The Whole Truth?
Generating and Suppressing Hard Evidence
Job Market Paper
Work in Progress

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November 14, 2018

Link to current version of the paper

1 Introduction

Song et al. (2010) estimate that nearly half of all clinical trials of medical treatments that are currently in use have been left unpublished. Furthermore, trials with positive outcomes are twice as likely to be published (Song et al., 2010, Goldacre, 2013). This suggests a strong publication bias for tests of drugs, even those drugs that are accepted by regulators. There has been a growing public concern over the effects of this bias (Chalmers, 2006). In response some pharmaceutical companies have publicly committed to registering all their trials in advance and to publish all the results (Kmietowicz, 2013).

The aim of this paper is to show that increasing transparency in this way can have a detrimental effect on public welfare, if the optimal strategic response of the pharmaceutical companies to the enforced transparency is taken into account. In my model, if suppressing bad evidence is possible, drugs will be accepted more often, but the average quality of accepted drugs will not decrease compared to the case when all evidence must be provided. On the other hand, the amount of testing needed to persuade a regulator is higher, when suppressing bad evidence is possible. When the benefit of a higher rate of acceptance outweighs the cost of more testing, the possibility of suppression increases welfare. I show that this happens when the expected quality of a drug is already close to the exogenously required level before testing starts.

*I want to thank Piero Gottardi for striking an almost impossible balance between unwavering support and unrelenting criticism. Words don't do justice to my gratitude. I thank Nina Bobkova for being a force of nature and directing some of her irrepressible intelligence and energy towards my problems. I thank Andrea Galeotti for helpful and detailed comments on early versions of this work. I gratefully acknowledge helpful comments from Giacomo Calzolari, Matteo Escudé, David Levine, Andrea Mattozzi, Aschkan Mery, Ana Moreno-Maldonado and Simon Skipka.

I construct a stylized model of the interaction between a sender (pharmaceutical company) that tries to persuade a receiver (regulator) to accept (a new drug). The sender always wants to be accepted, the receiver only wants to accept a sufficiently high quality (drug). Ex ante, the expected quality of the drug is such that the receiver does not want to accept. The sender tries to persuade the receiver by conducting repeated trials that may yield either a success or a failure. I compare the situation of *no suppression*, where the sender has to report the results of all trials he conducts, to the situation of *suppression*, where he can cherry pick the results, but he cannot fabricate results. Three aspects characterize the interaction. First, the sender has an informational advantage compared to the the receiver, even at the start of testing. I.e., he is informed of the quality before testing. Second, conducting trials is costly. Third, trials are conducted sequentially, i.e. the sender can observe the result of a trial before deciding to conduct the next one. These three factors introduce a signalling aspect in the communication. Evidence is not just informative about quality because higher quality yields more successes, but also because the sender is more inclined to conduct a costly trial when the probability of a success is higher. I decompose the total learning effect of reported trial outcomes for the receiver into a *statistical effect* and a *strategic effect* to distinguish these effects. Then I compare suppression to no suppression when the cost of running a test is small but positive.

In the case of suppression, reported evidence is only informative to the receiver because of the strategic effect, because any report can be generated by a sender with a dug of any quality in principle. However, for sender with a low quality, the expected cost of generating a report that is required for acceptance is too high. Therefore lower quality drugs will not be tested in equilibrium. A report is only informative because the expected total cost of testing to obtain it was too high for a sender with a low quality. I show that in the sender preferred equilibrium, the average quality of a drug that is tested and accepted is such that the receiver is approximately indifferent between accepting and rejecting. The total cost of testing is used to separate the qualities. For this reason, if the cost of running a single test decreases, the total cost of testing and the probability of acceptance do not change. The receiver will require a higher number of successes, in order to leave the expected total cost of testing unchanged and hence achieve the same separation. When the ex ante expectation of the receiver of the quality is far from the level that she needs to accept, the receiver will set a high requirement on the number of successes. This will lead the sender to begin testing only for higher qualities, but it will also increase the expected total cost of testing when he does test. Conversely, when the receiver is already close to accepting ex ante, the requirements she sets will be lower, leading the sender to be accepted more often and at a lower expected total cost of testing.

In the case of no suppression, reports also have a statistical effect, because different qualities can only generate a given report with different likelihoods. The receiver learns from a report because when evidence cannot be suppressed, more successes mean a higher quality is more likely. I focus on the case when the cost of running a single test is small. I show that then good quality drugs are accepted almost surely, while even very bad quality drugs are accepted with positive probability. Because the cost of testing is low, the sender can afford to run many tests. If he does, the receiver will be able to learn the quality exactly from the report, because the information in the report becomes very precise. Therefore, if the quality is high enough, the sender will be accepted almost surely. On the other hand, if the quality is not high enough, the sender can only appear to be of high enough quality by looking better than he really is in the early testing. This occurs with a positive probability that is strictly less than one. When the unit cost of testing becomes small, so does the total cost of testing. In the sender preferred equilibrium, the sender stops testing when the receiver is approximately indifferent between accepting and rejecting. Because it is not possible to run fractions of a test, the receiver typically strictly prefers accepting, in contradistinction to the case where suppression is possible. I argue that this is a second order effect.

The probability of a drug being accepted is higher when suppression possible than when it is not. The reason for this is that when suppression is possible, the lowest quality drugs do not test and are never accepted. In contrast, when suppression is not possible, even the lowest quality drugs are sometimes accepted. Since the expected quality of accepted drugs is not lower under suppression (even strictly higher, but approximately the same), the probability of acceptance is lower. On the other hand, when suppression is possible, the expected total cost of testing is higher.

Because the receiver is approximately indifferent between accepting and rejecting in either case, the relevant comparison for welfare is the sender pay-off. When suppression is possible, he faces a higher cost of testing, but also a higher probability of acceptance. Which of these effects dominates depends on the ex ante expectation of quality of the receiver. When this expectation is such that the receiver is ex ante close to accepting a drug, when suppression is possible, she will require little evidence in order to accept and the expected total cost of testing will be small. In that case, the higher probability of acceptance is the dominating effect and allowing suppression yields higher welfare. On the other hand, when the receiver is ex ante far from accepting, when suppression is possible, she will require a lot of evidence in order to accept and the total cost of testing will be high. In that the case the cost of testing is the dominant effect and not allowing suppression yields higher welfare.

The rest of this paper is organised as follows. In section 2, I describe the related literature. In section 3, I describe the model. In section 4, I consider the Bayesian Persuasion game specified to my environment as a useful benchmark. In section 5, I discuss the case of suppression. In section 6, I discuss the case of no suppression. In section 7, I carry out the comparison between the two cases. Finally, in section 8, I conclude.

2 Related literature

There are three strands in the literature that are related to my work. First there is the disclosure literature, building on Milgrom (1981), where the evidence is given exogenously and the effect of the possibility of suppression is studied. Second, in the Bayesian Persuasion model of Kamenica and Gentzkow (2011) testing is endogenous. There is a strand of the literature that builds on this model by imposing more structure on the testing process. Third, there is a literature that deals directly with the possibility of suppression with endogenous testing.

Of course, the cornerstone of the game theoretic communication literature is the beautiful paper on cheap talk by Crawford and Sobel (1982). There an informed sender can costlessly choose messages from an arbitrary message space. They show that if the ideal action of the sender and the receiver moves in the same direction with the state, then it is possible for the sender to communicate some of his information about this state. In my model, the preferences of the sender are state independent, i.e. he wants to be accepted even if the quality of the drug is low. Therefore, there would be no communication via cheap talk in my environment.

In my model the disclosure is of endogenously obtained test results. There is a rich literature on the disclosure of exogenously given information. The starting point of this literature is Milgrom (1981). He allows the sender state any set of types, as long as his actual type is contained in it. He shows that the receiver will learn the type exactly, a result known as unravelling. Crucial for the result is that revealing a higher type yields a higher pay-off, which is not true in my environment, sine the receiver can only accept or reject. In Dziuda (2011) exogenous evidence on two alternatives is known to the sender and relevant for the choice of the receiver. The sender also has a preference for one of the alternatives or is unbiased, this preference is private information. She shows that unravelling does not hold, since the receiver does not know how much evidence exists. Furthermore, she shows that even a biased type may reveal evidence that goes against his preference. Closely related to my work in both application and set-up is the paper by DiTillio, Ottaviani and Sørensen (2018). They treat the reporting strategy of the sender as given and compare the informativeness of a report when the sender reports the correct sample average to the informativeness of a report where the sender is known to report only the highest values of a sample.

A canonical model is Bayesian Persuasion by Kamenica and Gentzkow (2011). They consider a sender that tries to persuade a receiver by committing to an experiment without having private information. The class of experiments they allow is general in the sense that any distribution over signals conditional on type is allowed. Experiments are costless. In two follow-up papers, they allow costly experiments (Kamenica and Gentzkow, 2014) and suppression (Kamenica and Gentzkow (2017). There is a substantial literature that builds on the canonical model by imposing more structure on the set experiments. Gill and Sgroi (2012) consider a sender who tries to persuade a sequence of receivers by running a public test. The sender can choose the toughness of the test and generally prefers a tougher test that, although less likely to be passed, is more convincing when it is passed. Li and Li (2013) consider competing politicians who can choose the informativeness of their campaign and can choose to focus on their own quality (positive campaign) or the quality of the opponent (negative campaign). Perez-Richet (2014) investigates an abstract environment where the sender chooses the experiment from a restricted set after becoming informed, so that the choice of the experiment itself signals about this information. Experiments are costless. He shows that several natural refinements select the sender-preferred equilibrium.

Most closely related to my work are models in which testing is endogenous and where suppression is compared to no suppression. Felgenhauer and Loerke (2017) analyze a model in which the sender can run tests sequentially and at a cost. Each individual test is completely flexible, but when the sender reveals a test result, he also reveals the underlying experiment. They show that the sender runs only one experiment and that the receiver is always better off under observable testing, i.e. when suppression is not possible. Brocas and Carillo (2007) discuss a model of persuasion that is very similar in the testing structure to mine. Tests are binary and sequential, they allow for the possibility that testing is costly. However, they do not allow the possibility of suppression. Furthermore, the sender is not informed. Henry (2009) has a model in which an uninformed sender can acquire verifiable evidence at a cost. However, the acquisition is one shot: the sender decides a quantity of research and then observes a number that is the “net favourableness” of the research outcomes. In the case of suppression, the quantity of research acquired is not revealed. He shows that the sender is always worse off under suppression and the receiver always better off. In DeMarzo et al. (2015), the sender is uninformed and tries the maximize the price of an asset on a competitive market with risk neutral buyers. He can conduct one test of a set of exogenously given tests. After observing the test, he can disclose the results, which means giving both the test chosen and the outcome of the test. Each test can also produce one non-verifiable “null” result, which is the same for every test. The sender can therefore choose to not disclose information by instead reporting this outcome, even when it did not obtain, when the buyers would learn neither the test, nor the outcome. They show that in equilibrium the test is chosen that yields the lowest price conditional on reporting the null result. The sender gains nothing from the testing, but nonetheless, the equilibrium test is unique. Herresthal (2017) has a testing structure that is very similar to mine. However, the sender is uninformed, testing is costless and the number of tests cannot exceed an arbitrary value. The sender has state-based preferences and does not necessarily strictly prefer acceptance to rejection. She shows that when the preferences of the sender and receiver are misaligned, then the receiver is strictly better of when suppression is possible.

3 Set-up

The game is designed to capture in a very simple way an environment of strategic communication. A sender is privately informed of some parameter of interest. A receiver can choose to accept or reject the sender. The sender always wants to be accepted, the receiver only wants to accept if the parameter is high enough. Because of this conflict of interests, the sender cannot communicate the value of the parameter directly (he would not be believed). Instead, the sender can generate evidence that is verifiable, in order to convince the receiver to accept. I wish to compare the cases where the sender can or cannot suppress evidence that is generated.

There are two players, a sender and a receiver. The sender is privately informed of $\vartheta \in \mathbb{R}$. The receiver holds prior belief that ϑ is distributed according to a cumulative distribution function F. This distribution contains no atoms. $Supp(F)$ denotes the support of the distribution. $\vartheta_{min} := \min Supp(F)$, $\vartheta_{max} := \max Supp(F)$.

There are two stages in the game, first a testing stage and then a decision stage. In the decision stage, the receiver chooses $d \in \{a, r\}$, where a denotes acceptance and r rejection. The decision stage pay-off to the sender is:

$$\Pi^2_{Sender}(d, \vartheta) = W \mathbb{1}_{\{d=a\}}.$$

$W > 0$ is the benefit of being accepted. The sender always prefers acceptance to rejection. The decision stage pay-off to the receiver is:

$$\Pi^2_{Receiver}(d, \vartheta) = (\vartheta - \bar{\vartheta}) \mathbb{1}_{\{d=a\}},$$

with $E_F[\vartheta] < \bar{\vartheta} < 1$. The receiver therefore only wants to accept if his expectation of ϑ is above $\bar{\vartheta}$. Ex ante this is not the case.

In the testing stage, the sender generates evidence and presents a report of this evidence to the receiver. The aim of the testing and reporting is to increase the expectation of the receiver of ϑ.

The testing works as follows. The sender can sequentially run tests indexed by i. Each test is a random variable that is binary in that it yields one of two outcomes, success, $X_i = 1$, or failure $X_i = 0$. The probability of a success depends on ϑ: $Pr(X_i = 1|\vartheta) = 1 - Pr(X_i = 0|\vartheta) = p(\vartheta)$. $p(.)$ is a strictly increasing continuous function that is commonly known. Conditional on ϑ, X_i are independent.

Running a test costs $c \geq 0$ to the sender. After observing the result of test X_i, the receiver can choose to stop or to continue and run test X_{i+1}, again at cost c. Thus, if he stops testing after τ tests, the testing stage pay-off to the sender is

$$\Pi^1_{Sender}(\tau, \vartheta) := -c\tau$$

.

The receiver's pay-off is unaffected by the testing.

Associated with the underlying stochastic process $(X_i)_{i \in \mathbb{N}}$, I define $H_n := (X_i)^n_{i=1}$ as the the history of testing up to time n. Denote the set of all possible testing histories $\mathbb{H} := \bigcup_{n \in \mathbb{N}} \{0, 1\}^n$.[1]

If the sender stops testing after τ tests, having obtained testing history H_τ, he submits a report R to the receiver. Here I distinguish two cases, "suppression" and "no suppression".

[1] A typical element of $\mathbb{H}$ is H, or H_n, if I want to explicit about the number of tests in H.

In the case of "no suppression", if the sender stops after τ tests, he submits the entire history H_τ as a report. $R = H_\tau$.

In the case of "suppression", if the sender stops after τ tests, he can submit any report R that is a subsequence of H_τ. Intuitively, this means that he can omit any test result, but he cannot change the order of test results. Formally, $(X_i)_{i=1}^n \in \mathbb{H}$ is a subsequence of $(Y_i)_{i=1}^m \in \mathbb{H}$ if there exists an increasing function $k : \{1, \ldots, n\} \to \{1, \ldots, m\}$ such that $X_i = Y_{k(i)}$. I write $X|Y$ in that case. [2] Therefore $R \in \{H \in \mathbb{H} : H|H_\tau\}$.

I say that a report R is *feasible* with respect to a testing history $H \in \mathbb{H}$ if it can be submitted given that testing history. That is, in the case of "no suppression" R is feasible if and only if $R = H$, while in the case of "suppression", it is feasible if and only if $R|H$.

A strategy for the sender is now a stopping time τ and a reporting rule R. The stopping specifies when he stops testing, the reporting rule what he reports in that case. Both are random variables.

Formally, the profile of the sender is $\omega = (\vartheta, X)$, with $\vartheta \in \mathbb{R}$ and $X \in \{0,1\}^{\mathbb{N}}$. The set of all profiles is:

$$\Omega := \mathbb{R} \times \{0,1\}^{\mathbb{N}}$$

I equip Ω with the σ-algebra $\mathcal{F} := \mathcal{B}(\mathbb{R}) \times \mathcal{P}(\{0,1\}^{\mathbb{N}})$, where $\mathcal{B}(.)$ is the Borel σ-algebra and $\mathcal{P}(.)$ is the power set. I associate with the process $(X_i)_{i \in \mathbb{N}}$ the natural filtration $\mathcal{F}_n := \sigma(\vartheta, X_1, \ldots, X_n)$. Clearly $\mathcal{F}_n \subseteq \mathcal{F}_{n+1} \subseteq \mathcal{F}$.

