Collusion-Proof Vote Buying

Charles Louis-Sidois and Leon Musolff*

November 12, 2019

Abstract

We study a vote buying setup where a committee votes on a proposal important to the vote buyer. We characterize the cheapest combination of bribes that guarantees the passing of the proposal under different voting environments. We find that the optimal strategy consists in publicly offering small bribes to a large majority of members. Each member accepts this offer because he anticipates the proposal to pass regardless of his vote. We discuss the committee design that maximizes cost of capture: demanding majority requirements combined with diversity among members make the committee more expensive. On the other hand, larger committees, transparent voting rules and sequential voting lower the cost.

JEL Classification: D71, D72

When Robert Kennedy was appointed Attorney General in 1961, he waded into a campaign against organized crime. One of the main targets of this campaign was Jimmy Hoffa, a famous trade union leader entangled in several illegal activities. Kennedy got personally involved in Hoffa's case and even

*Louis-Sidois: University of Mannheim, Collaborative Research Center 884 "Political Economy of Reforms", B6, 30-32, 68131 Mannheim, Germany (e-mail: charles.louissidois@uni-mannheim.de). Musolff: Department of Economics, Princeton University, Julis Romo Rabinowitz Building, Princeton NJ 08544 (e-mail: lmusolff@princeton.edu).

gathered a "Get Hoffa" squad of prosecutors and investigators. In spite of those efforts, Hoffa managed to avoid conviction for several years. Before a trial in 1962, Kennedy was absolutely sure that Hoffa would be convicted. When a reporter asked what he would do if Hoffa was acquitted, he answered "I'll jump off the Capitol". However, Hoffa managed to prove him wrong one more time and left the tribunal a free man. During the press conference after the trial, Hoffa's lawyer boldly declared "I'm going to send Bobby Kennedy a parachute" (Schlesinger 1978).

What was Hoffa's secret to avoid conviction? He eventually went to jail for the very conduct that facilitated his prior acquittals: jury tampering. During this final trial, Ed Partin – a former associate of Hoffa who testified against him – revealed how Hoffa corrupted a jury in a previous case. Partin's testimony highlights several striking patterns of Hoffa's strategy[1]. Firstly, Hoffa was trying to approach as many jurors as possible. Partin was just one of the middlemen used by Hoffa and he was asked to get in touch with several jurors. Secondly, the bribes proposed were not as big as one would expect. The amount at stake during the trial was at least several millions, but one juror considered the promise that Hoffa would support his promotion as sufficient payment for his vote. Finally, and perhaps most surprisingly, Hoffa did not seem to care much about the secrecy of his offers. He even kept making offers after one of the jurors reported that he had attempted to bribe him! If those elements are somewhat surprising, the following example unveils a mechanism that explains why Hoffa's strategy turned out to be so successful.

A Motivating Example

Consider a committee of three members voting simultaneously on a proposal with a simple majority rule. Committee members dislike the proposal and get a disutility $v_i \sim U[0,1]$ if the proposal is accepted. Payoffs are drawn independently and privately.

A vote buyer (feminine pronoun) is interested in making the proposal pass

[1] The case was brought to the Supreme Court; see Hoffa v United States (1966).

by publicly committing to paying a bribe b to a given number of members if they vote in favor. She knows the theoretical distribution of members' disutilities but does not observe the realization of types. Moreover, assume that the proposal is very important to the vote buyer so that she wants to guarantee that it will pass with certainty. Subject to this condition, the cost spent on bribes is minimized.

We compare two possible strategies for the vote buyer. First, suppose she tries to bribe the minimal winning coalition, i.e. she offers a bribe to 2 members. Then each member is certain to be pivotal if the proposal is passed. Thus, as long as $b < 1$, there exists an equilibrium where players vote against the proposal if v_i is large enough. So to guarantee certain approval she should offer a bribe of 1, which yields a total cost of 2.

Now suppose the vote buyer proposes a bribe b to all three members. We will show that as long as $b > \frac{8}{27}$ there is no equilibrium where bribed members vote against the proposal with positive probability; in other words, we will show that buying a third player is cheaper for the vote buyer. To do so, note that for each member, voting in favor of the proposal guarantees payment of the bribe. But if the member is pivotal (i.e. if exactly one other player votes for the proposal), it also leads to the adoption of the proposal. Denoting the pivotal probability by π, player i votes for the proposal if $b > v_i \times \pi$.

The equilibrium of the voting stage takes a cutoff form: players vote against the proposal if their disutility is larger than a threshold. For the time being, focus on symmetric strategies and call the common cutoff v^*. Then $\pi(v^*) = 2v^*(1 - v^*)$ and an interior equilibrium satisfies

$$b = v^* \pi(v^*).$$

In Figure 1, we plot the right hand side of this equation as a function of v^*. We see that for small bribes like b_1, there exist two equilibria with interior cutoffs v_1^* and v_2^*. Moreover, there is a third equilibrium where all players accept the bribe: committee members are not pivotal and have no incentive to deviate. Throughout the paper, we assume that the vote buyer expects bribed committee members to collude and play their favorite equilibrium in the voting subgame: e.g. faced with b_1 they would play v_1^* as this lower cutoff implies a lower probability of passing the proposal.

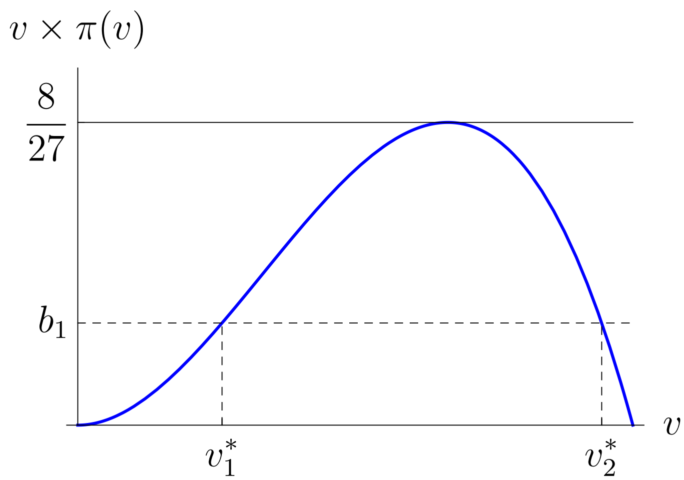

Figure 1: The Structure of Equilibrium in the Voting Subgame

When b is larger than the maximum of $v\pi(v)$, the third equilibrium – where the proposal is necessarily accepted – is the only equilibrium of the voting subgame. Here, this maximum is $\frac{8}{27}$. Thus, it is sufficient to pay $\frac{8}{9}$ to guarantee there is no equilibrium where members vote against the proposal. Intuitively, bribing a supermajority reduces players' pivotal probability, which induces them to accept smaller bribes in exchange for their vote.

Going back to Hoffa's bribing strategy, our simple example helps us to understand why it was so successful. From Partin's testimony, it appears that Hoffa was actually trying to convince jurors that he had already bought the committee. In the simple example, the optimal bribing scheme satisfies the three properties of Hoffa's strategy: the vote buyer bribes a supermajority, offers are visible and bribes are relatively small with respect to what is at stake.

Our setup primarily applies to any voting body that could be influenced by a vote buyer. It includes juries as in the above example or committees of experts (like FDA committees). It also has implications for legislatures. In particular, it can explain why lobbying is so successful. Our model provides an explanation for the so-called Tullock paradox, first introduced in Buchanan et al. (1980): lobbying expenses are typically very small compared to the benefits associated with lobbying activities. Pivotal considerations in collective decision making can explain this paradox: given that legislatures generally involve a large number of legislators, each of them might be willing to sell his vote at a small cost.

Another example is that of party discipline: our mechanism suggests that whips can impose party cohesion at a low cost when legislators do not expect to be pivotal. Our model has specific implications for the majority party: we predict that party discipline will be easy to impose if legislators expect a large margin of victory. This would be the case if the majority is large, if the members of the majority are expected to stick to the party line or if the opposition does not vote cohesively against the option supported by the majority party.

The mechanism we highlight can also explain why ruling parties are reelected with large majorities in non-democratic countries. Relying on a short majority would be expensive because each individual (or faction) can be decisive and thus the price of each vote would be large. Conversely, if citizens believe that the ruling party will be reelected with a large majority, they perceive themselves as having no chance of changing the outcome of the election and hence are willing to accept small benefits in exchange for their votes[2].

In the remainder of the paper, we stick to the vote buying terminology. However, our model is much more general and our results have implications for all the applications we have discussed.

The aim of this paper is to determine how the vote buyer's strategy and the resulting cost for her depend on the design of the committee. Crucially, all of our derivations assume that the committee is able to collude in the sense that members can coordinate on their preferred equilibrium. This assumption is in the spirit of Genicot and Ray (2006): it side-steps trivial cases where costless capture occurs because committee members play an equilibrium that

[2]A related argument can be found in Angeletos et al. (2007) or in Edmond (2013), where citizens play a coordination game to overthrow a regime.

favors the interest of the vote buyer. In our example, for instance, there exists an equilibrium where all members vote for the proposal without being bribed because no one is pivotal. Assuming that members collude seems to be natural in this setup.

First of all, we show that when committee members vote simultaneously and the vote buyer can contract on individual voting decisions, the optimal strategy consists in publicly bribing a supermajority. Moreover, the vote buyer should propose the same amount to all bribed committee members. At first glance, a possible strategy for the vote buyer could be to target specific members of the committee in order to break down their cooperation. Nevertheless, we show that this type of discrimination is not profitable.

Furthermore, a more demanding majority requirement unambiguously increases the cost of corruption: due to pivotal considerations, it is easier for committee members to coordinate on an equilibrium where some of them turn down the bribe when a supermajority is required to pass the proposal. Another key finding of our model is that large committees are not necessarily more expensive to capture. Increasing the number of members decreases the probability that a given vote is decisive and thus members accept lower bribes in exchange for their vote.

We also study the impact of the distribution of committee members' preferences. The vote buyer prefers to bribe larger supermajorities when voters' preferences are dispersed as this dispersion makes the pivotal channel easier to manipulate. Although we find that in general the effect of dispersion on cost is ambiguous, we show that combined with demanding majority requirements it increases the cost. This finding is of direct interest for the design of the committee: in order to make corruption as costly as possible, members with heterogenous backgrounds and different sensibilities should sit in the committee, as it increases the uncertainty of their assessment. Furthermore, this diverse recruitment should be combined with a supermajority requirement to pass the proposal.

Regarding the voting process, we find that moving to a sequential voting setup leaves our key results unchanged: the vote buyer still bribes a large supermajority and the comparative statics with respect to the cost are qualitatively similar. Institutions where members vote sequentially, such as the US Senate, are thus not necessarily more robust to outside influences.

However, other forms of contracts could lead to different predictions. For instance, if the vote buyer can only condition the payments of bribes on the outcome of the vote, the pivotal channel is severed. Nevertheless, if vote shares are disclosed and can be contracted on, we provide an example that highlights a novel mechanism through which the vote buyer can exploit pivotal considerations. Perhaps surprisingly, public deliberations might foster an outsider's attempts to corrupt the committee: more information on the voting process allows the vote buyer to design a more efficient bribing scheme as payments can be conditioned on a wider range of outcomes. This analysis suggests that a committee designer might want to keep the details of the vote secret.

Finally, we focus for most of our analysis on vote buyers with large valuations for the proposal so that they want to guarantee that the proposal passes. We provide a lower bound on the valuation for our analysis to apply at the end of the paper. We also briefly discuss the case of vote buyers with smaller valuations.

Our contribution relates first and foremost to the literature on vote buying in committees. Following Groseclose and Snyder (1996), a strand of the literature on vote buying has focused on two vote buyers moving sequentially. These papers include Banks (2000), Dekel et al. (2008), Morgan and Várdy (2011) and Iaryczower and Oliveros (2015). A key motivation of this literature is to explain why we observe supermajorities, as documented empirically in Mattila and Lane (2001) or experimentally in Fehrler and Schneider (2017). This literature shows that the first mover should bribe a large coalition in order to increase the cost for the follower to overbid her. In our model, we propose an independent explanation for the existence of supermajorities. As competition is hence not the focus of our paper, the discussion of it is relegated to Appendix D.

