Endogenous Lemon Markets: Risky Choices and Adverse Selection*

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Abstract

We study an adverse-selection model in which the distribution of the asset is affected by unobservable actions of the seller that are performed prior to the trade. We characterize the seller's equilibrium behavior by a risk-seeking property: the seller prefers second-order stochastically dominated distributions. We show that under rather general conditions, a riskier trade game results in lower equilibrium activity. That is, unfavorable conditions for trade in equilibrium are likely in environments in which the seller is able to manipulate the asset distribution. In addition, we study a normal-distribution specification of our model, in which the buyer observes a noisy signal of the asset before trade, and we show that our results carry through. We also show that our results hold if some sellers can verify the value of their asset and in the case where the buyer is a monopolist.

Keywords: Lemons Markets, Endogenous Distribution.
JEL Classification: C72, D83, L15

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1 Introduction

Adverse selection is commonly used as an explanation for the liquidity shocks observed in some financial markets during the crisis of 2007–2008; see Philippon and Skreta (2012) and Tirole (2012). In general, the severity of adverse selection in a competitive equilibrium is determined by the distribution of the seller's private information. That is, from Akerlof (1970) seminal work we can deduce the possibility of market failure, but it does not suggest that conditions particularly harmful to trade will predominate financial markets such as the market for mortgage-backed securities (MBS).

We show that introducing a distribution-related endogeneity into any adverse selection environment can have a dire effect on equilibrium trade. In contrast to Akerlof (1970) used-cars example, where the "lemons" are manufactured that way, the value of a structured investment is affected by private decision of the issuer. We show that the seller's ability to manipulate the asset distribution can exacerbate market unraveling

Financial markets are not the only trade environments subject to distribution-related endogeneity. Indeed, sellers are able to manipulate the asset distribution in many trade environments. For example, an entrepreneur's private decisions (regarding hiring strategy, project choice, etc.) can affect the value of a traded startup.

In light of the above, we study an extension of Akerlof (1970) in which, prior to the trade, the seller chooses the asset distribution where this choice is unobservable. Next, the asset value is realized according to the appropriate distribution and revealed to the seller privately. From that point, the game proceeds as in Akerlof's (1970) original model: the seller can either utilize the asset or she can offer it for sale in a competitive market. We show that the option value of the seller's equilibrium payoff leads her to pursue risky distributions. We then show that under quite general assumptions, equilibrium activity is lower given a riskier distribution. We also show that in the normal-distribution environment our results can be generalized to the case where the market observes a noisy signal of the asset value.

The following simple example captures the central take-away of this paper.

Example 1 Assume that the seller chooses between two projects, X_1 and X_2, where

$$X_1 = \begin{cases} \frac{1}{2} & \text{w.p. } 1, \end{cases} X_2 = \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2}. \end{cases}$$

Further assume that the seller's utility from consuming a good of type θ is θ, while the buyer's utility is $\theta + \Delta$ for some $\Delta \in \left(0, \frac{1}{2}\right)$.

The stage game of the first project results in an efficient competitive equilibrium, while the second project induces partial trade. However, there is no equilibrium in which the seller chooses project X_1. Given market-beliefs that are consistent with such a strategy, the competitive market price is $p = \frac{1}{2} + \Delta < 1$. Thus, the seller could profit from deviating to X_2 and then offer the project for sale only if the realization is 0. The seller's expected payoff would be $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot p > p$. It is easy to verify that in the unique equilibrium of this game the seller chooses X_2 and only "lemons" are traded.

Example 1 teaches us two lessons. First, the seller pursues risk. We show that as long as two distributions have the same expected value and are ranked according to a strict version of second-order stochastic dominance, there is no equilibrium in which the seller chooses the safe alternative with positive probability. The intuition behind this result is straightforward: in a trade game à la Akerlof (1970) the seller is paid at least the market price and has the option to consume realizations above this price, i.e., the seller's equilibrium payoff is convex in her type and thus she prefers risky distributions.

In addition, as we can see in Example 1, the seller's equilibrium choice is the project that induces minimal trade. There exist counterexamples in which a riskier distribution leads to more trade. However, we show that a riskier distribution induces less trade if and only if it is location-independently riskier than the other distribution, as defined by Jewitt (1989). Location-independent risk is a similar concept to second-order stochastic dominance in which the area below the commutative distribution function is compared at every *quantile*. Jewitt (1989) showed that this condition is useful in order to deal with a similar comparative statics issue in an insurance context. We discuss this concept and its equivalences, and derive some additional characterization of the environments in which there is a monotone link between risk and equilibrium trade.

In order to provide some intuition for the reason riskier distributions tend to perform worse in equilibrium, we consider the uniform-distribution example.

Example 2 Assume that the seller's choice set consists of two projects: $Y_1 \sim U[-1,1]$ and $Y_2 \sim U[-2,2]$. The utilities of the players are as defined in Example 1.

We can conclude that the Y_2 trade game results in lower equilibrium activity by observing that $Y_2 \overset{d}{=} 2Y_1$. Transforming the value distribution this way is equivalent to halving the gains from trade[1] Δ, thereby exacerbating adverse-selection effects. We show that a similar argument applies to the normal-distributions example since higher variance is equivalent to a linear expansion of the value also in normal distributions. Note however that the riskier distribution does not have to be a linear transformation of the safer one in order to meet the location-independent risk condition and to result in a decrease in trade.

Thus far, we have assumed that the buyer does not observe outside information. That is, the buyer cannot use public signals in order to settle some of the uncertainty regarding the asset value. Since Akerlof's (1970) model can be extended to allow such a signal, we analyze a normal-distribution model in which a noisy signal is publicly observed before trade. We show that the results of our baseline model follow. That is, the seller is incentivized to deviate to riskier distributions, causing trade to decrease.

We also study an extension that allows for some verifiability à la Dye (1985). Specifically, we give the seller the ability to verify her asset value with some probability. We show that our results hold, and the seller's equilibrium behavior results in a decrease in trade. In addition, we apply DeMarzo, Kremer and Skrzypacz's (2019) minimum principle in order to extend our statement to distributions that are incomparable in the sense of second-order stochastic dominance. Finally, we show that our results hold if the market power is in the hands of the buyer.

[1] An equilibrium (in the interior) of the Y_2-game is given by a type $\hat{\theta}$ that solves $\mathbb{E}\left[2Y_1 | 2Y_1 \leq 2\hat{\theta}\right] + \Delta = 2\hat{\theta}$.

Related Literature Conceptually, the seller's risk-seeking behavior in our model is mostly linked to that of Ben-Porath, Dekel and Lipman (2017), who study a generalized Dye (1985) disclosure game in which the asset is affected by the agent's preliminary choices. Since in both models the equilibrium payoff of the informed party has an option value, Ben-Porath, Dekel and Lipman's (2017) risk-seeking characterization is applicable to Akerlof's (1970) environment. Connected to this line of research are Chen (2015), who studies risky behavior in Holmstrom's (1979) model, and Barron, Georgiadis and Swinkels (2017), who study risk-seeking due to limited liability in a principal-agent context.

Our work is also related to the study of the role of the information structure in determining equilibrium trade and social welfare in an Akerlof (1970) model. Doherty and Thistle (1996) consider an Akerlof (1970) model in which the seller chooses whether to become one of two informed types or to remain uninformed. Note that a choice between a risky project and a safe one can be mathematically analogous to a choice between information types. Thus, comparative statics on equilibrium properties as a function of the information structure are relevant to our research. Levin (2001) studies welfare properties derived from different information structures and some of our characterizations are similar to his. Kessler (2001) compares different informativeness conditions. Ravid, Roesler and Szentes (2019) study information purchases by the buyer, and Athey and Levin (2018) study the value of information in general decision problems.

The necessary and sufficient condition for a monotone link between risk and trade in equilibrium appears in the literature in the context of insurance. Given any two SOSD-ranked lotteries and some risk-averse agent, there exists a maximal premium this agent is willing to pay in order to be faced with the safer lottery. Jewitt (1989) shows that the location-independent risk condition is necessary and sufficient to guarantee that every agent who is more risk-averse is also willing to pay this premium. See also Landsberger and Meilijson (1994) and Chateauneuf, Cohen and Meilijson (2004).

Another stream of related literature is the hold-up problem, formulated by Grout (1984) and Tirole (1986), where one party withholds an investment so as not to strengthen the other party's bargaining position. Gul (2001), Hermalin and Katz (2009), and Hermalin (2013) study the effect of observability of the investment decision in this problem. In contrast to our model, those models have a costly investment decision. Thus, there is a basic tension between the fact that in a trade equilibrium the seller is not incentivized to invest, but if she does not invest and hence there is no trade, the seller is better off investing for her own good; see Gul (2001). As a result, mixed equilibria characterization plays an important role in defining the equilibrium in the above models, while it is absent in toto from our analysis.

Finally, our results are somewhat different from those of Myers and Majluf (1984) and Nachman and Noe (1994). They show that debt contracts are the least vulnerable to asymmetric information and are therefore expected to be more common in practice. Like in our model, they have a seller selecting the distribution according to which the market is pricing the asset. However, in those models the distribution choice – the choice of the contract that defines the investor's payoff as a function of the state of the world – is observable. Thus, the seller necessarily chooses the payoff structure most immune to the forces of adverse selection. Note that observability of the distribution choice would promote efficiency in our model too. Since the seller pockets all social welfare in the competitive equilibrium, she will not detectably deviate to a subgame yielding inefficient results.

The rest of the paper is organized as follows. In Section 2 we introduce the model and provide a preliminary analysis. In Section 3 we characterize the seller's equilibrium behavior. In Section 4 we analyze the implications of the seller's risky choices for trade and social welfare. In Section 5 we consider an informative signal. In Section 6 we discuss a few extensions. Section 7 concludes.

2 Model and Preliminary Analysis

The model is a dynamic incomplete information game between a seller (she), Nature, and a competitive market of buyers. In the first stage of the game the seller chooses an alternative $X_i \in \{X_1, X_2, \dots X_n\}$ (e.g. a project), where $X_i \sim F_i$, $\mathcal{F} = \{F_1, \dots F_n\}$. Following the *unobservable* choice of the seller, the asset value is realized according to the appropriate distribution, and is privately observed by the seller. Finally, the seller decides whether to utilize the asset or to sell it in a competitive market. In the baseline model we assume that the seller's utility from consumption is

$$v_s(\theta) = \theta,$$

and the buyer's utility from consumption is

$$v_b(\theta) = \theta + \Delta,$$

for some $\Delta > 0$. Figure 1 illustrates the play of the game. As in Akerlof (1970), we assume that the market price p is the expectation of the buyer's value from consumption (given market beliefs).[2] That is, all market power is concentrated in the seller's hands.

Next, we analyze the equilibrium of the F trade game, i.e., an Akerlof (1970) game in which the asset distribution is given by F.

