The Cross-Section of Labor Leverage and Equity Returns*

A. DONANGELO, F. GOURIO, and M. PALACIOS†

ABSTRACT

We demonstrate theoretically in a standard production model that even if labor is fully flexible, it generates a form of operating leverage ("labor leverage"), provided that two conditions are satisfied: (1) wages are smoother than productivity; (2) the capital-labor elasticity of substitution is strictly lower than one. Moreover, our model supports using the labor share–the ratio of labor expenses to value added–as a proxy for labor leverage. We provide evidence for assumptions (1) and (2), and demonstrate the empirical importance of the labor leverage mechanism: high labor-share firms have operating profits that are more sensitive to aggregate shocks and have higher expected asset returns.

*This version: February 2016.

†Donangelo is with the Finance Department, the University of Texas at Austin, Gourio is with the Federal Reserve Bank of Chicago, and Palacios is with the Finance Department, Vanderbilt University. The views expressed here are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

Labor compensation is the largest expense for firms: despite its documented secular decline, labor costs still represents well over 50% of gross output in the United States.[1] Magnitude, however, is not the only distinguishing property of labor compensation. For asset pricing, an arguably equally important property of labor compensation is its smoothness relatively to firms' revenue, which is another well-known "stylized fact". In this paper, we demonstrate that this smoothness, combined with the complementarity of capital and labor leads to a labor-induced form of operating leverage (henceforth *labor leverage*), even in a frictionless environment. Labor leverage amplifies firm risk in a way analogous to financial leverage, but given the importance of labor, one might expect labor leverage to be as important as financial leverage. But while financial leverage can easily be measured, labor leverage is harder to measure. Empirical work has hence been hampered by the lack of theoretically supported measure of firm-level labor leverage in the literature. This paper fills this gap and provides theoretical support and empirical validation for labor share–the share of labor expenses in value added–as a measure of firm-level labor leverage. Moreover, this paper presents new evidence for the economic significance of labor leverage in explaining cross-sectional differences in the riskiness of cash flows and asset returns.

We first present a parsimonious production-based model that demonstrates the mechanism and justifies using labor share as a proxy for labor leverage. The model represents a firm that is exposed to systematic and idiosyncratic productivity shocks and fluctuations in economy-wide wages. The firm responds to these shocks by making frictionless adjustments to its labor input. The model highlights two conditions that are both necessary and sufficient for labor leverage: first, wages must be smooth relative to productivity; and second, the elasticity of substitution between capital and labor must be less than unity, i.e. labor and capital must be strict complements. The first assumption is intuitive: shocks to revenue

[1]For instance, Gollin (2002) finds that labor share is between 0.65 and 0.80 across most developed countries included in his sample. For a discussion of the global decline in labor share, see Piketty (2014) and Karabarbounis and Neiman (2014).

get amplified if costs do not move much. But labor costs can change either because the wage changes, or because the quantity of labor changes. The latter assumption is required to make sure that labor quantities are also sufficiently smooth. Both assumptions are supported in the data. Aggregate wages are less volatile than productivity; we also document that labor costs are significantly less variable than other costs: for instance, in our sample, a 1% reduction in sales leads on average to a .62% reduction in staff expenses, but to a 1.23% reduction in all other costs. We also provide evidence in favor of a low elasticity of substitution between capital and labor, consistent with a large literature in economics. Specifically, our model shows that a ratio of regression coefficients identifies the elasticity of substitution. For a wide range of specifications, this number is around 0.4 in our data.

For simplicity, this paper considers the simplest environment that generates a positive labor leverage that varies across firms. However, it is important to note that many other mechanisms can generate similar results.[2] We employ a constant elasticity of substitution (CES) production function that, consistent with empirical work, is restricted so that labor and capital are strict complements.[3] The strict complementarity between labor and capital imply that a firm with low labor share will have riskier cash flows, as we demonstrate.

Just like financial leverage, labor leverage is not a source of risk, it is merely an amplifier of existing risk. For this reason, we should only expect to see a positive relation between labor share and expected returns for firms with strictly positive systematic risk factor loading (i.e., almost all publicly listed firms). Our model shows that this condition is satisfied as long as a firm's productivity has a greater systematic risk loading than aggregate wages. An equivalent sufficient condition is the greater volatility and procyclicality of productivity with

[2]Examples of alternative mechanisms that drive labor cost smoothness are: labor contracts that insure workers (e.g., Danthine and Donaldson (2002), Berk and Walden (2013), and Favilukis and Lin (2015)), unionization (e.g., Chen, Kacperczyk, and Ortiz-Molina (2012)), and labor mobility (e.g., Donangelo (2014)).

[3]The widely used Cobb-Douglas production function does not allow for this flexibility since it constrains the elasticity of substitution between labor and capital to unity. As discussed by León-Ledesma, McAdam, and Willman (2010) and Klump, McAdam, and Willman (2012), there is strong empirical evidence in the literature that the elasticity of substitution is lower than one, especially at the firm-level.

respect to wages. We provide empirical evidence that supports this last condition.

The main contribution of our paper is empirical. We construct two novel alternative firm-level measures of labor share using Compustat data. These two measures are closely related to the one from our model. We validate our two measures of labor share by showing that these are in fact positively related to the sensitivity of operating profits to macroeconomic shocks. In particular, we show that the sensitivity of profits to real GDP and aggregate TFP shocks is positive for the average firm and cross-sectionally increasing in labor share. Consistent with the model, we also find that the sensitivity of profits to aggregate shocks to wages is negative for the average firm and increasing in magnitude in labor shares, albeit this result is not always significant at conventional levels.

After documenting the relation between labor share and cash-flows, we proceed to study the implications of our proposed mechanism for expected returns. To address the challenge that expected returns are not observable, we use two different types of proxies for them: realized asset returns and systematic risk loadings (i.e., betas on risk factors). We find supporting evidence that expected asset returns are increasing in labor share. In particular, we find that high-labor share firms earn, on average, higher realized asset returns and that these firms have higher betas.

This paper contributes to the literature that studies the relation between operating leverage and stock returns.[4] Within this literature, our paper is more closely related to the strand that discusses the relation between *labor-induced* forms of operating leverage and asset prices. Examples of this literature are Danthine and Donaldson (2002), Belo, Lin, and Bazdresch (2014), Donangelo (2014), Zhang (2014), and Favilukis and Lin (2015). Danthine and Donaldson (2002) discuss a mechanism where counter-cyclical capital-to-labor share leads to labor-induced operating leverage in a general equilibrium setting. In their model, wages are

[4]Some examples of this literature that focus on the traditional (i.e., non labor-induced) form of operating leverage are Lev (1974), Mandelker and Rhee (1984), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Garcia-Feijoo and Jorgensen (2010), and Novy-Marx (2011).

less volatile than profits, due to the limited market participation of workers, and firms insure workers through labor contracts against labor risk. Stable wages act as an extra risk factor for shareholders, as markets are incomplete in their model. Donangelo (2014) proposes a model that establishes the positive connection between labor mobility and labor leverage. Labor intensity and labor mobility are two complementary mechanisms that affect a firm's operating leverage. In a cyclical industry, the effect of labor mobility on firm risk is increasing in labor share, and the effect of labor share on firm risk is increasing in labor mobility. Most recently, Zhang (2014) derives predictions similar to our model based on the optimal implicit contract between workers and firms. Whereas our dynamics stems from the interaction of events in the wider economy and the firm, this line of the literature focuses on the interaction between a worker's productivity, wages, and the firm. In reality both variations in the worker's productivity and changing economic conditions affect firm risk. We thus we see our "spot" labor market analysis and the implicit contracts analysis as complementary to this literature.[5]

Related to the theoretical approach of this paper are studies of the cross-section of returns based on micro-level decisions through dynamic optimization. Examples of the literature are Berk, Green, and Naik (1999), Carlson et al. (2004), and Zhang (2005). The main departure point from the literature is that, in our work, labor decisions made by workers also affect firm risk, whereas the main determinant of firm risk are decisions made by equity holders on investments in Berk et al. (1999), Carlson et al. (2004), and Zhang (2005).

[5]Other papers that relate labor to finance issues are Peterson (1994), Santos and Veronesi (2006), Merz and Yashiv (2007), Chen and Zhang (2011), Chen et al. (2012), Eisfeldt and Papanikolaou (2013), Petrosky-Nadeau, Zhang, and Kuehn (2013), Kuehn, Simutin, and Wang (2014), Schmidt (2014), Favilukis, Lin, and Zhao (2015), and Favilukis and Lin (2015).

I. Theoretical Motivation for Labor Leverage

In this section, we provide theoretical support for the existence of the labor leverage mechanism and for the use of labor share as a valid proxy for it.[6]

The model represents a firm that operates over two dates. At date t, the firm produces value added Y according to

$$Y_t = X_t F[K_t, L_t], \tag{1}$$

where X denotes the firm's total factor productivity (TFP), L denotes labor, K denotes capital, and the function F satisfies the Inada conditions and has the constant returns-to-scale property. We consider a partial-equilibrium model and assume that the firm can hire any quantity of labor frictionlessly at wage rate W, but that adjustment costs for capital are sufficiently high as to make it constant in the instant considered, $K_t = K$.[7] The firm's profit maximization problem at time t defines optimized operating profits Π as given by

$$\Pi_t = \max_{L_t}\{X_t F[K, L_t] - L_t W_t\}, \tag{2}$$

where W denotes market wage, which is possibly correlated with the firm's TFP.[8]

We define labor leverage as the ratio of the elasticity of operating profit growth to productivity growth and the elasticity of value added growth to productivity growth minus one. Formally,

$$LL = \frac{\partial \Pi_t^{G} / \partial X_t^{G}}{\partial Y_t^{G} / \partial X_t^{G}} - 1, \tag{3}$$

[6]The last section of the paper presents a dynamic model of a firm exposed to labor leverage that represents an application of the theory developed in this section.

[7]Note that our setting abstracts away from any frictions affecting the labor demand or the labor supply and from any source of heterogeneity in labor markets.

[8]In the Appendix we briefly discuss the case where the firm is also subject to a fixed operating cost, which represents traditional operating leverage.

where the superscript G denotes log growth (e.g., $X_t^{\mathrm{G}} \equiv \log[X_t/X_{t-1}]$). This definition simply captures the extent to which a "top-line" shock (to value added, or more generally revenue) is transformed into a "bottom-line" shock, i.e. a shock to profits (operating income), and hence it is similar to operating leverage.

It is common to assume a Cobb-Douglas production, $F(K,L) = K^{\alpha}L^{1-\alpha}$. In this case, profits are a constant share of output, $\Pi = (1-\alpha)Y$, and hence the elasticity of profit growth equals the elasticity of value added growth, so that labor leverage $LL = 0$. However, as the next proposition shows, this case turns out to be knife-edge (and, we will argue later, not empirically relevant).

Proposition 1 (Labor Leverage and Labor Share)
For a general production function, labor leverage is given by

$$LL == \frac{\partial\Pi_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}} + \partial\Pi_t^{\mathrm{G}}/\partial W_t^{\mathrm{G}}\partial W_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}}{\partial\Pi_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}} + \partial\Pi_t^{\mathrm{G}}/\partial W_t^{\mathrm{G}}\partial W_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}} - 1 = \frac{1 + \frac{S_t}{1-S_t}\left(1 - \partial W_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}\right)}{1 + \gamma\frac{S_t}{1-S_t}\left(1 - \partial W_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}\right)} - 1, \tag{4}$$

where $\gamma \equiv \frac{F_{\mathrm{K}}[K,L_t]F_{\mathrm{L}}[K,L_t]}{F[K,L_t]F_{\mathrm{KL}}[K,L_t]}$ *is the elasticity of substitution between labor and capital,* $S \equiv L_tW_t/Y_t$ *is labor share.*[9]

Proposition (1) shows that labor leverage is a function of the firm's labor share, the elasticity of substitution of capital and labor, and the response of wages to productivity changes. In particular, note that if wages respond one-for-one with productivity, i.e. $\frac{\partial W_t^{\mathrm{G}}}{\partial X_t^{\mathrm{G}}} = 1$, then all firms have zero labor leverage. Hence, smooth wages are a necessary condition for labor leverage to exist. We will hence consider the following assumptions:

Assumption 1 (Smoothness of Wages and Strict Complementarity of Labor and Capital)

a. Wages are smooth relative to productivity: $\frac{\partial W_t^{\mathrm{G}}}{\partial X_t^{\mathrm{G}}} < 1$.

b. The elasticity of substitution between labor and capital is less than one: $\gamma < 1$.

[9]The subscripts K and L denote partial derivatives with respect to labor and capital, respectively.

The corollary that follows shows that Assumption 1 represents a set of sufficient conditions for the existence of labor leverage and for the validity of labor share as its proxy.