However, all of these papers disregard the potential for strategic interactions between voters. Exceptions are Henry (2008) and Felgenhauer and Grüner

(2008) who analyze how a vote buyer can manipulate the process of information aggregation, documented for instance in Feddersen and Pesendorfer (1996, 1997, 1998). In those papers, each member of the committee receives a signal about the quality of a common value proposal. In equilibrium, the vector of bribes determines the number of voters who vote informatively and shapes the inference drawn on others' signals when voters condition on being pivotal. In turn, those inferences affect the amount that voters are willing to accept in exchange for their vote. In our paper, strategies interact through the pivotal probability.

A different strand of the literature examines strategic interactions between voters in the absence of a vote buyer. These papers include Riboni (2013) as well as Casella et al. (2012), the latter of which provides an ex-ante competitive equilibrium concept in a model of *intra*-committee vote trading.

Closest to our analysis are Dal Bo (2007) and Genicot and Ray (2006) who propose models where an outsider manipulates agents' coordination to exploit them. In Dal Bo (2007), a vote buyer can condition the payment of bribes on the pivotal event: she proposes to pay an infinitesimal amount if voters are not pivotal and a large enough bribe if members turn out to be decisive. This allows the vote buyer to capture the committee at no cost.

The model of Genicot and Ray (2006), as ours, rules out contracts contingent on the simultaneous decisions of other players. Agents can be contracted by a principal or choose an outside option, the value of which depends positively on the number of non-contracted agents[3]. Key in their analysis is the fact that the principal approaches members sequentially and offers them a life-time wage if they accept the contract. Moreover, the authors assume that agents collude by ruling out equilibria in which a subset of agents could at some date improve payoffs by jointly deviating to a different profile of mutual best responses. However, their model allows the principal to break down agents' coordination

[3]Although Genicot and Ray (2006) mention the case of bribing a committee as an example, both the setup of Dal Bo (2007) as well as our setup differ because committee members value the outcome of the vote regardless of their acceptance decision. In their model, this would translate into agents deriving utility from the outside option even when contracted by the principal.

nevertheless, namely by exploiting the timing of the game: agents anticipate that the others will eventually succumb to the principal, thus lowering the value of the outside option in the future, and therefore accept contracts with wages smaller than the current value of the outside option.

The mechanism we discuss in our paper crucially differs from these models: unlike Dal Bo (2007), we follow Genicot and Ray (2006) in explicitly ruling out multilateral contracts as well as coordination failures; but unlike Genicot and Ray (2006), we follow Dal Bo (2007) in requiring a committee to vote simultaneously and thereby preventing the vote buyer from inducing a coordination breakdown by cleverly timing her offers. Dal Bo briefly examines a model without multilateral contracts, but as we highlight in the discussion of our model below, his assumptions lead him to starkly different results. He concludes that the best the vote buyer can do is to bribe the minimal winning coalition and give each member the disutility he would get if the proposal passes. We reach a very different conclusion: the optimal strategy for the vote buyer consists in bribing a supermajority, which drastically reduces the cost spent on bribes.

Finally, the bribes offered in our model differ from the solution described in Genicot and Ray (2006). There, the principal proposes contracts with varying values to exploit the timing of the game: earlier agents generally receive high wages, which allows capture of the remaining agents at a very cheap cost. Here, we reach the opposite conclusion: the vote buyer offers the same bribes to all members because it maximizes the uncertainty in the number of successes, thereby rendering players less likely to be pivotal.

In the next section, we present the main model and we characterize the optimal bribing scheme as well as the resulting cost for the vote buyer. In Section 2, we analyze the problem of the vote buyer when voting is sequential and study what the vote buyer should do when she can only contract on the outcome of the vote or on the vote share. Finally, we discuss the case of a vote buyer with limited interest for the proposal.

1 Main Model

We consider a committee of n members voting on a proposal favorable to the vote buyer. The proposal is accepted if at least m members vote for it. Committee member i privately draws the disutility $v_i \overset{iid}{\sim} U[0,1]$ he obtains if the proposal is accepted. We use the uniform distribution assumption as a benchmark: a general version of our results can be found in Appendix B and in the second part of this section we derive comparative statics with respect to the shape of the distribution. Prior to the voting stage, a vote buyer in favor of the proposal chooses a vector of bribes $(b_1, ..., b_n)$. Bribes are either public, i.e. each committee member observes the full bribe vector, or private, in which case each member is only informed about his own bribe. We assume that individual voting decisions are public and that the vote buyer is able to commit[4] to pay the bribe b_i if member i votes for the proposal. We explore other types of contracts in section 2.2. If we denote by W the vote buyer's valuation of the proposal, she chooses $(b_1, ..., b_n)$ to maximize:

$$\begin{aligned} U_{\text{VB}} &= W \times \mathbb{P}(\text{proposal passed} \,|\, (b_1, ..., b_n)) \\ &\quad - \sum_{i=1}^{n} b_i \mathbb{1}(\text{Player } i \text{ votes in favor}). \end{aligned}$$

Naturally, the voting subgame has multiple equilibria and the probability of passing depends on the equilibrium we consider. As we are interested in collusion-proof vote buying, we assume that the vote buyer expects bribed committee members to coordinate on their preferred equilibrium[5] in the voting subgame. This is generally be the equilibrium where the proposal is accepted with the smallest probability.

At first glance, the solution to the vote buyers' optimization problem seems to require knowledge of the probability of passing for any bribing scheme

[4]Modeling the commitment mechanism is beyond the scope of our paper. As in Rueda (2015), we could imagine that the committee and the vote buyer repeatedly interact.

[5]In the main section this is equivalent to saying *all* committee members coordinate on their preferred equilibrium. It is also equivalent to assuming that the vote buyer expects the worst (for her) equilibrium to be played, e.g. because she is ambiguity averse.

$(b_1, ..., b_n)$. However, the vote buyer ensures certain passing if $W > \overline{W}$ for some $\overline{W}$. For most of the analysis, we assume that this condition is satisfied so that the vote buyer seeks to guarantee the approval of the proposal for the smallest possible cost. We discuss this assumption in Section 2.3.

To recap the timing of the game, the vote buyer moves first and proposes a bribing scheme $(b_1, ..., b_n)$. She can make public offers or privately approach members. In a second stage, committee members learn the bribing scheme (or simply their own bribe if the vote buyer has made private offers), privately observe their type and simultaneously choose whether to vote in favor or against the proposal. Finally, the proposal is implemented if at least m members have voted for it and the vote buyer pays b_i to the bribed members who have supported the proposal.

1.1 Results

We solve the game backward and first consider the voting stage. We focus on players to whom the vote buyer proposes a positive bribe; unbribed players vote against the proposal in any equilibrium where their vote could change the result. Given a bribing scheme $(b_1, ..., b_n)$, if a player is not pivotal the payoff difference between voting in favor and voting against is simply the value of the bribe offered to him. If his vote is pivotal, however, he also has to account for the fact that a vote in favor is causing the proposal to pass. Thus, denoting by π_i the pivotal probability of committee member i, he accepts the bribe and vote for the proposal if

$$b_i \geq \pi_i v_i.$$

Our first lemma describes the outcome of the game if the vote buyer uses private offers.

Lemma 1. *If the vote buyer privately communicates the bribes, she must promise a payment of* 1 *to all bribed members and spend a total cost of* m *to guarantee the acceptance of the proposal.*

When offers are secret, the vote buyer cannot induce more than m players to vote for the proposal with certainty. If she did, she could secretly deviate and only offer bribes to exactly m players. This would be sufficient to pass the proposal. Anticipating this deviation, committee members need to be offered a bribe of 1 in order to support the proposal with certainty.

We now consider cases where the vote buyer bribes players publicly. At this stage we make two additional assumptions strictly to simplify the exposition (they are relaxed below). Firstly, we suppose that all positive bribes offered by the vote buyer are equal to the same value b. Calling k the number of positive bribes, a bribing scheme is now defined by (k, b). Secondly, we restrict our attention to type-symmetric equilibria: given a bribing scheme (k, b), all members who receive the same bribe and have the same type must choose the same strategy.

Under these assumptions, the equilibrium of the voting stage is as follows:

Lemma 2. *If $k \geq m$, in any type-symmetric equilibrium either*

1. *all bribed members vote in favor of the proposal, or*

2. *bribed members vote for the proposal if and only if their type is smaller than a cutoff v^* that satisfies*

$$\pi(v^*)v^* = b, \tag{1}$$

where

$$\pi(v) = \binom{k-1}{m-1} v^{m-1}(1-v)^{k-m}. \tag{2}$$

If $k = m$, the strategy profile described in Lemma 2.1 is the unique equilibrium only for $b \geq 1$. As in the case of private offers, the vote buyer must pay a total cost of m if she chooses to bribe a minimal winning coalition.

In Dal Bo's model, this is the best the vote buyer can do, but we show below that there exist cheaper strategies in our setup. This difference emerges because according to Dal Bo's solution concept, the vote buyer *'can induce or* *implement a decision "Yes" by the committee if and only if, given [her] offers, there are at least M members for whom voting "Yes" is a dominant strategy'* (Dal Bo, 2007, p.794). In the exact setup of Dal Bo, our solution concept (voter-preferred equilibrium) and Dal Bo's would lead to the same conclusion. However, the two concepts diverge once we introduce uncertainty. In particular, in our setup Dal Bo's concept still requires the vote buyer to bribe the minimal winning coalition and pay to each member the maximal disutility he would get if the proposal passes. We now show that this disadvantages the vote buyer in our setup because there exists a cheaper bribing scheme such that the proposal is accepted with certainty even in the least-preferred equilibrium of the vote buyer.

When $k > m$, the voting subgame is illustrated in Figure 1. The equilibrium where all bribed members vote for the proposal always exists. Members know that they are not pivotal and thus accept the bribe. In such a case, the proposal is accepted for sure. In the type of equilibrium described in Lemma 2.2, bribed members vote for the proposal if and only if $v_i \leq v^*$. When b is low enough, there are generically two equilibria of that type: $\pi(v^*)$ is a single peaked function equal to 0 for $v^* = 0$ and $v^* = 1$. By the intermediate value theorem, the equation $\pi(v^*)v^* = b$ admits two solutions if b is small enough, one if $b = \underset{v}{max}(v\pi(v))$ and none otherwise. If we let $v^{**} := \arg\max_v v\pi(v)$, the smallest bribe[6] such that the proposal is accepted for sure in any type-symmetric equilibrium is $b^*_k = v^{**}\pi(v^{**}) + \epsilon$. For the introductory example and hence in Figure 1, we have $b^*_{k=3} = \frac{8}{27}$. We now show that the restriction to type-symmetry is innocuous:

Lemma 3. *When the vote buyer offers b^*_k, the proposal is accepted with certainty in any equilibrium of the voting game.*

To establish Lemma 3, we need to show that if the vote buyer offers at least b^*_k to k players, there is no possible coordination on an asymmetric equilibrium. As any equilibrium strategy must be a cutoff-strategy, an asymmetric equilibrium

[6]This theoretically introduces open-set problems. These problems can be solved by assuming some minimum unit of currency ϵ.

amounts to committee members choosing different voting cutoffs. We use an iterated deletion of strictly dominated strategies to show that this cannot be an equilibrium. We define a function $\pi_{max}(\underline{v})$ that represents the maximal pivotal probability of a player given that other cutoffs must lie above $\underline{v}$. This can be interpreted as the maximal expectation that a player can form about his pivotal probability given that cutoffs below $\underline{v}$ have already been eliminated. Intuitively, member i's pivotal probability is maximized if the others use two extreme cutoffs and split suitably between them. For instance, before the first iteration, we can maximize the pivotal probability by making $m-1$ other (bribed) players always accept (cutoff at 1) and $k-m+1$ always reject (cutoff at 0). In such a case, player i is pivotal for sure: $\pi_{max}(0) = 1$. The first iteration eliminates all cutoffs below b_k^*. Once those strategies have been eliminated, players cannot anticipate being pivotal with certainty: $\pi_{max}(b_k^*) < 1$. As a result, the second iteration removes another set of cutoffs. This reasoning is illustrated in Figure 2: we plot the function

$$\underline{v}'(\underline{v}) = \frac{b_k^*}{\pi_{max}(\underline{v})}$$

as well as the 45 degree line. Given that cutoffs below $\underline{v}$ have already been removed and that other strategies must lie in the remaining set of cutoffs, $\underline{v}'(\underline{v})$ represents the lowest cutoff that could still be played. We show that the function $\frac{v^{**}\pi(v^{**})}{\pi_{max}(\underline{v})}$ is exactly tangent to the 45 degree line for $\underline{v} = v^{**}$ and lies above elsewhere. As $b_k^* > v^{**}\pi(v^{**})$, $\underline{v}'(\underline{v})$ is strictly above the 45 degree line, which implies that no cutoff below 1 is rationalizable.

