruptcy (time $t+\tau$).

Change of control With Poisson arrival rate λ_i an external entrepreneur can improve the firm technology by a factor $g > 1$, provided that she takes full control of the firm. The external entrepreneur is today liquidity constrained and value each unit of his today cash flow at $\kappa \in (1, g)$.[6] In acquiring the firm, the entrepreneur is also constrained by creditors rights, which limits the entrepreneur ability to finance the purchase of the firm by issuing new firm debt. Denote by B the initial firm debt and by B' firm debt after the change of control. Since the value of debt will depend on the debt-value ratio, $b \equiv \frac{B}{\mathcal{R}Z}$, we assume that the change of control is subject to the following *leveraged buyout constraint*:

$$B' \leq gB. \tag{12}$$

A buyout that violates (12) would breach fraudulent conveyance laws: it would defraud creditors, infringing their rights, and would be void.[7] In equilibrium with $\kappa > 1$, the external entrepreneur optimally chooses $B' = Bg$—the maximum debt level consistent with (12)—,which implies that upon a change of control, firm output discontinuously increases while its leverage ratio remains unchanged, roughly in line with the empirical evidence, see Figure 5 below.[8]

Let $V_{it}(B', Zg, \mathcal{R})$ denote the equity value of the firm after the change of control. Let $X^s_{it}(\mathbf{S})$ denote the price of bonds used to finance the leveraged buyout of the firm. The price paid by the external investor to acquire the control of the firm is

$$P_{it}(\mathbf{S}) = \frac{1}{\kappa}V_{it}(Bg, Zg, \mathcal{R}) + X^s_{it}(\mathbf{S})\left(g-1\right)B, \tag{13}$$

[6] Since a higher κ will imply a lower price for the acquisition of the firm, κ measures the bargaining power of the external entrepreneur.

[7] See Sherwin (1987), Goulet (1990) and Fox (2020), for a review of creditor's rights in the case of a leveraged buyout in the US; see Borsano (2015) for an analogous review for Italy. A dividend payment to shareholders is a fraudulent conveyance if it makes the corporation more likely to become insolvent, as it would happen if the inequality in (12) is violated. Both in the US and Italy, fraudulent conveyances are considered void.

[8] In equilibrium, the change of control has no effects on the debt price in (11): since the debt value depends just on the debt-value ratio, $b \equiv \frac{B}{\mathcal{R}Z}$, we have that $X_{it}(Bg, Zg, \mathcal{R}) = X_{it}(B, Z, \mathcal{R})$.

which is the net value of the acquisition for the external entrepreneur when choosing $B' = Bg$.

We normalize the coupon $\varkappa_i$ so that, absent default, the bond is worth one to the entrepreneur:

$$\overline{\varphi}_i = \frac{\varkappa_i + \rho_i}{r + \delta_i + \rho_i + \lambda_i \left(1 - \frac{1}{\kappa}\right) g} = 1, \qquad \forall i. \tag{14}$$

Firm creation Entrepreneurs can start-up a business by making the investment k_i. After the investment, the business starts producing with initial technology $Z_{i0} = e^z$ where z is a discrete random variable that assigns probability g_{iz} to $z \in \mathcal{Z}_i$, with $\sum_{z \in \mathcal{Z}_i} g_{iz} = 1$. The entrepreneur is liquidity constrained, cannot save, and finances k_i partly by pledging her tree income to financiers and partly by pledging B_{it}^0 bonds of the firm once productive. Given the convexity of the firm value function with respect to B_{it}^0 (see below), the entrepreneur wants to minimize B_{it}^0, so that

$$B_{it}^0 = \frac{1}{x_{it}^0} \cdot \max\left\{k_i - \frac{\varsigma_i}{r_c + \varrho_i}, 0\right\} \tag{15}$$

where x_{it}^0 is the bond price of a start-up at t in province i and $\frac{\varsigma_i}{r_c+\varrho_i}$ is the financiers evaluation of the tree income pledged by the entrepreneur to financiers. x_{it}^0 is equal to the expected bond value of the firm $X_{it}(B_{it}^0, e^z, \mathcal{R})$, which depends on the pledged bonds B_{it}^0, the firm technology Z and the net value in the province $\mathcal{R}$:

$$x_{it}^0 = \sum_{z \in \mathcal{Z}_i} X_{it}(B_{it}^0, e^z, \mathcal{R}) \cdot g_{iz}. \tag{16}$$

3 Equilibrium

We first characterize the equilibrium conditions of the model and then define the equilibrium.

3.1 Conditions

We focus on an equilibrium where there is positive business creation $\tilde{m}_{it} > 0 \, \forall t$ and $\forall i \in [0, 1]$. We guess and then verify that the net value $\mathcal{R}_i$ in (9) remain constant over time during the transition to the economy steady state. This implies firms are insulated from aggregate dynamics.

Firm problem Under the guess of a constant $\mathcal{R}_i$, given (10), the value added per technology unit is also constant and equal to

$$\mathcal{A}_i = \nu \left(\mathcal{R}_i + \chi_i\right). \tag{17}$$

Under the guess that $\mathcal{R}_i$ is constant, the equity value of a firm with outstanding debt B and technology Z operating with net value $\mathcal{R}$ in province i at time t, $V_{it}(\mathbf{S})$, satisfies the following Hamilton-Jacobi-Bellman (HJB) equation

$$(r + \delta_i)V_{it}(\mathbf{S}) = \max_L \mathcal{R}Z - (\varkappa_i + \rho_i) B + X_{it}(\mathbf{S}) L + \lambda_i \left[P_{it}(\mathbf{S}) - V_{it}(\mathbf{S})\right] + L\frac{\partial V_{it}(\mathbf{S})}{\partial B} + \mathbb{L}V_{it} \tag{18}$$

where $P_{it}(\mathbf{S})$ is the selling price of the firm given in (13) and $\mathbb{L}V_{it}$ is the following differential operator that characterizes how exogenous changes in $\mathbf{S}=(B,Z,\mathcal{R})$ and time (through changes in the aggregate states of the economy) affect the firm equity value:

$$\mathbb{L}V_{it} \equiv -\rho_i B \frac{\partial V_{it}(\mathbf{S})}{\partial B} + \frac{\sigma_i^2 Z^2}{2} \cdot \frac{\partial^2 V_{it}(\mathbf{S})}{\partial Z^2} + \frac{\partial V_{it}(\mathbf{S})}{\partial t}.$$

The subindex t in (18) refers to possible changes in the aggregate states of the economy that could affect the firm problem. The first term in (18) is the firm revenue net of labor and leisure costs. The second term is the payments for serving current debt B; the third term is the cash flow from issuing new debt L, chosen optimally by the firm. The last three terms in (18) are the capital gains due to a change in firm control–which happens with arrival rate λ_i—, to a change in debt value due to new debt L and to the exogenous changes in states, measured by $\mathbb{L}V_{it}$.

The problem in (18) is further characterized by a bankruptcy boundary $\bar{B}_{it}(Z,\mathcal{R})$: a firm with debt B greater or equal than $\bar{B}_{it}(Z,\mathcal{R})$ declares bankruptcy. After bankruptcy, with probability $1-\phi_i$ the firm pays the debt guarantees $\varphi_i B$ and exits, while with probability ϕ_i the firm restarts production with debt level $\alpha\bar{B}_{it}(Z,\mathcal{R})$. This implies that at $\bar{B}_{it}(Z,\mathcal{R})$, the following value matching condition holds:

$$V_{it}\left(\bar{B}_{it}(Z,\mathcal{R}),Z,\mathcal{R}\right) = -\left(1-\phi_i\right)\varphi_i \bar{B}_{it}(Z,\mathcal{R}) + \phi_i V_{it}\left(\alpha\bar{B}_{it}(Z,\mathcal{R}),Z,\mathcal{R}\right), \tag{19}$$

At $\bar{B}_{it}(Z)$ we also have the two following smooth pasting conditions

$$\left.\frac{\partial V_{it}}{\partial Z}\right|_{B=\bar{B}_{it}(Z,\mathcal{R})} = 0, \quad \text{and} \quad \left.\frac{\partial V_{it}}{\partial B}\right|_{B=\bar{B}_{it}(Z,\mathcal{R})} = -\left(1-\phi_i\right)\varphi_i + \phi_i\alpha \left.\frac{\partial V_{it}}{\partial B}\right|_{B=\alpha\bar{B}_{it}(Z,\mathcal{R})}. \tag{20}$$

Maximizing with respect to L in (18) implies that the firm issues bonds until the firm equity value V_{it} is unaffected at the margin by L so that

$$X_{it}\left(\mathbf{S}\right) = -\frac{\partial V_{it}(\mathbf{S})}{\partial B}. \tag{21}$$

Simplification To simplify the problem in (18), we define the firm *debt-value ratio*:

$$b \equiv \frac{B}{\mathcal{R}Z}.$$

In the Appendix, we guess and then verify that the value function V_i in (18) is time invariant because the price of debt $X_{it}\left(\mathbf{S}\right)$ and the selling price $P_{it}(\mathbf{S})$ are province specific function of firm states $\mathbf{S}=(B,Z,\mathcal{R})$, independent of time. We also show that $V_i\left(\mathbf{S}\right)$ can be written as

$$V_i\left(B,Z,\mathcal{R}\right) = v_i\left(b\right)\mathcal{R}Z$$

where the scaled, province specific value function $v_i(b)$ depends just on the debt-value ratio b. Let $\bar{b}_i \equiv \bar{B}_{it}(Z,\mathcal{R})/(\mathcal{R}Z)$ denote the threshold for the debt-value ratio, which triggers firm bankuptcy in province i. For $b \in [0, \bar{b}_i)$

$$v_i(b) = \frac{1}{r+\delta_i - \lambda_i\left(\frac{g}{\kappa}-1\right)} - b + \frac{1-\phi_i\alpha - (1-\phi_i)\varphi_i}{(1+\gamma_i)(1-\phi_i\alpha^{1+\gamma})}\left(\frac{b}{\bar{b}_i}\right)^{\gamma_i} b, \tag{22}$$

where γ_i is positive and equal to

$$\gamma_i = \frac{\rho_i + \lambda_i(g\text{-}1) - \frac{\sigma_i^2}{2} + \sqrt{\left[\rho_i + \lambda_i(g\text{-}1) - \frac{\sigma_i^2}{2}\right]^2 + 2\sigma_i^2\left[r+\delta_i+\rho+\lambda_i g\left(1\text{-}\frac{1}{\kappa}\right)\right]}}{\sigma_i^2}$$

For $b \geq \bar{b}_i$, the value function $v_i(b)$ can be evaluated recursively noticing that if $b \in \left[\alpha^{1-n}\bar{b}_i, \alpha^{-n}\bar{b}_i\right)$, $n = 1, 2, ...$, the firm restarts production only after n (successful) renegotiations, otherwise it pays the debt guarantees and exits, so that

$$v_i(b) = -(1-\phi_i^n)\varphi_i b + \phi_i^n v_i(\alpha^n b), \qquad \forall b \in \left[\alpha^{-n+1}\bar{b}_i, \alpha^{-n}\bar{b}_i\right) \tag{23}$$

The boundary conditions (19) and (20) determine the threshold $\bar{b}_i$ which depends just on structural parameters as follows

$$\bar{b}_i = \frac{1}{r+\delta_i-\lambda_i\left(\frac{g}{\kappa}-1\right)} \cdot \frac{\left(1+\frac{1}{\gamma_i}\right)(1-\phi_i)}{1-\phi_i\alpha-(1-\phi_i)\varphi_i}. \tag{24}$$

(22) together with (21) determine the equilibrium price of debt as a province-specific, constant-over-time-function of the debt-value ratio b. For $b \in [0, \bar{b}_i]$ we have that

$$x_i(b) = 1 - \frac{1-\phi_i\alpha-(1-\phi_i)\varphi_i}{1-\phi_i\alpha^{1+\gamma}}\left(\frac{b}{\bar{b}_i}\right)^{\gamma}. \tag{25}$$

For $b \geq \bar{b}_i$, the price of debt $x_i(b)$ can be evaluated recursively noticing that if $b \in \left[\alpha^{1-n}\bar{b}_i, \alpha^{-n}\bar{b}_i\right)$, $n = 1, 2, ...$:

$$x_i(b) = (1-\phi_i^n)\varphi_i + \phi_i^n x_i(\alpha^n b), \qquad \forall b \in \left[\alpha^{-n+1}\bar{b}_i, \alpha^{-n}\bar{b}_i\right)$$

Optimal debt In the Appendix, we show that a firm with debt-value ratio b in province i issues new debt per technology unit equal to

$$l_i(b) \equiv \frac{L}{Z} = (r-r_c)\frac{-v_i'(b)}{v_i''(b)} + \lambda_i\left[g\left(1-\frac{1}{\kappa}\right)\frac{-v_i'(b)}{v_i''(b)} - (g-1)b\right] \tag{26}$$

The three terms in the right hand side of (26) characterize the debt policy of the firm. The first term reflects the difference in discount rates between the firm and its financiers, equal to $r - r_c$. It arises from equating the financial gains of a new bond to its marginal cost in the absence of a market for a change of control ($\lambda_i = 0$): the marginal gain is equal to the cash flow from a new bond, $x_i(b) = -v_i'(b)$, times the difference in discount rates $r - r_c$; the marginal cost is equal to the fall in the bond value $x_i(b)$ due to a marginal increase in debt, equal to $l_i(b)\, v_i''(b)$—which is positive due to the convexity of the value function, see below. The last two terms in (26) measure the effects of the market for a change of control on firm incentive to issue debt. The term $\lambda_i g \left(1 - \frac{1}{\kappa}\right) \frac{-v_i'(b)}{v_i''(b)}$ is positive and it is a *leverage buy-out effect.* It arises because at low levels of debt B, higher B increases the value of the firm, making a leverage buy-out by the external entrepreneur possible through (12): when external entrepreneurs are financially constrained, $\kappa > 1$, firm debt B allows the external entrepreneur to finance the acquisition of the firm by issuing $(g-1)B$ new debt units. The third term in (26), equal to $-\lambda_i (g-1)\, b$, is negative and it is the *disciplinary effect* of equity markets. A change of control increases the firm equity value by a factor g. Since more debt reduces the firm equity value, the absolute increase of the equity value after a change of control is smaller when the firm is more indebted. Anticipating a possible future bid for the firm, the entrepreneur today refrains from issuing debt. The leverage buy-out effect dominate at debt levels close to zero, $b = 0$, while the disciplinary effect of equity markets dominates at debt levels close to the bankrutcy threshold $\bar{b}_{it}$. At $\bar{b}_{it}$, the leverage buy-out effect is literally nil with zero guarantees $\varphi_i = 0$ and zero renegotiation probability $\phi_i = 0$: in this case the smooth pasting condition in (20) implies that $v_i'\left(\bar{b}_{it}\right) = 0$ and the first two terms in (26) drop to zero. As a result, a higher probability of selling the firm (higher λ_i) gives the entrepreneur stronger incentives to move aways (in percentage terms) from the bankruptcy threshold $\bar{b}_{it}$, issuing less debt—i.e. the term $-\lambda_i (g-1)$ becomes more negative. As a result, in markets where a change of control is more likely (higher λ_i), firms operates with debt levels which are smaller relative to the bankruptcy threshold $\bar{b}_{it}$ and are less likely to go bankrupt.

