Reference-dependent choice bracketing

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November 9, 2020

Job market paper

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Abstract

I derive a theoretical model of choice bracketing from two behavioral axioms in an expected utility framework. The first behavioral axiom establishes a direct link between narrow bracketing and correlation neglect. The second behavioral axiom identifies the reference point as the place where broad and narrow preferences are connected. In my model the narrow bracketer is characterized by an inability to process changes from the reference point in different dimensions simultaneously. As a result, her tradeoffs between dimensions are distorted. While she disregards interactions between actual outcomes, she appreciates these interactions mistakenly with respect to the reference point. In addition to the theoretical contribution, I present an experiment which demonstrates the empirical testability of my model and provides preliminary evidence in support of its validity.

JEL code: D91
Keywords: narrow bracketing, correlation neglect, reference dependence, axiomatic foundation, experiment

*I thank Philipp Albert, Kai Barron, Yves Breitmoser, Dirk Engelmann, Julian Harke, Paul Heidhues, Frank Heinemann, Jan Henner, Lea Heursen, Steffen Huck, Alexander Koch, Johannes Leutgeb, Marco A. Schwarz, Joel Sobel, Alexander Teytelboim, Roel van Veldhuizen, Georg Weizsäcker, seminar participants in Berlin and Munich, as well as conference participants in Utrecht (IMEBESS 2019) for many helpful comments. Financial support of the DFG (CRC TRR 190) is greatly appreciated. Address: Spandauer Str. 1, 10099 Berlin, Germany, email: pauline.vorjohann@hu-berlin.de, Telephone: +49 30 2093 99547.

1 Introduction

The amount of decisions that we face and the interdependencies between all of these decisions force us to apply a simplified view of the world. We isolate decisions from one another to be able to make them at all. Following Read et al. (1999b) this mental procedure is referred to as choice bracketing. A decision maker who assesses all of her decisions jointly to find the optimal combination is referred to as a broad bracketer. A narrow bracketer takes some or all of her decisions in isolation, disregarding their interdependencies. As a result, the combination of decisions that a narrow bracketer makes is rarely optimal.

I present a theoretical model of choice bracketing. The model is derived from a choice-theoretic foundation in the context of expected utility. My model is applicable to a large variety of economic settings. In particular, it is the first theoretical model of choice bracketing that allows for multidimensional outcomes. Consequently, my model opens up the possibility to study the effects of narrow bracketing in many important economic settings ranging from basic consumption basket choice to complex multiattribute negotiations. Furthermore, I resolve the general incompatibility of narrow bracketing and budget balance. Finally, my model enables me to derive meaningful predictions for the behavior of a narrow bracketer who is not loss-averse at the same time, isolating the two behavioral biases from one antoher. In addition to the theoretical contribution, I present an experiment which demonstrates the testability of my model and provides preliminary evidence in support of its validity.

Empirical and experimental evidence suggests that narrow bracketing affects behavior in many important economic settings including, for example, labor supply decisions (Camerer et al., 1997), investment decisions (Kumar and Lim, 2008; Thaler et al., 1997; Gneezy and Potters, 1997), trade between agents (Kahneman et al., 1990), retirement savings decisions (Choi et al., 2009; Brown et al., 2008), consumption decisions (Abeler and Marklein, 2017; Read and Loewenstein, 1995), decisions under risk (Rabin and Weizsäcker, 2009), and intertemporal decisions (Koch and Nafziger, 2020; Andreoni et al., 2018; Read et al., 1999a). The prevalence of narrow bracketing is demonstrated by Ellis and Freeman (2020). Across three different contexts they find that only $0-15\%$ of subjects in their experiment are consistent with broad bracketing while $40-44\%$ of subjects are consistent with narrow bracketing. Furthermore, Mu et al. (2020) show that under the assumption of broad bracketing the principle of stochastic dominance is incompatible with the common observation that decision makers are risk-averse over small gambles, providing a theoretical argument for the importance of accounting for narrow bracketing when modeling decision making under risk.

Despite the ample evidence of both prevalence and relevance of narrow bracketing, we still lack a generally applicable theoretical model of this important behavioral bias. Providing such a model is the main contribution of my paper.

A decision maker (DM) faces a series of intermediate decisions. Together, these intermediate decisions comprise the prospect she receives. A prospect is a probability distribution on a multidimensional outcome set. Each prospect is decomposed into several subprospects representing the intermediate decisions. There is one subprospect for each dimension of the outcome set. The subprospect corresponding to a given dimension of the outcome set is the marginal distribution on that dimension induced by the prospect it comprises.

DM is characterized by two preference relations on prospects. Her *broad preference relation* captures her true preferences. If DM brackets broadly, she makes choices in line with her broad preference relation. If DM brackets narrowly, her choices are governed by her *narrow preference relation* instead. DM's narrow preference relation is characterized by a *system of brackets*. The system of brackets partitions the subprospects comprising an overall prospect into distinct groups (brackets). I take the system of brackets as given.[1] It determines the degree to which DM brackets narrowly. While a fully narrow DM puts each subprospect into a distinct bracket, a fully broad DM has only one bracket including all subprospects that comprise the overall prospect.

I derive a representation for DM's narrow preference relation from her broad preference relation and two behavioral axioms. I do so in the framework of expected utility. My first behavioral axiom specifies the mistake that a narrow bracketer makes. It identifies *correlation neglect*[2] as the central flaw of narrow decision making. A narrow DM considers the subprospects inside a given bracket in isolation, disregarding all subprospects outside of that bracket. Of course, if these other subprospects are entirely independent of the considered subprospects, there is no harm done in disregarding them. If, however, these other subprospects are correlated with the considered subprospects or there are important interdependencies between the subprospect outcomes, disregarding them becomes a problem.

My second behavioral axiom ties the narrow preference relation to its broad couterpart. The broad and narrow preference relations belong to one and the same DM. While the one captures DM's true preferences, the other captures the choices she makes. Therefore, the narrow preference relation may depart from the broad preference relation only if that departure can be rationalized by DM's bracketing behavior. In principle a narrow bracketer disregards all interdependencies between subprospects across brackets. However, I assert that the narrow bracketer is not entirely ignorant with respect to these across-bracket interdependencies. I assume that there exists a specific outcome, the *reference point*[3], at which she retains her ability to process all brackets simultaneously. Therefore, the narrow preference relation agrees with the broad preference relation for any two prospects that differ from each other and the reference point in at most one bracket. Intuitively, the reference point captures an outcome that DM is used to and therefore comfortably able to keep the overview of.

The derived expected utility representation of the narrow preference relation is additively separable across brackets. The narrow bracketer's expected utility from a given prospect can be decomposed into a sum of expected utilities from its bracketwise subprospects. Additive separability is implied by my correlation neglect axiom. To establish it I apply a theorem derived by Fishburn (1967) in the framework of multiattribute utility theory to my setting. The axiom that ties the narrow preference relation to its broad counterpart via the reference point imposes further structure on the narrow bracketer's bracketwise expected utilities. For a given bracket the expected utility function of the narrow bracketer is equivalent to the broad bracketer's expected utility function with all outside-bracket outcomes fixed at the reference point.

[1]For models of endogeneous bracket formation in the context of intertemporal decision making see, e.g., Galperti (2019); Hsiaw (2018); Koch and Nafziger (2016). Relatedly,Kőszegi and Matějka (2020) present a model of how people form mental budgets.

[2]For related papers on correlation neglect see, e.g., Enke and Zimmermann (2018); Ellis and Piccione (2017); Eyster and Weizsäcker (2016).

[3]The concept of a reference point was introduced by Kahneman and Tversky (1979) in the context of prospect theory.

My representation theorem reveals that when evaluating a prospect, the narrow bracketer can be modeled as using the same expected utility function as the broad bracketer. However, she applies that expected utility function separately to each bracket in her system of brackets. For each bracket she evaluates the broad expected utility function at the subprospects inside that bracket while keeping all other subprospects fixed at the reference point. Finally, she takes the sum of all of these bracketwise expected utilities. As a result, the narrow bracketer disregards any interactions between subprospects across brackets. However, she appreciates these interactions mistakenly with respect to her reference point.

My model of choice bracketing is simple in the sense that the derived representation of the narrow preference relation can be treated in exactly the same way as any broad expected utility representation. We can thus use the standard economics toolbox and the large body of existing results from microeconomic theory to study the choices of a narrow bracketer.

In particular, the model can be used in standard constrained (expected) utility maximization problems. One of the main obstacles towards formalizing the intuition of choice bracketing is that narrow bracketing is not readily compatible with the principle of budget balance. While narrow bracketing is associated with a decision maker's inability to think multidimensionally, budget balance requires her to make tradeoffs between dimensions. My model resolves this incompatiblity of narrow bracketing and budget balance by introducing the reference point. At the reference point the narrow bracketer retains her ability to think multidimensionally. However, since she is unable to process changes from the reference point in different dimensions simultaneously, her tradeoffs between dimensions are distorted.

Existing experiments on choice bracketing circumvent dealing with the general incompatibility of narrow bracketing and budget balance by design (see, e.g., Ellis and Freeman, 2020; Rabin and Weizsäcker, 2009; Tversky and Kahneman, 1981). They restrict attention to settings where the intermediate decisions that comprise an overall decision are not connected via a budget constraint. Then, a subject's choice in one intermediate decision has no influence on the choices available to her in any other intermediate decision.

Barberis and Huang (2009) and Barberis et al. (2001) present theoretical models of choice bracketing that remedy the incompatibility of narrow bracketing and budget balance by assuming that the narrow bracketer evaluates her utility function separately for each decision she takes and then maximizes the sum over all these individually evaluated utilities. Their model has been used to study choice bracketing in economic applications including portfolio choice (Barberis and Huang, 2009; Barberis et al., 2006; Benartzi and Thaler, 1995), asset pricing (Barberis and Huang, 2001; Barberis et al., 2001), and self-control problems (Koch and Nafziger, 2016; Hsiaw, 2018). My model of choice bracketing contributes to this literature in three respects. First, it provides a choice theoretic foundation for the additive formulation of Barberis and Huang (2009). Second, it extends the set of possible applications considerably by allowing for multidimensional outcomes. Third, by explicitly modeling the system of brackets, it allows for more subtle forms of partial narrow bracketing.

Barberis and Huang (2009) and Barberis et al. (2006) capture partial narrow bracketing through a global-plus-local utility function. This means that they model the partial narrow bracketer as evaluating the weighted sum of broad and fully narrow utility such that the weight attached to the fully narrow utility measures the extent of narrow bracketing. This formulation obviously has the advantage of being more simple than my approach. However, this simplicity comes at a cost. It blurs the very basic intution of choice *bracketing* and does not allow for investigations of the effects that a change in the system of brackets has on the behavior of a narrow bracketer. Furthermore, experimental results of Ellis and Freeman (2020) suggest that the costs of simplicity as imposed by the global-plus-local formulation may outweigh its benefits.

My model of choice bracketing reveals a tight relation between narrow bracketing and budgeting, which besides narrow bracketing is another important aspect of mental accounting as outlined by Thaler (1999). It is intuitively appealing to think of a consumer who chooses a complex consumption bundle as following a two-stage procedure (Gilboa et al., 2010). In the first stage, the budgeting stage, she optimally distributes her budget across general categories of goods like clothing, food, and entertainment. Then, in the second stage she decides separately for each good category how to allocate her category budget from the first stage across the individual goods belonging to that category. Such a budgeting procedure is generally admissibile if and only if the utility function is additively separable across good categories (Gorman, 1959; Strotz, 1957, 1959). Thus, akin to Blow and Crawford (2018)'s definition of boundedly rational mental accounting, additive separability of the narrow preference representation implies that a narrow bracketer can be interpreted as using the described budgeting procedure although her broad preferences do not allow it.

To demonstrate the effects that narrow bracketing has on behavior in basic economic settings, I apply my model to the economics 101 consumer's constrained utility maximization problem with two goods. Additive separability of the narrow preference representation implies that any interactions between the two goods in her bundle are disregarded by the narrow bracketer. This disregard is nicely illustrated by the shape of the narrow indifference curves in comparison to their broad counterparts. If the goods have negative interactions akin to substitutabilities, the narrow indifference curves are more convex than their broad counterparts. If the goods have positive interactions akin to complementarities, the narrow indifference curves are less convex than their broad counterparts. Intuitively, the more convex an indifference curve, the more complementary are the two goods. Thus, a narrow bracketer regards two substitutable goods as more complementary than they actually are and vice versa for two complementary goods.

However, while disregarding interactions for the consumption bundle she receives, the narrow bracketer is not fully ignorant of their existence. She mistakenly appreciates the interactions separately for each good dimension of her bundle with respect to her reference point. The narrow bracketer does not consider changes from the reference point in the two good dimensions simultaneously. Thus, when thinking about an alteration of her bundle away from the reference point in one good dimension, she keeps the respective other good dimension fixed at its reference point level. As a result, the tradeoffs she makes between the two good dimensions are distorted.

For illustration, suppose the two goods have positive interactions and the reference point is unbalanced towards the first good dimension. The higher reference point in the first good dimension implies that increases in the second good dimension are percieved by the narrow bracketer as more attractive than they actually are. At the same time, the lower reference point in the second good dimension makes increases in the first good dimension seem less attractive than they actually are. The narrow bracketer's mistaken attribution of interactions to the respective reference point levels instead of the amounts in her actual bundle move her optimum away from the reference point. In contrast, if the two goods have negative interactions, the narrow bracketer's chosen bundle is closer to the reference point than her optimal bundle.

I also study the implications of choice bracketing in an Edgeworth-box exchange economy assuming status-quo reference points. I find that, starting from any initial endowment structure, in the case of positive interactions the volume of trade is higher if the trading parties bracket narrowly. In contrast, in the case of negative interactions narrow bracketing results in a lower volume of trade. This result has important implications for how the procedures of negotiations affect their outcomes. Especially, it calls into question the general practice of splitting up multidimensional negotiations, negotiating every aspect of a deal separately, since this might induce the involved parties to bracket narrowly.

