

Time Inconsistency in Stress Test Design

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Abstract

We show that central banks face a time inconsistency problem when publishing bank stress test results. Before a stress test, they want to *appear* tough as the threat of letting banks fail the stress test incentivizes prudent behaviour. After the stress test, they want to *act* soft by releasing only partial information in order to reassure financial markets about bank health. We characterize an institutional design solution to this commitment problem: a social planner sets the framework within which the central bank communicates. We find that a hurdle rate framework, where all banks are judged to pass or fail relative to a common threshold, is optimal in many settings as it generates intermediate levels of both incentives and reassurance. With a hurdle rate framework, stress tests become an informational contagion channel, as changes in one bank's fundamental health affect the perceived health of other banks. Thus, informational contagion can be a feature of a socially optimal institutional design in the presence of a time inconsistency problem.

Keywords: *Bank Stress Tests, Strategic Communication, Contagion, Central Bank Design, Financial Stability*

JEL Classification: D83, E58, G21

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1 Introduction

“*Had we been fully open and fully transparent about what was going on during the financial crisis, it would, let me tell you, have been a lot, lot worse. That would have been shouting ‘fire’ in the theatre.*”

— Andy Haldane, Chief Economist, Bank of England¹

“*The stress test has been a catalyst for pressure to raise capital.*”

— Andrea Enria, Chair of the ECB Supervisory Board²

Following the recent financial crisis, the frequent publication of bank stress test results has become the centrepiece of central bank communication about bank stability. While central banks view the communication of bank stress tests as serving two purposes – incentivize banks to take prudent actions and reassure financial markets about bank stability – the optimal stress test design literature has largely focused on the latter (Goldstein and Leitner, 2018; Inostroza and Pavan, 2018).

This paper studies stress tests as serving both purposes. We find that a trade-off between these two purposes exists and that they create a time inconsistency problem which results in suboptimal combinations of incentives and reassurance. The trade-off arises because incentives for banks to improve their health are created by the central bank’s threat to reveal bank weakness, which to financial markets is the opposite of a reassuring message. Additionally, these two roles create a time-inconsistency problem: Before the stress test, when banks decide whether to take prudent actions, the central bank wants to *appear* tough, i.e. create a threat of revealing bank weakness. After the test, when banks cannot alter their health anymore, the central bank wants to *act* soft, i.e. not reveal bank weakness. Rational agents anticipate this. Banks know that the threat is not credible and therefore do not take prudent actions.

We propose an institutional design solution to this time inconsistency problem. A social planner chooses an institutional design before banks can take prudent actions. After banks have taken prudent actions and having observed bank health, the central bank (CB) communicates stress test results within the constraints set by the institutional design. A design consists of a communication framework which is either full disclosure, zero disclosure, or a hurdle rate framework where all banks are judged to pass or fail relative to a common level of stress.³ A design also specifies the CB’s objective function (henceforth: mandate). This approach echoes Rogoff’s (1985) work on time inconsistency in monetary policy.

We find that a hurdle rate framework is optimal for a large set of parameters as it provides an intermediate combination of both incentives and reassurance. A full disclosure framework

¹Quoted in *The Times*, 6 October 2017. Andy Haldane was the Bank of England’s Executive Director for Financial Stability during the financial crisis.

²Quoted in *Financial Times*, 15 July 2011. Andrea Enria was Chairperson of the European Banking Authority from 2011 to 2018.

³A full disclosure framework is currently used in the Euro Area whereas a hurdle rate framework is used by the Bank of England.

generates strong incentives but no reassurance. The reverse is true for a zero disclosure framework.

We show that, in a hurdle rate framework, *strategic delegation* provides additional benefits. This means the planner maximises ex-ante welfare by setting the central bank's mandate to differ from ex-post welfare maximisation. The optimal mandate can differ in either direction. If taking prudent actions has high costs for banks, the planner optimally specifies the CB's mandate to be "tougher" than welfare maximisation, so that the CB is less concerned about supporting weak banks than society. This makes it credible that the CB will reveal bank weakness by letting weak banks fail the stress test and thus generates strong incentives. If the costs of prudent actions are low, the optimal mandate is "softer" than welfare maximisation, so that the CB is very concerned about weak banks. This achieves high reassurance while still generating sufficient incentives to induce the prudent action.

Stress tests become an informational contagion channel when a hurdle rate framework is used. This arises because in some cases the central bank's optimal response to a deterioration in the health of one bank is to lower the severity of stress so that all banks continue to pass. For all banks, the resulting pass results are interpreted less favourably than at the initial higher severity of stress. Thus, a change in one bank's health leads the CB to send a different message, which in turn affects beliefs financial markets hold about all banks. More surprisingly, there are also cases where a deterioration of one bank's health improves the beliefs about other banks. This is the case when the CB raises the level of stress. As a result, the weak bank fails the test and beliefs about its health deteriorate. However, beliefs about other banks improve as they pass a tougher test. In a hurdle rate framework, both of these contagion patterns arise for all sender mandates, including "tough" mandates which focus on the average health in the banking system and place no extra weight on supporting weak banks.

Informational contagion can thus be a feature of an optimal design in the presence of a time-inconsistency problem; contagion need not reduce welfare. The mechanism that makes the hurdle rate framework optimal for some parameters is the same mechanism that turns stress tests into an informational contagion channel. In a hurdle rate framework, the CB is constrained by having to judge all banks against one common level of stress. From the planner's perspective, this constraint is valuable, as it results in some but less than full revelation of bank health and thus achieves intermediate levels of both incentives and reassurance. Without the constraint, there would be no contagion but also no resolution to the time inconsistency problem.

Our examination of stress tests has broader implications for economic theory. We contribute to the strategic communication literature by characterizing the equilibrium in a verifiable disclosure communication game with a cross-message constraint. In these games, the sender communicates about multiple banks, truthfully releases pass or fail results for each of them, but is constrained by having to judge all banks relative to a common level of stress, rather than using bank specific stress levels. The sender therefore faces a trade-off: While higher levels of stress result in more favourable messages for those banks that pass, they have the cost of more banks failing the test which is a very unfavourable message.

Our paper also contributes to the literature on incentives generate by communication, which has commonly been framed in the context of teachers grading exams (Boleslavsky and Cotton,

2015; Dubey and Geanakoplos, 2010). Our model can be reinterpreted in that setting: A class teacher (central bank) wants to incentivize students (banks) to study for an exam (take a prudent action) by threatening to let bad students fail the test. However, once the exam has been taken, the teacher wants to give out good marks to help students get good jobs. A head teacher (planner) tries to solve this time inconsistency problem by specifying the school’s grading policy (stress test design).⁴

The remainder of this paper is structured as follows: Section 2 reviews the related literature. Section 3 outlines the model and discusses key assumptions. Section 4 focuses on reassurance. First, it takes an ex-post perspective and characterises how much reassurance a central bank achieves for given health outcomes in equilibrium in a given design. For a hurdle rate framework, this involves solving the verifiable disclosure game with a cross-message constraint. Then, it takes the planner’s ex-ante perspective and characterises how much reassurance different designs achieve in expectation, i.e. across possible health outcomes. Section 5 compares designs in terms of incentives. Section 6 combines incentive and reassurance concerns to solve for the optimal institutional design. Section 7 shows that a hurdle rate framework implies that stress tests become an informational contagion channel. Section 8 concludes.

2 Literature Review

This paper is related to three different strands of the literature. Our idea of studying optimal institutional design solutions to a time inconsistency problem is related to the monetary economics and particularly the “central bank design” literature (Reis, 2013). Rather than focusing on a monetary policy maker who is tempted to boost output by creating surprise inflation we focus on a bank regulator tempted to shield weak banks from market pressure by not releasing information. Our modelling framework for this communication game builds on and contributes to the theoretical literature on strategic communication. Our topic of bank stress tests is rooted in the central bank communication literature.

Central Bank design and strategic delegation

In monetary economics, a time inconsistency problem exists when the policy maker would like to generate surprise inflation to increase output (Lucas, 1972, 1975; Kydland and Prescott, 1977; Barro and Gordon, 1983a,b). Agents with rational expectations anticipate this and therefore expect inflation to be so far above target that the monetary policy maker is indifferent between the cost of surprise inflation and the benefit of output gains. The result is “inflation bias”: an equilibrium with inflation above target and no output gains.⁵

⁴This paper differs from Boleslavsky and Cotton (2015) in not assuming commitment of the teacher (central bank) and in focusing on effort of a third party (students, banks) rather than the sender. It differs from Dubey and Geanakoplos (2010) in focusing on effort prior to and thus in anticipation of communication rather than afterwards.

⁵The CB is minimising losses which are quadratic in the deviation of inflation and output from target where the target level of output exceeds the natural rate. Then, if the economy were at the target level of inflation and at the natural rate of output, the CB would strictly prefer to generate surprise inflation. Due to quadratic losses, there exists a higher level of inflation at which the CB is indifferent between further inflation and the output

Institutional design solutions which reduce inflation bias range from strategic delegation to performance contracts. Rogoff (1985) argues that delegating policy to a “conservative central banker” who places more weight than society on inflation stabilisation relative to output stabilisation reduces the inflation bias and thus increases welfare. Walsh (1995) uses insights from principal agent theory to argue that inflation bias can be reduced by a performance contract which rewards the central banker for low inflation.

Both institutional design solutions have the common feature that the objective function of the policy maker which maximises ex-ante welfare differs from welfare maximisation.⁶ The general benefits of strategic delegation were outlined by Vickers (1985) who shows that in a strategic context the highest pay-off for the principal is achieved by an agent who maximises an objective function other than the principal’s pay-off. Specifically, in Cournot competition profit increases when decisions are delegated to a manager who places weight on market share, not just on profits, as this commits the firm to aggressive behaviour which in turn leads rivals to cut output. Effectively, strategic delegation allows the firm to become a Stackelberg leader.

We contribute to the CB design literature in three ways. We take CB design and delegation ideas to a new setting: focusing on communication rather than actions; on financial stability rather than monetary policy. In this setting, we formalise the existence of a time inconsistency problem, which to the best of our knowledge is a novel problem. By characterising solutions to this problem, we show benefits of delegation in a strategic communication game rather than a strategic action setting. We now turn to the communication literature on which our model builds.

Strategic communication theory

Communication theory models can be grouped according to two dimensions: Is the sender restricted to telling the truth, or is he allowed to lie? Does the sender communicate strategically based on private information or does he commit to a messaging rule before he becomes privately informed? In verifiable disclosure models (Grossman, 1981; Milgrom, 1981) a privately informed sender needs to state the truth but not the full truth (“ $x \geq 4$ ” rather than “ $x = 4$ ” is possible). In cheap talk models (Crawford and Sobel, 1982) a privately informed sender can lie. In models of Bayesian persuasion (Kamenica and Gentzkow, 2011) the sender commits to a messaging rule before becoming privately informed. A recent survey of the literature based on commitment is Bergemann and Morris (2019).

We assume that the regulator has inside information on bank resilience when deciding on the severity of the stress test and cannot lie. Hence, our model is based on verifiable disclosure. Readers more familiar with the macroeconomic literature may equally view our model as “communication with discretion”, i.e. where the regulator is not able to commit to a messaging rule. The opposite case of “communication with commitment” would be covered by a model of Bayesian persuasion. This paper contributes to the theoretical literature by solving a verifiable disclosure model with a cross-message constraint, i.e. where the sender communicates about two benefit. This is the equilibrium.

⁶In Rogoff, the objective function comes from intrinsic types. In Walsh, it results from a performance contract.

banks but where the pass/fail message for each bank is relative to a common level of stress.

Central bank communication

The literature on Central Bank communication can be divided according to a CB's two objectives: monetary and financial stability. For the former, which is not directly related to our paper, surveys are provided by Geraats (2002) and Blinder et al. (2008). Notable theoretical contributions include models of global games (Carlsson and van Damme, 1993; Morris and Shin, 2002; Svensson, 2006; Morris et al., 2006; Angeletos and Pavan, 2007) which show that coordination motives may lead agents to place too much weight on public relative to private messages;⁷ and more recently Morris and Shin (2018)⁸ who study public disclosure in “echo chambers”, i.e. when the policy maker is trying to infer information from a price which is itself affected by the policy maker's communication.

We focus on CB communication about financial stability and partition this literature further into two subfields: cases where the regulator cannot commit to a communication strategy (communication with discretion, i.e. strategic communication) and cases where the regulator is assumed to be able to commit (Bayesian persuasion). We first focus on the case with discretion, then review the literature with commitment.⁹

No commitment

Our key contributions to the hitherto small literature on the case without commitment are that we consider the incentive motive of stress test disclosure in addition to the reassurance motive and that we model the complexity arising from testing multiple banks simultaneously with a common stress scenario.

Bouvard et al. (2015) consider a regulator who communicates only because of the reassurance motive and faces a binary choice between full and zero disclosure. Intermediate levels of informativeness such as a hurdle rate framework are not considered. They find that regulators disclose information only in crisis when average health is so low that absent communication all banks would face a run. Disclosure then limits the run to those banks with health actually below the threshold that triggers runs.

Shapiro and Zeng (2018) argue that testing one bank repeatedly creates a trade-off: the regulator wants to acquire a reputation for being weak to encourage lending, but also for being tough to discourage excessive risk taking.¹⁰ We focus on two different trade-offs: the time-inconsistency problem created by the intertemporal conflict between the incentive and the reassurance motives; and the conflict of testing multiple banks simultaneously against the same stress scenario. Then higher levels of stress are favourable for those banks who continue to pass

⁷Specific applications to communication about monetary policy include Morris and Shin (2005, 2007).

⁸Similarly Goldstein and Yang (2019).

⁹A broad survey of arguments for and against disclosure of stress test results is provided by Goldstein and Sapra (2013). Relative to their review, we add a review of the more recent literature and focus in particular on the assumptions on communication, specifically whether it is strategic or not. This emphasize is, to the best of our knowledge, a novel contribution of our literature review.