Given (22), we calculate $v_i'(b)$ and $v_i''(b)$ and then substitute the resulting expressions in (26) to obtain the following expression for the firm debt policy $l_i(b)$:

$$\frac{l_i(b)}{b} = \frac{r - r_c + \lambda_i g \left(1 - \frac{1}{\kappa}\right)}{\gamma_i} \left[\frac{1 - \phi_i \alpha^{1+\gamma_i}}{1 - \phi_i \alpha - (1-\phi_i)\varphi_i} \left(\frac{b}{\bar{b}_i}\right)^{-\gamma_i} - 1 \right] - \lambda_i (g-1). \tag{27}$$

Start-ups Given (16) and after using (25), the bond price of a start-up x_i^0 satisfies

$$x_i^0 = \sum_{z \in \mathcal{Z}_i} x_i \left(\frac{B_i^0}{\mathcal{R}_i e^z} \right) \cdot g_{iz}. \tag{28}$$

The expected equity value of a start-up in province i is equal to

$$\mathcal{V}_i = \sum_{z \in \mathcal{Z}_i} v_i \left(\frac{B_i^0}{\mathcal{R}_i e^z} \right) \cdot \mathcal{R}_i e^z \cdot g_{iz} \tag{29}$$

Since only entrepreneurs with a business have access to external finance, the opportunity cost of the income forgone by the entrepreneur to finance the initial start-up investment is discounted at rate r and equal to $\frac{\varsigma_i}{r+\varrho_i}$. Under free entry with positive entry, $\tilde{m}_{it} > 0$, the value of forgone income should be equal to the expected equity value of the firm $\forall t$

$$\frac{\varsigma_i}{r+\varrho_i} = \mathcal{V}_i \tag{30}$$

Determination of $\mathcal{R}_i$ The function v_i in (22) together with $\bar{b}_i$ in (24) are function of structural parameters independent of $\mathcal{R}_i$. After noticing that (15) implies that B_i^0 is a negative function of x_i^0, (28) and (30) is a system of two equations in the two unknowns x_i^0 and $\mathcal{R}_i$. Since the function x_i in (25) is decreasing, the right hand side of (28) is increasing in x_i^0, which implies that for given $\mathcal{R}_i$ there could be multiple solution of x_i^0. To break possible multiplicities and since social welfare is decreasing in firm debt, we always select the solution with the highest value of x_i^0—i.e. with the lowest debt level B_i^0. We denote the resulting solution by $x_i^0(\mathcal{R}_i)$, which is an increasing function of $\mathcal{R}_i$. By substituting the function $x_i^0(\mathcal{R}_i)$ into (30) we conclude that the right hand side of (30) is globally increasing in $\mathcal{R}_i$ which guarantees that there is at most one value of $\mathcal{R}_i$ that makes (30) satisfied. That a solution is guaranteed by the fact that the function v_i is bounded, so that the right hand side of (30) is zero when $\mathcal{R}_i$ is zero and diverges to infinity when $\mathcal{R}_i$ goes to infinity. The value of $\mathcal{R}_i$ that solves (30) sustains the insulating equilibrium.

Kolmogorov forward equation Using the Ito's lemma and (5), we obtain that $\hat{b} \equiv \ln b$ and $\hat{z} \equiv \ln(\mathcal{R}Z)$ evolve according to

$$d\hat{b} = \mathrm{B}_i(\hat{b})dt - \sigma_i d\omega \tag{31}$$

$$d\hat{z} = -\frac{1}{2}\sigma_i^2 dt + \sigma_i \, d\omega. \tag{32}$$

where $\mathrm{B}_i(\hat{b})$ is a time invariant function of $\hat{b}$ obtained using the debt policy function l_i in (27):

$$\mathrm{B}_i(\hat{b}) = e^{-\hat{b}} \cdot l_i\left(e^{\hat{b}}\right) + \frac{\sigma_i^2}{2} - \rho_i.$$

The number of firms in province i at t is equal to

$$m_{it} = \int_{[0,\bar{b}_i]\times\mathcal{R}} f_{it}(\hat{\mathbf{s}})d\hat{\mathbf{s}}. \tag{33}$$

At entry the firm technology $\hat{z}$ is a random drawing. Given the bonds pledged by the start-up, B_i^0, the draw of $\hat{z}$ also determines the (logged) debt-value ratio $\hat{b}$. Let $\hat{\mathbf{s}}_{iz}^0 = (\hat{b}_{iz}^0, \hat{z}_{iz})$ with $\hat{b}_{iz}^0 = \ln B_i^0 - z - \ln \mathcal{R}_i$ and $\hat{z}_{iz} = z + \ln \mathcal{R}_i$. We denote by $\mathbf{\Delta}(x, y)$ the Dirac delta "function" which is zero everywhere except at $x = y$ where it is infinite. The inflow at $\hat{\mathbf{s}} = (\hat{b}, \hat{z})$ due to business creation is

$$f_{it}^0(\hat{\mathbf{s}}) = \tilde{m}_{it} \times \sum_{z\in\mathcal{Z}_i} g_{iz} \cdot \mathbf{\Delta}\left(\hat{b}, \ln B_{it}^0 - z - \ln \mathcal{R}_i\right) \times \mathbf{\Delta}(\hat{z}, z + \ln \mathcal{R}_i) \tag{34}$$

where $\tilde{m}_{it}$ denotes the mass of new firms in province i at t. For $\hat{b} < \ln \bar{b}_i$ with $\hat{b} \neq \ln \bar{b}_i - \ln \alpha$, the distribution $f_{it}(\hat{\mathbf{s}})$ in province i at time t solves the following Kolmogorov forward equation:

$$\begin{aligned}\frac{\partial f_{it}(\hat{\mathbf{s}})}{\partial t} &= f_{it}^{0}(\hat{\mathbf{s}}) + \lambda_i f_{it}(\hat{b}, \hat{z} - \ln g) - (\delta_i + \lambda_i)\, f_{it}(\hat{\mathbf{s}}) - \frac{\partial \left[\mathrm{B}_i(\hat{b}) f_i(\hat{\mathbf{s}})\right]}{\partial \hat{b}} + \frac{\sigma_i^2}{2}\frac{\partial f_{it}(\hat{\mathbf{s}})}{\partial \hat{z}} \\ &\quad + \frac{\sigma_i^2}{2}\left[\frac{\partial^2 f_{it}(\hat{\mathbf{s}})}{\partial \hat{b}^2} - 2\frac{\partial^2 f_{it}(\hat{\mathbf{s}})}{\partial \hat{b}\partial \hat{z}} + \frac{\partial^2 f_{it}(\hat{\mathbf{s}})}{\partial \hat{z}^2}\right] \end{aligned} \tag{35}$$

The left hand side is the change over time of the distribution $f_{it}(\hat{\mathbf{s}})$. The first two terms in the right hand side of (35) are the instantaneous inflow of firms into state $\hat{\mathbf{s}}$ due to entry $f_{it}^{0}(\hat{\mathbf{s}})$ and due to a change in control (arrival rate λ_i)—that increases firm technology by a factor g—of firms with (log) technology $\hat{z} - \ln g$ and debt-value ratio $\hat{b}$—unchanged after the change of control. The third term is the fall in $f_{it}(\hat{\mathbf{s}})$ due to entrepreneur death (arrival rate δ_i) or change of control (arrival rate λ_i). The last two terms in the first row of (35) is the change in $f_{it}(\hat{\mathbf{s}})$ due to the mean change in the debt-value ratio $\hat{b}$ in (31) and the mean change of $\hat{z}$ in (32). The last term in the second row is the (standard) second order term of the Kolmogorov forward equation for the two dimensional diffusion processes (31) and (32).

Equilibrium output Let $f_{it}(\hat{\mathbf{s}})$ denote the mass of firms in province i with state $\hat{\mathbf{s}} = (\hat{b}, \hat{z})$ where $\hat{b} \equiv \ln b$ and $\hat{z} \equiv \ln(\mathcal{R}Z)$. Let y_{it} denote the value added in province i at time t equal to

$$y_{it} = \mathcal{A}_i \int_{\left[0,\bar{b}_i\right]\times R} \frac{\exp(\hat{z})}{\mathcal{R}_i} f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}. \tag{36}$$

After using (8) and the labor force in the province ℓ_{it} is predetermined, the local labor market clears only if

$$w_{it} = \frac{\nu - 1}{\nu} \cdot \left(\mathrm{Y}_t \cdot \frac{y_{it}}{\mathcal{A}_i \ell_{it}}\right)^{\frac{1}{\nu}}. \tag{37}$$

Given $\mathcal{R}_i$, (10) implies that the wage rate in province i evolves according to

$$w_{it} = \frac{\nu - 1}{\nu}\left(\frac{\mathrm{Y}_t}{\mathcal{A}_i}\right)^{\frac{1}{\nu-1}} \tag{38}$$

Substituting (38) into the labor market clearing condition (37) and after using the definition of y_{it} in (36) we obtain that the per worker valued added in province i evolves according to

$$y_{it} = \ell_{it}\left(\frac{\mathrm{Y}_t}{\mathcal{A}_i}\right)^{\frac{1}{\nu-1}} \tag{39}$$

Given (36) and (41), over time the business creation $\tilde{m}_{it}$ should evolve to guarantee the following condition holds

$$\ell_{it}\left(\mathrm{Y}_t\right)^{\frac{1}{\nu-1}}\left(\frac{1}{\mathcal{A}_i}\right)^{\frac{\nu}{\nu-1}} = \int_{\mathcal{R}^2} \frac{\exp(\hat{z})}{\mathcal{R}_i} f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}} \tag{40}$$

Since aggregate output is equal to $Y_t = \int_0^1 y_{it} di$, using (39) we obtain that

$$Y_t = \left[\int_0^1 \ell_{it} \left(\frac{1}{\mathcal{A}_i} \right)^{\frac{1}{\nu-1}} di \right]^{\frac{\nu-1}{\nu-2}} \tag{41}$$

Y_t is a weighted averaged of the inverse firm value added per technology units $\mathcal{A}_i$, with weights equal to the province level workforce ℓ_{it} : maximizing output requires allocating the workforce to the province with the lowest $\mathcal{A}_i$'s, which in steady state are those with the highest wages. Y_t at t is fully determined given the current predetermined distribution of province level workforce ℓ_{it} and the value of $\mathcal{A}_i$'s.

Worker mobility The value to the worker from staying in province i is

$$U_{it} = \int_0^\infty e^{-rs} u_{it+s} ds \tag{42}$$

where u_{it} is the instantaneous utility of a worker in province i at t as given in (1). We denote by U_t^*, the maximum worker utility across provinces:

$$U_t^* = \max_{i \in [0,1]} U_{it}. \tag{43}$$

A proportion ψ of workers leave the province when $U_{it} < U_t^*$ so that

$$\frac{\dot{\ell}_{it}}{\ell_{it}} = -\psi \cdot \mathbb{I} \left(U_t^* - U_{it} > 0 \right). \tag{44}$$

where $\dot{\ell}_{it} = \frac{d\ell_{it}}{dt}$ denotes the time derivative of ℓ_{it}. [9]

Welfare Consumption in province i is equal to

$$c_{it} = y_{it} + \frac{\varsigma_i}{\varrho_i} \cdot \mathcal{E}_i - k_i \tilde{m}_{it} - \mathfrak{c}_{it} \tag{45}$$

The first two terms are province i output, due to firm output y_{it} and trees income $\frac{\varsigma_i}{\varrho_i} \cdot \mathcal{E}_i$. Th elast two terms are the costs due firm investment and the opportunity cost of entrepreneurs running a firm in the province equal to

$$\mathfrak{c}_{it} = \frac{\chi_i}{\mathcal{R}_i} \int_{[0,\bar{b}_i] \times \mathcal{R}} \exp\left(\hat{z}\right) f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}$$

Aggregate market consumption is obtained by aggregating consumption across provinces:

$$C_t = \int_0^1 c_{it} di \tag{46}$$

[9]This follows from the fact that at any point in time at least some workers remain in each province, $U_{it} > 0$ $\forall i, t$, and in equilibrium no worker switches province more than once.

We measure aggregate welfare $\mathbb{W}_t$ by the present value of the sum across provinces of the instantaneous utility flow of workers and entrepreneurs. After using (1) and (46) we obtain that

$$\mathbb{W}_t = \int_0^\infty e^{-rs} \left(C_{t+s} - H_{t+s} \right) ds \tag{47}$$

where H_t measures the aggregate amount of congestions in the economy

$$H_t = \int_0^1 h_i \left(\ell_{it} \right) \ell_{it} di. \tag{48}$$

3.2 Definition

Let $\mathbb{X}_{it} = \left[\mathbb{X}_{it}^S, \mathbb{X}_{it}^N \right]$, $i \in [0,1]$, be the province-i specific tuple obtained by combining the time invariant tuple

$$\mathbb{X}_i^S = \left[\mathcal{R}_i, \mathcal{A}_i, l_i(b), \bar{b}_i, x_i(b), x_i^0, B_i^0 \right]$$

with the time varying tuple

$$\mathbb{X}_{it}^N = \left[\tilde{m}_{it}, \ell_{it}, \dot{\ell}_{it}, U_{it}, f_{it}(\hat{\mathbf{s}}), f_{it}^0(\hat{\mathbf{s}}), m_{it}, w_{it} \right]$$

Let $\mathbf{X}_t = (U_t^*, Y_t, C_t, H_t, \mathbb{W}_t)$ characterize the integrated economy. An equilibrium is a combination of $\mathbb{X}_{it}$'s, $i \in [0,1]$ and $\mathbf{X}_t$ that satisfy the following conditions:

1. *Firm maximization* Given $\mathcal{R}_i$ that solves in (30) and $\mathcal{R}_i$ in (17), firms declare bankruptcy when their debt value ratio is above $\bar{b}_i$ in (24) and issue debt according to $l_i(b)$ in (27)

2. *Worker maximization* The emigration rate $-\dot{\ell}_{it}/\ell_{it}$ is (44) with U_{it} and U_t^* in (42) and (43), respectively.

3. *Province equilibrium* Given the distribution $f_{it}(\hat{\mathbf{s}})$ which satisfies (35) with $f_{it}^0(\hat{\mathbf{s}})$ in (34), clearing of the province-i's labor market implies that the wage w_{it} satisfies (37); free entry implies that business creation $\tilde{m}_{it}$ makes the free entry condition in (30) satisfied; clearing of financial markets requires that the bond price $x_i(b)$ is equal to (25), that the debt of a start-up B_i^0 satisfies (15) with x_i^0 that solves (28).

4. *Aggregate market clearing* : Clearing of the integrated labor market requires that (45) holds. Clearing of the goods market implies that aggregate gross output Y_t, consumption C_t and congestions H_t satisfy (41), (46) and (48). Aggregate welfare $\mathbb{W}_t$ is given by (47).

In an insulating equilibrium, the net value $\mathcal{R}_i$ and the value added per technology unit $\mathcal{A}_i$ are constant through time and equal to their steady state value. This makes firms insulated from aggregate dynamics, which implies that the firm value function $v_i(b)$, the debt policy

$l_i\,(b)$, the bankruptcy threshold $\bar{b}_i$ and the debt price of a start-up x_i^0, and its debt B_i^0 are all constant through time and independent of aggregate dynamics. The vector $\mathbb{X}_{it}^N$ and the vector of aggregate state of variables $\mathbf{X}_t$ vary over time. This implies that in each province the wage w_{it} and the labor force ℓ_{it} change over time, which determine the dynamics of aggregate output Y_t and welfare $\mathbb{W}_t$.

In response to shocks the condition (40) holds only if the shock the cost of credit is small. In particular it should be that the change in the cost of credit goes to zero at the same rate as the change in the time interval. We consider changes in the cost of credit r_c so that

$$\frac{dr_c}{dt} = \kappa$$

where κ is (positive or negative) constant bouned away from zero. Having m_{it} (a stock variable) as a jump variable, because of business entry and exit, is required to sustain an *insulating* equilibrium where firm optimal policies adjust instantaneously in response to shocks and therafter remain time invariant. The equilibrium is sustained through an instantaneous adjustment in business creation $\tilde{m}_{it}$ consistent with (40).

Shock The inflow at $\hat{\mathbf{s}}$ due to debt renegotiation happens with probability ϕ_i^n for firms that declared default ($\hat{b} \geq \ln \bar{b}_{it}$) with debt-value ratio in logs $\hat{b} \in [n \ln \bar{b}_{it}, \ln \bar{b}_{it} + (n+1) \ln \alpha]$, for $n = 1, 2, ...$, which implies that

$$f_{it}^R(\hat{\mathbf{s}}) = \sum_{n=1}^{\infty} (\phi_i)^n \, f_{it}(\hat{b} + n \ln \alpha, \hat{z}) \times \mathbb{I}\left(\ln \bar{b}_i \geq \hat{b} \geq \ln \bar{b}_i - \ln \alpha\right) \tag{49}$$

where $\mathbb{I}$ denotes the indicator function.

Overborrowing of mature firms The steady state value of a start-up $\mathcal{V}_i$ is decreasing in r_c : with a higher r_c start-ups are more indebted and thereby more likely to fail at birth. More expensive credit (higher r_c) reduces business creation as well as it makes new businesses more indebted and thereby more prone to bankruptcy, which are the *traditional channel* whereby tighter credit reduces production

The bankruptcy threshold $\bar{b}_i$ is increasing in the debt guarantees φ_i. The coefficient γ_i measures the effects of the stochastic process for b on the bankruptcy decision: greater idiosyncratic risk (greater σ_i) reduces γ_i and thereby lead to a higher bankruptcy threshold $\bar{b}_i$; a shorter debt maturity (higher ρ_i) increases γ_i which reduces $\bar{b}$.