A recent related literature shows how a consumer's limited attention to price or preference shocks provokes behavior akin to the narrow consumer's behavior in my model. For different definitions of limited attention, papers by Kőszegi and Matějka (2020), Lian (2020), and Gabaix (2014) show that in reaction to such a shock in one good dimension, the inattentive consumer behaves as if she (partially) disregards interactions of that good with the other goods in her bundle. I model narrow bracketing directly. Therefore, in contrast to models based on limited attention my model has bite also in settings with perfect information on prices and preferences. Indeed, experimental evidence suggests that narrow bracketing readily occurs even in such deterministic settings (see, e.g., Ellis and Freeman, 2020; Rabin and Weizsäcker, 2009).

Finally, I present the results of an online laboratory experiment. The main goal of the experiment is to demonstrate the empirical testability of my model. On a secondary note my experimental results provide preliminary evidence for the validity of my model. I show how to construct an experimental design that can test both the validity of my behavioral axioms and my model's predictions on the role of the reference point in narrow decision making. I compare behavior within subject in equivalent two-dimensional decision problems across two treatments. In the *braod treatment* subjects can access information on both dimensions of the decision problem jointly. In the *narrow treatment* subjects can access information on the two dimensions of the decision problem only separately. Furthermore, I impose a waiting time in-between accessing the information on each of the two dimensions of the decision problem that makes switching between the information on the dimensions costly.

A decision problem in the experiment is a multiple choice list between a portfolio and an increasing certain payment. I use the multiple choice list to elicit subjects' willingness to pay (WTP) for the portfolio. Each portfolio consists of two assets, a blue asset and an orange asset. The assets yield blue and orange point earnings respectively depending on the toss of a coin. Payments are determined by the combination of blue and orange point earnings. The payment rule induces interactions between blue and orange points to make the problem interesting in the context of my model.

To gain control over the reference point that subjects use in my experiment I introduce a base-portfolio. The base-portfolio is deterministic and kept constant over the course of the experiment. Every decision in the experiment is implemented with probability 0.5. If a decision is not implemented, the subject receives the base-portfolio instead. This approach of influencing the reference point that subjects use is inspired by Abeler et al. (2011).

The portfolios for which I elicit WTP are chosen such that my behavioral axioms and model predictions can be tested by comparing the WTP differences for pairs of portfolios across the two treatments. Despite observing a relatively small treatment effect, I find support for my model of choice bracketing. The experimental evidence is partially in line with my correlation neglect axiom. Furthermore, my second behavioral axiom on the connection of broad and narrow preferences via the reference point is fully supported. However, I do not find support for my model prediction on the role of the reference point. Overall, the results of my experiment serve as preliminary evidence for the validity of my model. More generally, the experiment demonstrates that my model of choice bracketing is empirically testable and provides a guideline for future experimental investigations, possibly amplifying the suggested treatment variation to induce a larger treatment effect.

The main respect in which my experiment departs from the experimental literature studying narrow bracketing is the treatment design. Existing apporaches to experimentally separate broad from narrow bracketing can be roughly categorized into two groups. First, broad and narrow treatments differ in whether subjects make decisions simultaneously or sequentially (see, e.g., Rabin and Weizsäcker, 2009; Read et al., 2001, 1999a). Second, broad and narrow treatments differ in whether subjects' rewards from their decisions are aggregated or separated (see, e.g., Koch and Nafziger, 2020; Stracke et al., 2017; Gneezy and Potters, 1997). I employ a treatment variation that allows for a direct test of my behavioral axioms. Instead of fully isolating the two dimensions of the decision problem in the narrow treatment, I preserve its multidimensional nature across treatments. I only vary the ease at which subjects can jointly access information on the two dimensions of the decision problem. As a side effect, my treatment variation is less convoluted with other factors such as, for example, reduced complexity, time preferences, and presentation effects.

The paper proceeds as follows. In Section 2 I present the model. In Section 3 I derive predictions of my model for constrained utility maximization and an exchange economy. In Section 4 I discuss the experimental test of my model. Section 5 concludes.

2 The model

2.1 Theoretical framework

The outcome set X is a Cartesian product $\prod_{i\in I} X_i$. I is a finite set $\{1,2,...,n\}$ indexing the dimensions of an outcome $x \in X$. Let $\mathcal{P}$ denote the set of all finite discrete probability distributions on the set of all subsets of X. A *prospect* $P \in \mathcal{P}$ is a probability distribution over the multidimensional outcomes assigning to each outcome $x \in X$ its probability $P(x)$. If $P \in \mathcal{P}$, then $0 \leq P(x) \leq 1$ for all $x \in X$ and $\sum_{x\in X} P(x) = 1$.

The domain of preference is the set of all prospects. A decision maker (DM) is characterized by two preference relations on the set of prospects. Her *broad preference relation* $\succcurlyeq_b$ and her *narrow preference relation* $\succcurlyeq_n$. Consider prospects $P,Q \in \mathcal{P}$. $[P \succcurlyeq_b Q]$ indicates that P is weakly preferred to Q according to $\succcurlyeq_b$. As usual, $[P \succ_b Q]$ indicates $[P \succcurlyeq_b Q$ and not $P \succcurlyeq_b Q]$ while $[P \sim_b Q]$ indicates $[P \succcurlyeq_b Q$ and $P \succcurlyeq_b Q]$. The indications apply analoguously to $\succcurlyeq_n$.

I interpret DM's broad preference relation as capturing her true preferences in the sense that if she brackets broadly, her choices are in line with $\succsim_b$. If DM brackets narrowly, her choices may not be in line with her true preferences. I interpret $\succsim_n$ as the preference relation that governs the narrow DM's choices.

Assumption 1 (Richness)**.** Every probability distribution over outcomes that takes only finitely many values is available in the preference domains of $\succsim_b$ and $\succsim_n$.

So far, my theoretical framework closely follows the literature on multiattribute utility theory (see e.g. Keeney and Raiffa, 1993; Fishburn, 1965, 1967). To accomodate the idea of choice bracketing I now carry the multiattribute nature of outcomes over to the prospect that generates them.

Let $\mathcal{P}_i$ be the set of all finite probability distributions on X_i. For every prospect $P \in \mathcal{P}$ there exists an element $P_i \in \mathcal{P}_i$ which is the marginal distribution on X_i induced by P. Refer to P_i as *subprospect* i of prospect P. I can now write each prospect P as a collection of subprospects corresponding to its outcome dimensions, $P = (P_1, P_2, ..., P_n)$.

The decomposition of prospects into subprospects captures that a DM's overall decision for a specific prospect is the result of several intermediate decisions. In each intermediate decision DM chooses a subprospect. Taken together these subprospects then generate the overall prospect. In the multidimensional outcome arising from this prospect each dimension represents the outcome of one subprospect.

As long as DM brackets broadly, i.e. makes choices in line with $\succsim_b$, the above decomposition of prospects is redundant. A broad bracketer chooses the same prospect independent of whether this choice is the result of just one or several intermediate choices. A narrow bracketer, however, does not keep track of the interdependencies between all intermediate decisions. Therefore, a narrow bracketer's overall decision for a specific prospect depends on whether it is decomposed into subprospects or not.

In its most extreme form, narrow bracketing means that DM decides about each subprospect in isolation disregarding its interdependencies with any other subprospect she chooses. I allow for less extreme forms of narrow bracketing in which DM retains her ability to process subsets of her intermediate decisions jointly. Therefore, I define a *system of brackets* characterizing the narrow preference relation. The system of brackets partitions the collection of subprospects that generate the overall prospect into distinct groups (brackets).

The *system of brackets* B characterizing $\succsim_n$ is a set $\{B_1, B_2, ..., B_m\}$ of nonempty subsets of the outcome dimension index set I with $\bigcup_{j=1}^{m} B_j = I$. We refer to B_j as *bracket* j of the system of brackets B. Let $\mathcal{P}^j$ be the set of all finite discrete probability distributions on the set of all subsets of the outcome set in bracket B_j, $X^j := \prod_{i \in B_j} X_i$. For every prospect $P \in \mathcal{P}$ there exists an element $P^j \in \mathcal{P}^j$ which is the marginal distribution on X^j induced by P. We refer to P^j as the *jth bracket prospect* of P. Given a system of brackets B, we can write each prospect P as a collection of bracketwise prospects, $P = (P^1, P^2, ..., P^m)$, and each outcome x as a collection of bracketwise outcomes, $x = (x^1, x^2, ..., x^m)$ where $x^j = (x_i)_{i \in B_j}$ for $j = 1, 2, ..., m$.

When a prospect $P \in \mathcal{P}$ is deterministic, i.e. $P(x) = 1$ for some $x \in X$, I refer to that prospect directly by its outcome x. Similarly, I refer to a deterministic subprospect $P_i \in \mathcal{P}_i$ by its outcome $x_i \in X_i$ and to a deterministic bracketwise prospect $P^j \in \mathcal{P}^j$ by its bracketwise outcome $x^j \in X^j$.

Given two prospects $P,Q \in \mathcal{P}$, denote by $(P^j, Q^{-j}) \in \mathcal{P}$ the prospect generated by combining the jth bracket prospect P^j of P with all but the jth bracket prospects of Q. Given two outcomes $x,y \in X$, denote by $(x^j, y^{-j}) \in X$ the outcome that combines the jth bracket outcome, x^j, in x with all but the jth bracket outcomes in y.

In gerneral, you can think of the multidimensional nature of outcomes in my framework in two ways. First, in line with what is normally thought of in the multiattribute utility literature, the outcomes of different subprospects may as such be qualitatively different from one another, naturally giving rise to a multiattribute formulation. For example, the overall outcome could be a consumption basket which is comprised of many individual goods, the different outcome dimensions, each of which was individually put into the basket by DM on her way through the supermarket.

Second, capturing the possibility of narrow bracketing in cases where outcomes do not have a multiattribute nature as such, I allow for a distinction between outcome dimensions that are qualitatively the same but are the result of distinct intermediate decisions. For example, the overall outcome could be total money earnings from a portfolio comprised of the earnings from a collection of assets, the outcome dimensions, each of which was puchased individually by DM.

2.2 Axiomatic foundation

In the following I derive a utility representation for the narrow preference relation $\succcurlyeq_n$ from the broad preference relation $\succcurlyeq_b$. I do so in the framework of expected utility (EU), implicitly assuming that the axioms underlying the EU representation are fulfilled for each of the two preference relations $\succcurlyeq_b$ and $\succcurlyeq_n$.[4]

Assumption 2 (EU)**.**

(1) There exists a function $u : X \to \mathbb{R}$, the *broad utility function*, such that for all prospects $Q,R \in \mathcal{P}$, $Q \succcurlyeq_b R \Leftrightarrow EU(Q) \geq EU(R)$ with $EU(P) := \sum_{x \in X} P(x)u(x)$. u is unique up to positive affine transformation.

(2) There exists a function $\tilde{u} : X \to \mathbb{R}$, the *narrow utility function*, such that for all prospects $Q,R \in \mathcal{P}$, $Q \succcurlyeq_n R \Leftrightarrow \widetilde{EU}(Q) \geq \widetilde{EU}(R)$ with $\widetilde{EU}(P) := \sum_{x \in X} P(x)\tilde{u}(x)$. $\tilde{u}$ is unique up to positive affine transformation.

My approach for finding a utility representation of the narrow preference relation proceeds as follows. I ask myself two basic questions about the behavior of a narrow bracketer. The answers to these questions are captured in my two behavioral axioms. Together with Assumption 2 (EU) these two behavioral axioms determine the shape of the narrow bracketer's preference representation.

What is the narrow bracketer's mistake? First, I restrict my attention to the narrow preference relation. The following behavioral axiom clarifies what exactly it is that the narrow bracketer misses when choosing between two prospects.

[4] For axiomatizations of EU see, for example, Fishburn (1970) and Wakker (2010).

Axiom 1 (correlation neglect)**.** For any two prospects $P, Q \in \mathcal{P}$, if all bracketwise prospects induced by P and Q on the system of brackets B are the same, i.e. $P^j = Q^j$ for all $j \in \{1, 2, ..., m\}$, then $P \sim_n Q$.

Axiom 1 states that the narrow bracketer is ignorant with respect to the correlation between the bracketwise prospects that comprise an overall prospect. When making a choice between two prospects, she only considers the individual bracketwise subprospects without keeping track of the overall prospects they comprise. Therefore, any two prospects that are comprised of the same subprospects, i.e. that induce the same marginal distributions on all bracketwise outcome sets, look exactly the same to her. This holds irrespective of whether the overall prospects, i.e. the joint distributions on the overall outcome set, are the same as well.

Of course, Axiom 1 only has bite in the sense that it harms the narrow bracketer, if there are meaningful interactions between the subprospects or their outcomes across brackets. Only then does the correlation structure of a prospect matter for the broad preference relation and only then does the correlation neglect axiom imply that the narrow preference relation deviates from its broad counterpart.

Axiom 1 is closely related to the concept of independence used in the multiattribute utility theory literature. In particular, Fishburn (1967) introduced an assumption equivalent to Axiom 1. I make heavy use of the results from that paper in the proof of my representation theorem. His assumption is a weaker version of mutual independence between the attributes of an outcome as defined in Fishburn (1965) allowing mutual independence to hold only between subsets of the attributes of an outcome.

Where are broad and narrow the same? Axiom 1 pins down the narrow bracketer's mistake. I now identify the instances in which the narrow bracketer's choice should not deviate from her true preferences. The following axiom considers the connection between the narrow preference relation and its broad counterpart.

Axiom 2 (Referene Point)**.** There exists an outcome $r \in X$, the *reference point*, such that for any two prospects $P, Q \in \mathcal{P}$, if the bracketwise prospects induced by P and Q differ from each other and r in at most one bracket, i.e. $P^j = Q^j = r^j$ for all but at most one $B_j \in \{B_1, B_2, ..., B_m\}$, then $P \succsim_b Q \Leftrightarrow P \succsim_n Q$.