The stopping time τ of the receiver is now a stopping time with respect to the natural filtration, i.e. $\tau : \Omega \to \mathbb{N}$ with $\{\omega : \tau(\omega) \leq n\} \in \mathcal{F}_n$. The reporting rule R satisfies $R : \Omega \to \mathbb{H}$, with $R(\omega) = R(\omega')$ if $\vartheta(\omega) = \vartheta(\omega')$ and $X_i(\omega) = X_i(\omega') \forall i \leq \tau(\omega)$.[3] In the case of "suppression", $R(\omega) \in \{H \in \mathbb{H} : H|H_{\tau(\omega)}(\omega)\}$. In the case of "no suppression", $R(\omega) = H_{\tau(\omega)}(\omega)$. Notice that τ and R depend on ϑ. Sometimes, if I want to be explicit about this dependence, I will write τ_ϑ and R_ϑ. I sometimes refer to the strategy of the sender as a stopping rule, since the reporting rule is often trivial even in the case of "suppression".

A pure strategy for the receiver specifies for each report $R \in \mathbb{H}$ if he accepts or rejects. Thus, it is a function $f : \mathbb{H} \to \{a, r\}$. I find it convenient to identify this function with the set $\mathbb{A} := f^{-1}(a) = \{H \in \mathbb{H} : f(H) = a\}$. I will refer to the strategy of the sender as the acceptance rule.

The equilibrium concept I use is the usual notion of Perfect Bayesian Equilibrium. Equilibrium is therefore a quadruple $\{\tau, R, \mathbb{A}, F^*(\vartheta|H)\}$, where $F^*(\vartheta|H)$ are the beliefs of the receiver conditional on any report. In equilibrium, (τ, R) are optimal given $\mathbb{A}$, $\mathbb{A}$ is optimal given (τ, R) and $F^*(\vartheta|H)$ (although optimality with respect to $F^*(\vartheta|H)$ alone is sufficient), and $F^*(\vartheta|H)$ satisfies Bayes' rule whenever it applies. Because multiplicity of equilibria occurs, I focus on the sender-preferred equilibrium.

[2] There appears to be no completely accepted notation in the mathematical literature for the subsequence relation. The notation I use can be found in "Structural Additive Theory" by David Grynkiewicz.

[3] Notice that this implies $\tau(\omega) = \tau(\omega')$

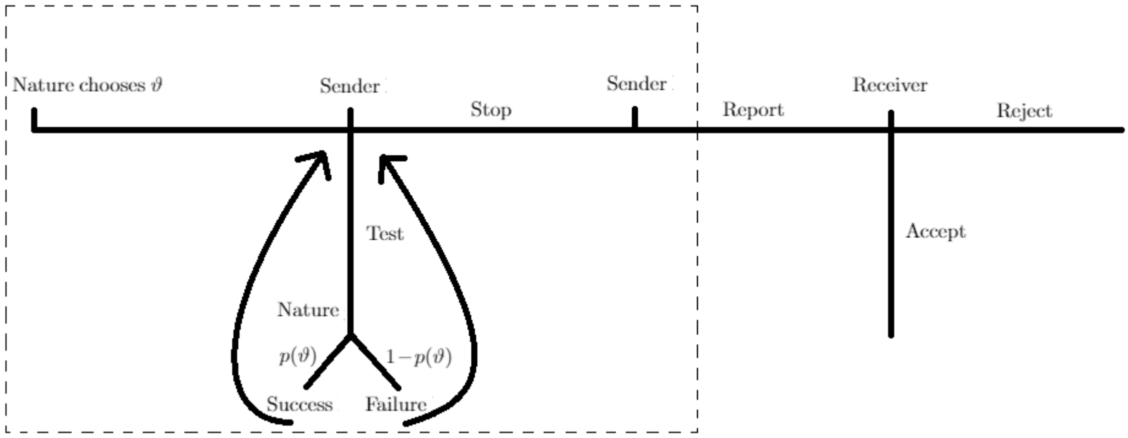


Figure 1: A graphic representation of the environment.

3.1 Statistical and Strategic Effect of Reports

Given a strategy (τ, R) of the sender, for any report $H \in \mathbb{H}$, $Pr_{(\tau,R)}(H|\vartheta)$, the probability of that report being submitted by ϑ, is determined and is independent of F. Then Bayes' rule becomes

$$F_{(\tau,R)}(\vartheta|H) = \frac{\int_{\vartheta_{min}}^{\vartheta} Pr_{(\tau,R)}(H|\vartheta')dF(\vartheta')}{\int_{\vartheta_{min}}^{\vartheta_{max}} Pr_{(\tau,R)}(H|\vartheta')dF(\vartheta')}$$

,

whenever the denominator is not 0.

I introduce a concept that will be useful in understanding why reports are informative in equilibrium. A report generated under the testing structure of $(X_i)_{i\in\mathbb{N}}$ will typically reveal information for two reasons. For example, if the sender reports that he obtained one success, this tells the receiver both that he tested at least once and that he obtained at least one success. The fact that the sender tested at least once is informative, because, knowing the strategy of the sender, the receiver might be able to rule out certain realisations of ϑ, i.e. those that do not test at least once in equilibrium. I refer to this as the strategic effect of the report, because it is derived from considering the strategy of the sender. On the other hand, the mere fact that there is a success in the test results is by itself also informative. After all, successes are more likely obtained by higher ϑ. I refer to this as the statistical effect on the report.

In order to precisely define these concepts, for a given report I define the statistical expectation conditional on that report as the expectation when the strategy of the sender, for every ϑ, is to keep testing until they can generate that report. This eliminates any strategic considerations as outlined above. So, for any report H, define

$$\tau_{stat}(H) := \min\{n : H_n \text{ is such that } H \text{ is a feasible report}\}^4$$

and

$$R_{stat}(H) := H.$$

Notice that this stopping time does not depend on ϑ, but it does depend on the report H. Then define

$$E_{stat}[\vartheta|H] := E_{F_{(\tau_{stat}(H),R_{stat}(H))}}[\vartheta|H] = \frac{\int_{\vartheta_{min}}^{\vartheta_{max}} \vartheta' Pr_{(\tau_{stat}(H),R_{stat}(H))}(H|\vartheta')dF(\vartheta')}{\int_{\vartheta_{min}}^{\vartheta_{max}} Pr_{(\tau_{stat}(H),R_{stat}(H))}(H|\vartheta')dF(\vartheta')}$$

as the expectation conditional on H, given this stopping time. I want to compare this expectation to the expectation under the optimal stopping times (in equilibrium), denoted by

$$E^*[\vartheta|H] := E_{F_{(\tau^*,R^*)}}[\vartheta|H] = \frac{\int_{\vartheta_{min}}^{\vartheta_{max}} \vartheta' Pr_{(\tau^*,R^*)}(H|\vartheta')dF(\vartheta')}{\int_{\vartheta_{min}}^{\vartheta_{max}} Pr_{(\tau^*,R^*)}(H|\vartheta')dF(\vartheta')},$$

where (τ^*, R^*) is the equilibrium strategy of the sender.

I define the statistical effect of the report as

$$\gamma_{stat}(H) = \frac{E_{stat}[\vartheta|H] - E[\vartheta]}{E^*[\vartheta|H] - E[\vartheta]}$$

and the strategic effect as

$$\gamma_{strat}(H) = \frac{E^*[\vartheta|H] - E_{stat}[\vartheta|H]}{E^*[\vartheta|H] - E[\vartheta]}$$

These measures allow me to say whether for a given report the persuasive effect comes primarily from strategic or statistical considerations. By construction $\gamma_{stat} + \gamma_{strat} = 1$. As we will see, perhaps unsurprisingly, in the case of suppression, reports will typically have only strategic effect, while in the case of no suppression, as the cost of testing becomes negligible, reports converge to having only statistical effect.

[4] Notice that τ is a Markov time, but not necessarily a stopping time, since it may be that $Pr(\tau_{stat} = \infty) > 0$. Indeed, it may be impossible for some ϑ to generate the report. This is however not relevant, since this is merely a technical definition and the stopping time is clearly not optimal anyway. The definition of τ_{stat} can easily be amended to a stopping time, by allowing the sender to stop testing as soon as it becomes impossible to generate the report. However, this would complicate (slightly) the definition.

4 Bayesian Persuasion, a Preliminary Observation

Before proceeding to the main analysis, I consider a useful benchmark, where the sender faces the costless information design problem, also known in the literature as the Bayesian Persuasion game (Kamenica and Gentzkow, 2011). In my environment, this means that game where the sender can, before learning ϑ and without cost, choose a conditional distribution of signals, from an unrestricted set of signals. The situation is thus different from my model in both the structure of testing and the fact that the sender is now uninformed. Otherwise the pay-off of the game is as above, i.e. the sender receives W if he is accepted and 0 otherwise, while the receiver obtains $\vartheta - \bar{\vartheta}$ if he accepts and 0 otherwise.

This is a useful benchmark, because it shows the best the sender can hope to do, from an ex ante perspective, when he is not constrained by the structure and cost of the testing that I impose. Since, later on, I will be interested in the sender-preferred equilibrium, which is always evaluated ex ante, this a relevant perspective.

The sender thus chooses a set S and for each $s_i \in S$ and each $\vartheta \in Supp(F)$, he chooses $g(s_i, \vartheta)$, so that $Pr(s = s_i|\vartheta) = g(s_i, \vartheta)$. The receiver then receives a signal drawn from S based on this conditional distribution and then chooses whether to accept. I again consider the Perfect Bayesian Equilibrium. Define $\underline{\vartheta}(\bar{\vartheta}) := \inf\{q : E_F[\vartheta|\vartheta \geq q] \geq \bar{\vartheta}\}$. Then the following is a standard result.

Proposition 0. ***Bayesian Persuasion***
The Bayesian Persuasion Equilibrium is (outcome equivalent to[5]) a conditional distribution with 2 signals, s_1, s_2, such that $Pr(s = s_1|\vartheta > \underline{\vartheta}) = Pr(s = s_2|\vartheta < \underline{\vartheta}) = 1$. The receiver accepts if and only if he receives signal s_1.

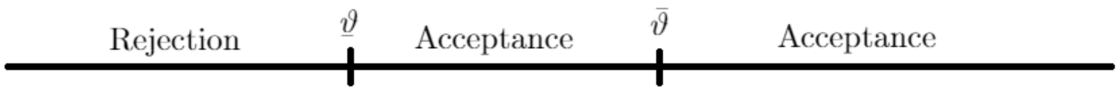


Figure 2: The outcomes induced in the Bayesian Persuasion game.

For future reference, denote $p_{BP}(\vartheta) := Pr(s = s_1|\vartheta)$ and $p_{BP} := Pr(s = s_1)$.

Proof. See Gentzkow and Kamenica (2016), section 5.1. □

To see that the probability of acceptance in the Bayesian Persuasion game indeed provides an upper bound on the unconditional probability of acceptance in any other communication game with the same pay-off to the receiver, consider any game **G** that has the same pay-off structure, i.e. in the final stage the receiver chooses $d \in \{a, r\}$ and the pay-off to the receiver is as above. Suppose that this game supports an equilibrium in which $Pr_{\mathbf{G}}(a|\vartheta)$ is such that $E[Pr_{\mathbf{G}}(a|\vartheta)] = Pr_{\mathbf{G}}(a) > p_{BP}$. Let $Pr(s = s_1|\vartheta) = Pr_{\mathbf{G}}(a|\vartheta)$

[5]Here outcome equivalence means that the ex ante probability of a different pay-off is 0.

and $Pr(s = s_2|\vartheta) = Pr_{\mathbf{G}}(r|\vartheta)$. Since in equilibrium of the game $\mathbf{G}$, whenever the receiver accepts, he is willing to do so, he is also willing to accept if he only knows he is in a state in which he accepts in $\mathbf{G}$, but not which state. Therefore, the receiver accepts upon signal s_1. This is a contradiction with the optimality of the Bayesian Persuasion outcome. In particular, also in the main model the probability of acceptance can never exceed p_{BP}.

5 Suppression

I now consider the game of communication with suppression, i.e. where the sender generates evidence from the process $(X_i)_{i\in\mathbb{N}}$ at a unit cost c each time and can submit any report $R|H_\tau$, where H_τ is the evidence gathered when the sender stops testing.

Notice that, if $c = 0$, any type can generate any report at no cost (other than ϑ_{min} if $p(\vartheta_{min}) = 0$ and ϑ_{max} if $p(\vartheta_{max}) = 1$, but these types occur with zero probability by assumption). Therefore this case reduces to cheap talk, which in this environment implies no communication. I therefore assume throughout the section on suppression that $c > 0$.

As a preliminary observation, note that reports will not have statistical effect when suppression is possible.

Lemma 1. ***Reports have no Statistical Effect***
For any report $H \in \mathbb{H}$, $\gamma_{stat}[H] = 0$.

Proof. Observe that $\tau_{stat}(H) = \min\{n : H|H_n\}$. Clearly, if $0 < p(\vartheta) < 1$, then $Pr(\tau < \infty|\vartheta) = 1$. Therefore $E_{stat}[\vartheta|H] = E[\vartheta|\vartheta_{min} < \vartheta < \vartheta_{max}] = E[\vartheta]$. □

5.1 Optimal Testing with Suppression

I first derive a simple result on the optimal stopping time in response to any acceptance rule $\mathbb{A} \subseteq \mathbb{H}$. If, for a given ϑ, the sender decides to test at least once, then he will test until he can send a report that will lead to acceptance. The intuition for this result is straightforward. If the sender decides to run a test and obtains a result that is not favourable (with respect to the acceptance rule, which may in principle require failures as well as successes), then he can simply suppress this result. The expected value of continuing to test is therefore not lower than before he obtained the unfavourable result. Formally, the continuation value is a submartingale. Therefore, if the sender was right in testing to begin with, he should keep testing until accepted.

Lemma 2. ***Optimal Stopping***
For any $\mathbb{A} \subseteq \mathbb{H}$ there is a stopping time τ that is an optimal response to $\mathbb{A}$ and has the property $Pr(a|\tau > 0) = 1$.

Proof. Denote by $V^\tau_{\mathbb{A}}(H)$ the value of stopping time τ given acceptance rule $\mathbb{A} \subseteq \mathbb{H}$ and after testing history $H \in \mathbb{H}$. Denote by $\tau_{\mathbb{A}}$ the optimal stopping time given $\mathbb{A}$. I write $V^*_{\mathbb{A}}(H) := V^{\tau_{\mathbb{A}}}_{\mathbb{A}}$ for the value of the optimal stopping time. I suppress the dependence of the value and stopping time on ϑ in this proof.

As a preliminary, I define the concatenation of 2 sequences. Let $X, Y \in \mathbb{H}$, $X = (X_i)_{i=1}^m$, $Y = (Y_i)_{i=1}^n$, then

$$X^\frown Y := (Z_i)_{i=1}^{n+m},\; Z_i := \begin{cases} x_i & \text{if } i \leq m \\ y_{i-m} & \text{if } i > m \end{cases}.$$

Next, for any $\mathbb{A} \subseteq \mathbb{H}$, $H \in \mathbb{H}$, I define the continuation acceptance rule as:

$$\mathbb{A} - \{H\} := \{H' \in \mathbb{H} : H^\frown H' \in \mathbb{A}\}$$

A sender facing $\mathbb{A}$ after testing result H faces the same problem as a sender facing $\mathbb{A} - \{H\}$ before any testing. $V^\tau_{\mathbb{A}}(H) = V^\tau_{\mathbb{A}-\{H\}}(H_0)$. This follows from the fact that, because of the conditional independence of the tests, the probability of any sequence H obtaining is independent of prior testing, and because $\forall H' \in \mathbb{H}$, $H'^\frown H \in \mathbb{A} \Leftrightarrow H \in \mathbb{A} - \{H'\}$.

