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University of Naples Federico II · University of Salerno · Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS – UNIVERSITY OF NAPLES
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unisa.it

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Daniela Fabbri♥ and Anna Maria C. Menichini♦

Abstract

The paper proposes a model of collateralized bank and trade credit. Firms use a two-input technology. Assuming that the supplier is better able to extract value from existing assets and has an information advantage over other creditors, the paper derives a series of predictions. (1) Financially unconstrained firms (with unused bank credit lines) take trade credit for a liquidation motive. (2) The reliance on trade credit does not depend on credit rationing, if inputs are liquid enough. (3) Firms buying goods make more purchases on account than those buying services, while suppliers of services offer more trade credit than those of standardized goods. (4) Suppliers lend inputs to their customers but not cash. (5) Greater reliance on trade credit is associated with more intensive use of tangible inputs. (6) Better creditor protection decreases both the use of trade credit and input tangibility.

JEL Classification: G32, G33, K22, L14.

Keywords: trade credit, collateral, financial constraints, asset tangibility, creditor protection ibility

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♥ University of Amsterdam. Contact address: Amsterdam Business School, University of Amsterdam, Roeterstraat 11 1018 WB Amsterdam. Tel.: +31 20 5254344. Fax: +31 20 525 5285. E-mail: d.fabbri@uva.nl

♦ University of Salerno, CELPE and CSEF. Contact address: Dipartimento di Scienze Economiche e Statistiche, University of Salerno, Via Ponte Don Melillo, 84084 Fisciano (SA), Italy. Tel.: +39 089 962174. Fax. +39 089 963169. E-mail: amenichi@unisa.it.

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Introduction

Firms procure funds not only from specialized financial intermediaries but also from suppliers, generally by delaying payments. The empirical evidence on trade credit raises questions that are hard to reconcile with existing theories. First, what justifies its widespread use by financially unconstrained firms that have access to seemingly cheaper alternative sources? Second, why is the reliance on trade credit not always increasing in the degree of credit rationing? Third, why do suppliers regularly extend credit by allowing delayed payment but seldom by lending cash? Last, does input lending have an impact on the borrower's choice of inputs? And, relatedly, are the financing and input choices affected by the degree of creditor protection? This paper addresses all these questions in a unified framework.

There is a general consensus that trade credit is most common among firms that face borrowing constraints. This follows from the assumption that trade credit is more expensive than bank loans.[1] According to this view, reliance on trade credit should increase in credit rationing, but the empirical evidence is not generally consistent with this common belief. Petersen and Rajan (1997) present evidence for the U.S. that large firms (presumably less likely to be credit-constrained) rely more heavily on trade credit than small firms, accounts payable averaging 11.6% and 4.4% of sales for large and small firms respectively.[2] Similarly, for the Italian manufacturing sector, Marotta (2005) documents that trade credit finances on average 38.1% of the input purchases of non-rationed firms, and 37.5% of rationed ones.[3]

A common feature in the use of trade credit, which is independent of the degree of credit rationing, is that the supplier's lending is tied closely to the value of the input. That is, suppliers readily lend inputs, but seldom cash. Given that not all inputs can be purchased on account,[4] trade credit is likely to go together with some bias in the input combination. This seems to be confirmed by scattered evidence on financing and technological choices. Some papers find greater use of trade credit in countries with less creditor protection, such as developing countries (see, among others, Rajan and Zingales, 1995; La

[1] The evidence on trade credit as a more expensive source of financing than bank loans is mostly anecdotal (Petersen and Rajan, 1997; Ng, Smith and Smith, 1999; Wilner, 2000). In support of this thesis, scholars generally cite the canonical "2/10 net 30" agreement (a 2% discount for payment within 10 days, with the net price charged for payment within 30 days), which implies an effective interest rate of more than 40% for those who do not take the discount. But it is not clear how widespread this kind of agreement actually is.

[2] Petersen and Rajan (1997) also find that firms that have been denied credit in the previous year receive more trade credit. However, the coefficient is not statistically significant.

[3] Marotta (2005) uses data from a survey conducted by the bank Mediocredito Centrale in 1994. Credit-constrained firms are identified by two questions: "In 1994, has the firm applied for, but not obtained, more bank loans?" and "in 1994, would the firm have accepted tighter terms (higher interest rates or higher collateral requirements) to obtain more bank loan?"

[4] For example, intangible assets cannot generally be financed by trade credit.

Porta *et al.*, 1998; Fisman and Love, 2003; Frank and Maksimovic, 2004). Further, there is evidence that firms in developing countries have a higher proportion of fixed assets and fewer intangibles than firms in developed countries (Demirguc-Kunt and Maksimovic, 2001). Although fragmented, these findings suggest the existence of a cross-country correlation between financing and input choices and identify the degree of creditor protection as a possible explanation.

To account for the foregoing stylized facts, we propose a model with collateralized bank and trade credit. Firms are opportunistic and face uncertain demand. They choose between two sources of external funding (bank and trade credit) and two types of input with different degrees of observability and collateral value (tangibles and intangibles). Firms may face borrowing constraints. Banks are specialized intermediaries and have a cost advantage in providing finance. Suppliers have both an information and a liquidation advantage. The former consists in observing input transactions costlessly, which enables them to provide credit to relax the firm's financial constraints, i.e. for incentive reasons. The second advantage derives from the supplier's ability to extract a greater liquidation value from the inputs collateralized in case of default. Uncertainty and multiple inputs in a model with moral hazard are the key notions used to address the open questions listed above.

An original feature of our model is the explanation of why firms with unused lines of bank credit may demand trade credit: even they may benefit from the liquidation advantage of their supplier. This advantage makes trade credit cheaper than bank loans, offsetting the banks' lower cost of funds.

The liquidation advantage is sufficient by itself to explain the demand for trade credit by financially unconstrained firms; the interaction between the liquidation and the information advantage helps show why reliance on trade credit does not always increase with the stringency of financing constraints. Financially constrained firms may take trade credit for both reasons. If it is for the incentive, credit-rationed firms finance a larger share of their inputs by trade credit than do non-rationed firms, as theoretical literature holds. Conversely, when the liquidation motive dominates, the share of inputs purchased on account remains constant across firms with different degrees of credit rationing.

Moreover, the relationship between the use of trade credit and financial constraints depends crucially on the characteristics of the inputs. Firms whose inputs are highly liquid (e.g., standardized inputs) or high collateral value (e.g., differentiated inputs) are more likely to use trade credit, to exploit the liquidation advantage of the supplier. Conversely, the incentive motive is more likely to dominate among financially constrained firms using illiquid inputs with low collateral value (e.g., services). We derive several testable predictions on how trade credit demand and supply vary across industries: buyers of goods (both differentiated and standardized) make more purchases on account than buyers of services, but suppliers of services offer more trade credit than suppliers of standardized goods.

Regardless of the motives underlying the use of trade credit, suppliers always finance the inputs they sell but they never lend cash. This result follows from the assumption that suppliers observe only their own transaction. If they could also observe the input purchases from other suppliers, cash lending would arise endogenously. To our knowledge, the only available evidence of cash lending concerns Japanese trading companies (Uesugi and Yamashiro, 2004), which typically feature a substantial involvement of suppliers in the firm's activity, owing to an organizational structure that guarantees continuous information flow from clients to suppliers. This feature is consistent with our theoretical findings.

The absence of cash lending by suppliers implies that trade credit can only be used to finance specific inputs, which in our setting are tangibles. It follows that whenever trade credit is used to relax financial constraints, a credit-rationed business can benefit from it only by distorting its input combination. This introduces a link between financing and input decisions, which we explore to derive new predictions. More intensive use of trade credit goes together with a technology biased towards tangible assets, and the bias increases as the legal protection of creditors weakens. These predictions reconcile the scattered international evidence discussed above (Rajan and Zingales, 1995; La Porta *et al.*, 1998; Demirguc-Kunt and Maksimovic, 2001).

The rest of the paper is organized as follows. Section 1 provides a sketch of the literature. Section 2 describes the model. Section 3 presents and discusses the results. Section 4 explores the effect on our predictions of considering bankruptcy and commercial laws. Section 5 concludes.

1 Related literature

The literature on trade credit has sought to explain why agents should want to borrow from firms rather than from financial intermediaries. The traditional explanation is that trade credit plays a non-financial role. That is, it reduces transaction costs (Ferris, 1981), allows price discrimination between customers with different creditworthiness (Brennan *et al.*, 1988), fosters long-term relationships with customers (Summers and Wilson, 2002), and even provides a warranty for quality when customers cannot observe product characteristics (Long *et al.*, 1993).

These non-financial theories can explain the existence of trade credit, but they do not offer any prediction on how borrowing constraints affect the demand for trade credit, since none of them explicitly models credit rationing. Financial theories have attempted to fill this gap (Biais and Gollier, 1997; Burkart and Ellingsen, 2004, among others), positing that in lending the supplier has an advantage over financial institutions. In Burkart and Ellingsen (2004), whose analysis is closest to ours, suppliers have an informational advantage that mitigates their exposure to borrowers' opportunism. Sufficiently rich firms, without incentive problems, never need trade credit. Poorer firms, which do have incentive problems, face credit rationing by banks, and here suppliers' informational advantage becomes relevant, as they can ease borrowing constraints by extending trade credit to their customers. Similarly, Biais and Gollier (1997) propose a screening model in which the provision of trade credit signals the creditworthiness of the buyer and thus mitigates credit rationing.

However, both of these papers, and financial theories of trade credit in general, fail to explain: (i) why trade credit is also used by financially unconstrained firms; and (ii) why resort to trade credit does not necessarily increase with the severity of financial constraints, as the empirical literature shows (Petersen and Rajan, 1997; Marotta, 2005). In order to distinguish between rationed and non-rationed firms, we model the information advantage as in Burkart and Ellingsen (2004) but interact it with a liquidation advantage, which can explain why even wealthy firms may wish to take up trade credit. The liquidation advantage of suppliers, when it exceeds the bank's intermediation advantage, justifies the use of trade credit by rationed and unrationed firms alike, which squares with the evidence that firms facing different degrees of credit rationing nevertheless tend to rely on trade credit to the same extent.

The thesis that trade credit is a means of exploiting the supplier's liquidation advantage has been tested in various empirical works (Mian and Smith, 1992; Petersen and Rajan, 1997, among others). Frank and Maksimovic (2004) have also modeled the effects of this advantage theoretically, showing that it makes trade credit cheaper than bank financing. In their framework, however, bank credit is never rationed, so that no prediction on the demand for trade credit by financially unconstrained firms can be derived.[5]

Finally, the literature has disregarded the relations between financing and input decisions and offered no explanation of why firms lend only inputs. The use of a multi-input technology allows us to fill these gaps.

2 The model

A risk-neutral entrepreneur has an investment project that uses a tangible and an intangible input. The tangible input can be interpreted as raw material and physical capital, intangibles as skilled labor.

[5] In their model, in order to extend trade credit suppliers must borrow from banks. This intermediary role of suppliers creates an adverse selection problem that induces banks to ration credit to suppliers. These, in turn, will ration creditworthy customers, who then turn to bank credit. Hence, banks will not ration credit to customer firms.

Let q_t and q_{nt} denote respectively the amount of tangible and intangible inputs purchased and $I_t \leq q_t$, $I_{nt} \leq q_{nt}$, the amount of such inputs invested. The purchase of inputs is observed only by their suppliers. The amount invested is totally unobservable and is converted into a verifiable state-contingent output y^σ, with $\sigma \in \{H, L\}$ and $y^H > y^L = 0$. The high state ($\sigma = H$) occurs with probability p. Uncertainty affects production through demand: at times of high demand, invested inputs produce output according to an increasing and strictly concave production function $f^H(I_t, I_{nt})$. At times of low demand, there is no output, and the firm's worth is only the scrap value of unused inputs. Inputs are substitutes, but a positive amount of each is essential for production.

The entrepreneur is a price-taker both in the input and in the output market. The output price is normalized to 1, and so are those of tangible and intangible inputs.[6]

To carry out the project, the entrepreneur uses observable internal wealth (A) as well as external funding from competitive banks ($L_B \geq 0$) and/or suppliers ($L_S \geq 0$). Banks and suppliers play different roles. Banks lend cash. The supplier of intangibles provides the input, which is fully paid for in cash. The supplier of tangibles sells the input, but can also act as a financier, lending both inputs and cash.[7]

Moral hazard. Unobservability of investment to all parties and of input purchases to parties other than the supplier raise a problem of moral hazard: the entrepreneur might not invest the funds raised, either in cash or in kind, in the venture, but divert them to private uses.[8] This problem limits the amount of credit the entrepreneur can obtain from financiers. However, the supplier can observe whether inputs have been purchased. This advantage together with the lesser liquidity of inputs than cash implies that moral hazard is less severe when funding comes from the supplier and not the bank. In particular, one unit of cash gives the entrepreneur a return $\phi < 1$ if diverted, where ϕ can be interpreted as the degree of vulnerability of creditor rights; one unit of the tangible input q_t gives a return $\phi\beta_t$ if diverted, where $\beta_t < 1$ denotes the tangible input liquidity. When β_t is close to 1, the input can be resold at near the purchase price and converted into a monetary benefit.[9] Lastly, diverting the intangible input q_{nt} gives a zero return. This implies that it is not possible to extract monetary benefits from workers

[6]This normalization is without loss of generality since we use a partial equilibrium setting.

[7]A remark on terminology is in order here. Henceforth, trade credit refers to credit, either in cash or in-kind, provided by the supplier. Strictly speaking, however, the term should be used only for in-kind finance and should not include any cash lending. We find that in equilibrium the supplier never lends cash but only inputs, which makes our terminology consistent. We will address this issue in Section 3.4.

[8]The assumption of full unobservability of input purchase to parties other than the direct supplier implies that the bank cannot condition the contract on q_t or on a share of that. This is a useful simplification but is not crucial to obtain our results. We only need to postulate that the supplier has some information/monitoring advantage relative to the bank. This can consist in getting more accurate information, or in getting the same information at a lower cost. Both situations are reasonable given the specific nature of the firm-supplier relationship.

[9]For example, standardized products, which can be used by many different customers, have high re-sale value (high β_t), while perishable goods, services and customized inputs (differentiated) are less liquid (low β_t).

by assigning them to tasks other than those they were hired for. In many countries, such practices are indeed prohibited by labor law.

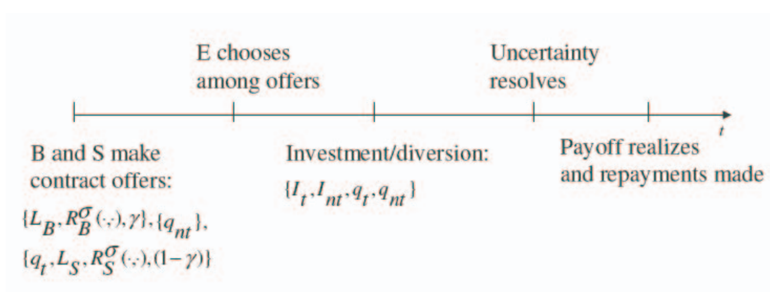
Collateral value. Inputs have value when repossessed in default.[10] We assume that only tangibles can be pledged, while intangibles have zero collateral value. Hence, the total value of pledgeable collateral is I_t. However, different financiers have different liquidation abilities. We define $\beta_i I_t$ as the liquidation value extracted by a given financier in case of default, with $i = B, S$ referring to bank or supplier. The supplier has a better knowledge of the resale market, so we assume $\beta_S > \beta_B$.