Axiom 2 states that there exists an outcome, the reference point, which ties together broad and narrow preference relation. At the reference point the narrow bracketer is perfectly able to consider all brackets jointly. She can properly process changes from the reference point as long as they only occur inside one bracket at a time. In a way, Axiom 2 tames the narrow preference relation. It allows for departures from the broad preference relation only if prospects differ from each other and the reference point in more than one bracket. The narrow bracketer is never fully ignorant of the existence of interactions between the subprospects across brackets since at and around the reference point she makes choices in line with her true preferences.

2.3 Representation theorem

I am now ready to state my representation theorem for the narrow preference relation.

Theorem 1 (Narrow Preference Representation)**.** *Under Assumptions 1 (Richness) and 2 (EU), Axioms 1 (Correlation neglect) and 2 (Reference point) hold if and only if for all prospects $P \in \mathcal{P}$ and corresponding bracketwise prospects $P^j \in \mathcal{P}^j$*

$$\widetilde{EU}(P) = \sum_{j=1}^{m} \widetilde{EU}_j(P^j) \quad \textit{with} \quad \widetilde{EU}_j(P^j) := \sum_{x^j \in X^j} P^j(x^j)\tilde{u}_j(x^j)$$

where $\tilde{u}_j : X^j \to \mathbb{R}$ for brackets $B_j \in \{B_1, B_2, ..., B_m\}$ are bracketwise utility functions with

$$\tilde{u}_j(x^j) := u(x^j, r^{-j}) \quad \forall x^j \in X^j \tag{1}$$

where $u(\cdot, r^{-j})$ denotes the broad utility function evaluated at the reference point for all brackets except bracket j, r^{-j}, which is treated as a fixed parameter of $\tilde{u}_j$.

Proof.

Step 1: The narrow utility function is additively separable across brackets. This result follows from Axiom 1 (Correlation neglect) using the results of Fishburn (1967). I restate his Theorem 1 translated to my framework:

Theorem (Fishburn, 1967)**.** *Under Assumptions 1 (Richness) and 2 (EU), Axiom 1 (Correlation neglect) holds if and only if there exist bracketwise utility functions $\tilde{u}_j : X^j \to \mathbb{R}$ for all brackets $B_j \in \{B_1, B_2, ..., B_m\}$ such that*

$$\widetilde{EU}(P) = \sum_{j=1}^{m} \widetilde{EU}_j(P^j) \quad \textit{with} \quad \widetilde{EU}_j(P^j) := \sum_{x^j \in X^j} P^j(x^j)\tilde{u}_j(x^j)$$

for all prospects $P \in \mathcal{P}$ and corresponding bracketwise prospects P^j. $\widetilde{EU}$ is unique up to positive affine transformation.

Step 2: The jth bracket utility function corresponds to the broad utility function evaluated at the reference point outside of bracket j. This result follows from Axiom 2 (Reference point). Consider any two prospects $P, Q \in \mathcal{P}$ with correspoinding bracketwise prospects P^j, Q^j such that $P^j = Q^j = r^j$ for all but at most one $B_j \in \{B_1, B_2, ..., B_m\}$. Without loss of generality take $B_j = B_1$ as the bracket for which P^j, Q^j and r^j may differ. By Assumption 2 (EU) for the broad preference relation, $P \succcurlyeq_b Q$ if and only if $EU(P) \geq EU(Q)$. We can rewrite P and Q as (P^1, r^{-1}) and (Q^1, r^{-1}), obtaining $EU(P^1, r^{-1}) \geq EU(Q^1, r^{-1})$. We thus have

$$P \succcurlyeq_b Q \quad \Leftrightarrow \quad \sum_{x^1 \in X^1} P^1(x^1)u(x^1, r^{-1}) \geq \sum_{x^1 \in X^1} Q^1(x^1)u(x^1, r^{-1}). \tag{2}$$

Similarly, by Assumption 2 (EU) for the narrow preference relation, $P \succcurlyeq_n Q$ if and only if $\widetilde{EU}(P) \geq \widetilde{EU}(Q)$. Rewriting P and Q as above, we obtain

$$P \succcurlyeq_n Q \quad \Leftrightarrow \quad \sum_{x^1 \in X^1} P^1(x^1)\tilde{u}(x^1, r^{-1}) \geq \sum_{x^1 \in X^1} Q^1(x^1)\tilde{u}(x^1, r^{-1}).$$

Now, by Step 1 we can rewrite the above expression as

$$P \succcurlyeq_n Q \quad \Leftrightarrow \sum_{x^1 \in X^1} P^1(x^1)\tilde{u}_1(x^1) + \sum_{j=2}^{m} \tilde{u}_j(r^j) \geq \sum_{x^1 \in X^1} Q^1(x^1)\tilde{u}_1(x^1) + \sum_{j=2}^{m} \tilde{u}_j(r^j)$$

and simplify it to

$$P \succcurlyeq_n Q \quad \Leftrightarrow \sum_{x^1 \in X^1} P^1(x^1)\tilde{u}_1(x^1) \geq \sum_{x^1 \in X^1} Q^1(x^1)\tilde{u}_1(x^1). \tag{3}$$

Now, by Axiom 2 (Reference point) $P \succcurlyeq_b Q \Leftrightarrow P \succcurlyeq_n Q$. Combining expressions 2 and 3 we therefore have

$$\sum_{x^1 \in X^1} P^1(x^1)u(x^1, r^{-1}) \geq \sum_{x^1 \in X^1} Q^1(x^1)u(x^1, r^{-1}) \quad \Leftrightarrow \quad \sum_{x^1 \in X^1} P^1(x^1)\tilde{u}_1(x^1) \geq \sum_{x^1 \in X^1} Q^1(x^1)\tilde{u}_1(x^1).$$

The above statement requires $\tilde{u}_1$ to be a positive affine transformation of u evaluated at r^{-1}. Now, by Axiom 2 (Reference point) this requirement holds for all bracketwise utility functions in the sequence $\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_m$. Furthermore, by Step 1 a transformation of a bracketwise utility function $\tilde{u}_j$ cannot be performed individually, i.e. without appropriately transforming all other bracketwise utility functions in accordance with the admissible transformations of $\widetilde{EU}$.[5] $\square$

The first part of Theorem 1 is essentially a restatement of Fishburn (1967)'s Theorem 1. Applied to my setting, his finding implies that under Assumptions 1 (Richness) and 2 (EU) Axiom 1 (Correlation neglect) holds if and only if the narrow utility function $\tilde{u}$ is additively separable across brackets. For each bracket B_j in the system of brackets characterizing the narrow preference relation, there exists a bracketwise utility function $\tilde{u}_j$, mapping the jth bracket outcome to the real numbers. The narrow utility function can be written as the sum of all bracketwise utility functions. This means that we can write the narrow expected utility of a prospect $P \in \mathcal{P}$ as a sum of bracketwise expect utilities from all bracketwise prospects P^j induced by P.

The important new insight of Theorem 1 is that the jth bracket utility function, $\tilde{u}_j$ is equivalent to the broad utility function keeping all outcomes except the jth bracket outcome fixed at the reference point. This means that we can interpret the narrow bracketer as actually using the same utility function she would use if she bracketed broadly. However, she applies that utility function separately to each bracket in her system of brackets. For a given bracket she evaluates her broad utility function at the outcomes inside that bracket while keeping all outside-bracket outcomes fixed at their reference point levels. Finally, her overall utility from a specific outcome is determined by the sum of all of these bracketwise evaluated utilities.

To illustrate the content of Theorem 1, consider the special case of $n = 2$ such that every prospect consists of two subprospects and suppose the system of brackets characterizing $\succcurlyeq_n$ separates these two subprospects into distinct brackets. Consider any prospect $P \in \mathcal{P}$. The expected utility of the broad bracketer is given by

$$EU(P) = \sum_{x \in X} P(x)u(x). \tag{4}$$

[5] For a detailed discussion of the admissible transformations on the sequence of functions $\widetilde{EU}_1, \widetilde{EU}_2, ..., \widetilde{EU}_m$ see Fishburn (1967).

Theorem 1 implies that the expected utility of the narrow bracketer can be expressed as

$$\widetilde{EU}(P)=\sum_{x_1\in X_1}P_1(x_1)u(x_1,r_2)+\sum_{x_2\in X_2}P_2(x_2)u(r_1,x_2)=\sum_{x\in X}P(x)[u(x_1,r_2)+u(r_1,x_2)] \tag{5}$$

with u equivalent across the two expected utility formulas.

The narrow bracketer's expected utility representation is an additively separable version of its broad counterpart. Consider the first formulation of $\widetilde{EU}(P)$ in (5) and compare it to the broad expected utility formula in (4). $\widetilde{EU}(P)$ is additively separable across brackets. It consists of the sum of two separate expected utility formulas, one evaluating the first subprospect P_1 and one evaluating the second subprospect P_2. This additive separability reflects the fact that any correlation between the two subprospects are disregarded by the narrow bracketer. By evaluating their expected utilities separately, she treats them as if they were entirely independent.

Furthermore, the narrow bracketer disregards any interactions between the outcomes of the two subprospects. This is nicely illustrated by the second formulation of $\widetilde{EU}(P)$ in (5). The utility that a narrow bracketer derives from an outcome x of the overall prospect P is, again, additively separable across brackets. Instead of evaluating the broad utility function at the overall outcome x as in (4), she evaluates the broad utility function separately for each bracket, once at the outcome of the first subprospect x_1 and once at the outcome of the second subprospect x_2. Since she never evaluates the broad utility at x_1 and x_2 jointly, she does not keep track of possible complementarities or substitutabilities between the two subprospect outcomes.

However, since the narrow bracketer uses the same utility function in her evaluation as the broad bracketer, she is never fully ignorant of the existence of interactions between the two outcome dimensions. She simply appreciates these interactions mistakenly with respect to the reference point. When the narrow bracketer evaluates the outcome of the first subprospect x_1, she keeps the outcome of the second subprospect fixed at r_2 and vice versa. Thus, while she considers the interdependencies between x_1 and r_2 as well as the interdependencies between r_1 and x_2, she fails to keep track of the interdependencies between x_1 and x_2. As a result, her tradeoffs between the outcome dimensions are distorted.

2.4 Discussion

Budget balance A major obstacle towards modeling narrow bracketing is that there exists a tension between the behavioral bias and the economic principle of budget balance. Intuitively, narrow bracketing is associated with "...making each choice in isolation" (Read et al., 1999b). Adhering to this basic intution, one might be drawn to model the narrow bracketer as sequentially making each decision in a set of concurrent decisions as if it were the only decision she faces overall. Such a modeling approach works nicely when applied to the specific environments studied in large parts of the experimental literature on choice bracketing. These experiments are designed such that the specific option a decision maker chooses in one decision does not influence the set of options that are available to her in any other decision (see, e.g., Ellis and Freeman, 2020; Rabin and Weizsäcker, 2009; Tversky and Kahneman, 1981). However, the approach of modeling narrow bracketing as fully isolated decision making runs into serious problems when applied to economically more relevant settings in which decision makers face resource constraints which tie together the option sets of concurrent decisions.

For illustration consider the constrained utility maximization problem of a consumer who has a fixed budget to spend on food and clothing. Suppose the consumer narrowly brackets these two good categories. As long as her budget is tight enough, full isolation of her decisions in these two categories implies that the consumer spends her whole budget on either one of the two categories leaving nothing for the respective other category. Once she enters a, say, clothing store she fully ignores that she might also want to get dinner later on and therefore spends her whole budget on a new outfit. Only later, when she passes by her favourite restaurant she realizes how hugry she is. Of course, the irrationality displayed by the consumer's behavior in this example is not what we observe in reality and goes far beyond what we actually think of when we talk about narrow bracketing.

The example demonstrates that a reasonable model of narrow bracketing needs to balance the isolated nature of narrow decision making with the integrated thinking required for making meaningful tradeoffs across brackets to satisfy budget balance. By defining the narrow preference relation on the same fully multidimensional prospects as the broad preference relation, I implicitly model the narrow bracketer's decision making as simultaneous. Therefore, my framework allows me to in principle cover the whole spectrum of isolation and integration in the narrow bracketer's decision making. Axiom 1 (Correlation neglect) imposes a limit on the ability of the narrow bracketer to integrate subprospects across brackets. This limit is balanced by Axiom 2 (Reference point) which retains the narrow bracketer's ability to integrate subprospects across brackets at and around the reference point. It is the combination of these two axioms that enables me to derive a representation of the narrow preference relation which captures the narrow bracketer's tendency to isolate intermediate decisions from one another and at the same time resolves the general incompatibility of this behavior with the principle of budget balance.

Mental accounting and budgeting Thaler (1999) defines mental accounting as "...the set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities". Choice bracketing is one component of such mental accounting. Another important component of mental accounting is *budgeting*. In the context of consumption choice budgeting describes the assignment of goods into categories with a fixed budget for each category. An important implication of budgeting is the violation of monetary fungiblity across categories.

Already long before behavioral economics was introduced into the scientific debate, economists contemplated how a general but sufficiently tractalbe utility function capturing consumer behavior should look like. Strotz (1957) argues that it is intuitively appealing to think of the consumer as follwing a two-stage maximization procedure akin to budgeting. In the first stage, the consumer allocates her overall budget across general good categories like, for example, food, clothing, and travel. Then, in the second stage she considers each category in isolation and allocates the previously determined category budget across the individual goods inside that category.

Gorman (1959) investigates the characteristics a utility function needs to have in order for the solution to a full constrained utility maximization problem to be equivalent to the solution obtained in the described two-stage-procedure. A necessary and sufficient condition for budgeting to be rational is that the consumer's utility is either additively separable across budget categories or separable with budgetwise utilities entering through an intermediate function that is homogeneous of degree one.