Suppose $\mathbb{A} \subseteq \mathbb{A}' \subseteq \mathbb{H}$. Then, for any $H \in \mathbb{H}$, $V^*_{\mathbb{A}}(H) \leq V^*_{\mathbb{A}'}(H)$. Indeed, $V^*_{\mathbb{A}}(H) \leq V^{\tau_{\mathbb{A}}}_{\mathbb{A}'}(H) \leq V^*_{\mathbb{A}'}(H)$. Here the first inequality follows from the fact that, for a given stopping time, $\mathbb{A}'$ leads weakly more often to acceptance, while the second inequality follows from the optimality of $\tau_{\mathbb{A}'}$.

Next, consider $\mathbb{A} \subseteq \mathbb{H}$. Let $\mathbb{A}' = \{H \in \mathbb{H} : \exists A \in \mathbb{A} \text{ s.t. } A|H\}$. $\mathbb{A}'$ induces the same behaviour in the sender, because he can submit a report that leads to acceptance under $\mathbb{A}$ if and only if he can submit a report that leads to acceptance under $\mathbb{A}'$. Indeed, if $H' \in \mathbb{A}'$ and H' is feasible, then there exists an $H|H'$ such that $H \in \mathbb{A}$ and H is feasible by construction. $\mathbb{A}'$ has the property that $A \in \mathbb{A}'$ and $A|A'$ imply $A' \in \mathbb{A}'$. It is therefore without loss of generality to assume that acceptance rules satisfy this property.

Finally, observe that for any $\mathbb{A} \subseteq \mathbb{H}$, $H \in \mathbb{H}$, $\mathbb{A} \subseteq \mathbb{A} - \{H\}$. Indeed, let $A \in \mathbb{A}$, then $A|H^\frown A$ and therefore $A \in \mathbb{A} - \{H\}$. Therefore $V^*_{\mathbb{A}}(H) = V^*_{\mathbb{A}-\{H\}}(H_0) \geq V^*_{\mathbb{A}}(H_0)$. Since the sender will prefer testing to stopping when $V^*_{\mathbb{A}}(H) > 0$, he will prefer testing to stopping after any history H, if he preferred it before testing, at H_0. Optimality then implies that the probability of acceptance must be 1. □

Lemma 2 implies that for any acceptance rule $\mathbb{A}$, there will be two sets of ϑ, those that get accepted with probability 1 and those that get rejected with probability 1.

5.2 Sender-preferred Simple Equilibrium with Suppression

Define

$$\mathbf{1}_S := \underbrace{(1, \ldots, 1)}_{S},$$

a sequence of S successes. In this section I consider equilibria where the acceptance rule is of the form $\mathbb{A} = \{\mathbf{1}_S\}$. In words, acceptance requires only some number of successes. Notice that this is not a restriction on the strategies of the receiver, but a selection within the class of equilibria. I call such an acceptance rule and the corresponding equilibrium, if it exists, *simple*. These equilibria are indeed simpler than the general class of equilibria, but their most important properties carry over to the general case. In the next section, I establish this connection to the more general case. In this section, my goal is to establish that there exist simple equilibria. Furthermore, I find the sender-preferred equilibrium within the set of simple equilibria. I call this equilibrium an *sender-preferred simple equilibrium.* In the next section I show that the sender-preferred simple equilibrium approximates the exact sender-preferred equilibrium.

Lemma 2 implies that there are sets of ϑ, one that tests in equilibrium and is accepted with probability one and an other that does not test and is never accepted. The requirements of a simple equilibrium are easier to meet, the higher is ϑ. Therefore, the two sets will be intervals, separated by a threshold on ϑ, with every ϑ above the threshold testing until accepted and every ϑ below the threshold not testing. To each $\mathbf{1}_S$ corresponds a threshold. If the threshold is above the Bayesian Persuasion threshold $\underline{\vartheta}(\bar{\vartheta})$, the receiver would be willing to accept and $\mathbf{1}_S$ induces an equilibrium. Lower thresholds lead to both a higher probability of acceptance and a lower cost of testing. Therefore the lowest threshold that is above $\underline{\vartheta}(\bar{\vartheta})$ is the sender-preferred simple equilibrium.

For any $S \in \mathbb{N}$, define $\vartheta_c(S) := p(cS/W)^{-1}$.

Proposition 1. ***Sender-preferred Simple Equilibrium***
Let $\mathbb{A} = \{\mathbf{1}_S\}$. If $\vartheta_c(S) \geq \underline{\vartheta}(\bar{\vartheta})$, this constitutes an equilibrium, with $Pr(a|\vartheta) = 1$ if and only if $\vartheta \geq \vartheta_c(S)$. The sender-preferred simple equilibrium has S such that $\vartheta_c(S-1) < \underline{\vartheta}(\bar{\vartheta}) \leq \vartheta_c(S)$. If $\vartheta_c(S) = \underline{\vartheta}(\bar{\vartheta})$, then $Pr(a) = p_{BP}$.

Proof. Since any sequence containing S successes permits the report $\mathbf{1}_S$, the task the sender faces is simply to obtain S successes in total. Denote by $\mathbf{T}_\vartheta(S)$ the expected waiting time until obtaining S successes. Clearly $\mathbf{T}_\vartheta(S) = p(\vartheta)\mathbf{T}_\vartheta(S-1) + (1-p(\vartheta))\mathbf{T}_\vartheta(S) + 1$, whence I conclude $\mathbf{T}_\vartheta(S) = S/p(\vartheta)$. If $c\mathbf{T}_\vartheta(S) \geq W$, the sender will test, otherwise not. This leads to the threshold $\vartheta_c(S) = p(cS/W)^{-1}$ By lemma 2, he will then test until accepted.

For this to constitute an equilibrium, it must indeed be the case that the receiver is willing to accept if and only if the report received is $\mathbf{1}_S$. Observe that for any off path report (any report other than $\mathbf{1}_S$ or H_0), I am free to specify any belief for the receiver. This is so, because any report can be generated by any ϑ. I therefore specify the beliefs to be such that $E[\vartheta|R] < \bar{\vartheta}$ for any $R \notin \{\mathbf{1}_S, H_0\}$. $E[\vartheta|H_0] = E[\vartheta|\vartheta < \vartheta_c(S)] \leq E[\vartheta] < \bar{\vartheta}$, so rejection is ensured there. Finally $E[\vartheta|\mathbf{1}_S] = E[\vartheta|\vartheta \geq \vartheta_c(S)]$. If and only if $\vartheta_c(S) \geq \underline{\vartheta}(\bar{\vartheta})$, this yields $E[\vartheta|\mathbf{1}_S] \geq \bar{\vartheta}$.

For sender-preferred simple equilibrium, consider that if $\mathbf{1}_S$ induces an equilibrium, then so does $\mathbf{1}_{S+1}$. However, $\mathbf{1}_S$ is then preferred by the sender, as it has a higher probability of acceptance and a lower cost of testing. Therefore, the smallest value of S that induces equilibrium is sender-preferred, yielding the characterization in the proposition. □

The sender-preferred simple equilibrium of proposition 1 does not always give $Pr(a) = p_{BP}$. This is because of integer problems. If it were possible to report fractions of a success, it always be possible to find values $S \in \mathbb{R}$ such that $\vartheta_c(S) = \underline{\vartheta}(\bar{\vartheta})$. Because it is not possible to let $S \in \mathbb{R}/\mathbb{N}$, I need to consider sender preferred equilibrium in the general class of (non-simple) equilibria. I do this in the next section, but first I establish an important feature of the sender-preferred simple equilibrium, namely that its outcomes are independent of c. By this I mean the following. Fix c and take a simple equilibrium, $\mathbb{A} = \{\mathbf{1}_S\}$ and $Pr(a|\vartheta) = 1$ if and only if $\vartheta \geq \vartheta_c(S)$. Now take $c' = c/2$. Then by taking $S' = 2S$, the same threshold is induced: $\vartheta_c(S) = \vartheta_{c'}(S')$. Furthermore, also the expected cost of testing is the same for any ϑ, since twice as many successes are required, at half the cost. Thus both the sender and the receiver get the same pay-off in these two equilibria.

Define $TC_c(\vartheta) := cE[\tau|\tau > 0, \vartheta]$, the total cost of testing, conditional on positive testing and $TC_c := E[TC_c(\vartheta)]$.

Lemma 3. ***The effect of*** c
Consider c, c' *and suppose* S, S' *are such that* $\vartheta_c(S) = \vartheta_{c'}(S') =: \hat{\vartheta}$. *Then*

$$TC_c(\vartheta) = TC_{c'}(\vartheta) = \frac{p(\hat{\vartheta})}{p(\vartheta)}W$$

and

$$TC_c = TC_{c'} = p_{BP}E[\frac{p(\hat{\vartheta})}{p(\vartheta)}|\vartheta \geq \hat{\vartheta}]$$

The establishes that in any simple equilibrium, and therefore in particular in the sender-preferred simple equilibrium, the total cost of testing is determined fully by the threshold on ϑ it induces and is independent of c.

Proof. Observe that given $\vartheta_c(S)$, $p(\vartheta_c(S))W = cS$, so that the expected cost of testing for any $\vartheta \geq \vartheta_c(S)$ is $TC_c(\vartheta) = cS/p(\vartheta) = p(\vartheta_c(S))W/p(\vartheta) = p(\vartheta_{c'}(S'))W/p(\vartheta) = TC_{c'}(\vartheta)$. TC_c is then straightforwardly derived. □

5.3 Exact Sender-preferred Equilibrium with Suppression

I now consider the sender-preferred equilibrium from the complete class of equilibria. I show that it behaves very much like the sender-preferred simple equilibrium, in the sense that whenever $\vartheta_c(S) = \underline{\vartheta}(\bar{\vartheta})$, the simple and the exact sender-preferred equilibrium coincide. I show that this happens for infinitely many positive values of c. Furthermore, when the sender-preferred simple equilibrium differs from the sender-preferred equilibrium, it is still close in outcomes, in the sense that the difference in outcomes converges to 0, as $c \to 0$.

Proposition 2. ***Sender-preferred Equilibrium***
The sender-preferred simple equilibrium approximates the exact sender-preferred equilib-

rium. In the exact sender-preferred equilibrium:

$$Pr_c(a) \leq p_{BP},$$
$$\frac{p(\underline{\vartheta}(\bar{\vartheta}))}{p(\vartheta)}W \leq TC_c(\vartheta),$$
$$TC_c \leq p_{BP}E\Big[\frac{p(\underline{\vartheta}(\bar{\vartheta}))}{p(\vartheta)}|\vartheta \geq \underline{\vartheta}(\bar{\vartheta})\Big]W + B(c),$$

where $B(c) \to 0$ *as* $c \to 0$*. For* $c \in \{p(\underline{\vartheta}(\bar{\vartheta}))W/n : n \in \mathbb{N}\}$*, these bounds hold with equality. For* $c \to 0$*, they hold with equality in the limit.*

Proof. $Pr(a)_c \leq p_{BP}$ is immediate from proposition 0.

Fix any acceptance rule $\mathbb{A} \subseteq \mathbb{H}$. Recall that $l(H)$ is the lenght of a sequence H, i.e. $l(H_n) = n$. Define $\mathbf{T}^{\mathbb{A}}_{p(\vartheta)}(H) := E[\tau_{\mathbb{A}}|\vartheta, H] - l(H)$, where $\tau_{\mathbb{A}} := \min\{n : \exists A \in \mathbb{A} \text{ s.t. } H_n|A\}$. In words, $\mathbf{T}^{\mathbb{A}}_{p(\vartheta)}(H)$ is the expected testing time until acceptance, after having obtained test results H.

I establish at the end of the proof, in lemma 4, that $p\mathbf{T}^{\mathbb{A}}_{p}(H)$ is non-decreasing in p. Consider now any equilibrium, with acceptance rule $\mathbb{A} \subseteq \mathbb{H}$. It must be that there is a $\vartheta' \geq \underline{\vartheta}$ that does not test (or is indifferent). Otherwise, by the continuity of $\mathbf{T}^{\mathbb{A}}_{p(\vartheta)}(H_0)$ in ϑ, there is also $\vartheta < \underline{\vartheta}$ willing to test. This would yield a probability of acceptance greater than p_{BP}, which is impossible. Suppose there is any $\vartheta'' > \vartheta'$ s.t. $c\mathbf{T}^{\mathbb{A}}_{p(\vartheta'')}(H_0) < p(\underline{\vartheta})W/p(\vartheta'')$. At the same time, $c\mathbf{T}^{\mathbb{A}}_{p(\vartheta')}(H_0) \geq W$. Then $p(\vartheta'')\mathbf{T}^{\mathbb{A}}_{p(\vartheta'')}(H_0) < p(\underline{\vartheta})\mathbf{T}^{\mathbb{A}}_{p(\vartheta')}(H_0) \leq p(\vartheta')\mathbf{T}^{\mathbb{A}}_{p(\vartheta')}(H_0)$, contradicting the fact that $p(\vartheta)\mathbf{T}^{\mathbb{A}}_{p(\vartheta)}(H_0)$ is non-decreasing in $p(.)$, since $p(.)$ is itself increasing. This establishes the lower bound on $TC_c(\vartheta)$.

To establish the upper bound on TC_c, consider the unique S such that $\vartheta_c(S-1) \leq \underline{\vartheta}(\bar{\vartheta}) \leq \vartheta_c(S)$. Since $p(\vartheta(S)) = cS/W$, it follows that $p(\underline{\vartheta}(\bar{\vartheta})) \leq p(\vartheta_c(S)) \leq p(\underline{\vartheta}(\bar{\vartheta})) + c/W$. Therefore the ex ante pay-off to the sender in the equilibrium induced by $\mathbb{A} = \{\mathbf{1}_S\}$ is bounded below by:

$$\Pi^{\mathbf{S}}_S = Pr^{\mathbf{S}}_c(a)W - TC^{\mathbf{S}}_c \geq$$
$$(p_{BP} - Pr(p(\underline{\vartheta}(\bar{\vartheta})) \leq p(\vartheta) \leq p(\underline{\vartheta}(\bar{\vartheta})) + \frac{c}{W}))W$$
$$-Pr(p(\vartheta) \geq p(\underline{\vartheta}(\bar{\vartheta})) + \frac{c}{W})E[\frac{p(\underline{\vartheta}(\bar{\vartheta})) + \frac{c}{W}}{p(\underline{\vartheta}(\bar{\vartheta}))}|p(\vartheta) \geq p(\underline{\vartheta}(\bar{\vartheta})) + \frac{c}{W}]W$$

In the sender preferred-equilibrium, we have for the pay-off to the sender:

$$\Pi^{SP}_S = Pr^{SP}_c(a)W - TC_c \leq p_{BP}W - TC_C$$

From the fact that $\Pi^{SP}_S \geq \Pi^{\mathbf{S}}_S$, I then obtain that:

$$TC_c \leq Pr(p(\vartheta) \geq p(\underline{\vartheta}(\bar{\vartheta})) + \frac{c}{W})E[\frac{p(\underline{\vartheta}(\bar{\vartheta})) + \frac{c}{W}}{p(\underline{\vartheta}(\bar{\vartheta}))}|p(\vartheta) \geq p(\underline{\vartheta}(\bar{\vartheta})) + \frac{c}{W}]W +$$
$$Pr(p(\underline{\vartheta}(\bar{\vartheta})) \leq p(\vartheta) \leq p(\underline{\vartheta}(\bar{\vartheta})) + \frac{c}{W})W \rightarrow$$
$$p_{BP}E[\frac{p(\underline{\vartheta}(\bar{\vartheta}))}{p(\vartheta)}|\vartheta \geq \underline{\vartheta}(\bar{\vartheta})]W \qquad \text{as } c \rightarrow 0$$

The claim for $c \in \{= p(\underline{\vartheta}(\bar{\vartheta}))W/n : n \in \mathbb{N}\}$ is immediate from the fact that in that case $\vartheta_c(n)$ induces those outcomes.

The claim for the limit is immediate.