Finally, the cost of raising one unit of funds on the market is assumed to be higher for the supplier than for the bank ($r_B < r_S$). This is consistent with the special role of banks. Moreover, suppliers are likely to be credit-constrained themselves and to face a higher cost of raising funds than banks.

Contracts. The entrepreneur-bank contract specifies: $\{L_B, R_B^\sigma (y_\sigma, L_B), \gamma\}$, where L_B is the loan, $R_B^\sigma \geq 0$ is the state-contingent repayment obligation and γ is the share of the collateral obtained in case of default. That with the supplier of the tangible input specifies $\{q_t, L_S, R_S^\sigma (y_\sigma, L_S), (1-\gamma)\}$, where q_t is the input provision, L_S is the amount of credit, $R_S^\sigma \geq 0$ is the repayment obligation and $(1-\gamma)$ is the share of the collateral. Notice that unlike the bank the supplier can condition the contract also on the input purchase q_t. Last, given that the intangible input is fully paid for when purchased, the contract between entrepreneur and supplier specifies the amount of the input purchased, q_{nt}.

All parties have limited liability protection.

The sequence of events is summarized in the following diagram:



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[10] We assume that the entrepreneur cannot divert unused inputs if the bad state realizes (i.e. ex-post diversion is not allowed). However, allowing for this case does not alter our qualitative results, as long as some minimal share of the assets cannot be hidden (e.g., the premises of the firm, or heavy machinery).

Optimization problem. Firms maximize profits, which can be split into two components: the return from production (EP) and from diversion (D). The expected return from production is

$$EP = p\left[f^H\left(I_t, I_{nt}\right) - R_B^H - R_S^H\right] + (1-p)\left[f^L\left(I_t, I_{nt}\right) - R_B^L - R_S^L\right]. \tag{1}$$

Since output is zero in the bad state, limited liability implies that the repayments to banks and suppliers in this state are both zero ($R_B^L = R_S^L = 0$).[11]

The return from diversion is

$$D = \phi\left\{\beta_t\left(q_t - I_t\right) + \left[A + L_B + L_S - q_t\right]\right\},$$

where the term in round brackets denotes the return from tangible input diversion, net of the amount invested in production, and that in square brackets denotes the return from residual cash diversion (the amount of cash not spent on the input purchase). Notice that an opportunistic entrepreneur only purchases tangibles ($q_t \geq I_t \geq 0$) and never intangibles for diversion ($q_{nt} = I_{nt} = 0$).[12] Moreover, the inefficient diversion technology ($\phi < 1$) implies that partial diversion is never optimal. Thus, either all funds (and inputs) are used for investment ($D = 0$), or they are diverted, in which case none of the purchased inputs is invested: $I_t = 0$.[13]

The entrepreneur's optimization problem is defined by programme $\mathcal{P}_{\mathcal{G}}$:

$$\max_{L_B, L_S, I_t, I_{nt}, R_B^H, R_S^H, \gamma} EP + \phi\left[\beta_t q_t + A + L_B - \left(q_t - L_S\right)\right] \tag{2}$$

$$\text{s.t. } EP \geq \phi\left(A + L_B\right), \tag{3}$$

$$EP \geq \phi\left[\beta_t q_t + A + L_B - \left(q_t - L_S\right)\right], \tag{4}$$

$$pR_B^H + (1-p)\gamma\beta_B I_t \geq L_B r_B, \tag{5}$$

$$pR_S^H + (1-p)(1-\gamma)\beta_S I_t \geq L_S r_S, \tag{6}$$

$$I_{nt} + I_t \geq A + L_B + L_S, \tag{7}$$

$$R_S^H \geq (1-\gamma)\beta_S I_t, \tag{8}$$

[11] Banks and suppliers can still get a repayment in the bad state by sharing in the scrap value of unused inputs.

[12] This is obvious since intangibles are postulated to have zero liquidity, but the result also holds for any positive liquidity of the intangible.

[13] Suppose the entrepreneur invests an amount sufficient to repay the loan in full. Diverting the marginal unit gives a return $\phi\beta_t$. Investing it in production, the firm gets the expected marginal product, which in the first-best case equals $r_B\left[1 - \beta_S/r_S\right]$. If $\phi < 1$, the return from diversion is lower than the return from production. Thus, the entrepreneur always prefers to invest the marginal unit, and more so for the inframarginal units. Suppose instead that the entrepreneur invests an amount not sufficient to repay the loan in full. Because output is observable, any return from production will be claimed by creditors. It is better then to divert all inputs.

where (3) is the incentive compatibility condition *vis- à-vis* the bank, which prevents the entrepreneur from diverting internal funds as well as the credit raised from the bank, and (4) is the incentive constraint *vis-à-vis* the supplier, preventing the entrepreneur from diverting inputs and cash. Under conditions (3) and (4), there is no diversion in equilibrium, so that $D = 0$ and $q_t = I_t$. (5) and (6) are the participation constraints of the bank and the supplier, respectively, requiring that the lenders' expected returns cover at least the opportunity cost of funds. Competition in banking and among suppliers implies that (5) and (6) are binding. The resource constraint (7) requires that input purchase cannot exceed available funds. Last, condition (8) requires repayment of the supplier to be non-decreasing in revenues.[14]

Notice that if creditor protection is high enough (ϕ small), even a zero-wealth entrepreneur has no incentive problems (constraints (3) and (4) are always slack) and can fund the optimal investment. To exclude this uninteresting case, we introduce the following assumption:

Assumption 1 *:* $\phi > \underline{\phi}$.[15]

The Liquidation Motive - LM. Assume constraints (3) and (4) are slack. Constraints (5), (6) and (8) identify the *liquidation motive* for trade credit demand. As $\beta_S > \beta_B$, pledging the collateral to the supplier relaxes his participation constraint more than the bank's. As a consequence, the total repayment due from the entrepreneur in the good state decreases and total surplus increases. However, $r_B < r_S$ implies that the entrepreneur prefers bank credit to trade credit, i.e., $L_S = 0$. Having the supplier acting as a liquidator without taking any trade credit implies, using constraint (6), $R_S^H < 0$. As we are interested in the supplier's role as financier, we do not allow for such contracts and require repayment to be non-negative. Solving (6) for R_S^H, condition (8) implies a lower bound on trade credit equal to the collateral value of the inputs pledged to the supplier:

$$L_S \geq (1-\gamma)\,\frac{\beta_S}{r_S} I_t. \tag{9}$$

However, supplier's finance is profitable ($L_S > 0$) only if his opportunity cost of funds r_S, *discounted* for the saving in repayment obtained by pledging the collateral to the supplier rather than to the bank, is lower than the opportunity cost of funds of the bank r_B:

Assumption 2 $r_S \leq r_B \left\{ \dfrac{\beta_S}{p\beta_S + (1-p)\,\beta_B} \right\}$.

When this condition holds, the higher opportunity cost of funds of the supplier is offset by the higher proceeds in case of liquidation. Under Assumption 2, we derive the following lemma (unless otherwise stated, proofs of lemmas and propositions are given in Appendix 2):

[14] This condition is standard in the literature (Innes, 1990).
[15] The value of $\underline{\phi}$ is defined in Appendix 2.

Lemma 1 *At equilibrium, $\gamma = 0$, i.e. the right to repossess and liquidate the collateral goes to the supplier.*

Under Lemma 1, condition (9) sets the trade credit demand for liquidation motives equal to the discounted value of the collateral to the supplier:

$$L_{S,LM} = \frac{\beta_S}{r_S} I_t. \tag{10}$$

The incentive motive - IM. In addition to extracting more value from assets, trade credit can also relax financial constraints on the entrepreneur. Since diverting inputs is less profitable than diverting cash, the supplier is less vulnerable than banks to borrowers' opportunism and may thus be willing to provide credit when the bank is not ((3) is binding). In this case, the demand for trade credit is above the level defined by (10) and trade credit is taken for *incentive motives.* However, suppliers are not willing to meet all possible requests, since supplying too many inputs on credit may induce the entrepreneur to divert them all. The maximum trade credit extended for incentive motives is

$$L_{S,IM_{max}} = (1 - \beta_t) I_t, \tag{11}$$

which obtains when both incentive constraints (3) and (4) are binding. Notice that $(1 - \beta_t)$ measures the extent to which the supplier's informational advantage reduces moral hazard. If inputs are as liquid as cash ($\beta_t = 1$), this advantage is ineffective. The supplier cannot offer any trade credit when banks ration cash. Conversely, if inputs are illiquid, the informational advantage becomes important. The maximum line of trade credit is positive, and is greater the less liquid the inputs.

From the foregoing it follows that there are two types of demand for trade credit. One derives from the liquidation advantage of the supplier and depends on the collateral value. The second arises from his informational advantage and depends on the borrowing constraints on the firm (hence on the entrepreneur's wealth) and on input liquidity. Two regimes may then arise, depending on whether or not the liquidation motive demand (10) exceeds the maximum credit line extended for incentive motives (11). This condition can be redefined exclusively in terms of the parameters of the model as follows:

$$\frac{\beta_S}{r_S} - (1 - \beta_t) \lesseqqgtr 0. \tag{12}$$

Having defined the determinants of trade credit use, let us now turn to the results.

3 Results

Our results are presented in five parts. Section 3.1 identifies two regimes and examines how trade credit varies with entrepreneur's wealth between regimes. Section 3.2 focuses on the trade credit demand of financially unconstrained firms. Section 3.3 links the dominance of each regime to observable industry characteristics. Section 3.4 discusses the issue of cash lending by suppliers, and Section 3.5 investigates the relation between financing, technology and borrowing constraints.

3.1 Trade credit and two alternative regimes

As shown in Section 2, trade credit may be taken for liquidation or for incentive reasons. The way these two motives interact across different levels of wealth depends on condition (12). When (12) is strictly negative, wealthy entrepreneurs take trade credit for liquidation motives (LM), the less wealthy for incentive motives (IM). The share of inputs purchased on credit is non-increasing in wealth and larger for entrepreneurs that are credit-rationed. We define this regime as *dominant incentive motive*. When (12) is positive or zero, all entrepreneurs, regardless of wealth, take trade credit for liquidation reasons and the share of inputs purchased on credit is the same for rationed and non-rationed firms. We define this regime as *dominant liquidation motive*.

Our theoretical results reconcile an apparent conflict between the theoretical literature and the empirical evidence. On the one hand, in arguing that trade credit mitigates credit rationing by banks, the theoretical literature (Biais and Gollier, 1997; Burkart and Ellingsen, 2004) has highlighted a positive relation between trade credit and borrowing constraints. On the other hand, some empirical literature finds that reliance on trade credit is practically unaffected by the degree of credit rationing (Petersen and Rajan, 1997; Marotta, 2005). This section accounts for both these cases.

Dominant incentive motive. This regime arises when condition (12) is strictly negative.

Proposition 1 *There exist three critical levels of wealth, $A_1 < A_2 < A_3$ such that:*

(i) *for $A \geq A_3$ entrepreneurs finance the first-best investment (I_t^{FB}, I_{nt}^{FB}) and take trade credit for liquidation motives and bank credit as a residual. The share of inputs purchased on credit is equal to the scrap value of tangible inputs ($\frac{\beta_S}{r_S}$);*

(ii) *for $A_2 \leq A < A_3$, entrepreneurs are credit-constrained by banks, invest $I_t(A) \in \left[I_t^*, I_t^{FB}\right)$ and $I_{nt}(A) \in \left[I_{nt}^*, I_{nt}^{FB}\right)$ and take trade credit for liquidation motives, with a share $\frac{\beta_S}{r_S}$;*

(iii) *for $A_1 \leq A < A_2$, entrepreneurs are credit-constrained by banks, invest $I_t^* < I_t^{FB}$, and $I_{nt}^* < I_{nt}^{FB}$, and take trade credit for incentive motives. The share of inputs purchased on credit is decreasing in wealth and within the interval $\left(\frac{\beta_S}{r_S}, 1-\beta_t\right]$;*

(iv) *for $A < A_1$, entrepreneurs are constrained on both credit lines, invest $I_t(A) < I_t^*$ and $I_{nt}(A) < I_{nt}^*$, and take trade credit for incentive motives. The share of inputs purchased on credit is constant and equal to the proportion that cannot be diverted, $1-\beta_t$;*

where $A_1 = \frac{1}{r_B}\left\{(\phi + r_B)(\beta_t I_t^* + I_{nt}^*) + (1-\beta_t) I_t^* r_S - p f^H(I_t^*, I_{nt}^*) - (1-p)\beta_S I_t^*\right\}$,
$A_2 = A_1 + \frac{1}{r_B}\left(1 - \beta_t - \frac{\beta_S}{r_S}\right)(\phi + r_B - r_S) I_t^*$, *and*
$A_3 = \frac{1}{r_B}\left\{(\phi + r_B)\left(I_{nt}^{FB} + I_t^{FB} - \frac{\beta_S}{r_S} I_t^{FB}\right) - p\left[f^H\left(I_t^{FB}, I_{nt}^{FB}\right) - \beta_S I_t^{FB}\right]\right\}$.

Proposition 1 is illustrated in Figure 1. The population of entrepreneurs is distributed into four wealth areas with different degrees of credit rationing. For each area, the figure shows the motive for trade credit demand (liquidation or incentive), and the share of inputs purchased on account. Sufficiently rich entrepreneurs ($A \geq A_3$) finance the first-best investment by taking a constant amount of trade credit, equal to the discounted value of collateralized assets, and a variable amount of bank credit.[16] Notice that each unit of trade credit below this amount costs less than bank credit, since the supplier exploits the greater liquidation revenues accruing in the bad state to decrease the repayment required in the good state. Using (5), (6) and (8), the price of one unit of trade credit and one unit of bank credit is given by r_S and r_B/p, respectively. Under Assumption 2, $r_S < r_B/p$. An extra unit of trade credit above the level set in (10) costs more than bank credit, since there is no more collateral to pledge. This level is thus the optimal amount of trade credit taken for *liquidation motives*. As wealth comes down towards A_3, the amount of trade credit stays constant while bank credit increases to compensate for the lack of internal wealth. For $A < A_3$, the loan needed to finance the first-best investment implies a large repayment obligation that leaves the entrepreneur with a return lower than the return from diversion. Banks must therefore ration the entrepreneur to prevent opportunistic behavior, whence credit rationing. Suppliers are still willing to sell inputs on credit because they face a less severe incentive problem. For $A_2 \leq A < A_3$, however, firms do not yet increase trade credit demand, since the cost of an extra unit is still higher than the cost of bank credit. Thus, they are forced to reduce the investment below the first-best level, and also the absolute amount of trade credit and bank finance, but they keep the share of inputs purchased on account constant. Only for wealth below A_2, does the shadow cost of bank credit

[16] We are implicitly assuming the entrepreneur's wealth is never so high to finance entirely the first-best investment.