This reveals how in my model narrow bracketing implies a boundedly rational form of budgeting as discussed by Blow and Crawford (2018). The narrow bracketer's expected utility representation is additively separable across brackets. Thus, she behaves as if she employed the described two-stage budgeting procedure with budgeting categories equivalent to the brackets in her system of brackets. However, her broad expected utility representation is not generally additively separable across brackets. Therefore, such budgeting behavior is not generally admissible according to the narrow bracketer's true preferences.

3 Model predictions

3.1 Constrained utility maximization

Consider economics 101 consumption bundle choice. DM faces the problem of allocating a given budget or wealth w across two goods. She chooses a consumption bundle $x \in \mathbb{R}^2_+$. We can write $x = (x_1, x_2)$ where x_1 denotes the amount of good 1 and x_2 denotes the amount of good 2. The per-unit prices of the two goods are p_1 and p_2 respectively. As benchmark consider the maximization problem solved by a broad bracketer:

$$\max_{x_1, x_2} u(x_1, x_2) \quad \text{subject to} \quad p_1 x_1 + p_2 x_2 \leq w. \tag{6}$$

Denote by $x^* = (x_1^*, x_2^*)$ the broad optimum, i.e. the argument that maximizes (6). I am interested in how a narrow DM's choice deviates from her broad optimum. Suppose DM brackets each good in her consumption bundle separately, i.e. $B = \{\{x_1\}, \{x_2\}\}$. She solves

$$\max_{x_1, x_2} u(x_1, r_2) + u(r_1, x_2) \quad \text{subject to} \quad p_1 x_1 + p_2 x_2 \leq w. \tag{7}$$

Denote by $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)$ the narrow optimum, i.e. the argument that maximizes (7). The direction in which the narrow optimum departs from its broad counterpart depends crucially on the type of interdependencies between the two goods captured by the sign of the broad utility function's cross-derivative.

Definition 1. Goods 1 and 2 have negative interactions if $\frac{\partial^2 u}{\partial x_1 \partial x_2} < 0$ for all $x \in \mathbb{R}^2_+$. They have positive interactions if $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$ for all $x \in \mathbb{R}^2_+$. The two goods have no interactions if $\frac{\partial^2 u}{\partial x_1 \partial x_2} = 0$ for all $x \in \mathbb{R}^2_+$.

Roughly, negative interactions are associated with substitutabilities between the two goods while postive interactions are associated with complementarities between the two goods.[6]

In Section 2.3 (Representation theorem) I alluded to the fact that the additive separability of the narrow utility function implies that the narrow bracketer disregards interactions. In the context of consumption bundle choice the following proposition illustrates this fact by comparing the indifference curves of the narrow bracketer to their broad counterparts. Like all further proofs, the proof of the proposition is relegated to the appendix.

[6] See Chambers and Echenique (2009) and Topkis (1998) for a detailed discussion on when a positive cross-derivative of the utility function implies complementarity.

Denote by $MRS(x)$ and $\widetilde{MRS}(x)$ the marginal rates of substitution between good 1 and good 2 at bundle $x = (x_1, x_2)$ for the broad and the narrow bracketer respectively, i.e. $MRS(x) = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2}$ and $\widetilde{MRS}(x) = \frac{\partial \tilde{u}}{\partial x_1} / \frac{\partial \tilde{u}}{\partial x_2}$.

Proposition 1 (Indifference curves)**.** *Assume goods 1 and 2 have either positive, negative, or no interactions. For any amount of good 1, x_1, there exists a corresponding amount of good 2, $f(x_1)$, such that $MRS(x_1, f(x_1)) = \widetilde{MRS}(x_1, f(x_1))$ where $f(r_1) = r_2$, $f(x_1) < r_2$ for $x_1 < r_1$ and $f(x_1) > r_2$ for $x_1 > r_1$. Furthermore,*

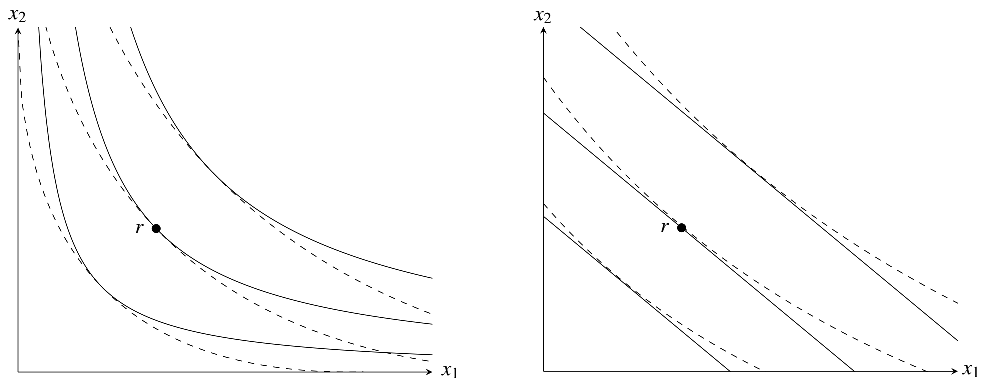
- *Positive interactions $\Rightarrow MRS(x) > \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$ with $x_2 > f(x_1)$ and $MRS(x) < \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$ with $x_2 < f(x_1)$*
- *Negative interactions $\Rightarrow MRS(x) < \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$ with $x_2 > f(x_1)$ and $MRS(x) > \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$ with $x_2 < f(x_1)$*
- *No interactions $\Rightarrow MRS(x) = \widetilde{MRS}(x)$ for all $x \in \mathbb{R}^2_+$.*

Proposition 1 states that at the reference point the slopes of broad and narrow indifference curves are the same. Furthermore, for every amount of good 1, there exists a corresponding amount of good 2 such that the slopes of broad and narrow indifference curves are the same at that bundle. If there are positive interactions between the two goods, the narrow indifference curve is flatter than the broad indifference curve to the left of that bundle and steeper than the broad indifference curve to the right of that bundle. Therefore, narrow indifference curves are less convex than their broad counterparts if the two goods have positive interactions. Conversely, narrow indifference curves are more convex than their broad counterparts if the two goods have negative interactions. Intuitively, the more convex the indifference curves, the more complementary are the two goods. Therefore, in the case of positive interactions, the narrow bracketer can be interpreted as behaving as if the two goods were less complementary than they actually are and vice versa for the case of negative interactions.

Figure 1 illustrates the content of Proposition 1 for two specific broad utility functions given the reference point r. Consider first Figure 1a. The figure shows the indifference curve maps of broad (solid) and narrow (dashed) bracketer for a broad utility function belonging to the Cobb-Douglas family. The utility function is characterized by complementarities which is reflected by the convex shape of the broad indifference curves. The corresponding narrow indifference curves are less convex than their broad counterparts, reflecting the fact that the narrow bracketer disregards the positive interactions between the two goods. However, at the reference point and at any bundle with a distribution of amounts between the two goods proportional to the reference point distribution, broad and narrow indifference curves have the same slope. This illustrates how the narrow bracketer's tradeoffs between the two goods remain undistorted at the reference point and proportional bundles.

In contrast, Figure 1b depicts the indifference curve maps of broad and narrow bracketer for a perfect substitutes broad utility function with negative interactions between the two goods[7]. Perfect substitutability between the two goods implies that the broad indifference curves are straight lines. The narrow bracketer, however, disregards the negative utility interactions between the two

[7]The utility function is widely used in the context of decision making under risk since it has the CRRA property.



(a) Positive interactions: $u(x_1,x_2) = \sqrt{x_1 x_2}$ (b) Negative interactions: $u(x_1,x_2) = \sqrt{x_1 + x_2}$

Figure 1: Comparison of broad (solid) and narrow (dashed) indifference curves with reference point r.

goods. As a result, her indifference curves are convex. She treats the two goods as more complementary than they are. Again, her tradeoffs at the reference point and at bundles proportial to the reference point remain undistorted.

The next proposition investigates how the narrow bracketer's chosen consumption bundle departs from her optimal consumption bundle.

Denote by $d(x,y)$ the Euclidean distance between two consumption bundles $x,y \in \mathbb{R}^2_+$, i.e. $d(x,y) := \sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$.

Proposition 2 (Narrow optimum)**.** *Assume $w = p_1 r_1 + p_2 r_2$ and $r \neq x^*$. The follwing two statements hold at any interior solutions x^* and $\tilde{x}$ to the maximization problems (6) and (7) respectively.*

- *Positive interactions $\Rightarrow d(r,x^*) < d(r,\tilde{x})$*
- *Negative interactions $\Rightarrow d(r,x^*) > d(r,\tilde{x})$*

Proposition 2 states that for budget balanced reference points, unless $r = x^*$, the narrow optimum $\tilde{x}$ is further away (in terms of Euclidean distance) from the reference point than the broad optimum x^* if the two goods have positive interactions. Conversely, the narrow optimum $\tilde{x}$ is closer to the reference point if the the two goods have negative interactions.

Considering Proposition 1 (Indifference curves) in isolation, one might expect that the narrow bracketer's disregard of interactions between the two goods and the resulting shape of her indifference curves imply that the narrow bracketer underdiversifies in the case of positive interactions and overdiversifies in the case of a negative interactions. However, while this intuition is not generally flawed, it does not take into account the role that the reference point plays for the narrow bracketer's decisions. The important role of the reference point is clarified by Proposition 2.

While the narrow bracketer disregards the interdependencies between the goods in her bundle, she is not fully ignorant of their existence. However, she does not consider changes from the respective reference quantities for the two goods simultaneously. Thus, when thinking about an alteration in the amount she might purchase of good 1, from r_1 to $x_1 \neq r_1$, she keeps the amount of good 2 fixed at the reference quantity of good 2, r_2. The reverse holds for alterations in the amount she purchases of good 2. Therefore, the narrow bracketer's appreciation of the interactions between the two goods only occurs separately for the two quantities she purchases and mistakenly with respect to the reference quantity of the respective other good. This implies that the reference point has a profound influence on the narrow bracketer's choice.

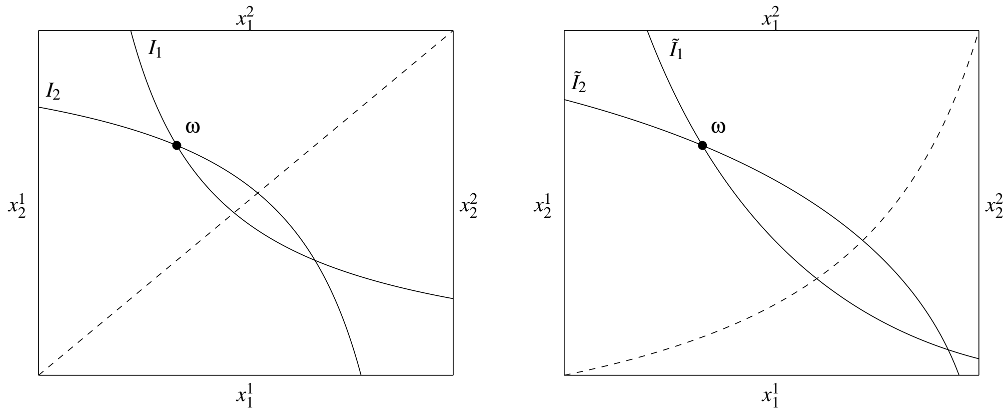
For example, if the goods have positive interactions, an unbalanced reference point with $r_1 > r_2$ pushes the narrow bracketer towards increasing her consumption of good 2 and decreasing her consumption of good 1. This happens because the high reference quantity of good 1, r_1, makes investments in good 2 seem more attractive than investments in good 1, which are in the narrow bracketers mind combined with the relatively low reference quantity of good 2, r_2. Now, if the optimal consumption basket of the broad bracketer x^* prescribes $x_1^* \leq x_2^*$, the fact that the narrow optimum $\tilde{x}$ is pushed further from the reference point r compared to the broad optimum x^* in this constellation always implies that the bundle chosen by the narrow bracketer is less diversified than the bundle chosen by the broad bracketer. If, however, the broad optimum x^* prescribes $x_1^* > x_2^*$, the extra push away from r might induce the narrow bracketer to choose a more diversified consumption bundle than the broad bracketer even though she disregards the positive utility interactions between the chosen quantities x_1 and x_2. Depending on the constellation of reference point and broad optimum, we might therefore observe a narrow bracketer overdiversifying her consumption bundle compared to the broad optimum although the goods have positive interactions. Similarly, we might observe a narrow bracketer underdiversifying her conusmption bundle compared to the broad optimum although the goods have negative interactions.

Interestingly, if the goods have positive interactions the effect of the reference point on the narrow bracketer's chosen bundle goes into the opposite direction of the effect that loss-aversion implies in this setting. The chosen bundle of a loss-averse narrow bracketer is always closer to the reference point than the chosen bundle of a narrow bracketer without loss-aversion. Thus, while narrow bracketing in the case of negative interactions exacerbates the effects of loss-aversion, in the case of positive interactions it actually dampens the effects of loss-aversion. My results reveal that the reference point plays an important role in the decision making of a narrow bracketer independent of whether she is loss-averse or not.

3.2 Exchange economy

Consider an exchange economy with two consumers $i = 1,2$ and two goods. Consumer i's consumption bundle is denoted by $x^i = (x_1^i, x_2^i)$. An allocation $x \in \mathbb{R}_+^4$ is an assignment of a consumption bundle to each consumer, i.e. $x = (x^1, x^2) = ((x_1^1, x_2^1), (x_1^2, x_2^2))$. The total endowments of goods 1 and 2 in the economy are given by $\omega_1 > 0$ and $\omega_2 > 0$ respectively. The initial endowment allocation is denoted $\omega = (\omega^1, \omega^2)$ with $\omega^1 = (\omega_1^1, \omega_2^1)$ denoting consumer 1's endowment such that consumer 2's endowment is given by $\omega^2 = (\omega_1 - \omega_1^1, \omega_2 - \omega_2^1)$. I assume $\omega_1^i, \omega_2^i \geq 0$ for $i = 1,2$. The systems of brackets for the two consumers are given by $B^i = \{\{x_1^i\}, \{x_2^i\}\}$ for $i = 1,2$.