Finally, I return to the promised technical lemma:

Lemma 4. *For any $\mathbb{A} \subseteq \mathbb{H}$ and any $H \in \mathbb{H}$, $p\mathbf{T}_p^{\mathbb{A}}(H)$ is non-decreasing in p.*

Proof. If $\mathbb{A} = \emptyset$, $\mathbf{T}_p^{\mathbb{A}}(H)$ is infinite and the statement is trivially true. Assume therefore that $\mathbb{A} \neq \emptyset$.

Now observe that $\mathbf{T}_p^{\mathbb{A}}(H^\frown(1)) - \mathbf{T}_p^{\mathbb{A}}(H^\frown(0)) \geq -1/p$. Indeed, $\mathbf{T}_p^{\mathbb{A}}(H) \geq \mathbf{T}_p^{\mathbb{A}}(H^\frown(0))$ and $\mathbf{T}_p^{\mathbb{A}}(H^\frown(1)) - \mathbf{T}_p^{\mathbb{A}}(H)$ is minimal when at least one more success is required after obtaining H. Then $\mathbf{T}_p^{\mathbb{A}}(H^\frown(1)) - \mathbf{T}_p^{\mathbb{A}}(H) = -\mathbf{T}_p^{\{(1)\}} = -1/p$.

The argument now proceeds by induction. Since the induction over $\mathbb{H}$ is non-standard, I lay out and prove explicitly the logic of the induction. For $H, I \in \mathbb{H}$, denote

$$I - H := \min_{\{J \in \mathbb{H}:\, \exists H' \in \mathbb{H} \text{ s.t. } H'|H \text{ and } H'^\frown J|I\}} l(J).$$

$I - H$ is the length of the shortest path from H that permits the report I. Observe that if $I|H$, $I - H = 0$.

I write $H \leq_{\mathbb{A}} H'$ if $H \in \mathbb{A}$ or $\forall A \in \mathbb{A}$, $A - H \leq A - H'$. If $H \leq_{\mathbb{A}} H'$ and $H' \not\leq_{\mathbb{A}} H$, I write $H <_{\mathbb{A}} H'$. I say H is *critical* if either $H \leq_{\mathbb{A}} H^\frown(0)$ or $H \leq_{\mathbb{A}} H^\frown(1)$. If $H \notin \mathbb{A}$ and H is critical, exactly one of $H^\frown(0) <_{\mathbb{A}} H$ or $H^\frown(1) <_{\mathbb{A}} H$ holds, since $\mathbb{A} \neq \emptyset$. For such H, denote by $H_{+1} \in \{H^\frown(0), H^\frown(1)\}$ such that $H_{+1} <_{\mathbb{A}} H$.

The induction now works as follows. Denote by $Q(H)$ any property of H.

Lemma 5. *If*

1. *If $H \in \mathbb{A}$, then $Q(H)$ holds.*
2. *If $Q(H^\frown(0))$ and $Q(H^\frown(1))$ hold, then $Q(H)$ holds.*
3. *If H is critical and $Q(H_{+1})$ holds, then $Q(H)$ holds.*

Then $P(H)$ holds $\forall H \in \mathbb{H}$.

Proof. Suppose $\exists H \in \mathbb{H}$ such that $\neg Q(H)$. Then $H \notin \mathbb{A}$. If H is not critical, there exists a successor history $H' \in \{H^\frown(0), H^\frown(1)\}$ such that $\neg Q(H')$. Since H is not critical, $H' <_{\mathbb{A}} H$. If H is is critical, then $\neg Q(H_{+1})$ and $H_{+1} <_{\mathbb{A}} H$. In this way, I construct a sequence of histories H_i with $\neg Q(H_i)$ and $H_i < H_{i+1}$ $\forall i \in \mathbb{N}$. But this sequence has $H_i \in \mathbb{A}$ for i sufficiently large. This is a contradiction and the lemma follows. □

I now apply lemma 5 to lemma 4, with $P(H) =$ "$\mathbf{T}_p^{\mathbb{A}}(H)$ is non-decreasing in p".

1. Since $\mathbf{T}_p^{\mathbb{A}}(H) = 0$ if $H \in \mathbb{H}$, this is clear.

2. Assume that $p\mathbf{T}_p^{\mathbb{A}}(H)$ is non-decreasing in p $\forall H$ s.t. $H'|H$ and $H \neq H'$. Observe that $\mathbf{T}_p^{\mathbb{A}}(H) = p\mathbf{T}_p^{\mathbb{A}}(H^\frown(1)) + (1-p)\mathbf{T}_p^{\mathbb{A}}(H^\frown(0)) + 1$. By an application of lemma 5 that is completely analogous to the current one, it is easy to prove that $\mathbf{T}_p^{\mathbb{A}}(H)$ is differentiable in p. Therefore:

$$\frac{d}{dp}p\mathbf{T}_p^{\mathbb{A}}(H) = p\frac{d}{dp}(p\mathbf{T}_p^{\mathbb{A}}(H^\frown(1))) + (1-p)\frac{d}{dp}(p\mathbf{T}_p^{\mathbb{A}}(H^\frown(0))) + p(\mathbf{T}_p^{\mathbb{A}}(H^\frown(1)) - \mathbf{T}_p^{\mathbb{A}}(H^\frown(0))) + 1 \geq$$
$$p\frac{d}{dp}(p\mathbf{T}_p^{\mathbb{A}}(H^\frown(1))) + (1-p)\frac{d}{dp}(p\mathbf{T}_p^{\mathbb{A}}(H^\frown(0))) \geq 0.$$

Here the first inequality follows from the fact that $\mathbf{T}_p^{\mathbb{A}}(H^\frown(1)) - \mathbf{T}_p^{\mathbb{A}}(H^\frown(0)) \geq -1/p$ and the second follows from the (differentiable version of) the induction hypothesis.

3. Assume that H is critical and $p\mathbf{T}_p^{\mathbb{A}}(H_{+1})$ is non-decreasing in p. Suppose first that $H_{+1} = H^\frown(1)$. Then $\mathbf{T}_p^{\mathbb{A}}(H) = \mathbf{T}_p^{\mathbb{A}}(H^\frown(0))$ and $\mathbf{T}_p^{\mathbb{A}}(H) = p\mathbf{T}_p^{\mathbb{A}}(H^\frown(1)) + (1-p)\mathbf{T}_p^{\mathbb{A}}(H^\frown(0)) + 1$ implies $\mathbf{T}_p^{\mathbb{A}}(H) = \mathbf{T}_p^{\mathbb{A}}(H^\frown(1)) + 1/p$. So $p\mathbf{T}_p^{\mathbb{A}}(H)$ is non-decreasing in p. Suppose instead that $H_{+1} = H^\frown(0)$. Then $\mathbf{T}_p^{\mathbb{A}}(H) = \mathbf{T}_p^{\mathbb{A}}(H^\frown(1))$ and $\mathbf{T}_p^{\mathbb{A}}(H) = \mathbf{T}_p^{\mathbb{A}}(H^\frown(0)) + 1/(1-p)$. Since $p/(1-p)$ is strictly increasing for $p \in [0, 1)$, once again $p\mathbf{T}_p^{\mathbb{A}}(H)$ is non-decreasing in p.

This completes the proof of lemma 4 and hence of proposition 2. □

□

Remark. Notice that the lower bound on the cost of testing is in terms of $TC_c(\vartheta)$, whereas the upper bound is in terms if TC_c. Unfortunately, no upper bound can be given on $TC_c(\vartheta)$ beyond $TC_c(\vartheta) \leq W$, which is trivial.

Similarly no lower bound exists on TC_c beyond $TC_c \geq 0$, which is trivial. Indeed, consider the case where $c > W$, then the only equilibrium is the pooling equilibrium with no testing and hence $TC_c = 0$.

The final result in the section is an almost immediate corollary of the above proposition. However, it is of independent importance, because it will shed light on the comparison between the cases of "suppression" versus "no suppression". The intuitive content of it is the following. Any acceptance rule works because it "kicks out" enough bad types to satisfy the receiver and is sender-preferred when it "kicks out" as few as possible. When $\bar{\vartheta}$ is close to $E[\vartheta]$, few types need to be kicked out to satisfy the receiver. The cost of testing is proportional to the highest type being kicked out. So, when $\bar{\vartheta}$ is close to $E[\vartheta]$, the cost of testing also becomes low. Similarly, when $\bar{\vartheta}$ is high, high types need to be kicked out and the cost of testing becomes high accordingly.

Recall that $\vartheta_{min} := \min Supp(F)$ and $\vartheta_{max} := \max Supp(F)$.

Corollary 1. ***The Effect of*** $\bar{\vartheta}$

Consider any sequence $(\bar{\vartheta}_i, c_i)$ *s.t.* $\bar{\vartheta}_i > E[\vartheta]\,\forall i$, $\bar{\vartheta}_i \to E[\vartheta]$, $c_i > 0\,\forall i$ *and* $c_i \to 0$, *then, in the sender-preferred equilibrium,* $Pr_c(a) \to 1$ *and* $TC_c(\vartheta) \to Wp(\vartheta_{min})/p(\vartheta)$.
Similarly, consider any sequence $(\bar{\vartheta}_i, c_i)$ *s.t.* $\bar{\vartheta}_i < \vartheta_{max}\,\forall i$, $\bar{\vartheta}_i \to \vartheta_{max}$, $c_i > 0\,\forall i$ *and* $c_i \to 0$, *then, in the sender-preferred equilibrium,* $Pr(a)_c \to 0$ *and* $TC_c(\vartheta) \to W$.

Proof. First, I show that as $\bar{\vartheta}_i \to E[\vartheta]$, $\inf\{q : E[\vartheta|\vartheta \geq q] \geq \bar{\vartheta}_i\} =: \underline{\vartheta}_i \to \vartheta_{min}$.

Since $Pr(\vartheta = \vartheta_{min}) = 0$, then it must be that there exists $\epsilon > 0$ such that $(\vartheta_{min}, \vartheta_{min} + \epsilon) \subseteq Supp(F)$, while $Pr(\vartheta = \vartheta') = 0\,\forall \vartheta' \in (\vartheta_{min}, \vartheta_{min} + \epsilon)$. This implies that f is continuous and strictly increasing on $[0, \frac{\epsilon}{2}]$. Furthermore, $f(\vartheta_{min}) = E[\vartheta]$. Now, if $f(\vartheta_{min} + \frac{\epsilon}{2}) < \bar{\vartheta}_i$, let $q_i = \vartheta_{min} + \frac{\epsilon}{2}$. If instead $f(\vartheta_{min} + \frac{\epsilon}{2}) \geq \bar{\vartheta}_i$, by the continuity of f and the intermediate value theorem, there exists a q such that $f(q) = \frac{E[\vartheta] + \bar{\vartheta}_i}{2} < \bar{\vartheta}_i$. Let $q_i = q$. For $\bar{\vartheta}_i$ sufficiently close to $E[\vartheta]$, the latter case obtains, so $q_i \to \vartheta_{min}$ as $\bar{\vartheta}_i \to E[\vartheta]$. Since $\vartheta_{min} \leq \underline{\vartheta}_i \leq q_i$, then also $\underline{\vartheta}_i \to \vartheta_{min}$ as $\bar{\vartheta}_i \to E[\vartheta]$.

Next, I show that as $\bar{\vartheta}_i \to \vartheta_{max}$, $\inf\{q : E[\vartheta|\vartheta \geq q] \geq \bar{\vartheta}_i\} =: \underline{\vartheta}_i \to \vartheta_{max}$.

There exists $\epsilon > 0$ such that $(\vartheta_{max} - \epsilon, \vartheta_{max}) \subseteq Supp(F)$. Then f is strictly increasing and continuous on $[\vartheta_{max} - \frac{\epsilon}{2}, \vartheta_{max}]$. Now, if $f(\vartheta_{max} - \frac{\epsilon}{2}) \geq \bar{\vartheta}_i$, let $q_i = -\infty$. If instead $f(\vartheta_{max} - \frac{\epsilon}{2}) < \bar{\vartheta}_i$, then by the continuity of f and the intermediate value theorem, there exists a q such that $f(\vartheta_{max} - \frac{\epsilon}{2}) < f(q) = \bar{\vartheta}_i - \frac{|\bar{\vartheta}_i - \vartheta_{max}|}{|\vartheta_{max} - f(\vartheta_{max} - \frac{\epsilon}{2})|}|\bar{\vartheta}_i - f(\vartheta_{max} - \frac{\epsilon}{2})| < \bar{\vartheta}_i$. Let $q_i = q$. For $\bar{\vartheta}_i$ sufficiently close to ϑ_{max}, the second case obtains, so $q_i \to \vartheta_{max}$ as $\bar{\vartheta}_i \to \vartheta_{max}$. Since $q_i \leq \underline{\vartheta}_i \leq \vartheta_{max}$, also $\underline{\vartheta}_i \to \vartheta_{max}$ as $\bar{\vartheta}_i \to \vartheta_{max}$.

The results are now immediate from proposition 2. □

Example. Suppose $\vartheta \sim U[\vartheta_{min}, \vartheta_{max}]$, with $0 \leq \vartheta_{min} < \vartheta_{max} \leq 1$ and $p(\vartheta) = \vartheta$. Then $\underline{\vartheta}(\bar{\vartheta}) = 2\bar{\vartheta} - \vartheta_{max}$ and $\Pi_S(\vartheta) \approx (\vartheta - \underline{\vartheta}(\bar{\vartheta}))W/\vartheta$ if $\vartheta \geq \underline{\vartheta}(\bar{\vartheta})$ and 0 otherwise. We see that as $\bar{\vartheta} \to \vartheta_{max}$, $\underline{\vartheta}(\bar{\vartheta}) \to \vartheta_{max}$ and no one is accepted, while if $\bar{\vartheta} \to E[\vartheta]$, $\underline{\vartheta}(\bar{\vartheta}) \to \vartheta_{min}$ and everyone will be accepted. If furthermore $\vartheta_{min} = 0$, the cost of testing vanishes and $\Pi_S(\vartheta) \to W$. I plot the sender utility in figure 3.

6 No Suppression

I will now consider the game of communication without suppression, i.e. the sender generates evidence from the process X_n at a cost c each time and submits the report H_τ, where τ is the stopping time. I will write S_H and F_H for the number of successes and failures associated with a history H.

In order to gain some first understanding of this environment, I eliminate the strategic effect of reports by investigating the case where testing is costless. Then I proceed to show in the next section how this case bears on the one where testing has a positive cost, but c is vanishing.

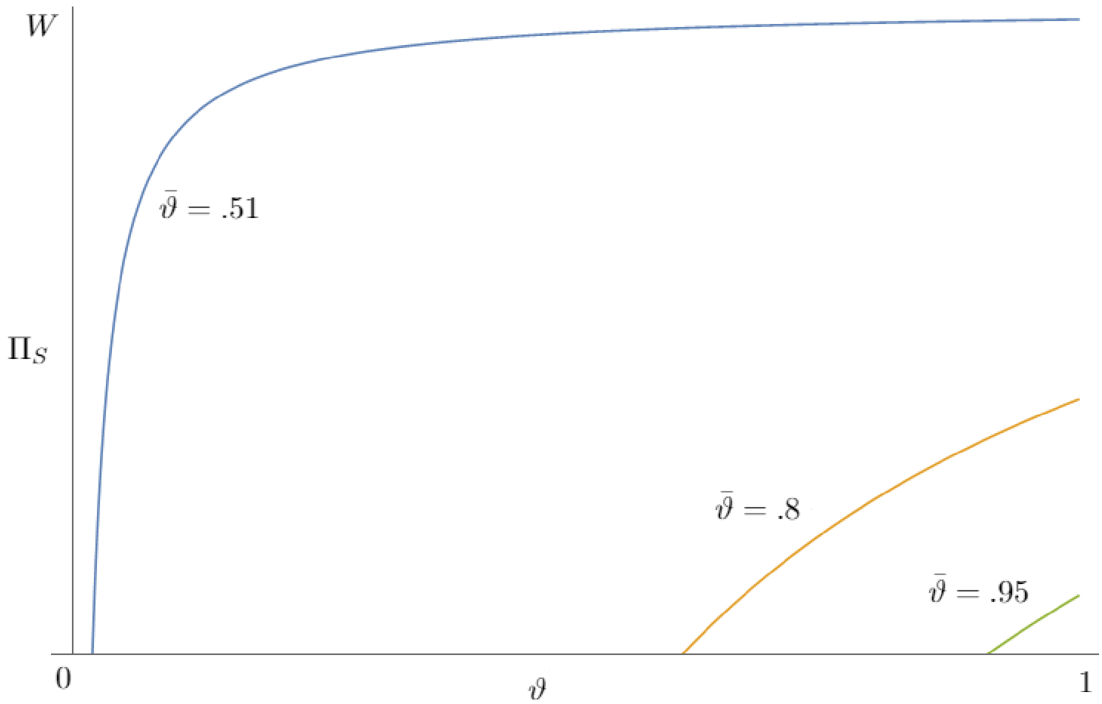


Figure 3: The sender pay-offs when $\vartheta \sim U[0, 1]$ for various values of $\bar{\vartheta}$.