We now turn to the problem of the vote buyer. When we relax the symmetry of bribes assumption, an interior[7] equilibrium – conceptually similar to the second type of equilibrium described in Lemma 2 – must satisfy

$$\begin{aligned} v_1^*\pi_1 &= b_1, \\ &\vdots \\ v_k^*\pi_k &= b_k, \end{aligned}$$

[7] This system of equation characterizes a fully interior equilibrium; the vote buyer must also guard against equilibria in which a subset of members choose $v_i^* = 1$.

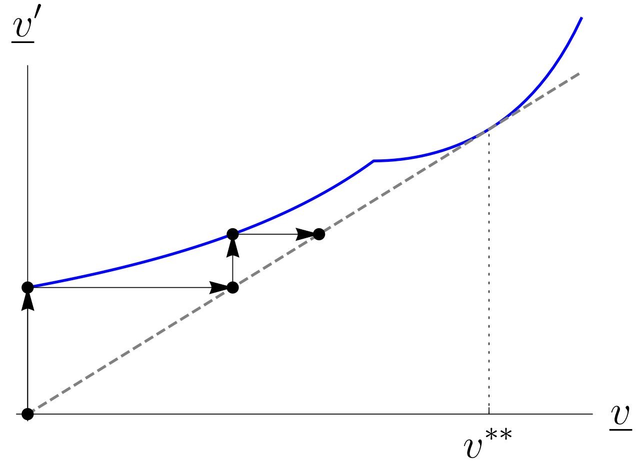

Figure 2: Iterated deletion of strictly dominated strategies

where $(v_1^*, \ldots, v_k^*)$ are the cutoffs played by all players. In equilibrium, the vote buyer proposes the cheapest combination of bribes such that there is no solution to the above system:

Proposition 1. *The cheapest bribing scheme inducing certain passing of the proposal is* $b_i = b_n^*$ *for all* i*.*

The optimal strategy of the vote buyer is to offer the same bribe b_n^* to all committee members. First of all, we can show that (at least one) solution to the above system exists if bribes are small enough. As in the case with symmetric offers, we can increase all bribes up to a point where no solution exists anymore. We define the *breakdown boundary* as the set of bribing schemes such that a solution exists, but increasing any bribe marginally would induce the system to have no solution. Clearly, a cost-minimizing vote buyer picks a bribing scheme inducing breakdown arbitrarily close to the cheapest point on the breakdown boundary.

With homogeneous bribes, we saw in Lemma 3 that breakdown occurs when the probability $v^*\pi(v^*)$ that a given player causes the proposal to pass is maximized. With heterogeneous bribes, pivotal probabilities can differ between committee members but the fundamental intuition remains unchanged: near the breakdown boundary, players are on average likely to be decisive. In general, one can lower the likelihood of the most likely events by increasing the spread of a distribution. We can thus lower the pivotal probabilities by moving the members' cutoffs v_i^* closer together, i.e. by offering them more homogeneous bribes. For a given number k of positive bribes, it hence turns out that the cheapest way to achieve breakdown is by offering each member the same bribe b_k^*.

Pushing this reasoning one step further, the vote buyer increases dispersion by offering bribes to as many players as possible. Indeed, offering k bribes equal to b_k^* and others equal to 0 induces heterogeneity in the bribes and cannot be optimal by the above argument. Therefore, the optimal strategy is to bribe the whole committee.

We discuss the robustness of this result to the distributional assumption below: while the vote buyer typically chooses not to bribe all committee members if the distribution is less dispersed, supermajorities remain optimal if we focus on symmetric strategies.

It should also be noted that the optimal coalition would be smaller if members care about voting decisions per se. Indeed, many papers on vote buying such as Groseclose and Snyder (1996) or Dekel et al. (2008) assume that members have only expressive preferences: they derive utility from their vote but do not take into account their impact on the outcome. This can be modeled by a fixed reputational or moral cost that each member incurs if he votes for the proposal. In a parliament for instance, legislators would be punished by their constituencies if they voted against public interests. In a setup where corruption is illegal, this cost could also represent the sanction if players are caught. Let us consider a mixed model where on top of the previous utility function we add a cost d for players who vote for the proposal[8]. For each possible coalition size, it is easy to see that the vote buyer should propose $b_k^* + d$ to each member of the coalition to ensure the proposal's acceptance. Thus, the cost of exploiting the pivotal channel is increasing and the size of the optimal coalition decreasing in the expressive voting cost d.

[8]This cost differs across players in Groseclose and Snyder (1996). Midjord et al. (2017) propose a model of reputation where the cost is determined in equilibrium.

In the next proposition, we consider the impact of the design of the committee on the cost for the vote buyer:

Proposition 2. *The cost for the vote buyer is:*

1. *decreasing in the number of committee members n,*
2. *increasing in the majority requirement m,*
3. *increasing slower than proportionally in committee scale.*

We have seen in Proposition 1 that the vote buyer always wants to bribe the largest possible coalition, but she is constrained by the number of committee members. Adding members without changing the majority requirement relaxes the constraint and allows her to exploit the pivotal channel even further, which decreases the cost.

However, increasing m without changing n raises the cost for the vote buyer. Intuitively, manipulating pivotal considerations becomes harder with a more demanding majority requirement. For instance, if we set $m = n$, the vote buyer cannot do better than bribing the minimal winning coalition and pays the cost described in Lemma 2.

Instead of increasing n or m separately, a potential committee designer could consider multiplying both parameters by the same factor. This would result in a larger committee with the same ratio $\frac{m}{n}$. Even though the cost for the vote buyer would increase, the last point of Proposition 2 shows that as long as $m < n$ pivotal considerations mitigate the impact of this strategy because members expect to be pivotal with a lower probability in a large committee. If $m = n$, the cost increases exactly proportionally.

1.2 Other Valuation Distributions

We now relax the uniform assumption and we analyze how the shape of the distribution affects the size of the optimal coalition and the resulting cost. Our results are illustrated in an example at the end of this section. We restrict attention to symmetric bribing strategies:

Assumption 1. *All bribed committee members are paid the same bribe.*

We consider a distribution with cumulative density function $F(\cdot)$. We also require the following technical assumption, which combines a differentiability requirement with the assumption that $F(\cdot)$ has an increasing generalized failure rate[9]:

Assumption 2. *$F(\cdot)$ is continuously differentiable and* $\frac{\partial}{\partial v}\left(\frac{vF'(v)}{1-F(v)}\right) \geq 0$.

The distribution of committee members' valuations for the proposal captures the uncertainty of their assessment. If we consider a committee where members have similar backgrounds, the assessment of players' valuation should be more accurate. Conversely, there will be more uncertainty in a committee where members come from different backgrounds and are renewed on a regular basis. Those elements are captured in the spread of the distribution and we use the following definition of dispersion as a comparison criteria:

Definition 1. *We say that $\tilde{F}(\cdot)$ is* more dispersed *than $F(\cdot)$ if $\tilde{F} \leq_* F$, i.e. if the ratio of the inverse CDFs, $\frac{\tilde{F}^{-1}(p)}{F^{-1}(p)}$, is nondecreasing in p.*

We exhibit examples of distribution functions that can be ranked in the dispersion ranking in Figure 3 to illustrate the concept. For our purposes, note that if $F \sim LogN(\mu, \sigma)$ and $\tilde{F} \sim LogN(\tilde{\mu}, \tilde{\sigma})$ then $\tilde{F}$ is more dispersed than F if and only if $\tilde{\sigma} > \sigma$, i.e. independently of the relationship between $\tilde{\mu}$ and μ. For more technical details on this order, we refer the interested reader to Shaked and Shanthikumar (2007, p.213).

Our analysis for general distributions also requires the following assumption:

Assumption 3. *We assume either*

(i) $F(\cdot)$ is sufficiently dispersed[10] to eliminate asymmetric equilibria, or

(ii) players play only type-symmetric strategies.

[9] This is a strictly weaker requirement than increasing failure rate; for more details see Lariviere (2006).

[10] A sufficient (but not necessary) condition for $F(\cdot)$ to be sufficiently dispersed is for it to be more dispersed than $U[0,1]$.

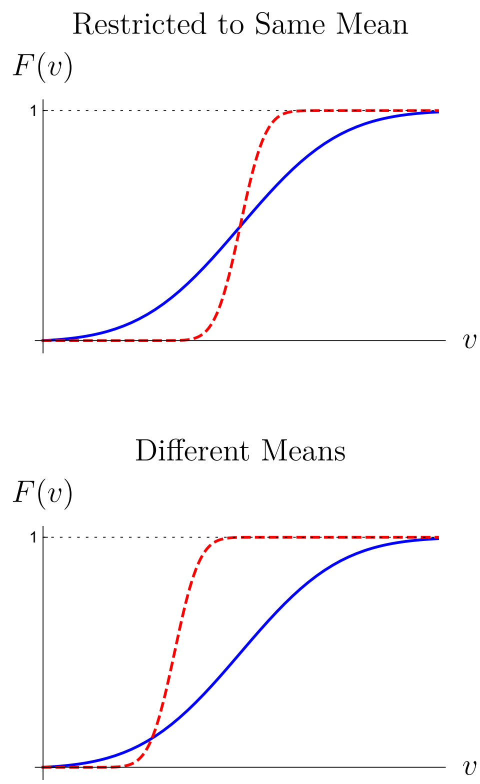

Figure 3: Dispersion Comparison: $\tilde{F}(v)$ blue, $F(v)$ red and dashed.

For the class of distributions satisfying Assumption 2 and Assumption 3, the results derived in the main section are qualitatively valid. We refer the interested reader to Appendix B, which states and proves Lemma 3 and Proposition 2 as well as a modified Proposition 1 in this environment. Furthermore, it provides examples where the assumptions are violated. We move on to describe how the size of the optimal supermajority for the vote buyer depends on the dispersion of the distribution.

Proposition 3. *The vote buyer bribes a larger supermajority when the distribution is more dispersed.*

Intuitively, it is easier to manipulate members' beliefs about the pivotal probability when the distribution is dispersed. The vote buyer therefore relies more on this channel when the dispersion is large, regardless of the fact that it implies paying a large number of bribes. However, even with little dispersion, the vote buyer bribes a substantial supermajority if we focus on type-symmetric strategies. For instance, we can show that if we remove dispersion (i.e the distribution converges to a single mass point), the vote buyer still bribes a substantial supermajority equal to roughly $\frac{3}{2}$ times the minimal winning coalition[11].

Proposition 3 raises the question of how the vote buyers' cost responds to an increase in dispersion. So far, our analysis suggested that it was easier for the vote buyer to exploit the pivotal channel when dispersion was large. However, it turns out that the impact of dispersion on the cost is ambiguous. On the one hand, increasing dispersion makes coordination harder for committee members, which benefits the vote buyer. On the other hand, it also means that extreme types are more likely, which could make high cutoffs sustainable.

We now identify a sufficient condition for the 'extreme values' channel to dominate, i.e. for dispersion to have a positive impact on cost. Let α be such that, for two distributions $F(v)$ and $\tilde{F}(v)$ where the latter is more dispersed, $\frac{\tilde{F}^{-1}(\alpha)}{F^{-1}(\alpha)} = 1$. Thus, α is the value of $F(v)$ at the crossing of $F(v)$ and $\tilde{F}(v)$. Given our definition of dispersion, this crossing needs to be unique.

Proposition 4. *Suppose $\frac{m-1}{n-1} \geq \alpha$. If $\tilde{F}(.)$ is more dispersed than $F(.)$, then the cost for the vote buyer is larger under $\tilde{F}(.)$.*

Dispersion increases the cost when $\frac{m-1}{n-1} \geq \alpha$, which holds when m is large with respect to n, that is, for demanding majority requirements. In such cases, all feasible coalitions are too small to fully exploit the pivotal channel. When dispersion increases, committee members with large disutility for the proposal are more likely to emerge, which makes high cutoffs easier to sustain. The vote buyer therefore needs to pay more to impede a possible coordination.