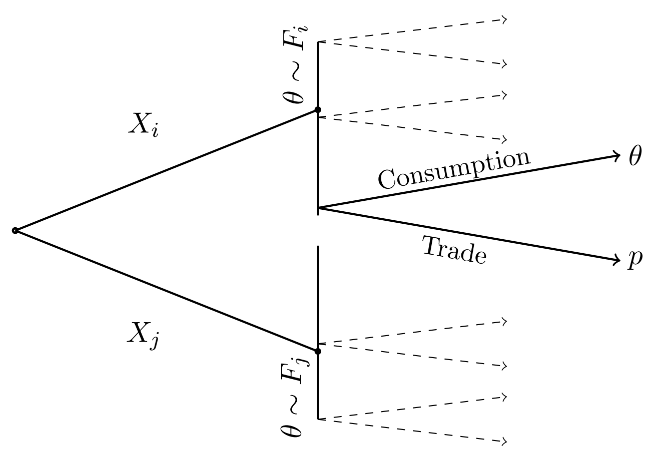

Figure 1: The Extended Trade Game

2.1 The Trade Stage Game

In order to simplify the presentation of our results we confine our analysis to continuous distributions over $\mathbb{R}_+$, in which there exists an equilibrium in the F trade game. In addition, we provide sufficient conditions for equilibrium uniqueness.

[2] This is a reduced-form presentation of the play in the trade stage. For the link to different structural assumptions on the trade game form, see Wilson (1980) and Mas-Colell, Whinston and Green (1995).

Assumption 1. *F is a CDF of a non-negative*[3] *random variable satisfying the following conditions:*

1. *The PDF $f(x) > 0$, $\forall x \in \mathbb{R}_+$.*

2. *F is log-concave.*

The unboundedness assumption implies that our discussion is limited to the Akerlof (1970) case. That is, for every distribution the equilibrium is in the interior and some types of the seller consume.[4] The log-concavity of the distribution implies equilibrium uniqueness, since the difference between the buyer's expected value up to a threshold and the threshold is a decreasing function.

A pure strategy of the seller in the F trade game is $s : \mathbb{R}_+ \to \{0,1\}$, specifying the seller's decision whether to sell the asset or not, where $s(x) = 1$ designates trade. The buyer's strategy is reduced to a price p, which is equal to the expectation of $v_b(x)$ with respect to the buyer's beliefs q.

A perfect Bayesian equilibrium (PBE) of the trade stage game is a pair (s,p) and a belief q that together satisfy (1)–(3) as defined below. The seller's strategy is

$$s(x) = \begin{cases} 1 & \text{if } x \leq p \\ 0 & \text{if } x > p, \end{cases} \tag{1}$$

where the price p is given by

$$p = \int_0^\infty v_b(x)\, q(x)\, dx. \tag{2}$$

In addition, the market belief q is consistent on the equilibrium path, i.e,

$$q(x) = \frac{f(x)\, s(x)}{\int_0^\infty f(y)\, s(y)\, dy}. \tag{3}$$

[3] When considering normal distributions, we extend our analysis to distributions unbounded from below.

[4] All of our results hold in the bounded case if some adverse selection is present in the market. As in Example 1, if there exists a distribution that has some realizations above the market price, then the seller is tempted to deviate from efficient choices.

Assumption 1 guarantees that $\int_0^\infty f(y)\,s(y)\,dy > 0$ on the equilibrium path. In addition, a deviation of the seller is undetectable; i.e., if the seller alters her trade strategy s the beliefs of the buyer remain unchanged. Therefore, limiting off-path beliefs will have no effect on equilibrium selection.

The stage game has a unique equilibrium for every $F \in \mathcal{F}$, defined by the price $\hat{p}_F$, where types below the price pool and sell the asset, and types above it consume. See Appendix B for more details. Let

$$PT(F) := F(\hat{p}_F) \tag{4}$$

denote the probability of trade in the unique equilibrium, and let

$$SW(F) := \int_0^{\hat{p}_F} v_b(\theta)\,d\theta + \int_{\hat{p}_F}^{\infty} v_s(\theta)\,d\theta \tag{5}$$

denote the social welfare in this equilibrium.

2.2 The Extended Trade Game

When considering the extended trad game we keep the same definitions as in the analysis of the stage game. We only need to add the first-stage seller's strategy $\alpha \in \Delta(\mathcal{F})$, where α_i denotes the probability that the seller's strategy assigns to the choice of F_i. A perfect Bayesian equilibrium (PBE) of the extended trade game is a triplet (α, s, p) and a belief q of the buyers, which together satisfy (6)–(9) as defined below.

Let $R_i(p) := \mathbb{E}_{F_i}[\max\{x, p\}]$ denote the seller's expected utility when choosing X_i given a market price p. The seller's first-stage strategy satisfies

$$\alpha_i > 0 \Longrightarrow R_i(p) \geq R_j(p)\,\forall j \in \{1, 2, \ldots n\}. \tag{6}$$

The seller's strategy in the trade stage is as defined above, i.e.,

$$s(x) = \begin{cases} 1 & \text{if } x \le p \\ 0 & \text{if } x > p, \end{cases} \tag{7}$$

where the price, p, is given by

$$p = \int_0^\infty v_b(x)\, q(x)\, dx. \tag{8}$$

In addition, the beliefs of the buyers are consistent on the equilibrium path, i.e,

$$q(t) = \frac{\sum_{i=1}^{n} \alpha_i f_i(t)\, s(t)}{\sum_{i=1}^{n} \alpha_i \int_0^\infty f_i(x)\, s(x)\, dx.} \tag{9}$$

Proposition 1. *There exists an equilibrium in the extended trade game that is either pure or involves mixing between exactly two distributions.*

The proof of Proposition 1 is standard, and, as with all other proofs, is deferred to Appendix A. In the next section we analyze the equilibria of the extended trade game.

3 Risky Choices

As we have shown above, the equilibrium of the stage game is characterized by an option value: the seller is guaranteed a payoff p, and will obtain a higher payoff if the realized value is above p. We now show how this aspect of the seller's equilibrium payoff implies risk-seeking.

Definition 1 (Strong Second-order Stochastic Dominance). *Given two distributions $F_i, F_j \in \mathcal{F}$, we say that F_i is strong second-order stochastically dominant over F_j, denoted by $F_i \succ_{SSOSD} F_j$, if*

$$\int_0^t F_i(x)\, dx < \int_0^t F_j(x)\, dx \tag{10}$$

for every $t > 0$.

We show that for any two distributions that have the same expectation, the seller strictly prefers the riskier one. Note that the regular definition of second-order stochastic dominance (with weak inequality) allows F_i and F_j to differ only in linear segments of $\max\{x, p\}$, and thus the seller might be indifferent between the two alternatives. Therefore, in order to claim that F_i is not a part of the seller's equilibrium strategy, we are required to use a stronger notion of stochastic dominance, adapted from Ben-Porath, Dekel and Lipman (2017) to our model.

Theorem 1. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$. If $F_i \succ_{SSOSD} F_j$, then in any equilibrium of the extended trade game $\alpha_i = 0$.*

The proof is very similar to Ben-Porath, Dekel and Lipman's (2017). The intuition for the seller's risk-seeking disposition can be best explained by considering the equivalent definition of second-order stochastic dominance from Rothschild and Stiglitz (1970). Since F_j is a mean-preserving spread of F_i, there exists a random variable ϵ such that $X_j \overset{d}{=} X_i + \epsilon$, where $\mathbb{E}[\epsilon | X_i = x_i] = 0$ for every x_i. For simplicity, let $X_j = X_i + \epsilon$, and assume to the contrary that the seller chooses X_i in equilibrium. Now, consider a deviation of the seller to X_j while following $s(x_i)$, i.e., $s(x_j) = 1$ if and only if $x_i \leq p$. Such a deviation does not affect the seller's expected utility since the conditional expectation of ϵ is 0. The utility of the seller can only increase if she adjusts her trade strategy in instances in which $x_i \leq p$ and $x_j > p$ or vice versa. The assumption that $F_i \succ_{SSOSD} F_j$ implies that such instances have positive probability. Thus, there is no equilibrium in which F_i is chosen.

4 Trade and Social Welfare

We now turn to analyze the implications of the seller's risk-seeking disposition on trade. We will demonstrate that the unobservability of the seller's actions increases adverse selection. When the seller's initial choice is observed by the market, her incentives are aligned with efficiency considerations. That is, since the market is exposed to an underlying adverse selection problem, the probability of trade in equilibrium is less than 1. However, given that every distribution gives rise to an adverse selection problem, the seller's choice maximizes social welfare. By contrast, as we have shown, when the seller's choice is unobserved, the seller prefers riskier distributions. We now show that under quite general conditions, a riskier distribution results in lower equilibrium trade and social welfare.

We first demonstrate our result with respect to normal distributions, where the proof is simple and intuitive. See Appendix B for the technical conditions allowing us to extend the analysis of the F trade game to distributions unbounded from below. We then provide a necessary and sufficient condition for the trade to be lower in the riskier distribution, and we apply it in order to derive a few more conclusions.

PROPOSITION 2. *Let* $F_i = N\left(\mu, \sigma_i^2\right)$*, and* $F_j = N\left(\mu, \sigma_j^2\right)$*. If* $\sigma_j^2 > \sigma_i^2$*, then* $SW\left(F_j\right) < SW\left(F_i\right)$*.*

Proposition 2 can be easily proved for every family of distributions in which $F_i \succ_{SOSD} F_j$ implies the existence of α and β such that[5] $X_j \overset{d}{=} \alpha X_i + \beta$. The intuition for the proof of Proposition 2 can be explained as follows. Consider an F_i trade game in which the seller's utility is given by $\hat{v}_s\left(x\right) = \alpha v_s\left(x\right) + \beta$, and the buyer's utility is given by $\hat{v}_b\left(x\right) = \alpha v_b\left(x\right) + \beta$. It is easy to see that since nothing of substance has been changed by this transformation, the equilibrium of this game is obtained at $\hat{p}_i$. Now consider the game in which the traded good is distributed according to $X_j = \alpha X_i + \beta$. When we compare the buyer's utility in both transformed games we can see that it is lower in the X_j game, whereas the seller's utility is the same in both games. Therefore, we know that the equilibrium is obtained at a lower realization of X_i; see Figure 2.

Note that in the baseline model, where we assume both players are risk-neutral and the gains from trade are constant, the minimization of $PT\left(F\right)$ is equivalent to the minimization of $SW\left(F\right)$. In Section 6 where we extend the model to general functional forms, we discuss the effects of the seller's risk-seeking disposition on social welfare.

Next we define a necessary and sufficient conditions, under which riskier distributions result in less trade in equilibrium.

[5] In the normal-distribution case, we have $X_j \overset{d}{=} \frac{\sigma_j}{\sigma_i} X_i - \frac{\sigma_j - \sigma_i}{\sigma_i} \mu$. In the uniform-distribution case, let $X_i \sim U\left[a_i, b_i\right]$ and $X_j \sim U\left[a_j, b_j\right]$; then we can present the riskier distribution as $X_j \overset{d}{=} \frac{b_2 - a_2}{b_1 - a_1}\left(X_i - 1\right)$.