Corollary 1 (Labor Leverage and Labor Share)
Assumption 1 implies

a. *the existence of labor-induced operating leverage:* $\frac{\partial \Pi_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}}{\partial Y_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}} > 1$

b. *labor-induced operating leverage to be increasing in labor share:* $\frac{\partial \left(\frac{\partial \Pi_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}}{\partial Y_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}} \right)}{\partial S_t} > 0$

The discussion so far shows that labor leverage makes operating profits relatively more sensitive to shocks. In order for labor leverage to also lead to higher expected returns, we should consider the relative systematic risk exposure of TFP X and wages W. For simplicity, assume that markets are effectively complete and that the economy has a single source of systematic shocks with zero autocorrelation and a constant price of risk. Let M denote the value of an asset that is only exposed to systematic risk and that has a risk loading $\beta^{\mathrm{M}} = 1$. Let $\beta_t^{\mathrm{X}} \equiv \frac{\partial X_t^{\mathrm{G}}}{\partial M_t^{\mathrm{G}}}$ and $\beta_t^{\mathrm{W}} \equiv \frac{\partial W_t^{\mathrm{G}}}{\partial M_t^{\mathrm{G}}}$ denote the systematic risk loadings of portfolios of securities that perfectly replicate TFP growth and wage growth.

Assumption 2 (Positive and high systematic risk loading of TFP relative to wages)
$\beta_t^{\mathrm{X}} > 0$ *and* $\beta_t^{\mathrm{X}} > \beta_t^{\mathrm{W}}$.

The proposition below shows that Assumption 2 implies that asset betas are increasing in labor share.

Proposition 2 (Asset Betas and Labor Share)

a. *Cash flow beta:* $\beta_t \equiv \frac{\partial \Pi_t^{\mathrm{G}}}{\partial X_t^{\mathrm{G}}} \frac{\partial X_t^{\mathrm{G}}}{\partial M_t^{\mathrm{G}}} + \frac{\partial \Pi_t^{\mathrm{G}}}{\partial W_t^{\mathrm{G}}} \frac{\partial W_t^{\mathrm{G}}}{\partial M_t^{\mathrm{G}}} = \beta_t^{\mathrm{X}} + \frac{S_t}{1-S_t} \left(\beta_t^{\mathrm{X}} - \beta_t^{\mathrm{W}} \right)$.

b. *Assumption 2 implies that* $\frac{\partial \beta_t}{\partial S_t} > 0$.

The corollary below shows how the capital-labor elasticity of substitution is related to the elasticities of value added growth and operating profits growth to shocks.

Corollary 2 (Useful Relation Involving Capital-Labor Elasticity of Substitution)
The elasticities of value added growth and operating profits growth to shocks are linearly related through the elasticity of substitution between labor and capital as given by

$$\partial Y_t^G / \partial X_t^G - 1 = \gamma(\partial \Pi_t^G / \partial X_t^G - 1).$$

We will later use the relation formalized in Corollary 2 to estimate the elasticity of substitution between labor and capital in the data.

II. Empirical Evidence

We first summarize our testable hypothesis. We then discuss how we construct the labor share variable. We then present evidence for the smoothness of labor costs, strict complementarity between labor and capital, and for the sensitivity of profits to aggregate shocks to be increasing in labor share. At the end of the section, we explore the cross-sectional relation between labor share and expected returns.

A. Testable Hypothesis

This section presents empirical support for two main testable implications of the theoretical discussion from the previous section: (1) firms with high labor share exhibit higher sensitivity of cash flows to aggregate shocks (Corollary (1)); and (2) firms with high labor shares have higher expected returns (Corollary (**??**)). As discussed, the relative smoothness of labor costs can be, together with the smoothness of wages, explained by the strict complementarity of labor and capital. We explore the theoretical prediction from Corollary (2) to estimate and confirm our assumption that the elasticity of substitution between labor and capital is lower than one.

B. Data

Our main empirical measure of labor share (hereafter LS) is given by the ratio of labor costs to value added. It is defined from Compustat items as follows:

$$\mathrm{LS}_{it} \equiv \frac{\mathrm{XLR}_{it}}{\mathrm{OIBDP}_{it} + \mathrm{XLR}_{it} + \Delta\mathrm{INVFG}_{it-1,t}}, \tag{5}$$

where *XLR* is Compustat variable "Staff Expense – Total" which we use as a proxy for labor costs[10], *OIBDP* is Compustat variable "Operating Income Before Depreciation", and $\Delta\mathrm{INVFG}_{t-1,t} \equiv \mathrm{INVFG}_t - \mathrm{INVFG}_{t-1}$ is the change in Compustat variable "Inventories – Finished Goods". We include change in inventories of final goods to make the empirical measure consistent with the theoretical one. The reason is that, unlike in our model, some of the goods produced over a given year are not sold during that year and, likewise, a portion of the goods sold by the firm in a given year were produced in previous years.[11]

A limitation of the LS measure is that, since the variable *XLR* is a supplementary income statement item, it is only available for roughly 12% of firm-year observations in our sample. To address this limitation, we use a second measure which we denote "extended" labor share (hereafter ELS). We define ELS as follows:

$$\mathrm{ELS}_{it} \equiv \begin{cases} \mathrm{LS}_{it} & \text{if } XLR \text{ is non-missing} \\ \frac{\mathrm{LABEX}_{it}}{\mathrm{OIBDP}_{it}+\mathrm{LABEX}_{it}+\Delta\mathrm{INVFG}_{it-1,t}} & \text{if } XLR \text{ is missing,} \end{cases} \tag{6}$$

where *LABEX* is a constructed variable defined as the product of Compustat variable *EMP* ("Number of Employees") and the average annual labor compensation per employee in the industry during that year. We estimate the average labor compensation per employee as the

[10]According to the U.S. GAAP definition, *XLR* is the sum of "salaries, wages, pension costs, profit sharing and incentive compensation, payroll taxes and other employee benefits".

[11]We set $\Delta\mathrm{INVFG}_{it}$ to 0 when either INVFG_{it} or INVFG_{it-1} are missing. The results presented in the paper are qualitatively unaffected by excluding the change in inventories from the measure of labor share.

average ratio of *XLR* and *EMP* in the industry, calculated using the firms that do report *XLR*.[12] We exclude from our sample firm-year observations where ELS is negative or greater than one.

Table I reports time series averages of median characteristics for portfolios of firms sorted on LS (Panel A) and ELS (Panel B). We present the statistics both for simple sorts and for within-industry sorts. (This is motivated by the evidence in Novy-Marx (2011) that intra- as opposed to inter-industry differences in book-to-market ratios are more closely related to cross-sectional variation in operating leverage intensity.) By construction, the second and third columns of Panel A are identical since ELS is defined as LS in the subsample of firms where the latter is non-missing. More telling is the fact that the second and third columns of Panel B are quite similar as well. We interpret this fact as evidence that the distribution of ELS conditional on missing LS is not significantly different from the distribution of ELS conditional on non-missing LS. The fourth column reports that the number of employees per unit of plant, property, and equipment (PPE), which represents an additional measure of labor intensity used in the literature, is increasing in both LS and ELS.

Columns five to eleven of the two panels how firm characteristics vary across labor market quintiles. High labor share firms tend to have higher book-to-market ratios than low labor share firms, in particular in industry-adjusted sorts. The table also shows a negative relation between labor share and both market value of equity and book value of assets. The negative trend in market value of equity is consistent with the hypothesized greater riskiness of high labor share firms. A possible explanation for the negative trend in asset values is a downward bias in asset value reporting, in particular since high labor share firms are both less capital intensive and have less tangible assets.[13] Consistent with a reporting bias, the panels report that the value of organizational capital, which is not considered in a firm's financial reports,

[12]We use the Fama-French 17-industry if available, otherwise we use the average ratio from the 2-digit SIC industry.

[13]See Damodaran (2011) for a discussion of the relation between intangibles and a bias in asset value reporting.

is increasing in labor share. Profitability ratios and to some extent financial leverage ratios seem fairly unrelated to labor share. All these patterns are qualitatively similar across our two measures of labor share.

<< Table I here >>

C. Evidence for the Labor Leverage Mechanism

In this section we present empirical support for the existence of the labor leverage mechanism. We start by verifying the two sufficient conditions discussed in the theoretical motivation section. The first condition, which is sufficient for the existence of the labor leverage mechanism, is for wages to be smoother and less procyclical than productivity. The second condition, which in addition to the first guarantee that labor leverage amplifies expected equity returns, is that labor share is countercyclical, or equivalently, that the capital-labor elasticity of substitution is less than one.

C.1. Evidence for Labor Cost Smoothness

Panel A of Table II gives some statistics that support the hypothesis that the wages are smoother and less procyclical than output, profits, and TFP. The table shows that the volatility of the growth rate of before-tax profits is 3.54 times the volatility of the growth rate of GDP, and the slope coefficient in a regression of profit growth on GDP growth, used as a proxy for procyclicality, is 2.22. On the other hand, the volatility of real wage growth is 0.51 times that of GDP growth, thus significantly smoother than profits. Moreover, the slope coefficient of wage on GDP growth is 0.14 which supports the assumption that wages are less procyclical than profits. TFP is slightly more volatile (volatility 0.57 times that of

GDP growth) and significantly more procyclical (slope coefficient of TFP growth on GDP growth is 0.49) than wages.[14]

Next we investigate the elasticity of total labor costs to changes in sales directly. The advantage of analyzing labor costs is that we can conduct the analysis at the firm level. Panel B of Table II shows that for each dollar change in sales, staff expenses change 9¢ while all other operating costs (i.e., the sum of costs of goods sold and sales, general, and administrative expenses minus staff expenses) change 72¢. The table also shows that for each percentage point change in sales, staff expenses change 0.43¢, which is half of the change in all operating expenses (1.07¢) and a third of that of non-labor operating expenses (1.46¢). These findings support the hypothesis that labor costs are relatively inelastic, which is consistent with the existence of the labor leverage mechanism.

<< Table II here >>

C.2. Evidence for Countercyclicality of Firm-Level Labor Share

The previous section shows that labor costs are relatively smoother than output and other types of costs. This section takes a step forward and investigates the direct implication of this finding, which is the counter cyclicality of firm-level labor share. In order to establish the cyclicality of labor share, we run the following panel data regressions with firm-fixed

[14]The GDP growth series is taken from Table 1.1.3 of the National Income and Product Accounts of the Bureau of Economic Analysis (www.bea.gov). The real wage series and total factor productivity growth series are annualized, based on the quarterly seasonally adjusted series from the Bureau of Labor Statistics Major Sector Productivity and Costs program (www.bls.gov/lpc). The series cover the non-farm business sector. Following Arias, Hansen, and Ohanian (2007), We compute TFP growth as $\Delta \log TFP = \Delta \log Y - \frac{2}{3} \Delta \log H$, where $\Delta \log Y$ is the real output series and $\log H$ is the hours of all persons series. For business cycle frequencies, taking into account capital does not affect the results. The real wage series is real hourly compensation. This measure is based on the BEA estimates for labor compensation, and includes benefits. As a result, our measures of real wage and productivity are comparable in sectoral coverage and in construction.

effects:

$$S^{g}_{i,t} = \beta_{i,0} + \beta_{i,1} x^{g}_{t} + \epsilon_{i,t} \tag{7}$$

where S^{g} is the annual percentage growth in the measure of labor share under consideration, LS or ELS, x^{g} is the percentage growth in our business cycle proxy (GDP growth, TFP growth, or market returns).

Table III documents the estimates from regression (7) in our samples of firms with non-missing LS and non-missing ELS. The table shows that our two measures of labor share are in fact time-varying and countercyclical. This result is consistent with the previous finding that wages are smooth and that the capital-labor elasticity of substitution is less than one, since in that case labor share and productivity are negatively related. Moreover, this result indicates that *labor leverage* is countercyclical and thus potentially significant for asset pricing. But before investigating the relation between labor share and expected returns, we investigate the hypothesis that labor and capital are strictly complements, which could, at least partially explain the relative smoothness of labor costs.

<< Table III here >>

C.3. Evidence for Strict Complementarity Between Labor and Capital

Recall from our theoretical motivation section that smoothness of wages alone does not guarantee smoothness of labor costs and thus the existence of the labor leverage mechanism proposed in this paper.[15] Moreover, our previous section shows that, when productivity is procyclical, smoothness of labor costs is equivalent to countercyclical labor share. Proposition (**??**) shows that in a frictionless setting with relatively smooth wages, perfectly elastic

[15] For instance, even with constant wages, labor costs can perfectly comove with operating profits in firm with a CRS Cobb-Douglas production function.

and homogeneous labor supply, labor share is countercyclical and labor costs are smoother than output, only if capital and labor are strict complements. Before proceeding, we should note that, while our theoretical motivation is based on perfect and heterogeneous labor markets, a strict complementarity between labor and capital should make labor and capital smoother even without this assumption. Note also what our theoretical motivation is *not* saying: the strict complementarity between labor and capital does not rule out that labor market imperfections or heterogeneity also explain the labor leverage mechanism.