We can measure overborrowing by

$$\Omega_i = \frac{l_i\left(\bar{b}_i\right)}{\bar{b}_i} = \frac{r - r_c + \lambda_i g\left(1 - \frac{1}{\kappa}\right)}{\gamma_i} \cdot \frac{\phi_i \alpha\left(1 - \alpha^{\gamma_i}\right) + \left(1 - \phi_i\right)\varphi_i}{1 - \phi_i \alpha - \left(1 - \phi_i\right)\varphi_i} - \lambda_i\left(g - 1\right)$$

We take Ω_i as a measure of the overindebtness of mature firms: higher Ω_i means that old firms end up operating with a debt level closer to their bankruptcy threshold which makes bankruptcy more likely.

A decrease in the cost of credit r_c increases overindebtness Ω_i since

$$\frac{d\Omega_i}{dr_c} = -\frac{1}{\gamma_i} \cdot \frac{\phi_i \alpha\left(1 - \alpha^{\gamma_i}\right) + \left(1 - \phi_i\right)\varphi_i}{1 - \phi_i \alpha - \left(1 - \phi_i\right)\varphi_i} < 0$$

An increase in the debt guarantees φ_i also makes mature firms more over-indebted, which follows from the fact that

$$\frac{d\Omega_i^1}{d\varphi_i} = \frac{1}{\gamma_i}\left[r - r_c + \lambda_i g\left(1 - \frac{1}{\kappa}\right)\right] \frac{\left(1 - \phi_i \alpha^{1+\gamma}\right)\left(1 - \phi_i\right)}{\left[1 - \phi_i \alpha - \left(1 - \phi_i\right)\varphi_i\right]^2}$$

As a result, firms tend to fluctuate more closely to their bankruptcy threshold when credit is cheaper (lower $\bar{r}$) and when financiers request more debt guarantees (higher φ_i). In both cases, the price of bond for given firm debt per technology $x_i(b)$ increases, giving firms incentives to accumulate more debt which thereby increases the risk of default. Idiosyncratic risk σ_i affect the default probability of mature firms indirectly by affecting the amount of overborrowing Ω_i and directly by increasing the default probability for given over-indebtedness, Ω_i. The effect of idiosyncratic risk σ_i on overborrowing Ω_i is non monotonic. Generally, the relation between Ω_i and σ_i is hump shaped: the derivative $\frac{d\Omega_i}{d\sigma_i}$ is positive at low values of σ_i and negative at high values of σ_i. At the calibrated values of σ_i's below, we are generally on the positively sloped arm of the Ω_i–σ_i relation. This implies that greater idiosyncratic risk σ_i increases the default probability of mature firms both indirectly because their overborrowing Ω_i increases and directly because default is more likely for given Ω_i. Notice that the leverage ratio of a firm is equal to $b/\mathcal{A}_i$ where $\mathcal{A}_i$ in (**??**) is firm value added per unit of technology. Since the firm policy function is independent of $\mathcal{A}_i$, the amount of overborrowing of mature firms Ω_i and thereby their default probability are independent of the average profitability of firms.

The effect of a change in the cost of credit on overborrowing, $\left|\frac{d\Omega_i}{d\bar{r}}\right|$, depends on idiosyncratic risk σ. Formally we have that

$$\frac{d\left|\frac{d\Omega_i}{dr_c}\right|}{d\sigma^2} = -\frac{A_i'\left(\sigma^2\right)}{\left(r - r_c\right)\left[A_i\left(\sigma^2\right)\right]^2} \tag{50}$$

where $A_i\left(\sigma^2\right) = \gamma_i + \frac{r-r_c}{\rho_i - \sigma^2}$ and $A_i'\left(\sigma^2\right)$ is its derivative with respect to σ^2. It is to check that $\lim_{\sigma^2 \to 0} A_i'\left(\sigma^2\right) = -\infty$ and $\lim_{\sigma^2 \to \rho_i} A_i'\left(\sigma^2\right) = \infty$. Given (50) this implies that the effect of a fall in the interest rate $\bar{r}$ on overborrowing, $\left|\frac{d\Omega_i}{d\bar{r}}\right|$, is generally hump-shaped in the amount of idiosyncratic risk σ^2.

The trade-off in partial equilibrium

A lower r_c makes new firms less indebted and thereby less prone to bankruptcy during their first years of life. Moreover, $\mathcal{V}_{it}$ in (29) is decreasing in b, $\mathcal{V}' = -v''(b)b < 0$, so entry also becomes more profitable, when r_c falls, which is welfare enhancing since the business creation rate is inefficiently low—quantitatively this latter effect will turn out to be small.

Changes in steady state interest rates

4 Quantitative analysis

We start describing the data sources, the definition of variables and we characterize North-South differences across Italian provinces in business exit rates, financial market conditions, business idiosyncratic risk, the market for change of control and the age profile of business exit rates, and leverage ratio. Then we the calibrate the model for North and the South provinces, and discuss what accounts for the North-South differences in business exit rates and leverage ratios. To validate the model we look at the firm response to an idiosycratic shock to firm demand.

4.1 Data and preliminary evidence

We use data from several sources. Unless otherwise specified, time averages are calculated over the years 2007-2015. We use data from the universe of Italian firms as collected by the Italian social security agency (INPS). We refer to this dataset as UNIMPS. Since 1990, UNIMPS samples the universe of Italian private businesses, organized as a legal entity or sole proprietorship, and reports the number of their employees. We define a firm as mature if it employs some workers and it is a new mature firm if it employs workers for its first time. A firm is exiting if its employment drops forever to zero. The age of a mature firm is the number of years since it has first employed some workers. The failure rate in province i in year t, denoted by f_{it}, is the fraction of mature firms in province i at the end of year $t-1$ that exit by the end of year t.

We use the Central Credit Register administered by the Bank of Italy to identify bad debts towards the banking and financial sector.[10] The Credit Register also reports information on debt guarantees. Data on firm total debt, and value added cover the universe of limited liability companies from the CERVED dataset. We match all firms in CERVED both with the Central Credit Register and with the Business Register administered by the Chambers of Commerce that contains a detailed classification of the reasons for why businesses drop out of the Business Register. A firm exits with "bankruptcy" if the firm exits with some bad loans, as recovered from the Credit Register, or with a formal bankruptcy procedure, as identified from the Business Register.

Data on interest rates on firm debt come from the TAXIA database collected by the Bank of Italy. TAXIA covers the universe of firm loans of at least €75,000 for all businesses. Interest rates include the total costs of debt inclusive of all fees paid by firms. Interest rates on loans are averaged using the outstanding debt of the firm as a weight. Data on aggregate GDP, working age population and CPI inflation come from the Italian Statistical Institute (ISTAT).

To identify business idiosyncratic shocks we rely on a representative sample of relatively big mature firms (INVIND database) over the sample period 2000-2020. INVIND is an annual survey of around 4000 firms (roughly 3,000 industrial firms and 1,000 service firms) in the private non-financial sector with at least 20 employees (representative of around 70% of total sales in the Italian economy); see Bank of Italy (2014) for a thorough description of INVIND. We match all firms in INVIND to CERVED, UNIMPS, the Credit Register and the Business Register to follow firms over time even after the firm leaves the INVIND database.[11] INVIND has information on: (i) firm sales and prices; (ii) expected and realized changes in prices and sales; and (iii) the elasticity of demand expected by firms. We use i-iii to identify (non-parametrically) unexpected shifts in firm demand by assuming, as in (7), that a firm faces the following log-linear demand for its goods:

$$\ln q_{jt} = \ln Y_{ist} + z_{jt} - \nu_j \ln p_{jt} \tag{51}$$

where q_{jt} is firm j demand, Y_{ist} is an aggregate demand shifter for province i in year t possibly varying according to the sector s where the firm operates, p_{jt} is the price set by firm j, z_{jt} is an idiosyncratic demand-shifter to firm demand and $\nu_j > 1$ is the price elasticity of firm j demand.

[10] All bad debts are reported in the Credit Register. Other outstanding debt of firms is reported only provided that the debt is greater or equal than €30,000; this threshold is lowered to €250 if the firm has some bad debt.

[11] This allows us to accurately identify whether an INVIND firm goes out of business after a shock, overcoming the sizeable attrition rate in INVIND: around 20 percent of firms in an INVIND wave are no longer present in the subsequent wave (D'Aurizio and Papadia 2016). The match with CERVED also allows us to recover the firm leverage ratio.

Firm j revenue is equal to $r_{jt} \equiv p_{jt} q_{jt}$.

For each firm present in two consecutive waves of INVIND we calculate the following Wold innovations (expectation errors) for revenue ϵ^r_{jt}, and prices ϵ^p_{jt}:

$$\nu^r_{jt} = \frac{r_{jt} - E_{jt-1}(r_{jt})}{r_{jt-1}} \qquad \text{and} \qquad \nu^p_{jt} = \frac{p_{jt} - E_{jt-1}(p_{jt})}{p_{jt-1}}.$$

Given (51), the definition of firm revenue, and the approximation $(x_{jt} - x_{jt-1})/x_{jt-1} \simeq \ln x_{jt} - \ln x_{jt-1}$, the Wold innovation on the demand shifter of firm j, ϵ^z_t, can be expressed as equal to

$$\epsilon^z_t \equiv \frac{z_{jt} - E_{jt-1}(z_{jt})}{z_{jt-1}} = \epsilon^r_{jt} + (\nu_j - 1)\,\epsilon^p_{jt} - \epsilon^A_{ist}, \tag{52}$$

where ϵ^A_{ist} is an aggregate shock common to all firms in the same province and sector:

$$\epsilon^A_{ist} = \ln Y_{ist} - E_{t-1}(\ln Y_{ist}).$$

ϵ^z_{jt} in (52) is an idiosyncratic shock to the demand of firm j. It can be calculated as the residual of the following regression:

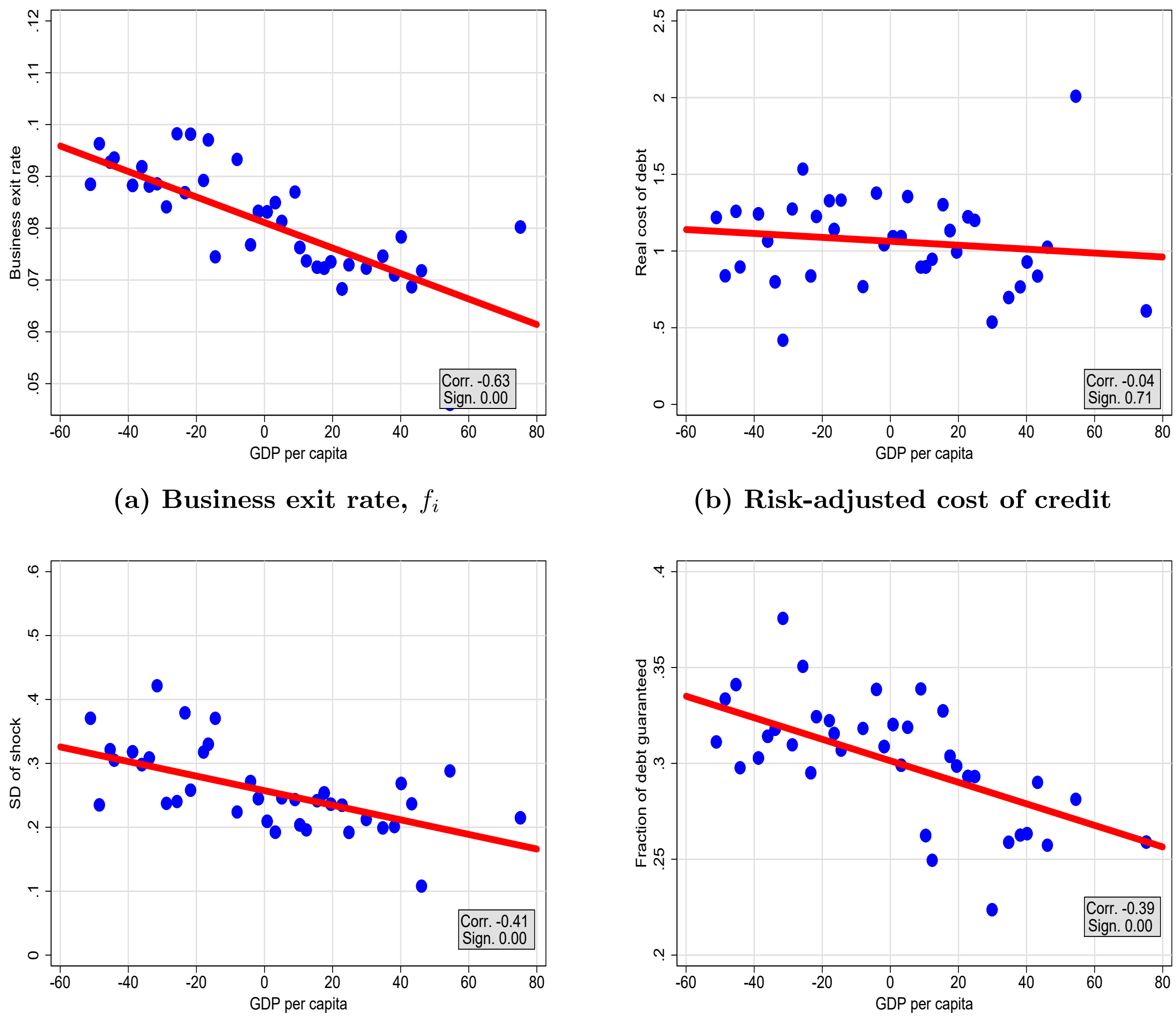
$$\epsilon^r_{jt} + (\nu_j - 1)\,\epsilon^p_{jt} = d_{st} + d_{it} + \epsilon^z_{jt} \tag{53}$$

In (53), d_{st}, and d_{it} are a full set of sector-time and province-time dummies, which control for the aggregate demand shock ϵ^A_{ist}. The elasticity of firm demand ν_j needed to calculate the dependent variable in (53) is calculated by relying on a unique feature of INVIND. Both in 1996 and in 2007, firm managers in INVIND were asked about the elasticity of their demand through the following question: "If your firm were to increase the selling prices by 10%, what percentage change in your nominal sales would be obtained, provided that all your competitors were to keep their prices unchanged and you were to leave all the other terms unchanged?". Answers to the question provides a direct estimate of $(1 - \nu_j) \times 0.1$, which we use to recover ν_j: we take the average sector-specific elasticity reported by firms as an estimate of the demand elasticity faced by firms in the sector.[12] We check that the residuals ϵ^z_{jt}'s from the regression (53) are serially uncorrelated over time, which is a key property of expectation errors.[13] For each province and year we calculate the province level standard deviation of ϵ^z_{jt} using the sample weights provided by INVIND. We take the time average of the resulting standard deviation of ϵ^z_{jt} as a measure of the standard deviation of idiosyncratic risk in the province σ_i. In Italy, the geographical North and the economic North overlap almost perfectly: the correlation between the latitude

[12] As discussed in Pozzi and Schivardi (2016) the reported elasticities ν_j's range between 1.2 and 5.5 and are in the order of magnitude estimated by the literature: they are similar to the elasticity for the textile sector estimated by De Loecker (2011) and similar to the IV estimates obtained by applying the methodology in Foster, Haltiwanger, and Syverson (2008).

[13] Our structural shock to firm idiosyncratic demand are therefore immune to the issue documented by Ma, Ropele, Sraer, and Thesmar (2022) that the Wold innovations on sales ϵ^r_{jt} in INVIND exhibit some serial correlation over time roughly equal to 10 percent.