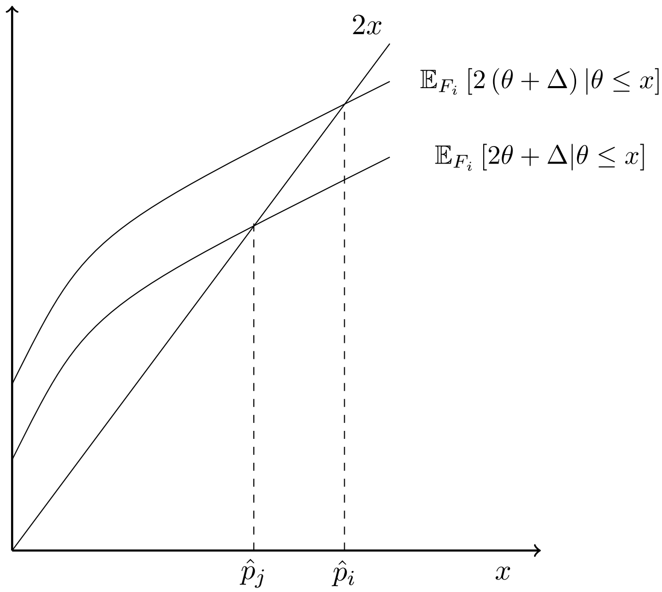

Figure 2: Higher variance implies a decrease in trade.

Definition 2 (Location-independent Risk). *We say that distribution F_i is location-independent less risky than F_j, denoted by $F_i \succ_{LIR} F_j$, if*

$$\int_0^{F_i^{-1}(q)} F_i(x)\,dx < \int_0^{F_j^{-1}(q)} F_j(x)\,dx, \tag{11}$$

for every $q \in (0,1)$. We denote a fulfillment of this inequality in a given quantile, $\tilde{q}$, by $F_i \succ_{LIR}(\tilde{q}) F_j$.

Theorem 2. *Let $F_i, F_j \in \mathcal{F}$, and $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$. Then $PT(F_i) > PT(F_j)$ for every $\Delta > 0$, if and only if $F_i \succ_{LIR} F_j$.*

As was shown by Landsberger and Meilijson (1994), in the case where $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$, $\succ_{LIR}$ implies $\succ_{SSOSD}$. That is, the seller necessarily chooses the distribution that induces less trade. In addition, let

$$\Phi_F(t) := \int_0^t F(x)\,dx. \tag{12}$$

Landsberger and Meilijson (1994) show that $F_i \succ_{LIR} F_j$ if and only if $\Phi_{F_j}^{-1}(t) - \Phi_{F_i}^{-1}(t)$ is non-decreasing in $t \in \mathbb{R}_+$.

In order to prove Theorem 2 we let $D_F(q)$ denote the difference between the expected value of the buyer and the price as a function of the quantile q, i.e., $D_F(q) := \mathbb{E}_F\left[v(x)\,|x \le F^{-1}(q)\right] - F^{-1}(q)$. The equilibrium of the F trade game is obtained at a point satisfying $D_F(q) = 0$. Our assumptions on the distributions in $\mathcal{F}$ imply that $D_F(q)$ is a decreasing function, and we show that if $F_i \succ_{LIR} F_j$ then $D_{F_i}(\hat{q}_j) > 0$. Thus, we conclude that F_j generates less trade.

We apply Theorem 2 in order to show that if the equilibrium of the safer distribution is obtained at a point $\hat{t}_i$ satisfying $F_i\left(\hat{t}_i\right) > F_j\left(\hat{t}_i\right)$ then F_j results in less trade.

Proposition 3. *Let $F_i, F_j \in \mathcal{F}$, $F_i \succ_{SSOSD} F_j$, and $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$. If $F_i\left(\hat{t}_i\right) > F_j\left(\hat{t}_i\right)$ then $SW(F_i) > SW(F_j)$.*

Note that if the antecedent in Theorem 2 is replaced with $F_i \succ_{LIR} (\hat{q}_i)\, F_j$, the consequent still follows. Thus, in order to prove Proposition 3 we need only to show that $F_i\left(\hat{t}_i\right) > F_j\left(\hat{t}_i\right)$ implies that $F_i \succ_{LIR} (\hat{q}_i)\, F_j$.

Consider now a unimodal symmetrical environment, where all distributions cross once at the median. If the equilibrium price of the riskiest distribution (which is chosen by the seller) is above the median, then, at this point, the CDF of every other distribution is above the CDF of the riskiest distribution. Thus, by Proposition 3, we have the following corollary.

Corollary 1. *Assume that $\mathcal{F}$ is SOSD-ordered and that for every $F \in \mathcal{F}$, $\mathbb{E}_F[\theta] = \mu$. Further assume that $\mathcal{F}$ consists of unimodal symmetric distributions. The probability of trade in equilibrium $PT\left(\hat{F}\right)$ satisfies at least one of the following conditions:*

- $PT\left(\hat{F}\right) = \min_{F \in \mathcal{F}} PT(F)$, *or*
- $PT\left(\hat{F}\right) \le \frac{1}{2}$.

We now examine the addition of a noise term to a distribution characterized by a decreasing density. We show that such an addition induces a decrease in trade.

Proposition 4. *Let F_i be a bounded distribution with weakly decreasing density f_i, and let $X_j \overset{d}{=} X_i + \epsilon$ for some $\epsilon \sim H[-k,k]$, where H is symmetric with zero expectation and independent of X_i. Denote by $\underline{\theta}_i$ the left bound of F_i. If $\hat{t}_i > \underline{\theta}_i + k$ then $SW(F_i) > SW(F_j)$.*

Note that F_j is not necessarily a log-concave distribution, and the stage game can accommodate a plethora of equilibria. However, the welfare in the best equilibrium of the F_j trade-game is below $SW(F_i)$.

Finally we show that the monotone cumulative probability ratio property, defined by Hopkins, Kornienko *et al.* (2003) in the context of auctions, implies that the seller's choice minimizes the equilibrium price. Minimization of the equilibrium price is not equivalent to minimization of social welfare since there are counterexamples in which a riskier distribution results in a lower price but trade volume increases. However, the probability of trade is usually lower when the price is lower, and in some cases (such as the normal-distribution one) minimization of price does imply minimization of social welfare. In addition, if the seller needs to satisfy some ex-post constraint on her income, low prices can push her out of the market.

Definition 3. *Two distributions, F_i, F_j, satisfy the monotone cumulative probability ratio (MCR) order, denoted by $F_i \succ_{MCR} F_j$, if the cumulative probability ratio*

$$C(x) := \frac{\int_0^x F_i(s)\, ds}{\int_0^x F_j(s)\, ds} \tag{13}$$

is strictly increasing.

Proposition 5. *Let $F_i, F_j \in \mathcal{F}$, and $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$. If $F_i \succ_{MCR} F_j$ then $p_i > p_j$.*

Note that this order implies SOSD, i.e., the seller prefers the dominated distribution. In addition, MCR is implied by the unimodal likelihood ratio property. That is, let $L(x) := \frac{f_i(x)}{f_j(x)}$; if $L(x)$ is unimodal then $F_i \succ_{MCR} F_j$. See Hopkins, Kornienko *et al.* (2003) for more details. Next, we introduce into our model an informative signal that allows the market to observe some information about the asset value before trade takes place.

5 An Informative Signal

Our baseline model does not allow the buyer to use outside information in order to resolve some of the uncertainty regarding the asset value. By contrast, one can easily extend Akerlof's (1970) "lemons" model to allow the buyer to update his beliefs after viewing the car: one simply needs to interpret the distribution of the asset as a description of the uncertainty unresolved by all public information. This reduction is not appropriate in our model since the joint distribution of the signal and the asset value typically depends on the seller's initial choice. Therefore, our option-value argument does not hold in the presence of a public signal and the seller might be dissuaded from perusing risk.

In order to demonstrate the general applicability of our results to Akerlof (1970) we study a normal-distribution model in which a signal $\rho = \theta + \epsilon$. We assume that $\epsilon \sim N\left(0, \sigma_\epsilon^2\right)$ and is independent of θ and the seller's choice. In this model, for each realization of the signal, the market updates its beliefs on the asset distribution and a standard Akerlof (1970) game is played.

The market price in this model is a function $p : \mathbb{R} \to \mathbb{R}$, where $p\left(s\right)$ is the equilibrium price of the stage game induced by a realization s of the signal. Let $\eta_i | \left(\rho = s\right)$ denote the market's beliefs given a realization s of the signal in the case where the buyer believes that the seller chooses F_i. It is easy to show that [6]

$$\eta_i\left(\theta\right) | \left(\rho = s\right) \sim N\left(\tilde{\mu}_i\left(s\right), \tilde{\sigma}_i^2\right). \tag{14}$$

Lemma 1 states that trade volume does not depend on the realization of the signal.

Lemma 1. *Let $F_i = N\left(\mu_i, \sigma_i^2\right)$ be the seller's equilibrium distribution choice. For every $s, s' \in \mathbb{R}$, $PT\left(N\left(\tilde{\mu}_i\left(s\right), \tilde{\sigma}_i^2\right)\right) = PT\left(N\left(\tilde{\mu}_i\left(s'\right), \tilde{\sigma}_i^2\right)\right)$.*

The variance of the buyer's beliefs does not depend on the realization of the signal, i.e., the players participate in shifts of the same trade game. Thus, the equilibrium price is obtained at the same quantile for every s. Next, we show that the seller is risk seeking also

[6] $\tilde{\mu}_i\left(s\right) = \frac{\sigma_\epsilon^2 \mu_i + \sigma_i^2 s}{\sigma_\epsilon^2 + \sigma_i^2}$, $\tilde{\sigma}_i^2 = \frac{\sigma_\epsilon^2 \sigma_i^2}{\sigma_\epsilon^2 + \sigma_i^2}$.

in this model.

Proposition 6. *Let* $F_i = N\left(\mu, \sigma_i^2\right)$, $F_j = N\left(\mu, \sigma_j^2\right)$. *If* $\sigma_j^2 > \sigma_i^2$, *then in any equilibrium of the extended trade game with the signal* ρ, $\alpha_i = 0$.

Let $X_1 \sim N\left(\mu, \sigma_1^2\right)$ and $X_2 \sim N\left(\mu, 2\sigma_1^2\right)$, and let $\pi\left(X_j, X_i\right)$ be a random variable denoting the seller's profits following a deviation to X_j under market beliefs X_i. To prove Proposition 6, let us assume to the contrary that the seller chooses X_1 in equilibrium. We show that $\mathbb{E}_{F_2}\left[\pi\left(X_2, X_1\right)|\rho \in \{-s, s\}\right] > \mathbb{E}_{F_1}\left[\pi\left(X_1, X_1\right)|\rho \in \{-s, s\}\right]$ for every s, and our claim follows. The intuition for this inequality is demonstrated in Figure 3.