I refer to the *broad economy* as the exchange economy in which both consumers bracket broadly and to the *narrow economy* as the exchange economy in which both consumers bracket narrowly. Furthermore, I refer to the *broad contract curve* as the set of Pareto optimal allocations of the broad economy and to the *broad core* as the set of Pareto optimal allocations that constitute Pareto improvements with respect to the initial endowment allocation in the broad economy.



(a) Broad economy

(b) Narrow economy

Figure 2: Edgeworth-box comparison of broad and narrow exchange economy with broad utilities $u^i(x_1^i, x_2^i) = \sqrt{x_1^i, x_2^i}$ for $i = 1,2$ (positive interactions) and reference points $r^i = \omega^i$ for $i = 1,2$. In each Edgeworth-box the lower left corner corresponds to consumer 1's origin and the upper right corner corresponds to consumer 2's origin. I_i and $\tilde{I}_i$ for $i = 1,2$ respectively denote consumer i's broad and narrow indifference curve reached at the initial endowment allocation ω. The dashed graph displays the contract curve of the respective economy. The part of the contract curve that is enclosed by the lense that opens up between the two indifference curves corresponds to the core of the economy.

Narrow contract curve and *narrow core* are defined analogously. It is a well known fact that any Walrasian equilibrium of an exchange economy is an element of its core (Mas-Colell et al., 1995).

The following proposition shows how choice bracketing systematically affects the volume of trade in the exchange economy.

Proposition 3 (Exchange economy)**.** *Assume that consumer i's reference point is equal to her initial endowment, i.e. $r^i = \omega^i$ for $i = 1,2$. For any inditial endowment allocation ω such that $MRS^1(\omega^1) \neq MRS^2(\omega^2)$, if two allocations x and $\tilde{x}$ are elements of the broad and narrow core respectively and they are not at the corner, then*

- *Positive interactions for both consumers $\Rightarrow d(\omega, x) < d(\omega, \tilde{x})$.*
- *Negative interactions for both consumers $\Rightarrow d(\omega, x) > d(\omega, \tilde{x})$.*

Proposition 3 states that starting from any inital endowment allocation there is more trade in the narrow exchange economy compared to its broad counterpart if the two goods have positive interactions. Conversely, there is less trade in the narrow exchange economy compared to its broad counterpart if the two goods have negative interactions.

Figure 2 illustrates the difference between a broad exchange economy and its narrow counterpart when there are positive interactions between the two goods. Consider first Figure 2a which shows the broad economy in an Edgeworth-box. At the initial endowment allocation ω consumer 1 holds a bundle that is unbalanced towards good 2 while consumer 2 holds a bundle that is unbalanced towards good 1. The indifference curves that the two consumers reach at this initial endowment allocation intersect. Any allocation inside the lense enclosed by the two indifference curves constitutes a Pareto improvement with respect to ω. In particular, redistributing a small amount of good 1 in exhange for a small amount of good 2 from consumer 2 to consumer 1 resulting in more balanced bundles makes both consumers better off. Performing a series of such small trades allows the consumers to arrive at the broad core which is located on the part of the contract curve that intersects with the lense. At the broad core the consumers have reached a Pareto optimal allocation. Since in this example the broad contract curve is on the 45° line, any such allocation has the property that it equalizes the amounts of good 1 and good 2 allocated to a given consumer. Thus, in the given broad economy we should expect the consumers to perform trades that move them from the initial endowment allocation towards an allocation that fully balances their consumption bundles.

Consider now the corresponding narrow exchange economy displayed in Figure 2b. As in the broad economy, the consumer's narrow indifference curves intersect at the initial endowment allocation. Furthermore, moving to an allocation which induces bundles that are more balanced between the two goods for both consumers constitutes a Pareto improvement. However, in the narrow economy the overall set of allocations constituting a Pareto improvement with respect to ω extends much further to the lower right corner of the Edgeworth-box than in the broad economy. This is a direct consequence of the narrow consumers' disregard of the positive interactions between the good dimensions in their bundles. As stated in Proposition 1 (Indifference curves) positive interactions between the two goods imply that the narrow indifference curves are less convex compared to their broad counterparts. The narrow consumers perceive the two good dimensions of their bundles as less complementary than they actually are.

Relatedly, the narrow contract curve is not on the 45° line but bent towards the lower right corner of the Edgeworth-box. As a result, the bundles in the narrow core allocations are not balanced between the two goods. Instead, any allocation in the narrow core has the property that consumer 1's bundle is unbalanced towards good 1 and consumer 2's bundle is unbalanced towards good 2. Interestingly, the imbalance in the consumers' bundles at the narrow core is exactly opposite to the imbalance in the consumers' bundles at the initial endowment allocation. This property of the narrow core mirrors the logic of Proposition 2 (Narrow optimum). The consumers appreciate the positive interactions between the two good dimensions mistakenly with respect to their reference points. Akin to status-quo based reference points, consumers' reference points are assumed to be equal to their respective bundles in the initial endowment allocation. Consider consumer 1. Her bundle in the initial endowment allocation is unbalanced towards good 2. Due to the complementarity between the two good dimensions, the resulting high reference point in the second good dimension makes increases in the amount of good 1 seem relatively more attractive than they actually are. Similarly, the low reference point in the first good dimension makes increases in the amount of good 2 seem relatively less attractive than they actually are. This constellation implies a push of narrow consumer 1's preference towards bundles that are characterized by an imbalance opposite to the imbalance in her initial endowment, i.e. towards good 1. Similarly, consumer 2's preferences is pushed towards bundles that are imbalanced towards good 2. As a result, the volume of trade predicted for the narrow economy is larger than the volume of trade predicted for the broad economy.

	6	12	18	20	30	32	36	40	42	44	50	52	54	74
6					0,24 €	0,47 €	0,90 €	1,29 €	1,47 €	1,64 €	2,09 €	2,22 €	2,34 €	2,99 €
12					0,90 €	1,10 €	1,47 €	1,80 €	1,95 €	2,09 €	2,45 €	2,55 €	2,64 €	
18			0,24 €	0,47 €	1,47 €	1,64 €	1,95 €	2,22 €	2,34 €	2,45 €	2,72 €	2,79 €	2,85 €	
20			0,47 €	0,69 €	1,64 €	1,80 €	2,09 €	2,34 €	2,45 €	2,55 €	2,79 €	2,85 €	2,90 €	
30	0,24 €	0,90 €	1,47 €	1,64 €	2,34 €	2,45 €	2,64 €	2,79 €	2,85 €	2,90 €	2,99 €	3,00 €		
32	0,47 €	1,10 €	1,64 €	1,80 €	2,45 €	2,55 €	2,72 €	2,85 €	2,90 €	2,94 €	3,00 €			
36	0,90 €	1,47 €	1,95 €	2,09 €	2,64 €	2,72 €	2,85 €	2,94 €	2,97 €	2,99 €				
40	1,29 €	1,80 €	2,22 €	2,34 €	2,79 €	2,85 €	2,94 €	2,99 €	3,00 €					
42	1,47 €	1,95 €	2,34 €	2,45 €	2,85 €	2,90 €	2,97 €	3,00 €						
44	1,64 €	2,09 €	2,45 €	2,55 €	2,90 €	2,94 €	2,99 €							
50	2,09 €	2,45 €	2,72 €	2,79 €	2,99 €	3,00 €								
52	2,22 €	2,55 €	2,79 €	2,85 €	3,00 €									
54	2,34 €	2,64 €	2,85 €	2,90 €										
74	2,99 €													

Figure 3: The payment table shown to participants in the experiment. The payment associated with a specific combination of blue and orange points can be found by choosing the row according to the amount of blue points and the column according to the amount of orange points.

4 Experiment

4.1 Design

In the experiment I elicit participants' willingnesses to pay (WTP) for portfolios. Each portfolio consists of two assets, a blue and an orange asset. The blue asset yields blue points and the orange asset yields orange points. Point earnings from a portfolio are determined by a coin toss performed by the computer. Importantly, it is the same coin toss that determines a participant's point earnings from the blue and the orange asset in a portfolio. Relating to my theoretical framework (Section 2.1) a portfolio in the experiment corresponds to a prospect while the two assets correspond to the subprospects comprising the prospect.

Preferences, or more accurately broad preferences, over portfolios are partly (risk preferences still matter) induced via a payment rule that translates any combination of blue and orange point earnings into payments. The payment rule induces negative interactions between blue and orange points, i.e. the more blue points a participant receives, the less valuable is an increase in orange points and vice versa. Throughout the experiment participants have access to a table stating the respective payments associated with all different combinations of blue and orange point earnings. The payment table is displayed in Figure 3.

The experiment has 20 rounds. In each round the participant is provisionally allocated a simple portfolio, the base-portfolio. The base-portfolio is deterministic, i.e. it yields the same point earnings irrespective of the result of the coin toss. It remains constant over the course of the experiment. I use the base-portfolio to induce participants' reference points. The base-portfolio is displayed in the first row of Table 1.

For every round of the experiment a random draw determines whether that round is a trade-round or a base-round. Both round types are equally likely. In base-rounds the participant keeps her base-portfolio. In trade-rounds she is offered another portfolio (trade-portfolio). Her WTP for the trade-portfolio is elicited via a multiple choice list. In each row of the choice list the participant has to make a decision between the offered trade-portfolio and an increasing certain payment. For each submitted choice list one row is randomly chosen and the participant's decision in the respective row is implemented. Since participants do not know whether a given round is a trade-round or a base-round, they fill out a multiple choice list in every round.

In each round the participant is displayed a decision-screen. At the top of the decision screen she sees the base-portfolio. Below, she can click a button to view the trade-portfolio of that round. The participant can switch back and forth between viewing the trade- and the base-portfolio anytime. Below the respective portfolio the multiple choice list for the offered trade-portfolio is displayed. I enforce a single switchpoint.

Overall, I elicit WTP for 10 different trade-portfolios in two treatments. The portfolios used in the experiment are displayed in Table 1. The experiment has a within-subject design. Therefore, each participant fills out a multiple choice list for each of the 10 trade-portfolios twice, once in the *broad treatment* and once in the *narrow treatment* (hence the 20 rounds). As suggested by their names, the treatments are designed to induce subjects to bracket broadly in the broad treatment and narrowly in the narrow treatment. The order in which participants see the different trade-portfolios in the two treatments is randomized.

The treatments differ in how the participant can access information about the contents of the trade-portfolio. In the broad treatment the participant views the blue and orange asset comprising the trade-portfolio jointly. In the narrow treatment the particpant can view the blue and orange asset comprising the trade-portfolio only separately. The view of the other asset is kept fixed at the respective asset in the base-portfolio. The participant can change between viewing the blue asset in the trade-portfolio and viewing the orange asset in the trade-portfolio anytime by clicking on a button. After clicking the button, the participant sees a waiting screen for 5 seconds and is then redirected to the respective view of the trade-portfolio. Importantly, the information available to the participant in the narrow treatment is the same as the information available to her in the broad treatment. The treatments only differ in how easy it is for the participant to jointly consider the two assets that comprise the trade-portfolio.

I conducted 10 online-sessions with roughly 18 subjects each. Overall 171 subjects took part in the experiment. The sessions took place in September 2020 with subjects from the WZB-Technical University laboratory subject pool in Berlin. Subjects were invited to participate in the experiment using ORSEE (Greiner, 2015). In conducting the sessions I closely followed the UCSC LEEPS Lab Protocol for Online Economics Experiments (Zhao et al., 2020). The experiment was programmed in oTree (Chen et al., 2016). I preregistered the experiment on the AEA RCT Registry, including a pre-analysis plan and power calculation (Vorjohann, 2020). The experimental sessions were preceded by two pilot sessions run in July 2020. I do not use the data from these pilot sessions in my analysis.

Portfolio	Blue asset		Orange asset		EV	Variance
	Heads	Tails	Heads	Tails		
Base	30	30	6	6	€0.24	0.00
Trade A1	30	42	32	6	€1.96	0.24
Trade A2	30	42	6	32	€1.57	1.77
Trade A3	52	30	12	30	€2.45	0.01
Trade A4	30	52	12	30	€1.95	1.10
Trade B1	30	74	6	6	€1.62	1.89
Trade B2	44	54	6	6	€1.99	0.12
Trade B3	30	30	6	50	€1.62	1.89
Trade B4	30	30	20	30	€1.99	0.12
Trade C1	52	44	30	32	€2.97	0.00
Trade C2	30	32	52	44	€2.97	0.00

Table 1: The portfolios used in the experiment. For each portfolio the table shows the number of blue and orange points the portfolio yields depending on the outcome of the coin toss for that portfolio. Furthermore, it shows expected value (EV) and variance of each portfolio rounded to two decimal places.

4.2 Hypotheses

Table 2 summarizes the correspondence between theory and experiment. Portfolios are probability distributions over combinations of blue and orange point earnings. A combination of blue and orange point earnings is a two-dimensional outcome. Thus, there is a direct correspondence between portfolios in the experiment and prospects as defined in my theoretical framework (Section 2.1). Furthermore, the blue asset in a portfolio is in effect the marginal distribution over blue points induced by the portfolio. Similarly, the orange asset in a portfolio is the marginal distribution over orange points induced by the portfolio. The assets in a portfolio therefore correspond to its subprospects. The payment rule translates combinations of blue and orange point earnings to money earnings. Following induced value theory (Smith, 1976) the broad utility associated with a combination of blue and orange point earnings can be measured by the payment it generates.

The reference point as defined by Axiom 2 (Reference point) in Section 2.2 plays a central role in my theory of choice bracketing. My model relies on the existence of a reference point but remains agnostic about which specific outcome constitutes the reference point. However, to design a meaningful test for the validity of my model I require additional knowledge about the reference points of subjects in my experiment. Therefore, I introduce the base-portfolio into my experimental design. The base-portfolio serves the purpose of inducing its deterministic outcome as reference point. This purpose is achieved in two ways, each of which builds on a prominent theory of the nature of reference points. First, by provisionally allocating the base-portfolio to subjects at the beginning of each round, the base-portfolio is established as the status-quo. Second, by implementing the base-portfolio instead of a subject's decision with a probability of 0.5 in each round, the base-portfolio enters the subject's expected outcome from a given decision. This design feature is inspired by Abeler et al. (2011) and builds on the theory of expectation-based reference points (Kőszegi and Rabin, 2006).