6.1 Costless testing

I start by considering the case where $c = 0$. In that case it is clear that it is clear that the sender, irrespective of ϑ, will keep testing until he is accepted. After all, there is no cost and a potential gain. Since there will be no on path reports leading to rejection, the strategic effect of a report disappears.

The main result of this section will be that, when testing is costless and evidence cannot be suppressed, $Pr(a|\vartheta) = 1$ if $\vartheta \geq \bar{\vartheta}$, while $0 < Pr(a|\vartheta) < 1$ if $\vartheta < \bar{\vartheta}$. The reason for this is as follows. As testing goes on long enough, eventually, by the law of large numbers, the test will reveal with probability 1 the true value of ϑ. This will lead to acceptance if and only if $\vartheta \geq \bar{\vartheta}$. $0 < \vartheta < \bar{\vartheta}$ can only get accepted by getting lucky and looking good in the early testing. Somewhat more formally, as evidence becomes abundant, meaning $l(H)$ becomes large, where $l(H)$ denotes the number of tests in H, the conditional expectation will depend less and less on the ex ante distribution and more and more on the evidence itself. In particular, as $l(H) \to \infty$, typically $E[\vartheta|H] \to p^{-1}(S_H/(S_H + F_H))$. Simply put, if the receiver sees a large sample with, say, 60% successes, then he will expect that $p(\vartheta) = .6$, so that ϑ can be deduced through $p(.)$. Now, if testing goes on long enough, eventually the proportion of successes in the sample will be roughly $p(\vartheta)$. Therefore, if $\vartheta \geq \bar{\vartheta}$, eventually acceptance will occur, while if $0 < \vartheta < \bar{\vartheta}$, acceptance can only occur if the sender "gets lucky" early in testing. This can happen (since any report generated by any ϑ can also be generated by another $0 < \vartheta < 1$ with positive probability), but it cannot happen with probability 1.

Recall that $E_{stat}[\vartheta|H]$ is defined as the expectation of ϑ conditional on the report H with respect to the stopping time $\tau_{stat}(H) := \inf\{n : H_n$ such that H is a feasible report$\}$, independent of ϑ and $R = H$. Notice that the stopping time $\tau_H = l(H)$ together with the reporting rule $R_H = H_{l(H)}$ produces the same expectation $E_{(\tau_H,R_H)}[\vartheta|H] = E_{stat}[\vartheta|H]$. What is required to compute the statistical expectation of a report, is simply that any ϑ keeps testing as long as they can still generate that report (and are unrestricted when they can no longer obtain the report). Then I find

$$E_{stat}[\vartheta|H] := E_{F_{(\tau_{stat}(H),R_{stat}(H))}}[\vartheta|H] = \frac{\int_{\vartheta_{min}}^{\vartheta_{max}} \vartheta' p(\vartheta')^{S_H}(1-p(\vartheta'))^{F_H} dF(\vartheta')}{\int_{\vartheta_{min}}^{\vartheta_{max}} p(\vartheta')^{S_H}(1-p(\vartheta'))^{F_H} dF(\vartheta')}$$

In comparison, the equilibrium expectation is:

$$E^*[\vartheta|H] := E_{F^*}[\vartheta|H] = \frac{\int_{\vartheta_{min}}^{\vartheta_{max}} \mathbb{1}_{\{Pr(\tau \geq l(H)|\vartheta,H)=1\}} \vartheta' p(\vartheta')^{S_H}(1-p(\vartheta'))^{F_H} dF(\vartheta')}{\int_{\vartheta_{min}}^{\vartheta_{max}} \mathbb{1}_{\{Pr(\tau \geq l(H)|\vartheta,H)=1\}} p(\vartheta')^{S_H}(1-p(\vartheta'))^{F_H} dF(\vartheta')}$$

Here $\mathbb{1}_{\{Pr(\tau \geq l(H)|\vartheta,H)=1\}}$ is the event that ϑ keeps testing along the path of H. Since the cost of testing is zero and the expected benefit is always positive, any ϑ will always keep testing and we have that for any $H \in \mathbb{H}$, $E_{stat}[\vartheta|H] = E^*[\vartheta|H]$.

One philosophical issue arises in the case of costless testing. In equilibrium, for some ϑ the optimal τ, while a Markov time, will not be a stopping time. Recall that the distinction is that a stopping time requires also that $Pr(\tau < \infty) = 1$. Clearly, it cannot be that all ϑ test until accepted and that for all ϑ, $Pr(\tau_\vartheta < \infty) = 1$, because it cannot be that all are accepted in equilibrium. Formally speaking, this is not much of a problem, although I would need to specify pay-offs in the case of infinite testing (naturally, they are 0). However, economically speaking, it is not entirely clear what infinite testing would mean. Rather than finding a direct economic interpretation, I will take a different approach. I will show below that, in a sense that I will make precise, sender-preferred equilibrium is continuous in c at $c = 0$ for the case of "no suppression" (in clear contrast to the case of "suppression"). For small but positive costs the issue of infinite testing does not arise. The costless testing equilibrium is then best understood as describing the limit behaviour of equilibria with a vanishing cost of testing.

Proposition 3. ***Sender-preferred Equilibrium with Costless Testing***
If $c = 0$, in the sender-preferred equilibrium $Pr(a|\vartheta) = 1$ if $\vartheta \geq \bar{\vartheta}$, while $0 < Pr(a|\vartheta) < 1$ if $\vartheta < \bar{\vartheta}$. The sender-preferred equilibrium is outcome equivalent to one where acceptance occurs if and only if $E_{stat}[\vartheta|H] \geq \bar{\vartheta}$. For any report H on the equilibrium path, $\gamma_{stat}(H) = 1$.

In light of the proposition, I will denote by $p_0(\vartheta) := Pr(a|\vartheta)$ the conditional probability of acceptance in the costless testing equilibrium, by $p_0 := Pr(a)$ the unconditional probability of acceptance and by $\mathbb{A}_0$ the costless equilibrium acceptance rule, i.e. $\mathbb{A}_0 := \{H \in \mathbb{H} : E_{stat}[\vartheta|H] \geq \bar{\vartheta}\}$. I will write $\tau_0 := \min\{n : H_n \in \mathbb{A}_0\}$ for the stopping rule in the costless equilibrium.

I phrase proposition 3 in terms of outcome equivalence. The reason for this is technical. The acceptance rule $\mathbb{A}_0$ contains many reports which are off path. If any of these reports were cut from the acceptance rule, nothing would change in the outcomes, but formally, this constitutes a different equilibrium.

Proof. It is clear that no matter what the acceptance rule $\mathbb{A}$ is, $\tau = \min\{n : H_n \in \mathbb{A}\}$. The only possible exception is $\vartheta = 0$ and there it does not matter what we specify for τ. From the remarks above it is also clear that $E_{stat}[\vartheta|H] = E[\vartheta|H]$. Since there is no cost, sender utility is maximized when the probability of acceptance is maximized. Clearly this occurs when $\mathbb{A} = \{H \in \mathbb{H} : E_{stat}[\vartheta|H] \geq \bar{\vartheta}\}$. [6] It remains to show that the probabilities of acceptance are as stated. I proceed in two steps. I start with a technical lemma.

Lemma 6. *As $n \to \infty$, $E_{stat}[\vartheta|H_n] \to p^{-1}(S_H/(S_H + F_H))$ if $p^{-1}((S_H/(S_H + F_H)) \in Supp(F)$, $E_{stat}[\vartheta|H_n] \to \vartheta_{min}$ if $p^{-1}((S_H/(S_H + F_H)) \leq \vartheta_{min}$ and $E_{stat}[\vartheta|H_n] \to \vartheta_{max}$ if $p^{-1}((S_H/(S_H + F_H)) \geq \vartheta_{max}$.*[7]

Proof. Since the set of discrete distributions on $[0, 1]$ is dense is the set of distributions on $[0, 1]$, it suffices to show the claim for discrete distributions. Let $Supp := \{\vartheta_i\}_{i\in\mathbb{N}}$ be the support of the discrete distribution and let p_i the corresponding probabilities. Then $E[\vartheta|(nS, nF)] = \sum_{Supp} \vartheta_i Pr(\vartheta = \vartheta_i|H_n)$. It suffices to show that all but the required conditional probabilities go to 0.

$$Pr(\vartheta = \vartheta_i|H_n) = \frac{p_i p(\vartheta_i)^{S_H}(1 - p(\vartheta_i))^{F_H}}{\sum\limits_{Supp} p_j p(\vartheta_j)^{S_H}(1 - p(\vartheta_j))^{F_H}}$$

Writing $q := S/(S+F)$, so that $F = S(1-q)/q$, this becomes (letting $n \to \infty$ for fixed q):

$$\frac{p_i p(\vartheta_i)^n (1 - p(\vartheta_i))^{n\frac{1-q}{q}}}{\sum\limits_{Supp} p_j p(\vartheta_j)^n (1 - p(\vartheta_j))^{n\frac{1-q}{q}}} = \frac{1}{1 + \sum\limits_{Supp\setminus\{\vartheta_i\}} \frac{p_j}{p_i}\left(\frac{p(\vartheta_j)(1-p(\vartheta_j))^{\frac{1-q}{q}}}{p(\vartheta_i)(1-p(\vartheta_i))^{\frac{1-q}{q}}}\right)^n}$$

Let us examine the properties of the function $p(1-p)^{(1-q)/q}$.

[6] Notice that it also occurs for, for example, $\tilde{\mathbb{A}} := \mathbb{A}/\{H_{n+1}\}$, where both $H_{n+1} \in \mathbb{A}$ and $H_n \in \mathbb{A}$. This is simply because the report H_{n+1} is then off the equilibrium path. Such acceptance rules clearly induce the same outcomes.

[7] Notice that these cases are not exhaustive, i.e. there may exist $\vartheta_{min} < \vartheta < \vartheta_{max}$ such that $\vartheta \notin Supp(\mathbb{P})$. However, the cases are sufficient for my needs.

$$\frac{d}{dp}p(1-p)^{\frac{1-q}{q}} = (1-p)^{\frac{1-q}{q}} - \frac{1-q}{q}p(1-p)^{\frac{1-2q}{q}} =$$
$$(1-p-\frac{1-q}{q}p)(1-p)^{\frac{1-2q}{q}} = \frac{q-p}{q}(1-p)^{\frac{1-2q}{q}}$$

Clearly the derivative is positive if $p < q$ and negative if $p > q$, so the function obtains its unique maximum at $q = p$. Furthermore, if $p_1 < p_2 < q < p_3 < p_4$, then $p_1(1-p_1)^{(1-q)/q} < p_2(1-p_2)^{(1-q)/q}$ and $p_4(1-p_4)^{(1-q)/q} < p_3(1-p_3)^{(1-q)/q}$. Returning our attention to the sum

$$\sum_{Supp\setminus\{\vartheta_i\}} \frac{p_j}{p_i}\left(\frac{p(\vartheta_j)(1-p(\vartheta_j))^{\frac{1-q}{q}}}{p(\vartheta_i)(1-p(\vartheta_i))^{\frac{1-q}{q}}}\right)^n,$$

it is clear that if $p(\vartheta_i) = q$, or $p(\vartheta_j) < p(\vartheta_i) < q \forall j$ or $q < p(\vartheta_i) < p(\vartheta_j) \forall j$, each element of the sum converges to 0 and the conditional probability goes to 1. This proves the lemma. □

Finally, I show that $Pr(a|\vartheta) > 0$ if $p(\vartheta) > 0$ and $Pr(a|\vartheta) = 1$ if and only if $\vartheta \geq \bar{\vartheta}$. It is clear that $\mathbb{A}$ is non-empty, so that $Pr(a|\vartheta) > 0$ for any $p(\vartheta) > 0$, since any report H can be generated with positive probability by any $p(\vartheta) > 0$. However, no report other than H_0 can be generated with certainty by any $0 < p(\vartheta) < 1$. However, it is also clear that for any finite n and $p(\vartheta) < 1$, the entire path $(H_i)_{i\in\{1,\dots,n\}}$ may have $E_{stat}[\vartheta|H_i] \leq \bar{\vartheta}$ with positive probability. Now observe that if $\vartheta \geq \bar{\vartheta}$, by the strong law of large numbers

$$Pr(\lim_{n\to\infty} H_n \in \mathbb{A}|\vartheta) = Pr(\lim_{n\to\infty} \frac{S_{H_n}}{S_{H_n}+F_{H_n}} \geq p(\bar{\vartheta})) = Pr(\lim_{n\to\infty} \frac{\sum_{i=1}^n X_n}{n} \geq p(\bar{\vartheta}))$$
$$\geq Pr(\lim_{n\to\infty} \frac{\sum_{i=1}^n X_n}{n} \geq p(\vartheta)) = E[X_1|\vartheta]) = 1,$$

while similarly for $\vartheta < \bar{\vartheta}$,

$$Pr(\lim_{n\to\infty} H_n \in \mathbb{A}|\vartheta) = Pr(\lim_{n\to\infty} \frac{S_{H_n}}{S_{H_n}+F_{H_n}} \geq p(\bar{\vartheta}))$$
$$\leq 1 - Pr(\lim_{n\to\infty} \frac{\sum_{i=1}^n X_n}{n} \leq p(\vartheta)) = E[X_1|\vartheta]) = 0.$$

□

Remark. Observe that although $\mathbb{A}_0 = \{H \in \mathbb{H} : E_{stat}[\vartheta|H] \geq \bar{\vartheta}\}$, in the case of no suppression, in contradistinction to the case of suppression, the receiver isn't exactly indifferent between accepting and rejecting for on path reports, even as c vanishes. For example, it may be that a report with two successes and no failures is not enough make the receiver accept, but three failures and no successes is more than enough. This is a kind of integer problem, as in the case of suppression, but, unlike in the case of suppression, this problem does not disappear as c vanishes. Therefore the receiver generically strictly prefers acceptance to rejection in the case of no suppression. It is an integer problem in the sense that this result would go away if it were possible to submit obtain half a success. I return to this point below, when I compare the two cases.

From proposition 3 the following result is now immediate. The probability of acceptance is lower under suppression than under no suppression. The intuition for the result is the following. In the Bayesian Persuasion game the lowest types are eliminated until the receiver is just willing to accept. In the present case even very low types have a positive probability of acceptance. Thus they dilute the pool of the accepted. Therefore the probability of acceptance must be lower. This effect is further strengthened by the fact that the receiver strictly prefers to accept.

Corollary 2. ***Acceptance under no Suppression***
$p_0 < p_{BP}$.

Proof. This is immediate from proposition 0. □

Example. Suppose the distribution of ϑ is uniform on $[0,1]$ and $p(\vartheta) = \vartheta$. As in the general case, it is immediate that the sender will not stop testing until accepted, $\tau = \min\{n : H_n \in \mathbb{A}\}$. This allows me to calculate the beliefs of the receiver for any report. Dropping the subscripts on S_H and F_H:

$$E[\vartheta|R] = \int_0^1 \frac{\vartheta^{S+1}(1-\vartheta)^F}{\int_0^1 \vartheta^S(1-\vartheta)^F d\vartheta} d\vartheta = \frac{\beta(S+2,F+1)}{\beta(S+1,F+1)} = \frac{\dfrac{(S+1)!F!}{(S+F+2)!}}{\dfrac{S!F!}{(S+F+1)!}} = \frac{S+1}{S+F+2}$$

Here $\beta(x,y)$ is the standard Beta function. The equilibrium will therefore have

$$\mathbb{A}_0 = \{H : \frac{S+1}{S+F+2} \geq \bar{\vartheta}\}$$

. I now calculate $Pr(a|\vartheta)$.