Figure 1: Geographical variation in financial conditions



(a) Business exit rate, f_i

(b) Risk-adjusted cost of credit

(c) Standard deviation of shocks

(d) Fraction of debt guaranteed

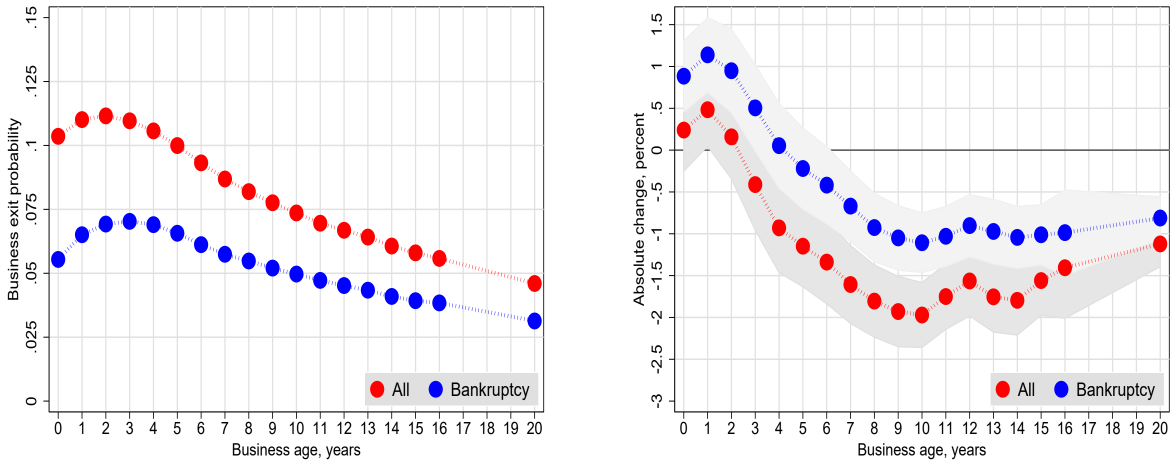
Bin scatter plots with 51 points. In panel (a) the business exit rate is for limited liability companies (CERVED) . In panel (b) the real risk-adjusted cost of credit in a province is calculated as $r_p + f_p(\varphi_p - 1)$ where r_p is the interet rate on term of loans of duration greater than 5 years over the period 2009-015 minus realized inflation over the next 5 years, f_p is the bankruptcy rate of firms older than 10 years of age in the province, φ_p are the contractual guarantees on long term loans multiplied by two thirds, which is the average debt recovery rate for guaranteed debt, see Fischetto, Guida, Rendina, Santini, and Scotto di Carlo (2018) . The overall number of provinces is 102. Panel (c) is an estimate of the Standard Deviation of shocks calculated using the idiosyncratic shock from INVIND. Debt guarantees in panel (d) are from the Credit Register and focus on the total debt of all legal business entities.

of a province and its GDP per capita in log is around 83%. We take the GDP per capita of a province as a measure of how further in the (economic) North the province is located. Figure 1 visually characterizes differences in the conditions of the credit market across Italian provinces. Panel (a) focus on differences in business exit rates, panel (b) on the risk-adjusted cost of credit r_c, panel (c) on the standard deviation of business idiosyncratic risk σ_i, and panel (d) on the fraction of outstanding debt guaranteed in case of default φ_i. The average failure rate of all limited liability businesses from CERVED in a province f_i is 1.5 percentage points higher in the South than in the North. The risk adjusted cost of credit is roughly constant across provinces at roughly a level of 1 percent in real terms. Idiosyncratic risk σ_i is 5 percentage poinst higher in the South than in the North. The fraction of debt guaranteed is 5 percentage points higher in the South than in the North.

The appendix also shows that firm employment size is strongly positively correlated with GDP per capita: a province with 40 percent higher GDP per capita than the national average has a firm size of 10 employees; a province with GDP per capita 20 percent smaller than the national average has an average firm size of 5 employees, implying a North-South difference in average employment size roughly equal to 66 percent (corresponding to 5/7.5).

Panel (a) of Figure 2 shows the average (across provinces) age profile of exit rates for all limited liability businesses from CERVED (blue line) and for all limited liability companies with bankruptcy (red line). To measure North-South differences in the age profile we use the cross

Figure 2: Exit and bankruptcy rates of Italian businesses



(a) Average profile of business exit rate **(b) N-S difference in business exit rates**

Red lines correspond to the exit rate of all limited liability companies from CERVED matched with UNIMPS, Credit Register and Business Register. Blue lines correspond to the business exit rate of limited liability companies with bankruptcy—i.e. leaving some bad loans to banks or exit after a formal bankruptcy procedure.

section of 102 provinces to run regressions of the type:

$$X_{ia} = cte_a^X + \beta_a^X \text{ North}_i + \text{error} \tag{54}$$

where X_{ia} corresponds to alternative definitions of the business exit rate in province i for firms of age a, North$_i$ is the average logged GDP per capita of province i over the period, cte_a^X is a constant and β_a^X measures how variable X_{ia} varies across provinces according to the GDP per capita of the province in log.[14] For expositional simplicity the North-South difference corresponds to two standard deviations differences in province level logged GDP per capita (equal to roughly 60 percent).

The North-South difference is measured by $0.6 \times \beta_a^X$ which corresponds to the "effect" of a 60 percent difference in GDP per capita on the cross sectional difference of X_{ia}. Grey areas correspond to 95 percent confidence intervals. In Panel (b) the red line corresponds to the values of the $0.6 \times \beta_a^X$ when X_{ia} is the exit rate for all limited liability companies; the blue line corresponds to the values of the $0.6 \times \beta_a^X$ when X_{ia} is the bankruptcy rate of all limited liability companies. Overall the business exit rate at birth and during the first 2 years of life of firms is not larger in the South than in North. It is only after that the firm has more than 5 years of age that firms in the South starts to fail with higher probability than firms in the North. After around 10 years of age, differences in failure rates across provinces reach a plateau with a difference in failure probability slightly above one percentage point for all limited liability companies, exiting with or without bankruptcy.[15]

Figure 3 characterizes the average age profile of the leverage ratio (panel a) and how it varies between the North and the South by running the regression in (54) having as dependent variable X_{ia} the leverage ratio (panel b). The average leverage ratio starts from a value of 2 at birth and then it falls to a value roughly equal to 1.6 after 16 years of life of the business. Firms in the North starts their life with a leverage ratio which is 40 percentage points higher than the analogous ratio for firms in the South. After more than 15 years of life, the leverage ratio of firms in the North is on average 20 percentage points lower than the analogous ratio for firms in the South.

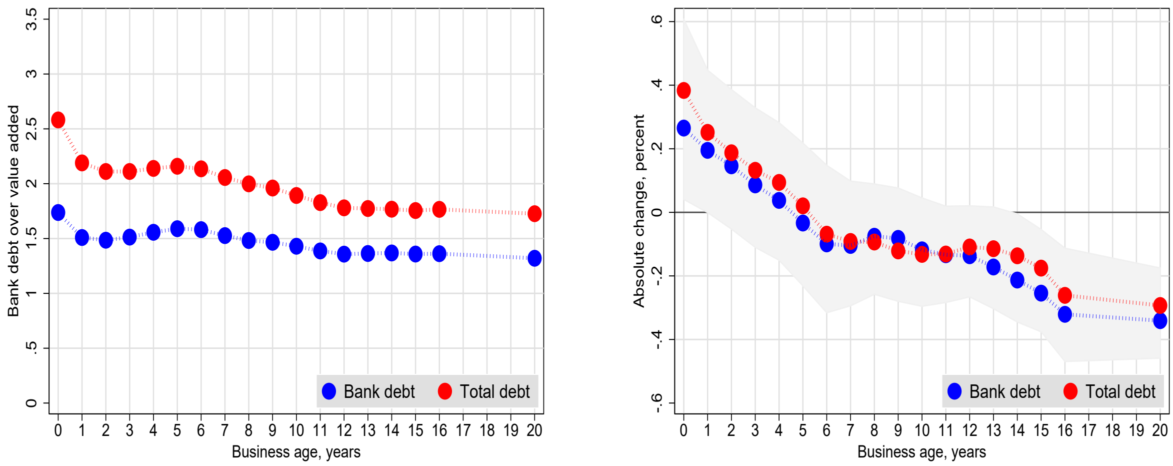
Figure **??** in the appendix shows the average (across provinces) age profile of logged employment size, and labor productivity for all limited liability businesses from CERVED. Employment size is normalized to one at birth—i.e. the average across provinces is divided by its average value for a newly created business. North-South differences in firm size are relatively stable as firms age. There are no pronounced differences in the age profile of firm logged employment size. Both in the South and in the North employment size increases as the firm ages. At each age, differences in average employment size and labor productivity are relatively constant between firms in the South and those in the North with a gap of around 40 percent for employment size.

Since 2010, from the business register, we have information on the identity of all shareholders of all Italian companies and the equity share owned by each of them. Shareholders are identified

[14]Results changes little if we used the latitude of the province rather than its GDP per capita.

[15]In the Appendix we also run panel regressions controlling for a full set of sector and province dummies as well as for firm employment and we find that the North-South difference in business failure rates do indeed increase with firm age. In Figure **??** in the Appendix we also show that the North-South difference in the age profile of businesses remains roughly unchanged when focusing just on businesses which are legal entities exiting with or without bankruptcy.

Figure 3: Age profile of leverage



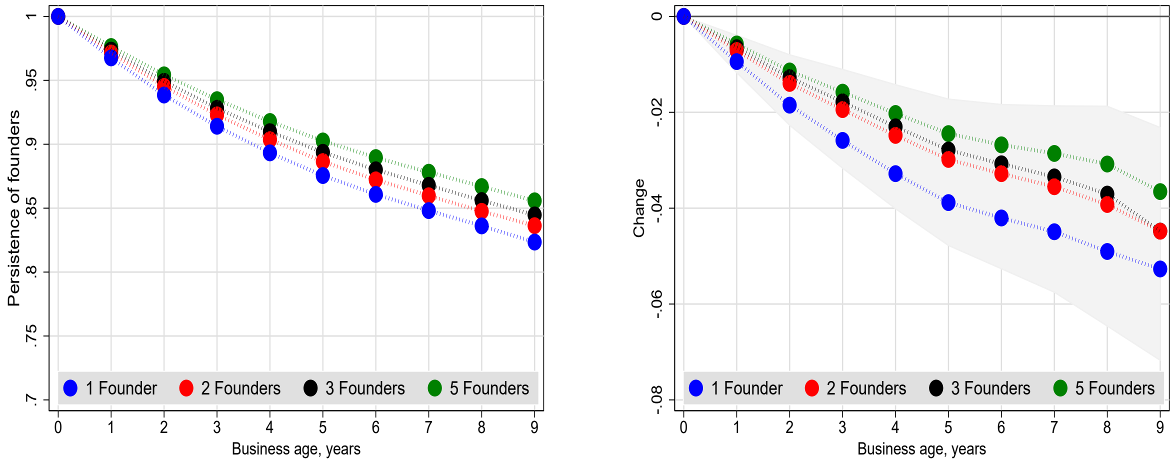
(a) Leverage ratio **(b) N-S difference in leverage ratio**

Universe of limited liability companies. Firm leverage is measured as firm debt over firm total value added as reported from CERVED; In panel (c) and (d) blue line uses bank debt, red line all firm debt The average leverage across firms is weighted by the firm value added.

using the first three letters of their fiscal code, which implies that individuals that belong to the same family (fathers, children, etc.) are considered as the same shareholder. For each new firm, we identify the x greatest shareholders: the (at most x) shareholders with an equity share of the company greater than $1/(1+x)$. This threshold guarantees that there cannot be any other (residual) shareholder of the company with an equity share equal to the equity share of one of the x greatest shareholders. In other words,the group of the x-greatest shareholders is uniquely identified. We refer to the x greatest shareholders of the firm as the x-founders of the firm. We focus on $x = 1, 2, 3, 5$. In each year of life of the firm, we calculate the equity share jointly own by the x-founders. We say that the firm is sold to other investors if the x-founders have all sold their equity shares of the firm. The firm has been sold at t, if t is the first date when all founders no longer own an equity share of the company. For all firms for which we could identify the x-founders, we calculate the date when the firm is sold. The x-founders can dilute their equity partecipation through the extensive margin (selling their entire equity share) or the intensive margin (reducing their equity share). Conditional on at least one of the x-founders still owning some equity share of the company, we calculate the equity share jointly own by the x-founders. This is the intensive margin of equity share held by the x-founders. Selling of the firm corresponds to the extensive margin.

To characterize the evolution of firm leverage after the firm is sold, we perform an event study analysis. We focus on the sample of CERVED of firms for which (i) the x-founders could be identified and (ii) founders have exited. This means that the sample change depending on the number of founders, $x = 1, 2, 3, 5$. This will allow us to fully saturate the model and identify

Figure 4: Retention Probability of founders



(a) Percentage of founders still in the firm **(b) N-S difference in owners turnover**

Notes: Panel (a) reports the probability that at least one of founders is still in the firm at different firm ages. Panel (b) is the North-South difference.

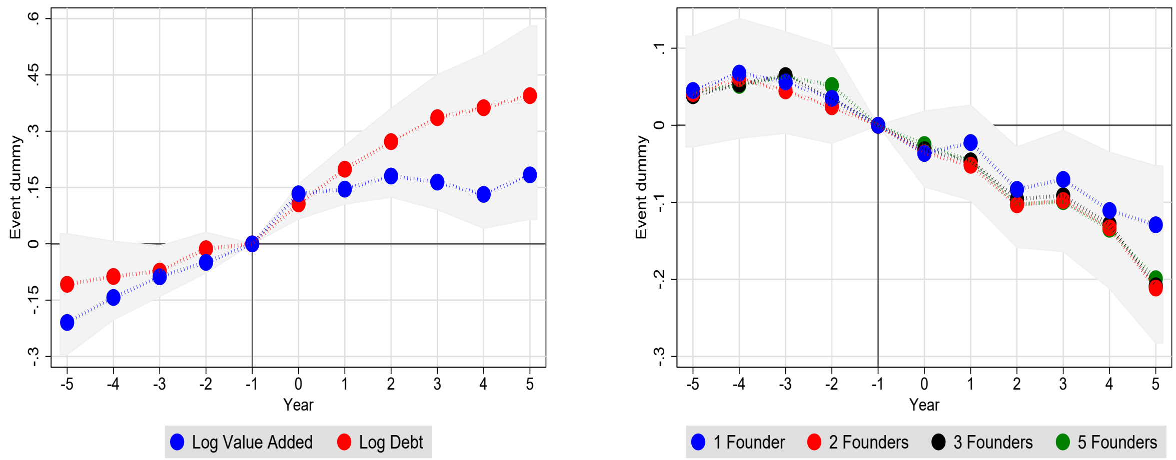
the effect of founders exit. Let d_{jt}^{τ} denote a dummy equal to one if the firm is sold at $t-\tau$ for $\tau = -5, -4, ...0, 1, ...5$. We also introduce the dummy d_{jt}^{-6} if the firm will be sold in more than 5 years. Similarly we introduce the dummy d_{jt}^{6} if the firm was sold more than 5 years ago. With this set of dummies the model is fully saturated. We drop the dummy for the year before selling of the firm: $d_{jt}^{-1} = 0$. Let $\mathbf{X}_{jt}$ denote firm variable we study around a change of control. We focus on the leverage ratio of firm j, logged debt and logged value added. We run the following regression:

$$\mathbf{X}_{jt} = \sum_{\tau=-6}^{6} \beta_{\tau} d_{jt}^{\tau} + d_j + \alpha_{jt} + \psi_t + \epsilon_{jt}, \tag{55}$$

where t is current year; d_{jt}^{τ} are the event dummies for the firm being sold at $t-\tau$ with $d_{jt}^{-1} = 0$ (because the model is fully saturated); d_j is a firm fixed effect; α_{jt} are firm-age dummies that control for the (decreasing) age profile of firm leverage and firm size and ψ_t are time dummies. $t = 1$ is the first year with the new ownership structure. The coefficient at $t = 0$ is normalized at zero. The regression is ran on the full sample of CERVED firms over the period 2010-2019 that experienced a change in ownership. Panel (a) of Figure **??** shows the event dummy coefficients β_{τ} for the leverage ratio. The leverage ratio remains relatively stable at the time of a change of control and increases in the years followings the change in ownership. Panel (b) shows the North-South differences in the time profile of the leverage ratio around the time of the change in ownership.

Figure 5 shows the event dummy coefficients β_{τ} for logged debt and logged value added. Panel a-c differ depending on whether we focus on an ownership composition with 1,2, 3 or 5 owners. The leverage ratio remains relatively stable at the time of a change in the identity of the owners despite an instantaneous sharp increase in value added of around 18 percentage points.