Since my axiomatization builds on the connection between broad and narrow preference relation, both characterizing one and the same decision maker, I employ a within-subject design.

Theory	Experiment
Prospect	Portfolio
Subprospects	Blue asset and orange asset
Outcome	Combination of blue and orange point earnings
Outcome dimensions	Blue and orange points
Broad utility of an outcome	Payment for a combination of blue and orange point earnings
Reference point	Blue and orange point earnings in the base-portfolio

Table 2: Correspondence between theory and experiment.

The idea is to elicit a subject's WTP for a given trade-portfolio twice, once when she brackets the two assets in the portfolio broadly and once when she brackets them narrowly. I manipulate how subjects bracket the assets in a trade-portfolio by varying the ease at which they can be considered jointly. In the broad treatment, subjects see the two assets on the same screen. This makes it relatively easy to integrate them and keep track of the overall portfolio they comprise. In contrast, to integrate the assets in the narrow treatment, subjects have to recall them since they are accessible only on separate screens. The waiting time imposed when switching between viewing each of the assets in the trade-portfolio further complicates joint consideration.

The main goal of my experiment is to test the validity of the behavioral axioms underlying my theoretical model. Additionally, I test one of my model's predictions concering the role of the reference point. The trade-portfolios for which I elicit subjects' WTP (Table 1) can be classified into three groups. Trade-portfolios A1-A4 are designed to test Axiom 1 (Correlation neglect). Trade-portfolios B1-B4 are designed to test Axiom 2 (Reference point). Trade-portfolios C1 and C2 are designed to test the model prediction.

Consider first trade-portfolios A1 and A2. The two portfolios induce the same marginal distributions over blue and orange points, i.e. a fifty-fifty chance between 30 and 42 blue points and a fifty-fifty chance between 32 and 6 orange points. Thus, if Axiom 1 (Correlation neglect) holds, subjects in the narrow treatment are expected to have the same WTP for the two portfolios. However, overall trade-portfolios A1 and A2 are not the same. They differ in the joint distribution over blue and orange points they induce. Trade-portfolio A1 induces a fifty-fifty chance between the overall outcomes (30 blue points, 32 orange points) and (42 blue points, 6 orange points). This is equivalent to a fifty-fifty chance between receiving €2.45 and €1.47 (see the payment table in Figure 3) and implies an expected value of €1.96. In contrast, trade-portfolio A2 induces a fifty-fifty chance between the overall outcomes (30 blue points, 6 orange points) and (42 blue points, 32 orange points). This is equivalent to a fifty-fifty chance between €0.24 and €2.9 and implies an expected value of €1.57. Thus, in the broad treatment a risk neutral subject should have a higher WTP for trade-portfolio A1 compared to trade-portfolio A2. Furthermore, since A2 has a higher variance than A1, the same should hold for a risk averse subject. An equivalent logic applies to the pair of trade-portfolios A3 and A4.

Denote by $WTP_i(P)$ the WTP for trade-portfolio P expressed by subject i. $|WTP_i(P) - WTP_i(Q)|$ denotes the absolute value of the WTP difference between portfolios P and Q expressed by subject i in a given treatment. Hypothesis 1 summarizes the theoretical predictions based on Axiom 1 (Correlation neglect).

Hypothesis 1 (Correlation neglect)**.**

(a) $|WTP_i(A1) - WTP_i(A2)|$ *is higher in the broad treatment than in the narrow treatment.*

(b) $|WTP_i(A3) - WTP_i(A4)|$ *is higher in the broad treatment than in the narrow treatment.*

Next, consider trade-portfolios B1 and B2 in Table 1. Both portfolios contain the same orange asset which yields 6 orange points independent of the outcome of the coin toss. Furthermore, the orange asset in the two portfolios is equivalent to the orange asset in the base-portfolio. B1 and B2 differ from each other and the base-portfolio only in the blue asset they contain. Thus, if Axiom 2 (Reference point) holds, we should observe the same ordering between the WTP for the two portfolios across treatments. Since trade-portfolio B2 has both a higher expected value and a lower variance than trade-portfolio B1, risk-neutral and risk-averse subjects should express a lower WTP for B1 than for B2.

Similarly, trade-portfolios B3 and B4 in Table 1 differ from each other and the base-portfolio only in the orange asset they contain. Therefore, based on Axiom 2 (Reference point) I expect that if a subject's WTP for B3 is lower than her WTP for B4 in the broad treatment, this also holds for the same subject in the narrow treatment. Again, based on expected value and variance the WTP difference between B3 and B4 should be negative for risk-neutral and risk-averse subjects. The theoretical predictions based on Axiom 2 are summarized in Hypothesis 2.

Hypothesis 2 (Reference point)**.**

(a) *The sign of* $WTP_i(B1) - WTP_i(B2)$ *is the same across treatments.*

(b) *The sign of* $WTP_i(B3) - WTP_i(B4)$ *is the same across treatments.*

Finally, consider trade-portfolios C1 and C2 in Table 1. The only difference between C1 and C2 is that the labeling of the assets they contain is interchanged. The asset that yields 52 points for heads and 44 points for tails is called the blue asset in C1 and the orange asset in C2. Similarly, the asset that yields 30 points for heads and 32 points for tails is called the orange asset in C1 and the blue asset in C2. However, since blue and orange points enter the payment rule symmetrically (see the payment table in Figure 3), the lottery over payments the two portfolios induce is exactly the same. Both portfolios are equivalent to a fifty-fifty chance between €3 and €2.94. Therefore, a subject in the broad treatment should have the same WTP for C1 as for C2.

Based on my model of narrow bracketing I do not expect the same to hold for subjects in the narrow treatment. A narrow bracketer evaluates the broad expected utility function separately for each asset in the portfolio keeping the respective other asset fixed at the reference point level (see Theorem 1). Suppose a subject is risk neutral such that her broad expected utility from a portfolio is equivalent to the expected value of that portfolio. Consider trade-portfolio C1. Regarding the blue asset in the portfolio, the narrow bracketer can be modeled as calculating the expected value of a fictitious portfolio that combines the blue asset in C1 with the orange asset in the base-portfolio. Such a portfolio would induce a fifty-fifty chance between the outcomes (52 blue points, 6 orange points) and (44 blue points, 6 orange points) associated with an expected value of €2.28. In turn, regarding the orange asset in C1, the narrow bracketer can be modeled as calculating the expected value of another fictitious portfolio that combines the blue asset in the base-portfolio with the orange asset in C1 which is €2.05. Similarly, the expected values resulting from such a separate evaluation of the assets in C2 are €1.62 and €1.69. Since according to my representation theorem the narrow bracketer's preferences over portfolios are representable by the sum of these separately evaluated expected values, I expect that in the narrow treatment a risk-neutral subject's WTP for C1 is higher than her WTP for C2.

The prediction derived for trade-portfolios C1 and C2 is a manifestation of the general intution concerning the role that an unbalanced reference point plays for the behavior of a narrow bracketer discussed in Section 3 (Model predictions). The base-portfolio yields considerably more blue points than orange points. It therefore induces a reference point that is unbalanced towards blue points. Since C1 yields more blue than orange points irrespective of the outcome of the coin toss, C1 is characterized by an imbalance towards blue points as well. Conversely, C2 is characterized by an imbalance towards orange points. Now, the negative interactions between blue and orange points induced by the payment rule push the narrow bracketer's preference towards the otherwise equivalent trade-portfolio which is characterized by an imbalance towards blue points, namely C1. Hypothesis 3 summarizes the experimental prediction on the role of the reference point derived from my model.

Hypothesis 3 (Role of the reference point)**.** $WTP_i(C1) - WTP_i(C2)$ *is lower in the broad treatment than in the narrow treatment.*

4.3 Results

A prerequisite for me to be able to use my experimental data to test my model of choice bracketing is that the treatment manipulation worked. I require that subjects bracket the blue and orange asset comprising a trade-portfolio jointly in the broad treatment and separately in in the narrow treatment. Suppose the treatment manipulation did not work. Then, a subject's WTP for a given trade-portfolio should be the same across treatments. For each trade-portfolio used in the experiment, Figure 4 shows the distribution of the within subject WTP difference between treatments. While the distributions are centered around zero for all portfolios, the plots also show considerable variation in the WTP for the same portfolio between treatments. Across the different portfolios 41-77% of subjects in my sample express a WTP in the broad treatment that differs by more than €0.1 from the WTP they express for the same portfolio in the narrow treatment. These results indicate that while the treatment manipulation may not have been successful for all subjects in my experiment, there still is a considerable share of subjects that behaves differently across the two treatments.

Consider first Hypothesis 1 (Correlation neglect). For each of the two trade-portfolio pairs A1&A2 and A3&A4 the hypothesis states that the difference in WTP between the two portfolios should be higher in the broad treatment compared to the narrow treatment. Hypothesis 1a concerning A1&A2 is fulfilled for roughly 50% of subjects in my sample. The share of subjects for whom Hypothesis 1b concerning A3&A4 is fulfilled is 42%. Figure 5 compares the means of the absolute value of the WTP difference between the portfolios in each of the two portfolio pairs across treatments. In line with Hypothesis 1a, the mean absolute WTP difference between trade-portfolios A1 and A2 is higher in the broad treatment than in the narrow treatment. Furthermore, this observation is confirmed to be statistically significant in a one-sided paired two-sample t-test (p=0.04). However, Hypothesis 1b is not supported in my experimental data. Figure 5 already

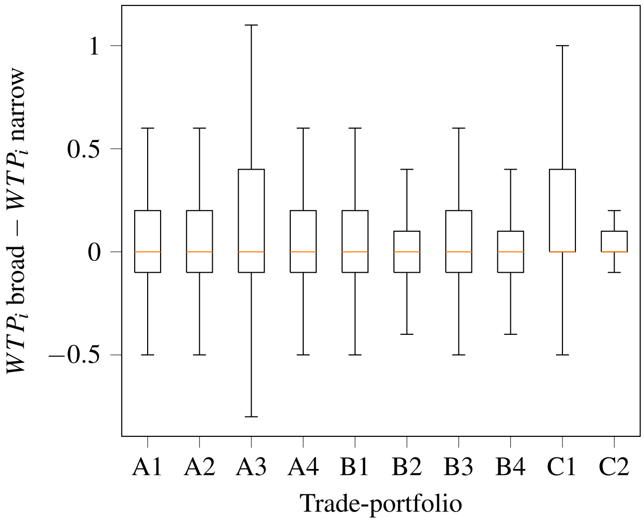


Figure 4: Comparison of WTP for trade-portfolios across treatments. Each boxplot visualizes the distribution of the difference between WTP for a trade-portfolio in the broad treatment and WTP for the same trade-portfolio in the narrow treatment. As usual, the box shows the interquartile range with the horizontal line between lower and upper quartile marking the median. The whiskers extend to the extrema of the distribution excluding outliers.

shows that the mean absolute WTP difference between A3 and A4 is virtually the same across the two treatments, a result which is confirmed by the corresponding t-test.

According to Axiom 1 (Correlation neglect), a subject that brackets the two assets in a portfolio narrowly should be indifferent between the portfolios in the two trade-portfolio pairs A1&A2 and A3&A4. Thus, if all subjects bracketed narrowly in the narrow treatment, we should observe a mean WTP difference of zero for the two pairs in that treatment. Even for A1&A2 this is clearly not the case. Only 20% of subjects in my sample show a WTP difference between A1 and A2 of at most €0.1 in the narrow treatment. With a mere 11% the share of subjects that can be accordingly classified as correlation neglecters in the narrow treatment is much lower for A3&A4. However, for both portfolio pairs I do observe a considerable drop in the share of correlation neglecters in the broad treatment compared to the narrow treatment. In the broad treatment only roughly 8% of subjects show a WTP difference of at most €0.1 between the portfolios in the respective pair. I interpret this drop in the share of correlation neglecters when moving from the narrow to the broad treatment as additional suggestive evidence that narrow bracketing is related to correlation neglect as asserted by Axiom 1 (Correlation neglect).

Result 1 (Correlation neglect)**.** *My experimental results concerning Hypothesis 1 provide preliminary evidence for the validity of Axiom 1.*

Consider now Hypothesis 2 (Reference point). For each of the two portfolio pairs B1&B2 and B3&B4, the hypothesis states that the ordering of WTP for the two portfolios in the pair should be the same across treatments. Hypothesis 2 is fulfilled for the majority of subjects in my sample. For 64 and 69% of subjects respectively I observe the same ordering of WTP within portfolio pairs B1&B2 and B3&B4. For each of the two portfolio pairs Table 3 shows a contingency table providing the respective frequencies of the feasible combinations of WTP orderings in the

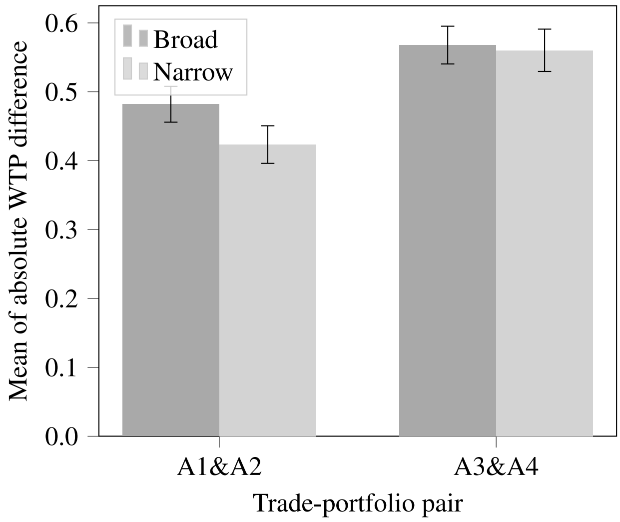


Figure 5: Visualization of the results for Hypothesis 1 (Correlation neglect). The graph shows the mean of the absolute value of the WTP difference between the portfolios in each of the two trade-portfolio pairs A1&A2 and A3&A4 separated by treatment. The error bars indicate the standard error of the respective mean measured by the standard deviation divided by the squareroot of the sample size.

broad and narrow treatment. Consider first Table 3a. As expected, for the majority of subjects $WTP_i(B1) - WTP_i(B2)$ is negative in both treatments. This means that the majority of subjects prefers the portfolio in the pair that is characterized by a higher expected value and a lower variance in both treatments. In a Pearson's chi-squared test the null hypothesis of independence of the signs of $WTP_i(B1) - WTP_i(B2)$ in the broad and narrow treatment is clearly rejected (p=0.007). This result is in line with Hypothesis 2a. Considering Table 3b a similar picture emerges. For 100 out of 171 subjects $WTP_i(B3) - WTP_i(B4)$ is negative in both treatments. Furthermore, a Pearson's chi squared test clearly rejects the null hypothesis of independence (p<0.001).