Since I want to be explicit about the length of history H, I write H_n and S_n and F_n for the number of successes and failures respectively obtained in the first n tests. I let $D_n := S_n - F_n$. Then $H_n \in \mathbb{A} \Leftrightarrow D_n \geq (2\bar{\vartheta} - 1)n + 4\bar{\vartheta}$. Notice that D_n is a random walk with drift $2\vartheta - 1$, I therefore normalize the drift away by defining $\tilde{D}_n = D_n - (2\vartheta - 1)n$, so that $H_n \in \mathbb{A} \Leftrightarrow \tilde{D}_n \geq 2(\bar{\vartheta} - \vartheta)n + 4\bar{\vartheta}$. It is well known that any random walk crosses any bound with probability 1 and therefore, a fortiori, $Pr(a|\vartheta) = Pr(\tilde{D}_n \geq 2(\bar{\vartheta} - \vartheta)n + 4\bar{\vartheta}) = Pr(\tau_\vartheta < \infty) = 1$ if $\vartheta \geq \bar{\vartheta}$. It remains to compute the corresponding probability $Pr(a|\vartheta)$ for $\vartheta < \bar{\vartheta}$. I approximate this probability in the following way.

Let $q > 1$ solve $f(q) := \vartheta q^{2(1-\bar{\vartheta})} + (1-\vartheta)q^{-2\bar{\vartheta}} = 1$. To see that for $\vartheta < \bar{\vartheta}$ such a $q > 1$ exists and is unique, observe that in that case $f(1) = 1, f'(1) < 0, f'' > 0, \lim_{q\to\infty} f(q) = \infty$. The strict convexity of f shows that there can be at most 2 solutions to $f(q) = 1$, the other facts and the intermediate value theorem show that there exist exactly 2 solutions, one greater than 1. Then, denoting by $x \wedge y := \min\{x, y\}$, define $M_n := q^{D_{n\wedge\tau_\vartheta} - (2\bar{\vartheta}-1)(n\wedge\tau_\vartheta) - 4\bar{\vartheta}}$. Observe that M_n is a martingale, indeed:

$$E[M_{n+1}|M_n] = E[q^{D_{(n+1)\wedge\tau_\vartheta} - (2\bar{\vartheta}-1)((n+1)\wedge\tau_\vartheta) - 4\bar{\vartheta}}|q^{D_{n\wedge\tau_\vartheta} - (2\bar{\vartheta}-1)(n\wedge\tau_\vartheta) - 4\bar{\vartheta}}] =$$
$$\mathbb{1}_{\tau_\vartheta > n}(\vartheta q^{2(1-\bar{\vartheta})} + (1-\vartheta)q^{-2\bar{\vartheta}})M_n + \mathbb{1}_{\tau_\vartheta \le n} M_n = M_n$$

By the law of large numbers, $D_n/n \to 2\vartheta - 1$ a.s. as $n \to \infty$, so that $D_n - (2\bar{\vartheta}-1)n - 4\bar{\vartheta} \to -\infty$ a.s. as $n \to \infty$ if and only if $\vartheta < \bar{\vartheta}$. Furthermore, $D_{\tau_\vartheta} - (2\bar{\vartheta}-1)\tau_\vartheta - 4\bar{\vartheta} \ge 0$. Hence, by dominated convergence:

$$q^{-4\bar{\vartheta}} = E[M_0] =$$
$$\lim_{n\to\infty} E[M_n] = Pr(\tau_\vartheta = \infty)E[\lim_{n\to\infty} q^{D_n - (2\bar{\vartheta}-1)n - 4\bar{\vartheta}}] + Pr(\tau_\vartheta < \infty)E[q^{D_{\tau_\vartheta} - (2\bar{\vartheta}-1)\tau_\vartheta - 4\bar{\vartheta}}] =$$
$$Pr(\tau_\vartheta < \infty)E[q^{D_{\tau_\vartheta} - (2\bar{\vartheta}-1)\tau_\vartheta - 4\bar{\vartheta}}] \ge Pr(\tau_\vartheta < \infty)$$

To obtain a lower bound on $Pr(\tau_\vartheta < \infty)$, observe that the last test before the stopping time always yields a success, since otherwise one could stop also before the last test. Hence $D_{\tau_\vartheta} - 1 - (2\bar{\vartheta}-1)(\tau_\vartheta - 1) - 4\bar{\vartheta} = D_{\tau_\vartheta} - (2\bar{\vartheta}-1)\tau_\vartheta - 2 - 2\bar{\vartheta} < 0$. Therefore, defining $L_n := q^{D_{n\wedge\tau_\vartheta} - (2\bar{\vartheta}-1)(n\wedge\tau_\vartheta) - 2 - 2\bar{\vartheta}}$, by an analogous argument, I find:

$$q^{-2-2\bar{\vartheta}} = E[L_0] = \lim_{n\to\infty} E[L_n] < Pr(\tau_\vartheta < \infty)$$

Hence I find for $\vartheta < \bar{\vartheta}$, $q^{-2-2\bar{\vartheta}} \le Pr(\tau_\vartheta < \infty) = Pr(a|\vartheta) \le q^{-4\bar{\vartheta}}$, or $z \le Pr(a|\vartheta) \le z'$, where $z < 1$ solves $\vartheta z^{\frac{-2(1-\bar{\vartheta})}{2\bar{\vartheta}+2}} + (1-\vartheta)z^{\frac{2\bar{\vartheta}}{2\bar{\vartheta}+2}} = 1$, and $z' < 1$ solves $\vartheta z^{\frac{-2(1-\bar{\vartheta})}{4\bar{\vartheta}}} + (1-\vartheta)z^{\frac{1}{2}} = 1$. See figure 4.

6.2 Costly testing

Now assume that $c > 0$. I make good on my earlier promise by showing that sender-preferred equilibrium converges to the costless equilibrium when $c \to 0$. As the unit cost of testing decreases, any ϑ will keep testing for a longer time before giving up. The probability of stopping before being accepted converges to 0, but does so at a slower rate than the unit cost of testing, so that the total cost of testing still converges to 0. Thus the costless testing equilibrium is the limit of equilibria with vanishing cost of testing in both the conditional probability of acceptance and the total cost of testing.

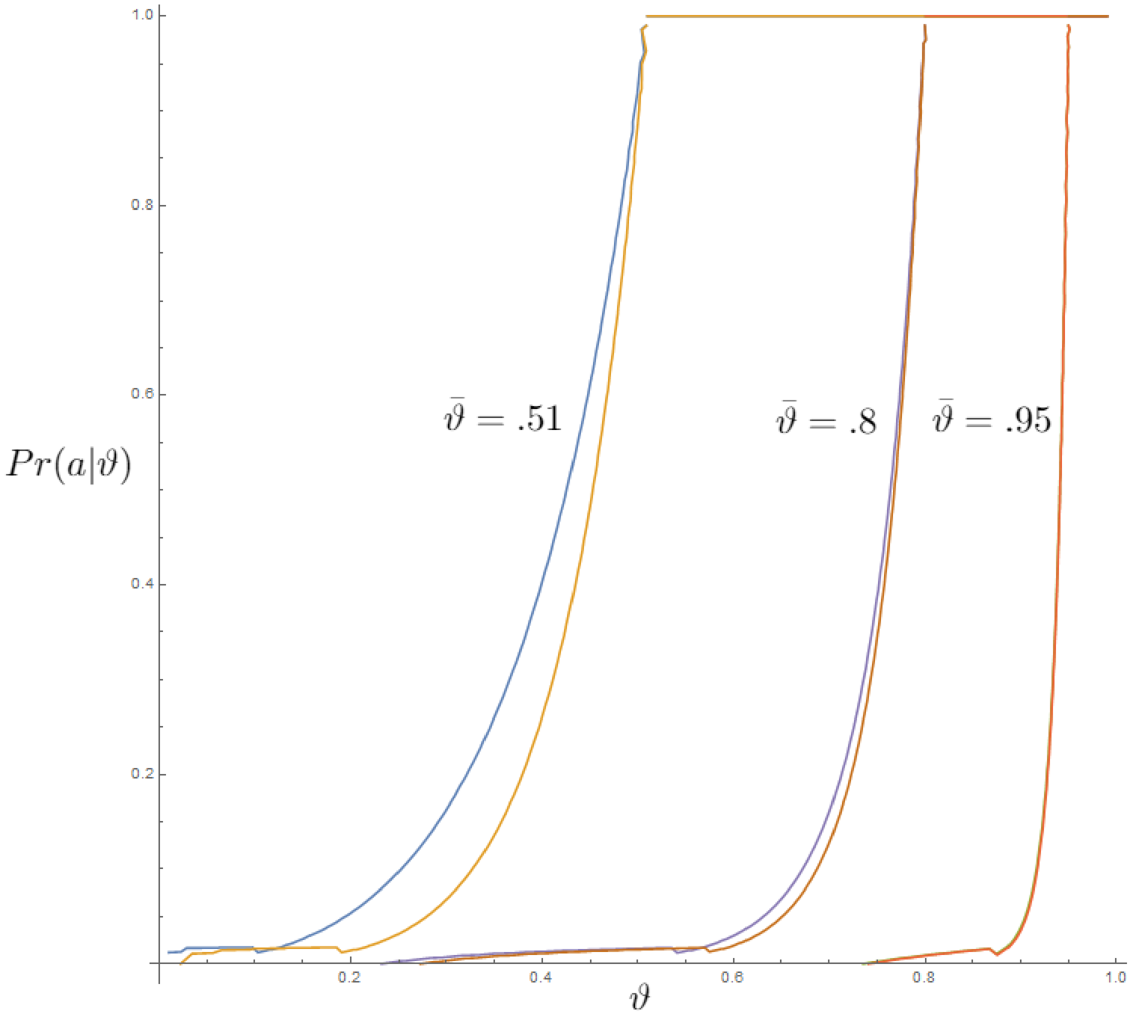


Figure 4: The probability of acceptance for 3 values of $\bar{\vartheta}$ for the example. I plotted both the upper and lower bound that I computed in the example. Notice that the probability of acceptance is always strictly positive, but not always high enough to permit proper calculation by my computer programme.

Recall that $p_0(\vartheta)$ is the conditional probability of acceptance in the costless equilibrium, $\mathbb{A}_0 := \{H \in \mathbb{H} : E_{stat}[\vartheta|H] \geq \bar{\vartheta}\}$ is the acceptance rule of the costless equilibrium and τ_0 is the stopping rule of that equilibrium. For any $c > 0$, I denote by $Pr_c(a), TC_c(\vartheta)$ etc. the outcomes in the sender-preferred equilibrium for that c.

For a sequence $\mathbb{A}_i \subseteq \mathbb{H}, i \in \mathbb{N}$, I say that $\mathbb{A}_i \to \mathbb{A}_{lim} \subseteq \mathbb{H}$, if for any $H \in \mathbb{H}$ there exists an $N(H)$ such that $H \in \mathbb{A}_{lim} \Leftrightarrow H \in \mathbb{A}_i$ if $i > N(H)$. This is a special case the standard mathematical notion of convergence in sets that is sufficient for my purposes. For a sequence of stopping times τ_i I say that $\tau_i \to \tau_{lim}$ if $\lim_{i\to\infty} Pr(\tau_i \neq \tau_{lim}) = 0$.

Proposition 4. ***Sender-preferred Equilibrium with Vanishing Costs***

The sender-preferred equilibrium with costly testing converges to the sender-preferred equilibrium with costless testing as the cost of testing vanishes. As $c \to 0$, $Pr_c(a|\vartheta) \to p_0(\vartheta)$, $TC_c(\vartheta) \to 0$, $\mathbb{A}_c \to \mathbb{A}_0$, $\tau_c \to \tau_0$ and $\gamma_{c,stat}(H) \to 1$ for any report H on the equilibrium *path.*

Proof. I start by constructing a sequence of equilibria that converges in the sense of the proposition as $c \to 0$. Let $\mathbb{A}_N = \{H \in \mathbb{H} : E_{stat}[\vartheta|H] \geq \bar{\vartheta} \wedge F_H \leq N\}$. For any ϑ with $p(\vartheta) > 0$, for c small enough, it would be optimal to keep testing until they are accepted, or until they have accumulated $N + 1$ failures, when acceptance is no longer possible. Therefore, for c sufficiently small for H with $S_H > 0$, $E[\vartheta|H] \to E_{stat}[\vartheta|H]$ and acceptance occurs if and only if $E_{stat}[\vartheta|H] \geq \bar{\vartheta}$. For H with $S_H = 0$, $E[\vartheta|H] \leq E[\vartheta] < \bar{\vartheta}$, so rejection occurs. This establishes that for any $N \in \mathbb{N}$, for c small enough, $\mathbb{A}_N$ induces an equilibrium.

Next I establish the convergence properties of the equilibria induced by $\mathbb{A}_N$ as N becomes large (and c small accordingly). It is immediate that the acceptance rule and the optimal stopping time converge in the sense of the proposition. It is then also clear that $Pr_c(a|\vartheta) \to p_0(\vartheta)$. To see that $TC_c(\vartheta) \to 0$, consider the stopping rule $\tau = \min\{n : H_n \in \mathbb{A}_{\mathbb{N}} \vee n \geq 1/\sqrt{c}\}$. Then, as $c \to 0$, $Pr(a|\vartheta) \to p_0(\vartheta)$, while $cE[\tau] \leq \sqrt{c}$ tends to 0. Since the optimal stopping time is weakly preferred to τ and leads to the same probability of acceptance in the limit, it must have a weakly lower expected cost of testing, and hence $TC_c(\vartheta) \to 0$.

I now show that, since a sequence of equilibria exists that converges to the costless equilibrium, then also the sender-preferred equilibrium must converge. This is done by showing that for any $\epsilon > 0$, there exists a c' such that if $c < c'$, for any equilibrium and any ϑ', $Pr(a|\vartheta') \leq p_0(\vartheta') + \epsilon$. Indeed, for any $N \in \mathbb{N}$, there exists a $c(N)$ such that, if $c < c(N)$ and $n \leq N$, then $H_n \in \mathbb{A}$ only if $E_{stat}[\vartheta|H_n] \geq \bar{\vartheta}$. On the other hand, for $n > N$, as N becomes large, by the law of large numbers, $Pr(|S_{H_n}/(S_{H_n} + F_{H_n}) - p(\vartheta')| > \delta) \to 0$ and, by lemma 6, if $S_{H_n}/(S_{H_n} + F_{H_n}) \to p(\vartheta')$, then $E[\vartheta|H_n] = \vartheta'$. Thus, if $\vartheta' < \bar{\vartheta}$, this would lead to rejection and $Pr(a|H_n, \vartheta', n > N) \to 0$. Since $Pr(a|\vartheta') = Pr(a|\tau \leq N) + Pr(a|\tau > N) \leq p_0(\vartheta') + (1 - p_0(\vartheta'))Pr(a|H_n, \vartheta', n > N)$, the result obtains for $\vartheta' < \bar{\vartheta}$. Since for $\vartheta' \geq \bar{\vartheta}$, $p_0(\vartheta') = 1$, I am done.

Since in any equilibrium, for c small enough, the cost of testing is non-negative and the probability of acceptance is bounded by $p_0(\vartheta)$, and, since the sequence of equilibria I constructed obtains these bounds as $c \to 0$, also the sender-preferred equilibrium must obtain these bounds. □

7 Comparison with Vanishing Cost

I compare the cases of suppression and no suppression when the cost of testing is vanishing. I start by comparing the sender pay-off, which I argue to be the main factor of interest. I show that the sender prefers suppression if $\bar{\vartheta}$ is close to $E[\vartheta]$ and $p(\vartheta_{min})$ is small, but that he prefers no suppression when $\bar{\vartheta}$ is close to ϑ_{max}. Then I show that the receiver always prefers the case of no suppression, but that this arises from what I consider to be an artifact of the model. In principle the pay-off of the sender and the receiver do not admit a direct comparison. However, by comparing their pay-off to the their maximal possible pay-off, I am able to illustrate the order of magnitude of the effects. I show that, even though I consider the preference of the receiver for the case of suppression to be artificial, the effect is nonetheless not negligible. In the discussion I return to this point.