¹⁰Their model is based on the implicit assumption that less well capitalised banks can lend more which is contested. For empirical evidence of the contrary see e.g. Gambacorta and Shin (2018).

but come at the cost of failing other banks.

Commitment

Several papers study stress tests when the regulator can commit to a communication rule before observing bank health (Bayesian persuasion) and when the sole purpose of stress tests is to reassure financial markets. Similar to Bouvard et al. (2015), Goldstein and Leitner (2018) find that the regulator will only release information when the average bank is perceived to be so low that, absent communication, all banks would experience a run. Goldstein and Leitner’s regulator can send partially pooling messages, e.g. reporting the average health of a pool of banks. This saves at least some banks from the run.¹¹ However, this is at odds with current communication frameworks in which central banks have committed to regularly running and disclosing stress test results. Silence thus seems more likely to be interpreted as a bad signal which is a feature of our model. While Goldstein and Leitner’s regulator can send any partially pooling message, including averages, we restrict attention to messages about the minimum health of a group. This rules out that banks with high and low health pass while medium banks fail the stress test.

Inostroza and Pavan (2018) add an explicit model of how receivers coordinate by introducing additional heterogeneous private information. Parlatore (2015) views stress tests as generating *private* signals in a Diamond and Dybvig (1983) model of bank runs. Faria-e-castro et al. (2016) study how a governments fiscal position and disclosure policy interact: When govt funds are available to bail-out weak banks, more information can be released. In their model, this reduces actual spending on bail-outs. Orlov et al. (2017) endogenise asset prices and reach the opposite conclusion of Goldstein and Leitner: They argue that the regulator wants to be uninformative when bank health is low. If the regulator revealed capital shortfalls, fear of fire sales in the future would depress asset prices today, making it more costly for banks to recapitalise.

Williams (2017) also models a regulator as able to commit and as disclosing information only to avoid runs¹² but considers an extension in which banks can adjust to the communication rule. Williams finds that banks reduce their liquidity buffers when stress tests are conducted since pass grades make runs less likely and thus act as substitute for liquidity buffers. This is a response to communication to which the regulator is committed. We study the effect of anticipated *strategic* communication to which the regulator is not committed.¹³

We now turn to our model of stress tests by a regulator who cannot commit.

3 Model

Our model decomposes stress test design into two steps which are potentially carried out by different agents thus capturing potential benefits of delegation. First, a welfare maximising

¹¹Goldstein and Leitner frame their paper as studying how disclosure affects risk sharing among banks (the Hirshleifer effect (1971) which is related to our term “reassurance”).

¹²He models runs explicitly based on Allen and Gale (1998).

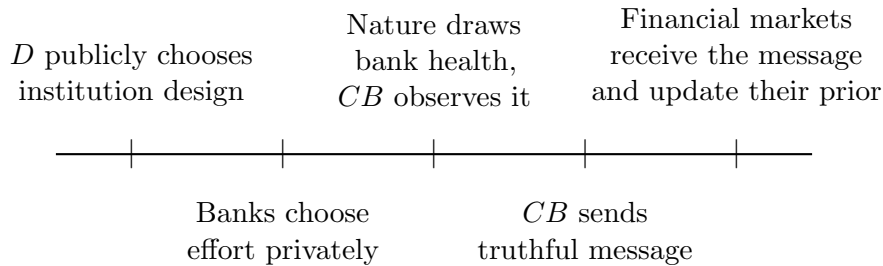
¹³An additional difference is that Williams views stress tests as having probabilistic outcomes while we view them as deterministic. In Williams a stress test results in a public message with an error probability. The regulator commits to an error probability. In our setting stress test messages are always correct, but need not be perfectly informative.

institution designer D (parliamentary committee, social planner) decides on a framework in which stress tests are conducted. Then, the CB or regulator communicates stress test results in the set framework to financial markets who update their belief on bank health. In between, banks can take a prudent action which improves their health and the CB observes health. We show that the difference in timing and information between the moves of D and CB creates a time-inconsistency problem: While a welfare maximiser wants to appear tough at the design stage to induce prudent actions by banks, a welfare maximiser wants to act soft at the communication stage to reassure financial markets as bank health is fixed. We show that welfare is maximised by delegating stress tests to a CB with a mandate that differs from welfare maximisation as this captures gains from strategic delegation. We are particularly interested in the difficulties and opportunities that arise from subjecting multiple banks to the same stress test simultaneously.

3.1 Description of the Model

At the first stage, an institution designer (D) chooses the regulator's mandate and specifies how he should communicate stress test results. Based on this stress test framework, banks decide how much to invest in stability. This investment makes higher health realisations more likely. The health outcomes ($H_i \in [0, 1]$) are privately observed by the regulator who then runs a stress test maximising his mandate and publishes results truthfully. Financial markets receive these messages and update their belief on bank stability. The sequence of moves is summarised in Figure 1 below:

Figure 1: Order of Moves



At the first stage, an institutional designer D chooses an institutional design $\mathcal{D} = \{\mathcal{C}, \mathcal{M}\}$ which consists of a communication framework \mathcal{C} and a mandate for the central bank \mathcal{M} . The set of possible communication frameworks includes a zero disclosure framework (ZDF), in which never any information is released, a full disclosure framework (FDF), in which each H_i is always fully revealed, and a hurdle rate framework (HRF) in which all banks are judged to pass or fail relative to a common level of stress. We focus on the case with two banks which is the simplest possible case that allows us to capture three general regimes: zero hurdles (ZDF), fewer hurdles than banks (HRF), at least as many hurdles as banks (FDF). A mandate \mathcal{M} is a mapping from a pair of beliefs about bank health $(\mu_1, \mu_2) \in [0, 1]^2$ to a value on the real line $V \in \mathbb{R}^1$, i.e. $\mathcal{M} : (\mu_1, \mu_2) \rightarrow V$. The designer is trying to maximise welfare $\mathcal{W} : (\mu_1, \mu_2) \rightarrow W$ where $W \in \mathbb{R}^1$. \mathcal{D} becomes public knowledge.

Banks simultaneously make a private choice whether to take a prudent action or not. This

can capture improved risk management practices or other unobserved aspects affecting bank resilience. In line with the moral hazard literature, we formalise this as a choice between high and low effort $e_i \in \{l, h\}$. High effort makes high health realisations more likely, formalised by a monotone likelihood ratio property (MLRP)¹⁴ and is costly $C(h) > C(l)$. The bank maximises expected profit $E(\pi) = E(\mu_i) - C(e_i)$. A bank's strategy is thus: $\mathcal{B}_i : \mathcal{D} \rightarrow e_i$.

Bank health realisations H_1, H_2 are drawn by nature from the distributions implied by e_i and observed only by the central bank and not by financial markets. Bank health is independent across banks.

The central bank privately observes bank health $H_i \forall i$, runs a stress test and communicates the results in the framework \mathcal{D} . In ZDF and FDF this is a mechanical exercise. In HRF, the central bank chooses a level of stress s strategically to maximise its mandate \mathcal{M} and must report truthfully whether each bank passed or failed relative to that common threshold s . The resulting message is a triplet $\{s, o_1, o_2\}$ where $s \in [0, 1]$ and $o_i \in \{p, f\}$ is the outcome for bank B_i . For example $\{0.2, p, f\}$ corresponds to “*Stress was 0.2 and B_1 passed while B_2 failed.*”. This does not only reveal the number of pass and fail marks but also reveals which bank passed or failed.

In the pure communication game where we treat \mathcal{D} and e_1, e_2 as exogenously fixed, a central bank's messaging rule S is a mapping from any health state pair $(H_1, H_2) \in [0, 1]^2$ to a message $\{s, o_1, o_2\} \in [0, 1]^4$ subject to the truth-telling constraint that $o_i = p$ if and only if $H_i \geq s$, otherwise $o_i = f$, i.e. $S : (H_1, H_2) \rightarrow \{s, o_1, o_2\}$. The sender cannot lie, but he could obfuscate by sending a universally true message such as $\{0, p, p\}$. In the overall institutional design game, the central bank's strategy \mathcal{S} is a profile of such messaging rules, i.e. $\mathcal{S} : \mathcal{D} \rightarrow S$.

Financial markets do not observe bank health H_i directly but observe the CB's message $\{s, o_1, o_2\}$ and update their prior belief accordingly, i.e. form $\mu_i \in [0, 1] \forall i$. Financial market participants are interested in bank health because they trade bank shares, credit default swaps or assess the risk of interbank loans. They are thus minimising a quadratic loss problem. In the pure communication game, we say that receivers (financial markets) follow an interpretation rule $R : \{s, o_1, o_2\} \rightarrow (\mu_1, \mu_2)$. In the overall institutional design game, the receiver's strategy \mathcal{R} specifies an R for every design \mathcal{D} , i.e. $\mathcal{R} : \mathcal{D} \rightarrow R$. Based on the beliefs (μ_1, μ_2) pay-offs are realised.

3.2 Equilibrium Concept

We focus on perfect Bayesian equilibria (PBE) as solution concept. This means that we restrict our attention to those weak perfect Bayesian equilibria of the overall institutional design game which are weak perfect Bayesian equilibria in every subgame. To build an understanding of this equilibrium concept, we first define weak PBE in the pure communication game and then define PBE in the overall institutional design game.

Definition 1 *A weak perfect Bayesian equilibrium in the communication game is a set (S, R) such that:*

¹⁴Milgrom (1981); Holmstrom (1979, 1982); Lambert (1983); Ottaviani and Sorensen (2006).

- Given R ; S maximises V at every health state pair (H_1, H_2) .
- Given S ; R is correct in the sense of using all available information.

In the overall game, every institutional design \mathcal{D} corresponds to a subgame. The only additional subgame is the overall game. Hence, our solution concept means:

Definition 2 A perfect Bayesian equilibrium (PBE) of the overall institutional design game is a set of strategy profiles $(\mathcal{D}, \mathcal{B}_1, \mathcal{B}_2, \mathcal{S}, \mathcal{R})$ such that

- Given $\mathcal{B}_1, \mathcal{B}_2, \mathcal{S}, \mathcal{R}$; \mathcal{D} maximises expected welfare.
- For every \mathcal{D}
 - Given $\mathcal{B}_{-i}, \mathcal{S}, \mathcal{R}$; \mathcal{B}_i maximises expected profits
 - Given $\mathcal{B}_1, \mathcal{B}_2, \mathcal{R}$; \mathcal{S} maximises V at every health state pair (H_1, H_2) .
 - Given $\mathcal{B}_1, \mathcal{B}_2, \mathcal{S}$; \mathcal{R} is correct in the sense of using all available information.

3.3 Assumptions

Welfare To capture the reassurance and incentive motive of stress tests, we assume that welfare is increasing in beliefs about bank health ($\frac{\partial W}{\partial \mu_i} > 0 \forall i$) and is particularly concerned with the weak bank as the risks of belief driven phenomena such as runs, higher funding costs, or problems to roll over short term debt are particularly acute for weak banks. Thus, we assume that welfare is concave. We assume that banks enter welfare symmetrically $W(\alpha, \beta) = W(\beta, \alpha)$.

An example welfare function is $W(\mu_1, \mu_2) = \lambda \min(\mu_1, \mu_2) + (1 - \lambda) \frac{\mu_1 + \mu_2}{2}$ where $\lambda \in [0, 1[$ indexes the concern for the weak bank. While fundamentals H_i do not enter welfare directly, they enter indirectly via beliefs μ_i . The designer knows that an equilibrium property of Bayesian beliefs is $\mathbb{E}[\mu_i] = \mathbb{E}[H_i]$ ¹⁵ and thus that he can only achieve high expected beliefs by increasing expected fundamentals.

Mandate The mandate is endogenously chosen by the designer. The mandate need not be identical to welfare but we restrict our attention to the same set of functions. Thus, as example we let $V(\mu_1, \mu_2) = \omega \min(\mu_1, \mu_2) + (1 - \omega) \frac{\mu_1 + \mu_2}{2}$ where $\omega \in [0, 1[$. We refer to the CB as being “tougher” than welfare if $\omega < \lambda$.

Bank’s action The bank’s action affects the distribution its health is drawn from. In line with the moral hazard literature, we assume that the improvement is according to MLRP¹⁶. Denoting the pdf resulting from low effort as $g(H)$ and from high effort as $f(H)$, this assumption can be stated as:

$$\frac{f(x_0)}{g(x_0)} \leq \frac{f(x_1)}{g(x_1)} \quad \forall \quad x_0 \leq x_1 \quad (1)$$

¹⁵This is referred to as ‘Bayes plausibility’.

¹⁶Milgrom (1981); Holmstrom (1979, 1982); Lambert (1983); Ottaviani and Sorensen (2006).

This assumption is equivalent to assuming that the conditional distribution over every subinterval can be ranked according to first-order stochastic dominance (Shaked and Shanthikumar, 2007) and therefore implies that:

$$\mathbb{E}_G[H \mid a \leq H \leq b] \leq \mathbb{E}_F[H \mid a \leq H \leq b] \quad \forall \quad a \leq b \quad (2)$$

Thus, when recipients learn that $H_i \in [a, b]$, the resulting posterior is higher if effort is believed to be high. However, as effort is not observed, interpretations are based on conjectured effort ϵ_i not on actual effort e_i . We elaborate on this issue in section 5. Functional forms we will use are $G(H) = H$; $F(H) = H^2$. We refer to this distributional assumption as (AD2).

Central bank: no commitment, information We assume that the regulator chooses the level of stress strategically after observing bank health and reports results truthfully. The assumption that the CB cannot commit is in line with CBs referring to their choice of stress scenarios as judgements¹⁷ indicating discretion, with frequent debates on how meaningful a test was¹⁸, and with the extreme experience of the 2011 European stress test.¹⁹ Our assumption that the regulator knows bank health when deciding on the message is motivated by regulators' access to private information in their role as bank supervisor.²⁰

4 Reassurance

Comparing institutional designs in terms of reassurance, we show that delegating to a softer sender can be socially beneficial. First, we show that frameworks in which more information is released result in less reassurance. Second, we show that within a hurdle rate framework, reassurance is increasing in the weight the mandate places on the weak bank. Thus, delegating to a soft sender has the benefit that he provides more reassurance than a sender with social preferences credibly could. To derive this result, we characterise the equilibrium in a hurdle rate game which is also a contribution to the strategic communication literature.