Figure 5: Log-debt vs log-value added around a change of control



(a) One founder, debt & value added **(b) N-S difference in leverage ratio**

t=1 is the first year with the new ownership structure. The coefficient at t=0 is normalized at zero. Th ergression i sran on the full sample of CERVED firms over the period 2010-2019 that experienced a change in ownership

See figure **??** for the response of the leverage ratio log debt and value added for the case of 2 3 or 5 founders

4.2 Calibration

Since the pioneering work by Blanchard and Katz (1992), there is a large literature estimating the elasticity of the local labor force to shocks. Beyer, Smets, and Pijoan-Mas (2015) document that these elasticity are similar in the US and Europe, a finding confirmed by Basso, D'Amuri, and Peri (2018). Michael and Manning (2018) suggest that half life of a shock is around 30 years, that suggest that 2.5% of a shock is absorbed each year. Monras (2018) estimate a half life response of 10 years. We set ψ to target a half life of the shock of 20 years in the range of values estimated by Michael and Manning (2018) and Monras (2018). Given the initial labor force in the north ℓ_0^N and the new steady state work force in the North ℓ_1^N we set

$$\psi = \frac{\left(\ln \ell_0^S - \ln \ell_1^S\right) \times \mathbb{I}(\ln \ell_0^S - \ln \ell_1^S > 0)}{2 \times 20}$$

The long run elasticity of the labor force to a permanent increase in wages is roughly equal to

$$\hat{\beta} = \frac{d \ln \ell_i}{d \ln w_i} = \frac{w_i}{\bar{h}_i \ell_i^\eta} \cdot \frac{1}{\eta} = \frac{1}{1 - \frac{w_S}{w_N}} \cdot \frac{1}{\eta}$$

where w_i and ℓ_i are the initial steady state values in the province of wages and labor force and in the second equality we used the fact that $\bar{h}_i \ell_i^\eta = w_N - w_S$. We set $\hat{\beta}$ roughly equal to 5/6 (elasticity of 5) in line with the evidence of Notowidigdo (2020). In Monras (2018) the long -run elasticity of population to a permament change in wages is around 7.8=20/2.56 As a term of comparison, the implied elasticity between city size and wages implied by the literature on the city size wage premium is roughly 10, see for example Glaeser and Marè (2001).

We normalize the costs of living parameter $\bar{h}_i$ in the least productive province to zero and target η to match the long run response of the workforce to wage changes.

We set the yearly discount rate of firms to $r = 0.04$ to match the long return on the stock market. The cost of credit is set to $\bar{r} = 0.015$ roughly in line with the average real cost of debt in Italy over the period (see panel a of Figure **??**). The elasticity of substitution across varieties is set to $\nu = 5$, consistent the average elasticity of demand reported by firms in INVIND. We consider two representative provinces, one for the North, $i = N$, and one for the South, $i = S$. The South matches moments from provinces with GDP per capita 30% lower than the national average; the North from provinces with GDP per capita 30% higher than the average. Table 1 summarizes the calibrated parameter values for the North and the South.

Figure **??** documents variation in business idiosyncratic risk across provinces. Panel (a) is the average age profile across provinces of the within-province variance of log-employment size of a given age group of firms using data from CERVED with employment measured using UNIMPS.

Table 1: Parameter values

Parameter	Value	Targeted Moment	Value
Firm discount rate, r	0.040	Yearly equity return	0.040
Financiers discount rate, $\bar{r}$	0.015	Average cost of debt in Italy	0.015
Debt repayment arrival rate, ρ_i	1/7	Average debt maturity	7
Idiosyncratic risk, North, σ_N	0.275	Standard deviation of shocks, GDP +30%	0.21
Idiosyncratic risk, South, σ_S	0.314	Standard deviation of shocks, GDP -30%	0.24
Demand drift, North, μ_N	0.020	Average aggregate yearly growth rate of firm size	0.02
Demand drift, South, μ_S	0.005	Average North-South log-difference in firm size	0.35
Debt guarantees, North, φ_N	0.40	Fraction of debt guaranteed, GDP +30%	0.40
Debt guarantees, South, φ_S	0.45	Fraction of debt guaranteed, GDP -30%	0.45
Entrepreneur's labor cost, North, χ_N	1.9	Average leverage ratio, GDP +30%	1.5
Entrepreneur's labor cost, South, χ_S	2.4	Average leverage ratio, GDP -30%	1.8
Debt coupon, $\varkappa_i$	$\psi_i + r$	Cost of bond in absence of default, $\overline{\varphi}$	1

The variance of log employment size increases with business age, but it increases less in provinces with higher GDP per capita, as indicated by the interaction coefficient of firm age and province level GDP per capita in logs, which is negative and significant: the slope of the relation between the variance of log-employment size and business age is one percentage point lower in the North than in the South. Panel (b) is the province level standard deviation of our firm level demand shocks from INVIND adjusted for the fact that only 80% of firms in INVIND are present in two consecutive waves of the survey. Both measures indicate that business idiosyncratic risk is higher in the South than in the North which is in accordance with the geographical variation in job reallocation rates documented in Figure **??**. Given panel (b) of Figure **??**, we set $\sigma_N = 0.275$ in the North and $\sigma_S = 0.314$ in the South. The drift parameters μ_N and μ_S are set to match an yearly growth rate of firm employment equal to 2% in the aggregate and an average log-difference in firm size between the North and the South of 0.35, in line with panels (a)-(b) of Figure **??**. We set the debt duration parameter to $\rho_i = 1/7$, equal in the North and the South, in line with the average debt duration in panel (c) of Figure 1, which is roughly constant across provinces. The debt coupon $\varkappa_i$ is set so that the cost of debt to the firm in the hypothetical case of no bankruptcy is normalized to one, $\bar{\varphi}_i = 1$, which implies that $\varkappa_i = \psi_i + r$. The firm leverage ratio is equal to $b/\mathcal{A}_i$ where $\mathcal{A}_i$ is given in (**??**). The parameters governing the entrepreneurs' cost of labor, χ_i are chosen to match an average leverage ratio of 1.5 in the North and of 1.8 in the South, consistent with panel (a) of Figure 1. The implied χ_i's are $\chi_N = 1.9$ in the North and $\chi_S = 2.4$ in the South, corresponding respectively to 17% and 15% of value added. The parameters determining the fraction of debt guaranteed, φ_i, are chosen consistent with panel (d) of Figure 1, implying that 40% and 45% of the notional value of a loan is guaranteed, respectively, in the North and in the South. Firm debt at birth in a province varies according to a symmetric province-specific distribution $f_{0i}(b)$ that assigns equal mass to two points $\bar{b}_i > \underline{b}_i$. We choose $\bar{b}_i$ and $\underline{b}_i$ so that new businesses i) exit with 14% probability in their first year of life both in the North and 12% in the South (see Figure 2), and ii) enter with an average leverage ratio equal to 1.9 in the North and 1.7 in the South (see panels e and f in Figure **??**). We set the death arrival rates to zero, $\psi_N = \psi_S = 0$, in both regions. Finally, the per capita flow of new businesses is set equal across provinces as indicated by panel (b) in Figure **??**. Firms in the North enter with a technology normalized to one $Z_{0N} = 1$. Firms in the South enter with a technology 30 percent lower than in the North, $Z_{0S} = 0.7$ in line with panel (f) of Figure **??**.[16]

Elasticity of newborn firm's leverage to the borrowing cost To measure the response of the leverage of new firms to the cost of credit we measure $\bar{r}_{pt}$ in province p and year t as equal to $\hat{r}_{pt} = R_{pt} + \mathrm{F}_{pt+1} \times (\mathrm{G}_{pt} - 1)$ where R_{pt} is the real (HPI deflated) interest rate on term loans with maturity between 1 and 5 years in province p and year t, G_{pt} is the corresponding fraction of term loans guaranteed in case of insolvency and F_{pt+1} is the bankruptcy rate of mature companies (10 to 12 years of age) in the following year in the province.[17] Then we run the following province-year panel regression

$$\ln\left(\mathbf{LevNew}_{pt}\right) = \beta\,\hat{r}_{pt} + d_p + \varepsilon_{pt}, \tag{56}$$

where $\mathbf{LevNew}_{pt}$ is the average leverage ratio of new companies in province p in year t and d_p is a full set of province dummies. $\hat{r}$ is measured in in percentage points. The coefficient β in (56) measures the semi-elasticity of the leverage ratio of new firms to the cost of credit. We estimate the regression in (56) over the sample period 2005-2019, separately for all provinces and the subset of provinces in the North: the 35 Italian provinces whose average GDP per capita is higher than the national average by between 15 and 45 percent. Since the real interest rate $\hat{r}_{pt}$ could be endogenously affected by the credit demand of businesses and by their default risk, we also instrument the cost of credit $\hat{r}_{pt}$ with credit supply shocks originated by unexpected changes in the stance of monetary policy. Since we are interested in persistent changes in the cost of credit,

[16] As already observed, the average profitability of firms $\mathcal{R}_i$ and $\mathcal{A}_i$ in (**??**) are irrelevant for firm dynamics and are normalized to one in both provinces.

[17] We focus on the exit rate of mature firms because the data on interest rate for term loans come from TAXIA that cover just a subset of relatively large bank loans for relatively mature firms in the Credit registry.

we use as instruments monetary policy shocks in the past and current year. Monetary policy shocks are from Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019) as downloaded from the Euro Area Monetary Policy Event-Study Database at `https://www.ecb.europa.eu/pub/pdf/annex/Dataset_EA-MPD.xlsx`. The shock in the year is equal to the sum of the year changes in the median quote of the Overnight Index Swap (a measure of expectations of interest rates) during the time window between 13:25 and 15:50 of each ECB policy event date. Given the unconventional nature of monetary policy over this period, we focus on the response of the Overnight Index Swap at an horizon of 1 month, 6 months, 1 year and 5 years. The estimation results are reported in Table 2. Columns 1 and 2 are for the full sample, columns 3 and 4 for the North only. Odd columns report the OLS estimates, even columns the IV estimates. A one percentage increase in the cost of credit increases leverage at entry by roughly 30 percentage points, both in the North and the South. The OLS estimates are slightly higher than the IV estimates in accordance with the view that an increase in the demand for credit increases leverage and the cost of credit. We target a response of the leverage ratio at entry to a one percentage point increase in $\bar{r}$ roughly equal to 0.3, both in the North and the South.

Table 2: Semi-elasticity of leverage at entry to the cost of credit $\bar{r}$

	All provinces (1)	All provinces (2)	North provinces (3)	North provinces (4)
Real interest rate, $\bar{r}$	0.25***	0.14***	0.24***	0.14***
	(0.02)	(0.03)	(0.04)	(0.05)
Observations	1423	1423	488	488
R^2	0.22	0.06	0.16	0.07
F-statistic, 1st stage		204		89
Province dummies	Y	Y	Y	Y
Method	OLS	IV	OLS	IV

Notes: results from estimating (56). The dependent variable is the average leverage ratio in logs $\ln \text{Lev}_{pt}^{\text{New}}$ of new firms in a province over the sample period 2005-2019 Standard errors in parentheses. $\bar{r}$ is in in percentage points. *** p<0.01, ** p<0.05, * p<0.10

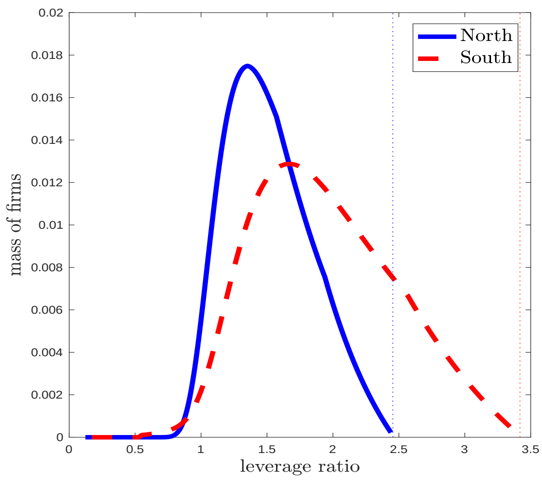
5 Quantitative properties of calibration

We now study the detreminants of difference sin the age profile sof firms in the North and South of Italy. WE also study whether the model matches the quantiative response of firms to an iisoyncratic shock in the North and the South

5.1 Decomposing differences in the age profiles of exit rates

Figure 6 shows the invariant distribution of the firm leverage ratio, $b/\mathcal{A}_i$, in the North (solid blue line) and the South (dashed red line). The vertical lines identifies the bankruptcy threshold $\bar{b}_i/\mathcal{A}_i$. The leverage ratio of firms in the South is higher on average and more dispersed than the leverage ratio of firms in the North. The leverage distribution is skewed to the right, with the South having a thicker right tail, which arises because firms in the North declare bankruptcy with a lower leverage ratio than firms in the South. This is partly due to the lower idiosyncratic risk σ_i in the North than in the South, that reduces the option value of waiting for a better technology draw once at the threshold, and to the higher debt guarantees φ_i in the South than in the North, which discourages firms in the South from declaring bankruptcy for given leverage.

Figure 6: Invariant distribution of leverage ratio



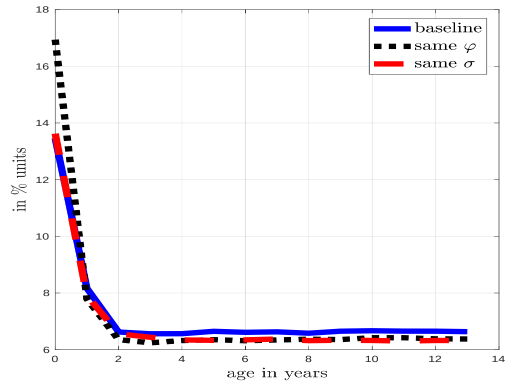
Invariant distribution of the firm leverage ratio, $b/\mathcal{A}_i$, in the North (solid blue line) and the South (dashed red line). The vertical lines identify the corresponding bankruptcy threshold $\bar{b}_i/\mathcal{A}_i$. The distribution is normalized so that it integrates to 1 in each region.

Figure 7 characterizes the age profile of the business exit rate (panel a and b), the leverage ratio (panels c and d) and business technology (panels e and f), which is proportional to firm size in employment or revenue. The firm technology is scaled by the technology at entry in the province Z_{0i}. The left column shows the average between the North and the South of the corresponding variable; the right column the North-South difference, as in Figures ??-??. The leverage ratio in the province firm-age group is calculated exactly as in Figure ??, equal to the ratio between the total debt and the total value added in the group. The solid blue lines correspond to the baseline calibration, the dotted black and dashed red lines to two counterfactuals corresponding to a parameter change of the baseline calibration: in the former the debt guarantee parameter φ in the South is set to its value in the North, in the latter idiosyncratic risk σ in the South is set to its value in the North. The exit rate of businesses decreases with business age as well as the North-South difference in business exit rates which becomes negative only for firms with more than 3 years of age. The exit rate of mature firms is on average 6% per year, a value consistent with our micro evidence and fully obtained through endogenous default decisions. Firms with more than 10 years of age declares bankruptcy with a yearly probability which is 1.4 percentage points higher in the South than in the North. At entry, firms in the North have a leverage ratio 20 percentage points higher than in the South. Over their life cycle, firms in the North deleverage more and after 10 years of life they end up with a leverage ratio which is 40 percentage points lower than the leverage ratio of mature firms in the South. The counterfactuals show that the North-South difference in the exit rate of mature firms is explained, roughly in equal proportion, by the higher idiosyncratic risk and debt guarantees of the South relative to the North. Differences in idiosyncratic risk and debt guarantees have an opposite effect on the leverage ratio at maturity: leverage in the South increases with lower risk σ, while it decreases with lower guarantees φ. The exit rate of new firms is equal in the South and the North because the higher risk in the South σ is compensated by the higher debt guarantees φ: for given leverage, a lower σ reduces exit, while a lower φ increases it. Panel (e) shows how firm size increases with business age partly because of the positive drift in technology and partly because of endogenous selection in exit: firms with lower technology Z are more leveraged and more likely to declare bankruptcy. Panel (f) shows that firms in the South are smaller by 35 percentage points at entry and that differences increases up to 45 percentage points after 10 years, roughly in line with the data.