7.1 Comparison of the Sender Pay-off

I investigate the sender pay-off under suppression and no suppression as $c \to 0$. First observe that the probability of acceptance in that case is higher under suppression than under no suppression. By proposition 2, under suppression $Pr^S(a) = p_{BP}$, while by corollary 2 $Pr^N(a) := p_0 < p_{BP}$. This is because the conditional probability of acceptance under suppression is the same as in the Bayesian Persuasion game. All the worst types are kicked out, until the receiver is just willing to accept. In contrast, under no suppression, even the worst types are accepted with a positive probability. Since they dilute the sample of the accepted, the probability of acceptance must be lower. This effect is further strengthened by the fact that the expected value of ϑ under no suppression is higher. See figure 5.

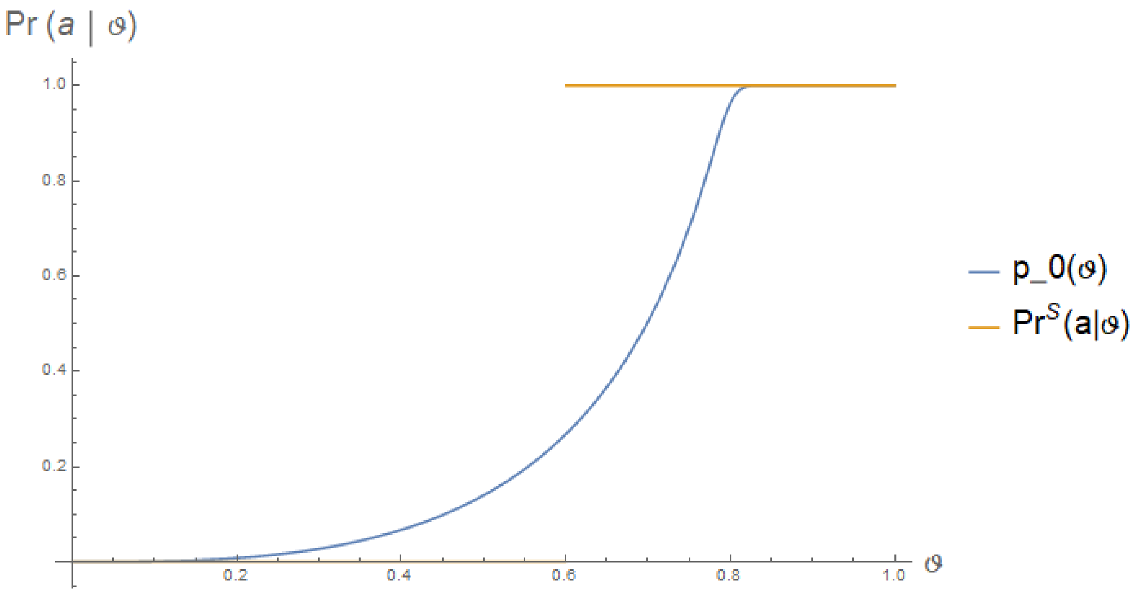


Figure 5: The conditional probability of acceptance under the two regimes.

On the other hand, the total cost of testing under suppression is always higher than under no suppression. Indeed, in the case of suppression it is strictly positive, even as $c \to 0$, while it becomes zero under no suppression. Which regime is preferred by the sender is determined by which of these forces dominates.

To understand which force dominates, consider three groups of ϑ. First, $\vartheta < \underline{\vartheta}$ strictly prefer no suppression, because they have a positive probability of acceptance and no cost of testing. Second, $\vartheta \geq \bar{\vartheta}$ strictly prefer no suppression, because they have a 0 cost of testing there and the probability of being accepted is 1 in either case. Finally, for $\underline{\vartheta} \leq \vartheta < \bar{\vartheta}$, the situation is ambiguous. On the one hand, in the case of suppression they have to pay a positive cost of testing, whereas in the case of no suppression the cost goes to 0.

On the other hand, the probability of acceptance in the case of suppression is 1, while it is strictly less than that in the case of no suppression. What allows me to make a comparison is to observe the effect of changing $\bar{\vartheta}$ on these three groups. By corollary 1, as $\bar{\vartheta} \to E[\vartheta]$, $\underline{\vartheta} \to \vartheta_{min}$, so the group $\vartheta < \underline{\vartheta}$ becomes small. Meanwhile, by the same corollary, $TC(\vartheta) \to p(\vartheta_{min})/p(\vartheta)$, which is small if $p(\vartheta_{min})$ is small. That means that $\underline{\vartheta} \leq \vartheta < \bar{\vartheta}$ will strictly prefer suppression, while $\vartheta \geq \bar{\vartheta}$ will become close to indifferent. Thus, as $\bar{\vartheta} \to E[\vartheta]$ and $p(\vartheta_{min})$ is close to zero, suppression will be preferred. On the other hand, as $\bar{\vartheta} \to \vartheta_{max}$, in the case of suppression the cost of testing goes to W for any $\vartheta < \vartheta_{max}$, so then no suppression becomes preferred for all ϑ.

Proposition 5. *Let $c \to 0$. There exists a $\vartheta' < \vartheta_{max}$ such that if $\bar{\vartheta} > \vartheta'$, no suppression is preferred.*
If $p(\vartheta_{min}) = 0$, there exists a $\vartheta'' > E[\vartheta]$ such that if $\bar{\vartheta} < \vartheta''$, suppression is preferred.

Proof. Denote by $\Pi_S^s(\vartheta)$ the pay-off to the sender conditional on ϑ under suppression and similarly $\Pi_S^n(\vartheta)$ the pay-off under no suppression. In the limit $c \to 0$, we have

$$\Pi_S^s(\vartheta) - \Pi_S^n(\vartheta) = \begin{cases} 0 - p_0(\vartheta) & \text{if } \vartheta < \underline{\vartheta}(\bar{\vartheta}); \\ (1 - \dfrac{p(\underline{\vartheta}(\bar{\vartheta}))}{p(\vartheta)})W - p_0(\vartheta)W & \text{if } \underline{\vartheta}(\bar{\vartheta}) \leq \vartheta < \bar{\vartheta} \\ (1 - \dfrac{p(\underline{\vartheta}(\bar{\vartheta}))}{p(\vartheta)})W - W & \text{if } \vartheta \geq \bar{\vartheta} \end{cases}$$

By corollary 1, as $\bar{\vartheta} \to \vartheta_{max}$, all three terms become negative. As $\bar{\vartheta} \to E[\vartheta]$, $\underline{\vartheta}(\bar{\vartheta}) \to \vartheta_{min}$. If furthermore $p(\vartheta_{min}) = 0$, then both the first and last term vanish, while the middle term becomes strictly positive.

□

Example. Consider again ϑ uniformly distributed on $[0, 1]$ and $p(\vartheta) = \vartheta$. The work is already done in the previous examples, so I simply plot the results together. See figures 6 and 7.

Example. Notice that proposition 5 requires that $p(\vartheta_{min}) = 0$. This is an indispensable requirement. In figures 8 and 9 I plot the sender pay-off under the two regimes for ϑ uniformly distributed and first $p(\vartheta) = \vartheta$, so that $p(\vartheta_{min}) = 0$, and second $p(\vartheta) = (\vartheta+1)/2$, so that $p(\vartheta_{min}) = 1/2 > 0$. In the second case, the sender prefers no suppression for any $\bar{\vartheta}$.

7.2 Comparison of the Receiver Pay-off

Under suppression, when $c \to 0$, the receiver is exactly indifferent between accepting and rejecting when he accepts. On the other hand, under no suppression, he typically strictly prefers acceptance, as discussed in the remark after proposition 3. The reason is that under no suppression, there are many reports that are on path, some of which contain only few

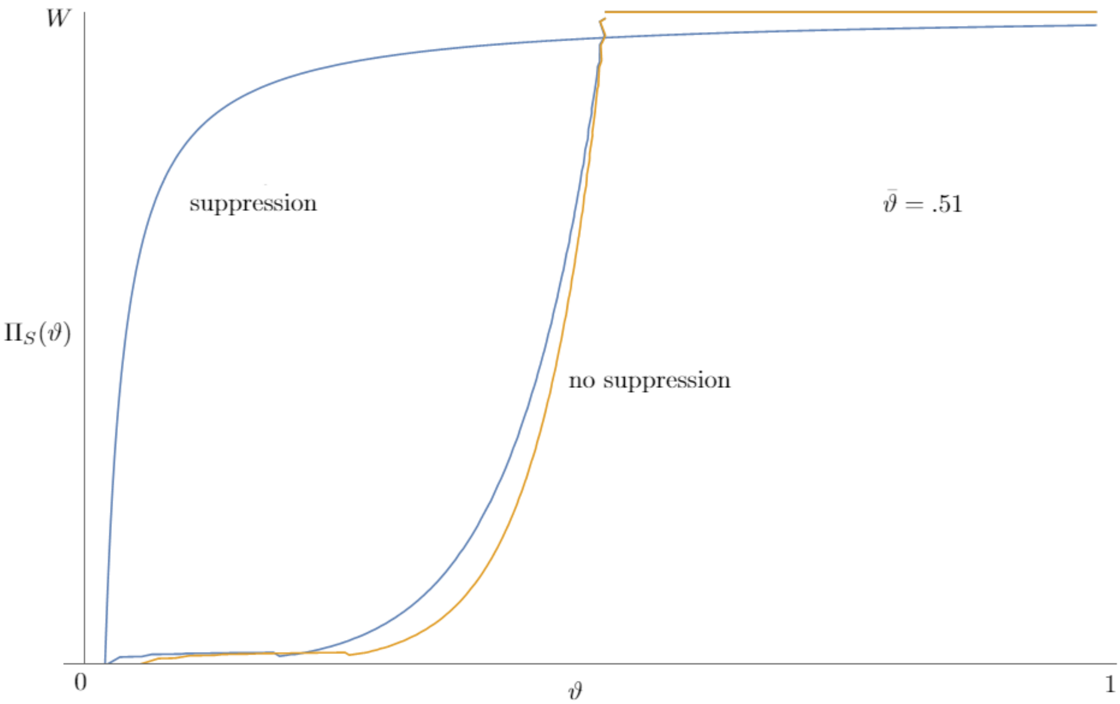


Figure 6: The pay-offs in both cases when $\bar{\vartheta}$ is close to $E[\vartheta]$

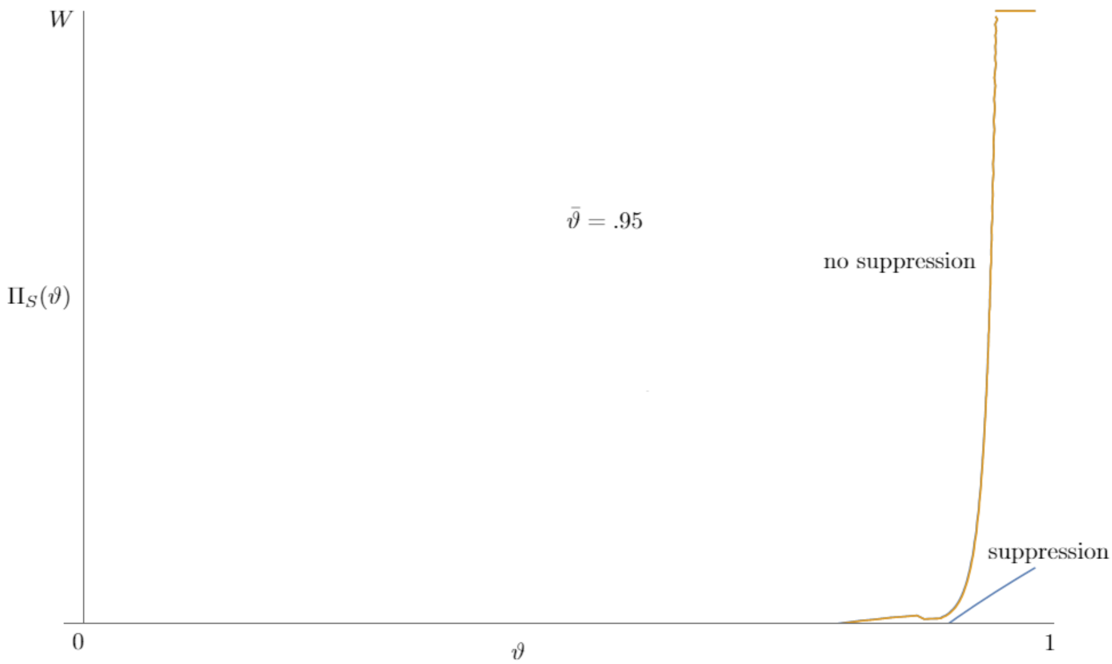


Figure 7: The pay-offs in both cases when $\bar{\vartheta}$ is close to ϑ_{max}

tests. Recall that $\mathbb{A}_0 = \{R \in \mathbb{H} : E_{stat}[\vartheta|R] \geq \bar{\vartheta}\}$. To illustrate the issue, take ϑ uniformly distributed on $[0, 1]$ and let $p(\vartheta) = \vartheta$. Then in the costless testing equilibrium under no suppression, the expectation of ϑ after 1 success is $E_{stat}[\vartheta|(1)] = 2/3$. If $\bar{\vartheta} < 2/3$, this means the sender will stop testing after 1 success and 0 failures and the receiver will have

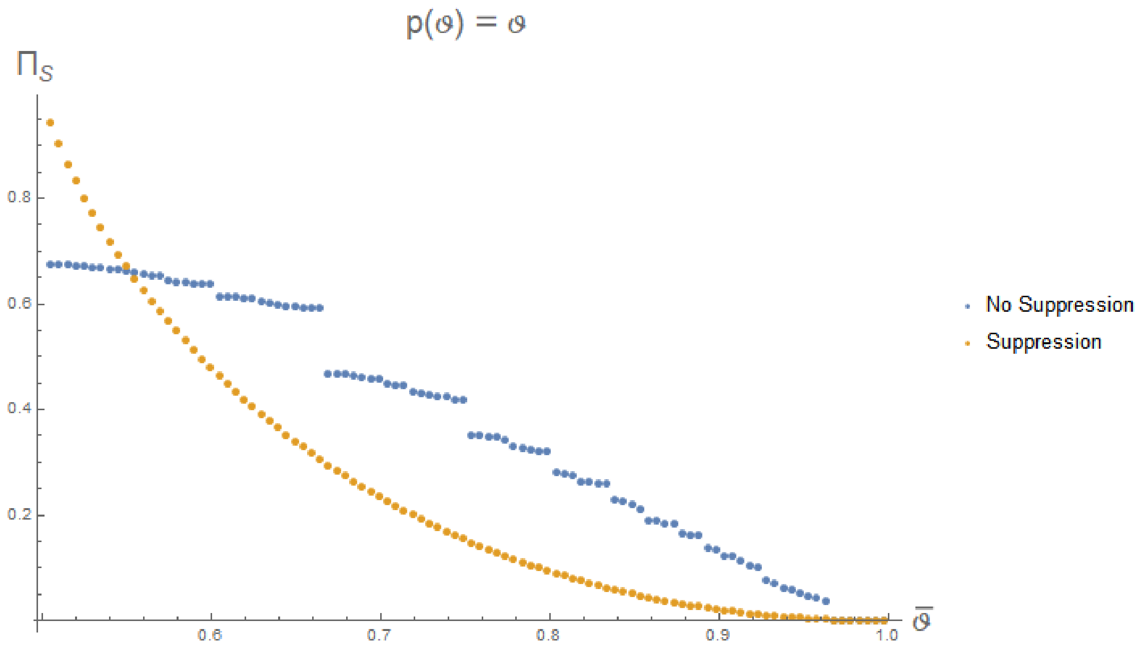


Figure 8: Proposition 5 applies and for small $\bar{\vartheta}$, the sender prefers suppression.