To estimate the capital-labor elasticity of substitution of firms in our sample, we use the theoretically motivate relation formalized in Corollary (2). In particular, we first estimate the elasticity of value added and operating profit growth to aggregate shocks. We use three proxies for aggregate sources of risk that affect the firm: GDP growth, TFP growth, and aggregate market returns. Specifically, we run the time-series regressions given by

$$prof^{\mathrm{g}}_{i,t} = \beta^{\Pi}_{i,0} + \beta^{\Pi}_{i,1} x^{\mathrm{g}}_t + \epsilon^{\Pi}_{i,t}, \tag{8a}$$

$$vadd^{\mathrm{g}}_{i,t} = \beta^{\mathrm{Y}}_{i,0} + \beta^{\mathrm{Y}}_{i,1} x^{\mathrm{g}}_t + \epsilon^{\mathrm{Y}}_{i,t}, \tag{8b}$$

where x is the aggregate shock (GDP growth, TFP growth, or market returns), $prof^{\mathrm{g}}$ is percentage growth operating profit before interest and depreciation and $vadd^{\mathrm{g}}$ is percentage growth in value added. The use of of percentage growth for operating profit restricts the sample to positive observations. We define value added using two different, consistent with the denominators of LS and ELS from (5) and (6). Note that β^{Π}_1 and β^{Y}_1 are conceptually similar to $\partial \Pi^{\mathrm{G}}_t / \partial X^{\mathrm{G}}_t$ and $\partial Y^{\mathrm{G}}_t / \partial X^{\mathrm{G}}_t$ from the theoretical section. This fact allows us to use the result from Corollary (2) to estimate the effective capital-labor elasticity of substitution from the data in the, cross-sectional second pass:

$$(\hat{\beta}^{\Pi}_{i,1} - 1) = \gamma_0 + \gamma(\hat{\beta}^{\mathrm{Y}}_{i,1} - 1) + \epsilon_i, \tag{9}$$

where $\hat{\beta}^{\Pi}_{i,1}$ and $\hat{\beta}^{Y}_{i,1}$ are the estimated slopes from (8a) and (8b).

Table IV shows the results of the two passes described below. The table shows results for the subsample of non-missing XLR-based value added (Panel A) and the non-missing LABEX-based value added (Panel B). We find that across the two panels and across the three different proxies for aggregate shock, that the estimated effective capital-labor elasticity of substitution ranges from 0.40 to 0.57. This result is consistent with existing literature and with our hypothesis that labor and capital are strictly complements, which at least partially explains the observed smoothness of labor costs and thus the existence of the labor leverage mechanism.

<< Table IV here >>

C.4. Sensitivity of Profits to Macroeconomic Shocks

So far, we presented evidence that supports labor share as a proxy for labor leverage. In this section, we take a step further and present evidence that operating profits of high labor-share firms are exposed to a higher level of operating leverage. A telltale sign that a firm has a high level of operating leverage (labor induced or otherwise) is a high sensitivity of operating profits (before interest and depreciation) to exogenous shocks. To investigate whether labor share is positively related to the sensitivity of operating profits to shocks, we use three proxies for aggregate sources of shocks that are exogenous to individual firms: GDP growth, TFP growth, and aggregate market returns. The hypothesis, which is formalized in Corollary (1), is that the sensitivity of profits to such shocks is increasing in labor share. To test this hypothesis we run the following panel data regressions with firm-fixed effects and interaction terms

$$prof^{g}_{i,t} = \beta_0 + \beta_1 x^{g}_t + \beta_2 x^{g}_t \times S_{i,t} + \beta_3 S_{i,t} + \epsilon_{i,t} \tag{10}$$

where x is the aggregate shock (GDP growth, TFP growth, or market returns), $prof^{\mathrm{g}}$ is percentage growth operating profit before interest and depreciation, and S is the proxy of labor share under consideration, LS or ELS.

Table V shows results generally consistent with the hypothesis. The positive exposure of profits to aggregate shocks is positive and increasing in magnitude in labor share. This finding suggests that operating profits of labor intensive firms are more sensitive to aggregate shocks and further supports the economic significance of the labor-induced operating leverage mechanism and also the validity of labor share as its proxy.

<< Table V here >>

D. Expected Asset Returns

Our theoretical model predicts that under relatively mild assumptions, expected returns should be increasing in labor share. In this section we investigate this prediction and explore the empirical relation between labor share and expected returns. To address the challenge that expected returns are not observable, we use two different types of proxies for them: realized stock returns and stock return loadings on risk factors (i.e., betas).

D.1. Realized Asset Returns

Table VI presents average post-ranking annual excess equity returns of quintile-portfolios of firms sorted on LS, and ELS, as well as a zero-investment portfolio (H-L portfolio). The H-L is a yearly rebalanced portfolio, long stocks in the highest LS or ELS quintile and short stocks in the lowest LS or ELS quintile. The H-L portfolio earned an excess returns of between 4.82% and 4.06% per year for LS-sorted portfolios and 3.29% and 3.25% per year for ELS-sorted portfolios. T-tests using Newey-West standard errors with four lags confirm that the LS-premia is statistically different from zero, although the ELS-premia is not statistically significant at conventional levels.

<< Table VI here >>

Table VII provides additional supporting evidence for this finding. The panel reports results of panel data regressions of annual returns on lagged values of LS and ELS. All independent variables are standardized so that they have mean zero and standard deviation in the sample. This standardization allows for a more direct comparison of the slopes across specifications. A one-standard deviation cross-sectional increase in LS and ELS leads to a cross-sectional increase in annual returns of 1.10% and 0.69%, respectively, after controlling for financial leverage and the size of the asset base. We do not control for book-to-market ratio and market value, since, as we show in the model, these variables subsume the effect of operating leverage on expected returns. Taken together, these results support the economic significance of the relation between labor share and expected asset returns.

<< Table VII here >>

D.2. Risk Factor Loadings

Under a rational expectation and full information setting, realized asset returns are an unbiased, albeit noisy, proxy for unobservable *expected* asset returns.[16] In this section we use loadings on traditional risk factors, i.e., risk factor betas, as an alternative proxy for expected asset returns. Note however that the use of empirical estimates of risk factor betas as proxies

[16]Despite its historical popularity and intuitive appeal, there is a growing concern in the literature is that average realized returns are very noisy and possibly biased proxies for expected returns. See Elton (1999) for a discussion of this concern.

for expected returns does not imply that this paper takes a stand on whether the empirical implementations of the CAPM or other traditional asset pricing models are well specified. In fact, our model is agnostic on the source of systematic risk in the economy, which is represented by $dZ_s u\lambda$ from Equation (11). The only extra required assumption needed in this section is for the empirical risk factors to be merely correlated to the true source(s) of risk in the economy. Under this assumption, empirical estimates of risk factor betas will be positively related to expected asset returns. And in that case, the hypothesis that expected returns are increasing in labor share is equivalent to the hypothesis that systematic risk loadings are increasing and labor shares.

Table VIII reports average conditional betas constructed as in Lewellen and Nagel (2006) for portfolios of firms sorted on both measures of labor shares. The table shows betas with respect to the market portfolio (MKT) as well as the SMB (small minus big) and HML (high minus low) risk factors related to size and value from Fama and French (1993). The table also includes betas with respect to the real macro variables described in Table II: GDP, TFP, and wage growth rates.

Panels A and B of the table show that average MKT, SMB, HML, GDP, and TFP betas are increasing in magnitude across the LS- and ELS-based portfolios, respectively. This finding is consistent with the existence of the labor-induced operating leverage mechanism that amplifies a firm's exposure to aggregate shocks. The difference in average wage growth beta between the highest and lowest labor share quintiles is positive but not statistically significant. The fact that HML betas are negative and increasing in magnitude across the LS-based (although not ELS-based) portfolios is also consistent with the proposed mechanism since it implies that loadings on -*HML* are positive and increasing. In fact, Kogan and Papanikolaou (2014) suggest that -*HML* is a risk factor that is related to investment-specific (IST) shocks and thus carries a negative price of risk.[17]

[17] IST shocks are shocks that affect the value of investment opportunities but not the value of assets in place. See Papanikolaou (2011), Garleanu, Panageas, and Yu (2012), and Kogan and Papanikolaou (2014)

<< Table VIII here >>

III. A Simple Model of Labor Leverage

In this section we present a specific application of the general theoretical predictions from Section I. We show that this simple model is able to explain the main findings presented in Section II.

A. Setup

The model represents a single firm and is partial equilibrium. Hence, we take the stochastic discount factor (SDF) as exogenous. The dynamics of the SDF, which we denote by Λ, are given by:

$$\frac{d\Lambda_t}{\Lambda_t} = -r dt - \eta dZ_t^{\lambda}, \tag{11}$$

where $r > 0$ is the instantaneous risk-free rate, dZ^{λ} is a Wiener process that represents the single source of systematic risk in the economy, and η represents the aggregate price of risk.

We assume perfect competition, so that the firm takes as given both its output price and the real wage it has to pay its employees. The dynamics of the real wage W are given by:

$$\frac{dW_t}{W_t} = \mu_{\text{w}} dt + \sigma_{\text{w}} \rho_{\text{w}} dZ^{\lambda} + \sigma_{\text{w}} \sqrt{1 - \rho_{\text{w}}^2} dZ^{\text{w}}, \tag{12}$$

where dZ^{w} is a Wiener process orthogonal to dZ^{λ} (i.e., $\mathrm{E}[dZ^{\text{w}} dZ^{\lambda}] = 0$), μ_{w} and σ_{w} are the drift and volatility of the wage growth process, respectively, and ρ_{w} is the priced portion of

for a discussion of the asset pricing implications of IST shocks.

the wage growth risk.

The firm's productive technology is represented by the following general constant elasticity of substitution (CES) production function:

$$Y_t = X_t \left(\alpha L_t^\rho + (1-\alpha) K^\rho\right)^{\frac{1}{\rho}}, \tag{13}$$

where L and K denote labor and capital employed in production, $\alpha \in (0,1)$ captures the relative importance of labor in total production, and X denotes the level of total factor productivity (TFP), and the parameter ρ determines the elasticity of substitution between capital and labor, $\gamma \equiv \frac{1}{1-\rho}$. The limit $\rho \to -\infty$ represents the case where capital and labor are perfect complements while the other extreme case, $\rho = 1$, represents the case where capital and labor are perfect substitutes. The case when $\rho \to 0$ represents the Cobb-Douglas production function. We focus on the empirically relevant case where labor and capital are strictly complements, $\rho < 0$.[18] To focus on the implications of the labor share for firm risk, we abstract away from investment and depreciation so that capital K is fixed.

It is convenient to decompose further the firm's TFP X into two components, aggregate TFP (X^{A}) and the idiosyncratic component of TFP (X^{I}), such that $X = X^{\text{A}} X^{\text{I}}$. Aggregate TFP X^{A} follows the diffusion process:

$$\frac{dX_t^{\text{A}}}{X_t^{\text{A}}} = \mu_{\text{x}} dt + \sigma_{\text{x}} \rho_{\text{x}} dZ^{\lambda}. \tag{14}$$

[18]Multiple studies estimates values for the elasticity of substitution between capital and labor γ of .7 or lower, which imply values for ρ lower than -0.4. See Klump et al. (2012) and references therein to studies that support the strict complementarity between labor and capital in a number of countries around the world, and see Oberfield and Raval (2014) for a recent study about the US manufacturing sector that finds an average elasticity of .5. As demonstrated in that paper (and following the insight of Houthakker (1955)), the micro-elasticity of substitution (which is relevant for our mechanism) may differ substantially from the macro-elasticity of substitution.

The idiosyncratic component of TFP X^{I} follows the diffusion process:

$$\frac{dX^{\mathrm{I}}_t}{X^{\mathrm{I}}_t} = \sigma_{\mathrm{x}}\sqrt{1-\rho_{\mathrm{x}}^2}dZ^{\mathrm{x}}, \tag{15}$$

where dZ^{x} is orthogonal to both dZ^{λ} and dZ^{w} (i.e., E$[dZ^{\mathrm{x}}dZ^{\lambda}]$= 0 and E$[dZ^{\mathrm{x}}dZ^{\mathrm{w}}]$= 0).

Profit maximization drives the firm to set its labor demand L^{D} such that the marginal profitability of labor ($\frac{dY_t}{dL_t}$) is equated to the real wage (W). Labor demand L^{D} is given by:

$$L^{\mathrm{D}}_t = \left(\frac{1-\alpha}{\left(\frac{W}{\alpha X}\right)^{\frac{\rho}{1-\rho}} - \alpha}\right)^{1/\rho}. \tag{16}$$

Equation (16) implies that, consistent with intuition, the firm will demand more labor when its productivity is high relative to the real wage. In what follows, we always assume that the firm sets labor optimally.