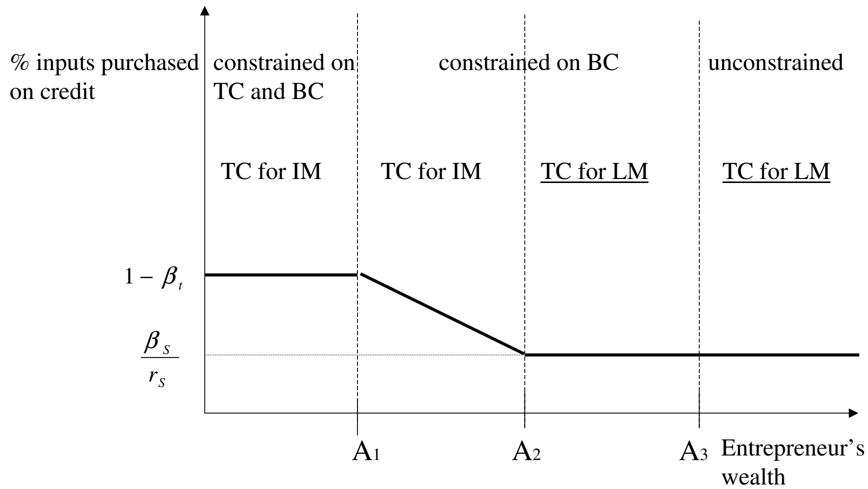

Figure 1: The regime where the incentive motive dominates

exceed the marginal cost of trade credit. Firms start demanding trade credit also for incentive motives, i.e., to relax financial constraints and keep the investment constant. Thus, the amount of bank credit stays constant, but both the absolute level of trade credit and the share of tangible inputs purchased on account rise to their maximum. This is reached at $A = A_1$, when the incentive constraint *vis-à-vis* the supplier also binds. For $A < A_1$, the entrepreneur is constrained on both credit lines and is forced to reduce investment further. Both trade and bank credit decrease, but the share of inputs purchased on credit stays constant at its maximum $(1 - \beta_t)$. In summary, across the wealth areas described in Figure 1, the share of inputs purchased on account is non-decreasing in credit rationing.

Corollary 1 *The bank gets a contract with repayments increasing in cash flows for any level of wealth, while the supplier gets a contract with flat repayments across states when $A \geq A_2$, and a contract with repayments increasing in cash flows when $A < A_2$.*

According to Corollary 1, the motivation for trade credit demand (incentive or liquidation) also affects the properties of the financial contract between the entrepreneur and the financiers. The proof is straightforward. By Lemma 1, the supplier always gets full priority in case of repossession of the collateral. Two cases may then arise. Trade credit may be demanded for liquidation motive ($A \geq A_2$ in Figure 1): the supplier gets the same return across states, equal to the scrap value of unused inputs.

Alternatively, trade credit is demanded for incentive motives ($A < A_2$ in Figure 1): the value of the unused scrap inputs is not sufficient to repay the supplier. An extra unit of trade credit can be provided only if higher repayment is promised in the good state. Therefore the supplier gets an increasing repayment contract, with an extra return for any unit of trade credit taken above the collateral value. Lastly, the bank always gets a contract with repayment increasing in cash flows. This is because it gets a positive return only in the good state, given that, by Lemma 1, the collateral is always repossessed by the supplier. ■

Dominant liquidation motive. This regime arises when condition (12) is positive or equal to zero.

Proposition 2 *There exists a critical level of wealth, $\hat{A}_1$, such that:*

(i) *for $A \geq \hat{A}_1$ entrepreneurs finance the first-best investment $(\hat{I}_t^{FB}, \hat{I}_{nt}^{FB})$ taking trade credit for liquidation motives and bank credit as a residual;*

(ii) *for $A < \hat{A}_1$, entrepreneurs are credit-constrained on both bank credit and trade credit. They invest $\hat{I}_t(A) < \hat{I}_t^{FB}$ and $\hat{I}_{nt}(A) < \hat{I}_{nt}^{FB}$ taking trade credit for liquidation motives;*

in either case, the share of inputs purchased on credit equals the scrap value of tangible inputs ($\frac{\beta_S}{r_S}$), and $\hat{A}_1 = \frac{1}{r_B}\left\{(\phi + r_B)\,\hat{I}_L^{FB} + \left[\left(1 - \frac{\beta_S}{r_S}\right) r_B + \phi\beta_t + p\beta_S\right]\hat{I}_t^{FB} - p\left[f^H\left(\hat{I}_t^{FB}, \hat{I}_{nt}^{FB}\right)\right]\right\}$.

Figure 2 illustrates Proposition 2 and has the same interpretation as Figure 1. In this case, there are only two wealth areas. For $A \geq \hat{A}_1$, firms are wealthy enough to finance the first-best investment without exhausting their credit lines. They use a constant amount of trade credit, equal to the scrap value of collateral assets, and, as wealth decreases, an increasing amount of bank credit. The funding from banks ceases when $A = \hat{A}_1$. At this level of wealth, because the amount of inputs financed on credit is already very large, the total funding obtained is so great that an extra amount of it, be it in cash or in kind, would induce the entrepreneur to divert all resources. Thus, for $A < \hat{A}_1$, being financially constrained on both credit lines, entrepreneurs are forced to reduce both sources of external financing as well as the investment level. In contrast with the previous regime, they keep financing a constant share of input by trade credit equal to β_S/r_S for any level of wealth. They have no incentive to alter it, since this would increase the total cost of financing: each unit of trade credit above the scrap value of collateral assets is more expensive than bank loans; similarly, each unit below this amount can only be replaced by more costly bank credit. Thus, in contrast with earlier financial theories, trade credit use

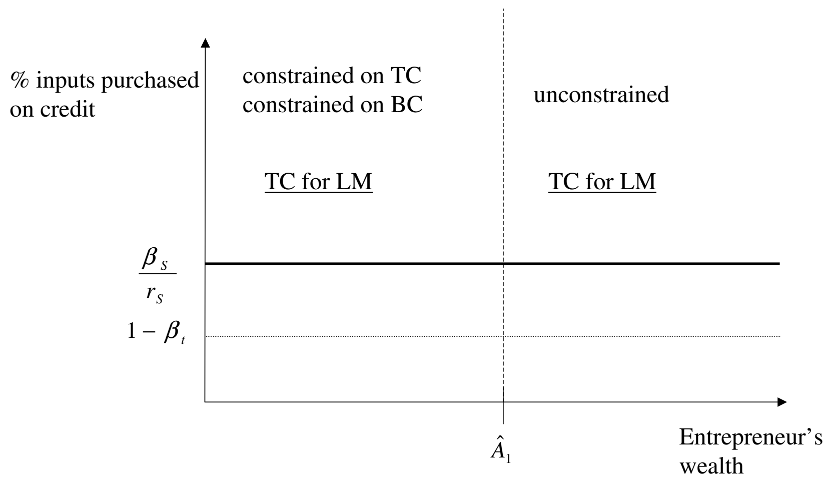

Figure 2: The regime where the liquidation motive dominates

is independent of financial constraints: both rationed and non-rationed firms purchase the same share of inputs on account, as the empirical evidence to date indicates. In this second regime, trade credit is never demanded to mitigate borrowing constraints but only for liquidation motives.

Corollary 2 *The bank gets a contract with repayment increasing in cash flows, while the supplier gets a contract with constant repayment across states for any level of wealth.*

Proof. Because trade credit is taken for liquidation motives and the share of inputs bough on credit stays constant across wealth and equal to β_S/r_S, by the same argument used in the proof of Corollary 1, the supplier gets a flat contract, while the bank gets an increasing repayment contract. ■

3.2 Trade credit demand of financially unconstrained firms

Points (i) of Propositions 1 and 2 focus on unconstrained firms and deliver a common prediction that is worth highlighting and discussing separately:

Prediction 1. *Financially unconstrained firms take trade credit in order to exploit their supplier's liquidation advantage. The amount of trade credit used equals the collateral value of tangible inputs pledged to the supplier (equation 10).*

This result fills a gap in the literature. As we saw in Section 1, earlier theories explain the existence of trade cedit but not its use by unconstrained firms, an empirical fact (see Petersen and

Rajan (1997) for the U.S. economy, Miwa and Ramseyer (2008) for Japan and Marotta (2005) for the Italian manufacturing sector).

Prediction 1 also posits that the use of trade credit is bound to the value of the inputs as collateral, in line with the evidence of Mian and Smith (1992) and Petersen and Rajan (1997).[17] This is because the supplier's liquidation advantage makes trade credit cheaper than bank loans only up to this collateral value.[18] Therefore, our liquidation story requires that: i) the input has a positive collateral value; ii) it is worth sufficiently more to the supplier than to the bank in case of default, which by Lemma 1 implies supplier's contractual seniority; iii) the bankruptcy law does not alter the contractually agreed claims held by creditors. Section 4 discusses these issues further.

Our result thus implies that even though the opportunity cost of funds is higher for input suppliers than for banks, trade credit can be cheaper than bank loans. This contrasts with the rather high interest rates implied by standard buyer-seller agreements, generally cited in the related litarature.

The lack of appropriate data has traditionally prevented econometricians from comparing the cost of funds borrowed from suppliers and from banks. This comparison requires information about the implicit trade credit rate charged by suppliers to their customers, which cannot be inferred from accounting data. More recently, however, rich survey data at the firm level became available.[19] Using this information, several recent papers document that trade credit can indeed be cheaper than bank loans. For example, Marotta (2005) shows that trade credit provided by Italian manufacturing firms is not more expensive than bank credit. Giannetti et al. (2006) document that the majority of the U.S. firms in their sample seems to receive cheap trade credit. Fabbri and Klapper (2008) find that for over 20% of Chinese firms in their sample, trade credit is cheaper than bank loans. Finally, Miwa and Ramseyer (2008) argue that firms borrow heavily from their suppliers at implicit rates that track the explicit rates banks would charge.

In short, the recent evidence on the cost of trade credit and the documented relationship between the liquidation value and trade credit use seem to be consistent with our story.

[17] Petersen and Rajan (1997) show that the supplier's liquidation advantage is one of the determinants of trade credit. They use the fraction of the firm's inventory not consisting of finished goods to proxy the liquidation advantage, based on the assumption that when the inputs are converted into finished goods, the supplier's liquidation advantage is lost.

[18] Other works have argued that trade credit may be cheap. Burkart and Ellingsen (2004), for example, say that if suppliers are unconstrained in the bank credit market, the trade credit interest rate might be as low as the bank rate. If there is a wedge between the banks' deposit rates and lending rates, the equilibrium trade credit interest rate may end up strictly below the bank rate. In Frank and Maksimovic (2004), as in our model, trade credit is cheap because of the liquidation advantage, but they do not derive any prediction for the trade credit demand by unconstrained firms.

[19] For example, firms are asked to report not only the interest rate on bank loans, but also very detailed information on trade credit terms received from their suppliers. Specifically, this information includes whether firms have been offered a discount on early payments, the extent of the discount and its application period, the number of days before imposing penalty.

3.3 The role of input characteristics

Let us extend the foregoing analysis to discuss the role of input characteristics in determining which regime dominates. This extension has a clear economic interpretation and provides several testable predictions. In our analysis (see inequality (12)), dominance depends on the liquidity of the tangible input (β_t) and its collateral value to the supplier (β_S). The incentive motive is more likely to arise among firms purchasing inputs that are illiquid or that have low collateral value (Figure 1). Conversely, the liquidation motive dominates among firms using relatively liquid or high-value inputs (Figure 2).

Since to some extent the two characteristics of the input reflect industry characteristics, we can use them to classify goods into categories/industries. One possible classification would distinguish services (low liquidity and low collateral value), standardized goods (high liquidity and low collateral value), and differentiated products (low liquidity and high collateral value). Using this classification, our theory provides three testable predictions on how the use of trade credit varies across industries.

Prediction 2. *Firms buying services make more purchases on account the tighter the credit constraints, while firms buying goods (both standardized and differentiated) finance the same share of their purchases on account independently of credit rationing.* This prediction can be derived by comparing the pattern of trade credit use across wealth areas between Figures 1 and 2.

Prediction 3. *Firms buying goods (both standardized and differentiated) make more purchases on account than firms buying services.* This prediction can be derived by focusing on the right hand sides of Figures 1 and 2, which isolate the use of trade credit by wealthy firms. Since these firms are unconstrained in the use of trade credit, we interpret them as the demand side of the trade credit market. This finding is consistent with the empirical evidence provided by Giannetti *et al.* (2006, p. 20) for firms taking trade credit.

Prediction 4. *Suppliers of services offer more trade credit than suppliers of standardized goods.* This prediction is derived by comparing the left hand sides of Figures 1 and 2, which isolate the maximum share of inputs purchased on account by poor firms. As they are constrained on trade credit, these firms are up against the supply side of the trade credit market. Since this prediction compares inputs with the same collateral value but different degrees of liquidity, it is useful to represent the maximum share of inputs purchased on account, namely $L_S/I_t = \max\left\{1-\beta_t, \frac{\beta_S}{r_S}\right\}$, as a function of input liquidity, β_t. This relation is represented in Figure 3. The pattern is weakly monotonic, in contrast to Burkart and Ellingsen (2004), who find a pattern always decreasing in the liquidity parameter. In particular, there is a threshold degree of liquidity, $\hat{\beta} = 1 - \frac{\beta_S}{r_S}$, such that if $\beta_t < \hat{\beta}$, the maximum

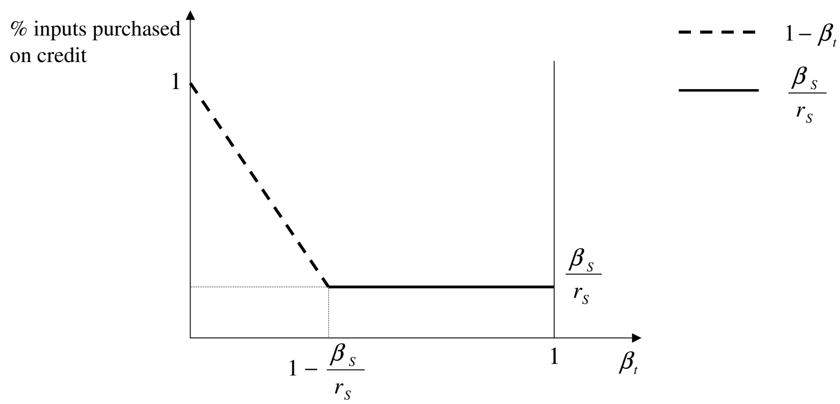

Figure 3: Trade credit line and input liquidity for constrained firms

share of tangible inputs financed by trade credit is decreasing in β_t. This situation corresponds to the dashed line in Figure 3. Conversely, if $\beta_t \geq \hat{\beta}$, the share is constant, which corresponds to the solid line. The two patterns capture the two motives for less wealthy firms to rely on trade credit. When the incentive motive dominates, this relation is negative because the supplier's information advantage becomes more important the more the inputs differ from cash, i.e. the less their liquidity in case of diversion. Conversely, when the liquidation motive is the driver, the use of trade credit does not depend on input liquidity. Producers of services are identified by the upper part of the dashed line ($\beta_t \simeq 0$), producers of standardized goods by the right side of the solid line ($\beta_t \simeq 1$). Thus, producers of services are willing to finance a larger share of inputs. This prediction is consistent with the empirical evidence presented by Giannetti *et al.* (2006, pp. 15, 17) for firms supplying trade credit.