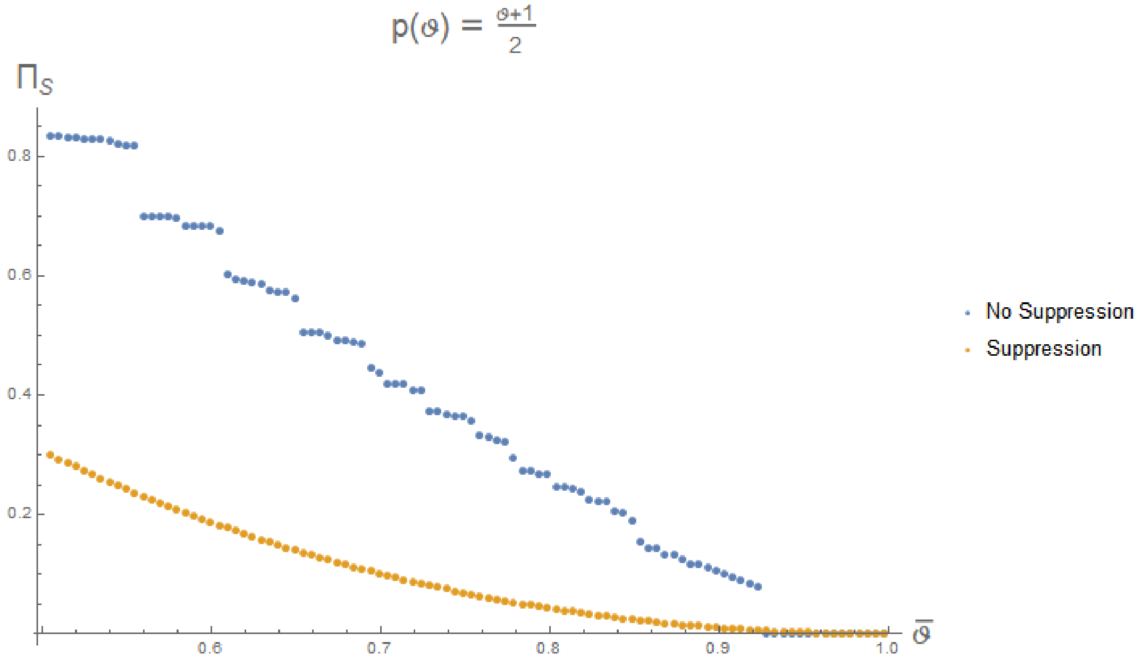


Figure 9: Proposition 5 does not apply and indeed no suppression is preferred for all $\bar{\vartheta}$.

a pay-off from accepting of $\Pi_{Receiver} = 2/3 - \bar{\vartheta} > 0$.

The effect has an artificial flavour, because the sender would prefer to make the receiver indifferent, but cannot do so because the number of tests is discrete. If it were possible to submit less than a full test result, the sender would like to do so. However, the example does illustrate that the effect is non-negligible, as the probability of obtaining a success on the first test is $1/2$. In this section I will take the integer problem as given and analyze its magnitude. In the discussion I describe an extension of the model that addresses the integer problem.

In order to make a meaningful statement about the magnitude of the receiver pay-off, I would like to compare it to the sender pay-off. However, the two are not directly comparable. I tackle this issue in the following way. The sender pay-off is always expressed as a fraction of W, the maximal utility the sender can hope to achieve - it is his ex ante pay-off if he is always accepted. I will similarly describe the pay-off of the receiver as a fraction of the maximal pay-off he could obtain, i.e. if only the $\vartheta \geq \bar{\vartheta}$ are accepted. This maximal pay-off is $\int_{\bar{\vartheta}}^{\vartheta_{max}} (\vartheta - \bar{\vartheta}) dF(\vartheta)$. Observe that this maximal pay-off depends on $\bar{\vartheta}$, in contrast to the maximal sender pay-off.

In figures 10 and 11 I plot the receiver pay-off as a fraction of his maximal pay-off and the difference between the sender pay-off under suppression and no suppression, as a function of $\bar{\vartheta}$. I take ϑ uniform on $[0, 1]$ and $p(\vartheta) = \vartheta$ in figure 10 and $p(\vartheta) = \vartheta/2$ in figure 11. We see that the sender prefers suppression for small $\bar{\vartheta}$, but the effect is always smaller than the preference of the receiver for no suppression. In other words, while the preference of the receiver for no suppression may be due to an artificial aspect of the model, the effect is not negligible. This finding has been robust to any specification of $p(.)$ that I tried.

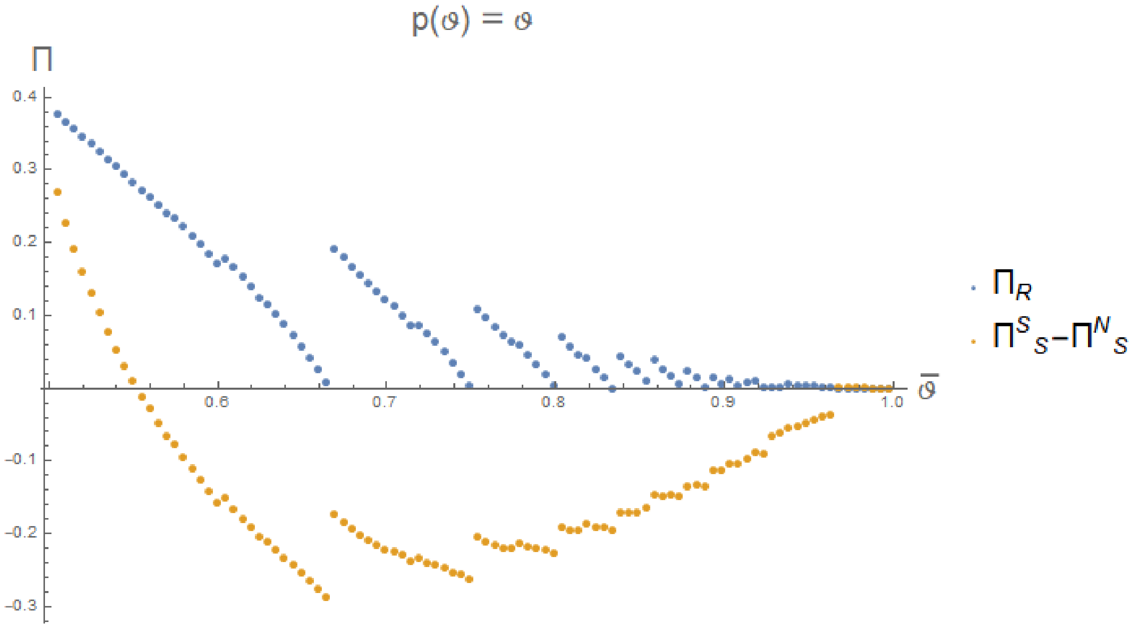


Figure 10: The pay-off of the receiver exhibits irregularities that are typical of an integer problem, dropping to zero when the value of $\bar{\vartheta}$ coincides with the on-path elements of $\mathbb{A}_0$.

8 Discussion

In the main model of the paper, I showed that the sender prefers suppression if $\bar{\vartheta}$ is close to $E[\vartheta]$ and $p(\vartheta_{min})$ is small, while he prefers no suppression when $\bar{\vartheta}$ is close to ϑ_{max}. I also showed that the receiver always prefers no suppression, but that this came about because of

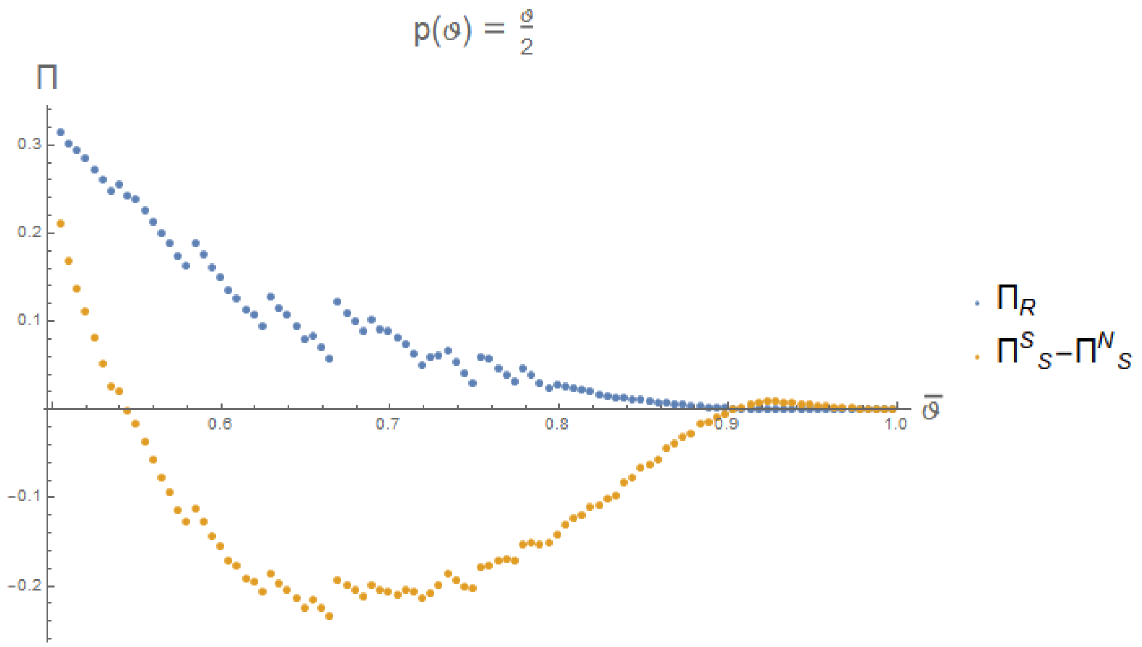


Figure 11: Notice that it appears in this graph that the sender prefers suppression for high $\bar{\vartheta}$. This is is because the approximation I use underestimates the probability of acceptance under no suppression and becomes inaccurate for high $\bar{\vartheta}$.

integer problems. When suppression is not possible, a report submitted for acceptance may contain only few tests. When the sample is small, there are only few values the conditional expectation can take, of which potentially none make the receiver indifferent. Therefore he must strictly prefer acceptance.

It seems artificial that the receiver would prefer acceptance because the only test the receiver could run was too informative, in the sense that the expectation conditional on one more success overshoots the required level. It would seem easy for the sender to be less informative. If this is also less costly for him, he would want to be less informative and the receiver preference would vanish.

I suggest the following model to capture this idea. The informativeness of a test is determined by the relative likelihoods of success of the different types. Let everything be as in the main model and in particular fix $p(.)$. Now, before running a test i, the receiver can choose to decrease its informativeness by a fraction α. This comes about through a transformation on $p(.)$ that leaves the ex ante probability of success unaffected, but decreases the likelihood ratio between different ϑ. Formally, the sender chooses $\alpha_i \in [0,1]$ at the start of each test i and the probability of success is $p_{\alpha_i}(\vartheta) = \alpha_i p(\vartheta) + (1-\alpha_i)p(E[\vartheta])$. This is a form of a truth-or-noise technology. The cost of running a test with α is $cf(\alpha)$ where $f(0) > 0$, $f' > 0$ and $f(1) = 1$. In other words, a less informative test is less costly, but never free. A testing history is now a sequence of pairs $(\alpha_i, X_i)_{i=1}^n$, containing the informativeness of each test and the outcome. Again, under no suppression the entire sequence is reported, under suppression any subsequence can be reported.

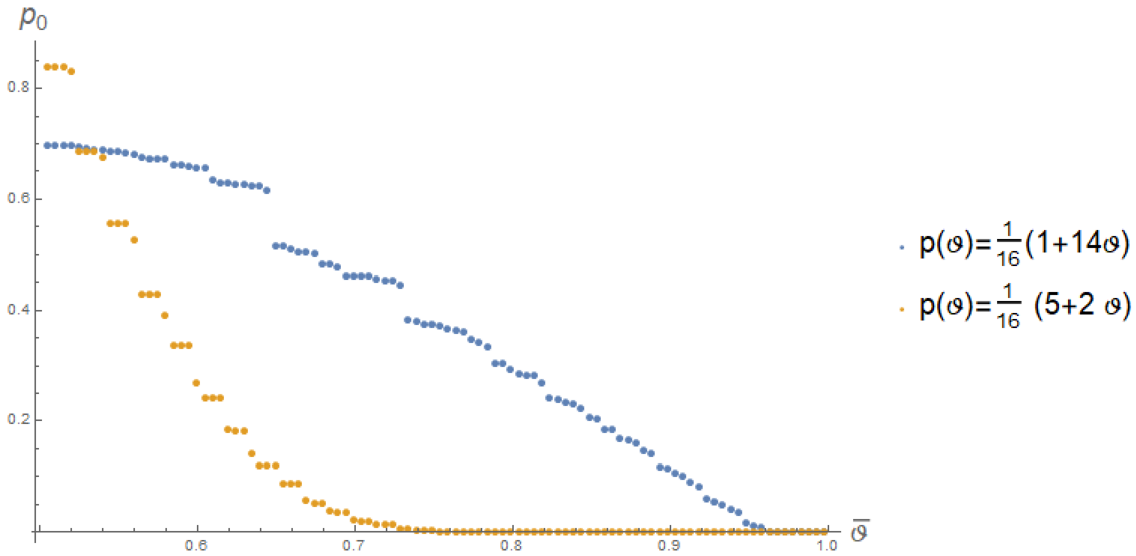
In the case of suppression, it is clear that as $c \to 0$, the sender-preferred equilibrium will be the same as in the main model with $p(.)$. The reason is that the probability of acceptance will then be maximal, while the expected total cost of testing is determined by $p(\underline{\vartheta}(\bar{\vartheta})/p(\vartheta)$, which is minimal when $\alpha = 1$ or the informativeness is maximal. The intuition is that the more informative the test is, the less costly it is for the ϑ to separate themselves.

In the case of no suppression, things are less clear. It may be intuitive to suppose that again the most informative test will be chosen. However, now this will also affect the probability of being accepted. Indeed consider figure 8. It shows p_0 for $p(\vartheta) = \vartheta$ and $\alpha = 1/16$ and for $\alpha = 15/16$. It thus compares the probability of acceptance when an informative test is chosen throughout to when a relatively uninformative test is chosen throughout. For low values of $\bar{\vartheta}$, the probability of acceptance is higher with the uninformative test. This suggests that the optimal choice of informativeness is not necessarily the highest level.

In a way, this is to be expected. If the sender in this extended game were to simply replicate his strategy from the main model, only to deviate to a less informative test when one more success would "push him over the top", this would not affect his probability of acceptance very much, while significantly reducing the pay-off of the receiver. It stands to reason that he can appropriate the surplus of the receiver in a more efficient way. In other words, while it may be artificial that some surplus accrues to the receiver because of the integer problem, the surplus that accrues is not itself artificial and it is natural that transferring it to the sender would require altering his behaviour.

Returning to the comparison between suppression and no suppression, this discussion does suggest that a version of proposition 5 survives. Indeed, no matter the choice of informativeness, $p_0 < p_{BP}$. Furthermore, in this extension it seems likely that the receiver would be indifferent between accepting and rejecting, at least for an appropriate choice of $f(\alpha)$, the cost of informativeness. Thus this extension can serve as a justification for focusing on the sender-pay-off. In the main model the integer problem makes it impossible for the sender to appropriate all the surplus. Therefore a comparison of only his pay-off across the two regimes misses an important welfare aspect. When we "solve" the integer problem as in this extension, the sender can appropriate all the surplus and thus an assessment of his pay-off alone is a real welfare comparison.

A final observation. If we allow the sender to pick $p(.)$ freely, possibly with some cost associated to the functional form chosen, he would simply pick $p(\vartheta) = \mathbb{1}_{\vartheta \geq \underline{\vartheta}(\bar{\vartheta})}$. Then the distinction between suppression and no suppression, and indeed the distinction between my model and the Bayesian Persuasion game, would vanish.



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