We define labor share S as the ratio of labor costs to value added, $S \equiv \frac{L^{\mathrm{D}}W}{Y}$. Intuitively, labor share is a measure of how value added is split between workers and the firm (capital) owners. Using Ito's Lemma we find the dynamics of S:

$$\frac{dS_t}{S_t} = \mu_{\mathrm{s}}dt + \sigma_{\mathrm{s}\lambda}dZ^{\lambda} + \sigma_{\mathrm{sw}}dZ^{\mathrm{w}} + \sigma_{\mathrm{sx}}dZ^{\mathrm{x}}, \tag{17}$$

where
$$\mu_{\mathrm{s}} \equiv -\left(\frac{\rho}{\rho-1}\right)\left(\mu_{\mathrm{a}} - \mu_{\mathrm{w}} - \sigma_{\mathrm{a}}^2\right) + \left(\frac{\rho}{\rho-1}\right)^2\left(\frac{\sigma_{\mathrm{x}}^2}{2\rho} - \rho_{\mathrm{w}}\rho_{\mathrm{x}}\sigma_{\mathrm{w}}\sigma_{\mathrm{x}} + \frac{\sigma_{\mathrm{w}}^2}{2\rho}\right), \tag{17a}$$

$$\sigma_{\mathrm{s}\lambda} \equiv -\left(\frac{\rho}{\rho-1}\right)\left(\rho_{\mathrm{x}}\sigma_{\mathrm{x}} - \rho_{\mathrm{w}}\sigma_{\mathrm{w}}\right), \tag{17b}$$

$$\sigma_{\mathrm{sw}} \equiv \left(\frac{\rho}{\rho-1}\right)\sigma_{\mathrm{w}}\sqrt{1-\rho_{\mathrm{w}}^2}, \tag{17c}$$

$$\sigma_{\mathrm{sx}} \equiv -\left(\frac{\rho}{\rho-1}\right)\sigma_{\mathrm{x}}\sqrt{1-\rho_{\mathrm{x}}^2}. \tag{17d}$$

Equation (17) implies that labor share is differently affected by shocks to wages and shocks to productivity. In the empirically relevant case where labor and capital are strictly complements ($\rho < 0$), labor share S is decreasing in idiosyncratic productivity (i.e., $\sigma_{\text{sx}} < 0$).[19] Equation (17) also shows that, despite the fact that labor demand decreases with wages, the labor share S is increasing in wages (i.e., $\sigma_{\text{sw}} > 0$) because the price effect dominates the quantity effect. Figure 1 illustrates the negative relationship between labor share and idiosyncratic productivity and the positive relationship between labor share and wages. Finally, the effect of aggregate productivity (i.e. the priced shock λ) on the labor share reflects a combination of the two above effects. On the one hand, higher aggregate productivity leads to a lower labor share, but on the other hand higher aggregate productivity is associated with a higher real wage (according to ρ_{w}) which increases the labor share. The overall effect is negative (i.e., $\sigma_{\text{s}\lambda} < 0$), provided that real wage response is not too large, which is the empirically relevant case as we discuss below.

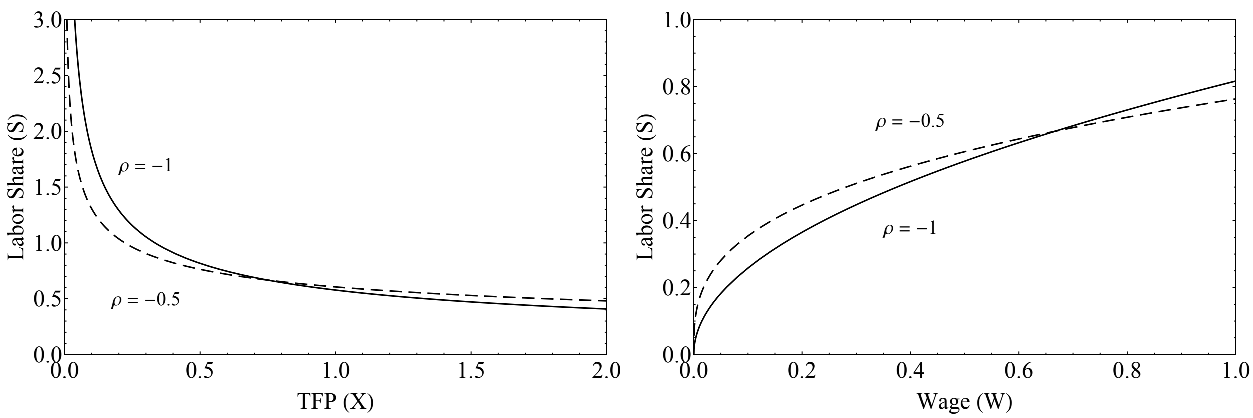

Figure 1

Determinants of Labor Share. Labor share as a function of productivity and wages in the production model. The figure shows the numerical solution for the firm's labor share as a function of productivity and wages. The top panel shows that labor share is decreasing in productivity. The bottom panel shows that labor share is increasing in economy-wide wages. The chosen values for ρ result in elasticities of substitution of .5 and .7, values in the range of what many empirical studies find for the elasticity of substitution between capital and labor. Parameter values used in numerical solution: $\alpha = 0.67$, $W = 0.5$ (left panel), and $X = 1$ (right panel).

[19]For completeness, it is worth mentioning the two cases not considered in this paper. Labor share is constant in the standard Cobb-Douglas production function (i.e., when $\rho \to 0$) and equals α. When labor and capital are strictly substitutes (i.e., when $\rho > 0$), labor share is decreasing in wages and increasing in productivity.

Operating profits are defined as the residual cash flows of the firm after labor expenses are paid, $\Pi \equiv Y - LW$. For simplicity, we assume that firms can frictionlessly suspend and resume production (and thus operating costs) over time.[20] Operating profits under at the optimal labor demand can then be expressed as a function of productivity X and labor share S:

$$\Pi_t = \begin{cases} (1-\alpha)^{\frac{1}{\rho}} X_t K \left(1 - S_t\right)^{\frac{\rho-1}{\rho}}, & \text{if} \quad S_t < 1, \\ 0, & \text{if} \quad S_t \geq 1. \end{cases} \tag{18}$$

where the second region reflects the fact that the firm will optimally suspend production before operating profits become negative, which happens when $S \geq 1$. Figure 2 shows the negative relation between labor share and operating profits (holding productivity X fixed). For instance, an increase in the real wage leads to an increase in labor share, so that a larger share of revenues is used to compensate labor, and operating profits decline.[21] On the other hand, higher productivity increases operating profits both by reducing the labor share and by changing the scale of the firm (according to (18)). The dynamics of profit growth are given by:

$$\frac{d\Pi_t}{\Pi_t} = \mu_{\pi}[S_t]dt + \sigma_{\pi\lambda}[S_t]dZ_t^{\lambda} + \sigma_{\pi\mathrm{w}}[S_t]dZ_t^{\mathrm{w}} + \sigma_{\pi\mathrm{x}}[S_t]dZ_t^{\mathrm{x}}, \tag{19}$$

where:

[20]If we did not allow this, we would have to allow shareholders to exit the industry, or to assume that limited liability is violated.

[21]The firm also reacts to the higher real wage by reducing labor demand, but the effect this has on operating profits is zero (to a first order) according to the Envelope theorem, i.e. labor is set optimally.

$$\mu_\pi[S_t] \equiv \left(\frac{1}{1-S_t}\right)\left(\mu_{\text{x}} - S_t\mu_{\text{w}} + \left(\frac{\rho}{1-\rho}\right)\left(\frac{S_t}{1-S_t}\right)\left(\frac{\sigma_{\text{x}}^2}{2} - \rho_{\text{w}}\rho_{\text{x}}\sigma_{\text{w}}\sigma_{\text{x}} + \frac{\sigma_{\text{w}}^2}{2}\right)\right), \tag{19a}$$

$$\sigma_{\pi\lambda}[S_t] \equiv \left(\frac{1}{1-S_t}\right)(\rho_{\text{x}}\sigma_{\text{x}} - \rho_{\text{w}}\sigma_{\text{w}}S_t), \tag{19b}$$

$$\sigma_{\pi\text{w}}[S_t] \equiv -\left(\frac{S_t}{1-S_t}\right)\left(\sqrt{1-\rho_{\text{w}}^2}\sigma_{\text{w}}\right), \tag{19c}$$

$$\sigma_{\pi\text{x}}[S_t] \equiv \left(\frac{1}{1-S_t}\right)\left(\sqrt{1-\rho_{\text{x}}^2}\sigma_{\text{x}}\right). \tag{19d}$$

Equation (19) shows that, since the capital stock is fixed, the dynamics of operating profits follow only from systematic and idiosyncratic TFP shocks and from shocks to the real wage. It also shows that the sensitivity of profit growth to the three shocks (dZ^λ, dZ^{w}, and dZ^{x}) are increasing in *magnitude* in labor share S. This fact, which we formalize next, is at the heart of the link between labor share and labor induced operating leverage.

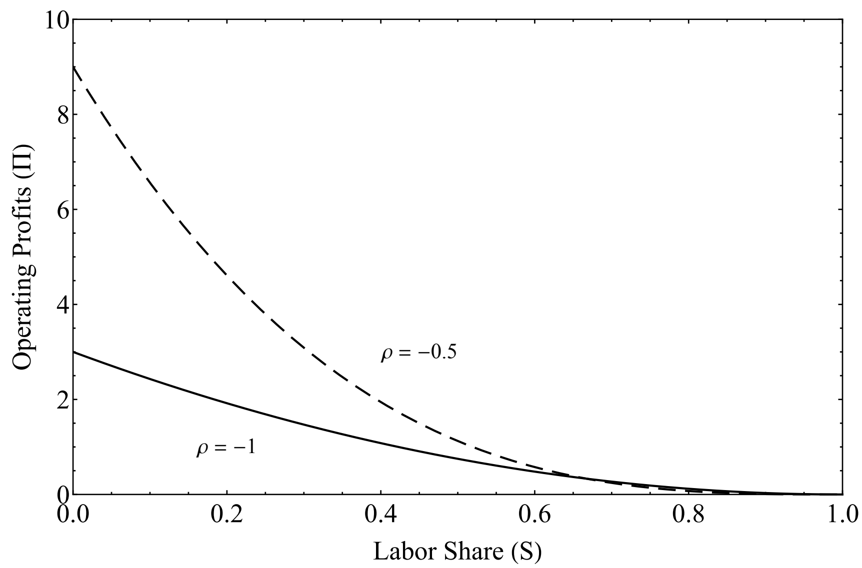

Figure 2

Operating Profits and Labor Share. Operating profits a function of labor share in the production model. Parameter values used in numerical solution: $W = 0.5$, $\alpha = 0.67$, $K = 1$, $\mu_{\text{x}} = 0$, $\sigma_{\text{x}} = 0.2$, $\rho_{\text{x}} = 0.5$, $\mu_{\text{w}} = 0$, $\sigma_{\text{w}} = 0.05$, $\rho_{\text{w}} = 0.1$, $r = 0.02$, and $\eta = 0.5$.

B. Labor Leverage

Having derived the dynamics of cash flows, we can now formalize the labor leverage mechanism. The "traditional" operating leverage arises from the existence of fixed operating expenses. In contrast, the labor leverage mechanism is not based on existence of fixed costs - note that all costs in the model are variable. The labor leverage mechanism is based instead on the relative smoothness of wages and the imperfect correlation between wages and productivity.

To see this, note that the response of profits to the aggregate productivity shock (i.e. the priced shock λ) equals $\left(\frac{1}{1-S_t}\right)(\rho_{\mathrm{x}}\sigma_{\mathrm{x}} - \rho_{\mathrm{w}}\sigma_{\mathrm{w}}S_t)$ according to (19), and hence in the special case where wages respond one-for-one to productivity, i.e. $\rho_{\mathrm{x}}\sigma_{\mathrm{x}} = \rho_{\mathrm{w}}\sigma_{\mathrm{w}}$, the response of operating profits to the shock equals one for all firms. In contrast, if wages respond less than one-for-one to productivity shocks, i.e. $\rho_{\mathrm{x}}\sigma_{\mathrm{x}} > \rho_{\mathrm{w}}\sigma_{\mathrm{w}}$, then the response of operating profits to the shock is greater than one for all firms. Firms "leverage" the smoothness of wages, making operating profits more procyclical. Moreover, this leverage effect is larger when the labor share S_t is larger.

The assumption $\rho_{\mathrm{x}}\sigma_{\mathrm{x}} > \rho_{\mathrm{w}}\sigma_{\mathrm{w}}$ is consistent with standard stylized facts. In aggregate data, corporate profits (or earnings) are highly procyclical, and more volatile than total factor productivity (TFP) or GDP. It is well understood that an important reason for this fact is that labor compensation is relatively smooth and weakly correlated with TFP or GDP growth.[22]

To quantify the effect of labor share on firm risk amplification, we define two measures of the sensitivity of operating profits to each of its two sources of shocks: productivity and wages. The first is a measure of the sensitivity of cash flow growth to TFP shocks, Θ, which we denote simply as "operating leverage." Operating leverage, Θ, is defined as in Donangelo

[22]For instance, Longstaff and Piazzesi (2004) hypothesize that the reason for the extreme volatility and procyclicality of corporate earnings is that stockholders are residual claimants to corporate cash ÂĞows. Thus, the compensation of workers is a senior claim to cash flows. See also Gomme and Greenwood (1995).