3.4 Do suppliers ever lend cash?

Propositions 1 and 2 show that the share of tangible inputs financed by suppliers is always less than one. This means that despite the information advantage suppliers extend credit for their products but do not lend cash. The empirical evidence that cash lending does occur in Japan (Uesugi and Yamashiro, 2004) thus raises two questions that must be addressed. First, is the information advantage concerning the supplier's input always insufficient to induce him to lend cash? And second, what peculiar features does the Japanese trade credit market have? This section addresses both questions.

The lack of cash lending depends crucially on the assumption that the information advantage concerns *only* the purchase of the inputs provided by the supplier, in our case tangibles.[20] If the advantage extends to the other input as well (for example both creditors can partially but asymmetrically observe the intangible input purchase), then cash lending will occur. Denoting by δ_B and δ_S the degree of observability of intangibles by bank and supplier respectively, with $\delta_S > \delta_B$, the incentive constraints (3) and (4) are replaced by

$$\begin{aligned} EP &\geq \phi\left(A + L_B - \delta_B I_{nt}\right), \\ EP &\geq \phi\left(\beta_t I_t + A + L_B + \left(L_S - I_t - \delta_S I_{nt}\right)\right). \end{aligned}$$

Using the resource constraint (7) and assuming that both incentive constraints are binding, the maximum credit lines offered by suppliers and banks are[21]

$$\begin{aligned} L_S &= \left(1 - \beta_t\right) I_t + \left(\delta_S - \delta_B\right) I_{nt}, \\ L_B &= I_t \beta_t + \left(1 - \delta_S + \delta_B\right) I_{nt} - A. \end{aligned}$$

The supplier not only provides the inputs and allows deferred repayment for a share equal to $1 - \beta_t$ of their value, but also provides an amount of cash to finance the intangibles equal to a fraction $\delta_S - \delta_B$ of their value. Hence, in order for there to be cash lending, the supplier must also have an information advantage over the bank on the intangible input. The bigger this advantage $\delta_S - \delta_B$, the larger the amount of cash lending.

Interestingly, Uesugi and Yamashiro (2004) show that in Japan cash is lent by trading companies. These are large integrated firms, dealing with a variety of commodities and carrying out a range of business transactions sometimes including all the stages of production and marketing. Thus, Japanese trading companies can supply raw materials to manufacturing firms but also work as sales agents for them. Commodity transactions are supported by a variety of financial service, from trade credit to long-term and short-term loans, loan guarantees and investment in equities.[22] The supplier therefore provides many types of service to the same buyer. This organization guarantees a continuous flow of information, enabling the supplier to better monitor its customer. In line with our intuition, the Japanese evidence suggests that cash lending arises when the supplier's information advantage extends to various aspects of the firm's activity and is not confined exclusively to the firm-supplier relationship.

[20] Parlour and Rajan (2001) show that with multiple cash lenders equilibria are typically not competitive. This implies that cash lending is naturally exclusive and provides a further reason why suppliers do not lend cash.

[21] Notice that cash lending can occur only when there is an incentive motive for trade credit. There is no scope for borrowing cash if liquidation motive dominates.

[22] Examples include Mitsubishi Companies, Mitsui and Toyota Tsusho Corporation. This kind of business organization is rare in the rest of the world, except in Korea and China. See Uesugi and Yamashiro (2004) for a detailed description.

3.5 Input tangibility, financial decisions and creditor protection

The lack of cash lending implies that trade credit finances only tangible inputs. It follows that when a constrained entrepreneur uses trade credit to relax a borrowing constraint, he also distorts the input mix towards tangibles. This implies a link between financing and input choices across different levels of wealth and thus across different degrees of borrowing constraint.

We now explore this link and investigate the impact of changes in creditor protection. Greater use of trade credit goes together with an input bias towards tangible assets, and the bias becomes stronger when creditor vulnerability increases. The intuition is that since bank credit is more sensitive than trade credit to moral hazard, weaker creditor protection raises the relative cost of bank financing. Rationed entrepreneurs consequently rely more heavily on trade credit and shift towards more intensive use of tangible inputs.

We develop this intuition in the next two propositions, which relate asset tangibility, I_t/I_{nt}, and trade credit intensity, $L_S/\left(A+L_B+L_S\right)$, to firm wealth, A, and to the degree of creditor vulnerability, ϕ.[23] We restrict our analysis to homothetic functions, which have the property that the optimal input combination depends only on the input price ratio, in our case P_t/P_{nt} (tangible over intangible).[24]

Proposition 3 *Both asset tangibility and trade credit intensity are non-increasing in wealth.*

Proposition 4 *Greater creditor vulnerability increases both asset tangibility and trade credit intensity for any $A < A_1$ and $A_2 \leq A < A_3$; it increases trade credit intensity and leaves asset tangibility constant for any $A_1 \leq A < A_2$; it has no effect on either for any $A \geq A_3$.*

Figures 4 and 5 display trade credit intensity and input tangibility for different wealth levels. Firms with $A \geq A_3$ are unconstrained on both credit lines, so both the price ratios between trade and bank credit and those between inputs are invariant in wealth. It follows that both trade credit intensity and input tangibility hold constant for levels of wealth above A_3. When wealth falls below A_3, the moral hazard problem vis à vis the bank becomes binding. Reductions in wealth within the interval $A_2 \leq A < A_3$ increase the shadow cost of bank credit and thus decrease the price ratio between the two sources of funding. Firms give up more bank credit than trade credit, increasing trade credit intensity (solid line in the interval $A_2 \leq A < A_3$ of Figure 5). The higher price of bank credit also affects the two input prices, but by a different amount. It is translated fully into a higher price of intangibles, as they

[23]Propositions 3 and 4 refer to the case in which condition (12) is strictly negative (dominating incentive motive). However, qualitatively similar results hold also for the complementary case (dominating liquidation motive).

[24]This property simplifies the comparative statics analysis used to derive our results.

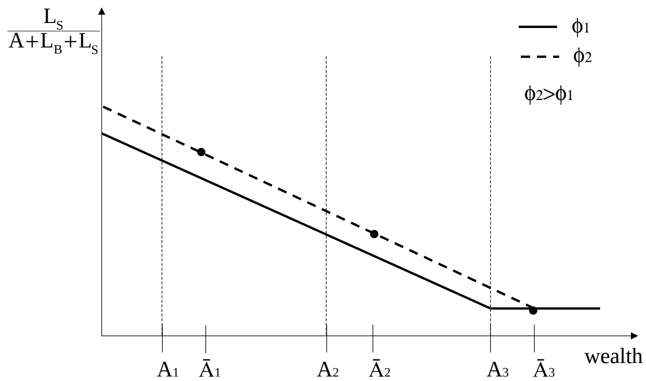

Figure 4: Trade credit intensity, wealth and creditor rights protection

are totally financed by bank credit, and only partially into a higher price of tangibles, given that only the share $(1 - \beta_S/r_S)$ is financed by bank credit. The input price ratio thus falls for decreasing levels of wealth, inducing entrepreneurs to increase input tangibility (solid line in the interval $A_2 \leq A < A_3$ of Figure 5).

When wealth falls below A_2, the shadow cost of bank credit equals the cost of trade credit. In the interval $A_1 \leq A < A_2$, firms are indifferent between sources of financing, but while they are constrained by banks, they are still unconstrained by suppliers and can therefore take trade credit at a constant price to compensate for their lesser wealth. Thus, trade credit intensity increases (solid line in the interval $A_1 \leq A < A_2$ of Figure 4). This extra credit is used to finance the purchase of tangibles, freeing resources to finance intangibles and leaving the input combination unchanged (solid line in the interval $A_1 \leq A < A_2$ of Figure 5). Finally, when wealth falls below A_1, entrepreneurs are financially constrained on both credit lines. The prices of both sources rise, but bank credit more than trade credit, given their differential exposure to moral hazard. As the tangible input is financed partly by trade credit, while the intangible is financed entirely by bank credit, the input price ratio decreases, increasing input tangibility (solid line in the area $A < A_1$ of Figure 5).

Consider now how trade credit intensity and input tangibility respond to an increase in creditor vulnerability (dotted lines in Figures 4 and 5). Notice first that any increase in ϕ moves all the threshold levels of A to the right, given that all the incentive constraints become binding at higher levels of wealth. Firms with $A \geq \bar{A}_3$ are unconstrained on both credit lines and neither trade credit intensity nor asset tangibility varies. When wealth decreases ($\bar{A}_2 \leq A < \bar{A}_3$), the incentive constraint on the bank becomes stringent and the shadow cost of bank credit rises. Thus, both the price ratio between bank and trade

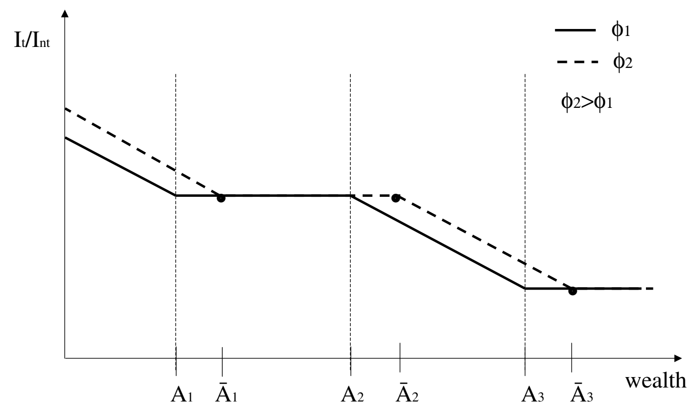

Figure 5: Input tangibility, wealth and creditor rights protection

credit and that between inputs increase, inducing entrepreneurs to rely more on trade credit and to shift towards a technology that relies more on tangible inputs (the dotted lines shift upwards in both graphs). When $\bar{A}_1 \leq A < \bar{A}_2$, the two sources of finance have the same price, but firms are not constrained by suppliers and can therefore use trade credit to keep investment and input combination constant (the dotted line does not shift upwards in Figure 5) and increase trade credit intensity (the dotted line shifts upwards in Figure 4). When $A < \bar{A}_1$, the change in ϕ makes the entrepreneur's moral hazard more severe *vis-à-vis* both bank and supplier. The prices of the two sources of finance and of the two inputs increase, but bank credit (intangibles) rises more than trade credit (tangibles), since only the fraction β_t of tangibles can be diverted. Thus, both trade credit intensity and asset tangibility increase, as is shown by the upward shift of the dotted lines in both figures.

The previous analysis allows us to obtain the following predictions:

Prediction 5. *Credit-constrained firms have higher trade credit intensity and use technologies more intensive in tangible assets than unconstrained ones.*

Moreover, if we assume that countries only differ in the degree of creditor protection:

Prediction 6. *When located in countries with weaker creditor protection, credit-constrained firms have higher trade credit intensity and a technological bias towards tangibles. Unconstrained firms have the same trade credit intensity and input tangibility across countries with different degrees of creditor protection.*

Taking into account that credit constrained firms are more widespread in countries with weaker creditor protection, Prediction 6 is consistent with two distinct sets of empirical evidence. First, there is a greater use of trade credit in countries with less creditor protection, including developing countries (see among others, Rajan and Zingales, 1995; La Porta *et al.*, 1998; Fisman and Love, 2003; Frank and Maksimovic, 2004). Second, firms in developing countries have a higher proportion of fixed to total assets and fewer intangible assets than those in developed countries (e.g., Demirguc-Kunt and Maksimovic, 2001). Our paper thus offers a theory that reconciles these distinct findings.

4 The role of the legal framework

The liquidation story discussed in Sections 3.1 and 3.2 presupposes that in case of bankruptcy priority should be assigned on efficiency basis, i.e. to the supplier ($\gamma = 1$ by Lemma 1). However, the legal system may prevent the supplier from seizing particular goods, thus eliminating the liquidation motive for trade credit and hence "contractual seniority." One way to obtain this outcome is to design debtor- rather than creditor-oriented bankruptcy codes, which subordinate all creditor rights, including suppliers' rights, to the firm's survival. A second, more specific, way is to establish priority rules that privilege certain creditors over suppliers. Although a thorough analysis is beyond the scope of this paper, we can discuss these two aspects of bankruptcy codes, to understand how they may alter our results.

Regarding the first aspect, the French bankruptcy law, for instance, has the stated objective of helping distressed firms (Biais and Mariotti, 2003) and favoring their reorganization, with an *automatic stay* against secured creditors that prevents them from removing their collateral during the reorganization period. The German bankruptcy law has similar provisions, with a greater role assigned to creditors in the decision to reorganize. These two systems can be seen as debtor-oriented, as opposed to the Anglo-Saxon codes that are traditionally creditor-oriented. The U.S. bankruptcy law gives managers the choice between filing for bankruptcy under Chapter 7 (liquidation) or under Chapter 11 (reorganization). Liquidation can therefore occur without prior reorganization, and according to Franks *et al.* (1996), the majority of U.S. bankruptcies are actually processed through Chapter 7.[25] In the U.K., there are two bankruptcy regimes: receivership and administration order. Under the first, if a firm defaults a creditor holding a general secured interest in its assets, known as a *floating charge*, may appoint a receiver with the right to sell any assets to repay the claim, except those that are subject to another creditor's lien.[26] To prevent the liquidation of the firm's assets, an administration order can be issued, appointing a bankruptcy official with the task of proposing a reorganization plan to the

[25] They report that in the Central District of the California Bankruptcy Court there were 57,752 Chapter 7 cases pending as compared with only 6,739 Chapter 11 cases.

[26] For example those subject to a *fixed charge*, i.e. with a security on a specific asset such as heavy machinery.

creditors' committee. However, unlike in the U.S., an administration order cannot block a receivership procedure that has already started, except with the consent of the holder of the floating charge.