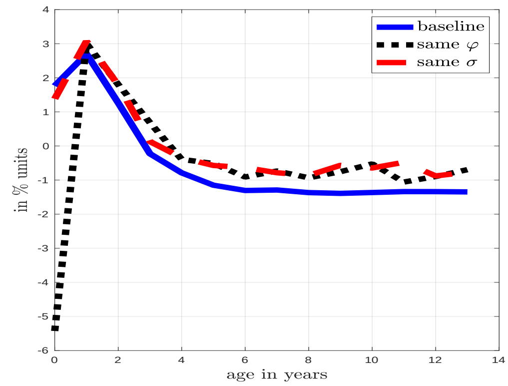
5.2 Validation: Firm responses to idiosyncratic demand shocks

To evaluat eteh model we characterize differences in the responses of firm exit to firm idiosyncratic shocks. We now use our sample of mature firms from INVIND matched with CERVED, UNIMPS, the Credit Register and the Business Register to analyze how the response of firm exit to idiosyncratic demand shocks varies across provinces and whether firm debt accounts for the observed differences. We look at the response of firm j in terms of firm prices p_{jt}, quantities q_{jt}, and business exit F_{jt}, equal to one if firm j exits at t and zero otherwise. We show that in response to a demand shock firms in low GDP provinces are more likely to exit, which is

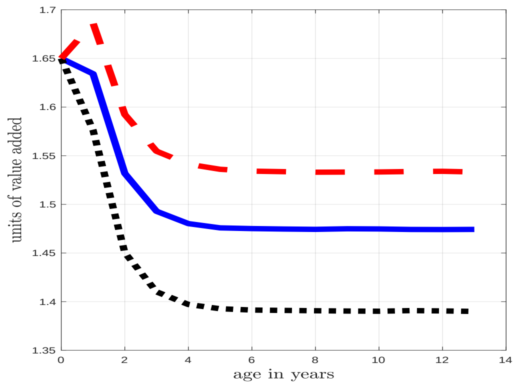
Figure 7: Age profiles of exit rate, leverage and size



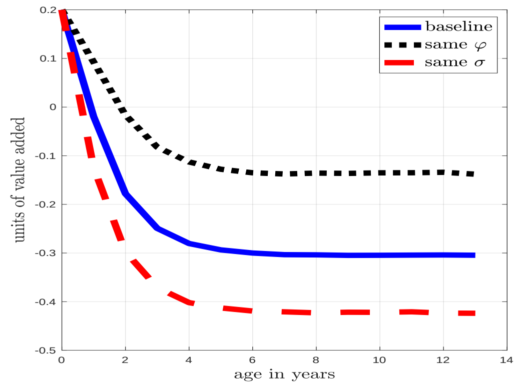
(a) Exit rate, North-South average



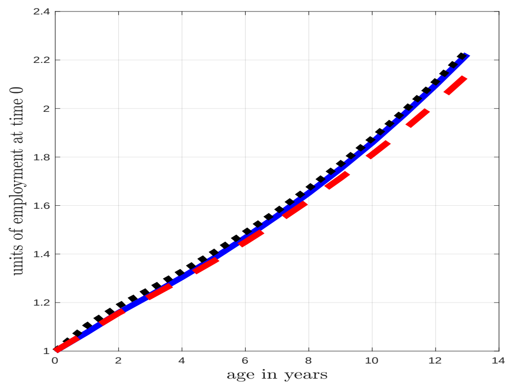
(b) Exit rate, North-South difference



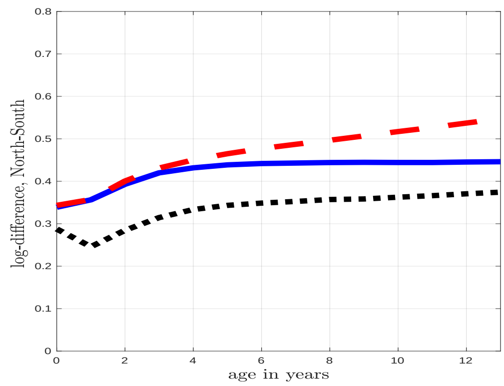
(c) Leverage ratio, North-South average



(d) Leverage ratio, North-South difference



(e) Labor productivity, North-South avg.



(f) Labor productivity, North-South diff.

The panels in the left column are the age profile of the North-South average of the corresponding variable; the panels in the right column are the North-South difference. The solid blue lines correspond to the baseline calibration; the dashed red line to a counterfactual where the South has the same idiosyncratic risk as the North $\sigma_S = \sigma_N = 0.275$; the dotted black lines to a counterfactual where the South has the same guarantees as the North $\varphi_S = \varphi_N = 0$. The average technology in the province firm-age group is scaled by Z_{0i} in the province. The leverage ratio in the province firm-age group is calculated as the ratio of the total debt over total value added in the group.

in line with the previous evidence for mature firms. We use a projection method to estimate the response of a variable x_{jt+n} $n \geq 0$ years after the shock ϵ^z_{jt} recovered after estimating the regression in (53). Given (51), the change in the demand shifter between $t+n-1$ and $t+n$ can be calculated as

$$\Delta \tilde{z}_{jt+n} = \Delta r_{jt+n} + (\nu_j - 1)\, \Delta p_{jt+n}$$

where Δ the growth rate operator: $\Delta x_{jt} = \frac{x_{jt} - x_{jt-1}}{x_{jt-1}}$. To estimate the response to the shock ϵ^z_{jt} of $x = \tilde{z}, p, q, F$ at $n = 0, 1, 2, 3$ we run the following regression:

$$\mathbf{R}^{\mathbf{x}}_{jt+n} = \beta^n_x \epsilon^z_{jt} + d_{st} + d_{it} + d_j + \gamma_x X_{jt-1} + \text{error}_{jt} \tag{57}$$

where $\mathbf{R}^{\mathbf{x}}_{jt+n}$ is the response of variable x, n periods after the shock, d_{st} and d_{it} are a full set of sector-time dummies and province-time dummies to control for the aggregate shocks ϵ^A_{ist}'s , d_j is a firm fixed effect and X_{jt-1} is a set of controls that contain only information available at time $t-1$—thereby orthogonal to ϵ^z_{jt}. The dependent variable $\mathbf{R}^{\mathbf{x}}_{jt+n}$ is equal to $\sum_{k=0}^{n} \Delta x_{jt+k}$ when $x = \tilde{z}, p, q$.[18] For failure probabilities, $\mathbf{R}^{\mathbf{F}}_{jt+n}$ is equal to one if the firm has failed by year $t+n$ and zero otherwise (exiting is an absorbing state), so that

$$\mathbf{R}^{\mathbf{F}}_{jt+n} = \max_{k=1,\ldots n} F_{jt+n}. \tag{58}$$

The coefficient β^n_x measures the response of variable x n periods after a doubling of demand—a demand shock of 100 percent.

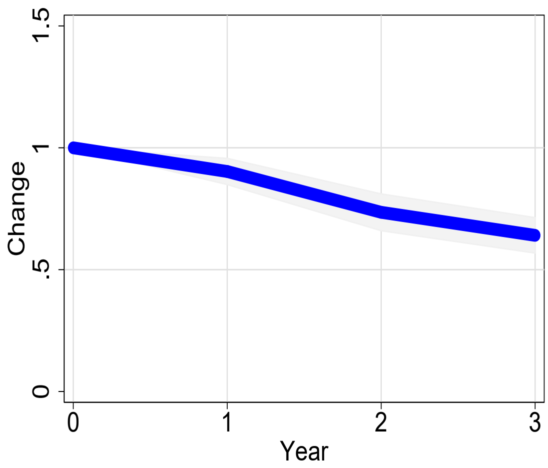
Figure 8 shows the average response of INVIND firms to the demand shock when estimating the regression (57) for different n's and x's. Grey areas correspond to 95 percent confidence intervals. Panel (a) shows the (percentage) response of the demand shifter z which is one on impact by construction, amounting to a doubling in firm demand. Panel (b) shows the (percentage) response of firm prices p, panel (c) of firm quantities q, panel (d) of the failure probability F. The observed firm responses are consistent with the predictions of a canonical demand shock: in response to a positive shift in demand, both prices and quantities increase while the probability of going out of business falls. The shocks are highly persistent. A doubling of demand is associated with an increase in prices of roughly 10 percent and an increase in quantities which averages 17 percent in the year of the shock and in the following three years. The probability of going out of business falls by roughly 1.5 percentage point.

Since firms in different sectors could respond differently to the same shock, to characterize differences across provinces, we estimate the following regressions where the impulse response coefficient β^n_x are allowed to vary by sector s and province i

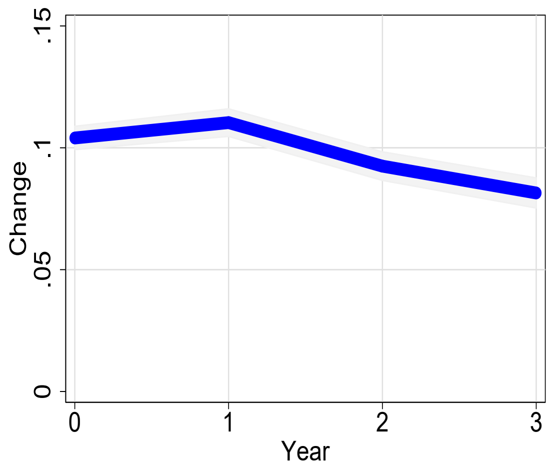
$$\mathbf{R}^{\mathbf{x}}_{jt+n} = (\beta_{xs} + \beta_{xi})\, \epsilon^z_{jt} + d_{st} + d_{it} + d_j + \gamma_x X_{xt-1} + \text{error}_{jt}, \quad n = 1, 2, 3 \tag{59}$$

[18]For this set of x's, the set of controls X_{t-1} includes the growth rate of x at $t-1$ Δx_{jt-1}, and the expected (at $t-1$) growth rate at t $E_{jt-1}(\Delta x_{jt})$, which is information available from INVIND.

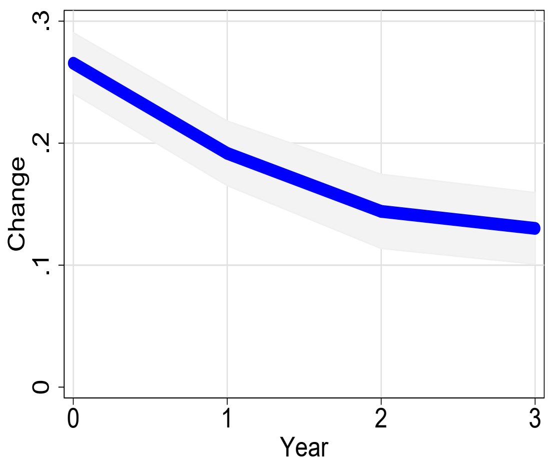
Figure 8: Impulse responses



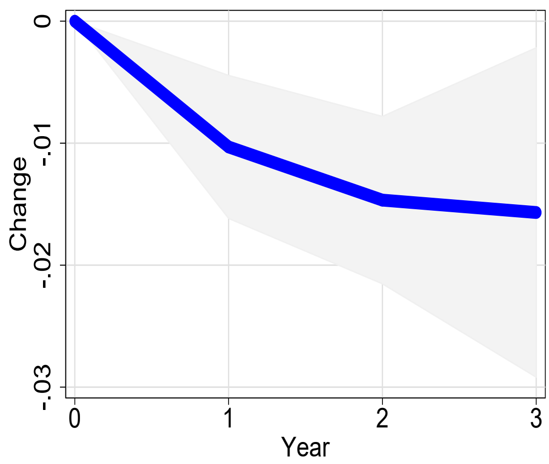
(a) Demand shifter



(b) Price



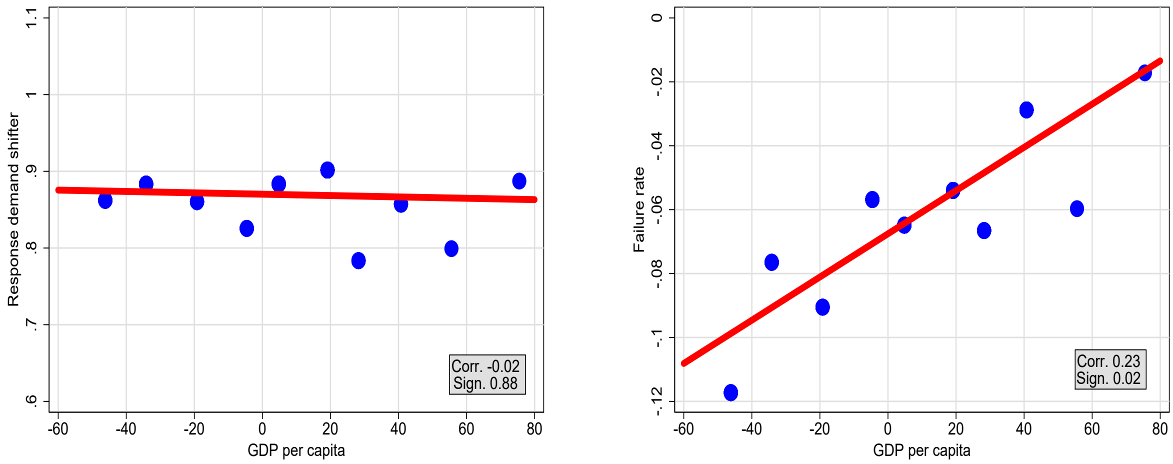
(c) Quantity



(d) Failure probability

We pooled together the responses at all n's. The coefficient β_{xs} measures the average response of variable x over the 3 periods after the shock in the sector s where firm j operates. The coefficient β_{xi} measures the average response of variable x in province i after controlling for sectoral differences across provinces. To identify the full set of province dummies (corresponding to our 102 provinces) we set to zero the value of β_{xs} for the reference sector (sector NACE3). Panel (a) of Figure 9 shows how the average response of the demand shifter β_{zi} varies across provinces according to the GDP per capita of the province. Panel (b) shows differences in the response of business exit. The demand shock is highly persistent with small differences across provinces. There is indication that in response to an increase in firm demand, the failure probability falls substantially more in low GDP per capita provinces than in high GDP per capita provinces. Quantitatively differences are large: the failure probability responds little in the provinces with the highest GDP per capita of Italy, while it falls by almost 4 percentage points in provinces with GDP per capita lower than 50 percent than the national average.

Figure 9: Variation in elasticities: 3 years average after demand shock



(a) Demand shifter **(b) Failure probability**

To evaluate whether firm debt accounts for the observed differences across provinces in the response of business exit, we run the following regression analogous to (59):

$$\mathbf{R}^{\mathbf{F}}_{jt+n} = \beta^n_y \times \text{GDP}_i \times \epsilon^z_{jt} + \beta^n_l \text{Leverage}_{jt-1} \times \epsilon^z_{jt} + \beta^n_s \times \epsilon^z_{jt} + d_{st} + d_{it} + d_j + \gamma_x X_{jt-1} + \text{error}_{jt}, \quad (60)$$

$\mathbf{R}^{\mathbf{F}}_{jt+n}$ is the response of firm exit in (58). We interact the shock ϵ^z_{jt} with the average GDP per capita in the province (in logs) GDP_i, the firm leverage ratio before the shock Leverage_{jt-1}, and a full set of sectoral dummies corresponding to the sector s where firm j operates. The specification in (60) contains a fixed effect for the firm and a full set of sector and province dummies both interacted with time dummies. The set of controls X_{jt-1} includes the leverage ratio of the firm at $t-1$. In running the regression, GDP_i and Leverage_{jt-1} are standardized by their cross-sectional dispersion. Information on firm leverage come from CERVED, so the regression is run on the sample of INVIND firms matched with CERVED. Standard errors are clustered at the province level. The coefficient β^n_y measures how the response of business exit n years after the shocks varies with the GDP of the province. We are interested in testing whether the value of β^n_y falls after controlling for firm leverage: if firm leverage explains differences in the response of firms across provinces the effect of GDP should drop after controlling for firm leverage. Table 3 shows the results from estimating (60) on impact $n = 1$, two years after the shock $n = 2$ and three years after the shock $n = 3$, with and without controlling for firm leverage.[19] After controlling for firm leverage, the effect of GDP per capita on business exit falls by two thirds and becomes statistically insignificant. This indicates that firms in low GDP provinces fails more in response

[19]Results are almost unchanged when we do not interact the shocks with the full set of sector dummies—i.e. we drop the regressors $\beta^n_s \times \epsilon^z_{jt}$ from the regression in (60).