(2014) as the scaled covariance of equilibrium operating profit growth and TFP growth, i.e., $\Theta \equiv \mathrm{Cov}\left[\frac{d\Pi}{\Pi}, \frac{dX}{X}\right] \big/ \mathrm{Var}\left[\frac{dX}{X}\right] - 1$.[23] Operating leverage then is given by

$$\Theta[S_t] = \frac{S_t}{1 - S_t}\left(1 - \frac{\rho_{\mathrm{w}}\rho_{\mathrm{x}}\sigma_{\mathrm{w}}}{\sigma_{\mathrm{x}}}\right). \tag{20}$$

Equation (20) shows that the sensitivity of operating profits to TFP shocks is positive and monotonically increasing in labor share S as long as TFP is more volatile than the component of wage growth correlated with TFP growth. [24] This result is summarized in the proposition below:

Proposition 3 (operating leverage is monotonically increasing in labor share)
The condition $\sigma_{\mathrm{x}} > \rho_{\mathrm{w}}\rho_{\mathrm{x}}\sigma_{\mathrm{w}}$ implies that operating leverage is positive and increasing in labor share S:

$$\Theta[S_t] > 0 \text{ and } \frac{d\Theta[S_t]}{dS_t} \geq 0.$$

The Proposition follows directly from Equation (20). The main message of Proposition 3 is that, under strict complementarity of labor and capital, labor share can be used as a proxy for the degree of labor leverage experienced by the firm.

We also define a related measure Θ^{W} as the sensitivity of operating profits to changes in economy-wide wages, i.e., $\Theta^{\mathrm{W}} \equiv \mathrm{Cov}\left[\frac{d\Pi}{\Pi}, \frac{dW}{W}\right] \big/ \mathrm{Var}\left[\frac{dW}{W}\right] - 1$. The measure Θ^{W} is given by

$$\Theta^{\mathrm{W}}[S_t] = -\frac{1}{1 - S_t}\left(1 - \frac{\rho_{\mathrm{w}}\rho_{\mathrm{x}}\sigma_{\mathrm{x}}}{\sigma_{\mathrm{w}}}\right). \tag{21}$$

Equation (21) shows that the sensitivity of operating profits to wages shocks is negative and

[23] Alternatively, Θ is defined as the slope of a regression of operating profit growth on TFP growth minus one. We remove one so that Θ is zero if there is no amplification.

[24] We anticipate that the assumption is fairly weak. For instance, we document that aggregate wage growth is less volatile and not highly correlated with aggregate TFP growth.

its magnitude is monotonically increasing in labor share S. This result is summarized in the corollary below:

Corollary 1 (sensitivity of operating profits to wage shocks)
The condition $\sigma_{\mathrm{x}} > \rho_{\mathrm{w}}\rho_{\mathrm{x}}\sigma_{\mathrm{w}}$ implies that the sensitivity of operating profit growth to wage growth is negative and increasing in magnitude in labor share S:

$$\Theta^{\mathrm{W}}(S) < 0 \text{ and } \frac{d\Theta^{\mathrm{W}}[S]}{dS} \leq 0.$$

The corollary follows directly from Equation (21).

Figure 3 illustrates the relation of labor share to the exposure of operating profits to the two sources of uncertainty: productivity and wages. The figure shows that the magnitudes of the positive sensitivity of operating profits to productivity and the negative sensitivity of operating profits to wage shocks is increasing in labor share. This effect, which is directly related to labor leverage, is an intuitive result: higher labor share is related to lower profit margins, which buffer the firm against either type of shocks. Productivity is positively related to operating profits, so that the exposure to productivity shocks is always positive and increasing in labor share. Labor expenses are negatively related to operating profits, so that the exposure to wages shocks is always negative and its magnitude increasing in labor share.

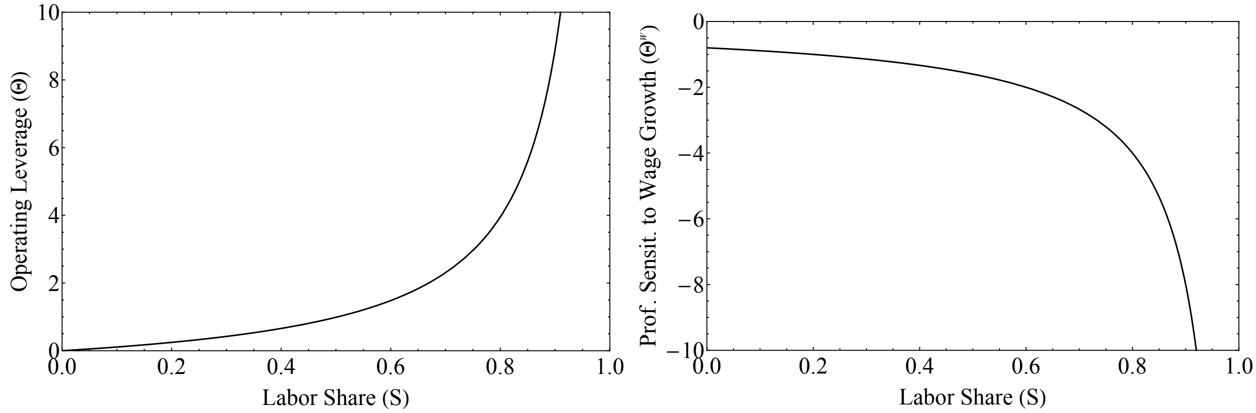

Figure 3

Labor Leverage and Labor Share. Sensitivity of operating profits to productivity and wage shocks in the production model. The figure shows the relation of labor share to the exposure of operating profits to the two sources of uncertainty: productivity and wages. The figure shows that the magnitudes of the positive sensitivity of operating profits to productivity and the negative sensitivity of operating profits to wage shocks is increasing in labor share. Parameter values used in numerical solution: $W = 0.5$, $\alpha = 0.67$, $K = 1$, $\sigma_\mathrm{x} = 0.2$, $\rho_\mathrm{x} = 0.5$, $\sigma_\mathrm{w} = 0.05$, and $\rho_\mathrm{w} = 0.1$.

C. Valuation and Expected Returns

In equilibrium, firm value (V) equals the value of the discounted stream of optimized operating profits:

$$V_t = \mathrm{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \Pi_s ds \right]. \tag{22}$$

Under technical conditions, the solution to equation (22) exists, and it is given by:

$$V_t = (1-\alpha)^{1/\rho} X_t K v[S_t], \tag{23}$$

where v is a monotonically decreasing function of labor share, such that $\lim_{S\to 0} v[S] = 1$ and $\lim_{S\to 1} v[S] = 0$. The explicit solution is given in the appendix.

The solution for the firm value is intuitive. First, when labor costs become negligible relative to the value added generated by the firm ($S \to 0$), the value of the firm converges to that of a firm with a perpetual dividend governed by a geometric Brownian motion, where the current dividend equals $(1-\alpha)^{1/\rho}AK$. As the cost of labor increases relative to the value added generated by the firm, the dividend falls and, consistently, the value of the firm falls. When labor costs equal value added ($S \to 1$) operating profits are zero so the firm shuts down production and all firm value arises from the option to resume production when operating profits become positive again.

The negative relation between labor share and firm value is driven by two complementary channels: a cash flow channel and a discount rate channel. The cash flow channel is that labor intensive firms have lower operating profits due to higher labor expenses per unit produced (i.e., higher marginal profitability of labor). The discount flow channel is related to the higher loading on systematic risk of a labor intensive firm relative to a capital intensive one. Figure 4 illustrates the negative relation between labor share and firm value.

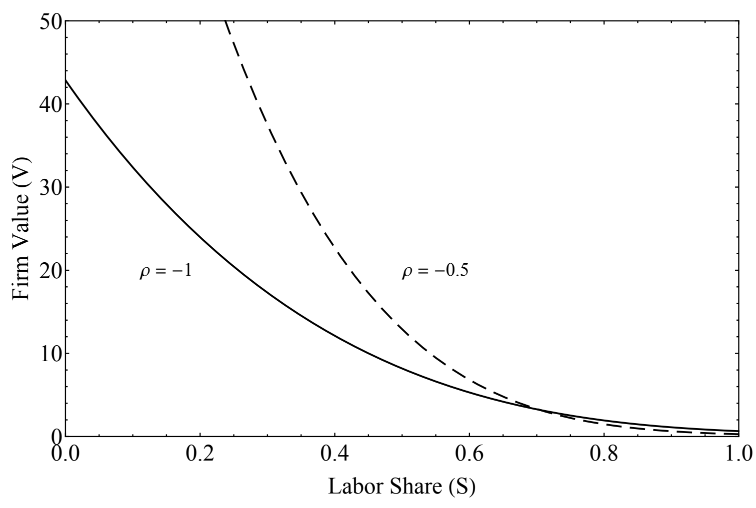

Figure 4

Firm Value and Labor Share. Firm value as a function of labor share in the production model. Parameter values used in numerical solution: $W = 0.5$, $\alpha = 0.67$, $K = 1$, $\mu_x = 0$, $\sigma_x = 0.2$, $\rho_x = 0.5$, $\mu_w = 0$, $\sigma_w = 0.05$, $\rho_w = 0.1$, $r = 0.02$, and $\eta = 0.5$.

Expected returns are defined as the instantaneous drift of the gains process that reinvests dividends, $\mathrm{E_t}[R_t] \equiv \mathrm{E_t}\left[\frac{dV_t + \Pi_t^* dt}{V_t}\right]$, and are given by:

$$\mathrm{E_t}[R_t] = r + \eta\sigma_{\mathrm{x}}\rho_{\mathrm{x}} + S_t \frac{v'[S_t]}{v[S_t]} \frac{\rho}{1-\rho} \left(\rho_{\mathrm{X}}\sigma_{\mathrm{X}} - \rho_{\mathrm{w}}\sigma_{\mathrm{w}}\right) \tag{24}$$

We show in the appendix that $v'(s) < 0$ holds for all parameter values where a feasible solution for Equation (23) exists. Thus, Equation (24), in conjunction with our assumption that $\rho < 0$, implies that the relationship between risk and labor share depends on the sign of $\rho_{\mathrm{X}}\sigma_{\mathrm{X}} - \rho_{\mathrm{w}}\sigma_{\mathrm{w}}$. This is formalized below:

Proposition 4 (Asset Returns and Labor Share)
For $S \in (0,1)$, $\rho_{\mathrm{x}}\sigma_{\mathrm{x}} > \rho_{\mathrm{w}}\sigma_{\mathrm{w}}$ is a sufficient condition for $\frac{d\mathrm{E_t}[R_t]}{dS} \geq 0$.

If the condition is satisfied, wages are less procyclical than productivity, and labor intensive firms have higher exposure to systematic risk (and narrower profit margins).

Equation (24) shows that the firm's excess returns over the risk free rate depends on two sources of priced risk. The first source is a premium paid for the riskiness coming from the covariance between the firm's productivity and the stochastic discount factor ($\rho_{\mathrm{X}}\sigma_{\mathrm{X}}$). We call this source of risk *productivity risk*. Productivity risk affects expected returns directly–through its impact on overall productivity–and indirectly through its impact on the relative productivity of capital and labor. It is this second, indirect, component that depends on the firm's labor share. The direct impact of productivity risk for an average firm will be positive, as the average firm produces more, or finds the prices of its products go up, in good times. The indirect impact is also positive, since our assumption about complementarity between labor and capital implies that a positive shock to the firm's productivity will amplify the impact on profits.

The second source of risk captures the the firm's exposure to aggregate wages ($\rho_{\mathrm{w}}\sigma_{\mathrm{w}}$). We call this component *wage hedge*, as it depends only on the firm's exposure to wages and will likely *reduce* risk for the average firm. The wage hedge is linked to the labor share, since variations in wages will have a larger effect on firms for which most of their value added is used to pay for labor. This component is a hedge because, on average, wages fall in bad times, which is precisely when firm's profits are falling due to systematic falls in their own productivity.

Combining the two sources of risk–one positive, the other a hedge–delivers the relation between the firm's labor share and expected returns. The relation will be positive if the firm's systematic component of productivity is procyclical enough relative to wages. For instance, the systematic risk loadings of a firm whose productivity is uncorrelated with the stochastic discount factor ($\rho_{\mathrm{X}} = 0$) is decreasing in its labor share. This is because in this case the hedge effect of wages is uncontested: in good times wages go up, so profits fall; in bad times wages go down and the firm's profits increase. The hedging impact of wages, though, is muted when the firm's productivity is sufficiently procyclical ($\rho_{\mathrm{X}}\sigma_{\mathrm{X}} > \rho_{\mathrm{w}}\sigma_{\mathrm{w}}$). In this case, even though wages are a hedge, the procyclical variation in the firm's sales price dominates, making the firm riskier as its labor share increases.

Figure 5 shows that asset betas are increasing in labor shares. The figure also shows that the positive relation between betas and labor shares implies a positive relation between betas and wages and a negative relation between betas and productivity. The last panel shows that betas are insensitive to productivity once we control for labor shares.