Regarding the more specific issue of the priority rule, it is generally true that trade credit is junior, unless it is secured, in which case the supplier can reclaim any good not yet transformed into output. This limits the types of good that can be secured, generally not intermediate goods or services, but rather durable goods. One might therefore expect the demand for trade credit to be driven, among other things, by the seller's ability to create a lien, hence by input characteristics. Notice that this prediction is fully in line with our analysis, where we find that the liquidation motive is stronger where the scrap value of the inputs is larger (β_S high). In countries like the U.K. and the U.S., trade creditors also have specific liquidation rights. In the U.K., suppliers can include a *Retention of Title* clause in the sale contract allowing them to reclaim all the goods supplied on credit in case of bankruptcy, as long as they are distinguishable from other suppliers' goods. Such Title makes them become first in the order of seniority along with the holders of fixed charges (Franks and Sussman, 2005). In the U.S., even when the firm is not under a bankruptcy procedure, the Uniform Commercial Code gives the seller the right to reclaim the goods sold to an insolvent buyer within ten days after delivery.[27]

If the effects of such legal provisions are incorporated into our model, the role played by the liquidation motive will depend on both input and bankruptcy code characteristics. In particular, we get the following prediction:

Prediction 7. *The liquidation motive is more important when the good is durable (β_S high), when it has not been transformed into finished products (β_S high), and when bankruptcy codes protect contractual rights in general and supplier debt claims in particular (γ high).*

This discussion acknowledges the effect of legal institutions on the demand for trade credit, in line with previous studies, but we also identify a new channel. Legal institutions affect the use of trade credit through the degree of legal protection granted to the supplier. The economic force is the liquidation motive. Conversely, in the related literature, legal institutions affect the reliance on trade credit through the legal protection granted to banks rather than suppliers.

5 Conclusion

The paper investigates the determinants of trade credit and its interactions with borrowing constraints, input combination and creditor protection. The paper proposes a model of collateralized bank and

[27] This deadline does not apply if misrepresentation of solvency has been made to the seller in writing. See Garvin (1996) for more details.

trade credit. Firms use a two-input technology.

We explain the use of trade credit by financially unconstrained firms as the effect of the seller's advantage in liquidating the inputs in case of default. This complements previous financial theories of trade credit, which explain its use only by rationed firms relying on an incentive motive.

By interacting liquidation and incentive motives, we also show that, as financial constraints tighten, the share of inputs purchased on credit may stay constant or increase. The dominance of one regime or the other depends on input characteristics, such as liquidity and collateral value. We then derive testable predictions on how trade credit varies among firms using differentiated or standardized inputs, rather than services.

In addition, we find that suppliers will only lend inputs that they sell and we identify the conditions under which they will lend cash. Finally, we show that input and financing decisions are strictly related and both react to changes in creditor protection. More intensive use of trade credit goes together with an input combination biased towards tangible assets. Weaker creditor protection increases both the reliance on trade credit and the degree of tangibility of inputs.

Our analysis could be extended in several directions. The most direct would be to test the predictions derived in Section 3.5 on the relation among input choices, financing decisions and legal institutions empirically. From a theoretical point of view, it would be interesting to explore the effects of the supplier's making input provision conditional on the purchase of complementary inputs. In our model this assumption would imply that the intangible input is partially observable. This generates cash lending by the supplier with potential implications for input choices and for financing decisions that are still unexplored.

Appendix 1

Table of notation
$y^\sigma = f^\sigma(\cdot,\cdot)$: state-contingent output with $\sigma \in \{H, L\}$
p : probability of state $\sigma = H$
q_t : purchase of tangible input
q_{nt} : purchase of intangible input
I_t : investment in tangible input
I_{nt} : investment in intangible input
β_t : degree of liquidity of the tangible input
ϕ : degree of creditor rights vulnerability
A : entrepreneur's wealth
L_B : bank credit
L_S : supplier credit
β_S : value of one unit of collateral asset to the supplier
β_B : value of one unit of collateral asset to the bank
r_B : bank's cost of raising one unit of funds on the market
r_S : supplier's cost of raising one unit of funds on the market
R_B^σ : state-contingent repayment due to the bank
R_S^σ : state-contingent repayment due to the supplier
$\gamma, (1-\gamma)$: share of collateral obtained in case of default by bank and supplier, respectively

Appendix 2

To prove our results, let us redefine programme $\mathcal{P}_\mathcal{G}$ as follows:

$$\max_{L_B, L_S, I_t, I_{nt}, R_B^H, R_S^H, \gamma} EP = p\left[f^H(I_t, I_{nt}) - R_B^H - R_S^H\right]$$

$$\text{st } EP \geq \max\left\{\phi(A+L_B), \phi\left[\beta_t I_t + A + L_B - (I_t - L_S)\right]\right\} \tag{13}$$
$$L_S \gtreqless (1-\beta_t) I_t \tag{14}$$
$$pR_B^H + (1-p)\gamma\beta_B C = L_B r_B \tag{15}$$
$$pR_S^H + (1-p)(1-\gamma)\beta_S C = L_S r_S \tag{16}$$
$$A + L_B + L_S = I_{nt} + I_t \tag{17}$$
$$R_S^H \geq (1-\gamma)\beta_S C \tag{18}$$

Notice that the return from diversion in the incentive constraint (13) is expressed as the maximum between cash-only diversion and input-and-cash diversion. Which one of the two is higher depends on how many inputs the entrepreneur buys on credit in equilibrium, i.e. how much trade credit he uses. If $L_S \leq (1-\beta_t) I_t$, the return from cash diversion is no less than the return from input-and-cash diversion. The relevant temptation facing the entrepreneur in this case is to divert all cash $(A + L_B \geq \beta_t I_t + A + L_B - (I_t - L_S))$. If $L_S > (1-\beta_t) I_t$, instead, the return from cash diversion is strictly less than the return from input-and-cash diversion. The entrepreneur may then be tempted to borrow cash from the bank, buy inputs on credit from the supplier and divert both cash and inputs. Whether in equilibrium $L_S \gtreqless (1-\beta_t) I_t$ depends on the amount of trade credit that is taken for liquidation motives.

Definition 1 *The threshold level of ϕ below which a zero-wealth firm can carry out the level of investment which is optimal by using both bank credit and trade credit is given by* $\underline{\phi} = \frac{pf^H(I_t,I_{nt})-(I_{nt}+I_t)r_B+(r_B-r_S)L_S+(1-p)(\gamma\beta_B+(1-\gamma)\beta_S)I_t}{I_{nt}+I_t-L_S}$, *where I_t, I_{nt} solve the first order conditions of programme $\mathcal{P}_{\mathcal{G}}$, with the incentive constraint (13) slack and* $L_S = \max\left\{(1-\beta_t) I_t, \frac{\beta_S}{r_S} I_t\right\}$.

Proof of Lemma 1. Under assumption 2, the entrepreneur takes trade credit to exploit the supplier's liquidation advantage. Solving (15) and (16) for R_B^H and R_S^H, (18) sets the minimum demand for trade credit as $L_S \geq (1-\gamma)\frac{\beta_S}{r_S} I_t$. Assuming this to be binding and using it in $\mathcal{P}_{\mathcal{G}}$, we get:

$$\max_{I_t,I_{nt},L_B,\gamma} EP = pf^H(I_t,I_{nt}) - L_B r_B - p(1-\gamma)\beta_S C + (1-p)\gamma\beta_B C \tag{19}$$

$$\text{st } EP \geq \max\left\{\phi(A+L_B), \phi\left(A+L_B+\tfrac{(1-\gamma)\beta_S}{r_S}C-(1-\beta_t)I_t\right)\right\} \tag{20}$$

$$A+L_B+\tfrac{(1-\gamma)\beta_S}{r_S}C = I_{nt}+I_t \tag{21}$$

Solving (21) for L_B, we define programme $\mathcal{P}_{\mathcal{F}}$:

$$\begin{aligned}\max_{I_t,I_{nt},\gamma} EP_F &= p\left[f^H(I_t,I_{nt})-(1-\gamma)\beta_S C\right] - \left(I_{nt}+I_t-A-\tfrac{1-\gamma}{r_S}\beta_S C\right)r_B + (1-p)\gamma\beta_B C &(22)\\ \text{st } EP_F &\geq \max\left\{\phi\left(I_{nt}+I_t-\tfrac{1-\gamma}{r_S}\beta_S C\right), \phi(I_{nt}+\beta_t I_t)\right\} &(23)\end{aligned}$$

Defining λ_1 as the multiplier of constraint (23), the Lagrangean is:

$$\Lambda_F = EP_F + \lambda_1\left[EP_F - \max\left\{\phi\left(I_{nt}+I_t-\tfrac{1-\gamma}{r_S}\beta_S C\right), \phi(\beta_t I_t + I_{nt})\right\}\right] \tag{24}$$

Differentiating Λ_F wrt γ :

$$\frac{\partial\Lambda_F}{\partial\gamma} : \left(p(\beta_S-\beta_B) - \tfrac{1}{r_S}(r_B\beta_S - r_S\beta_B)\right)(1+\lambda_1) - \lambda_1 \max\left\{\tfrac{\beta_S}{r_S}\phi, 0\right\} \leq 0 \tag{25}$$

Under Assumption 2, $\frac{\partial\Lambda_F}{\partial\gamma} \leq 0$, which implies that $\gamma = 0$ and proves the lemma.[28] ■

Proof of Proposition 1. (**Dominant incentive regime**) When condition (12) is strictly negative, $L_S \in \left[\frac{\beta_S}{r_S} I_t, (1-\beta_t) I_t\right]$ and the relevant incentive constraint in (13) is the one *vis-à-vis* the bank.

The proposition is proved in steps: we first prove that *a*) $I_i(A_{A<A_1}) < I_i^* < I_i(A_{A_2 \leq A < A_3}) < I_i^{FB}$, $i = t, nt$; then that *b*) the critical levels A_1, A_2 and A_3, exist and are unique. To establish part *a*), we first focus on $A \geq A_2$, where the entrepreneur demands trade credit only for liquidation motives, and then on $A < A_2$, where the entrepreneur demands trade credit also for incentive motives.

$\mathbf{A} \geq \mathbf{A}_2$.

[28] When $\frac{\partial\Lambda_F}{\partial\gamma} = 0$, $\gamma \in [0,1]$. We take it to be zero.

The entrepreneur takes trade credit only to exploit the supplier's liquidation advantage and $L_S = \frac{\beta_S}{r_S} I_t < (1-\beta_t) I_t$. The problem is to solve programme $\mathcal{P}_{\mathcal{F}}$, where the relevant incentive constraint in (23) is the one *vis-à-vis* the bank. Using Lemma 1, the FOC's are:

$$\frac{\partial \Lambda_F}{\partial I_t} \quad : \quad p\frac{\partial f^H}{\partial I_t} - r_B - \beta_S \left(p - \frac{r_B}{r_S}\right) = \frac{\lambda_1}{1+\lambda_1}\phi \left(1 - \frac{\beta_S}{r_S}\right) \tag{26}$$

$$\frac{\partial \Lambda_F}{\partial I_{nt}} \quad : \quad p\frac{\partial f^H}{\partial I_{nt}} - r_B = \frac{\lambda_1}{1+\lambda_1}\phi \tag{27}$$

$$\frac{\partial \Lambda_F}{\partial \lambda_1} \quad : \quad EP_F \geq \phi \left[I_{nt} + \left(1 - \frac{\beta_S}{r_S}\right) I_t\right] \tag{28}$$

Conditions (26) and (27) can also be written as:

$$\frac{r_S}{r_S - \beta_S}\left(\frac{\partial f^H}{\partial I_t} - \beta_S\right) = \frac{\partial f^H}{\partial I_{nt}} \tag{29}$$

Within $A \geq A_2$, we further distinguish two wealth areas: $A \geq A_3$ and $A_2 \leq A < A_3$.

$\mathbf{A} \geq \mathbf{A}_3$: In this case the incentive constraint (23) is slack and the firm invests I_t^{FB}, I_{nt}^{FB} solving (26) and (27) with $\lambda_1 = 0$. The optimal financial contract has the following properties:

$$\begin{aligned}
R_S^H &= \beta_S I_t^{FB} \\
L_S &= \frac{\beta_S}{r_S} I_t^{FB} \\
L_B &= I_{nt}^{FB} + \left(1 - \frac{\beta_S}{r_S}\right) I_t^{FB} - A \\
R_B^H &= \frac{r_B}{p}\left(I_{nt}^{FB} + \left(1 - \frac{\beta_S}{r_S}\right) I_t^{FB} - A\right)
\end{aligned}$$

$\mathbf{A}_2 \leq \mathbf{A} < \mathbf{A}_3$: The incentive constraint (23) is binding and the firm invests $I_i(A) \in [I_i^*, I_i^{FB})$, where $I_i(A)$, $i = t, nt$, solve (26) and (27) with $\lambda_1 > 0$.

To prove that $I_i^* < I_i^{FB}$, consider the FOC's (26) and (27). Relative to the first-best ($\lambda_1 = 0$), there is now an increase in the cost of both factors. In order to derive the implications of such rise on the levels of I_i, we totally differentiate (26) and (27), and get:

$$\begin{bmatrix} \frac{\partial^2 f(\cdot,\cdot)}{\partial I_t^2} & \frac{\partial^2 f(\cdot,\cdot)}{\partial I_{nt}\partial I_t} \\ \frac{\partial^2 f(\cdot,\cdot)}{\partial I_{nt}\partial I_t} & \frac{\partial^2 f(\cdot,\cdot)}{\partial I_{nt}^2} \end{bmatrix} \begin{bmatrix} dI_t \\ dI_{nt} \end{bmatrix} = \begin{bmatrix} dP_t \\ dP_{nt} \end{bmatrix} \tag{M_1}$$

where $dP_t > 0$, $dP_{nt} > 0$ are the changes in the cost of factors induced by a change in one of their determinants. Inverting (M_1), we can solve for the vector of unknowns :

$$\begin{bmatrix} dI_t \\ dI_{nt} \end{bmatrix} = \frac{1}{H} \begin{bmatrix} \frac{\partial^2 f(\cdot,\cdot)}{\partial I_{nt}^2} & -\frac{\partial^2 f(\cdot,\cdot)}{\partial I_{nt}\partial I_t} \\ -\frac{\partial^2 f(\cdot,\cdot)}{\partial I_{nt}\partial I_t} & \frac{\partial^2 f(\cdot,\cdot)}{\partial I_t^2} \end{bmatrix} \begin{bmatrix} dP_t \\ dP_{nt} \end{bmatrix} \tag{M_2}$$

where H is the determinant of the Hessian, which is positive assuming the Hessian to be negative semi-definite. Thus, if factors are substitutes, i.e. $\frac{\partial^2 f(\cdot,\cdot)}{\partial I_t \partial I_{nt}} > 0$, then

$$\begin{aligned}
dI_t &= \frac{1}{H}\left(\frac{\partial^2 f(\cdot,\cdot)}{\partial I_{nt}^2} dP_t - \frac{\partial^2 f(\cdot,\cdot)}{\partial I_{nt}\partial I_t} dP_{nt}\right) < 0 \\
dI_{nt} &= \frac{1}{H}\left(-\frac{\partial^2 f(\cdot,\cdot)}{\partial I_t \partial I_{nt}} dP_t + \frac{\partial^2 f(\cdot,\cdot)}{\partial I_t^2} dP_{nt}\right) < 0
\end{aligned}$$

which implies that both factors are under-invested.[29]

The optimal financial contract has the following properties:

$$\begin{aligned} R_S^H &= \beta_S I_t(A), \\ L_S &= \tfrac{1}{r_S}\beta_S I_t(A), \\ L_B &= I_{nt}(A) + \left(1 - \tfrac{\beta_S}{r_S}\right) I_t(A) - A, \\ R_B^H &= \tfrac{r_B}{p}\left(I_{nt}(A) + \tfrac{1}{r_S}(r_S - \beta_S) I_t(A) - A\right) \end{aligned}$$

where $I_t(A), I_{nt}(A)$ solve (26), (27) and (28) with $\lambda_1 > 0$.