Overall, the experimental results concerning Hypothesis 2 are in line with Axiom 2 (Reference point). The portfolios in the pairs B1&B2 and B3&B4 differ from each other and the base-portfolio in only one asset. Therefore, the observation of equal WTP orderings between the

	Narrow treatment			
Broad treatment	$-$	0	$+$	Total
$-$	96	9	15	120
0	15	3	6	24
$+$	13	3	11	27
Total	124	15	32	171

(a) Sign of $WTP_i(B1) - WTP_i(B2)$

	Narrow treatment			
Broad treatment	$-$	0	$+$	Total
$-$	100	4	24	128
0	6	6	1	13
$+$	15	3	12	30
Total	121	13	37	171

(b) Sign of $WTP_i(B3) - WTP_i(B4)$

Table 3: Visualization of the results for Hypothesis 2 (Reference point). Each contingency table displays the bivariate frequency distribution of the sign of the WTP difference between the trade-portfolios in the respective pair observed in the two treatments.

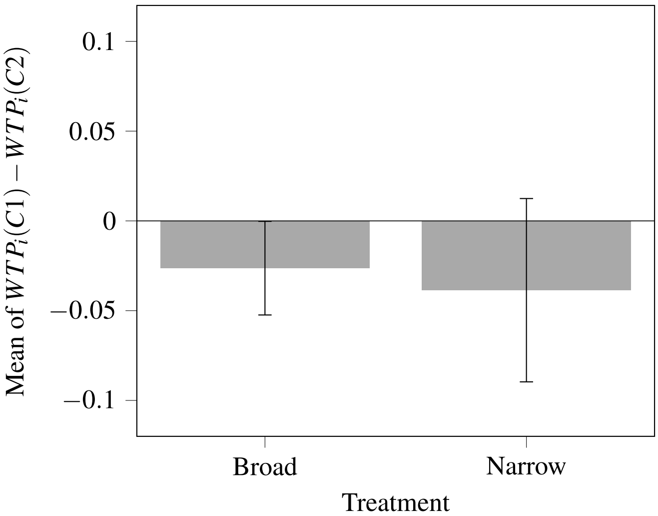


Figure 6: Visualization of the result for Hypothesis 3 (Role of the reference point). The graph shows the mean of $WTP_i(C1) - WTP_i(C2)$ in the broad an the narrow treatment. The error bars indicate the standard error of the respective mean measured by the standard deviation divided by the squareroot of the sample size.

portfolios within each pair across treatments is consistent with the existence of a reference point, in this case equal to the outcome of the base-portfolio, that ties together the broad and narrow preference relations on portfolios. However, in conjunction with the preceding analysis of my experimental data, this interpretation should be taken with a grain of salt. My analysis so far suggests that my treatment manipulation was only partially successful in inducing subjects to bracket narrowly in the narrow treatment. At the same time, Hypothesis 2 should equally hold for a subject who brackets broadly in both treatments. Therefore, in light of my relatively weak treatment effect, the presented test of this hypothesis is not entirely conclusive.

Result 2 (Reference point)**.** *My experimental results concerning Hypothesis 2 support the validity of Axiom 2.*

Finally, consider Hypothesis 3 (Role of the reference point). The hypothesis states that the difference in WTP between trade-portfolios C1 and C2 in the narrow treatment should be lower than the same WTP difference in the broad treatment. Hypothesis 3 is fulfilled for only 29% of subjects in my sample. For the vast majority of subjects (85%) the absolute values of the WTP difference between the two portfolios differ by at most €0.1 across the two treatments. Figure 6 compares the means of the WTP difference between C1 and C2 across treatments. In both treatments the mean of this WTP difference is close to zero. A paired two-sample t-test confirms that there is no significant difference between the mean WTP differences across treatments.

For the majority of subjects in my sample (60%) the WTP for portfolios C1 and C2 in the broad treatment differ by at most €0.1. This is as expected since the two portfolios are essentially equivalent. A subject who brackets the two assets in a portfolio broadly should be indifferent between C1 and C2. In the narrow treatment the share of subjects for whom the WTP for C1 and C2 differ by at most €0.1 drops to 47%. This drop in the share of subjects behaving consistent with broad bracketing in the narrow treatment is reflected by a higher variance in the distribution of the WTP difference between C1 and C2 in that treatment.

Result 3 (Role of the reference point)**.** *My experimental results concerning Hypothesis 3 do not support the validity of my model's prediction on the role of the reference point.*

Overall, I interpret the results of my experiment as providing preliminary evidence for the validity of the behavioral axioms underlying my model of choice bracketing. However, since the treatment effect I observe in the experiment is relatively weak, I am not able to present a conclusive assessment of the validity of my model as a whole. In particular concerning my model's prediction on the role of the reference point, further empirical research will be needed to determine whether the lack of support for the prediction provided by my experiment is an artifact of my relatively weak treatment manipulation or a more general flaw of my modeling approach.

Fortunately, my experimental design can easily be adjusted to make the treatment stronger. For example, one could increase the waiting time that subjects need to endure in the narrow treatment when switching between the information on the two assets in a portfolio. This would make it more costly for subjects to repeatedly view each of the two assets and require a better memory for their joint consideration. Another possibility would be to increase the dimensionality of the decision problem by increasing the number of assets in a portfolio. This would make it harder for subjects to bracket broadly overall and especially so in the narrow treatment.

5 Conclusion

Narrow bracketing affects individual decision making. Individual decision making is the very basis of almost all economic activity. Therefore, the potential implications of this behavioral bias go through the whole economy. Indeed, empirical evidence suggests that narrow bracketing adversely affects behavior in a vast variety of important economic settings. In this paper I present a generally applicable theoretical model of choice bracketing. Previous models of choice bracketing are restricted to one-dimensional outcome spaces. Therefore, these models can accomodate only a small subset of the relevant economic applications. Allowing for multidimensional outcome spaces, my model opens up the possibility to systematically study the effects of narrow bracketing in new economic applications ranging from complex contract negotiations to basic consumption bundle choice. Furthermore, I derive my model from basic behavioral assumptions. In contrast to a model that is designed to generate specific predictions in a given setting my model is therefore more likely to make accurate predictions when applied across a variety of different settings. Finally, my model provides a theoretical framework that can inspire and organize future empirical research on choice bracketing.

An essential component of my model of choice bracketing is the reference point. It ties the narrow preference relation to its broad counterpart. However, my model takes the reference point as given and stays agnostic about where it comes from. In my applications I show that the direction and extent of the deviation of a narrow bracketer's choices from her broad optimum crucially depends on the specific form of the reference point. Future research investigating the nature of reference points in narrow bracketing is therefore essential to further our understanding of this behavioral bias.

Another important component of my model is the system of brackets. It characterizes the degree to which a decision maker brackets narrowly. For a given system of brackets my model fully characterizes the representation of the narrow preference relation. A promising direction for future research would be to identify a way to elicit a decision maker's system of brackets from choice data. My experimental results suggest that the system of brackets characterizing the narrow preference relation is not set in stone. Instead, the extent to which a decision maker brackets narrowly depends on how easy it is for her to access information on the different dimensions of her decision problem simultaneously. In that respect my experimental design can serve as a guideline for finding ways to improve individual decision making.

References

Abeler, J., Falk, A., Goette, L., and Huffman, D. (2011). Reference points and effort provision. *American Economic Review*, 101(2):470–92.

Abeler, J. and Marklein, F. (2017). Fungibility, labels, and consumption. *Journal of the European Economic Association*, 15(1):99–127.

Andreoni, J., Gravert, C., Kuhn, M. A., Saccardo, S., and Yang, Y. (2018). Arbitrage or narrow bracketing? on using money to measure intertemporal preferences. Technical report, National Bureau of Economic Research.

Barberis, N. and Huang, M. (2001). Mental accounting, loss aversion, and individual stock returns. *The Journal of Finance*, 56(4):1247–1292.

Barberis, N. and Huang, M. (2009). Preferences with frames: a new utility specification that allows for the framing of risks. *Journal of Economic Dynamics and Control*, 33(8):1555–1576.

Barberis, N., Huang, M., and Santos, T. (2001). Prospect theory and asset prices. *The quarterly journal of economics*, 116(1):1–53.

Barberis, N., Huang, M., and Thaler, R. H. (2006). Individual preferences, monetary gambles, and stock market participation: A case for narrow framing. *American economic review*, 96(4):1069–1090.

Benartzi, S. and Thaler, R. H. (1995). Myopic loss aversion and the equity premium puzzle. *The Quarterly Journal of Economics*, 110(1):73–92.

Blow, L. and Crawford, I. (2018). Observable consequences of mental accounting.

Brown, J. R., Kling, J. R., Mullainathan, S., and Wrobel, M. V. (2008). Why don't people insure late-life consumption? a framing explanation of the under-annuitization puzzle. *American Economic Review: Papers & Proceedings*, 98(2):304–09.

Camerer, C., Babcock, L., Loewenstein, G., and Thaler, R. (1997). Labor supply of new york city cabdrivers: One day at a time. *The Quarterly Journal of Economics*, 112(2):407–441.

Chambers, C. P. and Echenique, F. (2009). Supermodularity and preferences. *Journal of Economic Theory*, 144(3):1004–1014.

Chen, D. L., Schonger, M., and Wickens, C. (2016). otreeâĂŤan open-source platform for laboratory, online, and field experiments. *Journal of Behavioral and Experimental Finance*, 9:88–97.

Choi, J. J., Laibson, D., and Madrian, B. C. (2009). Mental accounting in portfolio choice: Evidence from a flypaper effect. *American Economic Review*, 99(5):2085–95.

Ellis, A. and Freeman, D. J. (2020). Revealing choice bracketing. *arXiv preprint arXiv:2006.14869*.

Ellis, A. and Piccione, M. (2017). Correlation misperception in choice. *American Economic Review*, 107(4):1264–92.

Enke, B. and Zimmermann, F. (2018). Correlation neglect in belief formation. *The Review of Economic Studies*, 0:1–20.

Eyster, E. and Weizsäcker, G. (2016). Correlation neglect in portfolio choice: Lab evidence.

Fishburn, P. C. (1965). Independence in utility theory with whole product sets. *Operations Research*, 13(1):28–45.

Fishburn, P. C. (1967). Interdependence and additivity in multivariate, unidimensional expected utility theory. *International Economic Review*, 8(3):335–342.

Fishburn, P. C. (1970). *Utility theory for decision making*. Research Analysis Corporation.

Gabaix, X. (2014). A sparsity-based model of bounded rationality. *The Quarterly Journal of Economics*, 129(4):1661–1710.

Galperti, S. (2019). A theory of personal budgeting. *Theoretical Economics*, 14(1):173–210.

Gilboa, I., Postlewaite, A., and Schmeidler, D. (2010). The complexity of the consumer problem and mental accounting.

Gneezy, U. and Potters, J. (1997). An experiment on risk taking and evaluation periods. *The Quarterly Journal of Economics*, 112(2):631–645.

Gorman, W. M. (1959). Separable utility and aggregation. *Econometrica*, 27(3):469–481.

Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with orsee. *Journal of the Economic Science Association*, 1(1):114–125.

Hsiaw, A. (2018). Goal bracketing and self-control. *Games and Economic Behavior*, 111:100–121.

Kahneman, D., Knetsch, J. L., and Thaler, R. H. (1990). Experimental tests of the endowment effect and the coase theorem. *Journal of political Economy*, 98(6):1325–1348.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47:278.

Keeney, R. L. and Raiffa, H. (1993). *Decisions with multiple objectives: preferences and value trade-offs*. Cambridge university press.

Koch, A. K. and Nafziger, J. (2016). Goals and bracketing under mental accounting. *Journal of Economic Theory*, 162:305–351.

Koch, A. K. and Nafziger, J. (2020). Motivational goal bracketing: An experiment. *Journal of Economic Theory*, 185:104949.

Kőszegi, B. and Matějka, F. (2020). Choice simplification: A theory of mental budgeting and naive diversification. *The Quarterly Journal of Economics*, 135(2):1153–1207.

Kőszegi, B. and Rabin, M. (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics*, 121(4):1133–1165.

Kumar, A. and Lim, S. S. (2008). How do decision frames influence the stock investment choices of individual investors? *Management Science*, 54(6):1052–1064.

Lian, C. (2020). A theory of narrow thinking.

Mas-Colell, A., Whinston, M. D., Green, J. R., et al. (1995). *Microeconomic theory*, volume 1. Oxford university press New York.

Mu, X., Pomatto, L., Strack, P., and Tamuz, O. (2020). Background risk and small-stakes risk aversion. *arXiv preprint arXiv:2010.08033*.

Rabin, M. and Weizsäcker, G. (2009). Narrow bracketing and dominated choices. *American Economic Review*, 99(4):1508–43.

Read, D., Antonides, G., Van den Ouden, L., and Trienekens, H. (2001). Which is better: Simultaneous or sequential choice? *Organizational Behavior and Human Decision Processes*, 84(1):54–70.