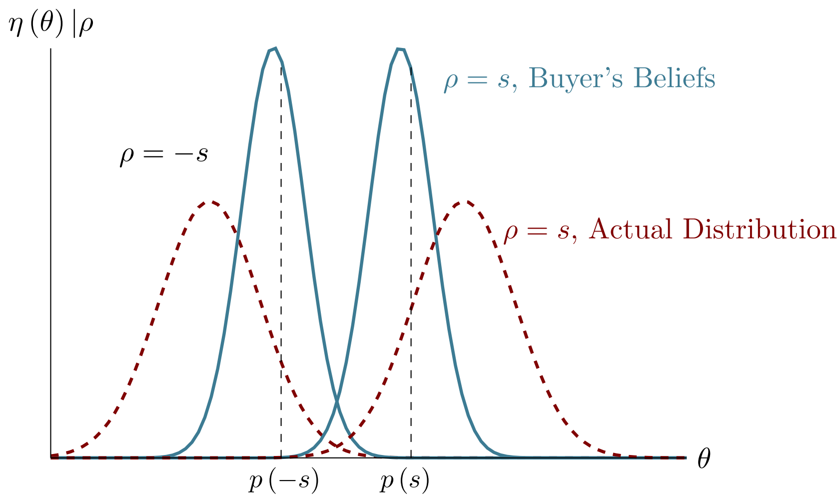

Figure 3: A Deviation to X_2

Conditional on every realization of the signal, the variance of to the asset value following an unobservable deviation of the seller to X_2, is higher than the buyer believes. In addition, the relative informativeness of ρ is higher and therefore the conditional expectation is weighted more toward the signal.

Note that we can divide this difference to two. First, conditional on each realization we add a noise term; see Figure 4. As we have already shown, such an addition strictly benefits the seller due to the option value of her equilibrium payoff.

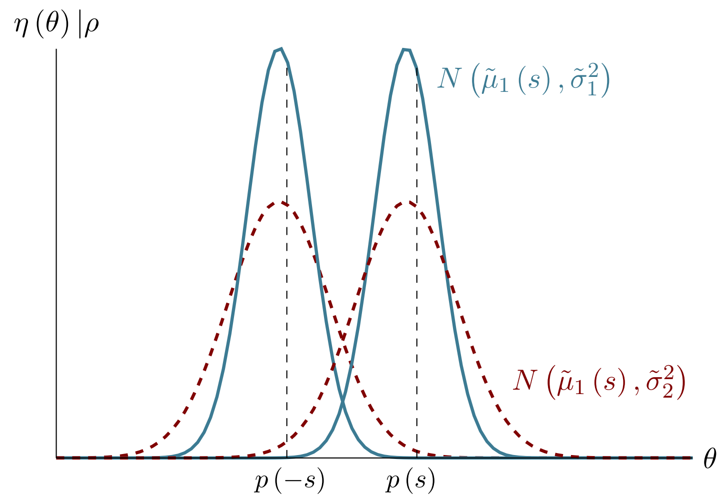

Figure 4: A Deviation to X_2: More Risk

In addition, the buyer's beliefs are obtained by a rightward shift of the good state distribution, which increases the seller's payoff, and a leftward shift of the bad state distribution, which generates losses for the seller. However, we can show that the profits are strictly higher than the losses. A rightward shift of a distribution F by a constant d generates additional profits larger than $(1 - PT(F))\,d$. Any realization consumed in the source distribution is consumed in the shifted distribution, and in each case the seller's profits increase by d. In contrast, the losses caused by a leftward shift of distribution F are smaller than $(1 - PT(F))\,d$. Any realization that is sold in the source distribution is sold in the shifted distribution too. In this case the seller incurs no losses, and for any realization that is consumed in the source distribution the seller looses at most d; see Figure 5.

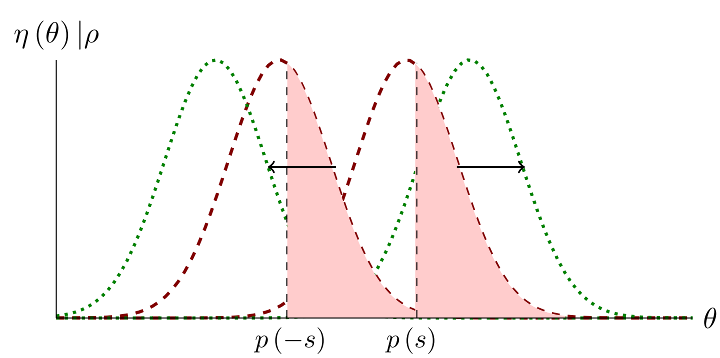

Figure 5: A Deviation to X_2 - Distributions Shift

It is easy to see that our social welfare characterization also follows and trade decreases due to the seller's risk-seeking disposition. Note however that the implications of the seller's behavior on trade become less and less meaningful as $\sigma_\epsilon^2 \to 0$. In the limit all asymmetric information considerations vanish since the asset value is observable, i.e., $\lim_{\sigma_\epsilon^2 \to 0} PT\left(N\left(\tilde{\mu}_i(s), \tilde{\sigma}_i^2\right)\right) = 1$.

6 Extensions

We study three extensions of our baseline model. First, we investigate the role of verifiability in our results. Second, we study an alternative model in which the market power is in the buyer's hands. Finally, we study general utility functional forms.

6.1 Disclosure and Adverse Selection

We apply Dye's (1985) model into the extended trade game by giving a mass $\beta \in (0, 1)$ of the sellers the opportunity to verify the value of their product before trade. In general, Dye's (1985) model has two possible interpretations of the agents' informativeness: the set of agents who cannot verify their product can be either informed or uninformed. Although in Dye (1985) this distinction is not important, when the model is extended to a trade model this distinction plays an important role, since an informed seller's strategy depends on the state of nature. In our context each reading of Dye (1985) can make sense, and we provide results for both.

Partly Informed Sellers

Assume now that the realization of the project is privately observed by the seller with probability $\beta \in (0, 1)$, in which case she can disclose the value verifiably; otherwise, the seller knows only which distribution she has chosen in the first stage. As in Dye (1985) we assume that the seller's verifiability and the asset's value are independent. The stage game in this model has two candidate equilibria. In each one an informed seller trades since she can always prove the value of the asset and make a profit of $\Delta > 0$. The only question

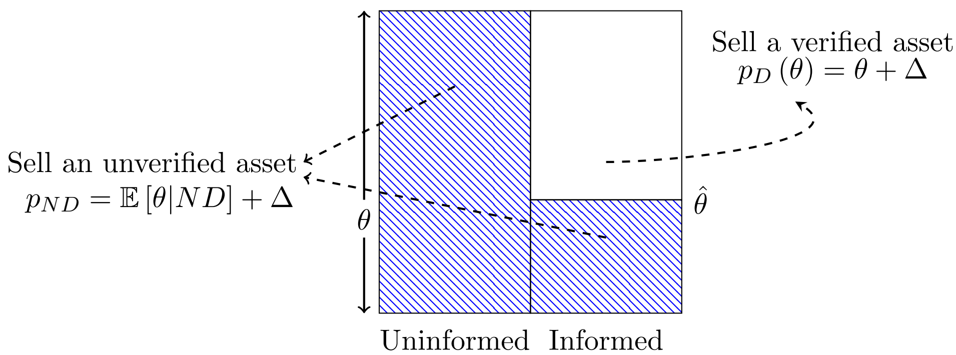

Figure 6: Full-Trade Equilibrium

is whether the price of an unverified asset is high enough to allow uninformed sellers to participate in the market.

In the first candidate equilibrium, sellers above some threshold type $\hat{\theta}$ disclose their value and are paid $\theta + \Delta$. Types below $\hat{\theta}$ are pooled with the uninformed types, and they sell an unverified asset; see Figure 6. In the other candidate equilibrium the market unravels. That is, all informed sellers disclose, and uninformed sellers are pushed out of the market. In this equilibrium a seller with an unverified asset is believed to be the lowest type, and uninformed sellers indeed prefer to consume. It is easy to see that, as long as the gains from trade are below the unconditional value, the unraveling equilibrium exists. That is, given a distribution F chosen by the seller according to market beliefs, if $\Delta < \mathbb{E}_F[\theta]$ then the market can unravel. Full trade may be part of an equilibrium only if the no-disclosure price is above the uninformed seller's value from self-consumption. Since the full trade equilibrium produces an option value for the seller, she is tempted to choose risky distributions. This lowers the equilibrium price of the unverified asset, thereby jeopardizing the existence of the full trade equilibrium.

Theorem 1 implies that the seller is risk-seeking, Thus, if the seller's hidden actions can introduce more risk to the asset distribution the equilibrium price is lower[7]. In addition, we show that DeMarzo, Kremer and Skrzypacz's (2019) minimum principle implies that the seller chooses the distribution with the lowest price for no-disclosure. That is, even if the

[7] Jung and Kwon (1988) already note that second-order stochastic dominance implies a lower no-disclosure prize in Dye's (1985) model; i.e., if $F_i \succ_{SOSD} F_j$ then $p_i^{ND} > p_j^{ND}$.

different distributions are noncomparable in the sense of second-order stochastic dominance, we can state that the seller's behavior brings about a decrease in trade.

Let

$$z_F(t) := \frac{\beta F(t)\,\mathbb{E}_F[\theta|\theta \le t] + (1-\beta)\,\mathbb{E}_F[\theta]}{\beta F(t) + (1-\beta)}. \tag{15}$$

In the second stage of the game, if the full-trade equilibrium is played, then the price of unverified good is

$$p_F^{ND} = z_F\left(\hat{\theta}_F\right) + \Delta. \tag{16}$$

Lemma 2 (DeMarzo, Kremer and Skrzypacz (2019)). *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$. If $p_i^{ND} > p_j^{ND}$, then in any full trade equilibrium $\alpha_i = 0$.*

Note that in order for the full trade equilibrium to exist, the uninformed seller should be incentivized to participate in the market. That is,

$$p_F^{ND} \ge \mathbb{E}_F[\theta]. \tag{17}$$

Thus, we obtain the following result.

Proposition 7. *Assume that $\mathbb{E}_F[\theta] = \mu$ for every $F \in \mathcal{F}$. If $\min_{F\in\mathcal{F}} \min_{t\in\mathbb{R}} z_F(t) + \Delta < \mu$, then in equilibrium uninformed sellers do not trade.*

Proposition 7 shows that if there exist a distribution and a disclosure strategy that induce a no-disclosure price below the unconditional mean, then there is no equilibrium in which uninformed sellers participate in the market. In such an equilibrium, the seller chooses the distribution that minimizes the no-disclosure price, and if this price is below the mean then the uninformed sellers prefer to not participate in the market.

Informed Sellers

We turn now to the second reading of Dye (1985) regarding the agents' informativeness. We can interpret Dye's (1985) model as one in which all sellers are informed while only some

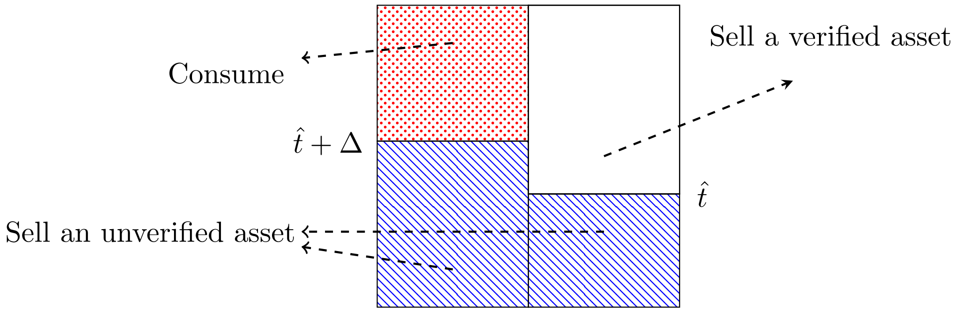

Figure 7: Equilibrium

of them hold hard evidence to back up their information. Unlike in Crawford and Sobel (1982) and other cheap-talk models, the seller's preferences do not depend on the state. Therefore, no information can be transmitted in equilibrium unless evidence is presented.