To isolate the reassurance effect of different institutional designs, this section removes the incentive motive of stress test communication by taking the bank health distribution as exogenously given. This equals the assumption that bank effort is exogenous and common knowledge. It reduces the model to a communication game. We now formally define reassurance.

Definition 3 *Institution Design \mathcal{D} is said to provide more reassurance than \mathcal{D}' if $E(\min(\mu_1, \mu_2))$ is higher in \mathcal{D} than in \mathcal{D}' .*

¹⁷“The FPC and PRC judge the stress scenario to be appropriate [...]” Bank of England, Key elements of the 2019 stress test, 5 March 2019.

¹⁸“Bank stress tests fail to tackle deflation spectre.” Financial Times, 27 October 2014.

¹⁹“The tests were long ago branded as flawed, so the results (most banks passed) were never going to serve their market-soothing purpose. The exams refused to countenance a sovereign default, even as such an event appears imminent. Investors understood that the tests had been overtaken by events, and that a ‘pass’ or ‘fail’ was largely meaningless. Instead, they dumped bank shares [...]” Financial Times, EU bank stress tests, 18 July 2011.

²⁰Moreover, stress tests as a diagnostic tool for regulators to learn about bank health existed had existed already prior to the financial crisis. The novelty of the post-crisis regime is that results are published in a specific frequency unaffected by economic events. The private information assumption allows us to focus on the communication part of stress tests.

The definition above may appear odd as it focuses purely on the weak bank and not on the average. We introduced the reassurance motive as a desire to increase the belief financial markets hold about both banks, sometimes with an added concern for the weak bank. An example functional form was $W = \lambda \min(\mu_1, \mu_2) + (1 - \lambda) \frac{\mu_1 + \mu_2}{2}$. The absence of the average term is explained by the ex-ante perspective this definition takes, i.e. the focus on expected reassurance across all possible health pairs. While for given bank fundamentals a regulator's choice of message can affect both the average ($\frac{\mu_1 + \mu_2}{2}$) and the dispersion of beliefs ($\mathbb{E}[\min(\mu_1, \mu_2)]$), the ex-ante choice of an institutional design \mathcal{D} only affects the dispersion of beliefs and not the average. This follows from a property of all Bayesian beliefs, called Bayes plausibility: beliefs must be correct on average $\mathbb{E}[\mu_i] = \mathbb{E}[H_i]$. While there are different possible dispersion measures, $\mathbb{E}[\min(\mu_1, \mu_2)]$ arises from the specific functional form we will use for welfare.²¹

With bank effort fixed, it follows that the more information is released in equilibrium, the less reassurance is provided. We now turn to a result ranking designs in terms of information released in equilibrium and thus also in terms of reassurance.

Proposition 1 *Institutional Designs \mathcal{D} can be ranked by how much reassurance they provide:*

1. *ZDF provides more reassurance than HRF which provides more than FDF.*
2. *Within HRF: reassurance is weakly increasing in the weight the mandate puts on the weak bank.*

Proof: See Appendix 9.2.

While the intuition behind the HRF result requires an understanding of the equilibrium in the communication game, which we turn to next, the ranking of ZDF and FDF is already intuitive. ZDF means zero disclosure and no updating of beliefs. Hence, there is no dispersion of posteriors and since $\mu_i = \mathbb{E}[H_i]$ reassurance is maximised. The full disclosure in FDF results in beliefs exactly reflecting fundamentals which means very low reassurance.

4.1 Hurdle Rate: Equilibrium Communication

We show that when the regulator is asked to use a hurdle rate framework, i.e. to judge banks to pass or fail relative to a common level of stress, an equilibrium exists in the resulting communication game. In this equilibrium, the regulator passes both banks if they are of similar health and passes one but fails the other if their health is very different. Financial markets thus interpret both banks passing to mean that banks are at the level of stress or slightly above, but not so far above that the regulator wanted to let the weaker bank fail the test. We show that messages where both banks pass are used more often when the mandate places more weight on the weak bank. Even if the mandate is only concerned with the average bank, the regulator in equilibrium still uses messages where both pass some of the time. This means that cases where most banks pass but financial markets are not reassured can arise in equilibrium even if the regulator places no extra weight on the weak bank.

²¹By Bayes plausibility: $\mathbb{E}[\frac{\mu_1 + \mu_2}{2}] = \mathbb{E}[H_i] \forall \mathcal{D}$ and thus $\mathbb{E}[W]$ reduces to $\mathbb{E}[\min(\mu_1, \mu_2)]$.

Solving for the equilibrium in a verifiable disclosure game where all banks are judged relative to a common level of stress (HRF) is challenging and has - to the best of our knowledge - not been done before. The difficulty comes from the regulator only having one strategic variable (s) which determines the message for both banks. This creates a trade-off in the regulator's strategic choice of s : Higher levels of stress have the benefit of increasing beliefs about those banks which continue to pass but have the cost of resulting in low beliefs about those banks who fail the stress test. Even more complexly, posteriors resulting from a messaging strategy are a function of that strategy as financial markets use all available information to infer the true state. Thus, the costs and benefits of different levels of stress depend on the strategy.

The Theorem below provides a solution to this problem, i.e. solves for the equilibrium. In this equilibrium, it is as if the regulator solved two strategic decisions sequentially: For every possible outcome pair $\{o_1, o_2\}$ and given beliefs, what is the optimal level of stress?²² Given these candidate solutions, which is the optimal outcome pair? Financial markets correspondingly interpret both the level of stress as strategic (analogous to the first question) and the outcome pair as strategic (second question). For example, in equilibrium, $\{s, p, f\}$ is interpreted to mean bank 1 has health exactly at s and bank 2 is not just below s but far below s . If banks had been close, the regulator would have sent $\{s', p, p\}$. The Theorem describes the equilibrium mathematically and the diagrams below show the equilibrium visually.

Theorem 1 *In the Hurdle Rate Framework communication game there exists a Perfect Bayesian Equilibrium (PBE). This equilibrium is characterised by two indifference frontiers $x(H_2)$ and $y(H_2)$ such that the sender plays (visualised in Figure 2a):*

$$\begin{aligned} s = H_1 & \quad \text{iff } H_1 > x(H_2) & \rightarrow \{H_1, p, f\} \\ s = \min(H_1, H_2) & \quad \text{iff } x(H_2) \geq H_1 \geq y(H_2) & \rightarrow \{\min(H_1, H_2), p, p\} \\ s = H_2 & \quad \text{iff } y(H_2) > H_1 & \rightarrow \{H_2, f, p\} \end{aligned}$$

where $x(0) = y(0) = 0$; $x(H_2) \geq y(H_2) \forall H_2$, $x(H_2) \leq 1 \forall H_2$; and $x(H_2)$ and $y(H_2)$ are both continuous and monotonically increasing ($\frac{dx(H_2)}{dH_2} > 0$, $\frac{dy(H_2)}{dH_2} > 0$).

Recipients form beliefs accordingly (visualised in Figure 2b). Denoting the cdf of H_1 as F and of H_2 as G .

$$\begin{aligned} \{s, p, f\} & \text{ is interpreted as } \mu_1 = s; \quad \mu_2 = \mathbb{E}_G[H \mid H < x^{-1}(s)] \\ \{s, f, p\} & \text{ is interpreted as } \mu_2 = s; \quad \mu_1 = \mathbb{E}_F[H \mid H < y(s)] \\ \{s, p, p\} & \text{ is interpreted as } \mu_1 = \alpha s + (1 - \alpha) \mathbb{E}_F[H \mid s \leq H \leq x(s)] \\ & \quad \mu_2 = (1 - \alpha) s + \alpha \mathbb{E}_G[H \mid s \leq H \leq y^{-1}(s)] \\ \{s, f, f\} & \text{ is interpreted as } \mu_1 = \mu_2 = 0 \forall s > 0 \end{aligned}$$

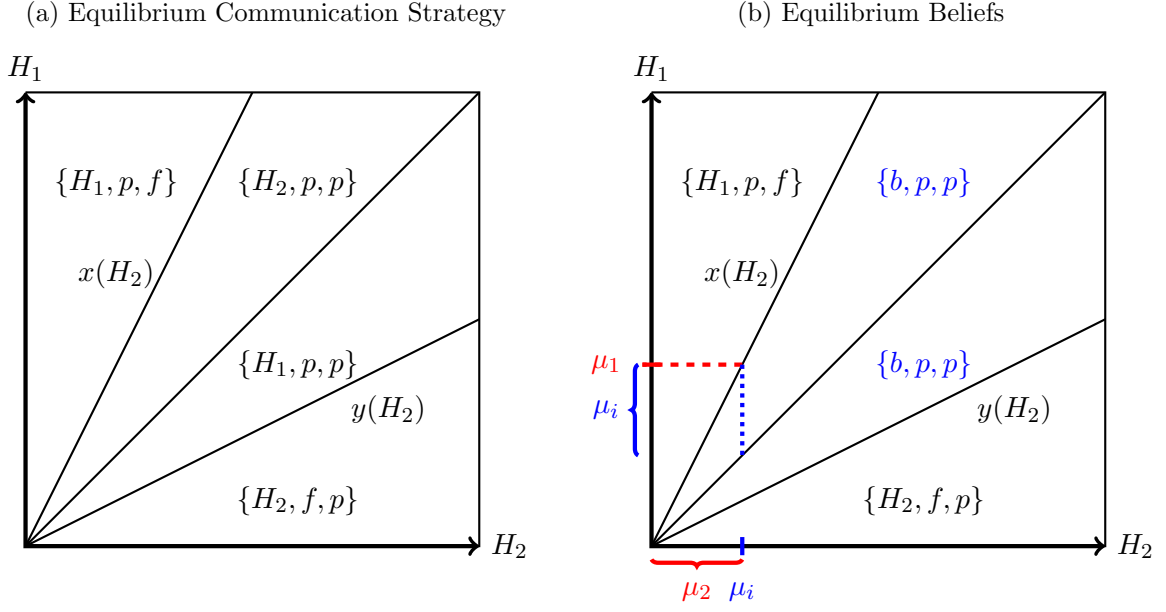
where

$$\alpha = \frac{f(s) [G(y^{-1}(s)) - G(s)]}{f(s) [G(y^{-1}(s)) - G(s)] + g(s) [F(x(s)) - F(s)]} \quad (3)$$

Proof: See Appendix 9.1.

²²Outcome pairs are $\{p, p\}$, $\{p, f\}$, $\{f, p\}$, and $\{f, f\}$.

Figure 2: Equilibrium in a Hurdle Rate Framework



Note: Example with $H_i \sim U[0, 1] \forall i = 1, 2$; $V(\mu_1, \mu_2) = \frac{\mu_1 + \mu_2}{2}$. Then $x(H_2) = 2H_2$; $y(H_2) = \frac{1}{2}H_2$.

The sender's actions are depicted in Figure 2a, the receivers interpretation is depicted in Figure 2b.

The two indifference frontiers $x(H_2)$ and $y(H_2)$ are defined by the following set of indifference equations where $\mu_i | \{s, o, o\}$ denotes the posterior formed upon receiving $\{s, o, o\}$. On $x(H_2)$:

$$V(\mu_1 | \{H_1, p, f\}, \mu_2 | \{H_1, p, f\}) = V(\mu_1 | \{H_2, p, p\}, \mu_2 | \{H_2, p, p\}) \quad (4)$$

On $y(H_2)$:

$$V(\mu_1 | \{H_1, p, p\}, \mu_2 | \{H_1, p, p\}) = V(\mu_1 | \{H_2, f, p\}, \mu_2 | \{H_2, f, p\}) \quad (5)$$

When bank health is identically distributed, the indifference frontiers are symmetric, i.e. $y(H_2) = x^{-1}(H_2)$ and $\alpha = 0.5$.

Example: Let $H_i \sim U[0, 1] \forall i = 1, 2$ and let $V(\mu_1, \mu_2) = \omega \min(\mu_1, \mu_2) + (1 - \omega) \frac{\mu_1 + \mu_2}{2}$. Then the equilibrium is characterised by:

$$x(H_2) = \frac{2 - \omega}{1 - 2\omega} H_2 \quad ; \quad y(H_2) = x^{-1}(H_2) \quad (6)$$

This holds for $\omega < 0.5$. For $\omega \geq 0.5$ the regulator sends $\{s, p, p\}$ messages in all cases apart from those exactly on the axis, i.e. when $\min(H_1, H_2) = 0$. In these cases he sets $s = \max(H_1, H_2)$ which results in one bank passing and one bank failing the stress test.

The equilibrium described in Theorem 1 is not a unique weak PBE, but it is the only weak

PBE which satisfies the following notion of monotonicity.

Definition 4 *An equilibrium is said to satisfy monotonicity if beliefs satisfy the following condition:*

$$\mu_i | \{s, \bar{o}_i, \bar{o}_j\} > \mu_i | \{s', \bar{o}_i, \bar{o}_j\} \forall s > s', \forall i = 1, 2, \forall \bar{o}_i = p, f, \forall \bar{o}_j = p, f \quad (7)$$

This means that given any outcome of the test, the resulting belief about this bank is higher if the test was more severe, i.e. more severe stress increases beliefs both condition on passing and conditional on failing. This seems close to commentary of stress tests where passing weak tests is seen as uninformative while passing tough tests is viewed as a strong signal. While this condition imposes a theoretical restriction, it seems plausible in the context of real world bank stress tests. For the remainder of the paper, in a HRF we focus on the equilibrium as described in Theorem 1.

To build an understanding of how delegation affects the equilibrium, the following proposition studies a comparative static in the sender's mandate.

Proposition 2 *The more weight the mandate places on the weak bank, the steeper the $x(H_2)$ -frontier is in equilibrium, i.e. the larger the set of states where $\{H_2, p, p\}$ is sent relative to the states where $\{H_1, p, f\}$ is sent. Correspondingly, the $y(H_2)$ -frontier is flatter.*

Proof: See Appendix 9.3.

