Mathematics 1:
Linear Algebra and Normed Vector Spaces

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Syllabus

1 Preliminaries of Real Analysis

1.1 Real Numbers
Natural numbers, integers, rationals, irrationals. Properties of real numbers. Absolute value. The ordered field $(\mathbb{R}, +, \cdot, \geq)$.

1.2 Real Sequences and Series
Sequences of real numbers: limits and properties, monotonic sequences, Cauchy sequences, subsequences, upper limit and lower limit. Series of real numbers: definitions and properties, geometric and harmonic series, convergence tests, rearrangement of a series.

1.3 Riemann Integration
Area and definite integral. Primitives and indefinite integral. Fundamental theorem and fundamental formula of calculus. Mean value theorem for definite integrals. Generalized integrals.

1.4 Review of Static Optimization
Local and global minimizers and maximizers, local and global minimum and maximum. Optimization problems with inequality constraints: the theorem of Kuhn-Tucker and procedures to solve. Implications of convexity and concavity in optimization.

Optional further topics: trigonometric functions, complex numbers.

2 Linear Algebra

2.1 Vector Spaces
Starting point: operations in $\mathbb{R}^n$. Definition, properties, and examples of vector space. Vector subspaces. Linear combination and linear independence of vectors. Generated subspaces. Bases and dimension of a vector space.

2.2 Linar Functionals and Operators
Linear functionals: definition and properties. Dual space and the Riesz theorem. Linear operators: definitions, properties, and operations. Operators between Euclidean spaces, matrices and the generalization of the Riesz theorem. Kernel and Image of a linear operator, rank–nullity theorem.

2.3 Isomorphisms

Bijective linear operators and isomorphic vector spaces. Implications of the rank-nullity theorem for injective and surjective linear operators. Invertible operators: definitions and properties. Inverse operators between Euclidean spaces, inverse matrix and applications to linear systems of equations.

Optional further topics: extensions of linear functionals and Hahn-Banach theorem.

3 Metric and Normed Spaces

3.1 Metric and Topology

Distance, definition and examples of metric spaces. Topology: neighborhood, open, closed, bounded sets, interior, isolated, frontier, accumulation points. Sequences of points in metric spaces and closure of sets. Cauchy sequences and complete metric spaces. Compact sets, Heine-Borel theorem, subsequences and sequential compactness.

3.2 Norm

Definition of norm and examples. Distance induced by a norm. Properties on norms.

Optional further topics: Correspondences and the Berge Maximum Theorem.

References

de la Fuente, A. *Mathematical Methods and Models for Economists*. Cambridge University Press. 2000.

Ok, E.A. *Real Analysis with Economic Applications*. Princeton University Press. 2007

Sundaram R.K. *A First Course in Optimization Theory*. Cambridge University Press. 1996.

Simon, C.P. and Blume, L. *Mathematics for Economists*. W.W. Norton & Company. 1994.

Sydsaeter, K., Hammond, P., Seierstad, A. and Strom, A. *Further Mathematics for Economic Analysis*. Pearson College. 2008.

Takayama, A. *Mathematical Economics*. Cambridge University Press. 2010.

Lectures slides.