We now provide an example that illustrates the results of this section.

[11] We do not have strict equality because of integer problems. If we set $m = \alpha n$ for some α and let $n \to \infty$ (so that integer problems disappear), then $k^*/n \to 3/2$.

Example 1. Suppose $F \sim \text{LogN}(\frac{1}{2}, 1)$ and $\tilde{F} \sim \text{LogN}(\frac{1}{2}, 1.2)$. Then $\tilde{F}$ is more dispersed than F. Furthermore, both distributions satisfy the requirements of Assumptions 2 and 3. In particular, Assumption 3(i) applies so that we do not need to assume type-symmetric strategies. If $m = 2$ and $n = 9$, then the vote buyer targets $k = 8$ players under F and $\tilde{k} = 9$ under $\tilde{F}$. As $\alpha = \frac{1}{2}$, we cannot conclude that cost is greater under $\tilde{F}$ and indeed at 1.81 the cost under F is less than the 2.11 the vote buyer would have to pay under $\tilde{F}$. The conclusion reverses if we lower n to 3, in which case the cost under F is 4.39 and that under $\tilde{F}$ is 3.68.

2 Other Environments

We first study sequential voting and then consider alternative payment schemes. Finally, we discuss the case of a vote buyer with a limited valuation for the proposal. From now on, we reintroduce the assumption that $v_i \overset{iid}{\sim} U[0, 1]$ and further assume that the vote buyer makes the same offer to all bribed players. We also focus on type-symmetric strategies.

2.1 Sequential Voting

In the main section, we have assumed that committee members voted simultaneously. In many committees (for instance the US Senate), votes take place sequentially. We thus consider a variation of our model where the vote buyer still moves first and proposes a bribe b to k members. An ordering of members is then drawn at random and we assume that all orderings are equally likely. Finally, voters observe the ordering, announce their vote sequentially and the proposal is implemented if at least m members vote in favor.

We first consider the voting game. In such a voting process, committee members use backward induction to infer their pivotal probability as in Spenkuch et al. (2018). First of all, it is easy to see that in a subgame perfect Bayesian equilibrium, non-bribed members vote against the proposal in any subgame where it can still be rejected. As a result, we can focus on the k members who receive a positive bribe. Define $S_i(x, y)$ as the subgame where member i is to play, x votes are still needed to pass the proposal and y (bribed) players remain to play after i. Table 1 is a useful representation of the game where each cell represents a subgame. When a player votes for, the subgame located North-West along the diagonal is reached while if he votes against we move to the subgame just to the North. In each cell, we write the subgame perfect Bayesian equilibrium strategy. A '+' indicates that the member to play should accept the bribe regardless of his type. When a player uses a cutoff rule (i.e. votes for the proposal if his disutility is smaller than a cutoff), we simply display the cutoff used.

The first row is easy to fill: it represents the strategy of the last player to vote as a function of the number of votes in favor still needed to pass the proposal. Whenever $x \neq 1$, the last player is not pivotal and accepts the bribe. When $x = 1$, the last member to cast a vote supports the proposal if the bribe is larger than his disutility: $b > v_i$. This implies that from the perspective of other players, the last member accepts the proposal with probability b if he happens to be pivotal. Moving one row up, it turns out that the second to last player to vote accepts if and only if there is only one vote needed to pass the proposal. To see that, first notice that if $x < 1$ or $x > 2$, this player has no impact on the acceptance decision and always accepts. If $x = 2$, the player would get 0 if he votes against and

$$b - v_i \mathbb{P}\Big(\text{proposal accepted} \mid \text{vote in favor at } (x, y) = (2, 1)\Big) = b - bv_i$$

if he votes for. As $v_i \leq 1$, this player accepts regardless of his type[12]. Finally, in the subgame $S(1, 1)$, the member accepts if $v_i < \frac{b}{1-b}$. Iterating the reasoning, we show that the subgame perfect equilibrium of the voting game is the following:

Lemma 4. *A subgame perfect equilibrium of the sequential game must satisfy the following conditions:*

1. *In any subgame $S_i(x, y)$ with $x \neq 1$, the member to play votes in favor.*

[12]Member of type $v_i = 1$ would be indifferent, but this event has zero mass and can be neglected.

2. In any subgame $S_i(1, y)$, the member to play accepts if $v_i < min\{1, \frac{b}{1-yb}\}$.

Table 1: Sequential voting.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$	...	$x = m-1$	$x = m$
$y = 0$	+	$\min\{b, 1\}$	+	+	...	+	+
$y = 1$	+	$\min\{\frac{b}{1-b}, 1\}$	+	+	...	+	+
$y = 2$	+	$\min\{\frac{b}{1-2b}, 1\}$	+	+	...	+	+
...			...				
$y = i$	+	$\min\{\frac{b}{1-ib}, 1\}$	+	+	...	+	+
...			...				
$y = k-2$					...	+	+
$y = k-1$					...		+

Note: Only bribed members are considered. x is the number of votes required to pass the proposal, y the number of players still to go. Each cell gives the SPNE strategy. We do not display all the subgames where the proposal is already accepted (i.e. x negative) as all bribed members accept for sure. Greyed out cells correspond to nonexistent subgames.

We can now consider the problem of the vote buyer. She wants to choose the combination (b, k) which makes the proposal accepted with certainty for the minimal possible cost. The game begins at $S(m, k-1)$; thus, choosing k amounts to choosing the number of rows in Table 1. Given the equilibrium structure of the game, the $m-1$ first members vote for the proposal until the subgame $S(1, k-m)$ is reached. Once the game arrives at the first column, the proposal is rejected with positive probability if for all $S(1, y), y \leq k-m$, some types reject the proposal. For the vote buyer, it is necessary and sufficient to choose b such that the player to move at $S(1, k-m)$ accepts the proposal with certainty. This implies that when the vote buyer decides to bribe k players, she offers a bribe $b_k^* = \frac{1}{k-m+1}$.

To find the optimal k, notice that the total cost as a function of k is $\frac{k}{k-m+1}$, which is strictly decreasing in k. As a result, the vote buyer will always buy the full committee when the vote is sequential.

Proposition 5. *When voting is sequential, the vote buyer bribes all committee members and offers them* $b_n^* = \frac{1}{n-m+1}$. *The cost for the vote buyer is decreasing in the number of committee members* n, *increasing in the majority requirement* m *and increasing slower than linearly with committee scale.*

We can therefore conclude that the main mechanism of the paper also applies to sequential voting: the vote buyer bribes a supermajority in order to make a pivotal event unlikely. By doing so, she makes sure that all members support the proposal even if the bribes offered are small. Given that the vote buyer pays $b_n^* = \frac{1}{n-m+1}$ to n members, the resulting cost is $\frac{n}{n-m+1}$. The comparative statics result from a direct inspection of this function and are qualitatively similar to the simultaneous voting case.

2.2 Other Visibility Setups

Until now we were assuming that the voting process was public and that the vote buyer could contract on the individual voting decisions. We now explore alternative mechanisms where the voting process is not fully transparent. More precisely, we first consider a committee where only the final decision is public. Furthermore, we also analyze a setup where the number of votes in favor is disclosed. In both cases, we assume that the vote buyer can only condition the payment of bribes on the information that is publicly disclosed after the vote.

2.2.1 Final Decision

Suppose that the only information disclosed after the vote is the decision of the committee. In such a case, the vote buyer pays the bribes if the proposal is accepted. We stick to the assumptions made in the main section: bribed committee members collude on their preferred equilibrium and the vote buyer wants to induce the proposal to be accepted with certainty. We have the following proposition:

Proposition 6. *When the vote buyer can only condition the bribes on the outcome of the vote, she offers a bribe of 1 to* m *players.*

In such a setup, a pivotal voter would induce the proposal to pass and the bribe to be paid if he votes in favor. As a result, bribed members support the proposal if $b > v_i$ and the vote buyer needs to offer $b = 1$ to guarantee that all bribed members vote in favor of the proposal. Knowing this, she cannot do any better than bribing the minimal winning coalition and ends up paying m. This result is in line with the third Proposition of Dal Bo (2007) and shows that the pivotal channel is severed when the vote buyer can only contract on the outcome of the vote.

2.2.2 Tally

Now let us suppose that the committee discloses the number of votes in favor of the proposal and that the vote buyer conditions the payment of bribes on this information. This problem is also considered in Dal Bo (2007). He concludes that the cheapest way to achieve certain acceptance is to promise a payment of 1 to m members if the proposal receives m votes, which is also the solution proposed in Proposition 6 above. This bribing scheme guarantees that voting for the proposal is a dominant strategy for at least m voters and is therefore consistent with the solution concept of Dal Bo (2007).

Nevertheless, it turns out that in our setup there exist cheaper ways to prevent members from colluding on an equilibrium where the proposal could be rejected. Those strategies also imply bribing a supermajority and we now present an example that illustrates the mechanism.

Let's consider a bribing scheme consisting of a number of bribed members k and a sequence of bribes $\{b^1, b^2, ..., b^m, ..., b^k\}$ where b^p is the bribe that the vote buyer commits to pay to all k bribed members if p players vote in favor. We assume that payments cannot be negative: $b^p \geq 0$ for all p.

Considering a given bribed player, we denote by π^p the probability that exactly $p - 1$ other members vote for the proposal. In such a case, voting in favor increases the payoff to each bribed member by $b^{p+1} - b^p$. Interior equilibria of the voting game take a cutoff form and must satisfy

$$v^* = \frac{\sum_{p=1}^{k} \pi^p(v^*)(b^p - b^{p-1})}{\pi^m(v^*)}.$$

The vote buyer needs to propose a bribing scheme such that no interior equilibrium exists. All bribed members vote for the proposal and the vote buyer pays $k \times b^k$. The problem of the buyer is thus to minimize b^k such that there is no solution in $[0,1]$ to the above equation.

In order to prevent a possible coordination on a strategy profile where all members reject the proposal, the vote buyer must propose $b^1 > 0$. Moreover, we must have $b^k > b^{k-1}$, i.e. bribes must increase right at the end: if not, bribed players would reject the proposal with a positive probability.

We can now study the following example, which shows that in a committee of 7 players with a majority of 4, the vote buyer can pay strictly less than 4 and still prevent any possible collusion.

Example 2. *Suppose $m = 4$ and $n = 7$. If the vote buyer proposes the bribing scheme depicted in Figure 4 to all members of the committee. She pays a total cost $3.86 < m$ and in exchange every agent votes in favor of her proposal.*

With the bribing scheme depicted in the upper panel of Figure 4, a vote in favor increases the bribes paid to all members if there are 3, 4 or 6 other votes in favor. However, in case exactly 5 other players have supported the proposal, a vote in favor reduces the bribes given.

In the lower panel of Figure 4, we express the expected utility gain from a vote in favor as a function of the voting cutoff v^* (and considering that other players also pick the same cutoff). An equilibrium of the voting game must satisfy $\triangle U(v^*) = 0$. We can see that the bribing scheme proposed guarantees that this condition never holds, which implies that $v^* = 1$ is the only equilibrium of the voting subgame.