Assume the seller is choosing a distribution F in equilibrium. The stage-game equilibrium in this model is unique, and is characterized by a partition of the seller's type set into three. High types with evidence disclose and sell the asset for a price of $\theta+\Delta$. High types without evidence consume. The remaining types pool and sell their good in the "unverified market" for the equilibrium price; see Figure 7. That is, the equilibrium of the stage-game is defined by two threshold seller's types. A seller that cannot verify her produce has two alternatives: she can either consume their unverified asset or she can sell it. Thus, in equilibrium the highest such type who sells is defined by $\tilde{t}_F=\tilde{p}_F^{ND}$. A seller that can verify her product has an additional alternative that dominates self-consumption. Therefore, the highest type who can verify the asset and sells an unverified good is defined by $\hat{t}_F+\Delta=\tilde{p}_F^{ND}$. As we can see, $\tilde{t}_F=\hat{t}_F+\Delta$. That is, if the market believes that the seller chooses distribution F, then the equilibrium price of an unverified asset is

$$\tilde{p}_F^{ND}=\frac{\beta F\left(\hat{t}_F\right)\mathbb{E}\left[\theta|\theta\leq\hat{t}_F\right]+(1-\beta)\,F\left(\hat{t}_F+\Delta\right)\mathbb{E}\left[\theta|\theta\leq\hat{t}_F+\Delta\right]}{\beta F\left(\hat{t}_F\right)+(1-\beta)\,F\left(\hat{t}_F+\Delta\right)}+\Delta. \tag{18}$$

We now state a result on equilibrium price minimization analogous to Proposition 5 of the baseline model. Denote by F^t the distribution F truncated at t, i.e., $F_t\left(x\right):=\frac{F(x)}{F(t)}$.

Proposition 8. *Let* $F_i,F_j\in\mathcal{F}$, $\mathbb{E}_{F_i}\left[\theta\right]=\mathbb{E}_{F_j}\left[\theta\right]$. *If* $F_i^t\succ_{SOSD}F_j^t$ $\forall t\in\mathbb{R}$, *then* $\tilde{p}_i^{ND}>$

$\tilde{p}_j^{ND}$.

Let $P(x) := \frac{F_i(x)}{F_j(x)}$. It is easy to show that if $P(x)$ is unimodal, then $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$ and $F_i \succ_{SOSD} F_j$, imply that $F_i^t \succ_{SOSD} F_j^t$ $\forall t \in \mathbb{R}$. Finally, as was noted above, price minimization is not equivalent to welfare minimization, but can imply welfare minimization under additional assumptions such as normal distribution.

6.2 A Monopolistic Buyer

Following Akerlof (1970), our baseline model assumes that all market power is concentrated in the seller's hands. We now extend our analysis to the case in which the buyer is a monopolist. We show that also under this assumption the seller's risk-seeking disposition generates "lemon" markets.

It is easy to see that also in this case the seller is risk-loving. The seller's equilibrium payoff has an option value for any price, and the way in which the price is set is irrelevant to the seller's considerations. Before we show the detrimental effects of the seller's choices on trade, we provide a concise analysis of the F trade game with a monopolistic buyer.

A monopoly sets a price in order to maximize its expected profit. That is, in the F trade game the price is given by

$$\max_p F(p)\left(\mathbb{E}_F[v_b(x)|x \leq p] - p\right). \tag{19}$$

From our previous analysis, we have

$$\mathbb{E}_F[v_b(x)|x \leq p] = v_b(p) - \frac{\int_0^p v_b'(x)F(x)dx}{F(p)}. \tag{20}$$

Plugging (20) into (19), we can write the buyer's maximization problem as

$$\max_p F(p)\left(v_b(p) - p\right) - \int_0^p v_b'(x)F(x)dx. \tag{21}$$

Thus, an interior solution $p_F^\star$ to the buyer's maximization problem satisfies

$$\frac{f(p_F^\star)}{F(p_F^\star)} = \frac{1}{v_b(p_F^\star) - p_F^\star}. \tag{22}$$

The reversed hazard rate of a distribution $\frac{f(x)}{F(x)}$ is decreasing if and only if the distribution F is log-concave (see for example Nanda and Sengupta, 2005.) That is, log-concavity of the distribution F is a sufficient condition for the uniqueness of the buyer's maximization problem.

Denote the unique solution to the monopolist's problem by $p_F^\star$, and let $\widetilde{SW}(F)$ denote the social welfare obtained by this price. For every distribution F, the monopoly's optimal choice is a price below the equilibrium price and thus trade decreases. We show that the seller's risky choices exacerbates adverse selection also in this case.

First, we provide a condition under which the riskier distribution results in less trade.

Definition 4. *F_j is more dispersed than F_i if the horizontal distance*

$$F_i^{-1}(p) - F_j^{-1}(p) \tag{23}$$

is a non-increasing function on the interval $(0,1)$. If F_j is more dispersed than F_i, and both distributions have the same mean, F_j is said to be monotone increasing in risk with respect to F_i, which is denoted by $F_i \succ_{MIR} F_j$.

Dispersion is location free; i.e., it is preserved if each random variable is shifted by the addition of a positive constant. Let $\hat{u}$ be a non-decreasing utility function that displays a higher Arrow–Pratt coefficient than a non-decreasing utility function u. Landsberger and Meilijson (1994) proved that if a decision maker whose preferences are represented by u is willing to buy a partial insurance contract that makes the insured position less dispersed than the uninsured position, then any decision maker with a utility function $\hat{u}$ is willing to buy it too.

Proposition 9. *Let F_i, F_j be two distributions such that $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$. Then $\widetilde{SW}(F)$ for every $\Delta > 0$ if and only if $F_i \succ_{MIR} F_j$.*

Finally, we show that if the equilibrium of a riskier distribution is obtained at a price in which the safer distribution CDF is above the riskier one, then the seller's profits are lower in the riskier distribution. Let $\Pi(F)$ denote the seller's profits in the equilibrium of the F trade game with a monopolistic buyer.

Proposition 10. *Let F_i, $F_j \in \mathcal{F}$, $F_i \succ_{SSOSD} F_j$, and $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$. If $F_i\left(p_j^\star\right) > F_j\left(p_j^\star\right)$ then $\Pi(F_i) > \Pi(F_j)$.*

6.3 General Utility Functions

Thus far, we have assumed that both players are indifferent to risk and that the gains from trade are constant. We now discuss more general functional forms.

First, note that the assumption $v_s(\theta) = \theta$ can be viewed as a normalization, and whichever functional form the seller's utility takes, all assumptions on the distributions in $\mathcal{F}$ can be read as assumptions on the distribution of the seller's utility. In this context we can focus our discussion on two cases: either the buyer is more risk-averse than the seller, i.e., v_b is a concave function, or the converse holds and v_b is convex.

Risk-averse Buyer

It is easy to show that our main results hold in this case. First, the seller's risk-seeking property is not affected by the buyer's risk aversion. In addition, we can show that as long as the gains from trade $(v_b(\theta) - \theta)$ are increasing, the seller's behavior minimizes social welfare.

Let the first-best be a counterfactual in which the asset's value is observed and therefore the seller chooses optimally and trade materializes for every realization. For every distribution F, let $V(F) := \mathbb{E}_F[v_b(\theta)]$ denote the social welfare in the case where the asset is traded with probability 1. Comparing the social welfare in equilibrium $(SW^\star)$ and the social welfare in the first-best (FB), we state the following result:

Proposition 11. *Let $F_i, F_j \in \mathcal{F}$, $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$ and $F_i \succ_{SSOSD} F_j$. If v_b is strictly concave, then $FB - SW^\star > V(F_i) - V(F_j)$.*

In the first-best $V(F_i)$ is attainable since there is no asymmetric information, while the social welfare in equilibrium is by definition below the potential social welfare of the chosen distribution.

Risk-averse Seller

If we reverse this assumption and assume that the buyer is less risk-averse then our results no longer carry through. First, note that in this case some risk-seeking by the seller is socially desirable. However, we can show that the seller still pursues unnecessary risk. No matter how risky a distribution is, the seller's utility in equilibrium is strictly convex around the price. Thus, the seller is still incentivized to choose unnecessarily risky projects and might bring about a decrease in trade.

7 Discussion and Conclusion

We conclude this paper by considering the implications of some changes to our assumptions and by discussing the significance of our results.

Sub-optimal Choices

Throughout this work we have refrained from showing that the seller makes a suboptimal choice. We next demonstrate that the conditions required in order to do so are minimal.

Proposition 12. *Let* $\tilde{F} = \arg\max_{F \in \mathcal{F}} SW(F)$. *If there exists* $F \in \mathcal{F}$ *for which* $F(p_{\tilde{F}}) < \tilde{F}(p_{\tilde{F}})$ *and* $\mathbb{E}_F[X|X \geq p_{\tilde{F}}] > \mathbb{E}_{\tilde{F}}[X|X \geq p_{\tilde{F}}]$, *then in any equilibrium of the extended trade game* $\alpha_{\tilde{F}} < 1$.

Note that if $\tilde{F}$ has a low enough variance such that in the equilibrium of the $\tilde{F}$ trade game there is trade with probability that is close to 1, then Proposition 12 states the following: if there is some other distribution with realizations above the price of the $\tilde{F}$ trade game, then there is no equilibrium in which the seller's choice is optimal.

Partly Observable Distribution Choice

We have assumed that the seller's choice is unobservable. If the seller's choice is observable then her equilibrium choice is efficient, as was noted above (Section 4). A natural question in this context is the robustness of the risk-seeking disposition to a partly informative signal regarding the seller's initial choice. First, note that the less risky distribution can be the seller's choice in equilibrium only if the signal does not have full support in all states. That is, assume, for example, that the seller chooses between X_1 and X_2 from Example[8] 1, and let the seller observe a signal $\gamma \in \{\gamma_1, \gamma_2\}$. If $\Pr[\gamma = \gamma_i | X_1] > 0$ for $i = 1$ and $i = 2$, then the seller does not choose X_1 equilibrium. A deviation from such a strategy is undetectable and has no effect on the buyer's beliefs, and thus our result carries through. If, however, $\Pr[\gamma = \gamma_2 | X_1] = 0$ and $\Pr[\gamma = \gamma_2 | X_2] > 0$, then the seller might be incentivized to choose X_1 in equilibrium. Denote the probability of detection ($\Pr[\gamma = \gamma_2 | X_2]$) by χ; then the effect of the signal increases with χ, as expected. In this example, if $\chi > \frac{1-2\Delta}{3-2\Delta}$, then there is an equilibrium in which the seller chooses X_1. Note that the threshold χ decreases when the gains from trade increase since the threat of losing them with some probability becomes more likely.