Table 3: GDP, leverage and the response of business exit to demand shocks

	Exit at								
	1 year	1 year	1 year	2 years	2 years	2 years	3 years	3 years	3 years
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\text{GDP}_i \times \epsilon^z_{jt}$	0.040*	0.021	0.024	0.038*	0.022	0.033	0.057**	0.033*	0.034*
	(0.02)	(0.021)	(0.022)	(0.019)	(0.022)	(0.023)	(0.025)	(0.016)	(0.015)
$\text{Leverage}_{jt-1} \times \epsilon^z_{jt}$		-0.022***	-0.014**		-0.035***	-0.026**		-0.041***	-0.031***
		(0.007)	(0.006)		(0.011)	(0.012)		(0.011)	(0.011)
$\text{ROA}_{jt-1} \times \epsilon^z_{jt}$			0.322***			0.246			0.446**
			(0.114)			(0.184)			(0.190)
$\ln(\text{Size}_{jt-1}) \times \epsilon^z_{jt}$			0.035			0.130***			0.086**
			(0.030)			(0.034)			(0.038)
Province FE × Year FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sector FE × Year FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sector FE × ϵ^z_{jt}	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	30,205	27,843	27,764	28,811	26,586	26,519	26,567	24,525	24,461
R^2	0.062	0.067	0.069	0.068	0.077	0.082	0.071	0.080	0.085

Coefficient on interaction coefficient of shock and province level GDP per capita. In all specification we allow the response coefficient to vary by sector. An observation is firm level observation in all waves of INVIND matched with CERVED. The sample period is 2000-2020. GDP_i is the average GDP per capita in logs in the province, Leverage_{jt-1} is the firm leverage ratio, ROA_{jt-1} is firm earnings over total assets and $\ln(\text{size})_{jt-1}$ is employment size in logs from UNIMP. Leverage_{jt-1}, ROA_{jt-1} and $\ln(\text{size})_{jt-1}$ are demeaned and are calculated in the year previous to the shock. In the overall sample the Standard deviation of GDP, Leverage, ROA and ln(size) are equal to 0.30, 0.77, 0.04 and 0.36 respectively. Standard errors robust to heteroscedasticity and clustered at the province level are in parentheses with p-value denoted by *** if p<.01, ** if p<.05, and * if p<.1.

to an unexpected fall in firm demand because they have accumulated larger level of firm debt, which is consistent with the hypothesis that firms in the South are more overindebted.

Leverage is total debt debt divided value added and it is calculated only for firms with positive value added, size is employment size from UNIMPS, ROA is earnings divided total assets.

5.3 Transition to a new normal

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Online Appendix “Overindebtness in the North and the South”

Appendix A further describes the data. Appendix B contains additional empirical results. Appendix ?? contains the derivation of the simple partial equilibrium model with overindebtness. Appendix ?? contains some derivations of the quantitative model and discusses the solution algorithm.

A Data appendix

We pool together *six* different sources of information to characterize how business failure affects outputacross Italian provinces and how firm responses to demand shocks: *i.* The Italian Central Credit Registry, *ii.* InfoCamere Data, *iii.* Italian National Social Security Institute Data, *iv.* Company Accounts Data Service, *v.* Survey of Industrial and Service Firms, and *vi.* Labor Force Survey. The *i.-iv.* are administrative data sets and *v.* and *vi.* are survey conducted respectively by the Bank of Italy and the Italian National Institute of Statistics.

A.1 Italian Central Credit Registry

The first data source on bank loans to businesses is drawn from the Italian Central Credit Registry (CCR).

The CCR is an information system operated by the Bank of Italy, the Italian central bank, which collects loans extended by banks and financial companies to their clients. It contains monthly detailed information on all individual loans granted by financial intermediaries to borrowers for which the overall exposure is above €75,000 toward a single intermediary (this threshold was lowered to €30,000 in January 2009). If a loan is in default (i.e. a bad loan), the overall credit exposure of the borrower is automatically registered in the CCR (even if not legally ascertained as insolvent), regardless of the loan amount and irrespective of any possible collateral or guarantee.

Loans are divided into three broad categories: overdraft loans (uncommitted credit lines), term loans (include leasing, mortgages, and committed credit lines), and loans backed by receivables. For each loan type the CCR records information on loan amount, date of origination, maturity, and guarantees. We also retrieve information on the interest rates that banks charge to individual borrowers. Specifically, we use Taxia, i.e. a subset of the CCR that covers information on more than 80% of total bank lending in Italy at quarterly frequency.[20] For each loan type, loan interest rate is the gross annual interest rate, inclusive of fees and commission charged.

A.2 InfoCamere Data

The second data source comes from the InfoCamere database, under the management of the Chambers of Commerce.

[20] Interest rates are available since 2004:Q1 only for bank clients whose total exposure exceed €75,000.

We use observations recorded for all corporations and partnerships (excluding sole proprietorships) on insolvency proceedings - which precedes the actual bankruptcy declaration - in the period between 2005 and 2019.

We do not use bankruptcy declaration because traditional Italian insolvency proceedings are extremely formal and require the involvement of courts and/or other public authorities, regardless of the size of the bankruptcy estate; consequently, they are usually lengthy and costly, the average duration being 7.5 years in 2019.[21]

A.3 Italian National Social Security Institute Data

The third data source comes from the Italian National Social Security Institute (INPS), which provides information on employment and wages at the firm level.

The INPS regularly compiles data archives on the national social security system and more generally on welfare-related issues by collecting administrative information that employers, operating in the private nonagricultural sectors, have to provide to pay pension contributions for their employees.

From this data set, we use firm-level monthly information on the number of employees available from 1990 to 2019 and - for workers employed by at least one firms surveyed by the Bank of Italy's Survey on Industrial and Services firms - we use their full working history available from 2005 to 2019.

A.4 Company Accounts Data Service

The fourth data source comes from Company Accounts Data Service (CADS), which contains balance sheet information on Italian limited liability firms.

Using data deposited by firms at the local Chambers of Commerce, as required by Italian law, CADS includes detailed information on balance sheet and income statements for almost all Italian limited liability companies since 1993. CADS is a proprietary database owned by Cerved Group S.p.A., a leading information provider in Italy and one of the major credit rating agencies in Europe. Each company's financial statement is updated annually.

From this data set, we collect yearly balance sheet information on financial debt and value added.

A.5 Survey of Industrial and Service Firms

The fifth data source comes from the Survey of Industrial and Service Firms (INVIND, henceforth), which is a large annual business survey conducted by the Bank of Italy on a representative sample of firms.

Since 2002, the reference universe in INVIND consists of firms with at least 20 employees operating in industrial sectors (manufacturing, energy, and extractive industries) and in non-financial private services, with administrative headquarters in Italy (representative of 70% of total sales in the Italian economy).[22] In recent years each wave has about 4,000 firms (3,000

[21] Statistics provided by the Italian Ministry of Justice.

[22] The survey adopts a one-stage stratified sample design based on an 11-sector classification, the number of employees, and the region in which the firm's head office is located. See (Bank of Italy 2014) for a thorough description of the INVIND data set.

Table O1: List of NACE codes in INVIND sample

Codes	Section/Subsection
CB	Mining and quarrying except energy producing materials
DA	Manufacture of food products; beverages and tobacco
DB	Manufacture of textiles and textile products
DC	Manufacture of leather and leather products
DD	Manufacture of wood and wood products
DE	Manuf. of pulp, paper & paper product; publishing & printing
DF	Manufac. of coke, refined petroleum products & nuclear fuel
DG	Manufac. of chemicals, chemical products and man-made fibres
DH	Manufacture of rubber and plastic products
DI	Manufacture of other non-metallic mineral products
DJ	Manufacture of basic metals and fabricated metal products
DK	Manufacture of machinery and equipment n.e.c.
DL	Manufacture of electrical and optical equipment
DM	Manufacture of transport equipment
DN	Manufacturing n.e.c.
E	Electricity, gas and water supply
G	Wholesale & retail trade; repair of motor vehicles, household ...
H	Hotels and restaurants
I	Transport, storage and communication
K	Real estate, renting and business activities

industrial firms and 1,000 service firms). Table O1 reports the list of all NACE codes included in our INVIND sample. There are 20 branches of activity. We removed firms operating in mineral extraction or construction and also firms affected by structural changes, namely split, incorporation, merger, spin-off, capital contribution or transfer os assets.

A.6 Italian Labor Force Survey

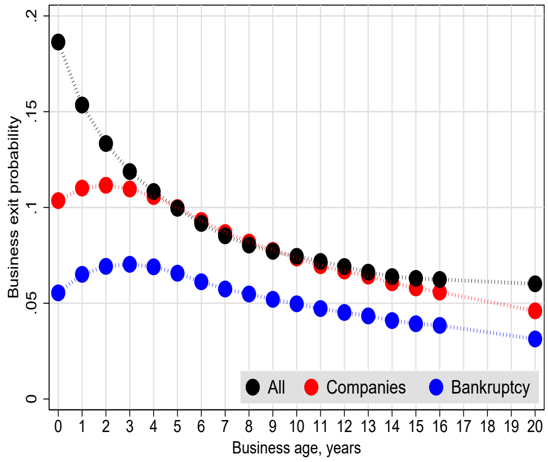
The sixth data source comes from the Italian Labour Force Survey (ILFS), which is a cross-sectional and longitudinal household sample survey, collated by the Italian National Institute of Statistics (ISTAT).

The database provides observations on labor market participation and person outside the labor force for about 250,000 households and 600,000 individuals per year. The ILFS is the main dataset used to provide the official statistics on the labor market and is part of the European Labor Force Survey.

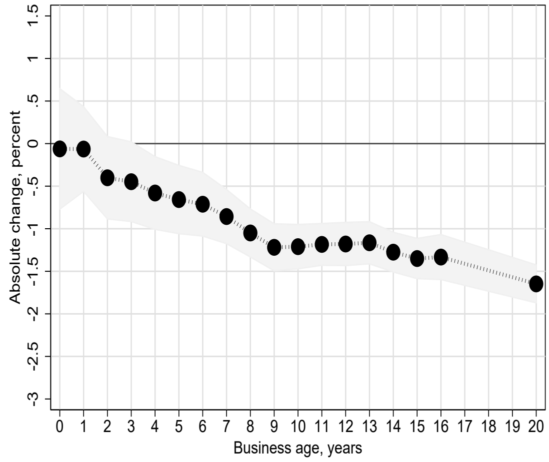
From this data set, we collect information at the province level to calculate GDP per capitas and worker inflows and outflows. *years?*

B Additional empirical evidence

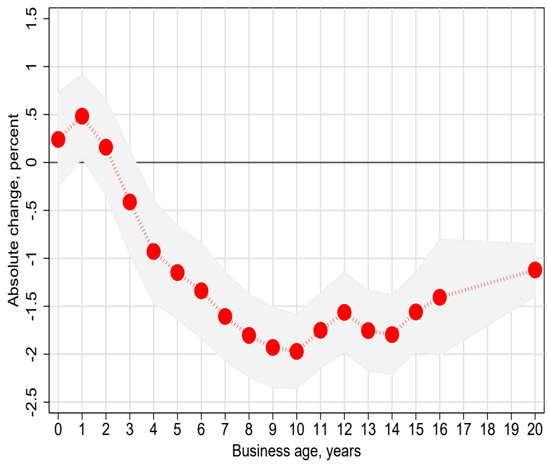
Figure O1: Exit and bankruptcy rates of Italian businesses



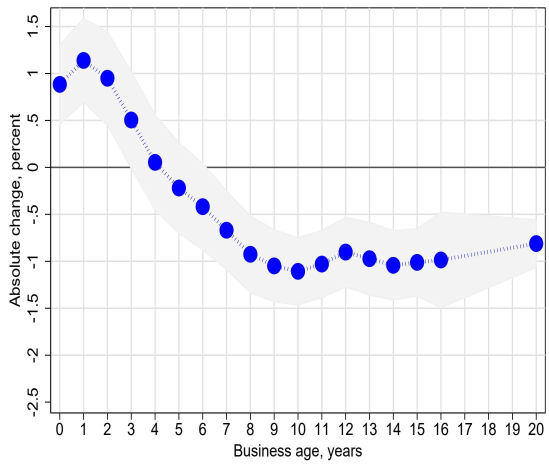
(a) Average profile of business exit rate



(b) N-S difference exit rate, all businesses



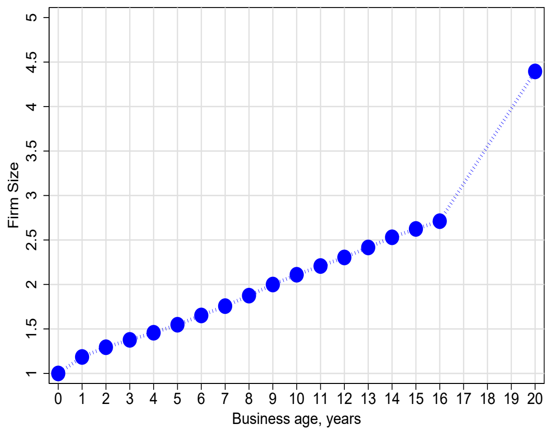
(c) N-S difference exit rate, CERVED



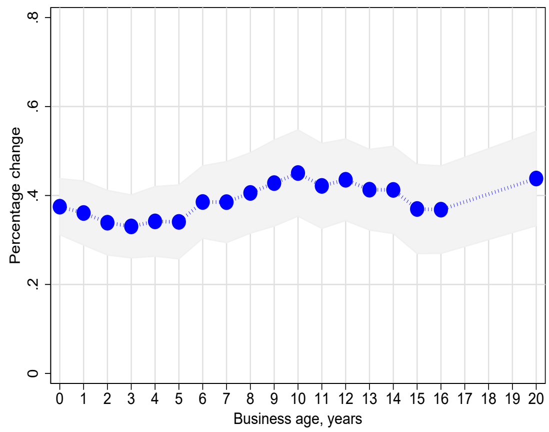
(d) N-S difference bankruptcy rate,CERVED

Black lines correspond to the business exit rates of all businesses (legal entities or sole proprietors) using UNINPS. Red lines correspond to the exit rate of all limited liability companies from CERVED matched with UNIMPS, Credit Register and Business Register. Blue lines correspond to the business exit rate of limited liability companies with bankruptcy—i.e. leaving some bad loans to banks or exit after a formal bankruptcy procedure.

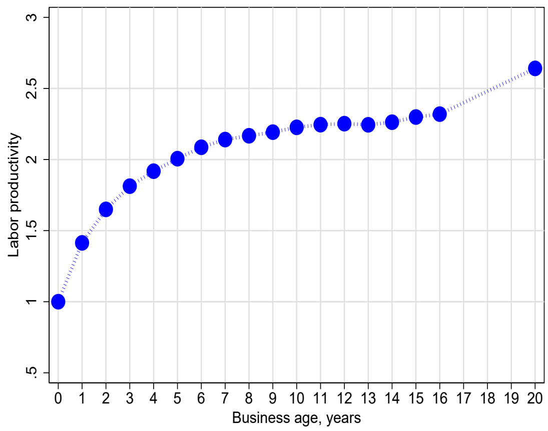
Figure O2: Firm life cycle of size and leverage