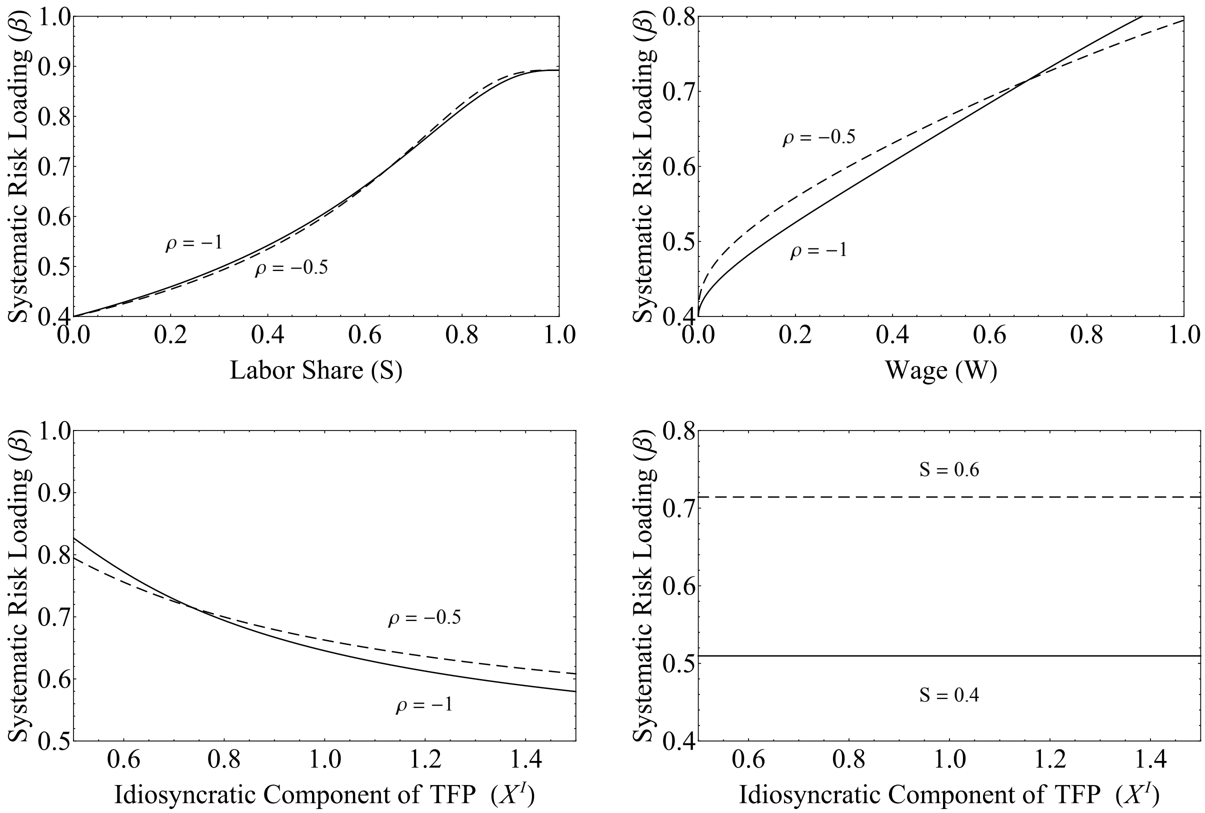

Figure 5

Betas, Productivity, Wages and Labor Share

Betas as a function of labor share, productivity, and wages in the production model. Parameter values used in numerical solution: $X^{\mathrm{I}} = 1$, $X^{\mathrm{A}} = 1$, $W = 0.5$, $\alpha = 0.67$, $K = 1$, $\mu_{\mathrm{x}} = 0$, $\sigma_{\mathrm{x}} = 0.2$, $\rho_{\mathrm{x}} = 0.5$, $\mu_{\mathrm{w}} = 0$, $\sigma_{\mathrm{w}} = 0.05$, $\rho_{\mathrm{w}} = 0.1$, $r = 0.02$, and $\eta = 0.5$.

Conclusion

This paper proposes labor share as a promising new firm characteristic that explains the cross-section of returns. We develop a simple production-based model of a firm to study the labor leverage mechanism. The model provides theoretical motivation for the use of labor share as a firm-level measure of the degree of labor leverage. The model shows that two sufficient conditions for the use of labor share as a proxy for labor leverage are: (1) labor and capital are strictly complements and (2) economy-wide wages to be smoother than aggregate productivity. These two sufficient conditions are generally supported in the data. Moreover, this paper provides model-agnostic empirical evidence that validates labor share as a measure of labor leverage. In particular, we document that the sensitivity of operating profits to shocks is cross-sectionally increasing in labor share. We further confirm a positive relation between labor leverage and expected asset returns. For instance, we show that average realized stock returns and average loadings on traditional systematic risk factors are increasing in labor share.

REFERENCES

Arias, Andres, Gary D. Hansen, and Lee E. Ohanian, 2007, Why have business cycle fluctuations become less volatile?, *Economic Theory* 32, 43–58.

Belo, Frederico, Xiaoji Lin, and Santiago Bazdresch, 2014, Labor hiring, investment and stock return predictability in the cross section, *Journal of Political Economy* 122, 129–177.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.

Berk, Jonathan B., and Johan Walden, 2013, Limited Capital Market Participation and Human Capital Risk, *Review of Asset Pricing Studies* 3, 1–37.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577–2603.

Chen, Huafeng, Marcin Kacperczyk, and Hernan Ortiz-Molina, 2012, Do nonfinancial stakeholders affect the pricing of risky debt? Evidence from unionized workers, *Review of Finance* 347–383.

Chen, L., and L. Zhang, 2011, Do time-varying risk premiums explain labor market performance?, *Journal of Financial Economics* 99, 385–399.

Damodaran, Aswath, 2011, *The little book of valuation: how to value a company, pick a stock and profit* (John Wiley & Sons).

Danthine, Jean-Pierre, and John B. Donaldson, 2002, Labor relations and asset returns, *Review of Economic Studies* 69, 41–64.

Donangelo, Andres, 2014, Labor mobility: Implications for asset pricing, *Journal of Finance* 68, 1321–1346.

Eisfeldt, Andrea L., and Dimitris Papanikolaou, 2013, Organization Capital and the Cross-Section of Expected Returns, *Journal of Finance* 68, 1365–1406.

Elton, Edwin J., 1999, Presidential address: Expected return, realized return, and asset pricing tests, *Journal of Finance* 54, 1199–1220.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Favilukis, Jack, and Xiaoji Lin, 2015, Wage Rigidity: A Solution to Several Asset Pricing Puzzles, *Review of Financial Studies, forthcoming* .

Favilukis, Jack, Xiaoji Lin, and Xiaofei Zhao, 2015, The Elephant in the Room: the Impact of Labor Obligations on Credit Risk, *working paper* .

Garcia-Feijoo, Luis, and Randy D. Jorgensen, 2010, Can Operating Leverage Be the Cause of the Value Premium?, *Financial Management* 39, 1127–1153.

Garleanu, Nicolae, Stavros Panageas, and Jianfeng Yu, 2012, Technological Growth and Asset Pricing, *Journal of Finance* 67, 1265–1292.

Gollin, Douglas, 2002, Getting income shares right, *Journal of Political Economy* 110, 458–474.

Gomme, Paul, and Jeremy Greenwood, 1995, On the cyclical allocation of risk, *Journal of Economic dynamics and control* 19, 91–124.

Gourio, François, 2007, Labor leverage, firms' heterogeneous sensitivities to the business cycle, and the Cross-Section of expected returns, *working paper* .

Houthakker, Hendrik S., 1955, The Pareto distribution and the Cobb-Douglas production function in activity analysis, *Review of Economic Studies* 27–31.

Karabarbounis, Loukas, and Brent Neiman, 2014, The Global Decline of the Labor Share, *Quarterly Journal of Economics* 129, 61–103.

Klump, Rainer, Peter McAdam, and Alpo Willman, 2012, The normalized CES production function: theory and empirics, *Journal of Economic Surveys* 26, 769–799.

Kogan, Leonid, and Dimitris Papanikolaou, 2014, Growth opportunities, technology shocks, and asset prices, *Journal of Finance* 69, 675–718.

Kuehn, Lars-Alexander, Mikhail Simutin, and Jessie Jiaxu Wang, 2014, A Labor Capital Asset Pricing Model, *working paper* .

León-Ledesma, Miguel A., Peter McAdam, and Alpo Willman, 2010, Identifying the elasticity of substitution with biased technical change, *American Economic Review* 100, 1330–1357.

Lev, Baruch, 1974, On the Association Between Operating Leverage and Risk, *Journal of Financial and Quantitative Analysis* 627–641.

Lewellen, Jonathan, and Stefan Nagel, 2006, The conditional CAPM does not explain asset-pricing anomalies, *Journal of Financial Economics* 82, 289–314.

Longstaff, Francis A., and Monika Piazzesi, 2004, Corporate earnings and the equity premium, *Journal of Financial Economics* 74, 401–421.

Mandelker, Gershon N., and S. Ghon Rhee, 1984, The Impact of the Degrees of Operating and Financial Leverage on Systematic Risk of Common Stock, *Journal of Financial and Quantitative Analysis* 19, 45–57.

Merz, Monika, and Eran Yashiv, 2007, Labor and the Market Value of the Firm, *American Economic Review* 97, 1419–1431.

Novy-Marx, Robert, 2011, Operating leverage, *Review of Finance* 15, 103–134.

Novy-Marx, Robert, 2013, The other side of value: The gross profitability premium, *Journal of Financial Economics* 108, 1–28.

Oberfield, Ezra, and Devesh Raval, 2014, Micro data and macro technology, *working paper* .

Palacios, Miguel, 2013, Are labor intensive assets riskier?, working paper.

Papanikolaou, Dimitris, 2011, Investment shocks and asset prices, *Journal of Political Economy* 119, 639–685.

Peterson, Mitchell A, 1994, Cash flow variability and firm's pension choice: A role for operating leverage, *Journal of Financial Economics* 36, 361–383.

Petrosky-Nadeau, Nicolas, Lu Zhang, and Lars-Alexander Kuehn, 2013, Endogenous disasters and asset prices, working paper.

Piketty, Thomas, 2014, *Capital in the Twenty-First Century* (Belknap Press).

Santos, Tano, and Pietro Veronesi, 2006, Labor Income and Predictable Stock Returns, *Review of Financial Studies* 19, 1–44.

Schmidt, Lawrence, 2014, Climbing and Falling Off the Ladder: Asset Pricing Implications of Labor Market Event Risk, *working paper* .

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.

Zhang, Mindy, 2014, Who bears firm risk? implications for cash flow volatility, working paper.

Appendix

Comparison Between Labor Leverage and Traditional Operating Leverage

Here we start with setting similar to that in our theoretical motivation section, but where the firm is also subject to a fixed operating cost f per unit of capital employed in production. The firm's optimization problem at time t, which defines optimized operating profits Π, is now given by:

$$\Pi_t = \max_{L_t}\{X_t F[K, L_t] - L_t W_t - fK\}. \tag{25}$$

The relative responses of operating profit growth and value added growth to shocks is given by

$$\frac{\partial \Pi_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}}{\partial Y_t^{\mathrm{G}}/\partial X_t^{\mathrm{G}}} = \frac{1 + \frac{S_t}{1-S_t}\left(1 - \frac{\partial W_t^{\mathrm{G}}}{\partial X_t^{\mathrm{G}}}\right)\left(1 + \frac{fK}{\Pi_t}\right)}{1 + \gamma\frac{S_t}{1-S_t}\left(1 - \frac{\partial W_t^{\mathrm{G}}}{\partial X_t^{\mathrm{G}}}\right)\left(1 + \frac{fK}{\Pi_t}\right)}, \tag{26}$$

where $S \equiv L_t W_t / Y_t$ is labor share and γ is the elasticity of substitution between labor and capital. The expression above shows how labor leverage, which amplifies firm risk through the term $\frac{S_t}{1-S_t}\left(1 - \frac{\partial W_t^{\mathrm{G}}}{\partial X_t^{\mathrm{G}}}\right)$ and traditional leverage, which amplifies firm risk through the term $\left(1 + \frac{fK}{\Pi_t}\right)$) interact and magnify each other.