Consider a sender with mandate \mathcal{M} who at a specific (H_1, H_2) on his $x(H_2)$ -indifference frontier is indifferent between $\{H_2, p, p\}$, which implies no dispersion of posteriors ($\mu_1 = \mu_2$) and $\{H_1, p, f\}$, which implies dispersion of posteriors ($\mu_1 > \mu_2$). Then at the same (H_1, H_2) a regulator whose mandate \mathcal{M}' places more weight on the weak bank, strictly prefers $\{H_2, p, p\}$. Therefore, the equilibrium under \mathcal{M}' exhibits a steeper $x(H_2)$ and a flatter $y(H_2)$ than under \mathcal{M} .

Interestingly, even when the mandate places no extra weight on the weak bank and is only concerned with the average belief, $\{s, p, p\}$ messages are still used in equilibrium.²³ This arises because strong messages where both pass, e.g. $\{0.9, p, p\}$ are only feasible in a set of states with high H_1 and high H_2 , guaranteeing high $\frac{\mu_1 + \mu_2}{2}$, while strong messages where only one bank passes, e.g. $\{0.95, p, f\}$ are also feasible when one bank is extremely weak. Thus, very strong interpretations of $\{0.95, p, f\}$ cannot be an equilibrium, as then the regulator would want to send $\{0.95, p, f\}$ also at $H_1 = 0.95, H_2 = 0$ and similarly low states, undermining the high interpretation. Thus, as health state pairs with low average health cannot feasibly pool with the high average pairs that could send $\{0.9, p, p\}$, $\{0.9, p, p\}$ will in equilibrium be used even if the mandate places no extra weight on the weak bank.

The sender almost always sends $\{s, p, p\}$ messages even for mandates that place some weight on average health. An extreme equilibrium in HRF is when the sender sends $\{s, p, p\}$ in all cases

²³This can be seen in the example used to construct Figures 2a and 2b. When $H_i \sim U[0, 1] \forall i = 1, 2$; $V(\mu_1, \mu_2) = \frac{\mu_1 + \mu_2}{2}$, then $x(H_2) = 2H_2$; $y(H_2) = \frac{1}{2}H_2$. Thus, a sender who places no extra weight on the weak bank sends $\{s, p, p\}$ in 50% of the states.

apart from when $\min(H_1, H_2) = 0$. In these cases, he chooses $s = \max(H_1, H_2)$. This messaging pattern arises for a mandate which is purely concerned with the weak bank, but it also arises for mandates that place some weight on the average. In the example, $\{s, p, p\}$ is almost always sent for all $\omega \geq 0.5$ which corresponds to 0.75 weight on the weak and 0.25 weight on the strong bank.

4.2 Hurdle Rate: Reassurance

While the previous subsection characterised what messages are sent and beliefs are formed for a given health pair, this subsection returns the focus to the ex-ante measure of expected reassurance. We show that reassurance provided in a HRF lies between reassurance in ZDF and FDF and is increasing in the softness of the mandate. This provides a rationale for delegation: Delegating stress tests to a softer sender provides more reassurance which increases welfare.

There are two challenges to characterising reassurance in a HRF. First, comparing reassurance in a HRF to ZDF and FDF requires computing the level of reassurance, i.e. $E(\min(\mu_1, \mu_2))$. The numerous thresholds in the HRF communication equilibrium complicate this. Second, comparing reassurance for different mandates within a HRF is not trivial as there are two opposing forces: On the one hand, a softer sender results in steeper $x(H_2)$, i.e. sends $\{s, p, p\}$ more often, which increases reassurance since $\{s, p, p\}$ creates less dispersion in posteriors than $\{s, p, f\}$. On the other hand, a steeper $x(H_2)$ frontier means that a given $\{s, p, f\}$ message results in more dispersion when it is sent.

Mathematically, reassurance in a HRF is:

$$\mathbb{E}[\min(\mu_1, \mu_2)] = \int_0^1 \int_0^1 \min(\mu_1, \mu_2) f(H_1) dH_1 g(H_2) dH_2 \quad (8)$$

As beliefs are based on the sender's strategy which is characterised by thresholds, this becomes:

$$\begin{aligned}
\mathbb{E}[\min(\mu_1, \mu_2)] &= \int_0^{x^{-1}(1)} \left[\int_0^{y(H_2)} \mathbb{E}(H \mid H < y(H_2),) f(H_1) dH_1 \right. \\
&\quad + \int_{y(H_2)}^{H_2} \frac{1}{2} H_1 + \frac{1}{2} \mathbb{E}(H \mid H_1 < H < x(H_1),) f(H_1) dH_1 \\
&\quad + \int_{H_2}^{x(H_2)} \frac{1}{2} H_2 + \frac{1}{2} \mathbb{E}(H \mid H_2 < H < x(H_2),) f(H_1) dH_1 \\
&\quad \left. + \int_{x(H_2)}^1 \mathbb{E}(H \mid H < x^{-1}(H_1),) f(H_1) dH_1 \right] g(H_2) dH_2 \tag{9} \\
&+ \int_{x^{-1}(1)}^1 \left[\int_0^{y(H_2)} \mathbb{E}(H \mid H < y(H_2),) f(H_1) dH_1 \right. \\
&\quad + \int_{y(H_2)}^{H_2} \frac{1}{2} H_1 + \frac{1}{2} \mathbb{E}(H \mid H_1 < H < 1) f(H_1) dH_1 \\
&\quad \left. + \int_{H_2}^1 \frac{1}{2} H_2 + \frac{1}{2} \mathbb{E}(H \mid H_2 < H < 1,) f(H_1) dH_1 \right] g(H_2) dH_2
\end{aligned}$$

We find that reassurance is increasing in the softness of the sender, i.e. the more frequent use of $\{s, p, p\}$ outweighs the more dispersed interpretation of $\{s, p, f\}$. We also find that reassurance in a HRF lies between ZDF and FDF for all possible mandates. These results, which form proposition 1 above, are shown in Appendix 9.2 based on assumption (AD2). This result has implications for institutional design.

Institutional Design: Benefit of Delegation 1 *Delegating stress tests to a softer sender can be optimal, as it has the benefit of making higher reassurance credible.*

More formally, suppose the bank health distribution is exogenous and the institutional designer D is restricted to choosing a mandate within HRF, i.e. ZDF, FDF are not available. Then every designer D who places a strictly positive extra weight on the weak bank (e.g. $\lambda > 0$), optimally delegates stress tests to a regulator who implements the softest possible communication rule, i.e. who almost always sends $\{\min(H_1, H_2), p, p\}$.²⁴

This benefit of delegation arises from a commitment problem. Ex-ante, before learning bank health, any regulator with some concern about the weak bank wants recipients to believe that he will always send $\{\min(H_1, H_2), p, p\}$ as this maximises reassurance.²⁵ Once the regulator learns bank health, there are cases where he is tempted to deviate and increase average posteriors at the expense of the weak bank, i.e. at the expense of creating dispersion by sending $\{H_1, p, f\}$. The larger the regulator's weight on the weak bank, the larger the set of states where sending $\{\min(H_1, H_2), p, p\}$ is actually credible and thus the larger reassurance is.

²⁴This is not necessarily the same as delegating to the softest possible sender. As argued in the context of proposition 2, there exists a range of mandates which result in almost always sending $\{\min(H_1, H_2), p, p\}$. In our example, $\omega \in [0.5, 1]$. The designer is indifferent between these mandates as they all imply the same level of reassurance.

²⁵This is the equilibrium in a HRF which achieves the highest reassurance possible. Reassurance would be even higher if the sender could commit to always sending $\{0, p, p\}$ but this is analogous to ZDF, which we have ruled out to focus on HRF.

To understand why it is not credible for every sender to always send $\{\min(H_1, H_2), p, p\}$ even though this would maximise reassurance, consider the example message $\{0.2, p, p\}$. While if $H_1 = 0.2, H_2 = 0.2$ this message will be sent by all regulators regardless of their preference, this is not true if $H_1 = 1, H_2 = 0.2$. Then, sending the feasible alternative $\{1, p, f\}$ achieves an average posterior above sending $\{0.2, p, p\}$. Even under the worst possible interpretation of $\{1, p, f\}$, i.e. $\mu_1 = 1, \mu_2 = 0$, we have $\frac{\mu_1 + \mu_2}{2} = 0.5$ which exceeds $\frac{\mu_1 + \mu_2}{2} = 0.4$ from $\{0.2, p, p\}$.²⁶ Hence, at states with disperse fundamentals only senders with a strong concern for the weak bank actually send $\{\min(H_1, H_2), p, p\}$, thus accepting lower average posteriors but achieving a higher minimum belief than under the alternative of deviating to $\{H_1, p, f\}$.

Since the social planner D cannot commit to and is not credible to provide reassurance at all states, i.e. send $\{\min(H_1, H_2), p, p\}$, the social planner can benefit from delegating stress tests to a softer sender. This results in a steeper $x(H_2)$ frontier and thus in more reassurance.

While this section showed that delegation to a softer sender can be beneficial as it increases reassurance, the next section shows that delegation to a tougher sender can be beneficial as it increases incentives.

5 Incentives

Comparing institutional designs in terms of incentives, we show that delegating to a tougher sender can be socially beneficial as it makes the threat of letting weak banks fail the stress test more credible. This incentivizes banks to take prudent actions. To derive this, we first show that communication creates incentives for banks to take prudent actions. Then, we rank communication frameworks according to incentives created, finding that FDF implies stronger incentives than HRF which implies more than ZDF. Within HRF, tougher mandates create stronger incentives. Then, we characterise equilibria in the game with endogenous effort and find that stronger incentives ensure that the equilibrium with high effort occurs even for higher effort costs. Thus, delegating stress tests to a tougher sender can be beneficial as it makes the threat of letting weak banks fail the stress test more credible which incentivises banks to take prudent actions.

5.1 Costs and Benefits of Bank Effort Choice

When deciding whether to exert high or low effort, a bank weighs up the certain cost of effort $C(h) - C(l)$, against the expected benefit of higher beliefs about its health. While effort improves the distribution of bank fundamentals according to MLRP, how this improvement translates into a change in expected beliefs depends on the communication framework. We refer to a communication framework as “generating incentives” if it results in expected beliefs and not just expected fundamentals responding to effort choice. Denoting the vector of low conjectured efforts $\mathbf{l} = (l_1, l_2)$ and $\mathbf{h} = (h_1, h_2)$ correspondingly:

²⁶Numbers are calculated assuming a uniform distribution.

Definition 5 *A communication framework \mathcal{C} generates effort incentives if and only if:*

$$\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] - \mathbb{E}_G[H] > 0 \quad (10)$$

since by Bayes plausibility of beliefs $\mathbb{E}_G[H] = \mathbb{E}[H_1 \mid l_1] = \mathbb{E}[\mu_1(\mathbf{l}) \mid l_1, l_2]$.

Incentives capture how expected posteriors change if a bank deviates in its effort choice from a candidate equilibrium. In this subsection, we study this deviation in which conjectured effort is held fixed. In the next subsection, we turn to the fixed point problem, i.e. characterise actual equilibria where such deviations do not occur.

While in equilibrium, expected posteriors are correct ($\mathbb{E}_F[H] = \mathbb{E}[\mu(\mathbf{h}) \mid h_1, h_2]$, see also the discussion of Bayes plausibility in section 4), this need not be true in a deviation from the equilibrium. Thus, the benefit to deviating is not always $\mathbb{E}_F[H] - \mathbb{E}_G[H]$, but depends on the communication framework. We now turn to ranking frameworks by the benefit of deviating by increasing effort, i.e. by incentives.²⁷

Proposition 3 *Ranking of communication frameworks in terms of incentives generated:*

1. *FDF provides more incentives than HRF which provides more than ZDF*
2. *Within HRF: the lower the sender's concern for the weak bank, the more incentives are generated.*

Proof: See Appendix 9.4.

While calculating incentives in a HRF poses several challenges, the results for the extreme frameworks are clear and help build the intuition for incentives in a HRF. ZDF provides zero incentives. Since no information is released, financial markets can never update their prior. The prior is based purely on conjectured effort. Hence, in ZDF beliefs are unaffected by actual effort which creates no incentive for banks to invest in their health. FDF provides strong incentives. Since true bank health is always perfectly revealed, beliefs track fundamentals one-for-one. We refer to this as full incentives.

Incentives in a HRF are difficult to understand as several partially offsetting forces interact and mathematically complex to express due to the thresholds in the communication equilibrium. The conceptual difficulty arises because of the divergence of actual and conjectured effort. While a bank's effort choice affects effort and thus the frequency of messages occurring, conjectured effort and thus the interpretation of a given message are unaffected.

Fixed conjectures mute, keep unchanged, or magnify the effect of changed fundamentals. As a tough experiment, suppose that both banks are conjectured to play low effort and let H_2 be

²⁷The definition of incentives considers deviations from a low effort equilibrium. Analogously, considering deviations from a high effort equilibrium, incentives are captured by the effect of reducing effort: $\mathbb{E}_h[H] - \mathbb{E}[\mu(\mathbf{h}) \mid l_1, h_2] > 0$. The ordering of communication frameworks by how strong incentives they create is the same in both cases. The magnitudes can differ. This affects the equilibrium characterisation in the next subsection, but not the ordering which we focus on in this subsection.

some realisation. Ex-post, how much will bank 1 benefit from having put in higher effort? The effect can be muted, i.e. less than on fundamentals. E.g. if effort leads to an improvement in H_1 from an H_1 that led to $\{H_2, f, p\}$ to a still low H_1 that also leads to $\{H_2, f, p\}$, then beliefs are unaffected. The effect can be in line with fundamentals. E.g. if initially $\{H_1, p, f\}$ is sent and effort results in a stronger $\{H'_1, p, f\}$ with $H'_1 > H_1$, then the belief about bank health has shifted in line with fundamentals since $\mu_i = H_i$ for this message. The effect can be magnified. If higher effort induces a switch of message from $\{H_2, f, p\}$ to $\{H_1, p, p\}$, then μ_1 has increased more strongly than H_1 .