A Counterexample

As discussed above, a riskier distribution does not necessarily lead to a decrease in trade in equilibrium. We now characterize the types of mean-preserving spreads that generate counterexamples, i.e., situations in which adding a random variable with expectation 0 to the realization of some distribution induces more trade. Let $X_i \sim F_i$ be a random variable, and let $X_j = X_i + \epsilon$. Assume, for simplicity, that for every realization x_i, it is the case that $\epsilon(x_i) = \epsilon^-(x_i) \leq 0$ with probability $\beta(x_i)$, and $\epsilon(x_i) = \epsilon^+(x_i) \geq 0$ with the complementary probability. One can construct a counterexample by assuming that ϵ^- and ϵ^+ are non-null only for realizations above the price, for which the "bad" realization $\epsilon^-(x_i) = p_i$. The equilibrium price in the spread distribution is the same as in source one, and thus the

[8] Recall that $X_1 = \begin{cases} \frac{1}{2} & \text{w.p. } 1, \end{cases}$ $X_2 = \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2}. \end{cases}$

probability of trade is higher. In fact, as long as ϵ^- is between the buyer's and the seller's valuations, the probability of trade is higher in the equilibrium of the spread distribution.

Trade and the State of the Economy

The failure of many financial markets to provide liquidity was well documented following the 2007–2008 financial crisis; see, Gorton (2009) and Chui, Domanski, Kugler and Shek (2010). Similar episodes were documented in other crises, e.g., Noyes (1909). The link between this phenomenon and adverse selection in an increasing gains from trade environment is immediate: if in a bad state of the world the distribution of the asset is shifted to the left, and thus the gains from trade are smaller, trade volume in equilibrium will be lower; see Philippon and Skreta (2012) and Tirole (2012) for an analysis of optimal intervention in such a model. Under reasonable assumptions the divergence of trade volumes in the different states of the economy will be larger if the seller chooses a riskier distribution, and our analysis suggests that a seller, with more discretion with respect to her actions prior to the trade, will bring about a higher dispersion of trade in the different states of the economy. This line of modeling can be employed in order to empirically substantiate our findings in future research.

Conclusion

In this paper we have provided reasons to suspect that lemon markets are generated endogenously. The option-value structure of the seller's equilibrium payoff in an Akerlof (1970) model implies that, as long as her choices prior to the trade are unobservable, the seller makes risky choices. As we have shown, under quite general assumptions the seller's equilibrium behavior implies a substantial decrease in trade.

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A PROOFS

Proof of Proposition 1 There is a finite set of projects (random variables) $X = \{X_1, X_2, ..., X_n\}$. For every $X_i \in X$, denote by p_i the price in the unique equilibrium of the trade stage game when F_i is chosen, i.e., $E[x_i + \Delta \mid x_i \leq p_i] = p_i$. Rearrange the set $\mathcal{F}$ such that if $i < j$ then $p_i < p_j$. For every $F_i \in \mathcal{F}$ define the function $u_i(p) := F_i(p)p + (1 - F_i(p))E[x_i \mid x_i > p]$. This function takes as an input a price p and returns the utility of a seller who's asset is distributed according to F_i and has the option of selling it at the price p. It is easy to verify that for every $i \in \{1, 2, ..., n\}$ u_i is continuous. For every price $p \in [p_1, p_2]$ define the set $Max(p) := \{i \mid u_i(p) \geq u_j(p) \forall j \in \{1, 2, ..., n\}\}$, in addition define $i(p) := min[Max(p)]$. Lastly, define the function $H : [p_1, p_n] \longmapsto \{p_1, p_2, ..., p_n\}$ to be $H(p) := p_{i(p)}$. Note that every discontinuity of H corresponds to an intersection between u_i functions at the frontier. It is clear that for every $i \in \{1, 2, ..., n\}$ if $H(p_i) = p_i$ then the project X_i is a pure equilibrium of the extended trade game. Let us assume that this is not the case, that is, for every $i \in \{1, 2, ..., n\}$ $H(p_i) \neq p_i$. Notice that it follows that $H(p_1) > p_1$ and $H(p_n) < p_n$. Under this assumption we prove the next Lemma.

LEMMA 3. *There exist a discontinuity point of H, $p \in (p_1, p_n)$ such that there exists a left neighborhood of p with $H(p') > p'$ for all p' in this neighborhood, and there exists a right neighborhood of p with $H(p') < p'$ for all p' in this neighborhood.*

Proof. Because we have that $H(p_1) > p_1$ and $H(p_n) < p_n$ and we assumed that there is no pure equilibrium it must be that the step function H and the identity function "cross" in the manner stated in the Lemma. This follows directly from the Intermediate Value Theorem when one of the functions is a step function and the other one is continuous. ■

Denote the price from the previous Lemma by p^*. From the assumption that there is no pure equilibrium it must be the case that $p^* \neq p_i$ for every $i \in \{1, 2, ..., n\}$. We also know that there exist a left neighborhood of p^* in which $H(p') > p^*$ for every p' in this neighborhood, and that H is a step function so it is constant on a small enough neighborhood. There is an index $i \in \{1, 2, ..., n\}$ such that $H(p') = p_l$ in this neighborhood.

In the same manner there exists an index $k \in \{1,2,...,n\}$ such that $H(p') = p_k$ at a small enough right neighborhood of p^*. It is clear that $l > k$. We now argue that there is a mix between the projects X_l, X_k that is an equilibrium of the extended trade game.

LEMMA 4. *There exists an $\alpha \in (0,1)$ such that the price in the trade game with the project $\alpha X_l + (1-\alpha)X_k$ is p^*.*

Proof. We know $p_l > p^* > p_k$, and it follows that $E[x_l \mid x_l \leq p^*] > p^*$ and $E[x_k \mid x_k \leq p^*] < p^*$. It is clear that there exists an $\alpha \in (0,1)$ such that $\alpha E[x_l \mid x_l \leq p^*] + (1-\alpha)E[x_k \mid x_k \leq p^*] = p^*$. ∎

From the first Lemma we also have that $u_l(p^*) = u_k(p^*)$ and that $\{l,k\} \in argmax_i u_i(p^*)$. This ends the proof.

Proof of Theorem 1 We need to show $\alpha_i = 0$. Assume by the way of contradiction $\alpha_i > 0$, and let H_α denote the cumulative distribution of θ given the first-stage strategy α. That is, $H_\alpha(\theta) := \sum_i \alpha_i F_i(\theta)$.

The utility of the seller in equilibrium is

$$R_{H_\alpha}(p) = H_\alpha(p)\,p + \int_p^\infty x h_\alpha(x)\,dx \tag{24}$$

where (integration by parts)

$$R_{H_\alpha}(p) = \int_0^p x h_\alpha(x)\,dx + \int_0^p H_\alpha(x)\,dx + \int_p^\infty x h_\alpha(x)\,dx. \tag{25}$$

Put

$$\mathbb{E}_{H_\alpha}[x] = \int_0^p x h_\alpha(x)\,dx + \int_p^\infty x h_\alpha(x)\,dx \tag{26}$$

with (24) and (25) and you have:

$$R_{H_\alpha}(p) = \mathbb{E}_{H_\alpha}[X] + \int_0^p H_\alpha(x)\,dx. \tag{27}$$

Now, consider the strategy α' obtained by moving all mass of play from F_i to F_j. That is, $\alpha'_i = 0$ and $\alpha'_j = \alpha_i + \alpha_j$. Since α is an equilibrium strategy we have

$$R_{H_\alpha}(p) \geq R_{H_{\alpha'}}(p) \tag{28}$$

That is,

$$\mathbb{E}_{H_\alpha}[x] + \int_0^p H_\alpha(x)\,dx \geq \mathbb{E}_{H_{\alpha'}}[x] + \int_0^p H_{\alpha'}(x)\,dx \tag{29}$$

Thus, we have obtained a contradiction, since we assumed $\mathbb{E}_{H_{\alpha'}}[X] = \mathbb{E}_{H_\alpha}[X]$, and by strong second order stochastic dominance we know $\int_0^p H_\alpha(x)\,dx < \int_0^p H_{\alpha'}(x)\,dx \ \forall x$.

Proof of Proposition 2 We prove Proposition 2 for any weakly concave v_b for which the gains from trade are increasing in x.

In equilibrium of the F_i trade game the following holds:

$$\mathbb{E}_{F_i}[v_b(x)\,|x \leq \hat{p}_i] = \hat{p}_i \tag{30}$$

Consider the affine transformations to both players' utility function, $\tilde{v}_l(x) = kv_l(x) - (k-1)\mu$ for $l \in \{s, b\}$.

The equilibrium of the F_i trade game in which those are the players' utilities is described by

$$\mathbb{E}_{F_i}[kv_b(x) - (k-1)\mu|x \leq \hat{p}_i] = k\hat{p}_i - (k-1)\mu \tag{31}$$

That is, $\hat{p}_i$ does not change since this transformation does not change any aspect of economic importance. Now, since F_i and F_j are normal distribution, $X_j \overset{d}{=} kX_i - (k-1)\mu$, i.e., the equilibrium of the F_j trade game is defined by

$$\mathbb{E}_{F_i}[v_b(kx - (k-1)\mu)\,|x \leq \hat{p}_i] = k\hat{p}_i - (k-1)\mu \tag{32}$$

Claim 3. *$v_b(kx - (k-1)\mu) < kv_b(x) - (k-1)\mu$ for every x.*

Proof. $v_b(x) - x$ is an increasing function by assumption, and therefore $v_b(kx) - kx >$

$v_b(kx-(k-1)\mu)-(kx-(k-1)\mu)$. Rearrange and obtain

$$v_b(kx-(k-1)\mu) < v_b(kx)-(k-1)\mu \tag{33}$$

We are left to show that $v_b(kx)-(k-1)\mu < kv_b(\theta)-(k-1)\mu$. Since v_b is concave we know $\frac{v_b(kx)+v_b(0)}{k} < v_b(x)$, and $v_b(0) > 0$ as we assumed that the gains from trade are positive. ■

Claim 3 implies that the LHS in equation (32) is smaller than the LHS in equation (31), while the RHS is the same in both. Hence, the equilibrium of the F_j trade game results in a smaller realization of X_i. Applying Lemma 11 we have proven Proposition 2.