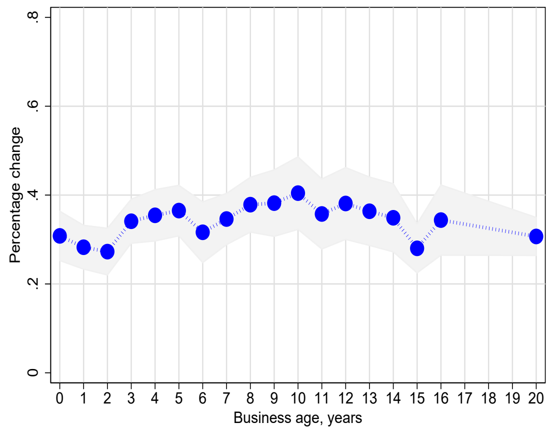
(a) Firm employment size



(b) Firm size and GDP



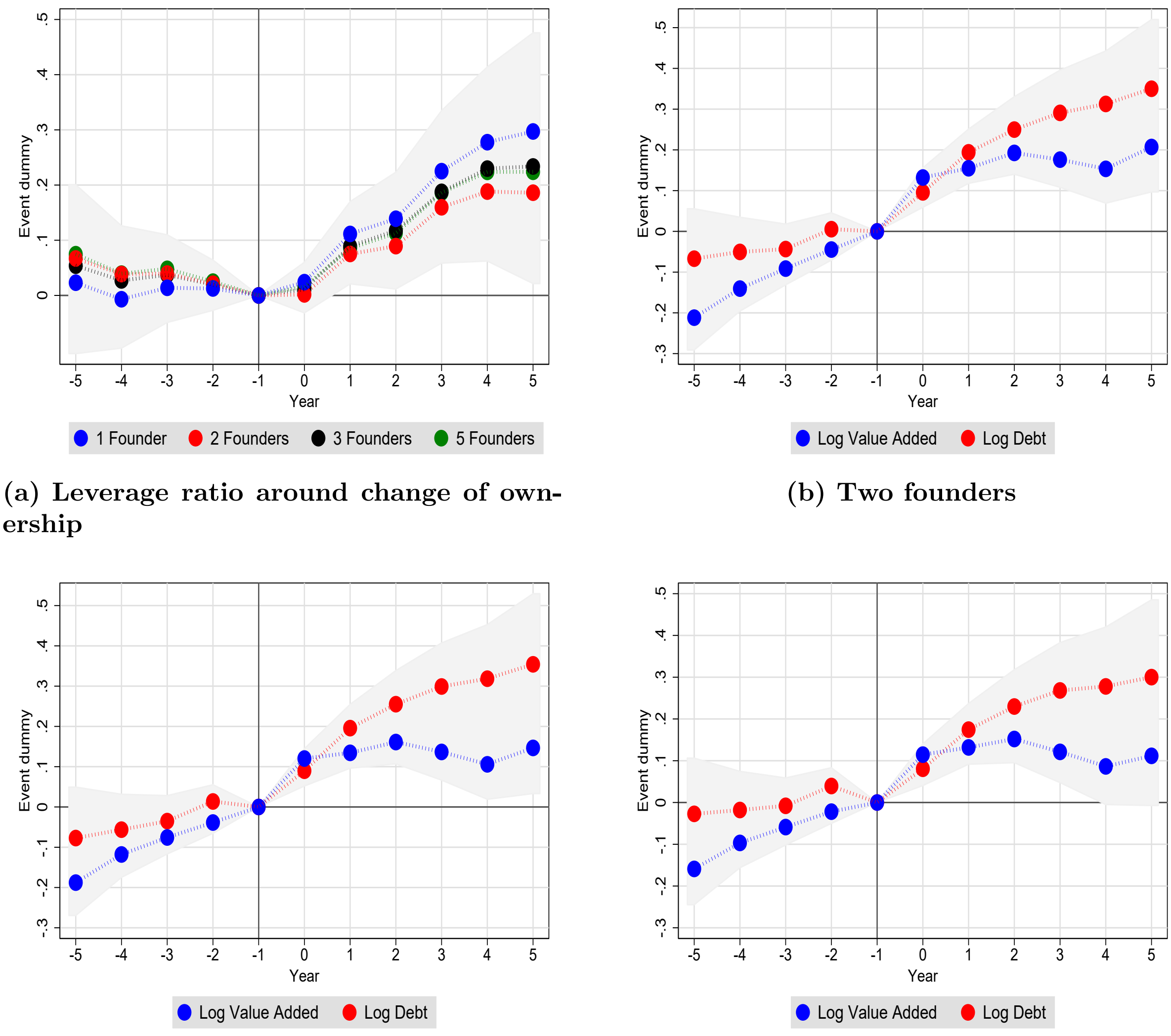
(c) Labor Productivity



(d) Productivity and GDP

Universe of limited liability companies. Employment is in logs. In Panel (a) is normalized to one at entry.

Figure O3: Log-Leverage vs log-value added dynamics around a change in firm owners



(a) Leverage ratio around change of ownership

(b) Two founders

(c) Three founders

(d) Five founders

t=1 is the first year with the new ownership structure. The coefficient at t=0 is normalized at zero. Th ergression i sran on the full sample of CERVED firms over the period 2010-2019 that experienced a change in ownership

C Verifying the guess for the standaridized value function

The HJB equation of the firm problem is given in (**??**). We define

$$V_{it}(B, Z, \mathcal{R}) = v_{it}(b)\mathcal{R}Z$$

with $b = B/(\mathcal{R}Z)$ and

$$\begin{aligned}
\frac{\partial V_{it}(B,Z,\mathcal{R})}{\partial B} &= v'_{it}(b) \\
\frac{\partial^2 V_{it}(B,Z,\mathcal{R})}{\partial B^2} &= v''_{it}(b)\frac{1}{\mathcal{R}Z} \\
\frac{\partial V_{it}(B,Z,\mathcal{R})}{\partial Z} &= v_{it}(b)\mathcal{R} - v'_{it}(b)b\mathcal{R} \\
\frac{\partial^2 V_{it}(B,Z,\mathcal{R})}{\partial Z^2} &= -v'_{it}(b)b\frac{\mathcal{R}}{Z} + v'_{it}(b)b\frac{\mathcal{R}}{Z} + v''_{it}(b)b^2\frac{\mathcal{R}}{Z} = v''_{it}(b)b^2\frac{\mathcal{R}}{Z} \\
\frac{\partial V_{it}(B,Z,\mathcal{R})}{\partial \mathcal{R}} &= v_{it}(b)Z - v'_{it}(b)bZ
\end{aligned}$$

Under the guess with (21) and (13) we have that (18) reads as

$$\begin{aligned}
(r+\delta_i)v_{it}(b) &= 1 - (\varkappa_i + \rho_i)\, b - \rho_i v'_{it}(b)b \\
&+ \lambda_i \left[\left(\frac{g}{\kappa} - 1\right) v_{it}(b) + x_{it}(b)b\,(g-1)\right] + \frac{\sigma_i^2}{2} v''_{it}(b)b^2 + \varrho_{it}\left[v_{it}(b) - v'_{it}(b)b\right] + \frac{\partial v_{it}}{\partial t}.
\end{aligned} \tag{61}$$

where

$$\varrho_{it} \equiv \frac{\dot{\mathcal{R}}_{it}}{\mathcal{R}_{it}}$$

Rearranging (61) we obtain that

$$\left[r + \delta_i - \varrho_{it} - \lambda_i\left(\frac{g}{\kappa} - 1\right)\right] v_{it}(b) = 1 - (\varkappa_i + \rho_i)\, b - \left[\rho_i + \varrho_{it} + \lambda_i\,(g - 1)\right] v'_{it}(b)b + \frac{\sigma_i^2}{2} v''_{it}(b)b^2 + \frac{\partial v_{it}}{\partial t}$$

which coincides to (**??**) in the main text.

D Verifying the guess for the value function in steady state

We guess that in steady state

$$v(b) = \bar{v}_0 - \bar{v}_1 b + \frac{\bar{v}_2}{1+\gamma}\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} b$$

Under the guess we have that

$$v'(b) = \bar{v}_2\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} - \bar{v}_1 \tag{62}$$

$$v''(b) = \bar{v}_2\gamma\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} \times \frac{1}{b} \tag{63}$$

By substituting the guess for $v(b)$ into the HJB in (**??**) in steady state $\frac{\partial v_{it}}{\partial t} = 0$, we obtain that the guess is verified if

$$\left[r + \delta - \lambda\left(\frac{g}{\kappa} - 1\right)\right]\left[\bar{v}_0 - \bar{v}_1 b + \frac{\bar{v}_2}{1+\gamma}\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} b\right] = \mathcal{R} - (\varkappa + \rho)\, b$$
$$+ \left[\rho + \lambda\,(g-1)\right]\left[\bar{v}_1 b - \bar{v}_2\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} b\right] + \frac{\sigma_i^2}{2}\bar{v}_2\gamma\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} b, \tag{64}$$

Since the value matching has to be satisfied it must also be that

$$\bar{v}_0 - \bar{v}_1\bar{b} + \frac{\bar{v}_2}{1+\gamma}\bar{b} = -\left(1-\phi\right)\varphi\,\bar{b} + \phi\left[\bar{v}_0 - \bar{v}_1\alpha\bar{b} + \frac{\bar{v}_2\alpha^{1+\gamma}}{1+\gamma}\bar{b}\right] \tag{65}$$

Finally, since smooth pasting condition has also to be satisfied it must be that

$$\bar{v}_2 - \bar{v}_1 = -\left(1-\phi\right)\varphi + \phi\alpha\left(\bar{v}_2\alpha^{\gamma} - \bar{v}_1\right). \tag{66}$$

By using (64), we obtain that under the guess $\bar{v}_0$ should be equal to

$$\bar{v}_0 = \frac{\mathcal{R}}{r + \delta - \lambda\left(\frac{g}{\kappa} - 1\right)} \tag{67}$$

and $\bar{v}_1$ should be equal to

$$-\left[r + \delta - \lambda\left(\frac{g}{\kappa} - 1\right)\right]\bar{v}_1 = -\left(\varkappa + \rho\right) + \left[\rho + \lambda\,(g-1)\right]\bar{v}_1$$

which implies that

$$\bar{v}_1 = \overline{\varphi} = \frac{\varkappa + \rho}{r + \delta + \rho + \lambda\left(1 - \frac{1}{\kappa}\right) g} \tag{68}$$

Moreover it has to be the case that

$$\left[r + \delta - \lambda\left(\frac{g}{\kappa} - 1\right)\right] = -\left[\rho + \lambda\,(g-1)\right](1+\gamma) + \frac{\sigma^2}{2}\gamma\,(1+\gamma),$$

which requires that γ should satisfy the equation

$$\frac{\sigma^2}{2}\gamma^2 - \left[\rho + \lambda\,(g-1) - \frac{\sigma^2}{2}\right]\gamma - \left[r + \delta + \rho + \lambda g\left(1 - \frac{1}{\kappa}\right)\right] = 0,$$

Remembering that the solution to $ay^2 + by + c = 0$ has the form

$$y_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

There are two solutions, but we are interested in the solution $\gamma > 0$, which guarantees that $\bar{b} > 0$. We conclude that

$$\gamma = \frac{\rho + \lambda\,(g\text{-}1) - \frac{\sigma^2}{2} + \sqrt{\left[\rho + \lambda\,(g\text{-}1) - \frac{\sigma^2}{2}\right]^2 + 2\sigma^2\left[r + \delta + \rho + \lambda g\left(1 - \frac{1}{\kappa}\right)\right]}}{\sigma^2} > 0 \tag{69}$$

The smooth pasting condition in (66) together with (68) implies that $\bar{v}_2$ should be equal to

$$\bar{v}_2 = \frac{-(1-\phi)\,\varphi + (1-\phi\alpha)\bar{v}_1}{1-\phi\alpha^{1+\gamma}} = \frac{(1-\phi\alpha)\overline{\varphi} - (1-\phi)\,\varphi}{1-\phi\alpha^{1+\gamma}} \tag{70}$$

Given (67), (68) and (70), $\bar{b}$ is determined by making the value matching condition in (65) satisfied, which requires that

$$(1-\phi)\,v_0 + \frac{\bar{v}_2}{1+\gamma}\bar{b} = \left[\frac{\phi\bar{v}_2\alpha^{1+\gamma}}{1+\gamma} - (1-\phi)\,\varphi + (1-\phi\alpha)\bar{v}_1\right]\bar{b} \tag{71}$$

After using the fact that (66) implies that

$$(1-\phi\alpha)\,\bar{v}_1 = (1-\phi)\,\varphi + \left(1-\phi\alpha^{1+\gamma}\right)\bar{v}_2$$

the condition in (71) reads as

$$(1-\phi)\,\bar{v}_0 = \left[\left(1-\phi\alpha^{1+\gamma}\right) - \frac{(1-\phi\alpha^{1+\gamma})}{1+\gamma}\right]\bar{v}_2\bar{b}$$

which implies that

$$\begin{aligned}\bar{b} &= \frac{\left(1+\frac{1}{\gamma}\right)(1-\phi)\,\bar{v}_0}{(1-\phi\alpha^{1+\gamma})\,\bar{v}_2} \\ &= \frac{\mathcal{R}}{r+\delta-\lambda\left(\frac{g}{\kappa}-1\right)} \cdot \frac{\left(1+\frac{1}{\gamma}\right)(1-\phi)}{(1-\phi\alpha)\overline{\varphi} - (1-\phi)\,\varphi} \end{aligned} \tag{72}$$

E Derivation of the optimal leverage policy in (26)

For simplicity we drop reference to the province where the firm operates. Notice that

$$\begin{aligned}
V_B(B,Z) &= v_t'(b)\\
V_{BB}(B,Z) &= v_t''(b)\frac{1}{Z}\\
V_Z(B,Z) &= v_t(b) - v_t'(b)b\\
V_{ZZ}(B,Z) &= -v_t'(b)b\frac{1}{Z} + v_t'(b)b\frac{1}{Z} + v_t''(b)b^2\frac{1}{Z} = v_t''(b)b^2\frac{1}{Z}
\end{aligned}$$

We have argued that that the optimal choice of L (in absence of debt renegotiation) requires that

$$X_t(B,Z) = -V_B(B,Z)$$

holds. The HJB of (11) implies that

$$\begin{aligned}
[r_c+\delta+\rho]\,X_t(B,Z) &= (\varkappa+\rho) + [L_t(B,Z) - \rho B]\frac{\partial X_t(B,Z)}{\partial B}\\
&\quad+\frac{\sigma^2 Z^2}{2}\frac{\partial^2 X_t(B,Z)}{\partial Z^2} + \frac{\partial X_t(B,Z)}{\partial t} \qquad (73)
\end{aligned}$$

which incorporates the fact that upon a change of control the price of the firm is given (13) with $X_t^s = X_t(B,Z)$. By taking the partial derivative of (18) with respect to B and after using the fact that $X_t(B,Z) = -V_B(B,Z)$ and that $V_t(Bg,Zg) = gV_t(B,Z)$ we obtain that

$$\begin{aligned}
-(r+\delta)X_t(B,Z) &= -(\varkappa+\rho) + \rho X_t(B,Z)\\
&\quad+\lambda\left[g\left(1-\frac{1}{\kappa}\right)X_t(B,Z) + \frac{\partial X_t(B,Z)}{\partial B}(g-1)B\right]\\
&\quad+\rho B\frac{\partial X_t(B,Z)}{\partial B} - \frac{\sigma^2 Z^2}{2}\frac{\partial^2 X_t(B,Z)}{\partial Z^2} - \frac{\partial X_t(B,Z)}{\partial t}.
\end{aligned}$$

which simplifies to

$$\begin{aligned}
-(r+\delta)X_t(B,Z) &= -(\varkappa+\rho) + \rho X_t(B,Z)\\
&\quad-\lambda\left[g\left(1-\frac{1}{\kappa}\right)\frac{\partial V_t(B,Z)}{\partial B} + \frac{\partial^2 V_t(B,Z)}{\partial B^2}(g-1)B\right]\\
&\quad+\rho B\frac{\partial X_t(B,Z)}{\partial B} - \frac{\sigma^2 Z^2}{2}\frac{\partial^2 X_t(B,Z)}{\partial Z^2} - \frac{\partial X_t(B,Z)}{\partial t} \qquad (74)
\end{aligned}$$

After adding side by side (74) to (73) we obtain

$$(r_c - r)\,X_t(B,Z) = L_t(B,Z)\frac{\partial X_t(B,Z)}{\partial B} - \lambda\left[g\left(1-\frac{1}{\kappa}\right)\frac{\partial V_t(B,Z)}{\partial B} + \frac{\partial^2 V_t(B,Z)}{\partial B^2}(g-1)B\right]$$

which can be rewritten as

$$\begin{aligned}
(r - r_c)\,\frac{\partial V_t(B,Z)}{\partial B} &= -L_t(B,Z)\frac{\partial^2 V_t(B,Z)}{\partial B^2}\\
-\lambda\left[g\left(1-\frac{1}{\kappa}\right)\frac{\partial V_t(B,Z)}{\partial B} + \frac{\partial^2 V_t(B,Z)}{\partial B^2}(g-1)B\right]&
\end{aligned}$$

which yields

$$-L_t\left(B,Z\right)\frac{\partial^2 V_t(B,Z)}{\partial B^2} = \left(r-r_c\right)\,\frac{\partial V_t(B,Z)}{\partial B}$$
$$+\lambda\left[g\left(1-\frac{1}{\kappa}\right)\frac{\partial V_t(B,Z)}{\partial B}+\frac{\partial^2 V_t(B,Z)}{\partial B^2}\left(g-1\right)B\right]$$

which can be finally be written as

$$L_t\left(B,Z\right)=\frac{-\left(r-r_c\right)V_B}{V_{BB}}+\lambda\left[g\left(1-\frac{1}{\kappa}\right)\frac{-V_B}{V_{BB}}-\left(g-1\right)B\right] \tag{75}$$

This implies that

$$l_t\left(b\right)\equiv\frac{L_t}{Z}=\left(r-r_c\right)\frac{-v_t'(b)}{v_t''(b)}+\lambda\left[g\left(1-\frac{1}{\kappa}\right)\frac{-v_t'(b)}{v_t''(b)}-\left(g-1\right)b\right] \tag{76}$$