Proof of proposition 1

Solution to the value of the firm

The discounted value of the gains portfolio that reinvests the firm's dividends is a Martingale leads to a partial differential equation (PDE). The solution to the value of a firm can be expressed as a function its TFP X and its labor share S, $V[X_t, S_t]$. Given that operating profits (Equation (18)) are homogeneous of degree one in X and S, we guess and later verify that the value of the firm is also homogeneous of degree one in X and S. That is, we assume the existence of a function $v[S]$ such that $V[X,S] = XKv[S]$. The homogeneity of the value of the firm allows us to simply the PDE into the following ordinary differential equation (ODE):

$$h[S_t] - \delta_0 v[S_t] + \delta_1 S v'[S_t] + \delta_2 S^2 v''[S_t] = 0, \tag{27}$$

where:

$$h[S_t] \equiv \frac{\Pi_t}{X_t K} = \begin{cases} (1-\alpha)^{\frac{1}{\rho}}(1-S_t)^{1-\frac{1}{\rho}}, & \text{if} \quad S_t < 1, \\ 0, & \text{if} \quad S_t \geq 1, \end{cases}$$

$$\delta_0 \equiv r + \eta\rho_{\text{x}}\sigma_{\text{x}} - \mu_{\text{x}},$$

$$\delta_1 \equiv \frac{\rho\left(2\eta\rho(\rho_{\text{x}}\sigma_{\text{x}} - \rho_{\text{w}}\sigma_{\text{w}}) + 2\eta\rho_{\text{w}}\sigma_{\text{w}} - 2\eta\rho_{\text{x}}\sigma_{\text{x}} - 2\mu_{\text{w}}(1-\rho) + 2\mu_{\text{x}}(1-\rho) - 2\rho_{\text{w}}\rho_{\text{x}}\sigma_{\text{w}}\sigma_{\text{x}} + \sigma_{\text{w}}^2 + \sigma_{\text{x}}^2\right)}{2(1-\rho)^2},$$

$$\delta_2 \equiv \frac{\rho^2\left(\sigma_{\text{w}}^2 + \sigma_{\text{x}}^2 - 2\rho_{\text{w}}\rho_{\text{x}}\sigma_{\text{w}}\sigma_{\text{x}}\right)}{2(1-\rho)^2}.$$

The value of the firm as $S \to 0$ converges to the value of a firm with a dividend of $X_t K(1-\alpha)^{\frac{1}{\rho}}$, a growth rate of μ_x, and a discount rate of $r + \eta\rho_{\text{x}}\sigma_{\text{x}}$, which results in $\lim_{S\to 0} v[S] = \frac{(1-\alpha)^{\frac{1}{\rho}}}{\delta_0}$. There are three other boundary conditions. The first one corresponds to $\lim_{s\to\infty} v[S] = 0$, since the value of a firm that goes further away from the region where produces positive operating profits should approach 0. The other two conditions are the smooth-pasting conditions when $s = 1$. At this point $\lim_{S\to 1^-} v[S] = \lim_{S\to 1^+} v[S]$ and $\lim_{S\to 1^-} v'[S] = \lim_{S\to 1^+} v'[S]$. In what follows we start solving the problem assuming $v(0) = 0$, and then adjust the function upwards to its true intercept.

The solution to Equation (27) in each of the two regions, $S < 1$ and $S \geq 1$, has the general form:

$$v[S] = v_h[S] + v_p[S], \tag{29}$$

where $v_h[S]$ and $v_p[S]$ are the homogeneous and particular solutions to ODE (27).

We start by finding two linearly independent solutions to the corresponding homogeneous differential equation:

$$v_h[S] = c_1 S^{x_1} + c_2 S^{x_2}, \tag{30}$$

where x_1 and x_2 are given by:

$$x_1 = \frac{\delta_2 - \delta_1 + \sqrt{(\delta_2 - \delta_1)^2 + 4\delta_0\delta_2}}{2\delta_2}, \tag{31a}$$

$$x_2 = \frac{\delta_2 - \delta_1 - \sqrt{(\delta_2 - \delta_1)^2 + 4\delta_0\delta_2}}{2\delta_2}. \tag{31b}$$

Since by assumption $\delta_0\delta_2 > 0$, $x_1 > 0 > x_2$. This observation will be used below.

We are looking for a particular solution of the type:

$$v_p[S] = g_1[S]S^{x_1} + g_2[S]S^{x_2}. \tag{32}$$

Without loss of generality, assume $g_1'[S]S^{x_1} + g_2'[S]S^{x_2} = 0$, then plugging the particular solution into the ODE, we obtain the following system of equations:

$$g_1'[S]S^{x_1} + g_2'[S]S^{x_2} = 0 \tag{33a}$$

$$g_1'[S]x_1 S^{x_1 - 1} + g_2'[S]x_2 S^{x_2 - 1} = \frac{(1-\alpha)^{\frac{1}{\rho}}\left(1 - (1-S)^{1-\frac{1}{\rho}}\right)}{\delta_2 S^2}. \tag{33b}$$

Solving for $g_1'[S]$ and $g_2'[S]$ we find:

$$g_1'[S] = \frac{(1-\alpha)^{\frac{1}{\rho}} \left(1-(1-S)^{1-\frac{1}{\rho}}\right) S^{-1-x_1}}{\delta_2(x_1-x_2)}, \tag{34}$$

$$g_2'[S] = -\frac{(1-\alpha)^{\frac{1}{\rho}} \left(1-(1-S)^{1-\frac{1}{\rho}}\right) S^{-1-x_2}}{\delta_2(x_1-x_2)}. \tag{35}$$

The solution to the particular equation therefore is:

$$v_p[S] = \frac{(1-\alpha)^{\frac{1}{\rho}}}{x_1-x_2} \left(S^{x_1} \int_{k_1}^{S} (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{1-x_1} d\tau - S^{x_2} \int_{k_2}^{S} (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{1-x_2} d\tau \right), \tag{36}$$

for arbitrary constants k_1 and k_2. What remains is a choice of k_1 and k_2 so that the solution is well defined and the boundary conditions are satisfied. An easy choice is to take $k_1 = 1$ and $k_2 = 0$, then the particular solution is:

$$v_p[S] = \frac{(1-\alpha)^{\frac{1}{\rho}}}{x_1-x_2} \left(-S^{x_1} \int_{S}^{1} (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{1-x_1} d\tau - S^{x_2} \int_{0}^{S} (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{1-x_2} d\tau \right) \tag{37}$$

The general solution will be the sum of the homogeneous solution and the particular solution. Since the value of the homogeneous solution can not grow without bound as $S \to 0$ or as $S \to \infty$ the constants in the homogeneous solution associated with S^{x_2} when $S < 1$ and S^{x_1} when $S \geq 1$ have to be zero. Thus, the solution in the region $S < 1$ is:

$$\begin{aligned} v[S] = {} & CS^{x_1} - \\ & \frac{2(1-\alpha)^{\frac{1}{\rho}}}{\sigma_s^2(x_1-x_2)} \left(S^{x_1} \int_{S}^{1} (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{-1-x_1} d\tau + S^{x_2} \int_{0}^{S} (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{-1-x_2} d\tau \right), \end{aligned} \tag{38}$$

and the solution in the region $S \geq 1$ is:

$$v[S] = DS^{x_2}. \tag{39}$$

What is left is to find the constants C and D such that the smooth-pasting conditions hold. The limit of $v_p[S]$ as $S \to 0$ is 0, so meeting the boundary condition for $S = 0$ will come from the solution to the homogeneous differential equation.

Define $A_2 \equiv \int_0^1 (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{-1-x_2} d\tau$. It is easy to see that $v_p[1] = \frac{-2(1-\alpha)^{\frac{1}{\rho}} A_2}{2\delta_2(x_1-x_2)}$ and $v_p'[1] = \frac{-2(1-\alpha)^{\frac{1}{\rho}} x_2 A_2}{2\delta_2(x_1-x_2)}$. Thus from the smooth-pasting conditions we obtain:

$$\frac{-2(1-\alpha)^{\frac{1}{\rho}} A_2}{2\delta_2(x_1-x_2)} + C + \frac{(1-\alpha)^{\frac{1}{\rho}}}{\delta_0} = D \tag{40}$$

$$\frac{-2(1-\alpha)^{\frac{1}{\rho}} x_2 A_2}{2\delta_2(x_1-x_2)} + Cx_1 = Dx_2 \tag{41}$$

Solving for C and D,

$$C = \frac{\frac{(1-\alpha)^{\frac{1}{\rho}}}{\delta_0} x_2}{x_1 - x_2} \tag{42}$$

$$D = \frac{(1-\alpha)^{\frac{1}{\rho}}}{2\delta_2(x_1-x_2)} \left(-2A_2 + \frac{2\delta_2 x_1}{\delta_0}\right) \tag{43}$$

The complete solution to the value of the firm is, therefore:

In the region $S < 1$:

$$\begin{aligned} v[S] = \frac{2(1-\alpha)^{\frac{1}{\rho}}}{2\delta_2\delta_0(x_1-x_2)} \Bigg(& (x_1 - x_2)\frac{2\delta_2}{2} + \frac{2\delta_2}{2} x_2 S^{x_1} - \\ & \delta_0 S^{x_1} \int_s^1 (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{1-x_1} d\tau - \delta_0 S^{x_2} \int_0^S (1-(1-\tau)^{\frac{\rho-1}{\rho}})\tau^{1-x_2} d\tau \Bigg) \end{aligned} \tag{44}$$

In the region $S \geq 1$:

$$v[S] = \frac{2(1-\alpha)^{\frac{1}{\rho}}}{2\delta_2\delta_0(x_1-x_2)} \left(-A_2\delta_0 + \frac{2\delta_2 x_1}{2}\right) S^{x_2} \tag{45}$$

Table I
Summary Statistics

Panels A and B report time series averages of median characteristics of portfolios of firms sorted on labor share (LS) and the extended measure of labor share (ELS), respectively. *LS* is ratio of labor expenses over the sum of labor expenses, operating profits, and change in inventories of final goods. *ELS*'s construction is identical to *LS* except that, for firms that do not report labor expenses, we proxy them by the product of the number of employees in the firm and the average wage in the industry. *Log. L/K* is the logarithm of the ratio of the number of employees over PPE. *B/M* is shareholders book value of equity divided by market value of equity. *Log Size* is the logarithm of market value of equity. *Log. Asset* is the logarithm of book value of assets. *Tang.* is tangibility and is defined as the ratio of plant, property, and equipment (PPE) over assets. *Org. Cap* is organizational capital, constructed as in Eisfeldt and Papanikolaou (2013). *Lev.* is leverage and is defined as the ratio of book value of debt minus cash and marketable securities over book value of assets minus cash and marketable securities. *Prof.* is the measure of gross profitability of Novy-Marx (2013). Firms are assigned to industries following the 17-industry classification system from Eugene F. Fama and Kenneth R. French. All variables are adjusted for inflation as measured by the Consumer Price Index. The sample covers all industries in Compustat, except Financials, over the period 1963–2012.

Panel A: LS											
Col.	1	2	3	4	5	6	7	8	9	10	11
Port.	LS	ELS	Log. L/K	B/M	Log. Size	Log. Asset	Tang.	Org. Cap.	Lev.	Prof.	Obs/ Year
Sample With Non-Missing LS											
All	0.62	0.62	2.47	0.68	7.18	7.56	0.50	0.98	0.54	0.24	285
Simple Sorts											
L	0.33	0.33	0.52	0.72	7.73	8.19	0.71	0.51	0.62	0.16	57
2	0.47	0.47	1.32	0.67	7.45	7.86	0.62	0.84	0.58	0.20	57
3	0.62	0.62	2.69	0.57	7.47	7.47	0.45	1.02	0.47	0.33	57
4	0.72	0.72	3.10	0.69	7.01	7.26	0.43	1.06	0.49	0.30	57
H	0.83	0.83	3.49	0.83	6.39	7.02	0.35	1.07	0.56	0.23	57
Within-Industry Sorts											
L	0.43	0.43	1.67	0.56	7.52	7.67	0.59	0.77	0.53	0.25	51
2	0.57	0.57	2.19	0.60	7.46	7.68	0.52	0.94	0.51	0.27	60
3	0.66	0.66	2.45	0.66	7.45	7.73	0.48	1.04	0.52	0.26	60
4	0.73	0.73	2.76	0.74	7.11	7.66	0.48	1.01	0.55	0.23	60
H	0.82	0.82	3.14	0.82	6.48	7.19	0.44	1.14	0.58	0.19	54

Table I
Summary Statistics (Cont.)

Panel B: ELS											
Col.	1	2	3	4	5	6	7	8	9	10	11
Port.	LS	ELS	Log. L/K	B/M	Log. Size	Log. Asset	Tang.	Org. Cap.	Lev.	Prof.	Obs/ Year
					Sample With Non-Missing ELS						
All	0.62	0.65	3.01	0.62	6.48	6.69	0.31	1.23	0.50	0.34	1632
					Simple Sorts						
L	0.33	0.32	0.81	0.63	7.22	7.43	0.59	0.63	0.58	0.21	326
2	0.53	0.53	2.60	0.49	6.89	6.83	0.32	1.17	0.47	0.39	327
3	0.65	0.65	3.09	0.57	6.56	6.64	0.30	1.28	0.47	0.40	327
4	0.74	0.74	3.31	0.69	6.22	6.53	0.28	1.37	0.49	0.37	327
H	0.85	0.85	3.63	0.82	5.74	6.28	0.27	1.43	0.52	0.33	326
					Within-Industry Sorts						
L	0.41	0.41	2.21	0.46	7.00	6.88	0.32	0.92	0.47	0.34	320
2	0.56	0.58	2.75	0.54	6.87	6.89	0.33	1.15	0.48	0.37	330
3	0.65	0.67	3.02	0.62	6.59	6.79	0.31	1.29	0.50	0.36	329
4	0.73	0.75	3.22	0.70	6.27	6.65	0.32	1.36	0.52	0.34	330
H	0.83	0.85	3.53	0.81	5.80	6.37	0.29	1.43	0.53	0.31	323

Table II
Smoothness and Cyclicality of Labor Costs

Panel A reports estimates and standard errors of panel data regressions of measures of profit growth on aggregate GDP, TFP, and wage growth. gdp^g is annualized growth calculated as the change of the logarithm of real GDP. tfp^g is annualized growth calculated as the change of the logarithm of TFP. $wage^g$ is annualized growth calculated as the change of the logarithm of real wages. Standard errors clustered by year are shown in parentheses. Panel B reports estimates of panel data regressions of changes of costs on changes in sales. Δlc and lc^g(%) are the $ and % changes of staff expenses. Δnlc and nlc^g(%) are the $ and % changes of the sum of operating expenses (SG&A and COGS) minus staff expenses. Δtc and tc^g(%) are the $ and % changes of the sum of operating expenses (SG&A and COGS). Standard errors clustered by firm are shown in parentheses. Significance levels are denoted by (* = 10% level), (** = 5% level) and (*** = 1% level). The sample covers all industries in Compustat, except Financials, over the period 1963–2012 (both panels). The sample in Panel B is restricted to firm-year observations with non-missing values for SG&A and COGS.