Proof of Theorem 2 We prove the theorem in two steps:

LEMMA 5. *$PT(F_i) \geq PT(F_j)$ For every $\Delta > 0$ if and only if $E_{F_i}[x \mid x \leq F_i^{-1}(q)] - E_{F_j}[x \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q)$ for every $q \in (0,1)$.*

Proof. Denote by $q_k(\Delta) := PT(F_k)(\Delta)$. It is clear that for every log-concave CDF F_k and every $q \in (0,1)$ there exist $\Delta > 0$ such that $q_k(\Delta) = q$. Assume that for every $q \in (0,1)$ we have that:

$$E_{F_i}[x \mid x \leq F_i^{-1}(q)] - E_{F_j}[x \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q) \tag{34}$$

This is true if and only if for every $q \in (0,1)$ we have that:

$$E_{F_i}[x \mid x \leq F_i^{-1}(q)] - F_i^{-1}(q) \geq E_{F_j}[x \mid x \leq F_j^{-1}(q)] - F_j^{-1}(q) \tag{35}$$

In particular we have that:

$$E_{F_i}[x \mid x \leq F_i^{-1}(q_j(\Delta))] - F_i^{-1}(q_j(\Delta)) \geq E_{F_j}[x \mid x \leq F_j^{-1}(q_j(\Delta))] - F_j^{-1}(q_j(\Delta)) = 0 \tag{36}$$

It follows that $q_i(\Delta) \geq q_j(\Delta)$. Now assume that there exists $\hat{q} \in (0,1)$ such that:

$$E_{F_i}[x \mid x \leq F_i^{-1}(\hat{q})] - F_i^{-1}(\hat{q}) < E_{F_j}[x \mid x \leq F_j^{-1}(\hat{q})] - F_j^{-1}(\hat{q}) \tag{37}$$

We mentioned earlier that there exist $\hat{\Delta} > 0$ such that $q_j(\hat{\Delta}) = \hat{q}$. It follows that $q_i(\hat{\Delta}) < q_j(\hat{\Delta})$. ■

We now move to the second step:

Lemma 6.

$$E_{F_i}[x \mid x \leq F_i^{-1}(q)] - E_{F_j}[x \mid x \leq F_j^{-1}(q)] \geq F_i^{-1}(q) - F_j^{-1}(q)$$

if and only if

$$\int_0^{F_i^{-1}(q)} F_i(x)dx \leq \int_0^{F_j^{-1}(q)} F_j(x)dx$$

Proof. From our analysis above we have that for every $K \in \{F_i, F_j\}$ and every $q \in [0,1]$:

$$E_K[x \mid x \leq K^{-1}(q)] = K^{-1}(q) - \frac{\int_0^{K^{-1}(q)} K(x)dx}{q} \tag{38}$$

This allows us to rewrite the condition in Lemma 6:

$$F_i^{-1}(q) - \frac{\int_0^{F_i^{-1}(q)} F_i(x)dx}{q} - (F_j^{-1}(q) - \frac{\int_0^{F_j^{-1}(q)} F_j(x)dx}{q}) > F_i^{-1}(q) - F_j^{-1}(q) \tag{39}$$

Rearranging we get:

$$\int_0^{F_i^{-1}(q)} F_i(x)dx \leq \int_0^{F_j^{-1}(q)} F_j(x)dx \tag{40}$$

■

Proof of Proposition 3 We show that the sufficient condition in Theorem 2 holds, that is,

$$\int_0^{\hat{t}_1} F_i\left(x\right) dx < \int_0^{F_j^{-1}\left(F_i\left(\hat{t}_1\right)\right)} F_j\left(x\right) dx. \tag{41}$$

Since we have $F_i\left(\hat{t}_1\right) > F_j\left(\hat{t}_1\right)$ it follows that $F_j^{-1}\left(F_i\left(\hat{t}_1\right)\right) > \hat{t}_1$. In addition, $F_j(x) > 0$ for every x, thus

$$\int_0^{\hat{t}_1} F_j(x)\,dx < \int_0^{F_j^{-1}(F_i(\hat{t}_1))} F_j(x)\,dx. \tag{42}$$

Finally, since F_j is second-order stochastically dominated by F_i we have

$$\int_0^{\hat{t}_1} F_i(x)\,dx < \int_0^{\hat{t}_1} F_j(x)\,dx \tag{43}$$

Put (42) and (43) together we obtain (41), and thus we have proven Proposition 3.

Proof of Proposition 4 We prove the theorem for the more general case in which v_b is concave and $v_b(x) - x$ is strictly increasing. We prove the proposition with the help of two lemmas.

LEMMA 7. *For every* $x \in X_i$ *with* $\hat{t}_i \geq \underline{\theta}_i + k$ *it holds that* $F_i(x) \geq F_j(x)$.

Proof. Select some $x^* \in X_i$ with $x^* \geq \underline{\theta}_i + k$, we need to prove that:

$$Pr_i := \int_{x^*}^{x^*+k} H(x^* - x) f_i(x) dx \leq \int_{x^*-k}^{x^*} (1 - H(x^* - x)) f_i(x) dx := Pr_o \tag{44}$$

Because H is symmetric and because $x^* \geq \underline{\theta}_i + k$ we have that for every x_2 with $x^* < x_2 \leq x^* + k$ we can find x_1 such that $x^* - k \leq x_1 < x^*$ and $x^* - x_1 = x_2 - x^*$. Because f_i is decreasing density we can conclude that for such couple $\{x_1, x_2\}$ the contribution of x_1 to the RHS in (44) is larger than the contribution of x_2 to the LHS in (44). This coupling argument ends the proof of the Lemma. ■

LEMMA 8. *For every* $x^* \in X$ *with* $x^* \geq \underline{\theta}_i + k$ *it holds that* $E_{F_i}[v_b(x) \mid x \leq x^*] > E_{F_j}[v_b(x) \mid x \leq x^*]$.

Proof. It is easy to verify that we can wright $E_{F_j}[v_b(x) \mid x \leq x^*]$ in the following way:

$$E_{F_j}[v_b(x) \mid x \leq x^*] = E_{F_i}[v_b(x) \mid x \leq x^*] + \int_{x^*}^{x^*+k} H(x^*-x)f_i(x)(E_H[v_b(x+\epsilon) \mid \epsilon \leq x^*-x])dx - \\ - \int_{x^*-k}^{x^*} (1-H(x^*-x))f_i(x)(E_H[v_b(x+\epsilon) \mid \epsilon \geq x^*-x])dx$$

We want to show that:

$$\int_{x^*}^{x^*+k} H(x^*-x)f_i(x)(E_H[v_b(x+\epsilon) \mid \epsilon \leq x^*-x])dx - \\ - \int_{x^*-k}^{x^*} (1-H(x^*-x))f_i(x)(E_H[v_b(x+\epsilon) \mid \epsilon \geq x^*-x])dx < 0 \tag{45}$$

We can use the coupling argument again (we couple every $x_2 \in (x^*, x^*+k)$ with $x_1 \in (x^*-k, x^*)$ such that $x^*-x_1 = x_2-x^*$) in order to rewrite (45):

$$\int_{x^*}^{x^*+k} (H(x^*-x)f_i(x)(E_H[v_b(x+\epsilon) \mid \epsilon \leq x^*-x]) - \\ -(H(x^*-x)f_i(2x^*-x)(E_H[v_b(2x^*-x+\epsilon) \mid \epsilon \geq x-x^*])))dx < 0 \tag{46}$$

Rearranging we get:

$$\int_{x^*}^{x^*+k} (H(x^*-x)[f_i(x)(E_H[v_b(x+\epsilon) \mid \epsilon \leq x^*-x]) - \\ -f_i(2x^*-x)(E_H[v_b(2x^*-x+\epsilon) \mid \epsilon \geq x-x^*])])dx < 0 \tag{47}$$

Now because the density f_i is decreasing we have that for every $x \in (x^*, x^*+k)$:

$$f_i(2x^*-x) = f_i(x-2(x-x^*)) > f_i(x) \tag{48}$$

In addition we have that for every $x \in (x^*, x^*+k)$:

$$E_H[v_b(x+\epsilon) \mid \epsilon \leq x^*-x]) \leq v_b(x^*) \tag{49}$$

$$E_H[v_b(2x^*-x+\epsilon) \mid \epsilon \geq x-x^*]) \geq v_b(x^*) \tag{50}$$

It follows that (47) is true and therefore we get that:

$$E_{F_j}[v_b(x) \mid x \leq x^*] < E_{F_i}[v_b(x) \mid x \leq x^*] \tag{51}$$

■

From the last Lemma we get that $\hat{t}_i > \hat{t}_j$ and from the first Lemma we get that $F_i(\hat{t}_i) > F_j(\hat{t}_i)$, it follows that $F_i(\hat{t}_i) > F_j(\hat{t}_j)$. From Lemma 11 we get that $SW(F) > SW(G)$.

Proof of Proposition 5 We have that $\frac{\int_0^x F_i(s)ds}{\int_0^x F_j(s)ds}$ is strictly increasing. It follows that for every $x \in \mathbb{R}_+$ we have that:

$$\frac{\int_0^x F_j(s)ds}{F_j(x)} > \frac{\int_0^x F_i(s)ds}{F_i(x)} \tag{52}$$

We already know that for every distribution F and every cutoff t it holds that

$$E_F[x \mid x \leq t] = t - \frac{\int_0^t F(s)ds}{F(t)} \tag{53}$$

It follows that

$$E_{F_j}[x \mid x \leq t] = t - \frac{\int_0^t F_j(s)ds}{F_j(t)} < t - \frac{\int_0^t F_i(s)ds}{F_i(t)} = E_{F_i}[x \mid x \leq t] \tag{54}$$

In addition, since

$$E_{F_i}[x \mid x \leq p_j] > E_{F_j}[x \mid x \leq p_j] = p_j - \Delta, \tag{55}$$

it follows that $p_i > p_j$.

Proof of Proposition 6 Let $m(F, F')$ denote the seller's utility in the case where the buyer believes that the asset is distributed according to F while the asset is distributed according to F'. We need to show that

$$\mathbb{E}\left[m\left(N\left(\tilde{\mu}_i(\rho), \tilde{\sigma}_i^2\right), N\left(\tilde{\mu}_j(\rho), \tilde{\sigma}_j^2\right)\right) | \rho \in \{-s, s\}\right] > \mathbb{E}\left[m\left(N\left(\tilde{\mu}_i(\rho), \tilde{\sigma}_i^2\right), N\left(\tilde{\mu}_i(\rho), \tilde{\sigma}_i^2\right)\right) | \rho \in \{-s, s\}\right].$$

First, it is easy to see that $m\left(N\left(\tilde{\mu}_i\left(s\right),\tilde{\sigma}_i^2\right),N\left(\tilde{\mu}_i\left(s\right),\tilde{\sigma}_j^2\right)\right)>m\left(N\left(\tilde{\mu}_i\left(s\right),\tilde{\sigma}_i^2\right),N\left(\tilde{\mu}_i\left(s\right),\tilde{\sigma}_i^2\right)\right)$. If the actual variance is higher, the option value of the seller's payoff implies that her profits are larger.

Second, we argue that

$$\mathbb{E}\left[m\left(N\left(\tilde{\mu}_i\left(\rho\right),\tilde{\sigma}_i^2\right),N\left(\tilde{\mu}_j\left(\rho\right),\tilde{\sigma}_j^2\right)\right)|\rho\in\{-s,s\}\right]>\mathbb{E}\left[m\left(N\left(\tilde{\mu}_i\left(\rho\right),\tilde{\sigma}_i^2\right),N\left(\tilde{\mu}_i\left(\rho\right),\tilde{\sigma}_j^2\right)\right)|\rho\in\{-s,s\}\right].$$

In the case where the signal is positive the difference is larger than $\left(\tilde{\mu}_j\left(s\right)-\tilde{\mu}_i\left(s\right)\right)\Delta$, while for a negative signal the difference is smaller than $\left(\tilde{\mu}_i\left(-s\right)-\tilde{\mu}_j\left(-s\right)\right)\Delta$, and since both signals are equally likely the inequality is implied.

Finally, note that $\mathbb{E}\left[m\left(N\left(\tilde{\mu}_i\left(\rho\right),\tilde{\sigma}_i^2\right),N\left(\tilde{\mu}_i\left(\rho\right),\tilde{\sigma}_i^2\right)\right)|\rho\in\{-s,s\}\right]=\mu_i+PT\left(\tilde{\sigma}_i^2\right)\Delta$. That is, for every pair of realizations of the signal ρ the seller's expected profit are strictly higher. Thus, a deviation to F_j is necessarily profitable.