We find that incentives implied by a HRF lie between those of ZDF and FDF. Moreover, the tougher the sender, i.e. the less weight his mandate puts on the weak bank and thus the more fail grades are given out in equilibrium, the stronger the incentives. These results, which form Proposition 3, are proved in Appendix 9.4, making use of the following expression for posteriors in a HRF when effort is changed away from conjectured effort:

$$\begin{aligned}
\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] &= \int_0^{x^{-1}(1)} \left[\int_0^{y(H_2)} \mathbb{E}_G(H \mid H < y(H_2)) f(H_1) dH_1 \right. \\
&\quad + \int_{y(H_2)}^{H_2} \frac{1}{2} H_1 + \frac{1}{2} \mathbb{E}_G(H \mid H_1 < H < x(H_1)) f(H_1) dH_1 \\
&\quad + \int_{H_2}^{x(H_2)} \frac{1}{2} H_2 + \frac{1}{2} \mathbb{E}_G(H \mid H_2 < H < x(H_2)) f(H_1) dH_1 \\
&\quad \left. + \int_{x(H_2)}^1 H_1 f(H_1) dH_1 \right] g(H_2) dH_2 \\
&+ \int_{x^{-1}(1)}^1 \left[\int_0^{y(H_2)} \mathbb{E}_G(H \mid H < y(H_2)) f(H_1) dH_1 \right. \\
&\quad + \int_{y(H_2)}^{H_2} \frac{1}{2} H_1 + \frac{1}{2} \mathbb{E}_G(H \mid H_1 < H < 1) f(H_1) dH_1 \\
&\quad \left. + \int_{H_2}^1 \frac{1}{2} H_2 + \frac{1}{2} \mathbb{E}_G(H \mid H_2 < H < 1) f(H_1) dH_1 \right] g(H_2) dH_2
\end{aligned} \tag{11}$$

Even the softest possible sender in a HRF creates strictly more incentives than ZDF.²⁸ The reason is that even if only $\{s, p, p\}$ messages are sent, then the level of s is informative and creates incentives. In ZDF, there is absolutely no information and thus no incentives.

5.2 Equilibrium with Bank Effort Choice

A benefit of delegating stress tests to a tougher sender is that this makes giving out fail marks more credible and therefore results in stronger incentives. We now turn to the equilibrium and show that stronger incentives ensure that equilibria with high effort occur even for higher effort costs. Welfare can be higher as a result.

Restricting our attention to pure strategy symmetric equilibria and to the case where effort

²⁸For a proof of this limit result see Appendix 9.4, specifically Sublemma 9.4.3.

is socially beneficial, i.e. costs lie below the change in expected fundamentals $\mathbb{E}_F[H] - \mathbb{E}_G[H] \geq C(h) - C(l)$ and where costs are strictly positive $C(h) - C(l) > 0$ we show the following proposition:

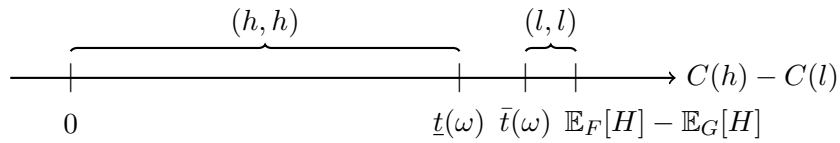
Proposition 4

1. In ZDF, there always exists a unique pure strategy equilibrium. In this equilibrium both banks choose low effort.
2. In FDF, there always exists a unique pure strategy equilibrium. In this equilibrium both banks choose high effort.
3. In HRF, there exist two thresholds which are functions of the sender's mandate $\underline{t}(\mathcal{M})$, $\bar{t}(\mathcal{M})$.
 - (a) For all $C(h) - C(l) \leq \underline{t}(\mathcal{M})$, there exists a unique pure strategy equilibrium. In this equilibrium, both banks choose high effort.
 - (b) For all $C(h) - C(l) \geq \bar{t}(\mathcal{M})$, there exists a unique pure strategy equilibrium. In this equilibrium, both banks choose low effort.
 - (c) For all $\underline{t}(\mathcal{M}) < C(h) - C(l) < \bar{t}(\mathcal{M})$, there does not exist a pure strategy equilibrium.

Proof: See Appendix 9.5.

Figure 3 illustrates the existence results for a HRF. The thresholds $\underline{t}(\mathcal{M})$ and $\bar{t}(\mathcal{M})$ are both increasing in the toughness of the mandate, i.e. are higher if the mandate places less concern on the weak bank. For a given mandate \mathcal{M} : $\bar{t}(\mathcal{M}) = \mathbb{E}[\mu_1(l) \mid h_1, l_2] - \mathbb{E}_G[H]$ and $\underline{t}(\mathcal{M}) = \mathbb{E}_F[H] - \mathbb{E}[\mu_1(h) \mid l_1, h_2]$.

Figure 3: Equilibrium in a Hurdle Rate Framework



Note: Thresholds depicted for the case where $G(H) = H, F(H) = H^2, V(\mu_i, \mu_2) = \frac{\mu_1 + \mu_2}{2}$.

The extreme frameworks are again useful to build the intuition which we can later apply to the HRF. As argued in the previous subsection, ZDF provides no incentives, i.e. banks have zero benefit of effort. As costs of effort are strictly positive, banks never find it optimal to exert effort which results in the low effort equilibrium. FDF provides full incentives. Since we assume that effort is socially beneficial, full incentives will always induce effort which results in the high effort equilibrium.

The intuition for the equilibrium in HRF is best understood in two steps. First, we explain why the equilibrium has a threshold structure. Then, we investigate why there are two thresholds and not one. A given mandate implies a strength of incentives $\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] - \mathbb{E}_G[H]$. If costs exceed incentives, banks optimally choose low effort and a low effort equilibrium exists. For lower costs banks would prefer to deviate to high effort. Hence, a mandate implies a certain fixed benefit of effort and then the equilibrium is determined by whether costs are above or below that benefit. The tougher the mandate is, the more incentives the regulator generates, and thus the higher the threshold is below with a high effort equilibrium exists.

The intuition for why there are two thresholds instead of one, i.e. why we do not have $\underline{t}(\mathcal{M}) = \bar{t}(\mathcal{M})$, does not follow from an understanding of the extreme frameworks. Rather, this is driven by the importance of conjectures.²⁹

Based on the equilibrium effect of incentives, we can now formulate the benefit of delegating to a tougher sender.

Institutional Design: Benefit of Delegation 2 *Delegating stress tests to a tougher sender increases incentives, as the threat of giving out fail marks is more credible and can thus shift the equilibrium from a low to a high effort equilibrium.*

A soft sender cannot credibly implement a tough communication rule. If financial markets believed the sender that he intends to implement a rule $\tilde{x}(H_2)$ which is tougher, i.e. flatter, than his equilibrium rule $x(H_2)$, then there exist states where the sender should send $\{H_1, p, f\}$ if he follows $\tilde{x}(H_2)$, but actually prefers to send $\{H_2, p, p\}$ as he is concerned about the weak bank. Thus, a soft sender cannot credibly implement a tough communication rule as his threat to give out fail marks is not credible. He prefers to give out pass marks instead.

This section established that a benefit of delegating to a tougher sender exists as a tougher sender can induce effort and thus achieve a higher average posterior in equilibrium. This comes at the cost of less reassurance, which we showed in the previous section as a benefit of delegating to a softer sender. We now turn to the case where *both* these benefits and costs are considered, characterise when which effect dominates, and show that delegation can be optimal in either direction depending on how costly effort is.

6 Central Bank Design

Taking both reassurance and incentives into account, we show that the optimal institutional design depends on the cost of effort but need not depend on the strength of society's reassurance motive. For low cost of effort, the optimal design is a hurdle rate framework with the softest possible mandate which generates just enough incentives to induce high effort. When costs of effort are so high that no hurdle rate framework can provide sufficient incentives to induce effort, the optimal design is a full disclosure framework. This gives rise to our delegation result: As

²⁹We conjecture that for $\underline{t}(\mathcal{M}) < C(h) - C(l) < \bar{t}(\mathcal{M})$ a mixed strategy equilibrium exists. This is work in progress.

the optimal mandate in a HRF does not depend on social preferences, there are cases where delegating to a tougher sender is optimal while in others delegating to a softer sender is optimal.

6.1 The Optimal Communication Framework

This subsection characterises the optimal institutional design and focuses on the intuition for the delegation result and particularly on why delegating to a softer sender can increase welfare.

We denote the mandate which places no extra weight on the weak bank as $\mathcal{M}(0)$, e.g. $\omega = 0$. Then, the following proposition arises.

Proposition 5 (AD2):

1. When $C(h) - C(l) \leq \underline{t}(\mathcal{M}(0))$, i.e. costs of effort are sufficiently low such that there exists a mandate in a HRF which induces high effort in equilibrium, then the optimal institutional design is a HRF with the softest possible mandate which just achieves high effort in equilibrium, i.e.

$$\{HRF, \tilde{\mathcal{M}}\} \text{ where } \tilde{\mathcal{M}} \text{ is defined by } \underline{t}(\tilde{\mathcal{M}}) = C(h) - C(l).$$

2. When $C(h) - C(l) > \underline{t}(\mathcal{M}(0))$, i.e. costs of effort are so high that there does not exist a mandate in a HRF which induces high effort in equilibrium, then the optimal institutional design is FDF.

Proof: See Appendix 9.6.

To understand this result, it is useful to first compare ZDF and FDF and in a second step ask when a HRF improves on the socially preferred extreme framework. Since for the distributions in (AD2) $\mathbb{E}_h[\min(H_1, H_2)] > \mathbb{E}_l[H_1]$, a designer prefers FDF to ZDF regardless of how much weight his mandate places on the weak bank. The benefit of effort on the bank health distribution is so large that even expected reassurance under high effort exceeds the expected fundamental under low effort. Can HRF achieve even higher expected welfare than FDF? If there exist HRF mandates which achieve high effort, these mandates result in even higher expected welfare than FDF as they achieve the benefits of generating incentives and additionally have the benefit of providing reassurance. When costs of effort are low, such HRF mandates exist and the optimal mandate is then the one that provides as much reassurance as possible while still inducing high effort. When costs of effort are high, no HRF mandate can provide enough incentives to induce effort. Then, FDF is optimal.³⁰ This has implications for optimal institution design.

Institutional Design: Benefit of Delegation 3 *In a HRF, the optimal mandate depends on the cost of effort, but does not depend on social preferences. Thus,*

1. *there are cases where it is optimal to delegate to a tougher sender than welfare.*

³⁰A generalisation to also allow $\mathbb{E}_h[\min(H_1, H_2)] \leq \mathbb{E}_l[H_1]$ yields similar but different results and is currently work in progress.

2. *there are cases where it is optimal to delegate to a softer sender than welfare.*

The first part arises because delegation to a tough sender is a partial substitute for committing to being tough and thus reduces the time inconsistency problem. A tougher sender is more credible to let banks fail the test, and thus give out more fail marks in equilibrium which creates stronger incentives.

The second part is more surprising. We first show that stress test can be excessively tough, i.e. incentives can be excessively strong, and then explain why a sender cannot commit to acting like a soft sender. Decreasing effects of incentives generate the possibility of excessively fierce stress tests. In our binary effort setting the result is stark. Once incentives result in the high effort equilibrium, further incentives have no benefit but reduce reassurance and thus harm expected welfare. This intuition applies more generally: Society values not the incentives as such, but only the effect they ultimately have on the distribution of fundamentals. If, for example, the cost of effort is convex, or the effect of effort on the bank health distribution decreases at higher levels of effort, then the cost of incentives (reduced reassurance) means that full incentives (FDF) are not optimal. Instead, incentives should be just strong enough to reach high effort and subject to that provide as much reassurance as possible. Why can a tough sender not act like a soft sender? As argued in the context of section 4.2 on reassurance, a sender who is tough would prefer to deviate from soft messaging equilibria, i.e. prefers $\{H_1, p, f\}$ messages in some cases where a softer sender would prefer $\{H_2, p, p\}$. Thus, acting softer is also not credible.

This institutional design result implies that the reassurance based rationale for delegating to a softer sender (see section 4) and the incentive based rationale for delegating to a tougher sender (see section 5), need not cancel out. The more costly effort is, the tougher the optimal sender is. For high costs, FDF is preferred to any HRF.

7 Informational Contagion

We show that in a HRF, stress tests become an informational contagion channel, i.e. the health of one bank affects beliefs about the health of the other bank. Changes in one bank's fundamental affect what message the regulator sends and thus which information financial markets receive. This in turn affects the beliefs financial markets hold, not just on the bank itself, but also on the other bank. Informational contagion can take different forms: There are cases where a deterioration in the fundamental health of one bank results in lower beliefs about the other bank. More surprisingly, there are also cases where a deterioration in the health of one bank *increases* the belief about the other bank. As a HRF is the socially optimal design for low costs of effort, this implies that contagion can be a feature of a socially optimal design and thus need not reduce welfare.

7.1 Contagion Patterns

A deterioration in the health of one bank *decreases* the belief about the health of another bank when the regulator responds by lowering the level of stress. Compare (H_1, H_2) close to but above the 45-degree line³¹ where $\{H_2, p, p\}$ is sent, to a (H_1, H'_2) with $H'_2 < H_2$. There are (H_1, H'_2) at which the regulator sends $\{H'_2, p, p\}$.³² In this case, lowering the level of stress has allowed the regulator to continue to send a pass message about the weak bank instead of revealing its weakness, but has the cost of resulting in H_1 passing a weaker stress test. As a result, the beliefs about both banks are lower.