Panel A: Smoothness and Cyclicality of Macroeconomic Variables

Variable	gdp^g	tfp^g	$wage^g$	$profit^g$
gdp^g	1.000	.	.	.
tfp^g	0.862	1.000	.	.
$wage^g$	0.275	0.480	1.000	.
$profit^g$	0.628	0.621	-0.063	1.000
σ	0.030	0.017	0.015	0.105
σ / σ^{gdp}	1.000	0.573	0.513	3.538
Slope on gdp^g*	1.000	0.494	0.141	2.222

Panel B: Elasticity of Firm-Level Costs to Sales

	Cost (Dependent Variable)					
	Δlc	Δnlc	Δtc	lc^g(%)	nlc^g(%)	tc^g(%)
Δsale	0.09***	0.72***	0.81***			
	(0.01)	(0.03)	(0.03)			
$sale^g$(%)				0.43***	1.46***	1.07***
				(0.16)	(0.28)	(0.12)
Firm FE	Y	Y	Y	Y	Y	Y
R-sq. (%)	19.23	72.88	76.69	0.00	9.96	59.25
Obs.	8,173	8,173	8,173	8,173	8,173	8,173

* Slope β_0 from regression $x_{it} = \beta_0 + \beta_1 gdp^g_t$.

Table III
Cyclicality of Labor Share

This table reports estimates and standard errors of panel data regressions of labor share growth on growth in business cycle indicators. gdp^g is annualized growth calculated as the change of the logarithm of real GDP. tfp^g is annualized growth calculated as the change of the logarithm of TFP. MKT is the excess market return described in Fama and French (1993). Standard errors clustered by year are shown in parentheses. Significance levels are denoted by (* = 10% level), (** = 5% level) and (*** = 1% level). The sample covers all industries in Compustat, except Financials, over the period 1963–2012.

	Proxy for Labor Share (S)					
	LS			ELS		
Ind. Variable	I	II	III	I	II	III
gdp^{g}_t	-0.33***			-0.46***		
	(0.12)			(0.10)		
tfp^{g}_t		-0.43*			-0.52**	
		(0.25)			(0.23)	
MKT_t			-0.03*			-0.06***
			(0.02)			(0.02)
Firm FE	Y	Y	Y	Y	Y	Y
R-sq. (%)	0.54	0.30	0.16	0.34	0.14	0.25
Obs.	13,508	13,508	13,508	75,720	75,720	75,720

Table IV
Elasticity of Substitution Between Labor and Capital

The table presents estimates of the elasticity of substitution between labor and capital. The methodology is based on two stages. In the first stage, we estimate the elasticity to shocks of operating profit growth (Θ^{Π}) and value added growth (Θ^{Y}). The table shows estimates of time-series regressions of firm-level measures of real operating profit Π growth and real value added Y growth on aggregate GDP growth, TFP growth, and the returns on the market portfolio. In the second stage, we regress $\hat{\Theta}^{\Pi}$ on $\hat{\Theta}^{Y}$ from the first pass to obtain estimates of the elasticity of substitution between labor and capital. A value added is constructed as the sum of labor expenses, operating profits before interest and depreciation, adjusted for changes in the inventories of final goods. Labor expenses used are staff expenses (XLR) in Panel A and, in Panel B, the product of number of employees (EMP) and the industry average of (XLR/EMP) if XLR is missing. gdp^{g} is annualized growth calculated as the change of the logarithm of real GDP. tfp^{g} is annualized growth calculated as the change of the logarithm of TFP. MKT is the excess market return factor from Kenneth French's website. Standard errors clustered by firm are shown in parentheses. Significance levels are denoted by (* = 10% level), (** = 5% level) and (*** = 1% level). The sample covers all industries in Compustat, except Financials, over the period 1963–2012.

	First Stage						Second Stage		
	$\hat{\Theta}^{\Pi}$			$\hat{\Theta}^{Y}$			L/K Elasticity of Subs.		
	I	II	III	I	II	III	I	II	III
Panel A: Sample with Non-Missing XLR-Based Value Added									
gdp_t^g	5.66***			3.21***			0.40***		
	(0.25)			(0.12)			(0.03)		
tfp_t^g		10.19***			5.68***			0.40***	
		(0.44)			(0.21)			(0.03)	
MKT_t			0.79***			0.47***			0.40***
			(0.04)			(0.02)			(0.03)
R-sq. (%)	0.13	0.11	0.04	0.15	0.13	0.04	0.67	0.70	0.62
Obs.	10,536	10,536	10,536	10,536	10,536	10,536	10,536	10,536	10,536
Panel B: Full Sample									
gdp_t^g	9.29***			7.55***			0.57***		
	(0.17)			(0.15)			(0.02)		
tfp_t^g		16.01***			12.50***			0.54***	
		(0.30)			(0.23)			(0.02)	
MKT_t			1.18***			0.85***			0.42***
			(0.02)			(0.01)			(0.02)
R-sq. (%)	0.09	0.07	0.04	0.07	0.05	0.03	0.43	0.47	0.44
Obs.	54,406	54,406	54,406	54,406	54,406	54,406	54,406	54,406	54,406

Table V
Labor Share and Sensitivity of Operating Profits to Macroeconomic Shocks

This table reports estimates and standard errors of panel data regressions of measures of real operating income before depreciation growth (OP^g) on aggregate GDP, TFP, wage growth, standardized labor share, and interaction terms. gdp^g is annualized growth calculated as the change of the logarithm of real GDP. tfp^g is annualized growth calculated as the change of the logarithm of TFP. $wage^g$ is annualized growth calculated as the change of the logarithm of real wages. LS and ELS are standardized every year. Standard errors clustered by year are shown in parentheses. Significance levels are denoted by (* = 10% level), (** = 5% level) and (*** = 1% level). The sample covers all industries in Compustat, except Financials, over the period 1963–2012.

	Proxy for Labor Share (S)					
	LS			ELS		
Ind. Variable	I	II	III	I	II	III
gdp^g_t	1.96^{***}			2.37^{***}		
	(0.23)			(0.26)		
$S_{it-1} \times gdp^g_t$	1.15^{***}			0.54^{***}		
	(0.21)			(0.16)		
tfp^g_t		2.83^{***}			2.79^{***}	
		(0.59)			(0.67)	
$S_{it-1} \times tfp^g_t$		1.53^{***}			0.90^{***}	
		(0.45)			(0.26)	
MKT_t			0.18^{***}			0.28^{***}
			(0.05)			(0.06)
$S_{it-1} \times MKT_t$			0.12^{***}			0.06^{*}
			(0.04)			(0.03)
S_{it-1}	0.13^{***}	0.14^{***}	0.16^{***}	0.16^{***}	0.16^{***}	0.18^{***}
	(0.01)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)
Firm FE	Y	Y	Y	Y	Y	Y
R-sq. (%)	10.89	9.11	6.80	8.59	6.98	7.21
Obs.	13,530	13,530	13,530	68,873	68,873	68,873

Table VI
Asset Returns of Firms Sorted by Labor Share

This table reports report two-years ahead post-ranking mean annual excess stock returns over annualized one-month Treasury bill rates of equally- and value-weighted portfolios of firms sorted on twice lagged LS and ELS. H-L is the zero net-investment portfolio long high labor share (H) stocks and short low labor share (L) stocks. Newey-West standard errors estimated with five lags are shown in parentheses. The sample covers all industries in Compustat, except Financials, over the period 1964–2012.

	Portfolio					
Sort Var.	L	2	3	4	H	H-L
			Equally-Weighted Returns			
LS_{t-1}	6.91***	8.94***	9.18***	8.77***	11.72***	4.82**
	(1.88)	(1.89)	(2.05)	(2.22)	(2.70)	(2.25)
ELS_{t-1}	8.49***	9.73***	10.02***	10.81***	11.78***	3.29*
	(1.95)	(1.86)	(2.03)	(2.24)	(2.72)	(1.91)
			Value-Weighted Returns			
LS_{t-2}	6.11***	7.80***	6.26***	5.73**	10.18***	4.06*
	(1.91)	(1.90)	(2.01)	(2.67)	(2.46)	(2.20)
ELS_{t-2}	6.98***	7.36***	7.00***	7.47***	10.23***	3.25*
	(1.79)	(1.78)	(1.74)	(2.11)	(2.54)	(1.92)

Table VII
Stock Returns and Labor Share

This table shows estimates and standard errors of panel data regressions annual stock returns on twice lagged measures of labor share and controls for leverage and assets. Standard errors clustered by firm are shown in parentheses. Significance levels are denoted by (* = 10% level), (** = 5% level) and (*** = 1% level). The sample covers all industries in Compustat, except Financials, over the period 1964–2012.

	Specification					
Ind. Var.	I	II	III	IV	V	VI
LS_{t-2}	1.21***	1.37***	1.18***			
	(0.31)	(0.35)	(0.35)			
ELS_{t-2}				0.82***	0.84***	0.70***
				(0.20)	(0.20)	(0.20)
Lev_{t-2}		4.53*	6.44**		4.85***	6.36***
		(2.35)	(2.56)		(1.22)	(1.39)
$Assets_{t-2}$			-0.71***			-0.44***
			(0.23)			(0.13)
Year FE	Y	Y	Y	Y	Y	Y
R-sq. (%)	0.10	0.13	0.21	0.02	0.05	0.06
Obs.	14,291	14,291	14,291	78,719	78,719	78,719

Table VIII
Risk Factor Loadings

The table reports average conditional betas of portfolios stocks sorted on lagged measures of Labor Share (LS and ELS). MKT, SMB, and HML are the market, size, and value risk factors described in Fama and French (1993). TFP, WAG, and GDP are total factor productivity, wages, and gross domestic product growth described in Table II. H-L is the zero net-investment portfolio long high labor share (H) stocks and short low labor share (L) stocks. Newey-West standard errors estimated with one lag are shown in parentheses. Significance levels are denoted by (* = 10% level), (** = 5% level) and (*** = 1% level). The sample covers all industries in Compustat, except Financials, over the period 1964–2012.

	Portfolio					
Factor	L	2	3	4	H	H-L
Panel A: Average Betas of Portfolios Sorted on LS						
MKT	0.69***	0.81***	1.08***	1.21***	1.37***	0.68***
	(0.09)	(0.05)	(0.05)	(0.04)	(0.08)	(0.10)
SMB	0.32*	0.49***	0.87***	1.03***	1.39***	1.08***
	(0.17)	(0.18)	(0.16)	(0.16)	(0.16)	(0.08)
HML	-0.01	-0.11	-0.32	-0.37*	-0.50*	-0.49**
	(0.10)	(0.15)	(0.20)	(0.22)	(0.26)	(0.20)
TFP	2.21	2.72	3.44	4.30*	6.06**	3.85***
	(1.79)	(2.14)	(2.14)	(2.37)	(2.90)	(1.41)
GDP	0.34	0.34	1.44	2.11	4.19	3.86**
	(1.86)	(1.69)	(2.08)	(2.11)	(2.91)	(1.62)
WAG	1.69	-1.67	4.75*	3.89	3.48	1.79
	(1.53)	(3.56)	(2.57)	(3.28)	(3.02)	(2.59)
Panel B: Average Betas of Portfolios Sorted on ELS						
MKT	1.05***	1.31***	1.37***	1.44***	1.52***	0.47***
	(0.07)	(0.05)	(0.05)	(0.05)	(0.07)	(0.07)
SMB	0.73***	1.05***	1.21***	1.32***	1.56***	0.83***
	(0.13)	(0.12)	(0.13)	(0.14)	(0.13)	(0.05)
HML	-0.43***	-0.67***	-0.60**	-0.57**	-0.55**	-0.12
	(0.15)	(0.21)	(0.23)	(0.23)	(0.24)	(0.10)
TFP	3.81	4.93*	5.15**	5.38**	5.93**	2.12**
	(2.41)	(2.44)	(2.26)	(2.39)	(2.43)	(0.80)
GDP	1.78	2.17	2.86	2.88	3.56	1.78**
	(1.93)	(2.13)	(2.00)	(1.96)	(2.20)	(0.83)
WAG	0.84	0.18	4.02	3.14	3.66	2.83
	(1.82)	(3.38)	(3.19)	(2.93)	(3.36)	(2.09)