The bribing scheme that we consider exploits the fact that close pivotal events become salient nearly simultaneously, i.e. whenever an agent places

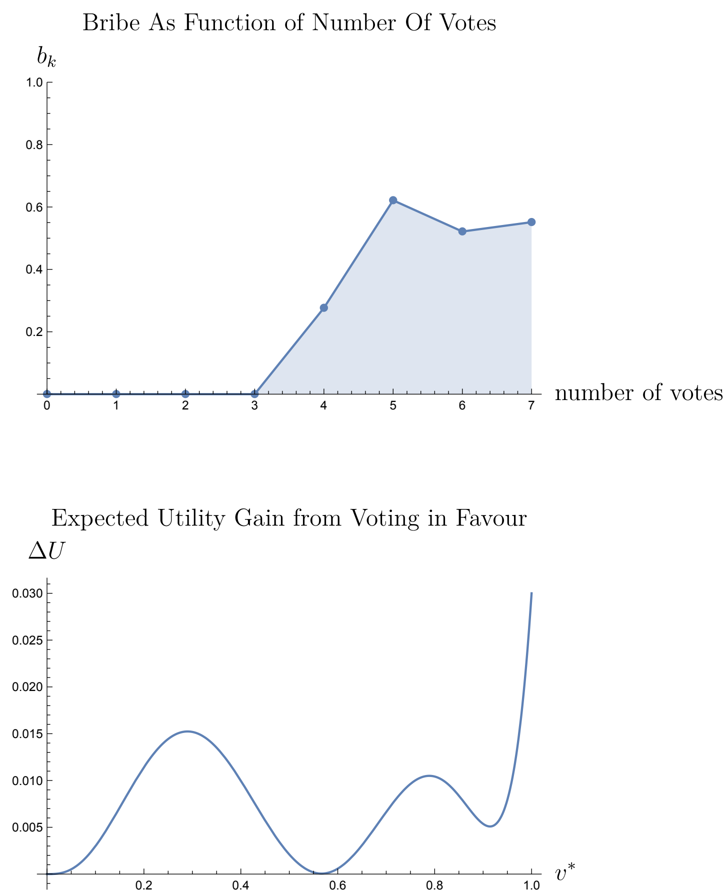

Figure 4: Saving money by bribing a supermajority.

a high probability on being at a specific pivotal event she also places a high probability on being at nearby events. By exploiting these interactions between the pivotal events, it is possible to decrease the bribe locally and still prevent any potential coordination by re-increasing it at a higher number of votes in favor.

The mechanism that we highlight can only be exploited for large enough committees: it requires lowering and re-increasing the promised bribes with more than m votes in favor. This is of course not possible for small committees or when unanimity is required. Moreover, we see that this mechanism also requires bribing a supermajority.

To our knowledge, the idea presented in Example 2 is new. It shows that when members' preferences are uncertain and when we focus on symmetric strategies, the cost derived in Dal Bo (2007) can be lowered by offering multiple thresholds to a supermajority. In his setup, the fact that players have known valuations for the proposal combined with a potential collusion on asymmetric equilibrium would make the bribing scheme of our example ineffective[13]. Other papers also consider payments conditioned on vote shares[14] but they assume that the bribes are paid if the number of votes in favor is above a unique threshold. Such a strategy typically induces multiple equilibria: e.g. there always exists an equilibrium where the proposal is rejected with positive probability.

2.3 Vote Buyer's Valuation

In this section, we relax the certain passing assumption and we consider the general problem of the vote buyer. As in the main setup, payments are conditioned on individual voting decisions. Recall that the objective function of the vote buyer is

$$\begin{aligned} U_{\mathrm{VB}} &= W \times \mathbb{P}(\text{proposal passed} \,|\, b \text{ offered to } k \text{ members}) \\ &\quad - b \times (\#\text{votes for}), \end{aligned}$$

where W is her valuation of the proposal. The vote buyer may now prefer to induce an interior equilibrium, i.e an equilibrium where bribed members have cutoffs smaller than 1 and where the proposal is rejected with a positive probability. As a result, the vote buyer could end up paying positive bribes even if the proposal is rejected. In spite of this risk, it turns out that she always offers positive bribes, even when her valuation for the proposal is small:

[13]Let $v_i = 1/2$ for all players. There exists an equilibrium where 3 players vote for with certainty and 4 vote against.

[14]See for instance Morgan and Várdy (2011), Smith and De Mesquita (2012), Gingerich and Medina (2013) and Rueda (2015, 2017).

Proposition 7. *If $W > 0$, the vote buyer offers strictly positive bribes.*

This result is driven by pivotal considerations: when the bribes offered go to zero, committee members always reject the proposal and are never pivotal. The marginal impact of the bribes on the voting cutoffs – and thus on the probability of acceptance – is therefore very large and the vote buyer always attempts to bribe the committee.

As we discussed above, there exists $\overline{W}$ such that for $W > \overline{W}$ the vote buyer makes sure that the proposal is always accepted. In our next result, we propose an upper bound for $\overline{W}$:

Lemma 5. *It is optimal for the vote buyer to guarantee certain acceptance if $W > \overline{W}$ where $\overline{W} < m + 1$.*

This bound guarantees that for any coalition size, the utility of the vote buyer plotted in Figure 5 is strictly increasing in v^*. However, while this is sufficient to ensure the vote buyer wants to guarantee certain acceptance, it is not necessary. In particular, it does not account for the payoff discontinuity induced by the equilibrium breakdown. This discontinuity might provide enough incentives to induce certain approval even when $W < m + 1$. For instance, in the setup of our introductory example – $n = 3, m = 2$ – we have $\overline{W} \approx 1.14 < m + 1 = 3$.

Let us now consider cases where the vote buyer wants to induce an interior equilibrium. With respect to the optimal number of bribes, the effect of an additional player is now ambiguous and the mechanism is different from the main model. Suppose that the vote buyer spends a given amount that she splits between k players. Moreover, suppose that W is small so that bribes offered and cutoffs are small. What would happen if the vote buyer spent the same amount but split it between $k + 1$ players?

Ignoring pivotal probabilities for the moment, the direct effect of this move is to increase the spread of the distribution of the number of successes. This benefits the vote buyer: when cutoffs are small, getting at least m votes in favor is unlikely. By making the number of successes more uncertain, the probability of obtaining enough votes increases. However, there is now a countervailing

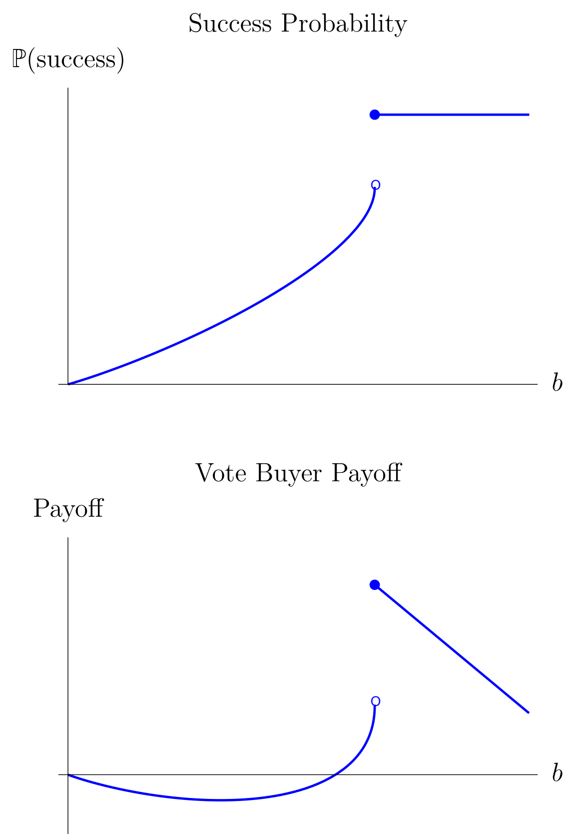

Figure 5: The General Vote Buyer's Problem

effect: when we increase uncertainty in the number of successes, we incidentally increase the probability of unlikely outcomes, including pivotal events. As a result, players are more reluctant to vote for the proposal and play lower cutoffs. When W is small, supermajorities are therefore not necessarily optimal. It is indeed possible to build an example where the vote buyer prefers to bribe m players than $m + 1$[15].

[15]For instance, if $n = 17$ and $W = 0.2$.

Conclusion

We have shown that a vote buyer can shape committee members' assessments of their pivotal probabilities. As a result, it is generally cheaper to bribe a supermajority for an outsider who wants to manipulate a vote. We have shown that this strategy can be used in a large variety of voting setups.

Our paper is of interest for a potential committee designer and we have derived multiple policy recommendations. First of all, if the results of the vote need to be published, large majority requirements are suitable to make the committee more expensive to bribe. However, increasing the size of the committee is of limited interest. Increasing proportionally n and m leads to a less than proportional increase in cost and increasing only n can even make the committee cheaper to bribe. Instead of hiring new members, the committee designer can diversify the recruitment of committee members: increasing the dispersion of members' valuation for the proposal combined with a demanding majority requirement increases the cost.

With respect to the voting rule, we have seen that sequential voting is of little interest. A vote buyer could still manipulate the pivotal channel and would end up facing the same considerations. However, and perhaps surprisingly, a less transparent voting process can make the committee more robust to corruption. The vote buyer's ability to manipulate pivotal considerations relies on her capacity to monitor (and contract on) individual voting decisions: if she only observes the outcome of the vote, there always exists an equilibrium in which the proposal is rejected. However, we show that observing the number of votes in favor is sufficient to exploit the pivotal channel, especially in large committees.

Our discussion was framed in terms of committee members voting on a proposal, but the mechanism we highlight is much more general. To begin with, it can explain why ruling parties are reelected with large majorities in non-democratic countries. Relying on a short majority would be expensive because each individual (or faction) can be decisive and thus the price of each vote would be large. Conversely, if people believe that the ruling party will be reelected with a large majority, they perceive themselves as having no chance of changing the outcome of the election and hence are willing to accept even small benefits in exchange for their votes. A related argument can be found in Angeletos et al. (2007) or in Edmond (2013), which have citizens playing a coordination game to overthrow a regime.

References

(1966). Hoffa v. United States. `supreme.justia.com/cases/federal/us/385/293/case.html`.

Angeletos, G.-M., C. Hellwig, and A. Pavan (2007). Dynamic global games of regime change: Learning, multiplicity, and the timing of attacks. *Econometrica 75*(3), 711–756.

Banks, J. S. (2000). Buying supermajorities in finite legislatures. *American Political Science Review 94*(03), 677–681.

Buchanan, J. M., R. D. Tollison, and G. Tullock (1980). *Toward a theory of the rent-seeking society*. Number 4. Texas A & M Univ Pr.

Casella, A., A. Llorente-Saguer, and T. R. Palfrey (2012). Competitive equilibrium in markets for votes. *Journal of Political Economy 120*(4), 593–658.

Chen, C. (2005). Inequalities for the polygamma functions with application. *Scientia Magna 1*(2), 91–95.

Dal Bo, E. (2007). Bribing voters. *American Journal of Political Science 51*(4), 789–803.

Dekel, E., M. O. Jackson, and A. Wolinsky (2008). Vote buying: General elections. *Journal of Political Economy 116*(2), 351–380.

Edmond, C. (2013). Information manipulation, coordination, and regime change. *Review of Economic Studies 80*(4), 1422–1458.

Feddersen, T. and W. Pesendorfer (1997). Voting behavior and information aggregation in elections with private information. *Econometrica: Journal of the Econometric Society*, 1029–1058.

Feddersen, T. and W. Pesendorfer (1998). Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *American Political science review 92*(01), 23–35.

Feddersen, T. J. and W. Pesendorfer (1996). The swing voter's curse. *The American economic review*, 408–424.

Fehrler, S. and M. T. Schneider (2017). Buying supermajorities in the lab.

Felgenhauer, M. and H. Grüner (2008). Committees and special interests. *Journal of Public Economic Theory 10*(2), 219–243.

Genicot, G. and D. Ray (2006). Contracts and externalities: How things fall apart. *Journal of Economic Theory 131*(1), 71–100.

Gingerich, D. W. and L. F. Medina (2013). The endurance and eclipse of the controlled vote: a formal model of vote brokerage under the secret ballot. *Economics & Politics 25*(3), 453–480.

Groseclose, T. and J. M. Snyder (1996). Buying supermajorities. *American Political Science Review 90*(02), 303–315.

Henry, E. (2008). The informational role of supermajorities. *Journal of Public Economics 92*(10), 2225–2239.

Iaryczower, M. and S. Oliveros (2015). Competing for loyalty: The dynamics of rallying support.

Lariviere, M. A. (2006). A note on probability distributions with increasing generalized failure rates. *Operations Research 54*(3), 602–604.

Mattila, M. and J.-E. Lane (2001). Why unanimity in the council? a roll call analysis of council voting. *European Union Politics 2*(1), 31–52.

Midjord, R., T. R. Barraquer, and J. Valasek (2017). Voting in large committees with disesteem payoffs: A 'state of the art'model. *Games and Economic Behavior*.

Morgan, J. and F. Várdy (2011). On the buyability of voting bodies. *Journal of Theoretical Politics 23*(2), 260–287.

Palfrey, T., H. Rosenthal, and N. Roy (2017). How cheap talk enhances efficiency in threshold public goods games. *Games and Economic Behavior 101*, 234–259.