A deterioration in the health of one bank *improves* the belief about the health of another bank when the regulator responds by changing the type of message, i.e. by increasing the level of stress. For example, suppose that initially H_1 and H_2 are far apart, i.e. the health state pair is close to but below the $x(H_2)$ -indifference frontier. Then already a small fall in H_2 to H'_2 results in the regulator switching message from $\{H_2, p, p\}$ to $\{H_1, p, f\}$. The regulator has increased the level of stress. This amplifies the effect on H_2 , i.e. beliefs μ_2 fall more strongly than fundamentals, but results in higher beliefs for the bank that continues to pass as it now passes a tougher stress test.

7.2 Contagion in the Optimal Design

First, we show that contagion is not socially undesirable but instead is a feature of the socially optimal design. Then, we show that the optimal design exhibits contagion for a large set of parameters.

While contagion effects are often viewed as an undesirable feature of the financial system in our model stress test becoming an information contagion channel is a feature of the socially optimal institution design. This positive interpretation of contagion as being part of a desirable design is driven by the no-commitment assumption. In our model, which assumes that the regulator cannot commit, a HRF is the unique framework which combines intermediate levels of incentives and reassurance. This makes HRF the optimal framework for many cost parameters. HRF achieves this intermediate combination by creating a trade-off for the regulator between supporting both banks or just one. Hence, the same trade-off that results in HRF combining some incentives and some reassurance also makes HRF have contagion comparative statics. To see this more clearly, suppose a regulator in the no commitment case had two hurdles available. This would remove contagion, but also no longer achieve a combination of incentives and insurance. It is in equilibrium equivalent to FDF, thus achieves full incentives but not reassurance. Suppose a regulator can commit. Then he could combine partial incentives and partial reassurance without causing contagion, e.g. by choosing $s = 0.5$ for all health realisations. Hence, the communication framework which creates contagion is the socially optimal design because the mechanism that creates the contagion comparative static is the same mechanism which allows the designer ex-ante to combine intermediate levels of both reassurance and incentives.

³¹ $H_1 > H_2; H_1 - H_2 > 0$ but small.

³²This is the result for small deteriorations of H_2 . How small they can be depends on the sender's mandate, i.e. on how often $\{s, p, p\}$ is sent in equilibrium.

Contagion arises in a HRF for all sender mandates. Contagion arises as long as both $\{s, p, p\}$ and $\{s, p, f\}$ messages are sent in equilibrium. We showed in section 4 that this is the case for all sender mandates including for a sender who places no extra weight on the weak bank. Thus, whenever HRF is the socially optimal design, i.e. for all $C(h) - C(l) \leq \underline{t}(\mathcal{M}(0))$, contagion is a feature of this design.

8 Conclusion

We showed that central banks face a time inconsistency problem when publishing bank stress test results. Before a stress test, they want to *appear* tough as the threat of letting banks fail the stress test incentivizes prudent behaviour. After the stress test, they want to *act* soft by releasing only partial information in order to reassure financial markets about bank health.

We characterised an institutional design solution to this commitment problem: a social planner specifies the central bank’s mandate and sets the framework within which the central bank communicates. We find that a hurdle rate framework, where all banks are judged to pass or fail relative to a common threshold, is optimal in many settings as it generates intermediate levels of both incentives and reassurance.

In a hurdle rate framework, *strategic delegation* provides additional benefits. This means the planner maximises ex-ante welfare by setting the central bank’s mandate to differ from ex-post welfare maximisation. The optimal mandate can differ in either direction. If taking prudent actions has high costs for banks, the planner optimally specifies the CB’s mandate to be “tougher” than welfare maximisation, so that the CB is less concerned about supporting weak banks than society. This makes it credible that the CB will reveal bank weakness by letting weak banks fail the stress test and thus generates strong incentives. If the costs of prudent actions are low, the optimal mandate is “softer” than welfare maximisation, so that the CB is very concerned about weak banks. This achieves more reassurance while still generating sufficient incentives to induce the prudent action.

Informational contagion can be a feature of an optimal design in the presence of a time-inconsistency problem and thus need not reduce welfare. This arises because, in a hurdle rate framework in some cases, the central bank’s optimal response to a deterioration in the health of one bank is to lower the severity of stress so that all banks continue to pass. For all banks, the resulting pass results are interpreted less favourably than at the initial higher severity of stress. Thus, a change in one bank’s health leads the CB to send a different message, which in turn affects beliefs financial markets hold about all banks. More surprisingly, there are also cases where a deterioration of one bank’s health improves the beliefs about other banks. This is the case when the CB raises the level of stress. As a result, the weak bank fails the test and beliefs about its health deteriorate. However, beliefs about other banks improve as they pass a tougher test.

These results lead to a reinterpretation of past stress tests: While stress tests by the EBA or

its predecessor (the CEBS) were repeatedly criticised for being too soft,³³ which leads some to question the regulator's competence, our model suggests that these soft stress tests may have been the optimal response of a regulator facing an unhealthy banking system and subject to a time inconsistency problem. Importantly, when the banking system is unhealthy, low levels of stress are optimal for all mandates, including "tough" mandates which focus on the average health in the banking system and place no extra weight on supporting weak banks.

This paper adds a new dimension to the debate on stress test communication by highlighting that in addition to the communication framework also the central bank's mandate affects welfare. Current central bank designs do not specify what objective the central bank should pursue when communicating stress test results. Absent an explicit mandate, central banks are likely to maximise ex-post welfare. Thus, our paper suggests that unexploited welfare gains exist: Central banks should be given an explicit mandate which differs from ex-post welfare maximisation as this difference generates gains from strategic delegation.

In practice, the optimal mandate also depends on whether the authority conducting the stress test is simultaneously the bank supervisor or whether the two tasks are carried out by different authorities. If one authority carries out both tasks, this creates implicit incentives, i.e. this authority would have additional reasons to avoid giving out fail grades as this would reflect negatively on its past performance in the role as supervisor. The authority would therefore act "softer" than implied by its mandate. Two remedies are possible. Stress testing can be assigned to an independent authority which is not the supervisor.³⁴ In that case, our results on the optimal mandate apply. If stress testing and supervision are to be carried out by one institution,³⁵ the mandate should be "tougher" than implied by our results such that the additional toughness of the mandate offsets the softness created by implicit incentives. Future research should further explore the interplay of explicit and implicit incentives in central bank communication.

³³"Which part of 'stress test' do the eurozone's policy makers not understand? That so many European banks passed the annual exams in July yet still had their shares trashed by investors says it all: the pass mark was too low and the questions were too narrow." Financial Times, Lex, European stress tests: a grim backdrop, 6 October 2011.

³⁴This is the case in the Euro Area where the EBA is responsible for stress tests while the SSM supervises banks. Our model suggests that this institutional separation has the benefit of creating stronger incentives for banks to behave prudently than stress tests run by the SSM would achieve. This is not an argument against data sharing or cooperation. Rather, our model suggests that it is beneficial if the EBA interprets its role in stress testing as an independent check on the SSM. In particular, it is important that the EBA is responsible for the severity of the adverse scenario in the stress test.

³⁵This is the case in the UK where the Bank of England is responsible for stress tests and bank supervision.

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9 Appendix

9.1 Proof of Theorem 1

This Appendix proves Theorem 1 in the general case where bank health can follow any distribution and where different banks potentially follow different distributions.

The strategy profile (S, R) specified in Theorem 1 is an equilibrium of the Hurdle Rate Framework communication game if and only if:

1. Given R ; S maximises the sender's mandate \mathcal{M} at every health state pair (H_1, H_2) .
2. Given S ; R is correct in the sense of using all available information.

We prove these in turn, using the logic of revealed preference.

Part 1: Given the interpretation rule R , for a given outcome pair $\{o_1, o_2\}$ higher levels of stress result in higher beliefs μ_i for both banks. Since \mathcal{M} is strictly increasing in $\mu_i \forall i$, this gives rise to the following Lemma:

Lemma 9.1.1 *Given the interpretation rule R , the CB always prefers the highest level of stress possible for a given outcome pair to any alternative level of stress for that outcome pair.*

Formally, this means that the CB strictly prefers $\{\min(H_1, H_2), p, p\}$ to all $\{s', p, p\}$ where $s' < \min(H_1, H_2)$; the CB strictly prefers $\{H_1, p, f\}$ to all $\{s'', p, f\}$ where $s'' < H_1$; the CB strictly prefers $\{H_2, f, p\}$ to all $\{s''', f, p\}$ where $s''' < H_2$.

Proof: Since \mathcal{M} is strictly increasing in μ_i and R specifies that μ_i are increasing in s for a given outcome pair, this Lemma holds. *q.e.d.*

Lemma 9.1.1 means that the CB chooses either $s = \max(H_1, H_2)$ or $s = \min(H_1, H_2)$ in response to interpretation rule R . We proceed by showing that the sets of health state pairs (H_1, H_2) where a given $\{s, o_1, o_2\}$ is sent can be expressed via indifference frontiers $x(H_2)$ and $y(H_2)$ as in Theorem 4.2.

Lemma 9.1.2 *If at a health state pair (H_1, H_2) with $H_2 > 0$ sending $\{H_1, p, f\}$ is the message which maximises the CB's mandate \mathcal{M} , then for all (H_1, H'_2) with $H'_2 < H_2$ it is mandate maximising to send $\{H'_1, p, f\}$.*

Proof: At (H_1, H_2) , $\{H_1, p, f\}$ was revealed preferred to $\{H_2, p, p\}$, i.e. $V(\{H_1, p, f\}) > V(\{H_2, p, p\})$. Given the interpretation rule R specified in Theorem 1, $\{H_2, p, p\}$ results in higher beliefs μ_i about both banks and therefore in a higher sender pay-off than $\{H'_2, p, p\}$, i.e. $V(\{H_2, p, p\}) > V(\{H'_2, p, p\})$. Thus $\{H_1, p, f\}$ must be preferred to $\{H'_2, p, p\}$, i.e. we must have $V(\{H_1, p, f\}) > V(\{H'_2, p, p\})$. *q.e.d.*

Lemma 9.1.3 *If at a health state pair (H_1, H_2) sending $\{H_1, p, f\}$ is the message which maximises the CB's mandate \mathcal{M} , then for all (H'_1, H_2) with $H'_1 > H_1$ it is mandate maximising to send $\{H'_1, p, f\}$.*

Proof: Analogous to the proof of Lemma 9.1.2: Revealed: $V(\{H_1, p, f\}) > V(\{H_2, p, p\})$. Since higher levels of stress for a given outcome pair result in higher posteriors and higher posteriors result in a higher pay-off V : $V(\{H'_1, p, f\}) > V(\{H_1, p, f\})$. Hence, we must have: $V(\{H'_1, p, f\}) > V(\{H_2, p, p\})$. *q.e.d.*

Lemma 9.1.4 *There never exists a perfectly vertical part of the $x(H_2)$ indifference frontier.³⁶*

Proof by contradiction: Suppose a perfectly vertical part of the $x(H_2)$ indifference frontier existed. Then on this part there would be at least two health state pairs, (H_1^b, H_2) and (H_1^a, H_2) with $H_1^a > H_1^b$, at which the sender is both times indifferent between $\{H_2, p, p\}$ and $\{H_1, p, f\}$. At (H_1^b, H_2) , the sender is indifferent between $\{H_1^b, p, f\}$ and $\{H_2, p, p\}$ if $V(\{H_1^b, p, f\}) = V(\{H_2, p, p\})$. At (H_1^a, H_2) , the sender is indifferent between $\{H_1^a, p, f\}$ and $\{H_2, p, p\}$ if $V(\{H_1^a, p, f\}) = V(\{H_2, p, p\})$. This would imply that $V(\{H_1^a, p, f\}) = V(\{H_1^b, p, f\})$. But this is not possible given the interpretation rule R . Specifically, this would violate the monotonicity property of beliefs resulting from R which ensures that $\{s, p, f\}$ results in higher μ_i than $\{s', p, f\} \forall s > s'$ and $\forall \mu_i$. Hence, $\{H_1^a, p, f\}$ must be strictly preferred to $\{H_1^b, p, f\}$. Therefore, no perfectly vertical part of the $x(H_2)$ indifference frontier can exist. *q.e.d.*

Lemma 9.1.1 means that the CB chooses either $s = \max(H_1, H_2)$ or $s = \min(H_1, H_2)$. Lemma 9.1.2 - 9.1.3 showed that the health state pairs where $\{H_1, p, f\}$ is sent form a connected set in the top left part of the state space $(H_1, H_2) \in [0, 1]^2$. Lemma 9.1.4 showed that this connected set does not have a perfectly vertical boundary. It also cannot be decreasing. Thus, it is without loss of generality to describe the CB's optimal strategy as sending $\{H_1, p, f\}$ for all (H_1, H_2) above an $x(H_2)$ indifference frontier which is monotonically increasing, i.e. as described in Theorem 1. This proves part 1. *q.e.d.*

Part 2: Given the strategy S , the interpretation rule is correct in the sense of using all available information. Naive interpretations of $\{s, p, p\}$ would result in $\mu_1 = \mathbb{E}[H \mid s \leq H \leq 1]$. Partially sophisticated interpretations would realise that $s = \min(H_1, H_2)$ and result in $\mu_1 = \alpha s + (1 - \alpha)\mathbb{E}_F[H \mid s \leq H \leq 1]$. Fully sophisticated beliefs realise that $s = \min(H_1, H_2)$ and additionally that the sender preferred $s = \min(H_1, H_2)$ to $s = \max(H_1, H_2)$. Thus, fully sophisticated beliefs result in $\mu_1 = \alpha s + (1 - \alpha)\mathbb{E}_F[H \mid s \leq H \leq x(s)]$. These beliefs correspond to the beliefs in Theorem 1. *q.e.d.*

9.2 Proof of Proposition 1: Ordering designs by reassurance

This Appendix proves that institution designs \mathcal{D} can be ordered by how much reassurance $\mathbb{E}[\min(\mu_1, \mu_2)]$ they provide as described in Proposition 1.

³⁶A similar logic applies to $y(H_2)$.