Proof of Proposition 8 First, we show that the equilibrium is unique.

LEMMA 9. *Let $f(x),g(x)$ be non-decreasing functions with $f'(x)<1$ and $g'(x)<1$ and $f(x)\geq g(x)$ for every x, and let $\alpha(x)$ be a function such that $\alpha(x)\in[0,1]$ for every x. Define $h(x):=\alpha(x)f(x)+(1-\alpha(x))g(x)$. If $\alpha(x)$ is non increasing then $h'(x)<1$ for every x.*

Proof.

$$h'(x)=\alpha(x)f'(x)+\alpha'(x)f(x)+(1-\alpha(x))g'(x)-\alpha'(x)g(x)<1 \tag{56}$$

If and only if

$$\alpha(x)f'(x)+(1-\alpha(x))g'(x)+\alpha'(x)(f(x)-g(x))<1 \tag{57}$$

Because $\alpha(x)\in[0,1]$ for every x We know that:

$$\alpha(x)f'(x)+(1-\alpha(x))g'(x)<1 \tag{58}$$

Because for every x we have that $f(x)\geq g(x)$ and $\alpha'(x)\leq 0$ we get that:

$$\alpha'(x)(f(x)-g(x))\leq 0 \tag{59}$$

This ends the proof of the Lemma. ■

For every log-concave distribution F define the next function $k : \mathbb{R}_+ \longmapsto \mathbb{R}$:

$$k_F(t) := \frac{(1-\beta)F(t+\Delta)E_F[\theta \mid \theta \leq t+\Delta] + \beta F(p)E_F[\theta \mid \theta \leq t]}{(1-\beta)F(t+\Delta) + \beta F(t)} \tag{60}$$

An equilibrium of the stage game with log-concave distribution F is characterized by a cutoff type $\hat{t}$ such that $k_F(t) = t$. It is clear that equilibrium exists, we want to show that it is unique. In order to show that, it is sufficient to prove that $k'_F(t) < 1$ for every t. This follows from the Lemma by making the following postings: $f(t) = E_F[\theta \mid \theta \leq t+\Delta]$, $g(t) = E_F[\theta \mid \theta \leq t]$ and $\alpha(t) = \frac{(1-\beta)F(t+\Delta)}{(1-\beta)F(t+\Delta)+\beta F(t)}$.

Now we prove that the seller's strategy minimizes the unique equilibrium price. Define the function $g : \mathbb{R}_+ \longmapsto \mathbb{R}$:

$$g_F(t) := \frac{(1-\beta)F(t+\Delta)E_F[\theta \mid \theta \leq t+\Delta] + \beta F(p)E_F[\theta \mid \theta \leq t]}{(1-\beta)F(t+\Delta) + \beta F(t)} - t = k_F(t) - t \tag{61}$$

Because we know that there is a unique solution to the stage game for every log-concave distribution F we get that for every such F there exists a unique type $\hat{t}_F$ such that the function g is positive for types below $\hat{t}_F$ and negative for types above $\hat{t}_F$. Note that if we think about a Dye (1985) game with the distribution F_i truncated at $\hat{t}_{F_i} + \Delta$ we get that $\hat{t}_{F_i}$ is the unique equilibrium, that is, all informed types above $\hat{t}_{F_i}$ disclose and all types below $\hat{t}_{F_i}$ do not disclose and sell for the price $\hat{t}_{F_i}$. Jung and Kwon (1988) already show that second-order stochastic dominance implies a lower no-disclosure price in a Dye (1985) model, i.e., if F_i SOSD F_j then $p^N_{F_i}D \geq p^N_{F_j}D$. Note that we have that the distribution F_i truncated at $\hat{t}_{F_i} + \Delta$ second-order stochastic dominance the distribution F_j truncated at $\hat{t}_{F_i} + \Delta$. It follows (from Jung and Kwon (1988)) that the type $\hat{t}_{F_i}$ is weakly above the equilibrium threshold type of the Dye game with distribution F_j truncated at $\hat{t}_{F_i} + \Delta$. It follows that:

$$g_{F_j}(\hat{t}_{F_i}) \leq 0 \tag{62}$$

From this we deduce that $\hat{t}_{F_i} \geq \hat{t}_{F_j}$ and the proposition follows.

Proof of Proposition 9 It is clear from the definition of MIR that for every quantile $q \in (0,1)$ it holds that $F_i'(F_i^{-1}(q)) \geq F_j'(F_j^{-1}(q))$. That is,

$$f_i(F_i^{-1}(q)) \geq f_j(F_j^{-1}(q)). \tag{63}$$

Denote by q_{F_j} the probability of trade in the monopolistic buyer stage game with distribution F_j. From the FOC we know that $\frac{q_{F_j}}{f_j(F_j^{-1}(q_{F_j}))} = \Delta$. From (63) we get that $\frac{q_{F_j}}{f_i(F_i^{-1}(q_{F_j}))} \leq \Delta$. It follows that $q_{F_i} \geq q_{F_j}$. We turn to prove the "only if" part of the proposition. It is clear that if $D(p)$ is not non-increasing then there exists a quantile $\hat{q} \in (0,1)$ such that (63) does not hold. Now we can choose $\hat{\Delta} = \frac{\hat{q}}{f_j(F_j^{-1}(\hat{q}))}$, so $q_{F_j} = \hat{q}$. It follows that under this $\hat{\Delta}$ we get that $q_{F_i} < q_{F_j}$.

Proof of Proposition 10 Denote the profit of the seller as a function of the distribution to be:

$$\Pi_F(p) := F(p)(E_F[v(x) \mid x \leq p] - p) \tag{64}$$

We have that:

$$E_F[v(x) \mid x \leq p] = v(p) - \frac{\int_0^p v'(s)F(s)ds}{F(p)} \tag{65}$$

It follows that:

$$\Pi_F(p) = F(p)(v(p) - p) - \int_0^p v'(s)F(s)ds$$

We need one technical lemma now.

Lemma 10. *For every p the next inequality holds*

$$\int_0^p v'(s)F(s)ds \leq \int_0^p v'(s)G(s)ds$$

We have that $G(p_G^*) < F(p_G^*)$ so we can wright the following inequality:

$$\Pi_F(p_G^*) = F(p_G^*)(v(p_G^*) - p_G^*) - \int_0^{p_G^*} v'(s)F(s)ds \geq G(p_G^*)(v(p_G^*) - p_G^*) - \int_0^{p_G^*} v'(s)F(s)ds$$

Now we use the lemma and we have the following equality:

$$G(p_G^*)(v(p_G^*) - p_G^*) - \int_0^{p_G^*} v'(s)F(s)ds > G(p_G^*)(v(p_G^*) - p_G^*) - \int_0^{p_G^*} v'(s)G(s)ds > \Pi(G)$$

This ends the proof because we showed that the buyer in the F stage game can get (by choosing the price p_G^*) a larger profit than profit that the seller in the G stage game gets.

B Supplementary Material

Proposition 13. *If a distribution F satisfies Assumption 1 then the equilibrium of the F-trade game exists and is unique. The equilibrium is defined by a threshold type $\hat{t}_F$ such that the types below $\hat{t}_F$ sell the object for a price $\hat{t}_F$ and types above it consume.*

Proof. Let

$$\beta_F(t) := \mathbb{E}_F[v(\theta)|\theta \leq t]$$

The equilibrium is obtained at $t^\star$ for which $\beta_F(t) - t = 0$. Note that $v(0) > 0$ and $\lim_{t\to\infty} \beta_F(t) - t = -\infty$. Hence, there exists a point $t^\star$ for which $\beta_F(t^\star) - t^\star = 0$. Since $\beta_F(t)$ is a concave function there is a single crossing of t and $\beta_F(t)$, i.e, the equilibrium is unique.

■

Lemma 11. *Let $F_i, F_j \in \mathcal{F}$, $F_i \succ_{SOSD} F_j$ and $\mathbb{E}_{F_i}[\theta] = \mathbb{E}_{F_j}[\theta]$. Assume $v_s(x) = x$, $v_b(x)$ is weakly concave, and that the gains from trade ($v_b(x) - x$) are weakly increasing. $PT(F_i) > PT(F_j)$ then $SW(F_i) > SW(F_j)$.*

Proof. Since we are dealing in this section with Normal distributions we provide a proof that applies also to unbounded from below random variables.

For every $q \in (0,1)$ let

$$\widetilde{F}_q(\theta) := \begin{cases} \frac{F(\theta)}{q} & \theta \leq F^{-1}(q) \\ 1 & \theta > F^{-1}(q) \end{cases}$$

Claim 4. *If $F \succ_{SOSD} G$, then $\widetilde{F}_q \succ_{SOSD} \widetilde{G}_q$ $\forall q \in (0,1)$.*

Proof. $F \succ_{SOSD} G$: $\int\limits_{-\infty}^{x} G(\theta)\,d\theta > \int\limits_{-\infty}^{x} F(\theta)\,d\theta \;\forall x$, hence, $\int\limits_{-\infty}^{x} \frac{G(\theta)}{q} d\theta > \int\limits_{-\infty}^{x} \frac{F(\theta)}{q} d\theta \;\forall x$. ■

Having this result we turn to prove that if $F_i \succ_{SOSD} F_j$ and $F_i\left(\hat{t}_1\right) > F_j\left(\hat{t}_2\right)$ then $SW(F_i) > SW(F_j)$.

Let $\hat{q} = F_i\left(\hat{t}_1\right)$

If the seller chooses F_j the social welfare obtained in equilibrium is

$$SW(F_j) := \int\limits_{-\infty}^{\hat{t}_2} v_b(\theta)\, F_j(x)\, dx + \int\limits_{\hat{t}_2}^{\infty} x F_j(x)\, dx \tag{66}$$

$$= \int\limits_{-\infty}^{\hat{t}_2} \left(v_b(\theta) - x\right) F_j(x)\, dx + \mathbb{E}_{F_j}[X] \tag{67}$$

And, since we assume $t_2 < F_j^{-1}(\hat{q})$ (67) is smaller than

$$= \int\limits_{-\infty}^{F_j^{-1}(\hat{q})} (-x)\, F_j(x)\, dx + \mathbb{E}_{F_j}[X] \tag{68}$$

Now, let $m(\theta) := v_b(\theta) - \theta$. An agent whose preferences over lotteries are represented by the VNM utility function m is risk averse since $m' = v_b' - 1 > 0$ and $m'' = v_b'' < 0$. Hence, we can write (68) as

$$= \int\limits_{-\infty}^{F_j^{-1}(\hat{q})} u(x)\, F_j(x)\, dx + \mathbb{E}_{F_j}[X] \tag{69}$$

$$< \int\limits_{-\infty}^{F_i^{-1}(\hat{q})} u(x)\, F_i(x)\, dx + \mathbb{E}_{F_i}[X] \tag{70}$$

$$= SW(F_i)$$

where (69) is smaller than (70) since $\widetilde{F}_{iq} \succ_{SOSD} \widetilde{F_j}_q$ and $\mathbb{E}_{F_i}[X] = \mathbb{E}_{F_j}[X]$.

■