Palfrey, T. R. and H. Rosenthal (1991). Testing for effects of cheap talk in a public goods game with private information. *Games and economic behavior 3*(2), 183–220.

Qi, F. and B.-N. Guo (2016). An inequality involving the gamma and digamma functions. *Journal of Applied Analysis 22*(1), 49–54.

Riboni, A. (2013). Ideology and endogenous constitutions. *Economic Theory 52*(3), 885–913.

Rueda, M. R. (2015). Buying votes with imperfect local knowledge and a secret ballot. *Journal of Theoretical Politics 27*(3), 428–456.

Rueda, M. R. (2017). Small aggregates, big manipulation: Vote buying enforcement and collective monitoring. *American Journal of Political Science 61*(1), 163–177.

Samuels, S. M. (1965). On the Number of Successes in Independent Trials. *The Annals of Mathematical Statistics 36*(4), 1272–1278.

Schlesinger, A. M. (1978). Robert kennedy and his times, vol. 2.

Shaked, M. and J. Shanthikumar (2007). *Stochastic Orders*. Springer Series in Statistics. Springer New York.

Smith, A. and B. B. De Mesquita (2012). Contingent prize allocation and pivotal voting. *British Journal of Political Science 42*(2), 371–392.

Spenkuch, J. L., B. P. Montagnes, and D. B. Magleby (2018, July). Backward Induction in the Wild? Evidence from Sequential Voting in the US Senate. *American Economic Review 108*(7), 1971–2013.

Appendix A: Proofs

Lemma 3. *When the vote buyer offers b_k^*, the proposal is accepted with certainty in any equilibrium of the voting game.*

Proof. Consider at no loss of generality the strategy of (bribed) member 1. To begin the iterated elimination, note that choosing any cutoff below b_k^* is a dominated strategy for him: even if he expects to be pivotal with probability one, he should vote in favour if his bribe exceeds her disutility.

In general, if for all other bribed players i the smallest rationalizable cutoff after iteration t is $\underline{v}_i^{*(t)}$, his smallest rationalizable cutoff $\underline{v}_1^{*(t+1)}$ at iteration $t+1$ solves

$$\underline{v}_1^{*(t+1)} \times \left(\max_{v_i^* \in [\underline{v}_i^{*(t)}, 1]} \pi(v_2^*, \dots, v_k^*) \right) = b_k^*. \tag{3}$$

Given symmetry, we have $\underline{v}_i^{*(t)} = \underline{v}_j^{*(t)}$ for all i, j, n and hence refer simply to $\underline{v}^{*(t)}$. The remainder of the proof shows that if $\underline{v}^{*(t)} < 1$, then $\underline{v}^{*(t+1)} > \underline{v}^{*(t)}$.

1. The first committee member is pivotal if exactly $m-1$ of the $k-1$ bribed players vote in favor. If $S := \sum_{i=2}^{k} \text{Bernoulli}(F(v_i^*))$, then

$$\begin{aligned} \pi(v_2^*, \dots, v_k^*) &= \mathbb{P}(S = m-1) \\ &= f_{k-1}(m-1; \mathbf{v}^1), \end{aligned}$$

   where the last object is the PMF of a Poisson-Binomial random variable with $k-1$ trials and success probability vector given by $\mathbf{v}^1 = (v_2^*, \dots, v_k^*)$. For any i,

$$f_{k-1}(m-1; \mathbf{v}) = F(v_i^*) f_{k-2}(m-2; \mathbf{v}^{1i}) + (1 - F(v_i^*)) f_{k-2}(m-1; \mathbf{v}^{1i}),$$

   whence $\frac{\partial f_{k-1}}{\partial F(v_i^*)}$ is independent of $F(v_i^*)$. Thus, there is a solution to the maximization problem in (3) in which $v_i^* \in \{\underline{v}^{*(t)}, 1\}$ for all i.

2. In light of this, let $\pi_h(\underline{v})$ be the value of the pivotal probability if exactly h of the $k-1$ agents choose a cutoff of $v_i^* = 1$ and $k-1-h$ choose a cutoff of $\underline{v}$; then the value of the maximization problem is just $\max_h \pi_h(\underline{v}^{*(n)})$.

3. For any h, we can derive a critical bribe $b_{k,h}^*$ as in the main text that satisfies $\underline{v}\pi_h(\underline{v}) < b_{k,h}^*$ for all $\underline{v}$ and is defined by

$$b_{k,h}^* = max_v\{v \underbrace{\binom{k-h-1}{m-h-1}[F(v)]^{m-1-h}[1-F(v)]^{k-m}}_{\pi_h(v)}\}.$$

4. We now show that $b_{k,h}^* < b_k^*$ for all $h \in \{1, ..., m\}$ (indeed, we have $b_{k,0}^* = b_k^*$) by establishing

$$\begin{aligned} \frac{\partial b_{k,h}^*}{\partial h} &= -v\binom{k-h-1}{m-h-1}(1-F(v_h^{**}))^{k-m}F(v_h^{**})^{m-h-1} \\ &\quad \times \left\{Log[F(v_h^{**})] + \psi^{(0)}(k-h) - \psi^{(0)}(m-h)\right\} < 0. \qquad (4) \end{aligned}$$

If $v_i \sim U[0,1]$, we have $v_h^{**} = \frac{m-h}{k-h}$ and require

$$\log(m-h) - \psi^{(0)}(m-h) - [\log(k-h) - \psi^{(0)}(k-h)] \geq 0.$$

By Chen (2005), Theorem 1 we have $\psi^{(1)}(x) > \frac{1}{x} + \frac{1}{2x^2}$, whence we know that $\frac{\partial}{\partial x}(\log(x) - \psi^{(0)}(x)) < 0$; but then as $k-h > m-h$, we are done for the uniform case.

5. Finally, return to (3) and note that[16] for $\underline{v}_1^{*(t)} < 1$,

$$\begin{aligned} \underline{v}_1^{*(t)} \times \left(\max_{v_i^* \in [\underline{v}_i^{*(t)}, 1]} \pi(v_2^*, \ldots, v_k^*) \right) &=_{(1,2)} \underline{v}_1^{*(t)} \max_h \pi_h(v^{*(t)}) \\ &\leq_{(3)} \max_h b_{k,h}^* \\ &<_{(4)} b_k^*, \end{aligned}$$

whereby $\underline{v}_1^{*(t+1)} > \underline{v}_1^{*(t)}$.

[16]The index on equalities/inequalities refers to the relevant step in the proof.

$\square$

Proposition 1. *The cheapest bribing scheme inducing certain passing of the proposal is* $b_i = b_n^*$ *for all* i.

Proof. We proceed in four steps. Firstly, we define the notion of a 'breakdown-boundary' bribe. Secondly, we provide a necessary condition breakdown-boundary bribes must satisfy. Thirdly, we show this condition allows us to form some conclusions about the potential cutoff combinations on the breakdown-boundary. Finally, we use these conclusions to show that from any breakdown-boundary bribe there is a path to $(m/k, \ldots, m/k)$ such that moving along this path reduces bribing costs.

1. A breakdown bribe is a bribe vector $(b_1, \ldots, b_k)$ such that there is no solution to the following system of equations:

$$\begin{aligned} v_1^* \pi_1 &= b_1, \\ &\vdots \\ v_k^* \pi_k &= b_k. \end{aligned}$$

   We define the breakdown *boundary* to be the set

$$\begin{aligned} B \;:=\; & \{b \in \mathbb{R}^k \,|\, \forall \epsilon \in \mathbb{R}^{++} \; \exists db \in \mathbb{R}^k : b + db \text{ is a breakdown bribe,} \\ & \text{and } ||db|| < \epsilon\}. \end{aligned}$$

   So a bribe lies in the breakdown-boundary if increasing it by an arbitarily small amount in some direction causes breakdown. Clearly, a cost-minimizing vote buyer never induces breakdown by choosing a bribe vector above the breakdown *boundary* as he can achieve the same outcome by bribing arbitrarily close to it.

2. By Lemma C.2 below, the breakdown set is open. Thus, if there is some direction in which an arbitrarily small increase in bribes causes breakdown, then any arbitrarily small increase in bribes causes breakdown. So any

cutoff vector $(v_1^*, \ldots, v_k^*)$ for which there exists $dv = (dv_1, \ldots, dv_k)$ which induces $db = (db_1, \ldots, db_k) \geq 0$ (with $db \neq 0$) cannot possibly support a breakdown-boundary bribing vector.

3. Assume wlog that $v_1^* \leq v_2^* \leq \cdots \leq v_k^*$. If we are on the breakdown-boundary:

   - There must be s and r such that

     $$\begin{aligned} f_{k-1}(m-1; \mathbf{v}^s) &\geq f_{k-1}(m; \mathbf{v}^s), \text{ and} \\ f_{k-1}(m-1; \mathbf{v}^r) &\leq f_{k-1}(m; \mathbf{v}^r). \end{aligned}$$

     Say there is no such r. Then setting $dv_i = v_i^*$ for all i yields

     $$\begin{aligned} db_i &= \pi_i v_i^* + v_i^* \sum_{j \neq i} [f_{k-2}(m-2; \mathbf{v}^{ij}) - f_{k-2}(m-1; \mathbf{v}^{ij})] v_j^* \\ &= m v_i^* \Big\{ f_{k-1}(m-1; \mathbf{v}^i) - f_{k-1}(m; \mathbf{v}^i) \Big\} > 0, \end{aligned}$$

     which cannot be. If there is no such s, use $dv_i = -v_i^*$. Finally, by (7) in Samuels (1965) we can choose $s = k$ and $r = 1$.

   - We must have $\sum v_i^* \geq m - 1 + v_1^*$. Say not; then $\sum_{i \neq j} v_i^* < m - 1$ for all j. So set $d_k = 1$. This implies $db_k = \pi_k > 0$ and furthermore for $j \neq k$,

     $$db_j = v_j^* [f_{k-2}(m-2; \mathbf{v}^{jk}) - f_{k-2}(m-1; \mathbf{v}^{jk})] \geq 0,$$

     where the inequality follows by Theorem 1 in Samuels (1965).

   - We must have $\sum v_i^* < m + v_k^*$. Say not; then $\sum_{i \neq j} v_i^* \geq m$ for all j. So $f_{k-1}(\cdot; \mathbf{v}^j)$ has mean at least m and therefore mode at least m. But this contradicts the first bullet.

4. If we are on the breakdown-boundary, we can save costs as follows:

- If $\sum v_i^* < m$: Set $dv_1 > 0$ and $dv_j = 0$ for $j \neq 1$. This yields

$$\begin{aligned}\sum_i db_i &= dv_1 \left[\pi_1 + \left(\sum_{j=1}^n v_j^* \frac{\partial \pi_j}{\partial v_1}\right)\right] \\ &= m dv_1 [f_{k-1}(m-1; \mathbf{v}^1) - f_{k-1}(m; \mathbf{v}^1)] \leq 0.\end{aligned}$$

  This expression does not depend on v_1: thus while v_1 does not (necessarily) remain the smallest cutoff after an increase, increasing it continues to save costs. Furthermore, this path eventually hits $\sum v_i^* = m$: as $\sum v_i^* \geq m - 1 + v_1^*$ we will not run into the constraint that $v_1^* \leq 1$ when increasing v_1^*.

- If $\sum v_i^* > m$: Set $dv_s < 0$ and $dv_j = 0$ for $j \neq s$. This yields

$$\begin{aligned}\sum_i db_i &= dv_s \left[\pi_s + \left(\sum_{j=1}^n v_j^* \frac{\partial \pi_j}{\partial v_s}\right)\right] \\ &= m dv_s [f_{k-1}(m-1; \mathbf{v}^s) - f_{k-1}(m; \mathbf{v}^s)] \leq 0.\end{aligned}$$

  This expression does not depend on v_k^*: thus while v_k^* does not (necessarily) remain the largest cutoff after a decrease, decreasing it continues to save costs. Furthermore, this path eventually hits $\sum v_i^* = m$: as $\sum v_i^* \leq m + v_k$, we will not run into the constraint that $v_k^* \geq 0$ when decreasing it.

