Probability Theory I & II

Naples Ph.D. in Economics

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1 Overview

Description. This course provides a graduate-level introduction to Probability Theory and its foundations in Measure Theory. Throughout the course, we will explore several connections to Mathematical Analysis. Although we will not compromise on formalism, we will cover the course's content in such a way that the concepts and techniques you will learn are broadly applicable to a variety of ideas and problems in Economic Theory, Econometrics, and neighboring fields (such as Statistics, Computer Science, and Operations Research). By doing so, I hope to make the course valuable to all students, regardless of where they believe their research interests are or will be.

Prerequisites. Basic knowledge of Mathematical Analysis, Linear Algebra, and Matrix Analysis. Some prior knowledge of Probability and Statistics helps but is not strictly necessary. Mathematics I, which runs (almost) in parallel to our course, is engineered to provide you with all the prerequisite knowledge you need to progress with our course in real time. However, I will try to make the course's content as self-contained as possible.

2 Course Material

I will provide you with detailed and self-contained Lecture Notes. You will be responsible only for the material in the Lecture Notes. However, studying good textbooks (see Section 6 for suggestions) improves your understanding of the subject.

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3 Evaluation

To learn the material of this course, you must do more than just read Lecture Notes and listen to lectures. Some tips are the following.

- *Active reading:* work through each line of the Lecture Notes, be sure you know how to get from one line to the next, re-do and complete the proofs made in class.
- *Active listening:* follow each step as we work through arguments in class.
- *Working problems:* solving the assigned exercises (and, if you like, more) is the most valuable part of the course.
- *Working together:* working in groups is strongly encouraged, but always try to work through each exercise by yourself before talking to others.

Grading. Participation (10%), Problem Sets (25%), Final Exam (65%).

- *Participation.* I expect you to ask questions in class and give me feedback on the Lecture Notes (e.g., spotting typos and suggesting how to improve the exposition).
- *Problem Sets.* I will assign Problem Sets weekly. Each Problem Set consists of solving the Exercises in the part of the Lecture Notes we covered during the week. I will share all relevant instructions via email.

4 Office Hours

You can stop by my office whenever I am there, or contact me via email to schedule a Zoom meeting.

5 Detailed Syllabus

1. Sample Spaces, σ-Algebras, and Probability Measures
2. Random Variables and Distributions
3. Independence
4. Integration, Mathematical Expectation, and Related Theorems
5. Generating Functions and Their Applications
6. Probabilistic Inequalities
7. Convergence of Random Variables
8. Laws of Large Numbers and Central Limit Theorems
9. Conditional Expectation and Prediction
10. Basics of Information Theory
11. Introduction to Stochastic Processes

6 Main References

- Billingsley (2012). *Probability and Measure*. Anniversary Edition, Wiley.
- Grimmett and Stirzaker (2020). *Probability and Random Processes*. 4th Edition, Oxford University Press.
- Gut (2005). *Probability: A Graduate Course*. Springer.
- Jacod and Protter (2000). *Probability Essentials*. Springer.
- Ok (2023). *Measure and Probability Theory with Economic Applications*. Available on the author's website.
- Rosenthal (2006). *A First Look at Rigorous Probability Theory*. 2nd Edition, World Scientific.