$\mathbf{A} < \mathbf{A}_2$.

The entrepreneur is still constrained on bank credit, but, unlike the case in which $A_2 \leq A < A_3$, the shadow price of bank credit is so high that he finds it worthwhile to take trade credit not only for liquidation, but also for incentive motives. Thus $L_S > \frac{\beta_S}{r_S} I_t$.[30] However, to persuade the supplier to increase financing, the entrepreneur has to offer him a contract with repayments increasing in cash flows. Thus, the non-decreasing repayments condition (18) is slack. The optimal contract solves programme $\mathcal{P}_{\mathcal{G}}$ subject to the binding incentive constraint (13) *vis-à-vis* the bank and to constraint (14) as $L_S \leq (1-\beta_t) I_t$. Solving the resource constraint (17) for L_S, programme $\mathcal{P}_{\mathcal{G}}$ can be written as:

$$\max_{I_t, I_{nt}, L_B} EP_I = pf^H(I_t, I_{nt}) - L_B r_B - (I_t + I_{nt} - A - L_B) r_S + (1-p)\beta_S I_t \tag{30}$$

$$\text{s.t. } EP_I = \phi(A + L_B) \tag{31}$$

$$L_B \geq I_{nt} + \beta_t I_t - A \tag{32}$$

which, using binding (31), becomes:

$$\max_{I_t, I_{nt}} EP_{GI} = pf^H(I_t, I_{nt}) - (I_{nt} - A) r_B - [(1-\beta_t) r_S + \beta_t r_B - (1-p)\beta_S] I_t$$

$$\text{s.t. } EP_{GI} \geq \phi\{\beta_t I_t + I_{nt}\} \tag{33}$$

where (33) is the global incentive constraint. Setting up the Lagrangean $\Lambda_G = EP_{GI} + \lambda_2 [EP_{GI} - \phi(\beta_t I_t + I_{nt})]$, the FOC's are:

$$\frac{\partial \Lambda_G}{\partial I_t} : p\frac{\partial f^H}{\partial I_t} + \beta_S(1-p) - r_S(1-\beta_t) - r_B\beta_t = \phi\beta_t \frac{\lambda_2}{1+\lambda_2} \tag{34}$$

$$\frac{\partial \Lambda_G}{\partial I_{nt}} : p\frac{\partial f^H}{\partial I_{nt}} - r_B = \frac{\lambda_2}{1+\lambda_2}\phi \tag{35}$$

$$\frac{\partial \Lambda_G}{\partial \lambda_2} : EP_{GI} - \phi(\beta_t I_t + I_{nt}) \geq 0 \tag{36}$$

where λ_2 is the multiplier of the global incentive constraint. Conditions (34) and (35) can also be written as:

$$p\frac{\partial f^H}{\partial I_t} + \beta_S(1-p) - r_S(1-\beta_t) = p\beta_t \frac{\partial f^H}{\partial I_{nt}}. \tag{37}$$

Within $A < A_2$, we can further distinguish between two wealth areas: $A_1 \leq A < A_2$ and $A < A_1$.

[29] Notice that this result holds also for the case in which factors are complements, provided the Hessian has a dominant diagonal.

[30] This is feasible since the amount of trade credit taken for liquidation does not exhaust the maximum credit line offered by the supplier to a rationed entrepreneur (11) (recall that we are in the case in which $1 - \beta_t > \frac{\beta_S}{r_S}$).

$\mathbf{A}_1 \leq \mathbf{A} < \mathbf{A}_2$: The incentive constraint (33) is slack ($\lambda_2 = 0$). This implies that the entrepreneur can keep investing I_t^*, I_{nt}^* even for decreasing levels of wealth until (33) becomes binding. The properties of the optimal contract are defined by:

$$\begin{aligned}
L_S &\in \left(\tfrac{\beta_S}{r_S} I_t^*, (1-\beta_t) I_t^*\right] \\
L_B &\in \left[(I_{nt}^* + \beta_t I_t^* - A), I_{nt}^* + I_t^* \left(1 - \tfrac{\beta_S}{r_S}\right) - A\right) \\
R_S^H &\in \left(\left(\tfrac{\beta_S}{r_S} - (1-p)\beta_S\right) \tfrac{I_t^*}{p}, \left[(1-\beta_t) r_S - (1-p)\beta_S\right] \tfrac{I_t^*}{p}\right] \\
R_B^H &\in \left[\tfrac{r_B}{p}(I_{nt}^* + \beta_t I_t^* - A), \tfrac{r_B}{p}\left[I_{nt}^* + I_t^* \left(1 - \tfrac{\beta_S}{r_S}\right) - A\right]\right)
\end{aligned}$$

where I_t^*, I_{nt}^* solve (34) and (35) with $\lambda_2 = 0$.[31]

$\mathbf{A} < \mathbf{A}_1$: The incentive constraint *vis-à-vis* the supplier becomes binding ($\lambda_2 > 0$) and (34) and (35) imply that $I_t(A) < I_t^*$, and $I_{nt}(A) < I_{nt}^*$.[32] The contract has the following properties:

$$\begin{aligned}
L_S &= (1-\beta_t) I_t(A) \\
L_B &= I_{nt} + \beta_t I_t(A) - A \\
R_S^H &= \tfrac{1}{p}\left((1-\beta_t) r_S - (1-p)\beta_S\right) I_t(A) \\
R_B^H &= \tfrac{1}{p}\left(I_{nt}(A) + \beta_t I_t(A) - A\right) r_B
\end{aligned}$$

where $I_t(A)$, $I_{nt}(A)$ solve (34)/(35) and (36). ■

Part (b) is proved using the following lemma:

Lemma 2 *For any $r_B + \phi > r_S$ and $1 - \beta_t > \frac{\beta_S}{r_S}$ there exists a triple of threshold values A_i, $i = 1, 2, 3$, such that:*

1. $pf^H\left(I_t^{FB}, I_{nt}^{FB}\right) - \bar{L}_B r_B - p\beta_S I_t^{FB} - \phi\left(A + \bar{L}_B\right) = 0$ *for* $A = A_3(p, r_B, r_S, \phi, \beta_S)$, *with* $\bar{L}_B = I_t^{FB}\left(1 - \frac{\beta_S}{r_S}\right) + I_{nt}^{FB} - A$;

2. $pf^H\left(I_t^*, I_{nt}^*\right) - \bar{L}_B r_B - p\beta_S I_t^* - \phi\left(A + \bar{L}_B\right) = 0$ *for* $A = A_2(p, r_B, r_S, \phi, \beta_S)$, *with* $\bar{L}_B = I_t^*\left(1 - \frac{\beta_S}{r_S}\right) + I_{nt}^* - A$;

3. $pf^H\left(I_t^*, I_{nt}^*\right) - \bar{L}_B r_B - \bar{L}_S r_S + (1-p)\beta_S I_t^* - \phi\left(A + \bar{L}_B\right) = 0$ *for* $A = A_1(p, r_B, r_S, \phi, \beta_S, \beta_t)$, *with* $\bar{L}_B = \beta_t I_t^* + I_{nt}^* - A$ *and* $\bar{L}_S = (1-\beta_t) I_t^*$;

4. $A_3 > A_2 > A_1 > 0$.

Proof.

1. : The threshold $A_3(p, r_B, r_S, \phi, \beta_S)$ is the minimum wealth that allows the entrepreneur to invest I_t^{FB}, I_{nt}^{FB} fully exploiting the bank credit line and taking trade credit only for liquidation motives.[33]

[31] Notice that, while the level of the two inputs is constant in the above interval, the repayments due to bank and supplier in the two states vary with wealth.

[32] The proof of this result is analogous to the one obtained for the case in which $A_2 \leq A < A_3$ and thus omitted.

[33] This amounts to say that $A_3 + \bar{L}_B + L_{S,LM} = A_3 + \bar{L}_B + \dfrac{\beta_S}{r_S} I_t^{FB} = I_t^{FB} + I_{nt}^{FB}$.

Thus A_3 must satisfy:

$$A_3 = \frac{1}{r_B}\left\{(\phi + r_B)\left(I_{nt}^{FB} + I_t^{FB} - \frac{\beta_S}{r_S}I_t^{FB}\right) - p\left[f^H\left(I_t^{FB}, I_{nt}^{FB}\right) - \beta_S I_t^{FB}\right]\right\} \tag{38}$$

To prove that this threshold exists and is unique we need to show that: $(1a)$ $0+\bar{L}_B+L_{S,LM} < I_t^{FB}+I_{nt}^{FB}$, which follows from Assumption 1 ($\phi > \underline{\phi}$); $(1b)$ $\bar{L}_B$ is continuously increasing in A. To establish part $(1b)$, it is useful to define the following functions, obtained by taking the derivatives of constraint (23) wrt I_t and I_{nt} respectively:

$$\begin{aligned} h_{t1} &= p\frac{\partial f^H}{\partial I_t} - (r_B + \phi)\left(1 - \frac{\beta_S}{r_S}\right) - p\beta_S & (39)\\ h_{nt1} &= p\frac{\partial f^H}{\partial I_{nt}} - r_B - \phi & (40)\end{aligned}$$

Constraint (23) is only binding if $h_{t1}, h_{nt1} < 0$, otherwise I_t and I_{nt} could be further increased without violating the constraint.

Using (7) and (10), we deduce that $I_t = \frac{A+L_B-I_{nt}}{(r_S-\beta_S)}r_S$. The maximum bank credit line $\bar{L}_B$, given by the binding constraint (20),[34] can therefore be written as a function of I_{nt}, $\bar{L}_B$ and A : $pf^H\left(\left(\frac{A+\bar{L}_B-I_{nt}}{(r_S-\beta_S)}r_S\right), I_{nt}\right) - \bar{L}_B r_B - p\beta_S r_S\left(\frac{A+\bar{L}_B-I_{nt}}{r_S-\beta_S}\right) - \phi\left(A+\bar{L}_B\right) = 0$. Totally differentiating:

$$\begin{gathered}\left\{p\frac{\partial f^H(\cdot,\cdot)}{\partial I_{nt}} + \frac{p\beta_S r_S}{r_S-\beta_S} - \frac{pr_S}{(r_S-\beta_S)}\frac{\partial f^H(\cdot,\cdot)}{\partial I_t}\right\}dI_{nt} + \\ \left\{\frac{pr_S}{(r_S-\beta_S)}\frac{\partial f^H(\cdot,\cdot)}{\partial I_t} - r_B - \frac{p\beta_S r_S}{r_S-\beta_S} - \phi\right\}d\bar{L}_B + \left\{\frac{pr_S}{(r_S-\beta_S)}\frac{\partial f^H(\cdot,\cdot)}{\partial I_t} - \frac{p\beta_S r_S}{r_S-\beta_S} - \phi\right\}dA = 0\end{gathered}$$

and noting that the multiplier of dI_{nt} is null by (29), we can solve for $\frac{d\bar{L}_B}{dA} = -\frac{\frac{p}{(r_S-\beta_S)}\frac{\partial f^H(\cdot,\cdot)}{\partial I_t} - \frac{p\beta_S r_S}{r_S-\beta_S} - \phi}{\frac{p}{(r_S-\beta_S)}\frac{\partial f^H(\cdot,\cdot)}{\partial I_t} - r_B - \frac{p\beta_S r_S}{r_S-\beta_S} - \phi}$. The denominator is negative whenever constraint (23) binds, i.e. when $h_{t1} < 0$[35] (otherwise it would be possible to raise the credit limit $\bar{L}_B$, and thus raise either I_t or I_{nt}, without violating it). The sign of the numerator can be inferred by rearranging condition (26) as follows: $\frac{pr_S}{(r_S-\beta_S)}\frac{\partial f^H(\cdot,\cdot)}{\partial I_t} - \frac{p\beta_S r_S}{r_S-\beta_S} - \phi = r_B - \frac{1}{1+\lambda_1}\phi$. Because the right hand side is positive ($\frac{\phi}{1+\lambda} < 1$), the numerator of $\frac{d\bar{L}_B}{dA}$ is also positive and the whole expression is positive.

2. : The threshold $A_2(p, r_B, r_S, \phi, \beta_S)$ is the minimum wealth that allows the entrepreneur to invest $I_t^* < I_t^{FB}, I_{nt}^* < I_{nt}^{FB}$ fully exploiting the bank credit line and taking trade credit still for liquidation motives. The level A_2 must satisfy:

$$A_2 = \frac{1}{r_B}\left\{(\phi + r_B)\left(I_{nt}^* + I_t^* - \frac{\beta_S}{r_S}I_t^*\right) - p\left[f^H\left(I_t^*, I_{nt}^*\right) - \beta_S I_t^*\right]\right\} \tag{41}$$

The proof of existence and uniqueness of A_2 is analogous to the proof of point **1** and is omitted.

3. : The threshold $A_1(p, r_B, r_S, \phi, \beta_S, \beta_t)$ is the minimum wealth that allows the entrepreneur to invest still I_t^*, I_{nt}^* fully using both credit lines. The level A_1 must satisfy:

$$A_1 = \frac{1}{r_B}\left\{(\phi + r_B)\left(\beta_t I_t^* + I_{nt}^*\right) + (1-\beta_t)I_t^* r_S - pf^H\left(I_t^*, I_{nt}^*\right) - (1-p)\beta_S I_t^*\right\}. \tag{42}$$

To prove that A_1 exists and is unique we need to show that: $(3a)$ at zero wealth the amount of funding raised by the bank and the supplier is strictly less than the second-best investment, i.e.

[34] Recall that we are in the case in which the relavant incentive constraint is the one *vis-à-vis* the bank.

[35] It can be deduced by rearranging (39).

$0+\bar{L}_B+\bar{L}_S = I_t^*+I_{nt}^*$; (3$b$) $\bar{L}_B$ and $\bar{L}_S$ are continuously increasing in A. Part (3a) follows from Assumption 1 ($\phi > \underline{\phi}$). To establish part (3b) it is helpful to define the following functions, obtained taking the derivative of (33) wrt I_t and I_{nt} :

$$\begin{aligned} h_{t2} &= p\frac{\partial f^H}{\partial I_t} - \beta_t r_B - (1-\beta_t) r_S + (1-p)\beta_S - \phi\beta_t \quad (43) \\ h_{nt2} &= p\frac{\partial f^H}{\partial I_{nt}} - (r_B+\phi). \quad (44) \end{aligned}$$

Constraint (33) is only binding if $h_{t2}, h_{nt2} < 0$, otherwise I_t and I_{nt} could be further increased without violating it.