- If $\sum v_i^* = m$: By Theorem 1 in Samuels (1965) we have

$$\begin{aligned}f_{k-1}(m-1; \mathbf{v}^1) &\leq f_{k-1}(m; \mathbf{v}^1), \text{ and} \\ f_{k-1}(m-1; \mathbf{v}^k) &\geq f_{k-1}(m; \mathbf{v}^k).\end{aligned}$$

  Thus we can set $dv_1 = 1$, $dv_k = -1$ and $dv_j = 0$ for $j \notin \{1, k\}$. This keeps $\sum v_i^*$ constant, so once we hit this region we do not leave it.

Furthermore,

$$\begin{aligned}
\sum_i db_i &= dv_1 \left[\pi_1 - \pi_k + \left(\sum_{j=1}^{k} v_j^* \left[\frac{\partial \pi_j}{\partial v_1} - \frac{\partial \pi_j}{\partial v_k} \right] \right) \right] \\
&= m dv_1 \left[\left(f_{k-1}(m-1; \mathbf{v}^1) - f_{k-1}(m; \mathbf{v}^1) \right) \right] \\
&\quad - m dv_1 \left[\left(f_{k-1}(m-1; \mathbf{v}^k) - f_{k-1}(m; \mathbf{v}^k) \right) \right] \\
&\leq 0.
\end{aligned}$$

Note that this time the value of the expression depends on v_1^* and v_k^*; but as we can employ Samuels' Theorem this is not an issue.

Thus, for any point on the breakdown-boundary we have found a cost-saving path from this point to $\mathbf{v} = (m/k, \dots, m/k)$, which we know lies on the breakdown-boundary from the discussion of the symmetric case. $\square$

Proposition 2. *The cost for the vote buyer is:*

1. *decreasing in the number of committee members n,*
2. *increasing in the majority requirement m,*
3. *increasing slower than proportionally in committee scale.*

Proof. We prove each comparative static in turn.

1. Given k and m, pivotal probabilities do not depend on n; thus, the only way n can affect cost is by being a constraint on how high a k can be chosen.

2. The cost of bribing a committee with majority requirement m is given by

$$C(m) = \min_k \left\{ k \times \max_v \left[v \binom{k-1}{m-1} F(v)^{m-1} (1 - F(v))^{k-m} \right] \right\}.$$

By the envelope theorem $\frac{dC(m)}{dm} = \frac{\partial C(m)}{\partial m}$ and hence algebra yields

$$\frac{dC(m)}{dm} \propto \log(F(v^{**})) - \log(1 - F(v^{**})) + \psi^{(0)}(k^* - m + 1) - \psi^{(0)}(m),$$

where k^* and v^{**} are the relevant solutions in the nested optimization problems. As $v_i \sim U[0,1]$, $v^{**} = \frac{m}{k}$ and $\frac{dC(m)}{dm} > 0$ if

$$\log(m) - \log(k^* - m) + \psi^{(0)}(k^* - m + 1) - \psi^0(m) > 0.$$

By Qi and Guo (2016) we have $\psi^{(0)}(m) - \log(m) < -1/(2m)$ and

$$\psi^{(0)}(k - m + 1) - \log(k - m) > \frac{1}{2(k - m)} - \frac{1}{12(k - m)^2}.$$

We can solve the resulting inequality to confirm[17] that $\frac{dC(m)}{dm} > 0$.

3. Say the vote buyer optimally offers b^* to k^* members when the committee has size n and majority rule m; she pays $C = k^* \times b^*$. An upper bound for her bribing cost if the committee is scaled by factor s is $\overline{C} = k^* \times s \times \overline{b}^*$, where $\overline{b}^*$ refers to the new optimal bribe given fixed k^*. It is sufficient to show that $\overline{C} < sC$, i.e. that $\overline{b}^* < b^*$. But $\overline{b}^*$ can be seen as the largest rectangle in the plane $(\bar{\pi}(v), v)$ where $\bar{\pi}(v)$ is the pivotal probability in the new setup:

$$\bar{\pi}(v) = \binom{s \times k - 1}{s \times m - 1} F(v)^{s \times m - 1}(1 - F(v))^{s \times (k-m)}.$$

By Lemma C.3, we have $\frac{\partial \bar{\pi}(v)}{\partial s} < 0$, so $\overline{b}^* < b^*$. □

Proposition 3. *The vote buyer bribes a larger supermajority when the distribution is more dispersed.*

Proof. The optimal choice of k is the solution to:

$$\min_k \left\{ k v^{**} \binom{k-1}{m-1} F(v^{**})^{m-1}[1 - F(v^{**})]^{k-m} \right\}$$

where we consider v^{**} as an implicit variable of k. Using the envelope theorem,

[17] The specified bounds on $\psi^{(0)}(\cdot)$ only allow us to conclude this if $k > 3$, but tighter bounds are easily available from the same paper.

we can write a simplified FOC

$$1/k + \psi^{(0)}(k) - \psi^{(0)}(k-m+1) + Log(1 - F(v^{**})) \;\; = \;\; 0$$

where we have multiplied by strictly positive terms to eliminate their inverse. But now from Lemma C.1 below we know that $\tilde{F}(\tilde{v}^{**}) > F(v^{**})$. Thus, the FOC under the more dispersed distribution lies below the FOC under the old distribution at the (old) optimal k. Given that it can be verified that the cost function admits at most one local minimum, this is sufficient to conclude that the (new) optimal k needs to be larger. □

Proposition 4. *Suppose $\frac{m-1}{n-1} \geq \alpha$. If $\tilde{F}(.)$ is more dispersed than $F(.)$, then the cost for the vote buyer is larger under $\tilde{F}(.)$.*

Proof. To begin with, $\pi(v)$ and $\tilde{\pi}(v)$ exhibit the following properties:

1. The two functions have the same value at their maximum.
2. The maximum of $\tilde{\pi}(v)$ is reached for smaller v than that of $\pi(v)$.
3. At the right of the max of $\tilde{\pi}(v)$, the two functions cross once at $\bar{v}$.
4. $\pi(v)$ lies above $\tilde{\pi}(v)$ for $v > \bar{v}$.

Now suppose that the optimal bribe when the distribution is $F(.)$ is $b^* = v^{**}\pi(v^{**})$. To see that the optimal bribe is necessarily larger when the distribution is $\tilde{F}(.)$, notice that there is (at least) one $v' \geq v^{**}$ such that $\tilde{\pi}(v') = \pi(v^{**})$. This implies that the optimal bribe when the distribution is $\tilde{F}(.)$ is at least $v'\pi(v^{**}) \geq v^{**}\pi(v^{**})$. A similar reasoning implies that all b_k^* are larger under $\tilde{F}(.)$ than under $F(.)$. As a result, the vote buyer has to pay more under $\tilde{F}(.)$. □

Lemma 4. *A subgame perfect equilibrium of the sequential game must satisfy the following conditions:*

1. *In any subgame $S_i(x, y)$ with $x \neq 1$, the member to play votes in favor.*
2. *In any subgame $S_i(1, y)$, the member to play accepts if $v_i < min\{1, \frac{b}{1-yb}\}$.*

Proof. Consider Table 1.

- In any subgame where $x \leq 0$, the proposal will pass and any bribe must be accepted.

- In any subgame where $x > y+1$, the proposal cannot pass and any bribe must be accepted.

- Consider the subgames where $x = 1$. Let $P^+(y)$ be the probability that the proposal is accepted by the remaining members if he votes against it. The member to play at subgame $S(1, y)$ faces the following arbitrage:

$$\begin{aligned} a = 1 \quad &\rightarrow \quad b - v_i \\ a = 0 \quad &\rightarrow \quad - v_i \times P^+(y) \end{aligned}$$

He thus accepts if $v_i < min\{\frac{b}{1-P^+(y)}, 1\}$. If $\frac{b}{1-P^+(y)} > 1$, $P^+(y+1) = 1$: the member playing before knows that the proposal will be accepted for sure by the next player if he votes against. If $P^+(y+1) < 1$ we have:

$$\begin{aligned} P^+(y+1) &= \frac{b}{1-P^+(y)} + \left(1 - \frac{b}{1-P^+(y)}\right) P^+(y) \\ &= P^+(y) + b \end{aligned}$$

We also know that $P^+(1) = b$. As a result, $P^+(y) = yb$. We can therefore reformulate the tradeoff at subgame $S(1, y)$ and see that the member to play accepts if $v < min\{\frac{b}{1-yb}, 1\}$.

- It remains to check that in any subgame $S(x, y), 1 < x < y$, the member to play prefers to accept. Consider a subgame $S(2, y)$ where $P^+(y) < 1$. Moreover, suppose that at $S(2, y-1)$, the player accepts for sure.

$$\begin{aligned} a = 1 \quad &\rightarrow \quad b - v_i \times P^+(y) \\ a = 0 \quad &\rightarrow - v_i \times P^+(y-1) \end{aligned}$$

Substituting the value for P^+, it implies that the member to play at

$S(2, y)$ accepts if $v < 1$, which is true (potentially, type $v = 1$ could mix but this event has 0-mass and can be neglected). It remains to check that the member to play at $S(2, y - 1)$ would accept. Notice that at $S(2, 1)$, the member to play would get $b - v_i P^+(1) = b - v_i b$ if he votes for and 0 if he votes against (the proposal would be rejected for sure). As a result, he votes for if $v_i < 1$, which is also true. Iterating this implies that any member to play in a subgame $S(2, y)$ votes for the proposal. The same argument applies to any subgame $S(x, y), 1 < x \leq y + 1$ and the member to play must accept the proposal. $\square$

Proposition 7. *If $W > 0$, the vote buyer offers strictly positive bribes.*

Proof. Suppose that the vote buyer offers positive bribes to exactly m players (bribing the minimal winning coalition is not necessarily the optimal strategy but the focus is sufficient for the proof). Her expected utility is:

$$U_{\text{VB}}(v) = W \times v^m - b(v) \times \mathbb{E}(\#\text{votes for}|v, m).$$

For the minimal winning coalition, $b(v) = v\pi(v) = v^m$. Furthermore, the number of votes in favor is distributed binomially with parameters m and v. Thus, the expected number of successes is mv and we conclude

$$U_{\text{VB}}(v) = v^m \{W - m \times v\}.$$

As a result, for any $W > 0$, $U_{\text{VB}}(v) > 0$ if $v < \frac{W}{m}$. $\square$

Lemma 5. *It is optimal for the vote buyer to guarantee certain acceptance if $W > \overline{W}$ where $\overline{W} < m + 1$.*

Proof. We can write the VB objective function as

$$\begin{aligned} \Pi(v, k) \quad &:= \quad W \times \mathbb{P}(\text{proposal passed}) \\ & \qquad -kb^*(v) \times \mathbb{P}(\text{a bribed member votes in favour}) \\ &= \quad W \sum_{x=m}^{n} \binom{k}{x} v^x (1 - v)^{k-x} - kv^2 \binom{k-1}{m-1} v^{m-1} (1 - v)^{k-m}. \end{aligned}$$

Taking the first-order conditions wrt v yields local extrema at

$$v_{ext} = \frac{m + 1 + W \pm \sqrt{-4(1+k)W + (m+1+W)^2}}{2(1+k)}.$$

The sign of $\frac{\partial \Pi}{\partial v}$ does not depend on v if the discriminant is negative. Thus, as long as

$$W \in \left[\begin{array}{l} 1 + 2k - m - 2\sqrt{k + k^2 - m - km}, \\ \quad 1 + 2k - m + 2\sqrt{k + k^2 - m - km} \end{array} \right],$$

the vote buyer either does not want to bribe at all or bribes breakdown. Furthermore, if the lower of the v_{ext} exceeds $\frac{m}{k}$, again the derivative never changes sign on the region of interest. This happens if

$$W > 1 + 2k - m + 2\sqrt{k + k^2 - m - km},$$

whence we can conclude that $W > 1 + 2k - m - 2\sqrt{k + k^2 - m - km}$ is sufficient for the vote buyer to employ a breakdown strategy conditional on bribing at all. We have to check this inequality separately for each k to ensure that for no k does the vote buyer prefer an interior strategy. Thus, the true lower bound is the maximum of the RHS of the inequality wrt k. It is easy to show that the RHS is strictly decreasing in k; the lowest admissible k is m. $\square$

Appendix B: Extension to General Distributions

Assume $v_i \stackrel{iid}{\sim} F(\cdot)$ and impose $b_i = b_j$ for all i, j as well as Assumptions 2 & 3 stated in the main text. We restate and prove Lemma 3 as well as Propositions 1 and 2 in this environment.