Lemma 9.2.1 *The Zero Disclosure Framework generates $\mathbb{E}[\min(\mu_1, \mu_2)] = \mathbb{E}[H]$. We term this full reassurance.*

Proof: Since in ZDF never any information is released, recipients can never update their prior belief $\mathbb{E}[H_i]$. Thus, we always have $\mu_i = \mathbb{E}[H_i]$ and therefore $\mathbb{E}[\min(\mu_1, \mu_2)] = \mathbb{E}[\min(\mathbb{E}(H), \mathbb{E}(H))] = \mathbb{E}[H]$. We refer to this as ‘full reassurance’ as it is the highest possible level of reassurance achievable. *q.e.d.*

Lemma 9.2.2 *The Full Disclosure Framework generates $\mathbb{E}[\min(\mu_1, \mu_2)] = \mathbb{E}[\min(H_1, H_2)]$. We term this zero reassurance.*

Proof: Since in FDF health realisations H_i are always fully revealed, recipients always update their belief to the true fundamental: $\mu_i = H_i$. Since this is the case for all health states, the ex-ante *distribution* of fundamentals and beliefs coincide, which is stronger than Bayes plausibility which states that expected fundamental and expected belief coincide. Mathematically: $\mathbb{E}[\min(\mu_1, \mu_2)] = \mathbb{E}[\min(H_1, H_2)]$. *q.e.d.*

ZDF always generates more reassurance than FDF as $\mathbb{E}[H] \geq \mathbb{E}[\min(\mu_1, \mu_2)]$ holds for all distributions. In the uniform example used to construct the diagrams in the paper: $\mathbb{E}[H] = \frac{1}{2} > \frac{1}{3} = \mathbb{E}[\min(H_1, H_2)]$.

Lemma 9.2.3 *For all mandates \mathcal{M} , the Hurdle Rate Framework generates a level of reassurance that lies between the levels of FDF and ZDF. The level of reassurance in a HRF is monotonically increasing in the weight the mandate places on the weak bank.*

Proof: We know from (6) that the $x(H_2)$ indifference frontier can be expressed as $x(H_2) = \frac{2-\omega}{1-2\omega} H_2$ when we assume that $V(\mu_1, \mu_2) = \omega \min(\mu_1, \mu_2) + (1 - \omega) \frac{\mu_1 + \mu_2}{2}$ where $\omega \in [0, 1[$ as well as that $H_i \sim U[0, 1] \forall i$. For ease of exposition, we define $m \equiv \frac{2-\omega}{1-2\omega}$ and use m to characterise the sender’s preferences. Since $\omega \geq 0$ our parameter space is $m \geq 2$. For any m we can back out the underlying ω via $\omega = \frac{m-2}{2m-1}$. Hence, there is a one-for-one mapping between m and ω . Equation (9) describes $E(\min(\mu_1, \mu_2))$ in a HRF. Using $x(H_2) = mH_2$ this becomes:

$$\begin{aligned} \mathbb{E}[\min(\mu_1, \mu_2)] &= \int_0^{\frac{1}{m}} \left[\frac{H_2^2}{2m^2} + \frac{3+m}{8} \left(\frac{m^2-1}{m^2} \right) H_2^2 + \frac{3+m}{4} (m-1) H_2^2 + \frac{1}{4m} - \frac{m}{4} H_2^2 \right] g(H_2) dH_2 \\ &+ \int_{\frac{1}{m}}^1 \left[\frac{H_2^2}{2m^2} + \frac{3}{8} \left(\frac{m^2-1}{m^2} \right) H_2^2 + \frac{1}{4} \left(\frac{m-1}{m} \right) H_2 + \frac{1}{4} - \frac{3}{4} H_2^2 + \frac{1}{2} H_2 \right] g(H_2) dH_2 \end{aligned} \quad (12)$$

And further to:

$$\mathbb{E}[\min(\mu_1, \mu_2)] = \frac{1}{2} + \frac{-7m^3 + m^2 + 3m - 1}{24m^4} \quad (13)$$

This gives rise to three sublemmata which together prove Lemma 9.2.3.

Sublemma 9.2.1 *Even when the sender's mandate is only concerned with the average bank health, a hurdle rate framework implies more reassurance than the full disclosure framework.*

Proof: We know from Lemma 9.2.2 that in FDF $\mathbb{E}[\min(\mu_1, \mu_2)] = \mathbb{E}[\min(H_1, H_2)]$ which for these distributional assumptions is $\frac{1}{3}$. Using that the sender's mandate is only concerned with the average bank ($\omega = 0$ thus $m = 2$) in (13), we obtain $\mathbb{E}[\min(\mu_1, \mu_2)] = \frac{145}{384} \approx 0.378$. Hence, reassurance in FDF exceeds reassurance in a HRF with mandate $V(\mu_1, \mu_2) = \frac{\mu_1 + \mu_2}{2}$. *q.e.d.*

Sublemma 9.2.2 *The stronger the weight the mandate places on the weak bank, the more reassurance is provided in equilibrium.*

Proof: We aim to show that $\frac{d\mathbb{E}(\min(\mu_1, \mu_2))}{d\omega} \geq 0 \forall \omega \in [0, 1[$. This corresponds to showing that $\frac{d\mathbb{E}(\min(\mu_1, \mu_2))}{dm} \geq 0 \forall m \geq 2$. From (13) we obtain:

$$\frac{d\mathbb{E}(\min(\mu_1, \mu_2))}{dm} = \frac{7m^3 - 2m^2 - 9m + 4}{24m^5} \quad (14)$$

and

$$\frac{d^2\mathbb{E}(\min(\mu_1, \mu_2))}{dm^2} = \frac{-7m^3 + 3m^2 + 18m - 10}{12m^6} \quad (15)$$

Solving $\max_m \mathbb{E}[\min(\mu_1, \mu_2)]$ yields the FOC $\frac{d\mathbb{E}(\min(\mu_1, \mu_2))}{dm} \stackrel{!}{=} 0$ which is the case for $m = 1$ and for two lower values of m . The SOC confirms that $m = 1$ is indeed a minimum. Thus, for all $m > 1$ we have $\frac{d\mathbb{E}(W)}{dm} > 0$. Hence, insurance is monotonically increasing in m also for all $m \geq 2$. *q.e.d.*

Sublemma 9.2.3 *In the limit case where the sender is purely concerned with the weak bank ($m \rightarrow \infty$), i.e. when almost always $\{b, p, p\}$ is sent³⁷, the amount of reassurance provided approaches that of the zero disclosure framework.*

Proof: Using (13) and applying limit theorems:

$$\lim_{m \rightarrow \infty} \mathbb{E}[\min(\mu_1, \mu_2)] = \lim_{m \rightarrow \infty} \left(\frac{1}{2}\right) + \lim_{m \rightarrow \infty} \left(\frac{-7m^3 + m^2 + 3m - 1}{24m^4}\right) = \frac{1}{2} \quad (16)$$

From Lemma 9.2.1 we know that for the zero disclosure framework $\mathbb{E}[\min(H_1, H_2)] = \mathbb{E}[\min(H_1, H_2)] = \frac{1}{2}$. Thus, $\mathbb{E}[\min(\mu_1, \mu_2) \mid HRF, m \rightarrow \infty] = \mathbb{E}[\min(\mu_1, \mu_2) \mid ZDF]$. *q.e.d.*

Intuition: SubLemma 9.2.3 does not mean that as $m \rightarrow \infty$ the minimum posterior is always exactly $\frac{1}{2}$. Rather, it says that the *expected* minimum posterior is $\frac{1}{2}$, or more generally equals the expected fundamental³⁸, but there still is a distribution of minimum posteriors. Thus, while the first moment of the distribution of $\min(\mu_1, \mu_2)$ under a HRF approaches the first moment under ZDF as $m \rightarrow \infty$, higher moments remain different. Our definition of reassurance as $\mathbb{E}[\min(\mu_1, \mu_2)]$ focuses only on the first moment of the distribution of $\min(\mu_1, \mu_2)$. This definition

³⁷Always except when $\min(H_1, H_2) = 0$, exactly zero.

³⁸ $\mathbb{E}(\min(\mu_1, \mu_2)) = \mathbb{E}(H_i)$, i.e. $\mathbb{E}(\min(\text{posterior})) = \mathbb{E}(\text{fundamental})$

gives rise to the equality in the limit result. We conjecture that a broader definition of reassurance that includes higher moments of $\min(\mu_1, \mu_2)$ would continue to find that reassurance in a HRF is below reassurance in ZDF, in this case strictly below. The ranking of frameworks in terms of reassurance would then be unaffected.

Proof of Lemma 9.2.3: Combining Sublemmata 9.2.1, 9.2.2, 9.2.3 proves Lemma 9.2.3. *q.e.d.*

Proof of Proposition 1: Combining Lemmata 9.2.1, 9.2.2, 9.2.3 proves Proposition 1. *q.e.d.*

9.3 Proof of Proposition 2: Comparative Static on the sender's mandate

Let there be two mandates or sender pay-off functions, $V(\mu_1, \mu_2)$ and $U(\mu_1, \mu_2)$, where $U(\mu_1, \mu_2)$ is more concerned with the weak bank, i.e. is more concave.³⁹ Let the equilibrium in the communication game for $V(\mu_1, \mu_2)$ be characterised by indifference frontier $x(H_2)$ and the interpretation rule R while the equilibrium for $U(\mu_1, \mu_2)$ is characterised by $\tilde{x}(H_2)$ and \tilde{R} . On $x(H_2)$ we have by definition of $x(H_2)$ that $V(\{H_1, p, f\}) = V(\{H_2, p, p\})$. Given the interpretation rule R , $\{H_1, p, f\}$ results in dispersed posteriors $\mu'_1 \neq \mu'_2$, while $\{H_2, p, p\}$ results in both banks having the same posterior $\mu_1 = \mu_2$. Moreover, $\mu'_2 < \mu_1 = \mu_2 < \mu'_1$. Therefore, a sender with mandate $U(\mu_1, \mu_2)$ which places more weight on the weak bank than $V(\mu_1, \mu_2)$ strictly prefers $\{H_2, p, p\}$ on $x(H_2)$ for interpretation R . For interpretation rule R , $U(\mu_1, \mu_2)$ strictly prefers $\{H_2, p, p\}$ to $\{H_1, p, f\}$ for all (H_1, H_2) between the 45-degree line and the $x(H_2)$ -frontier. Thus, the hypothetical frontier $\bar{x}(H_2)$ at which $U(\{H_1, p, f\} | R) = U(\{H_2, p, p\} | R)$ must lie strictly above $x(H_2)$. However, $\bar{x}(H_2)$ does not characterise an equilibrium as it is based on R rather than the appropriate \tilde{R} . Allowing the interpretation rule to adjust to \tilde{R} results in $U(\mu_1, \mu_2)$ strictly preferring $\{H_2, p, p\}$ also on $\bar{x}(H_2)$. Thus, the resulting equilibrium indifference frontier $\tilde{x}(H_2)$ must lie above $\bar{x}(H_2)$. thus, since $\tilde{x}(H_2) > \bar{x}(H_2) \forall H_2$ and $\bar{x}(H_2) > x(H_2) \forall H_2$, we have that $\tilde{x}(H_2) > x(H_2) \forall H_2$. *q.e.d.*

9.4 Proof of Proposition 3: Ordering designs by incentives

This Appendix proves that institution designs \mathcal{D} can be ordered by the strength of incentives they generate ($\mathbb{E}[\mu_1(\mathbf{l}) | h_1, l_2] - \mathbb{E}_G[H] > 0$) as described in Proposition 3.

Lemma 9.4.1 *The Zero Disclosure Framework generates no incentives.*

Proof: Since in ZDF never any information is released, recipients can never update their prior belief $\mathbb{E}_G[H]$. Thus, we always have $\mu_i = \mathbb{E}_G[H_i]$. This means that the expected posterior is fully determined by conjectured effort (low, leading to distribution G) and completely isolated from actual effort. Formally $\mathbb{E}[\mu_1(\mathbf{l}) | h_1, l_2] = \mathbb{E}[\mu_1(\mathbf{l}) | l_1, l_2] = \mathbb{E}_G[H]$. *q.e.d.*

Lemma 9.4.2 *The Full Disclosure Framework generates full incentives.*

³⁹For our example functional forms this corresponds to $\omega_v < \omega_u$. In this proof we do not rely on these functional form assumptions.

Proof: Since in FDF health realisations H_i are always fully revealed, recipients always update their belief to the true fundamental: $\mu_i = H_i$. Thus, $\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] = \mathbb{E}[H_1 \mid h_1] = \mathbb{E}_F[H]$. Thus, incentives are $\mathbb{E}_F[H] - \mathbb{E}_G[H]$ which is strictly positive by assumption that effort improves the bank health distribution according to an MLRP property. *q.e.d.*

Lemma 9.4.3 *For all mandates \mathcal{M} , the Hurdle Rate Framework generates positive but muted effort incentives, i.e. incentives with a strength that lies between the strength of incentives generated ZDF and FDF. The strength of incentives in a HRF is monotonically decreasing in the weight the mandate places on the weak bank.*

Proof: We know from (6) that the $x(H_2)$ indifference frontier can be expressed as $x(H_2) = \frac{2-\omega}{1-2\omega} H_2$ when we assume that $V(\mu_1, \mu_2) = \omega \min(\mu_1, \mu_2) + (1-\omega) \frac{\mu_1 + \mu_2}{2}$ where $\omega \in [0, 1[$ as well as that $G(H) = H$ and $F(H) = H^2$ and the regulator conjectures that banks choose low effort. As in Appendix 9.2, for ease of exposition we define $m \equiv \frac{2-\omega}{1-2\omega}$ and use m to characterise the sender's preferences. Equation (11) describes $\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2]$ in a HRF. Using $x(H_2) = mH_2$ this becomes:

$$\begin{aligned} \mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] &= \int_0^{\frac{1}{m}} \left[\frac{H_2}{2m} F\left(\frac{H_2}{m}\right) + \frac{3+m}{4} \int_{\frac{H_2}{m}}^{H_2} H_1 f(H_1) dH_1 \right. \\ &\quad \left. + \frac{3+m}{4} H_2 [F(mH_2) - F(H_2)] + \int_{x(H_2)}^1 H_1 f(H_1) dH_1 \right] g(H_2) dH_2 \\ &\quad + \int_{\frac{1}{m}}^1 \left[\frac{H_2}{2m} F\left(\frac{H_2}{m}\right) + \int_{\frac{H_2}{m}}^{H_2} \frac{1}{4} + \frac{3}{4} H_1 f(H_1) dH_1 \right. \\ &\quad \left. + \left(\frac{1}{4} + \frac{3}{4} H_2\right) (1 - F(H_2)) \right] g(H_2) dH_2 \end{aligned} \tag{17}$$

and

$$\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] = \frac{1}{48m^6} (27m^6 + 15m^5 - 13m^4 - m^3 + 4m - 2) \tag{18}$$

This gives rise to Sublemmata which together prove Lemma 9.4.3.