We first prove that $\frac{d\bar{L}_B}{dA} > 0$. Using (7) and (11), it follows that $I_t = \frac{A+L_B-I_{nt}}{\beta_t}$. Substituting out in the incentive constraint (31), this can be written as a function of I_{nt}, $\bar{L}_B$ and A :

$$pf^H\left(\frac{A+\bar{L}_B-I_{nt}}{\beta_t}, I_{nt}\right) - \bar{L}_B r_B - ((1-\beta_t) r_S - (1-p)\beta_S)\left(\frac{A+\bar{L}_B-I_{nt}}{\beta_t}\right) = \phi\left(A+\bar{L}_B\right) \quad (45)$$

Totally differentiating, we obtain the following:

$$pf^H\left(\frac{A+\bar{L}_B-I_{nt}}{\beta_t}, I_{nt}\right) - \bar{L}_B r_B - ((1-\beta_t) r_S - (1-p)\beta_S)\left(\frac{A+\bar{L}_B-I_{nt}}{\beta_t}\right) = \phi\left(A+\bar{L}_B\right)$$

$$\begin{aligned} &\frac{1}{\beta_t}\left\{p\frac{\partial f^H}{\partial I_t} - \beta_t(r_B+\phi) - (1-\beta_t) r_S + (1-p)\beta_S\right\}\left(d\bar{L}_B + dA\right) + r_B dA + \\ &\quad + \left\{-\frac{p}{\beta_t}\frac{\partial f^H}{\partial I_t} + p\frac{\partial f^H}{\partial I_{nt}} + \frac{1}{\beta_t}((1-\beta_t) r_S - (1-p)\beta_S)\right\} dI_{nt} = 0 \end{aligned}$$

Using (37), the multiplier of dI_{nt} is zero, while the multiplier of $d\bar{L}_B$ is h_{t2}. Solving for $\frac{d\bar{L}_B}{dA}$ and rearranging, we obtain $\frac{d\bar{L}_B}{dA} = -\left(p\frac{\partial f^H}{\partial I_t} - r_S(1-\beta_t) + (1-p)\beta_S - \beta_t\phi\right)/h_{t2}$, whose sign depends on the sign of the numerator, given that the denominator is negative. Using the FOC on I_t, (34), and $r_B > \phi$, we deduce that the sign of the numerator is always positive, whence $\frac{d\bar{L}_B}{dA} > 0$.

To complete the proof we need to show that $\frac{d\bar{L}_S}{dA} > 0$. To prove this, we use the same procedure used to show that $\frac{d\bar{L}_B}{dA} > 0$. Using (7) and (11), it follows that $I_t = \frac{L_S}{1-\beta_t}$ and $L_B = \frac{\beta_t}{1-\beta_t}L_S - A + I_{nt}$. The incentive constraint (31) can therefore be written as a function of I_{nt}, $\bar{L}_S$ and A :

$$pf^H\left(\frac{\bar{L}_S}{(1-\beta_t)}, I_{nt}\right) - \left(\frac{\beta_t}{1-\beta_t}(r_B+\phi) + r_S - \frac{(1-p)\beta_S}{1-\beta_t}\right)\bar{L}_S + Ar_B - (r_B+\phi) I_{nt} = 0. \quad (46)$$

Totally differentiating, we obtain:

$$\frac{1}{1-\beta_t}\left\{p\frac{\partial f^H}{\partial I_t} - \beta_t(r_B+\phi) + (1-p)\beta_S - r_S(1-\beta_t)\right\} d\bar{L}_S + \left\{p\frac{\partial f^H}{\partial I_{nt}} - (r_B+\phi)\right\} dI_{nt} + r_B dA = 0$$

which, using h_{t2} and h_{nt2}, we write as:

$$\frac{1}{1-\beta_t}h_{t2}d\bar{L}_S + h_{nt2}dI_{nt} + r_B dA = 0 \quad (47)$$

Totally differentiating (37), we obtain: $\frac{p}{1-\beta_t}(f_{tt} - \beta_t f_{nt.t}) d\bar{L}_S + p(f_{t.nt} - \beta_t f_{nt.nt}) dI_{nt} = 0$. Solving for $dI_{nt} = -\frac{1}{1-\beta_t}(f_{tt} - \beta_t f_{nt.t})(f_{t.nt} - \beta_t f_{nt.nt})^{-1} d\bar{L}_S$, and substituting out in (47), we can solve for $\frac{d\bar{L}_S}{dA} = -r_B(1-\beta_t)\left(h_{t2} - \frac{f_{tt}-\beta_t f_{nt.t}}{f_{t.nt}-\beta_t f_{nt.nt}}h_{nt2}\right)^{-1} > 0$.

4. $\mathbf{A}_3 > \mathbf{A}_2 > \mathbf{A}_1 > \mathbf{0}$.

To prove that $A_3 > A_2$, we have to confront the levels of wealth obtained from the binding incentive constraint (23) when $I_i = I_i^{FB}$ and $I_i = I_i^*$, $i = t, nt$, respectively. This amounts to calculate the effect of a change in I_t or I_{nt} on A_3 leaving the incentive constraint unaltered. Totally differentiating the incentive constraint (23), we obtain:

$$p\left(\frac{\partial f^H}{\partial I_t}dI_t + \frac{\partial f^H}{\partial I_{nt}}dI_{nt}\right) - (r_B + \phi)\left(1 - \frac{\beta_S}{r_S}\right)(dI_t + dI_{nt}) - p\beta_S dI_t + r_B dA = 0$$

whence $\frac{dA}{dI_t} = -\frac{1}{r_B}\left\{p\frac{\partial f^H(\cdot,\cdot)}{\partial I_t} - \left[(r_B + \phi)\left(1 - \frac{\beta_S}{r_S}\right) + p\beta_S\right]\right\}$ and $\frac{dA}{dI_{nt}} = -\frac{1}{r_B}\left(p\frac{\partial f^H(\cdot,\cdot)}{\partial I_{nt}} - r_B - \phi\right)$. Whenever the incentive constraint binds, the terms in brackets, h_{t1}, h_{nt1} are negative, which implies that $\frac{dA}{dI_t}, \frac{dA}{dI_{nt}} > 0$. Thus, as I_t, I_{nt} decrease, A decreases, which proves that $A_3 > A_2$.

To prove that $A_2 > A_1$, we compare (23) and (33). Since within this wealth area the level of investment is unchanged and equal to I_t^*, I_{nt}^*, we only need to compare parameters. This leads to $A_2 - A_1 = \frac{1}{r_B}\left(1 - \beta_t - \frac{\beta_t}{r_S}\right)(r_B + \phi - r_S)\, I_t^* > 0$. Hence, $A_2 > A_1$.

Finally, $A_1 > 0$ follows from Assumption 1 ($\phi > \underline{\phi}$). ■

Proof of Proposition 2. (**Dominant liquidation regime**) When $\frac{\beta_S}{r_S} \geq (1 - \beta_t)$, $L_S = \frac{\beta_t}{r_S}I_t$ and the relevant incentive constraint in (13) is the one *vis-à-vis* the supplier.

The line of the proof is similar to that followed in the proof of Proposition 1. Given that (18) is binding, the maximization problem for any level of wealth is the one given by programme $\mathcal{P}_{\mathcal{F}}$. Setting up the Lagrangean (24) with $\gamma = 0$ gives the following FOC's:

$$\frac{\partial \Lambda_F}{\partial I_t} : p\frac{\partial f^H}{\partial I_t} - r_B - \beta_S\left(p - \frac{r_B}{r_S}\right) = \frac{\lambda_1}{1+\lambda_1}\phi\beta_t \tag{48}$$

$$\frac{\partial \Lambda_F}{\partial I_{nt}} : p\frac{\partial f^H}{\partial I_{nt}} - r_B = \frac{\lambda_1}{1+\lambda_1}\phi \tag{49}$$

$$\frac{\partial \Lambda_F}{\partial \lambda_1} : EP_F \geq \phi\,(I_{nt} + \beta_t I_t) \tag{50}$$

where (48) and (49) can also be written as:

$$\frac{1}{\beta_t}\left\{p\frac{\partial f^H}{\partial I_t} - r_B - p\beta_t\beta_S + \frac{r_B}{r_S}\beta_S\right\} = p\frac{\partial f^H}{\partial I_{nt}} - r_B \tag{51}$$

We can distinguish between two cases, according to whether $A \geq \hat{A}_1$ or $A < \hat{A}_1$.

$\mathbf{A} \geq \hat{\mathbf{A}}_1$: The incentive constraint (23) is slack and $\hat{I}_t^{FB}$, $\hat{I}_{nt}^{FB}$ solve (48) and (49) with $\lambda_1 = 0$. The optimal financial contract has the following properties:

$$\begin{aligned}
R_S^H &= \beta_S \hat{I}_t^{FB}, \\
L_S &= \frac{1}{r_S}\beta_S \hat{I}_t^{FB}, \\
L_B &= \hat{I}_{nt}^{FB} + \left(1 - \frac{\beta_S}{r_S}\right)\hat{I}_t^{FB} - A, \\
R_B^H &= \frac{r_B}{p}\left[\hat{I}_{nt}^{FB} + \left(1 - \frac{\beta_S}{r_S}\right)\hat{I}_t^{FB} - A\right]
\end{aligned}$$

Thus, the supplier gets flat repayments across states for the funding provided, getting the collateral in case of default, while the bank gets an increasing repayment contract.

$\mathbf{A} < \hat{\mathbf{A}}_1$: The incentive constraint (23) becomes binding and $\hat{I}_k$, $\hat{I}_N$ solve (48)/(49) and (50). Under the assumption that factors are substitutes, (48) and (49) imply that $\hat{I}_k < \hat{I}_t^{FB}$, and $\hat{I}_N < \hat{I}_{nt}^{FB}$.[36] The

[36] The proof of this result is analogous to the one obtained for the case in which $A_2 \leq A < A_3$ for Proposition 1 and thus omitted.

contract has the following properties:

$$\begin{aligned}R_S^H &= \beta_S \hat{I}_k,\\ L_S &= \tfrac{1}{r_S}\beta_S \hat{I}_k,\\ L_B &= \hat{I}_N + \left(1-\tfrac{\beta_S}{r_S}\right)\hat{I}_k - A,\\ R_B^H &= \tfrac{r_B}{p}\left(\hat{I}_N + \left(1-\tfrac{\beta_S}{r_S}\right)\hat{I}_k - A\right)\end{aligned}$$

■

Part (b) is proved using the following lemma:

Lemma 3 *For any* $1-\beta_t \leq \frac{\beta_S}{r_S}$ *there exists a unique threshold value* $\hat{A}_1(p, r_B, r_S, \phi, \beta_t, \beta_S)$ *such that* $pf^H\left(\hat{I}_t^{FB}, \hat{I}_L^{FB}\right) - \bar{L}_B r_B - \bar{L}_S r_S + (1-p)\beta_S \hat{I}_t^{FB} - \phi\left\{A + \bar{L}_B + \bar{L}_S - (1-\beta_t)\hat{I}_t^{FB}\right\} = 0$, $\bar{L}_B = \hat{I}_{nt}^{FB} + \left(1-\frac{\beta_S}{r_S}\right)\hat{I}_t^{FB} - A$ *and* $\bar{L}_S = \frac{\beta_S}{r_S}\hat{I}_t^{FB}$.

Proof. The threshold $\hat{A}_1(\cdot)$ is the minimum wealth that allows the entrepreneur to invest $\hat{I}_t^{FB}, \hat{I}_L^{FB}$ fully exploiting both credit lines.[37] This level must satisfy:

$$\hat{A}_1 = \tfrac{1}{r_B}\left\{(\phi + r_B)\hat{I}_{nt}^{FB} + \left(\left(1-\tfrac{\beta_S}{r_S}\right)r_B + \phi\beta_t + p\beta_S\right)\hat{I}_t^{FB} - p\left[f^H\left(\hat{I}_t^{FB}, \hat{I}_{nt}^{FB}\right)\right]\right\}$$

To prove that this threshold exists and is unique we need to show that: (i) $0 + \bar{L}_B + \bar{L}_S < \hat{I}_t^{FB} + \hat{I}_{nt}^{FB}$, which follows from assumption 1 ($\phi > \underline{\phi}$); (ii) $\bar{L}_B$ and $\bar{L}_S$ are continuously increasing in A.

To establish part (ii), it is useful to define the following functions, obtained by taking the derivatives of (23) wrt I_t and I_{nt} respectively:

$$h_{t3} = p\tfrac{\partial f^H(\cdot,\cdot)}{\partial I_t} - \tfrac{1}{r_S}\left[(r_S - \beta_S)r_B + pr_S\beta_S + \phi r_S \beta_t\right] \tag{52}$$

$$h_{nt3} = p\tfrac{\partial f^H(\cdot,\cdot)}{\partial I_{nt}} - r_B - \phi \tag{53}$$

Constraint (23) is only binding if $h_{t3}, h_{nt3} < 0$, otherwise I_t and I_{nt} could be further increased without violating the constraint.

We first prove that $\frac{\partial \bar{L}_B}{\partial A} > 0$. Using (7) and (10), we deduce that $I_t = \frac{A + L_B - I_{nt}}{(r_S - \beta_S)} r_S$. The binding incentive constraint (23) can be written as a function of I_{nt}, $\bar{L}_B$ and A : $pf^H\left(\frac{A+\bar{L}_B - I_{nt}}{(r_S-\beta_S)}r_S, I_{nt}\right) - I_{nt}(r_B + \phi) + Ar_B = \left\{\left(1-\frac{\beta_S}{r_S}\right)r_B + p\beta_S + \phi\beta_t\right\}\frac{A+\bar{L}_B - I_{nt}}{r_S - \beta_S}r_S$. Totally differentiating

$$\begin{aligned}&\left\{p\left(1-\tfrac{\beta_S}{r_S}\right)\tfrac{\partial f^H}{\partial I_{nt}} - p\tfrac{\partial f^H}{\partial I_t} - \phi\left(1-\beta_t-\tfrac{\beta_S}{r_S}\right) + p\beta_S\right\}dI_{nt} + \\ &\left\{p\tfrac{\partial f^H}{\partial I_t} - \left[\left(1-\tfrac{\beta_S}{r_S}\right)r_B + p\beta_S + \phi\beta_t\right]\right\}d\bar{L}_B + \left\{p\tfrac{\partial f^H}{\partial I_t} - \left[p\beta_S + \phi\beta_t\right]\right\}dA = 0\end{aligned} \tag{54}$$

Adding and subtracting $\left(1-\frac{\beta_S}{r_S}\right)r_B$ and using (52) and (53), the multiplier of dI_{nt} can also be written as

$$\left(1-\tfrac{\beta_S}{r_S}\right)h_{nt3} - h_{t3} \tag{55}$$

[37] This amounts to say that $\hat{A}_1 + \bar{L}_B + \bar{L}_S = \hat{A}_1 + \bar{L}_B + \dfrac{\beta_S}{r_S}\hat{I}_t^{FB} = \hat{I}_k^{FB} + \hat{I}_{nt}^{FB}$.