Read, D. and Loewenstein, G. (1995). Diversification bias: Explaining the discrepancy in variety seeking between combined and separated choices. *Journal of Experimental Psychology: Applied*, 1(1):34.

Read, D., Loewenstein, G., and Kalyanaraman, S. (1999a). Mixing virtue and vice: Combining the immediacy effect and the diversification heuristic. *Journal of Behavioral Decision Making*, 12(4):257–273.

Read, D., Loewenstein, G., and Rabin, M. (1999b). Choice bracketing. *Journal of Risk and Uncertainty*, 19(1-3):171–97.

Smith, V. L. (1976). Experimental economics: Induced value theory. *The American Economic Review*, 66(2):274–279.

Stracke, R., Kerschbamer, R., and Sunde, U. (2017). Coping with complexity–experimental evidence for narrow bracketing in multi-stage contests. *European Economic Review*, 98:264–281.

Strotz, R. H. (1957). The empirical implications of a utility tree. *Econometrica*, pages 269–280.

Strotz, R. H. (1959). The utility tree–a correction and further appraisal. *Econometrica*, pages 482–488.

Thaler, R. H. (1999). Mental accounting matters. *Journal of Behavioral decision making*, 12(3):183–206.

Thaler, R. H., Tversky, A., Kahneman, D., and Schwartz, A. (1997). The effect of myopia and loss aversion on risk taking: An experimental test. *The Quarterly Journal of Economics*, 112(2):647–661.

Topkis, D. M. (1998). *Supermodularity and complementarity*. Princeton university press.

Tversky, A. and Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science*, 211(4481):453–458.

Vorjohann, P. (2020). Reference-dependent choice bracketing. AEA RCT Registry. September 10. https://doi.org/10.1257/rct.6419-1.0.

Wakker, P. P. (2010). *Prospect theory: For risk and ambiguity*. Cambridge university press.

Zhao, S., López Vargas, K., Friedman, D., and Gutierrez Chavez, M. A. (2020). Ucsc leeps lab protocol for online economics experiments. Available at SSRN: https://ssrn.com/abstract=3594027 or http://dx.doi.org/10.2139/ssrn.3594027.

A Proofs of Section 3

A.1 Proof of Proposition 1 (Indifference curves)

Proof. The marginal rates of substitution for the broad and the narrow bracketer are

$$MRS(x_1,x_2) = \left.\frac{\partial u}{\partial x_1}\right|_{(x_1,x_2)} \left(\left.\frac{\partial u}{\partial x_2}\right|_{(x_1,x_2)}\right)^{-1} \quad \text{and} \quad \widetilde{MRS}(x_1,x_2) = \left.\frac{\partial u}{\partial x_1}\right|_{(x_1,r_2)} \left(\left.\frac{\partial u}{\partial x_2}\right|_{(r_1,x_2)}\right)^{-1}.$$

Thus, we obviously have $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$. In this proof I focus on the case $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$. The other two cases can are proven analogously. Consider pairs (r_1,x_2) with $x_2 > r_2$. The above expressions for the broad and narrow marginal rates of substitution reveal that the numerator of $\widetilde{MRS}(r_1,x_2)$ is equal to the numerator of $\widetilde{MRS}(r_1,r_2)$ and the denominator of $MRS(r_1,x_2)$ is equal to the denominator of $\widetilde{MRS}(r_1,x_2)$. Furthermore, by $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$ we have that the numerator of $MRS(r_1,x_2)$ is larger than the numerator of $MRS(r_1,r_2)$. Together with $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$ this implies that $MRS(r_1,x_2) > \widetilde{MRS}(r_1,x_2)$ for all $x_2 > r_2$. Similar reasoning reveals that $MRS(r_1,x_2) < \widetilde{MRS}(r_1,x_2)$ for all $x_2 < r_2$. Now, consider pairs (x_1,r_2) with $x_1 > r_1$. The above expressions for the broad and narrow marginal rates of substitution reveal that the denominator of $\widetilde{MRS}(x_1,r_2)$ is equal to the denominator of $\widetilde{MRS}(r_1,r_2)$ and the numerator of $MRS(x_1,r_2)$ is equal to the numerator of $\widetilde{MRS}(x_1,r_2)$. Furthermore, by $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$ we have that the denominator of $MRS(x_1,r_2)$ is larger than the denominator of $MRS(r_1,r_2)$. Together with $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$ this implies that $MRS(x_1,r_2) < \widetilde{MRS}(x_1,r_2)$ for all $x_1 > r_1$. Similar reasoning reveals that $MRS(x_1,r_2) > \widetilde{MRS}(x_1,r_2)$ for all $x_1 < r_1$. Finally, the full claim presented in the proposition follows by convexity of preferences as implied by positive interactions. □

A.2 Proof of Proposition 2 (Narrow optimum)

Proof. Focus on the case $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0$. The proof for $\frac{\partial^2 u}{\partial x_1 \partial x_2} < 0$ proceeds analogously. Since x^* and $\tilde{x}$ are interior solutions and $r \neq x^*$, it must hold that $MRS(x_1^*,x_2^*) = \frac{p_1}{p_2}$, $\widetilde{MRS}(\tilde{x}_1,\tilde{x}_2) = \frac{p_1}{p_2}$, and $MRS(r_1,r_2) \neq \frac{p_1}{p_2}$.

Now, suppose $MRS(r_1,r_2) < \frac{p_1}{p_2}$. Since $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$, this holds iff $\widetilde{MRS}(r_1,r_2) < \frac{p_1}{p_2}$. Since x^* and $\tilde{x}$ are interior solutions and $w = p_1 r_1 + p_2 r_2$, $MRS(r_1,r_2) < \frac{p_1}{p_2}$ and $\widetilde{MRS}(r_1,r_2) < \frac{p_1}{p_2}$ imply that $x_1^*,\tilde{x}_1 < r_1$ and $x_2^*,\tilde{x}_2 > r_2$. Thus, by Proposition 1 $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0 \Rightarrow MRS(x_1^*,x_2^*) > \widetilde{MRS}(x_1^*,x_2^*)$ and $MRS(\tilde{x}_1,\tilde{x}_2) > \widetilde{MRS}(\tilde{x}_1,\tilde{x}_2)$. As $\frac{p_1}{p_2} = MRS(x_1^*,x_2^*) > \widetilde{MRS}(x_1^*,x_2^*)$ and $MRS(\tilde{x}_1,\tilde{x}_2) > \widetilde{MRS}(\tilde{x}_1,\tilde{x}_2) = \frac{p_1}{p_2}$ it must therefore hold that $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0 \Rightarrow d(r,x^*) < d(r,\tilde{x})$.

Suppose instead $MRS(r_1,r_2) > \frac{p_1}{p_2}$. Since $MRS(r_1,r_2) = \widetilde{MRS}(r_1,r_2)$ this holds iff $\widetilde{MRS}(r_1,r_2) > \frac{p_1}{p_2}$. Since x^* and $\tilde{x}$ are interior solutions and $w = p_1 r_1 + p_2 r_2$, $MRS(r_1,r_2) > \frac{p_1}{p_2}$ and $\widetilde{MRS}(r_1,r_2) > \frac{p_1}{p_2}$ imply that $x_1^*,\tilde{x}_1 > r_1$ and $x_2^*,\tilde{x}_2 < r_2$. Thus, by Proposition 1 $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0 \Rightarrow MRS(x_1^*,x_2^*) < \widetilde{MRS}(x_1^*,x_2^*)$ and $MRS(\tilde{x}_1,\tilde{x}_2) < \widetilde{MRS}(\tilde{x}_1,\tilde{x}_2)$. As $\frac{p_1}{p_2} = MRS(x_1^*,x_2^*) < \widetilde{MRS}(x_1^*,x_2^*)$ and $MRS(\tilde{x}_1,\tilde{x}_2) < \widetilde{MRS}(\tilde{x}_1,\tilde{x}_2) = \frac{p_1}{p_2}$ it must therefore hold that $\frac{\partial^2 u}{\partial x_1 \partial x_2} > 0 \Rightarrow d(r,x^*) < d(r,\tilde{x})$. □

A.3 Proof of Proposition 3 (Exchange economy)

Proof. For any elements x and $\tilde{x}$ of the respective broad and narrow cores, we have $MRS^1(x^1) = MRS^2(x^2)$ and $\widetilde{MRS}^1(\tilde{x}^1) = \widetilde{MRS}^2(\tilde{x}^2)$.

Focus first on $\frac{\partial^2 u^i}{\partial x_1^i \partial x_2^i} > 0$ for $i = 1,2$ and ω such that $MRS^1(\omega^1) > MRS^2(\omega^2)$. From Proposition 1 we know that since $r^i = \omega^i$, $MRS^i(\omega^i) = \widetilde{MRS}^i(\omega^i)$ for $i = 1,2$. Therefore, $MRS^1(\omega^1) > MRS^2(\omega^2)$ implies $\widetilde{MRS}^1(\omega^1) > \widetilde{MRS}^2(\omega^2)$.

By $MRS^1(\omega^1) > MRS^2(\omega^2)$ and $\widetilde{MRS}^1(\omega^1) > \widetilde{MRS}^2(\omega^2)$ it must hold for any interior broad and narrow core allocations x and $\tilde{x}$, that $x_1^1 \geq \omega_1^1$, $x_2^1 \leq \omega_2^1$, $\tilde{x}_1^1 \geq \omega_1^1$, and $\tilde{x}_2^1 \leq \omega_2^1$, with one of the two inequalities concerning x and $\tilde{x}$ holding strictly.

Now, consider any allocation y with $y_1^1 \geq \omega_1^1$ and $y_2^1 \leq \omega_2^1$, implying $y_1^2 \leq \omega_1^2$ and $y_2^2 \geq \omega_2^2$, where one of the two inequalities holds strictly. By Proposition 1 we have $MRS^1(y^1) < \widetilde{MRS}^1(y^1)$ and $MRS^2(y^2) > \widetilde{MRS}^2(y^2)$.

Thus, starting from the initial endowment allocation ω, increasing the amount of good 1 allocated to person 1 while decreasing the amount of good 2 allocated to person 1 reduces the difference between the broad marginal rates of substitution of persons 1 and 2 faster than the difference between the narrow marginal rates of substitution of persons 1 and 2. Therefore, it must hold that at any allocation in the broad core $x = (x^1, x^2)$, $\widetilde{MRS}^1(x^1) > \widetilde{MRS}^2(x^2)$ while at any allocation in the narrow core $\tilde{x} = (\tilde{x}^1, \tilde{x}^2)$, $MRS^1(\tilde{x}^1) < MRS^2(\tilde{x}^2)$, such that the Euclidean distance between the initial endowment allocation ω and any allocation in the broad core x, $d(x, \omega) = \sqrt{(\omega_1^1 - x_1^1)^2 + (\omega_2^1 - x_2^1)^2}$, is smaller than the Eucleadian distance between the intital endowment allocation ω and any allocation in the narrow core $\tilde{x}$, $d(\omega, \tilde{x}) = \sqrt{(\omega_1^1 - \tilde{x}_1^1)^2 + (\omega_2^1 - \tilde{x}_2^1)^2}$.

Focus now on $\frac{\partial^2 u^i}{\partial x_1^i \partial x_2^i} > 0$ for $i = 1,2$ and ω such that $MRS^1(\omega^1) < MRS^2(\omega^2)$. From Proposition 1 we know that since $r^i = \omega^i$, $MRS^i(\omega^i) = \widetilde{MRS}^i(\omega^i)$ for $i = 1,2$. Therefore $MRS^1(\omega^1) < MRS^2(\omega^2)$ implies $\widetilde{MRS}^1(\omega^1) < \widetilde{MRS}^2(\omega^2)$.

By $MRS^1(\omega^1) < MRS^2(\omega^2)$ and $\widetilde{MRS}^1(\omega^1) < \widetilde{MRS}^2(\omega^2)$ it must hold for any interior broad and narrow core allocations x and $\tilde{x}$, that $x_1^1 \leq \omega_1^1$ and $x_2^1 \geq \omega_2^1$, respectively $\tilde{x}_1^1 \leq \omega_1^1$ and $\tilde{x}_2^1 \geq \omega_2^1$, with one of each of the two inequalities holding strictly.

Now, consider any allocation y with $y_1^1 \leq \omega_1^1$ and $y_2^1 \geq \omega_2^1$, implying $y_1^2 \geq \omega_1^2$ and $y_2^2 \leq \omega_2^2$, where one of the two inequalities holds strictly. By Proposition 1 we have $MRS^1(y^1) > \widetilde{MRS}^1(y^1)$ and $MRS^2(y^2) < \widetilde{MRS}^2(y^2)$.

Thus, starting from the initial endowment allocation ω, decreasing the amount of good 1 allocated to person 1 while increasing the amount of good 2 allocated to person 1 reduces the difference between the broad marginal rates of substitution of consumers 1 and 2 faster than the difference between the narrow marginal rates of substitution of consumers 1 and 2. Therefore, it must hold that at any allocation in the broad core x, $\widetilde{MRS}^1(x) < \widetilde{MRS}^2(x)$ while at any allocation in the narrow core $\tilde{x}$, $MRS^1(\tilde{x}) > MRS^2(\tilde{x})$, such that the Euclidean distance between the initial endowment allocation ω and any allocation in the broad core x, $d(x, \omega) = \sqrt{(\omega_1^1 - x_1^1)^2 + (\omega_2^1 - x_2^1)^2}$, is larger than the Eucleadian distance between the intital endowment allocation ω and any allocation in the narrow core $\tilde{x}$, $d(\omega, \tilde{x}) = \sqrt{(\omega_1^1 - \tilde{x}_1^1)^2 + (\omega_2^1 - \tilde{x}_2^1)^2}$.

The proof for $\frac{\partial^2 u^i}{\partial x_1^i \partial x_2^i} < 0$ for $i = 1,2$ proceeds analogously. $\square$