Lemma B.1. *When the vote buyer offers b_k^*, the proposal is accepted with certainty in any equilibrium of the voting game.*

Proof. To show that (4) also holds for more dispersed distributions, notice that $\frac{\partial b_{k,h}^*}{\partial h}$ is decreasing in $F(v)$. Moreover, by Lemma C.1, $\tilde{F}(\tilde{v}_h^{**}) > F(v_h^{**})$ if $\tilde{F}(\cdot)$ is more dispersed than $F(v)$. It follows that ISDS eliminates all strategies below v^{**} if $F(v)$ is more dispersed than the uniform. □

Proposition B.1. *The cheapest bribing scheme inducing certain passing of the proposal always involves bribing more than m committee members.*

Proof. If we let $\hat{C}(k)$ denote the cost of bribing k members and define v_{max} to be the maximal value that v can take, we have

$$\hat{C}(m+1) \quad < \quad (m+1) \times v_{max} \times \max_{v \in [0, v_{max}]} \{m(F(v))^{m-1}(1-F(v))\},$$

we have $\hat{C}(m) - \hat{C}(m+1)$ bounded below by

$$\begin{aligned} Z &:= mv_{max} - (m+1)v_{max} \times \max_{v \in [0, v_{max}]} \{m \times (F(v))^{m-1}(1-F(v))\} \\ &= mv_{max} - (m+1)v_{max} \left(\frac{m-1}{m}\right)^{m-1} \end{aligned}$$

where the last line follows from solving the maximization problem. It is sufficient to show that $Z > 0$, which is true for $m \geq 2$. □

Proposition B.2. *The cost for the vote buyer is*

1. *decreasing in the number of committee members n,*

2. increasing in the majority requirement m,

3. increasing slower than proportionally in committee scale.

Proof. We only used $v_i \sim U[0,1]$ in the proof of (ii). But from Lemma C.1, we know that $\tilde{F}(\tilde{v}^{**}) > \frac{m}{k}$ for any distribution $\tilde{F}(\cdot)$ more dispersed than the uniform distribution. Thus, the proof of (ii) still applies in this case. □

We now provide examples to illustrate how violations of Assumptions 1 and 3 can cause our results to fail.

Example 3. When the distribution of types is not sufficiently dispersed, committee members can collude on an asymmetric equilibrium when offered the bribing scheme discussed in the main section. To see that, assume $n = 3$, $m = 2$ and $v = 1/2$ for all players.

To begin with, focus on symmetric equilibria and suppose the vote buyer proposes a bribe b to all 3 players. When $b > 0$, each member should accept with a positive probability x. In equilibrium, x^* satisfies

$$b = \frac{1}{2}\pi(x^*),$$

where $\pi(x) = 2x(1-x)$. Proceeding as in the introductory example, no interior and symmetric equilibrium exists when $b > 1/4$, and the vote buyer spends a total amount of $3/4$.

However, if the vote buyer proposes this bribing scheme, committee members can collude on an asymmetric equilibrium where the proposal is rejected. For instance, one player can accept the bribe while the other two decline it. The players refusing the offer are pivotal with certainty and have no incentive to deviate. Such an equilibrium was ruled out in Lemma 3 using Assumption 3, which requires that valuations are sufficiently dispersed.

Example 4. This example shows that for some distributions the vote buyer could prefer to propose asymmetric bribes. Assume $n = 2$, $m = 1$. Furthermore, $v = 1/4$ with probability $3/4$ and $v = 1$ with probability $1/4$.

To begin with, suppose the vote buyer proposes the same bribe to both players. When $b > 1/4$, there is no equilibrium where the proposal is rejected with positive probability. Thus, the vote buyer can ensure passing for a total cost of $1/2$; by contrast, ensuring passing while bribing only a single player would cost 1.

However, the vote buyer can pay even less if she uses asymmetric bribes. Suppose that member 1 receives a bribe slightly larger than $1/4$ while member 2 receives $1/16$. To see that the proposal will be accepted, we apply an iterated deletion of dominated strategies. Member 1 must accept his bribe if his type is $1/4$, thus member 2 is at most pivotal with a probability $1/4$. Knowing this, member 2 must accept a bribe of $1/16$ if his type is $1/4$. Member 1 should then also accept the bribe if his type is 1, which guarantees that player 2 is never pivotal and should also accept the bribe.

Appendix C: Technical Lemmas

Lemma C.1. *Suppose $\tilde{F}(\cdot)$ and $F(\cdot)$ have increasing generalized failure rates. Suppose further that $\tilde{F}(\cdot)$ is more dispersed than $F(\cdot)$. Then $\tilde{F}(\tilde{v}^{**}) > F(v^{**})$, where $\tilde{v}^{**}$ is the equilibrium cutoff if $v_i \sim \tilde{F}(\cdot)$ and v^{**} is the equilibrium cutoff if $v_i \sim F(\cdot)$.*

Proof. To find the equation implicitly defining v^{**}, we solve

$$\max_v v \binom{k-1}{m-1} F(v)^{m-1}[1-F(v)]^{k-m},$$

which yields the FOC defining v^{**}:

$$\frac{[F(v^{**})-1]F(v^{**})}{v^{**}F'(v^{**})} = (m-1)-(k-1)F(v^{**}). \tag{5}$$

We can rewrite (5) in terms of $x = F(v)$ to yield

$$\frac{x-1}{F'(F^{-1}(x))F^{-1}(x)} = \frac{m-1}{x} - (k-1).$$

Now notice that the RHS is unambiguously decreasing. The LHS is increasing by our assumption of increasing generalized failure rates. Furthermore, as $\tilde{F} \leq_* F$, the LHS lies lower at $\tilde{F}$ than at F:

$$\frac{x-1}{\tilde{F}'(\tilde{F}^{-1}(x))\tilde{F}^{-1}(x)} \quad < \quad \frac{x-1}{F'(F^{-1}(x))F^{-1}(x)} \iff \frac{\partial}{\partial x}\left(\frac{F^{-1}(x)}{\tilde{F}^{-1}(x)}\right) \geq 0.$$

But this must mean that $\tilde{x} = \tilde{F}(\tilde{v}^{**}) > F(v^{**}) = x$. $\square$

Lemma C.2. *The breakdown set is open.*

Proof. Consider the function $g : \mathbb{R}^k \to \mathbb{R}^k$ defined by

$$g_i(v_1^*, \ldots, v_k^*) = v_i^* \pi_i.$$

As g is continuous, and as continuous images of compact sets are compact,

$g([0,1]^k)$ is compact, so closed. But then its complement must be open. $\square$

Lemma C.3. *If $x \sim$ Binomial$(\alpha n, p)$, then $\mathbb{P}(x = \alpha k)$ is weakly decreasing for $\alpha > 1$.*

Proof. As $\mathbb{P}(x = \alpha k) = \binom{\alpha n}{\alpha k} p^{\alpha k}(1-p)^{\alpha(n-k)}$, we have

$$\begin{aligned}\frac{\partial \mathbb{P}(x = \alpha k)}{\partial \alpha} &= -(1-p)^{\alpha(n-k)} p^{\alpha k} \binom{\alpha n}{\alpha k} \Big[(n-k)\psi^{(0)}(\alpha(n-k)+1) \\ &\quad +(k-n)\log(1-p) - k\log(p) \\ &\quad +k\psi^{(0)}(\alpha k + 1) - n\psi^{(0)}(\alpha n + 1) \Big].\end{aligned}$$

If the term in square brackets is always weakly positive, we are done. But

$$\frac{\partial [\,\cdot\,]}{\partial p} = \frac{n-k}{1-p} - \frac{k}{p},$$

whence there is a unique local minimum of the square bracket term at $p^* = \frac{k}{n}$ (SOC confirms this). Now if $p \to 0$ or if $p \to 1$, $[\,\cdot\,] \to \infty$; thus, to ensure $[\,\cdot\,] > 0$ everywhere it suffices to check the minimum. At $p = \frac{k}{n}$ we have

$$\begin{aligned}[\,\cdot\,] &= (n-k)\left\{\psi^{(0)}(\alpha(n-k)+1) - \log(n-k)\right\} \\ &\quad +k\left\{\psi^{(0)}(\alpha k + 1) - \log(k)\right\} - n\left\{\psi^{(0)}(\alpha n + 1) - \log(n)\right\}.\end{aligned}$$

If we let $f(x) = \psi^{(0)}(\alpha x + 1) - \log(x)$, it suffices to show that

$$\frac{n}{k} \geq \frac{f(n-k) - f(k)}{f(n-k) - f(n)}$$

As $f(\cdot) > 0, f'(\cdot) < 0$, we have $\frac{f(n-k)-f(k)}{f(n-k)-f(n)} \leq 1$; thus, we are done. $\square$

Appendix D: Competition

Consider a setup with two competing vote buyers, an attacker A and a defender D. As in the main text, we assume that A has a large valuation for the proposal and hence seeks certain passing. D, however, is assumed to have resources limited by her budget constraint X. In a legislative setting, D can be a whip. In a court trial, the opposite party can also attempt to interfere with the jury.

Because of the deep pocket assumption, D cannot prevent passing of the proposal. Hence, we assume her objective is to maximize the overall bribing cost to A. We denote by $\{b_1^A, ..., b_n^A\}$ and $\{b_1^D, ..., b_n^D\}$ the bribes offered by A and D respectively. Thus, the voting subgame is equivalent to that in the main text with $b_i = b_i^A - b_i^D$.

As in Groseclose and Snyder (1996), vote buyers make offers sequentially, but we reverse the order and let D move first. The timing of offers is paramount, as explained for instance in Banks (2000). Suppose without loss of generality that $b_1^D \leq b_2^D \leq ... \leq b_n^D$. Then we have the following result:

Proposition 8. *Let* $\underline{X} = n[(n-1)b_{n-1}^* - nb_n^*]$. *Then*

1. *If* $X > \underline{X}$, *there is a unique equilibrium. In this equilibrium,* $b_i^D = \frac{X}{n}$ *and* $b_i^A = \frac{X}{n} + b_{k(X)}^*$ *for all* i *where* $k(X)$ *is decreasing in* X.

2. *If* $X < \underline{X}$, *there is an equilibrium for any possible bribe vector* $\{b_1^D, ..., b_n^D\}$ *for which* $b_n^D < (n-1)b_{n-1}^* - nb_n^*$. *In this equilibrium,* $b_i^A = b_i^D + b_n^*$ *for all* i.

Proof. We proceed by backward induction.

1. The cheapest way for A to ensure passing by employing a coalition of size k is to propose $b_i^A = b_i^D + b_k^*$ to members $i = 1, 2, \ldots, k$. Proposing $0 < b_i^A < b_i^D$ is never optimal as member i would still vote against the proposal with certainty. Thus $b_i^A \geq b_i^D$ for all bribed members. But then the problem reduces to the one in the main text and hence symmetric 'net' bribes must be optimal.

2. If $b_n^D < (n-1)b_{n-1}^* - nb_n^*$, A chooses $k = n$ and hence pay $nb_n^* + X$ no matter how D decides to allocate her budget (as long as she doesn't violate the inequality).

3. Otherwise, A chooses $k < n$ and hence bribe a coalition of the k cheapest members. If $b_1^D < b_N^D$ then D can increase the cost to A by setting increasing b_1^D and decreasing b_N^D. Hence, $b_i^D = \frac{X}{n}$.

4. Finally, given D's strategy, A will choose

$$k^* \in \arg\max kb_k^* + \frac{k}{n}X,$$

and hence $\frac{\partial k^*}{\partial X} \leq 0$.

$\square$

Thus, the existence of the second vote buyer decreases the size of the supermajority bribed by A. Because members now receive payments from the opposite party, there is now a fixed cost for including an additional member in the coalition. This fixed cost makes the pivotal channel more expensive to use.

Finally, we analyze how the cost for the attack depends on the resources available to the defense. According to Proposition 8, the number of players bribed by the attack is decreasing in X, but the defense needs to split her budget equally across all members. This implies that when X is large, increasing the budget of D leads her to increase some bribes that are eventually ignored by the attack. The marginal impact of X on the cost for the attack is therefore decreasing:

Corollary 1. *The cost for the attack is increasing, concave and piecewise linear in X.*