Sublemma 9.4.1 *If the mandate \mathcal{M} is only concerned with average beliefs, strictly positive but muted incentives are generated.*

Proof: we know from Lemma 9.4.2 that in FDF incentives are $\mathbb{E}_F[H] - \mathbb{E}_G[H]$ and from Lemma 9.4.1 that in ZDF incentives are zero. If the mandate is only concerned about average beliefs ($m = 2$) and places no extra weight on the weak bank, then (18) yields $\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] = \frac{333}{512}$. Since $\mathbb{E}_F[H] = \frac{2}{3}$ we have $\mathbb{E}_G[H] < \mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] < \mathbb{E}_F[H]$ and thus effort incentives are indeed strictly positive and muted. *q.e.d.*

Sublemma 9.4.2 *Incentives are monotonically decreasing in the weight the mandate places on the weak bank.*

Proof: We aim to show that $\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2] - \mathbb{E}_G[H]$ is decreasing in $\omega \in [0, 1[$. Since $\mathbb{E}_G[H]$ is independent of sender preferences and using notation $x(H_2) = mH_2$ to capture the mandate ω , this is equivalent to showing that $\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2]$ is decreasing in m for all $m \geq 2$. From (18) we obtain:

$$\frac{d\mathbb{E}[\mu_1(\mathbf{l}) \mid h_1, l_2]}{dm} = \frac{-15m^5 + 26m^4 + 3m^3 - 20m + 12}{48m^7} \quad (19)$$

For our entire parameter space ($m \geq 2$), this expression is negative. *q.e.d.*

Sublemma 9.4.3 *For all mandates \mathcal{M} , a Hurdle Rate Framework always creates strictly positive incentives. Even when the sender is purely concerned with the weak bank, incentives remain strictly positive and are never fully muted.*

Proof: If the mandate is purely concerned with the weak bank ($\omega = 1$), then $m \rightarrow \infty$. The limit of (18) thus becomes $\lim_{m \rightarrow \infty} \mathbb{E}(\mu_1(\mathbf{l}) \mid h_1, l_2) = \frac{9}{16}$. Thus, $\lim_{m \rightarrow \infty} \mathbb{E}(\mu_1(\mathbf{l}) \mid h_1, l_2) - E_G(H) > 0$ which means that incentives are strictly positive. Since by Sublemma 9.4.2 incentives are monotonically decreasing in ω , all $\omega \in [0, 1[$ must have even higher and thus strictly positive incentives. *q.e.d.*

Intuition for Sublemma 9.4.3: If the mandate is only concerned about the weak bank, then in equilibrium the CB almost always sends $\{\min(H_1, H_2), p, p\}$ except when $\min(H_1, H_2) = 0$. Some information is revealed which thus generates positive incentives. If bank 1 knew that $\min(H_1, H_2) = H_1$ will result, it would have full incentives while if it knew that $\min(H_1, H_2) = H_2$ would result it would face zero incentives. If the CB always sends $\{\min(H_1, H_2), p, p\}$ there is a chance of bank 1 being the weak bank and thus some incentives remain even in the limit.

Proof of Lemma 9.4.3: Combining Sublemma 9.4.1, 9.4.2, 9.4.3 proves Lemma 9.4.3. *q.e.d.*

Proof of Proposition 3: Combining Lemma 9.4.1, 9.4.2, 9.4.3 proves Proposition 3. *q.e.d.*

9.5 Proof of Proposition 4: Equilibrium in the endogenous effort game

Proof of Part 1: Proof by backward induction: In ZDF, recipients never receive a message and thus cannot update their prior belief. The posterior belief is therefore identical to the prior belief which is based on conjectured effort. Actual effort does not alter μ_i . When banks choose their effort level e_i to maximise $\mathbb{E}[\pi_i] = \mathbb{E}[\mu_i] - C(e_i)$ they anticipate that $\mathbb{E}[\mu_i]$ is not affected by e_i , hence, there is no benefit of effort, while effort is costly. Thus, in ZDF, low effort choice is the optimal strategy regardless of what effort level receivers conjecture. The unique equilibrium therefore has (l_1, l_2) as only this case satisfies the equilibrium requirement that conjectures are correct. *q.e.d.*

Proof of Part 2: Proof by backward induction: In FDF, health realisations H_i are always fully revealed and recipients thus always update their belief to the true fundamental: $\mu_i = H_i$. The expected posterior is therefore identical to the expected fundamental $\mathbb{E}[\mu_i] = \mathbb{E}[H_i | e_i]$. When banks choose their effort level e_i to maximise $\mathbb{E}[\pi_i] = \mathbb{E}[\mu_i] - C(e_i)$, they anticipate that $\mathbb{E}[\mu_i]$ is fully determined by e_i and thus face full effort incentives. They exert effort iff $\mathbb{E}_F[H] - \mathbb{E}_G[H] > C(h) - C(l)$ which is always true by our assumption that effort is socially beneficial. Hence, in FDF, high effort is the optimal action regardless of what effort level receivers conjecture. The unique equilibrium therefore has (h_1, h_2) as only this case satisfies the equilibrium requirement that conjectures are correct. *q.e.d.*

Proof of Part 3:

Lemma 9.5.1 *A low effort equilibrium exists if and only if $C(h) - C(l) \geq \bar{t}(\mathcal{M})$*

Proof: In a HRF, an equilibrium where both banks choose low effort exists only if a bank does not benefit from unilaterally deviating to high effort, i.e. $\mathbb{E}[\mu_1(\mathbf{l}) | h_1, l_2] - \mathbb{E}_G[H] \leq C(h) - C(l)$. We know from Proposition 3 that the strength of incentives generated in a HRF depends on the sender's mandate. Defining $\mathbb{E}[\mu_1(\mathbf{l}) | h_1, l_2] - \mathbb{E}_G[H] = \bar{t}(\mathcal{M})$, the condition above can be restated as $C(h) - C(l) \geq \bar{t}(\mathcal{M})$. All other equilibrium conditions are also satisfied, given how we constructed $\mathbb{E}[\mu_1(\mathbf{l}) | h_1, l_2]$. *q.e.d.*

Lemma 9.5.2 *A high effort equilibrium exists if and only if $C(h) - C(l) \leq \underline{t}(\mathcal{M})$.*

Proof: In a HRF, an equilibrium where both banks choose high effort exists only if a bank does not benefit from unilaterally deviating to low effort, i.e. $\mathbb{E}_F[H] - \mathbb{E}[\mu_1(\mathbf{h}) | l_1, h_2] \geq C(h) - C(l)$. The strength of incentives generated in a HRF depends on the sender's mandate. Defining $\underline{t}(\mathcal{M}) = \mathbb{E}_F[H] - \mathbb{E}[\mu_1(\mathbf{h}) | l_1, h_2]$, the condition can be restated as $C(h) - C(l) \leq \underline{t}(\mathcal{M})$. All other equilibrium conditions are also satisfied given how we constructed $\mathbb{E}[\mu_1(\mathbf{h}) | l_1, h_2]$. *q.e.d.*

Lemma 9.5.3 $\underline{t}(\mathcal{M}) < \bar{t}(\mathcal{M})$

When both banks play high effort which results in $F(H) = H^2$, then the hurdle rate communication game for $V(\mu_1, \mu_2) = \frac{\mu_1 + \mu_2}{2}$ results in $x(H_2)$ -indifference frontier $x(H_2) = \sqrt{3} H_2$. In the endogenous effort game, there exists an equilibrium where both banks play high effort if and only if $C(h) - C(l) \leq \mathbb{E}_F[H] - \mathbb{E}[\mu_1(\mathbf{h}) | l_1, h_2] \approx 0.1285$. Correspondingly, both banks playing low effort results in $x(H_2) = 2H_2$ and is an equilibrium if and only if: $C(h) - C(l) \geq \mathbb{E}[\mu_1(\mathbf{l}) | h_1, l_2] - \mathbb{E}_G[H] = \frac{77}{512} \approx 0.1504$. Clearly, $0.1285 < 0.1504$ which shows that Lemma 9.5.3 holds. *q.e.d.*

Proof of Part 3: Since the only candidate symmetric pure strategy equilibria have (l_1, l_2) or (h_1, h_2) , combining Lemma 9.5.1, 9.5.2, 9.5.3 proves Part 3. Lemma 9.5.1 and 9.5.2 show regions in which (l_1, l_2) or (h_1, h_2) equilibria exist and Lemma 9.5.3 shows that the regions of the cost space in which these equilibria exist are disjoint, leading to a region where no pure strategy symmetric equilibrium exists between $\underline{t}(\mathcal{M})$ and $\bar{t}(\mathcal{M})$. *q.e.d.*

9.6 Proof of Proposition 5: Central Bank Design

Lemma 9.6.1 *ZDF achieves higher welfare than all Hurdle Rate Frameworks which result in low effort.*

Mathematically: $W(\{ZDF\}) > W(\{HRF, \omega'\}) \forall \omega'$ which result in low effort, i.e. all ω' such that $\omega' > \bar{\omega}$ where $\bar{\omega}$ is defined by $C(h) - C(l) = \bar{t}(\bar{\omega}) = \mathbb{E}[\mu_1(\mathbf{l}) | h_1, l_2, \bar{\omega}] - \mathbb{E}_G[H]$.

Proof: All designs compared in Lemma 9.6.1 achieve low effort. Thus, the designs only differ in how much reassurance they provide. From Proposition 1 we know that ZDF provides the highest reassurance. For $W(\mu_1, \mu_2) = \lambda \min(\mu_1, \mu_2) + (1 - \lambda) \frac{\mu_1 + \mu_2}{2}$, welfare comparisons among these designs simplify to reassurance comparisons. Thus, ZDF achieves higher welfare than all $\{HRF, \omega'\}$ specified in Lemma 9.6.1. *q.e.d.*

Lemma 9.6.2 *If there exists a HRF which achieves a high effort equilibrium, then the HRF with the softest possible mandate which just induces high effort achieves higher welfare than all tougher HRF mandates and than FDF.*

Mathematically: If a design $\{HRF, \tilde{\omega}\}$ exists where $\tilde{\omega}$ is defined by $C(h) - C(l) = \underline{t}(\tilde{\omega}) = \mathbb{E}_F[H] - \mathbb{E}[\mu_1(\mathbf{h}) | l_1, h_2, \tilde{\omega}]$ than $\{HRF, \tilde{\omega}\}$ achieves higher welfare than $\{HRF, \omega\} \forall \omega < \tilde{\omega}$ and than FDF.

Proof: All designs compared in Lemma 9.6.2 achieve high effort. Thus, designs only differ in how much reassurance they provide. From Proposition 1 we know that FDF provides the lowest reassurance and that in a HRF, reassurance is monotonically increasing in the softness of the mandate (ω). For $W(\mu_1, \mu_2) = \lambda \min(\mu_1, \mu_2) + (1 - \lambda) \frac{\mu_1 + \mu_2}{2}$, welfare comparisons among these designs simplify to reassurance comparisons. Thus, $\{HRF, \tilde{\omega}\}$ achieves higher welfare than $\{HRF, \omega\} \forall \omega < \tilde{\omega}$ and than FDF. *q.e.d.*

Given our distributional assumptions $G(H) = H, F(H) = H^2$ we have $\mathbb{E}_G(\min(H_1, H_2)) = \int_0^1 \int_0^1 \min(H_1, H_2) dH_1 dH_2 = \frac{1}{3}$, $\mathbb{E}_F(\min(H_1, H_2)) = \int_0^1 \int_0^1 \min(H_1, H_2) 2H_1 2H_2 dH_1 dH_2 = \frac{8}{15} \approx 0.533$. This means that effort is so beneficial that $\mathbb{E}_F(\min(H_1, H_2)) > \mathbb{E}_G(H)$ and thus $W(\{FDF\}) > W(\{ZDF\}) \forall \lambda$, i.e. for all social preferences society would prefer FDF to ZDF.

Combining Lemma 9.6.2, 9.6.1 and $W(\{FDF\}) > W(\{ZDF\}) \forall \lambda$ we can prove Proposition 5:

If costs of effort are low enough that a hurdle rate framework can induce high effort, i.e. $\{HRF, \tilde{\omega}\}$ exists, then by Lemma 9.6.2: $W(\{HRF, \tilde{\omega}\}) > W(\{FDF\}) \forall \lambda$ and $W(\{HRF, \tilde{\omega}\}) > W(\{HRF, \omega\}) \forall \omega < \tilde{\omega}$. Since $W(\{FDF\}) > W(\{ZDF\}) \forall \lambda$ we must also have $W(\{HRF, \tilde{\omega}\}) > W(\{ZDF\})$ and by Lemma 9.6.1 $W(\{ZDF\}) > W(\{HRF, \omega'\})$. Thus, $\{HRF, \tilde{\omega}\}$ results in the symmetric pure strategy equilibrium which achieves the highest welfare.

If costs of effort are so high that no hurdle rate framework can induce high effort, i.e. $\{HRF, \tilde{\omega}\}$ does not exist, then from $W(\{FDF\}) > W(\{ZDF\})$ and $W(\{ZDF\}) > W(\{HRF, \omega'\})$ it follows that FDF results in the pure strategy equilibrium which achieves the highest welfare. *q.e.d.*