After some manipulations,[38] this reduces to

$$\left(1-\beta_t-\frac{\beta_S\beta_t}{r_S}\right)h_{nt3} \tag{56}$$

Given that $1-\beta_t<\frac{\beta_S}{r_S}$ and knowing that $h_{nt3}<0$ whenever the incentive constraint binds, we deduce that (55), and thus the multiplier of dI_{nt} in (54), is positive. Hence, (54) writes as

$$\left(1-\beta_t-\frac{\beta_S}{r_S}\right)h_{nt3}dI_{nt}+h_{t3}d\bar{L}_B+\left\{p\frac{\partial f^H}{\partial I_t}-[p\beta_S+\phi\beta_t]\right\}dA=0 \tag{57}$$

Totally differentiating (51) and recalling that $I_t=\frac{A+\bar{L}_B-I_{nt}}{(r_S-\beta_S)}r_S$, we get:

$$\frac{r_S}{(r_S-\beta_S\beta_t)}\left(f_{tt}-\beta_t f_{nt.t}\right)\left(dA+d\bar{L}_B-dI_{nt}\right)+\left(f_{t.nt}-\beta_t f_{nt.nt}\right)dI_{nt}=0.$$

Solving for dI_{nt}, we get $dI_{nt}=-\frac{\frac{r_S}{(r_S-\beta_S)}(f_{tt}-\beta_t f_{nt.t})(dA+d\bar{L}_B)}{den(dI_{nt})}$, where $den\left(dI_{nt}\right)=f_{t.nt}-\beta_t f_{nt.nt}-\frac{r_S}{(r_S-\beta_S)}\left(f_{tt}-\beta_t f_{nt.t}\right)>0$. Substituting out in (57), we get:

$$\left\{h_{t3}-\frac{\left(1-\beta_t-\frac{\beta_S}{r_S}\right)\frac{r_S}{(r_S-\beta_S)}(f_{tt}-\beta_t f_{nt.t})}{den(dI_{nt})}h_{nt3}\right\}d\bar{L}_B=$$

$$=\left\{\frac{\left(1-\beta_t-\frac{\beta_S}{r_S}\right)\frac{r_S}{(r_S-\beta_S)}(f_{tt}-\beta_t f_{nt.t})}{den(dI_{nt})}h_{nt3}-\left[p\frac{\partial f^H}{\partial I_t}-(p\beta_S+\phi\beta_t)\right]\right\}dA$$

whence

$$\frac{d\bar{L}_B}{dA}=\frac{\overbrace{\left(1-\beta_t-\frac{\beta_S}{r_S}\right)}^{<0}\frac{r_S}{(r_S-\beta_S)}\overbrace{(f_{tt}-\beta_t f_{nt.t})}^{<0}\overbrace{h_{nt3}}^{<0}-\overbrace{\left[p\frac{\partial f^H}{\partial I_t}-(p\beta_S+\phi\beta_t)\right]}^{>0}den(dI_{nt})}{h_{t3}den(dI_{nt})-\left(1-\beta_t-\frac{\beta_S}{r_S}\right)\frac{r_S}{(r_S-\beta_S)}(f_{tt}-\beta_t f_{nt.t})h_{nt3}}$$

Using h_{t3},h_{nt3}, $f_{ii}<0$ and $f_{ij}>0$ and using the FOC on I_t (48), we deduce that the numerator of $\frac{\partial\bar{L}_B}{\partial A}$ is negative. The sign of $\frac{\partial\bar{L}_B}{\partial A}$ depends on the sign of the denominator. Using the equality of (55) with (56), we can write the denominator as

$$den\left(\frac{d\bar{L}_B}{dA}\right)=h_{t3}den\left(dI_{nt}\right)-\frac{r_S}{(r_S-\beta_S)}\left(f_{tt}-\beta_t f_{nt.t}\right)\left(\left(1-\frac{\beta_S}{r_S}\right)h_{nt3}-h_{t3}\right).$$

This reduces to

$$den\left(\frac{d\bar{L}_B}{dA}\right)=\underset{<0}{h_{t3}}\underbrace{(f_{t.nt}-\beta_t f_{nt.nt})}_{>0}-\frac{r_S}{(r_S-\beta_S)}\underbrace{(f_{tt}-\beta_t f_{nt.t})}_{<0}\left(1-\frac{\beta_S}{r_S}\right)\underset{<0}{h_{nt3}}<0$$

which is unambiguously negative. This completes the proof that $\frac{\partial\bar{L}_B}{\partial A}>0$.

The last step is to show that $\frac{\partial\bar{L}_S}{\partial A}>0$. Using (7) and (10), we deduce that $L_B=I_{nt}-\left(1-\frac{r_S}{\beta_S}\right)L_S-A$ and $I_t=\frac{r_S}{\beta_S}L_S$. The incentive constraint *vis-à-vis* the supplier (20) can therefore be written as a function of I_{nt}, $\bar{L}_S$ and A :

$$pf^H\left(\frac{r_S}{\beta_S}\bar{L}_S,I_{nt},\right)-I_{nt},(r_B+\phi)-\frac{\bar{L}_S}{\beta_S}\left\{(r_S-\beta_S)r_B+p\beta_S r_S+\phi\beta_t r_S\right\}+Ar_B=0$$

[38] By adding and subtracting $\beta_t\left(p\frac{\partial f^H}{\partial I_{nt}}-r_B\right)$ to (55) and using (51).

Totally differentiating:

$$\left\{p\frac{\partial f^{H}(.,.)}{\partial I_{nt}}-(r_B+\phi)\right\}dI_{nt}+r_BdA+\frac{r_S}{\beta_S}\left\{p\frac{\partial f^{H}(.,.)}{\partial I_t}+\frac{1}{r_S}\left[(r_S-\beta_S)\,r_B+r_Sp\beta_S+\beta_tr_S\phi\right]\right\}d\bar{L}_S=0$$

which, using h_{t3}, h_{nt3}, we can write as:

$$h_{nt3}dI_{nt}+r_BdA+\frac{r_S}{\beta_S}h_{t3}d\bar{L}_S=0 \tag{58}$$

Totally differentiating (51), we get: $dI_{nt}=\frac{r_S}{\beta_S}\left(f_{tt}-\beta_tf_{nt.t}\right)\left(f_{t.nt}-\beta_tf_{nt.nt}\right)^{-1}d\bar{L}_S$. Plugging this in (58) and solving for $\frac{\partial\bar{L}_S}{\partial A}=r_B\left(h_{nt3}\frac{\frac{r_S}{\beta_S}(f_{tt}-\beta_tf_{nt.t})}{f_{t.nt}-\beta_tf_{nt.nt}}-\frac{r_S}{\beta_S}h_{t3}\right)^{-1}$. Given the assumptions on the production function, $\frac{\partial\bar{L}_S}{\partial A}$ is certainly positive whenever the incentive constraint (20) binds, i.e. when $h_{t3},h_{nt3}<0$. ■

Proof of Proposition 3. Under the assumption that the production function is homothetic, the input tangibility (I_t/I_{nt}) only depends on the input price ratio (P_t/P_{nt}). Using the proof of Proposition 1, we can write P_t/P_{nt} as a function of the parameters of the model. Let us consider the four wealth areas separately.

When $\mathbf{A}\geq\mathbf{A}_3$, $\frac{P_t}{P_{nt}}=\frac{\left[r_B-\beta_S\left(\frac{r_B}{r_S}-p\right)\right]}{r_B}$ and $\frac{L_S}{L_S+L_B+A}=\beta_Sr_S\left[\left(\frac{I_{nt}^{FB}}{I_t^{FB}}\right)+1\right]^{-1}$. Notice that trade credit intensity, $(L_S/\left(A+L_B+L_S\right))$, is increasing in input tangibility (I_t/I_{nt}). Since $\frac{\partial(P_t/P_{nt})}{\partial A}=0$ and trade credit intensity depends on wealth only through I_t/I_{nt}, both input tangibility and trade credit intensity are independent of A.

When $\mathbf{A}_2\leq\mathbf{A}<\mathbf{A}_3$: $\left(\frac{P_t}{P_{nt}}\right)_{A_2\leq A<A_3}=\frac{\left[r_B+\frac{\phi\lambda_1}{(1+\lambda_1)}(1-\frac{\beta_S}{r_S})-\beta_S\beta_t(\frac{r_B}{r_S}-p)\right]}{\left[r_B+\frac{\phi\lambda_1}{(1+\lambda_1)}\right]}<\left(\frac{P_t}{P_{nt}}\right)_{A>A_3}$ and $\frac{L_S}{L_S+L_B+A}=\frac{\beta_S}{r_S\left(\frac{I_{nt}}{I_t}\right)+r_S}$. Notice that trade credit intensity is increasing in input tangibility. Moreover, $\frac{\partial(P_t/P_{nt})_{A_2\leq A<A_3}}{\partial A}=\frac{\left[(1-\frac{\beta_S}{r_s})-\frac{P_t}{P_{nt}}\right]}{\left[r_B+\frac{\phi\lambda_1}{(1+\lambda_1)}\right]}\frac{\frac{\partial\lambda}{\partial A}\phi}{(1+\lambda_1)^2}>0$, since $\frac{\partial\lambda}{\partial A}\leq0$ and $\left[(1-\frac{\beta_S}{r_s})-\frac{P_t}{P_{nt}}\right]=-\frac{(\beta_Sp)}{r_B+\frac{\phi\lambda_1}{(1+\lambda_1)}}\leq0$. Given that $L_S/\left(A+L_B+L_S\right)$ depends on wealth only through I_t/I_{nt}, both input tangibility and trade credit intensity are decreasing in A.

When $\mathbf{A}_1\leq\mathbf{A}<\mathbf{A}_2$: $\left(\frac{P_t}{P_{nt}}\right)_{A_1\leq A<A_2}=\frac{[\beta_tr_B+(1-\beta_t)r_S-\beta_S(1-p)]}{r_S}<\left(\frac{P_t}{P_{nt}}\right)_{A_2\leq A<A_3}$ and $\frac{L_S}{L_S+L_B+A}=\frac{\mu}{\left(\frac{I^*_{nt}}{I^*_t}\right)}$, where $(\beta_S/r_S)\leq\mu\leq(1-\beta_t)$ and varies with A. Since $\frac{\partial(P_t/P_{nt})_{A_1\leq A<A_2}}{\partial A}=0$, $\dfrac{\partial\mu}{\partial A}\leq0$,[39] and input tangibility is independent of A, trade credit intensity is decreasing in A.

When $\mathbf{A}<\mathbf{A}_1$: $\left(\frac{P_t}{P_{nt}}\right)_{A_1<A}=\frac{\left[\beta_t\left(r_B+\frac{\lambda\phi}{1+\lambda}\right)+(1-\beta_t)r_S-\beta_S(1-p)\right]}{\left[r_B+\frac{\lambda\phi}{1+\lambda}\right]}<\left(\frac{P_t}{P_{nt}}\right)_{A_1<A<A_2}$ and $\frac{L_S}{L_S+L_B+A}=\frac{(1-\beta_t)}{\left(\frac{I_{nt}}{I_t}\right)+}$. Notice that trade credit intensity is increasing in asset tangibility. Moreover, $\frac{\partial(P_t/P_{nt})_{A<A_1}}{\partial A}=\frac{\left[\beta_t-\frac{P_t}{P_{nt}}\right]}{\left[r_B+\frac{\phi\lambda}{(1+\lambda}\right]}\frac{\frac{\partial\lambda}{\partial A}\phi}{(1+\lambda_1)^2}>0$, since $\frac{\partial\lambda}{\partial A}\leq0$ and $\left[\beta_t-\frac{P_t}{P_{nt}}\right]=\beta_S(1-p)-(1-\beta_t)r_S\leq0$ when $(1-\beta_t)\geq\frac{\beta_s}{r_s}$, which

[39] This follows from Proposition 2 and from Figure 2. The intuition is the following: when $A=A_2$, the firm uses a share of trade credit equal to β_S/r_S and the shadow cost of bank credit equals the cost of trade credit. Since the firm is constrained on bank credit but still unconstrained on trade credit, any reduction in wealth is compensated by a rise in trade credit, keeping investment at I^*_t,I^*_{nt}. Thus the share of trade trade credit increases until it reaches $(1-\beta_t)$ when $A=A_1$.

corresponds to the dominant incentive regime we are considering. Since $L_S/(A+L_B+L_S)$ depends on wealth only through I_t/I_{nt}, both input tangibility and trade credit intensity are decreasing in wealth. ■

Proof of Proposition 4. Notice that, from the proof of Proposition 3, trade credit intensity is an increasing function of input tangibility. Let us consider separately the four relevant wealth areas.

When $\mathbf{A} \geq \mathbf{A}_3$, $\frac{\partial(P_t/P_{nt})}{\partial\phi} = 0$. It follows that I_t/I_{nt} is independent of ϕ. However since ϕ affects trade credit intensity only through the input combination, also trade credit intensity is independent of ϕ.

When $\mathbf{A}_2 \leq \mathbf{A} < \mathbf{A}_3$, the sign of the derivative $\frac{\partial(P_t/P_{nt})}{\partial\phi}$ depends on the sign of $\left(\frac{\lambda_1}{1+\lambda_1} + \phi\frac{\partial\lambda_1/\partial\phi}{(1+\lambda_1)^2}\right)(-p\beta_S)$. Since $(\partial\lambda_1/\partial\phi) > 0$, the whole expression is negative. This implies that asset tangibility increases in ϕ. Since a change in ϕ affects trade credit intensity through the input combination, also $L_S/(A+L_B+L_S)$ is increasing in ϕ.

When $\mathbf{A}_1 \leq \mathbf{A} < \mathbf{A}_2$, $\frac{\partial(P_t/P_{nt})}{\partial\phi} = 0$, which implies that I_t/I_{nt} is independent of ϕ. However, since $\frac{\partial\mu}{\partial\phi} > 0$, $L_S/(A+L_B+L_S)$ is increasing in ϕ. When ϕ increases, the shadow cost of bank credit equals the cost of trade credit at $\bar{A}_2 > A_2$. For decreasing A, the firm substitutes bank credit with trade credit, thereby increasing the absolute level of trade credit from $\beta_S I_t^*/r_S$ at $A = \bar{A}_2$ to $(1-\beta_t)I_t^*$ at $A = \bar{A}_1 > A_1$.

When $\mathbf{A} < \mathbf{A_1}$, the sign of the derivative $\frac{\partial(P_t/P_{nt})}{\partial\phi}$ depends on the sign of $\left(\frac{\lambda}{1+\lambda} + \phi\frac{\partial\lambda/\partial\phi}{(1+\lambda)^2}\right)[(1-\beta_t)r_S - \beta_S(1-p)]$. Since $(\partial\lambda/\partial\phi) > 0$ and the term in square brackets is negative, the whole expression is negative.[40] This implies that asset tangibility increases. Since ϕ affects trade credit intensity only through the input combination, also $L_S/(A+L_B+L_S)$ is increasing in ϕ. ■

[40] Since we are considering the dominant incentive regime, $(1-\beta_t) > \frac{\beta_S}{r_S}$.

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